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Derivation of the Generalized, Average Euclidean Distance Function for the PDI Model

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PREFACE

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13. ABSTRACT *(Maximum 200 words)*

This report derives distance functions that form the basis for the Population Density Index (PDI) model, which is a three-parameter square-root model for measuring discrete spatial density in finite populations. The PDI and its methods have been applied to facilities layout methodologies in submarine environments at the Naval Undersea Warfare Center Division, Newport, RI, resulting in several U.S. patent applications. The emphasis here is on the "micro-population" model in which the linear units are "feet." The derivations relate Cartesian rectangular coordinate systems to uniform unit and nonunit lattices, as well as to the nonlattice distribution. Other proofs relate to the bounds of the calculated density measure and the density rate index called "effective distance." Alternative distance functions are discussed, and examples of the numerical calculations are provided. Also derived is the algorithm for selecting a rectangular lattice conformal to a quadrilateral area and for calculating interpoint distance in a PDI lattice. A table of computer-generated unit lattice average Euclidean distances for up to 10,000 density points is included.

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DERIVATION OF THE GENERALIZED, AVERAGE EUCLIDEAN DISTANCE FUNCTION FOR THE PDI MODEL

INTRODUCTION

Research has demonstrated spatial density (or crowding) to be a significant stressor in animal and human populations (Galle, Grove, and McPherson, 1972; Baum and Epstein, 1975). In previous papers, the author formulated and tested a mathematical model and methodology for measuring discrete spatial density in human populations (O'Brien (1989, 1990a, 1990b)). The model, called the population density index (PDI) model, was demonstrated to provide a more accurate and flexible approach for discrete spatial density measurement than the conventional formulation. The traditional approach to measuring human physical density involves two parameters: the number of persons (n) and the geometric area (A) in which the persons dwell. The equation $D = n/A$ serves as the conceptual and computational definition for "density," "congestion," "population density," or "physical crowding," each term used interchangeably. In contrast, the PDI model is based on three parameters: n, A, and inter-object distance. The derivation of the PDI model metrics is patterned on the "square-root law" of average distances used in the physical sciences. The capability to model inter-object distance within a defined geometric plane is a significant enhancement to discrete spatial density measurement. In O'Brien (1991a), the PDI model was generalized to any finite number of density points (i.e., people).

The motivation for developing the PDI formula and model was the need to be able to measure crowding among people from variable spatial configurations such as in a typical dynamic workplace environment. The conventional density model assumes that a static description is adequate without taking into account the way in which people use an environment over time.

The PDI model has been used at the Naval Undersea Warfare Center (NUWC) Division, Newport, for density measurement (O'Brien and Kanter, 1988; Kanter and O'Brien, 1989a; 1989b) in submarine attack center concept of operations experiments (Wallin, 1987). Practical applications of the PDI model resulting from research at NUWC have been documented for a variety of disciplines in several U.S. patent applications (O'Brien, 1991c, 1991d, 1991e, 1991f).

The purpose of this report is to provide a more rigorous derivation of the PDI model than currently exists. The basis of the PDI model is the distance function in Euclidean space. All of the measures in the model are related to distance. Thus, an attempt is made to characterize the PDI distance function in $\mathbf{R}^2$ (two-dimensional Euclidean space).

DERIVATION OF THE DISTANCE FUNCTION

GENERAL CASE LATTICES

The notation and structure of this section is patterned on Morrey (1962, Chapter 8, "The Definite Integral"), where the theory of area and concept of functional uniform continuity are developed in detail. Also, the ideas of inner and outer areas of bounded sets and the idea of a planar figure developed in Morrey are germane to the present development.

In the X-Y Euclidean plane (quadrant I) of figure 1, any two consecutive abscissa (horizontal) or ordinate (vertical) points (denoted by a large dot •) are assumed to be equidistant with interpoint spacing parameter δ. That is, the directed distances of the collinear point pairs $(P_1P_2) = [(x_k, y_\ell),(x_{k+1}, y_\ell)]$ and $(P_3P_4) = [(x_m, y_j)(x_m, y_{j+1})]$ are

$$\begin{aligned} \overline{P_1P_2} &= |x_{k+1} - x_k| = \delta , \\ \overline{P_3P_4} &= |y_{j+1} - y_j| = \delta , \end{aligned} \tag{1}$$

where x_k is a representative abscissa and y_j is a representative ordinate; $(x_k, y_j) > 0$. Generally, x_k, y_j will not be lattice (integer) points. In this report the units of the interpoint distance parameter δ for human populations are assumed to be feet ($\delta \geq 1$).

The interior rectangular lattice shown in figure 1 consists of n (a nonprime number) finite points arranged uniformly with R row (horizontal) and C column (vertical) points such that $n = RC$ ($n \geq 2$). The selection of an RC configuration and the computation of δ are explained in appendix A.

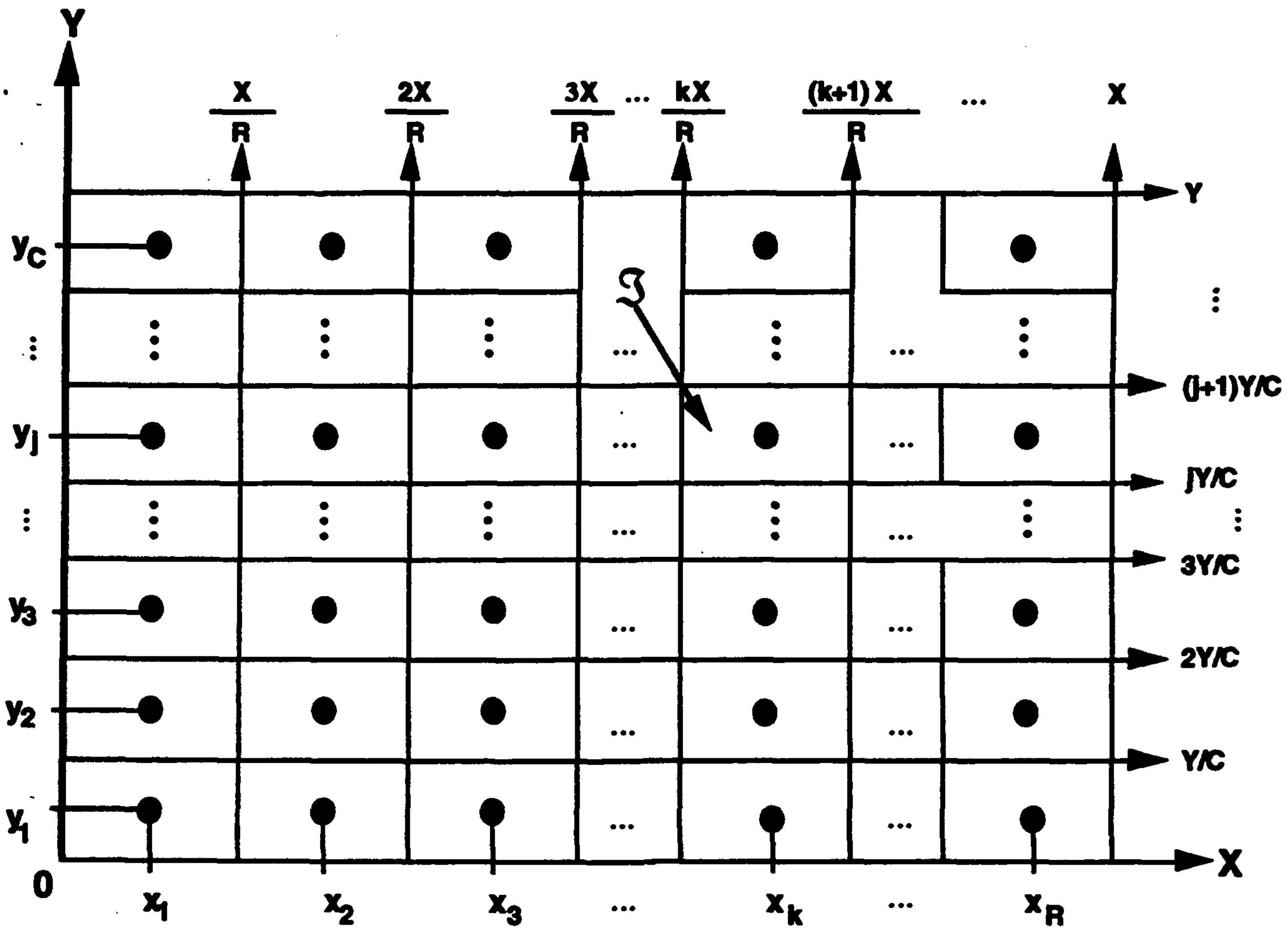

Figure 1. General Case Lattice

For a representative region $\mathfrak{S}$ bounded by the nonnegative curves (see figure 1)

$$
\begin{aligned}
& f_A(x) = (j+1)Y/C \\
& g_A(x) = jY/C\,,\ g_A(x) < f_A(x)\,, \\
& kX/R \le x \le (k+1)X/R\,,\quad 0 \le k \le R-1, 0 \le j \le C-1\,,
\end{aligned}
\tag{2}
$$

the area $A(\mathfrak{S})$ is defined as

$$
\begin{aligned}
A(\mathfrak{S}) &= \int_{kX/R}^{(k+1)X/R} [f_A(x) - g_A(x)]dx\,, \\
&= XY/RC = A/n\,,
\end{aligned}
\tag{3}
$$

which is seen to be a rectangle. For human populations with feet as the linear units, the restriction will be placed on the value of A/n; viz., $A \geq n$. When $A = n$, the RC rectangular or square uniform discrete distribution is referred to as a "unit lattice"; otherwise, the homogeneous distribution of points is called a "nonunit lattice." The distinction will be understood in context. Each such rectangle will be obtained by dividing the total study area $n[A(\mathfrak{S})]$ into $n = RC$ partitioned rectangles each, with area given by equation (3).

