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# A REEXAMINATION OF THE PLASTIC FLOW CRITERION FOR COPPER

NORRIS J. HUFFINGTON, JR.

JULY 1992

U.S. ARMY LABORATORY COMMAND

BALLISTIC RESEARCH LABORATORY
ABERDEEN PROVING GROUND, MARYLAND

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It was found impossible to reconcile discrepancies between uniaxial and torsion test data for copper within the framework of the von Mises yield function; use of a more general function employing both the second and third invariants of the stress deviator was studied and certain limitations on use of such functions are discussed.

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## 1. INTRODUCTION

While finite element hydrocodes can predict the contours of finitely deforming metals with reasonable accuracy, their ability to determine local strain and stress states leading to catastrophic failure by such mechanisms as adiabatic shear banding and void openings leaves much to be desired. An improvement in the ability to compute such local state histories would significantly enhance the design of warheads and predictions of armor penetration and behind-armor debris formation. In recognition that better material characterization was an essential ingredient of the desired improvement, a joint U.S. Army Ballistic Research Laboratory/Materials Technology Laboratory (BRL/MTL) program entitled Advanced Constitutive Models was initiated several years ago. This report discusses one facet of this investigation.

At BRL, it was decided to employ the DYNA3D hydrocode (Hallquist 1983) in connection with this study. This is a Lagrangian finite element computer program currently in widespread use which employs an adequate finite deformation formulation and features a choice of approximately 30 constitutive models.* However, only a few of these models are suitable for the large strain, low rate applications to be discussed in sequel. These models generally ** employ the von Mises yield condition $J_2 - k^2 = 0$, where $J_2$ is the second invariant of the stress deviator tensor, as the plastic potential function and use the Jaumann stress rate to account for the rigid body rotation of elements.

## 2. QUASI-STATIC MATERIAL PROPERTIES

For an evaluation of material constitutive behavior, it is necessary to have test data for the specific lot of material to be characterized. It was decided that this investigation would commence with a study of oxygen-free, high conductivity (OFHC) copper and MTL was tasked

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* This term is used to identify a mathematical function or algorithm which determines stress tensor components from the current rate-of-deformation components, some past history data, and possibly temperature through an incremental plasticity time marching process.

** An exception is the model of D. Bammann (Johnson and Bammann 1984) as incorporated by M. Chiesa of Sandia/ Livermore which is a multi-parameter micromechanically-based constitutive system which uses the Green-Naghdi stress rate.

to perform the necessary experimentation. Dr. Tusit Weerasooriya has reported (Weerasooriya and Swanson 1991) data from two types of quasi-static (isothermal) tests on annealed copper:

(1) Uniaxial compression stress-strain curves for the range $-1.30 < \varepsilon_{xx} < 0$ natural strain.

(2) Torsional shear stress-strain data from twist tests on modified Lindholm-type thin-walled tubular specimens for the range $0 < \varepsilon_{zx} < 1.4$ (tensor) shear strain. These tests were performed for two conditions of axial restraint: (a) almost total axial restraint in which case the induced axial force was recorded and (b) no applied axial restraint where the axial displacement was monitored.

Also, Dr. Weerasooriya provided the author a curve for reversed loading of a torsion specimen which permitted an assessment of the Bauschinger effect for this material.

## 3. CONSTITUTIVE MODELING CONSIDERATIONS

The compression test data cited above reveal that the stress-strain curve for annealed copper is nonlinear over the entire range, the elastic portion being of negligible size. For a curve of this form, a bilinear representation (such as DYNA3D Material 3) is unsatisfactory. Material 10 of the DYNA code, which permits input of up to 16 stress-strain points and interpolates linearly for intermediate values, is more appropriate but only treats isotropic hardening.

The Lindholm-type torsion specimen does not result in a homogeneous state of stress in the thin-walled test section; also, the shearing strain and plastic deformation extend into the transition section. For this reason, various investigators have resorted to 3-D finite element modeling of the whole specimen (using an assumed constitutive model) to provide a basis for interpretation of test data. Lipkin et al. (1987) reported a DYNA3D calculation using an earlier version of the Bammann constitutive model in which a twist rate high enough to predict adiabatic shear banding was used. The present author performed a DYNA3D analysis for the MTL geometry using Material 3 and found this geometry was prone to premature torsional buckling. It was recommended that a thicker wall be used since an interpretive analysis would be required in any event. Dr. C. S. White (1990) reported an ABAQUS analysis of a geometric configuration closely corresponding to that used by Weerasooriya and has concluded that "about 78% of the twist that is applied at the grips actually goes into the deformation in the gauge section." Although White's calculations were made for a different material, it was decided to multiply Weerasooriya's shear strain data by a factor of 0.78.