The connected density points in each of the horizontal and vertical intervals are defined by relations (or multiple-valued discrete constant functions):

$$\begin{aligned} f(R) &= X = (R-1)\,\delta + p, \quad p > 0, \\ f(C) &= Y = (C-1)\,\delta + q, \quad q > 0. \end{aligned} \tag{4}$$

Equations (4) indicate that each X or Y interval consists of two components: the length of the density points segment $[(R-1)\delta$ or $(C-1)\delta]$ and an excess factor (p or q). The region outside the perimeter of the uniform point arrangement [equal to $A - (R-1)(C-1)\delta^2$] is required to accommodate environmental objects (furniture, equipment, displays, etc.). Each of the CX intervals and RY intervals is defined by the constant functions in equations (4). The interval X will be partitioned into R subintervals, each subpartition of which will have the length shown in figure 1, and Y will be similarly divided and have the length shown in figure 1.

The derivation of the coordinate system for the general-case lattice will allow a precise graph to be drawn of any uniform rectangular distribution on a rectangular Cartesian X-Y coordinate system such that the interior RC lattice is contained within the XY exterior region. The coordinates of the density points derived from equation (4) will be generated by

$$\begin{aligned} x_k &= p/2 + (k-1)\delta, \quad 1 \leq k \leq R, \\ y_j &= q/2 + (j-1)\delta, \quad 1 \leq j \leq C. \end{aligned} \tag{5}$$

Then, the coordinate system for the general case will be defined as

$$\begin{aligned} (x_k, y_j) &= \left[(x_1, y_1), (x_2, y_1), \ldots, (x_k, y_j), \ldots, (x_R, y_C) \right] \\ &= \left[\left(\frac{p}{2}, \frac{q}{2}\right), \left(\frac{p}{2} + \delta, \frac{q}{2}\right), \ldots, \left(\frac{p}{2} + (k-1)\delta, \frac{q}{2} + (j-1)\delta\right), \ldots, \right. \\ &\qquad \left. \left(\frac{p}{2} + (R-1)\delta, \frac{q}{2} + (C-1)\delta\right) \right]. \end{aligned} \tag{6}$$

The coordinate system of equation (6) applies to either a unit or nonunit lattice because it is derived from the general case. An example of the use of equation (6) is depicted in figure 2. The coordinates were generated from the following assumptions: $n = 6$; $R = 3$, $C = 2$ (from equation (A-4) in appendix A); $A = X \times Y = 16 \times 6$; $p/2 = 4$, $q/2 = 1$ (from (4)); $\delta = 4$ (from equation (A-5) in appendix A). Then the coordinate points are generated by $x_k = 4 + 4(k - 1)$; $y_j = 1 + 4(j - 1)$. The plot points are obtained from all $k \times j$ combinations ($k = 1, 2, 3$; $j = 1, 2$).

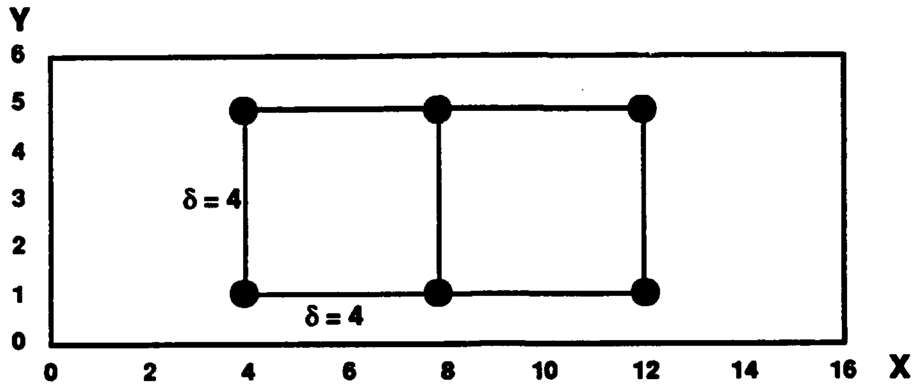

Figure 2. Example of General Case PDI Graph

SPECIAL CASE LATTICES

Each density point (•) is now assumed to be the centroid (center of mass) bounded by its respective planar region (see figure 3). Let a representative region be called S. The area of S can be determined by first defining the nonnegative curves as the boundaries of S:

$$\begin{aligned} &f_B(x) = (j + 1)\delta\,, \\ &g_B(x) = j\delta\,,\quad g_B(x) < f_B(x)\,, \\ &k\delta \le x \le (k + 1)\delta\,,\quad 0 \le k \le R - 1\,,\quad 0 \le j \le C - 1\,. \end{aligned} \tag{7}$$

The special case of equation (7) can be derived from equation (2) by assuming that $\delta = p = q = X/R = Y/C$ in equation (4) of the general case (i.e., proportionate commensurability between the dimensions of the outer and inner rectangular areas).

The area of S is then found by integrating between the curves $f_B(x)$ and $g_B(x)$ in the x interval, and applying the Fundamental Theorem of Calculus:

$$A(S) = \int_{k\delta}^{(k+1)\delta} [f_B(x) - g_B(x)]dx = \delta^2, \quad \delta^2 \geq 1 . \tag{8}$$

This is intuitively the area of a square figure. The figure will be obtained by dividing the total area n[A(S)] into n = RC partitions after determining which lattice configuration will accommodate best the n points into a rectangular configuration with associated interpoint spacing parameter δ (see appendix A). Note that for commensurate (unit or nonunit) lattices, the interpoint spacing parameter is related to the region in equation (8).

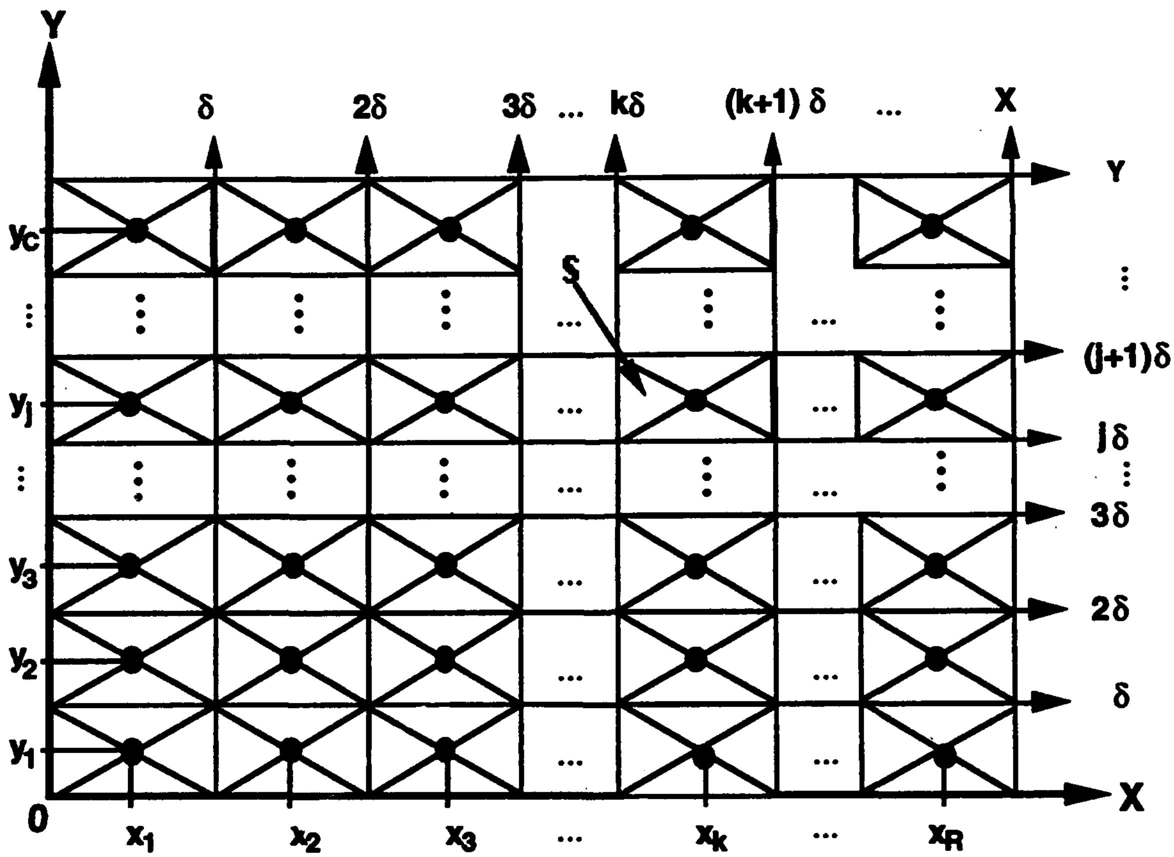

Figure 3. Special Case Lattice

Bers (1969, Vol. II, chapter 8, section 8, "Centroids of Plane Regions and Curves") gives the following definitional formulas for determining the coordinate points (x_k, y_j) of the centroid in region S:

$$(x_k, y_j): x_k = \frac{\int_{k\delta}^{(k+1)\delta} x\,[f_B(x) - g_B(x)]dx}{\int_{k\delta}^{(k+1)\delta} [f_B(x) - g_B(x)]dx} = \frac{\delta(2k+1)}{2},$$

$$(x_k, y_j): y_j = \frac{\int_{k\delta}^{(k+1)\delta} 1/2\,[f_B(x)^2 - g_B(x)^2]dx}{\int_{k\delta}^{(k+1)\delta} [f_B(x) - g_B(x)]dx} = \frac{\delta(2j+1)}{2}. \quad (9)$$

Here, (x_k, y_j) represents the rule for locating all and every density point (centroid) in the entire XY area, given concisely as

$$\begin{aligned}(x_k, y_j) &= \left[\left(\frac{\delta}{2} + (k-1)\delta\right), \left(\frac{\delta}{2} + (j-1)\delta\right)\right] \\ &= [(x_1, y_1), (x_2, y_1), (x_3, y_1), \ldots, (x_k, y_1), \\ &\quad (x_1, y_2), \ldots, (x_k, y_2), \ldots, (x_k, y_j), \ldots, (x_R, y_C)] \\ &= \{[\delta/2, \delta/2], [3\delta/2, \delta/2], [5\delta/2, \delta/2], \ldots, [\delta(2R-1)/2, \delta/2], \\ &\quad [\delta/2, 3\delta/2], \ldots, [\delta(2R-1)/2, 3\delta/2], \ldots, \\ &\quad [\delta/2 + (k-1)\delta, \delta/2 + (j-1)\delta], \ldots, [\delta(2R-1)/2, \delta(2C-1)/2]\}, \\ &\quad 1 \le k \le R,\ 1 \le j \le C.\end{aligned} \quad (10)$$

The coordinate system of equation (10) applies to unit lattices and commensurate nonunit lattices. Figure 4 is an example of equation (10) applied to a 3 x 2 unit lattice ($\delta = 1$ from equation (A-5) in appendix A). Note that $X/R = Y/C = p = q = \delta = \sqrt{XY/RC} = 1$ because all unit lattices are commensurate. The graph is plotted from equation (10) by $x_k = k - 0.5$; $y_j = j - 0.5$ ($k = 1,2,3; j = 1,2$).

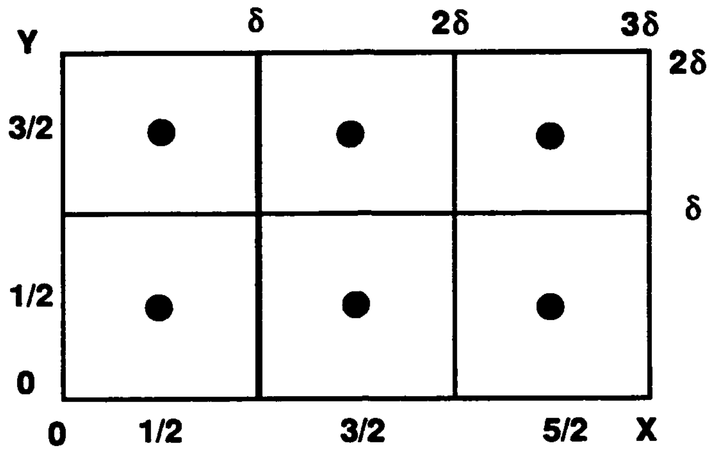

Figure 4. Example of Special Case PDI Graph (Unit Lattice)

Figure 5 is an example of a graph for a nonunit commensurate lattice with n = 15 points within area of 40 ft x 24 ft. Note that $X/R = Y/C = p = q = \delta = \sqrt{XY/RC} = 8$. Plot points are generated from equation (10): $x_k = 4 + 8(k - 1)$; $y_j = 4 + 8(j - 1)$.

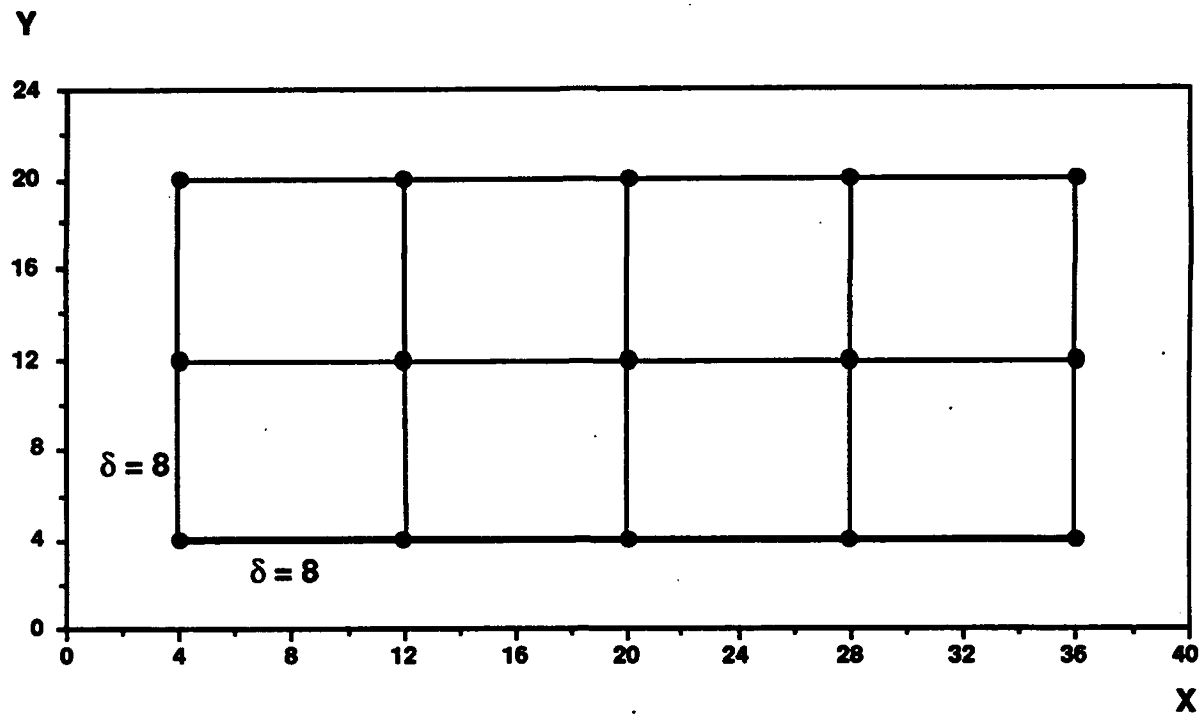

Figure 5. Example of Special Case PDI Graph (Commensurate Nonunit Lattice)

GENERALIZED DISTANCE FUNCTION IN A LATTICE

Since the coordinate points in the PDI lattice can now be specified completely, the PDI "exact" and "approximate" distance formulas can be derived (O'Brien, 1990b, 1991b). Here is derived the generalized, Euclidean distance formula for any PDI lattice (nonunit lattice and thereby the unit lattice as a special case) and any nonuniform distribution. First shown is the derivation for a lattice using the general case notation system. The derivation applies equally to the special case by assuming commensurability.