## 4. USE OF EXPERIMENTAL DATA; DISCREPANCIES

If the compression test data are used as input to DYNA3D Material 10 and a calculation is made for an element constrained to deform uniaxially with no lateral restraints, the code predictions are (naturally) in excellent agreement with the experimental values. (It is necessary to choose an equation-of-state which permits specifying the pressure to be proportional to the volumetric strain {proportionality constant = bulk modulus}, use the hourglass viscosity type 3 {Flanagan-Belytschko (1981) with exact volume integration}, and to set the Gruneisen coefficient = 0 for an isothermal calculation.) Similarly, when torsion test data are inserted in DYNA3D Material 10 and a simple shear problem is run for an element, there is no discrepancy.

However, when compression test data are used in Material 10 to predict the stresses in an element subjected to increasing simple shear the result shown in Figure 1 is obtained, where the overprediction of the shear stress $\sigma_{zx}$ is as great as 30%. Not surprisingly, the converse is also true: use of the torsion test data as input for a uniaxial compression calculation results in a significant underprediction of the axial stress $\sigma_x$ as shown in Figure 2.

This phenomenon is no new discovery and has been discussed in the literature by Prager (1945), Drucker (1949), Edelman and Drucker (1951), and many others. The basic problem is that use of the von Mises condition as a loading function for work hardening materials does not closely describe the plastic deformation of many materials (even though it is widely employed for this purpose in most currently used hydrocodes). According to the authors just cited, the problem can be resolved by use of a loading function depending on both the second and third invariants of the stress deviation; i.e., a $(J_2, J_3)$ theory. Although such theories seem to have fallen into disuse, perhaps this approach should be considered for applications involving large strains, where the discrepancies are greatest.

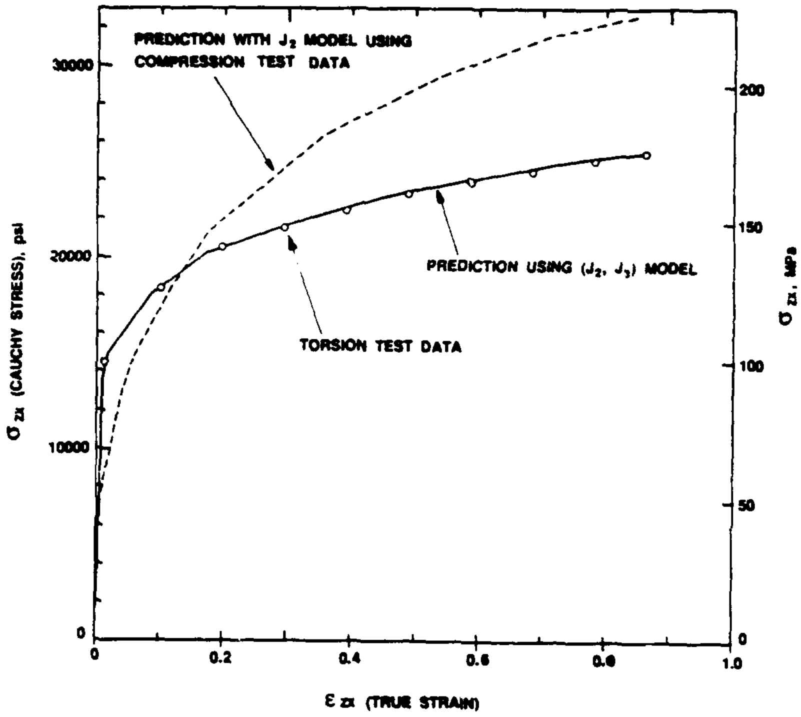

Figure 1. Simple Shear Calculation (Geometric Constraints, Material Type 10).