Let any density point in the X-Y plane be called (x_k, y_j) and let a second distinct point be called $(x_{k+i}, y_{j+\ell})$. Then, from equation (6),

$$(x_k, y_j) = \left(p/2 + (k-1)\delta,\ q/2 + (j-1)\delta\right),$$
$$(x_{k+i}, y_{j+\ell}) = \left(p/2 + (k+i-1)\delta,\ q/2 + (j+\ell-1)\delta\right), \qquad (11)$$
$$1 \le k \le R-1, \quad 1 \le j \le C-1,$$

$$2 \le k+i \le R, \quad 2 \le j+\ell \le C.$$

Bers (1969, Vol. I) shows that the classical Pythagorean distance formula for any two points in a Cartesian plane can generally be derived from the integral calculus arc length formula, given for our notation as

$$\mathcal{L} = \int_{x_k}^{x_{k+i}} \sqrt{1 + [f'(x)]^2}\,dx. \tag{12}$$

The quantity $f'(x)$ is the first derivative of the function $f(x)$, taken to be a generalized single-valued relation for two points in a Euclidean X-Y plane specified by the first degree equation $f(x) = a + bx$, for the slope intercept a and linear slope b.

Since $f'(x) = \frac{d[f(x)]}{dx} = D_x(a + bx) = b$, the constant slope of the points in equation (11) can be defined as $b = \frac{y_{j+\ell} - y_j}{x_{k+i} - x_k} = \frac{\delta\ell}{\delta i}$. Then,

$$\begin{aligned} \mathcal{L} &= \int_{x_k}^{x_{k+i}} \sqrt{1 + \left(\frac{\ell}{i}\right)^2}\,dx, \\ &= \delta\sqrt{i^2 + \ell^2}, \end{aligned} \tag{13}$$

which is seen to be of the form for the standard bivariate Pythagorean theorem scaled by a constant:

$$\mathcal{L} = \delta\sqrt{(x_{k+i} - x_k)^2 + (y_{j+\ell} - y_j)^2}\,. \tag{14}$$

Bers (1969, Vol. I, p. 279) terms equation (12) the "length formula." It may also be viewed as an average -- the average length of one pair of points. The length (distance) between any one pair of points in the uniform RC lattice can be generalized to an average among all possible pairs of RC points since each point pair defines a simple linear function each of which possesses a piecewise continuous first derivative. The average pair-to-pair distance, summed over all pairs of points, will be the average of all the line-to-line curves (total length), since the connected graph defines a multiple-valued relation (Bers, 1969, Vol. I, page 279). That is, the uniform average distance in the total lattice is

$$\bar{d} = \frac{\sum_{k=j=1}^{n} \ell_{kj}}{C(n,2)}, \tag{15}$$

where

$$C(n,2) = \frac{(RC)!}{2!(RC-2)!} = \frac{n(n-1)}{2}, \qquad (n \geq 2), \tag{16}$$

is the combinatorial expression specifying the total number of nonredundant pairwise-connected lines from n nodes and the exact summation index limits are given in equation (11). The uniform lattice distance equation (15) can be further expressed in a more computational convenient form as

$$\bar{d} = \delta\bar{\Delta}, \tag{17}$$

where δ is given in appendix A and $\bar{\Delta}$ is the unit lattice average distance, which has been derived in O'Brien (1991a) as

$$\bar{\Delta} = \frac{12\sum_{i=1}^{R-1}\sum_{j=1}^{C-1}(R-i)(C-j)\sqrt{i^2+j^2} + RC(R^2+C^2-2)}{3(RC)(RC-1)}, \tag{18}$$

where R is the number of horizontal points in each row of the unit lattice, C is the number of vertical points in each column of the unit lattice, and RC is the total number of density points in the unit lattice.

An accurate approximation to equation (18) exists when n is not small. This relation is derived under the assumption that there is a continuous uniform distribution within a rectangular plane. The objective is to find the average distance between any two randomly selected points of a convex set. The approximation formula* (Santalo, 1976, formula 4.18, page 49) is as follows:

$$\bar{\Delta}' = \frac{1}{15}\left\{\frac{R^3}{C^2} + \frac{C^3}{R^2} + d\left(3 - \frac{R^2}{C^2} - \frac{C^2}{R^2}\right) + \frac{5}{2}\left[\frac{C^2}{R}\ln\left(\frac{R+d}{C}\right) + \frac{R^2}{C}\ln\left(\frac{C+d}{R}\right)\right]\right\}, \tag{19}$$

---

* The author gratefully acknowledges an anonymous referee of *The American Mathematical Monthly* for suggesting equation (19) (in correspondence related to O'Brien, 1990c).

where $d = \sqrt{R^2 + C^2}$ and ln is the natural logarithm operator.

Calculations have shown equation (19) to be a good approximation to equation (18). For example, for n under 100, the maximum discrepancy is less than 10 percent. Equation (19) is an interesting example where a continuous distribution relation is applied to a discrete distribution to obtain an approximation to the latter. In the limiting case, as RC approaches infinity, the difference between equations (18) and (19) approaches zero.

In conclusion, for any finite, discrete, uniform distribution with distance between any two points δ, the generalized average Euclidean distance in any PDI lattice among all possible pairs of RC points will be $\bar{d} = \delta\bar{\Delta}$ or $\delta\bar{\Delta}' \approx \bar{d}$. If a unit lattice ($\delta = 1$), then $\bar{d} = \bar{\Delta}$ or $\bar{\Delta}' \approx \bar{d}$. Selected values of $\bar{\Delta}$ calculated from equation (18) are given in appendix B for all RC configurations from $R \times C = 2 \times 2$ to $R \times C = 100 \times 100$ (n = 10,000 density points).

GENERALIZED DISTANCE FUNCTION IN A NONLATTICE

Here, density points can fall anywhere within the X-Y geometric area, subject to restrictions specified earlier. The average Euclidean distance is calculated by equation (15) from known coordinate points as

$$\bar{d} = \frac{\sum_{i<j} \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}}{C(n,2)}, \tag{20}$$

where (x_i, y_i), (x_j, y_j) ($i = 1, 2, ..., n$; $j = 1, 2, ..., n$) denote the coordinate locations for the density points ($n > 1$) within the rectangular study area $X \times Y$ with arbitrary origin O. Equation (20) is the form used for calculating the population density measure of observed density points (i.e., PDI_{act}, defined below in equation (29)).

An approximation to equation (20) is useful because exact (x_i, y_i), (x_j, y_j) coordinates cannot always be obtained. Recently, O'Brien (1991b) derived an approximation PDI method by assuming knowledge of the relative location of the density objects when (x_i, y_i), (x_j, y_j) data were unavailable.

If one assumes that the study area $A = X \times Y$ has been partitioned into n = RC rectangles, each with subarea given by equation (3), then the following abbreviated calculation routines can be derived.

Define a cell density measure,

$$D_{jk} = n_{jk}/A_{jk}\,, \tag{21}$$

where n_{jk} is the number of objects observed to be within each of the subareas $A_{jk} = A/n$ ($j = 1, 2, ..., R$; $k = 1, 2, ..., C$), $0 \le n_{jk} \le A_{jk}$, $0 \le D_{jk} \le 1$. Then, define a cell indicator variable I:

$$I_{jk} = \begin{cases} 1 & \text{if } D_{jk} \neq 0\,, \\ 0 & \text{if } D_{jk} = 0\,. \end{cases} \tag{22}$$

Let

$$m = \sum_{k=1}^{R} \sum_{j=1}^{C} I_{jk}\,, \qquad nD \le m \le n\,, \tag{23}$$

where $D = n/A$ is obtained from equation (21) as an average cell density with weights spread over all cells; i.e., $D = \sum_{k=1}^{R} \sum_{j=1}^{C} D_{jk}\, n^{-1}$. The measure m represents the total number of RC partitions occupied by at least one object. In practice, n_{jk} is taken as the smallest integer value. Likewise, m is taken to be the largest integer value.

Hence, equation (17) can be redefined to give the following approximation to equation (20):

$$\bar{d}' = \delta'_{eff}\bar{\Delta}\,, \tag{24}$$

where

$$\delta'_{eff} = \left(\frac{\sum_{k=1}^{R} \sum_{j=1}^{C} D_{jk}}{m}\right)^{-1/2} = \left(\frac{mA_{jk}}{n}\right)^{1/2}, \qquad 1 \le \delta'_{eff} \le D^{-1/2}\;. \tag{25}$$

δ'_{eff} is obtained from equation (21) as an average cell density with weights spread over only the m occupied cells. The limits of equation (24) follow immediately by substituting the lower and upper limits of m given in equation (23); viz., $\bar{\Delta} \le \bar{d}' \le \bar{\Delta}\sqrt{A/n}$. Noting that $1 \le \delta \le \sqrt{A/n}$ (see appendix A) and assuming, in practice, that $1 \le \delta'_{eff} \le \delta$, it then follows that

$$\bar{d}_{max} \le \bar{d}' \le \bar{d}_{min}\,, \tag{26}$$

where $\bar{d}_{min}$ and $\bar{d}_{max}$ are, respectively, the lower and upper distance measures in the exact PDI model (O'Brien, 1990b). The relationship of (26) translates directly into a proof of the bounds of the approximate PDI measure ($PDI'_{act} = \sqrt{D}/\bar{d}' = [D\sqrt{n/m}\,]/\bar{\Delta}$; i.e., PDI'_{act}) is bounded by the PDI_{min} and PDI_{max} relations defined in O'Brien (1990b) and in equations (27) and (28) below.

ALTERNATIVE DISTANCE MODELS

Thus far, the distance function has been derived for a rectangular configuration of points by assuming a rectangular exterior region. Mathematically, there is good reason for doing this because a square or rectangle can be drawn around any closed curve (Steinhaus, 1969).

Occasionally, the environment of interest may be modeled by curved configurations such as ellipses or circles, the latter being the easier to work with. Circular distributions have two advantages. First, for regions nearly square, a circle offers a more compact concentration of points, which may provide more realistic bounds on the density measure for highly cluttered environments. Second, any number n of points (including prime numbers) can be placed uniformly on a circle of radius r with linear point-to-point distance $d = 2r \sin(180/n)$. Based on this chord length measure, the author recently constructed a PDI model for discrete spatial density for circular distributions (O'Brien, 1992).

SELECTED PROOFS

PROOF THAT $PDI_{min} \leq PDI_{act} \leq PDI_{max}$

First, a statement of the relationships involved in this proof is given as follows:

$$\text{Lower bound: } PDI_{min} = \frac{1}{\delta\overline{\Delta}}\sqrt{\frac{n}{A}}, \tag{27}$$

$$\text{Upper bound: } PDI_{max} = \frac{1}{\overline{\Delta}}\sqrt{\frac{n}{A}}, \tag{28}$$

$$\text{Actual PDI: } PDI_{act} = \frac{1}{\overline{d}_{act}}\sqrt{\frac{n}{A}}. \tag{29}$$

The terms n, A, $\overline{\Delta}$, and $\overline{d}_{act}$ (equivalent to equation (20)) are used here as defined in this report; δ is defined in appendix A.

Now, to the proof. From the relationships of equations (27), (28), and (29), a formal statement of the relationship to be proven is as follows:

$$\frac{1}{\delta\overline{\Delta}}\sqrt{\frac{n}{A}} \leq \frac{1}{\overline{d}_{act}}\sqrt{\frac{n}{A}} \leq \frac{1}{\overline{\Delta}}\sqrt{\frac{n}{A}}. \tag{30}$$

To prove that equation (30) is a true statement, three assumptions are required:

$$\overline{d}_{act}, \delta, \text{ and } \overline{\Delta} \text{ are measured in linear units of feet,} \tag{31}$$

$$\delta \geq 1, \tag{32}$$

$$\overline{d}_{act} \leq \delta\overline{\Delta}. \tag{33}$$

The first assumption (31) is self-explanatory. The second assumption (32) is deemed reasonable because it amounts to saying that if persons are positioned uniformly the head-to-head distance (δ) is about 1 foot. Although (32) would not be a reasonable assumption for areal units of, say, square miles, (32) is reasonable when the areal units are square feet. (See O'Brien, 1991f, for the finite "macro" PDI model when areal units are square miles.)

The third assumption (33) states that, for a given geometric area to be studied in a density analysis, the actual clustering of the density points (i.e., people) in that area (with associated density $\overline{d}_{act}$) will not be greater than the maximum theoretical dispersion provided by the relation $\delta\bar{\Delta}$ (equation (17)). The region outside $\delta\bar{\Delta}$ is assumed to contain physical objects such as furniture, equipment, etc., making it unlikely that density points will be observed in that region. Empirical evidence from Monte Carlo simulations in O'Brien (1989) is cited in support of (33). In effect, (33) assumes that the persons are maximally dispersed in accord with the relation $\delta\bar{\Delta}$. Figure 6 describes the essential meaning of (33).

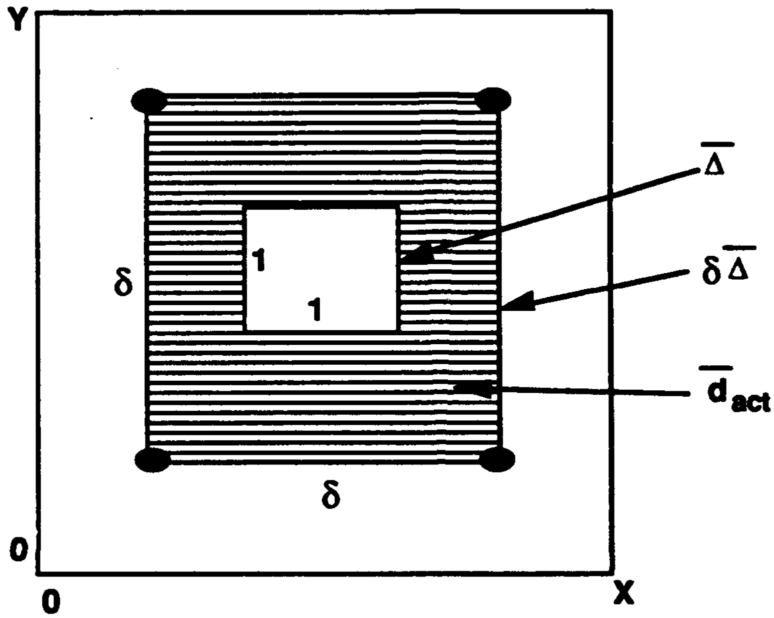

Figure 6. Intuitive Justification for Third Assumption

The formal proof of equation (30) can now be given in detail. The proof is presented in three parts. The first part states

$$\frac{1}{\delta\bar{\Delta}}\sqrt{\frac{n}{A}} \le \frac{1}{\bar{d}_{act}}\sqrt{\frac{n}{A}} \,. \tag{34}$$

Simplifying and rearranging the terms of equation (34) gives the following relationship:

$$\bar{d}_{act} \le \delta\bar{\Delta}\,, \tag{35}$$

which follows directly from (33).

The second part of the proof states that

$$\frac{1}{\bar{d}_{act}}\sqrt{\frac{n}{A}} \le \frac{1}{\bar{\Delta}}\sqrt{\frac{n}{A}}\,. \tag{36}$$

Simplifying and rearranging the terms of equation (36) gives the following relationship:

$$\bar{\Delta} \le \bar{d}_{act}\,. \tag{37}$$

From (33) the following relationship can be established:

$$\frac{\bar{d}_{act}}{\bar{\Delta}} \le \delta\,, \tag{38}$$

from which it can be deduced that

$$\frac{\bar{\Delta}}{\bar{d}_{act}} \le \frac{1}{\delta}\,. \tag{39}$$

Since, by (32), it follows that $1/\delta \le 1$, then it can be deduced that $\bar{\Delta}/\bar{d}_{act} \le 1$, from which it follows that $\bar{\Delta} \le \bar{d}_{act}$.

The third part of the proof asserts that

$$\frac{1}{\delta\bar{\Delta}}\sqrt{\frac{n}{A}} \le \frac{1}{\bar{\Delta}}\sqrt{\frac{n}{A}}\,. \tag{40}$$

The relationship between the lower and upper limits of equation (40) follows necessarily from the proofs given for equations (34) and (36) by the transitivity property of relations. It can be readily seen that equation (40) reduces algebraically to $\delta \ge 1$, which follows directly from (32). Thus, the statement of equation (30) has been shown to be true as derived from the stated definitions and assumptions.

The proof that the approximate PDI formula is bounded by the minimum and maximum bounds given in equations (27) and (28) follows from equation (26) and from the definition of the approximate PDI measure.

PROOFS FOR δ_{eff}

From O'Brien (1990b, equation(9)), δ_{eff} is defined as

$$\delta_{eff} = \frac{\bar{d}_{act}}{\bar{\Delta}}. \tag{41}$$

The objective is to show that $\delta_{eff} \geq 1$. Since $\frac{\bar{d}_{act}}{\bar{\Delta}} \geq 1$, as proven from equation (36), equation (41) follows.

The proof that $\delta/\delta_{eff} \geq 1$ is as follows:

By definition, $\delta_{eff} = \frac{\bar{d}_{act}}{\bar{\Delta}}$; then, $\delta/\delta_{eff} = \frac{\delta\bar{\Delta}}{\bar{d}_{act}} \geq 1$, which follows because it reduces to $\delta\bar{\Delta} \geq \bar{d}_{act}$, which was established previously in (33).

Because the quantity δ_{eff} is a "pure number" (i.e., it has no dimensions because they cancel out as in the above definition), it provides a pure measure of relative change in population density.

The reader may also note that in the approximation model $\delta'_{eff} \geq 1$ and $\delta/\delta'_{eff} \geq 1$ follows from the derived limits given in equation (25) and the relationship given in equation (26).

SUMMARY

This report has presented derivations of various distance functions that relate to the author's three-parameter square-root model for measuring discrete spatial density in finite populations. The model, called the Population Density Index (PDI) model, was developed to capture dynamic density relations among persons within a naturalistic environment. An "exact" model and an "approximate" model were presented.

The derivations related a generalized Euclidean distance function to the fundamental measures in the model (PDI_{act}, the approximation measure PDI'_{act}, their lower and upper bounds, and the density rate indices δ_{eff} and δ'_{eff}). Coordinate systems were derived for plotting graphs of the PDI lattices and calculating the distance measures.

Also derived was the algorithm required to select a conformal lattice and the average uniform distance among the lattice points based on the number of density points to be analyzed within the reference quadrilateral area.

Average Euclidean distance values ($\bar{\Delta}$) were presented for unit lattices up to a 100 x 100 matrix. Using these values, researchers will be able to compute lower and upper bounds of the PDI measures for up to 10,000 density objects.

APPENDIX A
SELECTING A UNIT LATTICE AND INTERPOINT DISTANCE PARAMETER

DERIVATION OF THE ALGORITHM

In this appendix, the algorithm is presented for (1) determining a unique finite, discrete, conformal RC lattice and (2) computing the average interpoint distance among the RC points.

To begin, it is assumed that n (sample size) and A = X x Y (the outer rectangular geometric area) are known. If n is a prime number (like 5 or 13 or 29), augment n by 1 before determining the rectangular/square dimensions of the unit lattice. The derivation of the algorithm for selecting an RC lattice is developed from concepts of number theory (Ore, 1967). In particular, interest is centered on sets and subsets of composite numbers that can be expressed as rectangular or square integers; i.e., positive (nonprime) integers that are two-integer products.

The value of n can be expressed in terms of the prime factors of the whole number:

$$n = \prod_{j=1}^{r} P_j^{\alpha_j}, \qquad \text{(A-1)}$$

where P_j represents the jth prime number and α_j is the number of occurrences of the jth prime number of n. For example, composite 60 can be decomposed into $P_1^{\alpha_1} P_2^{\alpha_2} P_3^{\alpha_3} = 2^2 \times 3 \times 5$. Next, it is desired to derive the total number of possible RC (n = R x C) product configurations of n in order to create the set of RC configurations; the latter will be a subset of the former. This number can be derived as follows.

Let $\tau(n)$ represent the number of all possible configurations of a composite integer n. Then it can be shown that this quantity is obtained from equation (A-1) by

$$\tau(n) = \prod_{j=1}^{r} (\alpha_j + 1). \qquad \text{(A-2)}$$

For example, 60 can be partitioned into $(2+1)(1+1)^2 = 12$ two-integer products.

Next, the set of the $\tau(n)$ configurations is examined to select only those nontrivial and/or nonredundant configurations. Let $\Phi(RC)$ represent the total number of nonredundant and nontrivial R x C configurations for composite n, $\tau(n) \supset \Phi(RC)$. The trivial configurations are those for which n = n x 1 or 1 x n, and the redundant configurations are the multiplicative, commutative equivalents of R x C; i.e., R x C = C x R ($R \geq C$) (e.g., 10 x 4 = 4 x 10). Then,

$$\Phi(RC) = \frac{\tau(n) - 2 + S}{2}, \qquad \text{(A-3)}$$

where $S = 0$ when n is a rectangular number, and $S = 1$ when n is a square number.* The set of all such specified configurations is denoted P of size $\Phi(RC) = m$; $P = \{R_1C_1, R_2C_2, ..., R_iC_i, ..., R_mC_m\}$, $(R_i \geq C_i)$. For example, if $n = 60$, then $\Phi(RC) = [(3 \times 2 \times 2) - 2 + 0]/2 = 5$; $P = \{30 \times 2, 20 \times 3, 15 \times 4, 12 \times 5, 10 \times 6\}$. Note that the trivial (60 x 1, 1 x 60) and redundant commutative equivalent configurations (2 x 30, 3 x 20, 4 x 15, 5 x 12, 6 x 10) have been eliminated from P. Likewise, for $n = 100$, $\Phi(100) = \Phi(2^2 \times 5^2) = [(3 \times 3) - 2 + 1)]/2 = 4$; $P = \{50 \times 2, 25 \times 4, 20 \times 5, 10 \times 10\}$.

Selection of a unique RC lattice with interpoint distance parameter δ is accomplished by the following guidelines.

Select the R x C lattice configuration (usually one) with dimensions most commensurate with the exterior X x Y dimensions; i.e., the one for which X/Y - R/C is a minimum absolute difference $(X \geq Y, R \geq C)$. Determine the uniform interpoint spacing parameter $\delta = \sqrt{A/n} = \sqrt{XY/RC}$ as defined in O'Brien (1990b, equation (3)). Next, test for conformity of the dimensions of the selected lattice to the study area dimensions by the quantities $(R - 1)\delta$ and $(C - 1)\delta$. If either of the R,C dimensions is nonconformal (i.e., $(R - 1)\delta \geq X$ or $(C - 1)\delta \geq Y$), then conform the lattice dimensions by adjusting δ by the relation $\delta = \min[X/(R - 1), Y/(C - 1)] - 0.1$. Finally, in the rarest of instances, when commensurability is achieved simultaneously by more than one lattice configuration, the researcher should approximate δ as above for each configuration, and then the R x C configuration will be that associated with the maximum δ value. If plural maxima δ occur, select the R x C configuration associated with the smallest value of $\overline{\Delta}$, given in appendix B.

The symbolic specification of the above guidelines can be stated as follows. Because the desired discrete R x C lattice must be unique, the selection mechanism requires a complex

* Equation (A-3) is not proven nor could a proof be found in the mathematical literature. Its correctness seems intuitively obvious. For example, for a number to be square, it is necessary and sufficient that all exponents in the prime factorization (equation (A-1)) be even (Ore, p. 42), which implies that $\tau(n)$ is odd, as is $\tau(n) - 2$, but adding 1 (S) makes $\Phi(RC)$ even. Finally, dividing by 2 eliminates the rectangular duplicates in $\tau(n) + S - 2$. The same logic applies to rectangular numbers, thus completing the proof outline.

two-step procedure. First, the following commensurability relation is determined from the dimensions of A and each element of the set P:

$$R_kC_k = \min_{1 \le i \le m} \left\| \left[\frac{\max(X,Y)}{\min(X,Y)} - \frac{\max(R_i, C_i)}{\min(R_i, C_i)} \right] \right\| \qquad (1 \le k \le m)\,. \tag{A-4}$$

Then, based on equation (A-4) above and equation (4) in the main body of the text, δ is determined from one of the following four mutually exclusive and exhaustive conditions:

$$\delta = \begin{cases} \sqrt{\frac{A}{n}} & \text{if } k = 1 \text{ and } p > 0 \text{ and } q > 0 \quad \text{(A-5)} \\ \min\left[\left(\frac{X}{R-1}, \frac{Y}{C-1}\right) - 0.1\right] & \text{if } k = 1 \text{ and } p \le 0 \text{ or } q \le 0 \quad \text{(A-6)} \\ \max\limits_{1 \le \ell \le k} \left\{ \min\limits_{2 \le k \le m} \left[\left(\frac{X}{R_k - 1}, \frac{Y}{C_k - 1}\right) - 0.1\right]\right\} & \text{if } k > 1 \text{ and } \ell = 1 \quad \text{(A-7)} \\ \min\limits_{2 \le \ell \le k} \left[\overline{\Delta}\,(R_\ell C_\ell)\right] & \text{if } k > 1 \text{ and } \ell > 1 \quad \text{(A-8)} \end{cases}$$

where p, q are defined in equation (4). In (A-5) through (A-8), $\delta \ge 1$ by definition. Also, it may be proven that $\delta \le \sqrt{A/n}$ based on equation (4) where it can be deduced that $(R - 1)\delta < X$, $(C - 1)\delta < Y$, and for commensurate lattices ($\delta \le \sqrt{A/n}$), $\delta = X/R = Y/C$. This relationship places an upper bound on δ that is important in the proofs and derivations of the text.

Figure A-1 summarizes the algorithm for the RC lattice selection and computation of δ. In summary, if k = 1, R_kC_k is the lattice selected from equation (A-4) and δ is selected from equation (A-5) or equation (A-6). If k > 1, δ is selected from equation (A-7) and $R \times C$ is selected as the lattice associated with the maximum δ in equation (A-7). Finally, if (A-7) provides a plurality of δ values, then (A-8) is used, which selects the $R_\ell C_\ell$ $(2 \le \ell \le k)$ lattice associated with the smallest $\overline{\Delta}$ value. Appendix B contains the required $\overline{\Delta}$ values computed to five decimal places. Note that for a unit lattice, or commensurate nonunit lattice, k = 1 and equation (A-5) computes the correct δ. Hansen et al. (1953, Vol. I) provides an interesting discussion of commensurate nonunit lattices related to a square-root law for distances in the field of discrete finite-population sampling theory when equation (A-5) applies.

Thus, equations (A-4) through (A-8) provide a unique, conforming lattice with associated interpoint distance parameter δ. A table of prime numbers and factorizations of composite numbers is an indispensable tool for implementing equation (A-4). See Lehmer (1941, 1961) for extensive tables and Abramowitz and Stegun (1964) for abbreviated tables.

These calculations assure that the lengths of the R and C line segments of the nonunit lattice, $(R - 1)\delta$ and $(C - 1)\delta$, containing human density points do not exceed the dimensions of the study area. The utility of adjusting δ (when so required) as recommended resides in plotting minimum/maximum dispersions of the RC density points in the study area as given in equations (6) and (10).

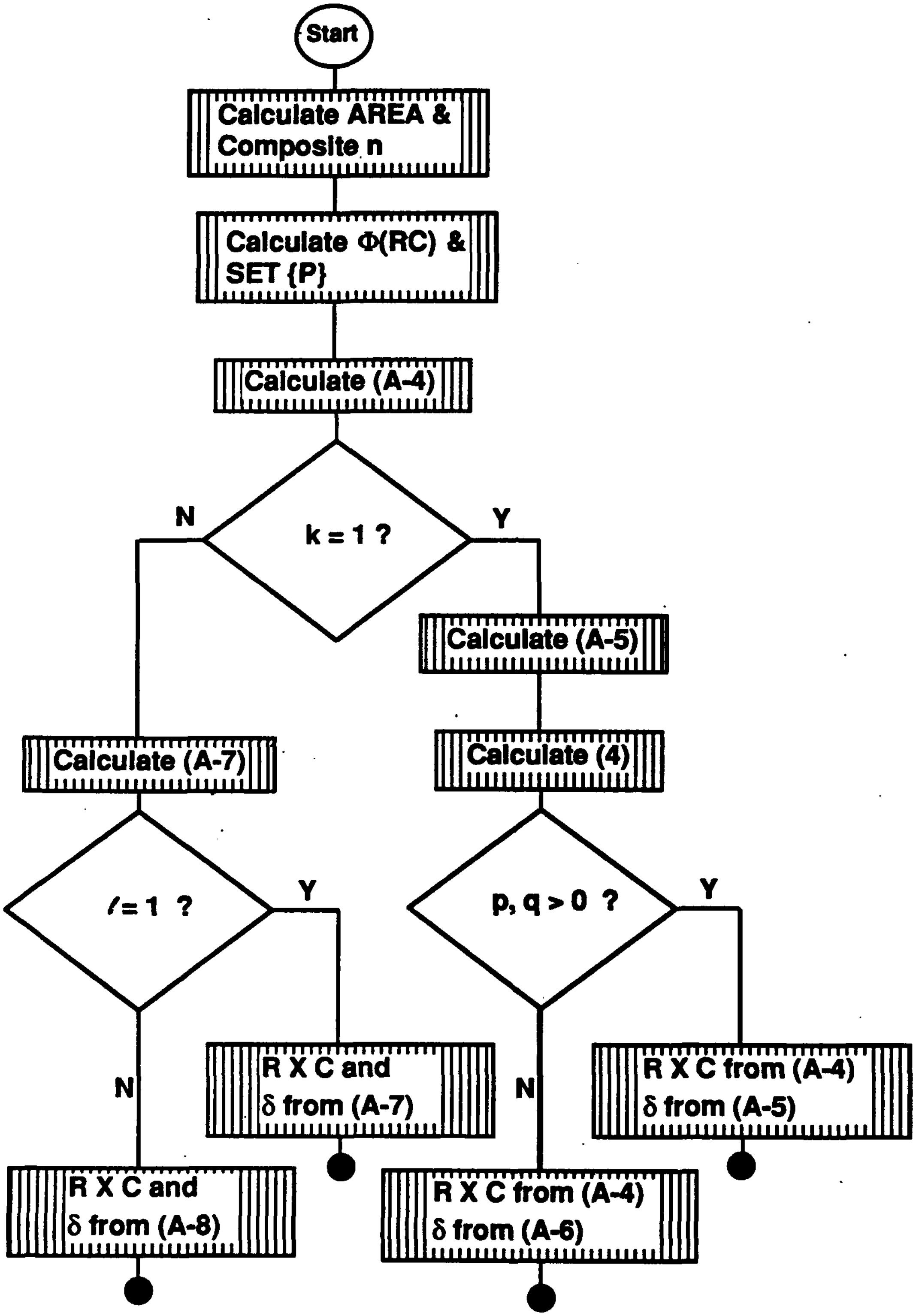

Figure A-1. Flowchart for Determining R × C Unit Lattice and Interpoint Distance Parameter δ

NUMERICAL EXAMPLES

Three artificial examples are selected to demonstrate the procedures. A complete setup is provided. The flowchart in figure A-1 is useful in tracing the decision logic.

In the first example, the data are as follows: $n = 12$, $A = 25 \times 25\ ft^2$. It is obvious that $n = 12$ provides two nontrivial, nonredundant choices ($\Phi(12) = 2$); viz., $R_1C_1 = 6 \times 2$ or $R_2C_2 = 4 \times 3$. Because $X/Y = 1$, $R_2C_2 = 4 \times 3$ comes closest to satisfying equation (A-4). Because $k = 1$, first compute $\delta = 7.22$ (from equation (A-5)); R,C is found to be conformal (each row/column "fits" inside the outside 25 ft^2 area in accord with equation (4)). Thus, $R = 4$, $C = 3$, and $\delta = 7.22$.

In the second example, $n = 64$ and $A = 50 \times 5\ ft^2$. This example is one of those rare possibilities. For $n = 64$, $\tau(64) = 7$; $\Phi(RC) = [7 + 1 - 2]/2 = 3$, and $P = \{32 \times 2, 16 \times 4, 8 \times 8\}$. Applying equation (A-4) shows that 32×2 and 16×4 are equally commensurate ($k > 1$); i.e., $|10 - 16| = |10 - 4|$. Thus, because $k = 2$ and ℓ is undetermined, apply equation (A-7), giving $\delta = \max\{1.57, 1.51\} = 1.57(\ell = 1)$. The configuration associated with the largest δ value is 16×4. Thus, $R = 16$, $C = 4$, and $\delta = 1.57$ for this data distribution.

As an example requiring equation (A-8) for determining $R \times C$ and δ, consider the data: $A = 80 \times 16\ ft^2$, $n = 32$, $P = \{16 \times 2, 8 \times 4\}$. Here, applying (A-4) to the above data distributions produces $|5 - 8| = |5 - 2|$ (i.e., $k = 2$), and (A-7) produces $\delta = \max\{(5.23, 5.23)\}$ ($\ell > 1$), which is clearly ambiguous. But $\min[\bar{\Delta}(16 \times 2), \bar{\Delta}(8 \times 4)] = \min(5.59, 3.27) = \bar{\Delta}(8 \times 4)$. Thus, $R \times C = 8 \times 4$, and $\delta = 5.23$.

In general, the reader will note that (A-7) or (A-8) will be required for determining δ whenever the study area ratio X/Y is equal to the average of the ratios of two equally commensurate lattices. The above examples bear out this relationship.

APPENDIX B
UNIT LATTICE AVERAGE EUCLIDEAN DISTANCE VALUES

Dimensions of Table: 100 by 100

B	C	Average Distance
2	2	1.13807
2	3	1.46666
2	4	1.76550
2	5	2.02300
2	6	2.28071
2	7	2.54750
2	8	2.80682
2	9	3.22846
2	10	3.61000
2	11	3.94430
2	12	4.27131
2	13	4.60648
2	14	4.93600
2	15	5.26766
2	16	5.60483
2	17	5.93637
2	18	6.26651
2	19	6.57683
2	20	6.90780
2	21	7.23043
2	22	7.56047
2	23	7.89067
2	24	8.22861
2	25	8.56346
2	26	8.88667
2	27	9.22077
2	28	9.55656
2	29	9.89043
2	30	10.22230
2	31	10.55443
2	32	10.88652
2	33	11.21967
2	34	11.55000
2	35	11.88016
2	36	12.21546
2	37	12.54784
2	38	12.88054
2	39	13.21269
2	40	13.54617
2	41	13.87769
2	42	14.21034
2	43	14.54263
2	44	14.87543
2	45	15.20806
2	46	15.54072
2	47	15.87341
2	48	16.20611
2	49	16.53864
2	50	16.87150
2	51	17.20436
2	52	17.53715
2	53	17.86978
2	54	18.20277
2	55	18.53561
2	56	18.86846
2	57	19.20133
2	58	19.53421
2	59	19.86710
2	60	20.20000
2	61	20.53288
2	62	20.86585
2	63	21.19879
2	64	21.53174
2	65	21.86470
2	66	22.19767
2	67	22.53065
2	68	22.86364
2	69	23.19663
2	70	23.52964
2	71	23.86265
2	72	24.19567
2	73	24.52870
2	74	24.86173
2	75	25.19477
2	76	25.52782
2	77	25.86087
2	78	26.19393
2	79	26.52700
2	80	26.86007
2	81	27.19315
2	82	27.52623
2	83	27.85932
2	84	28.19241
2	85	28.52551
2	86	28.85861
2	87	29.19172
2	88	29.52483
2	89	29.85795
2	90	30.19107
2	91	30.52419
2	92	30.85732
2	93	31.19045
2	94	31.52359
2	95	31.85673
2	96	32.18987
2	97	32.52302
2	98	32.85617
2	99	33.18932
2	100	33.52248
3	3	1.63496
3	4	1.90978
3	5	2.16848
3	6	2.43158
3	7	2.70437
3	8	3.00070
3	9	3.40633
3	10	3.72188
3	11	4.02968
3	12	4.35071
3	13	4.66107
3	14	4.98370
3	15	5.29740
3	16	5.60881
3	17	5.77740
3	18	6.26646
3	19	6.62018
3	20	6.95727
3	21	7.26646
3	22	7.61250
3	23	7.94128
3	24	8.26891
3	25	8.59696
3	26	8.92623
3	27	9.25773
3	28	9.58744
3	29	9.91738
3	30	10.24744
3	31	10.57760
3	32	10.90811
3	33	11.23806
3	34	11.56835
3	35	11.90015
3	36	12.23177
3	37	12.56210
3	38	12.89252
3	39	13.22644
3	40	13.55674
3	41	13.88712
3	42	14.21850
3	43	14.55011
3	44	14.88170
3	45	15.21336
3	46	15.54500
3	47	15.87666
3	48	16.20839
3	49	16.54007
3	50	16.87200
3	51	17.20447
3	52	17.53646
3	53	17.86854
3	54	18.20044
3	55	18.53277
3	56	18.86494
3	57	19.19714
3	58	19.52936
3	59	19.86164
3	60	20.19394
3	61	20.52626
3	62	20.85861
3	63	21.19098
3	64	21.52339
3	65	21.85582
3	66	22.18827
3	67	22.52074
3	68	22.85323
3	69	23.18574
3	70	23.51826
3	71	23.85083
3	72	24.18340
3	73	24.51599
3	74	24.84860
3	75	25.18121
3	76	25.51388
3	77	25.84650
3	78	26.17917
3	79	26.51186
3	80	26.84454
3	81	27.17725
3	82	27.50997
3	83	27.84270
3	84	28.17545
3	85	28.50821
3	86	28.84097
3	87	29.17375
3	88	29.50654
3	89	29.83934
3	90	30.17216
3	91	30.50496
3	92	30.83781
3	93	31.17064
3	94	31.50349
3	95	31.83636
3	96	32.16921
3	97	32.50209
3	98	32.83497
3	99	33.16786
3	100	33.50076
4	4	2.14100
4	5	2.40547
4	6	2.66682
4	7	2.97771
4	8	3.27108
4	9	3.57281
4	10	3.87583
4	11	4.18086
4	12	4.50942
4	13	4.81406
4	14	5.13132
4	15	5.44219
4	16	5.76484
4	17	6.08006
4	18	6.41066
4	19	6.73315
4	20	7.05644
4	21	7.38044
4	22	7.70866
4	23	8.02858
4	24	8.35400
4	25	8.68218
4	26	9.00978
4	27	9.33795
4	28	9.66909
4	29	9.98973
4	30	10.31647
4	31	10.64606
4	32	10.97534
4	33	11.30466
4	34	11.63218
4	35	11.96203
4	36	12.29132
4	37	12.62077
4	38	12.95020
4	39	13.28016
4	40	13.61007
4	41	13.94012
4	42	14.27030
4	43	14.60067
4	44	14.93096
4	45	15.26147
4	46	15.59208
4	47	15.92276
4	48	16.25354
4	49	16.58440
4	50	16.91536
4	51	17.24637
4	52	17.57746
4	53	17.90862
4	54	18.23985
4	55	18.57113
4	56	18.90248
4	57	19.23389
4	58	19.56534
4	59	19.89688
4	60	20.22841
4	61	20.56002
4	62	20.89168
4	63	21.22337
4	64	21.55511
4	65	21.88689
4	66	22.21871
4	67	22.55056

4	68	22.86246
4	69	23.21436
4	70	23.54634
4	71	23.87833
4	72	24.21036
4	73	24.54240
4	74	24.87446
4	75	25.20650
4	76	25.53872
4	77	25.87086
4	78	26.20307
4	79	26.53528
4	80	26.86751
4	81	27.19976
4	82	27.53204
4	83	27.86434
4	84	28.19666
4	85	28.52900
4	86	28.86136
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47	56	26.91091	48	69	30.76251
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47	58	27.45704	48	71	31.32977
47	59	27.73167	48	72	31.61453
47	60	28.00731	48	73	31.90002
47	61	28.28393	48	74	32.18622
47	62	28.56151	48	75	32.47311
47	63	28.84002	48	76	32.76069
47	64	29.11943	48	77	33.04893
47	65	29.39972	48	78	33.33782
47	66	29.68087	48	79	33.62735
47	67	29.96286	48	80	33.91749
47	68	30.24566	48	81	34.20824
47	69	30.52926	48	82	34.49958
47	70	30.81363	48	83	34.79150
47	71	31.09875	48	84	35.08398
47	72	31.38461	48	85	35.37702
47	73	31.67119	48	86	35.67060
47	74	31.95848	48	87	35.96472
47	75	32.24644	48	88	36.25935
47	76	32.53508	48	89	36.55449
48	90	36.85012	50	53	26.86383
48	91	37.14625	50	54	27.12869
48	92	37.44285	50	55	27.39505
48	93	37.73992	50	56	27.66238
48	94	38.03744	50	57	27.93126
48	95	38.33542	50	58	28.20104
48	96	38.63384	50	59	28.47190
48	97	38.93268	50	60	28.74381
48	98	39.23196	50	61	29.01674
48	99	39.53164	50	62	29.29066
48	100	39.83173	50	63	29.56555
49	49	25.55436	50	64	29.84139
49	50	25.81369	50	65	30.11814
49	51	26.07836	50	66	30.39578
49	52	26.34234	50	67	30.67430
49	53	26.60759	50	68	30.95367
49	54	26.87406	50	69	31.23387
49	55	27.14172	50	70	31.51488
49	56	27.41054	50	71	31.79668
49	57	27.68049	50	72	32.07925
49	58	27.95153	50	73	32.36257
49	59	28.22364	50	74	32.64663
49	60	28.49678	50	75	32.93140
49	61	28.77093	50	76	33.21688
49	62	29.04607	50	77	33.50304
49	63	29.32215	50	78	33.78987
49	64	29.59917	50	79	34.07735
49	65	29.87709	50	80	34.36547
49	66	30.15590	50	81	34.65422
49	67	30.43557	50	82	34.94358
49	68	30.71608	50	83	35.23354
49	69	30.99740	50	84	35.52409
49	70	31.27952	50	85	35.81520
49	71	31.56242	50	86	36.10688
49	72	31.84609	50	87	36.39911
49	73	32.13049	50	88	36.69187
49	74	32.41561	50	89	36.98516
49	75	32.70145	50	90	37.27897
49	76	32.98797	50	91	37.57328
49	77	33.27517	50	92	37.86808
49	78	33.56303	50	93	38.16337
49	79	33.85153	50	94	38.45914
49	80	34.14066	50	95	38.75537
49	81	34.43041	50	96	39.05206
49	82	34.72075	50	97	39.34919
49	83	35.01169	50	98	39.64676
49	84	35.30321	50	99	39.94477
49	85	35.59528	50	100	40.24319
49	86	35.88791	51	51	26.58696
49	87	36.18108	51	52	26.85427
49	88	36.47477	51	53	27.12087
49	89	36.76899	51	54	27.38472
49	90	37.06371	51	55	27.64980
49	91	37.35892	51	56	27.91606
49	92	37.65463	51	57	28.18347
49	93	37.95080	51	58	28.45200
49	94	38.24745	51	59	28.72162
49	95	38.54455	51	60	28.99231
49	96	38.84210	51	61	29.26403
49	97	39.14009	51	62	29.53675
49	98	39.43851	51	63	29.81045
49	99	39.73738	51	64	30.08511
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