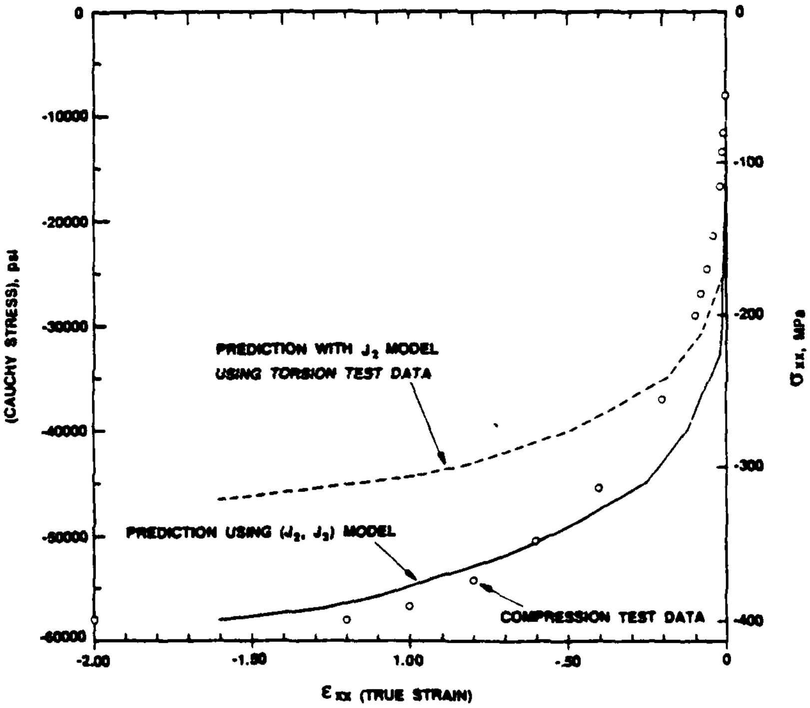

Figure 2. Uniaxial Compression Calculation (Geometric Constraints, Material Type 10).

## 5. IMPLEMENTATION OF A ($J_2$, $J_3$) THEORY

It was decided to employ the function quoted by Malvern (1969):

$$f(J_2,J_3) \equiv J_2\left[1 - \frac{c(J_3)^2}{(J_2)^3}\right] - k^2 = 0 \tag{1}$$

where

$$J_2 = \frac{1}{2}\left(S_{11}^2 + S_{22}^2 + S_{33}^2\right) + S_{12}^2 + S_{23}^2 + S_{31}^2 = \frac{1}{2}\left(S_1^2 + S_2^2 + S_3^2\right) \tag{2}$$

$$\begin{aligned} J_3 &= S_{11}\, S_{22}\, S_{33} + 2S_{12}\, S_{23}\, S_{31} - S_{11}\, S_{23}^2 - S_{22}\, S_{31}^2 - S_{33}\, S_{12}^2 \\ &= S_1\, S_2\, S_3 \end{aligned} \tag{3}$$

$k$ = Yield stress in pure shear, variable for a work hardening material

$c$ = nondimensional parameter to be adjusted to provide match of post yield flow data

$S_{ij}$ = deviatoric stress components

$S_k$ = principal deviatoric stresses

Installation of Equation 1 as the load function in DYNA3D was easily accomplished since current values of all required quantities are available in the stress evaluation subroutine. Because $J_3$ vanishes for pure shear, it is attractive to insert the tabular data from the shear tests in Material 10 and then determine the parameter c to provide correspondence to the compression test data for uniaxial stress calculations. For the present application, the peak stress was employed because the uniaxial stress-strain curve appears to level off at this value and the interest is in large strain plasticity. Of course, other matching criteria could be adopted for other strain ranges. Only a few trials were required to obtain $c = 2.64$; this value was used in DYNA3D to obtain the solid curves displayed in Figures 1 and 2. It may be seen in Figure 2 that there is a significant difference between experimental and predicted values in the small strain region. This is attributed to the much more rounded "knee" exhibited by copper in compression than in tension or shear.

## 6. GEOMETRIC REPRESENTATION

It is instructive to view the ($J_2$, $J_3$) yield function in principal stress space, where it may be recalled that the von Mises function is represented by a circular cylinder coaxial with the hydrostatic line $\sigma_1 = \sigma_2 = \sigma_3$. To achieve this, the following orthogonal coordinate transformation was made:

$$\begin{aligned}\sigma_1 &= -\frac{\tau_1}{\sqrt{6}} + \frac{\tau_2}{\sqrt{2}} + \frac{\tau_3}{\sqrt{3}} \\ \sigma_2 &= -\frac{\tau_1}{\sqrt{6}} - \frac{\tau_2}{\sqrt{2}} + \frac{\tau_3}{\sqrt{3}} \\ \sigma_3 &= \frac{2\tau_1}{\sqrt{6}} + \frac{\tau_3}{\sqrt{3}}\end{aligned} \tag{4}$$

With this transformation, the $\tau_3$ axis coincides with the hydrostatic line and the $\tau_1$ axis is aligned with the projection of the $\sigma_3$ axis on the deviatoric plane $\sigma_1 + \sigma_2 + \sigma_3 = 0$. Using Equations 4, one obtains

$$J_2 = \frac{1}{2}(\tau_1^2 + \tau_2^2) \tag{5}$$

$$J_3 = \frac{1}{\sqrt{6}}\left(\frac{\tau_1^3}{3} - \tau_1\tau_2^2\right) \tag{6}$$

and, substituting these values in Equation 1, there results

$$\begin{aligned}27\tau_2^6 + \left\{(81-36c)\tau_1^2 - 54k^2\right\}\tau_2^4 + \left\{(81+24c)\tau_1^4 - 108k^2\tau_1^2\right\}\tau_2^2 \\ + (27-4c)\tau_1^6 - 54k^2\tau_1^4 = 0\end{aligned} \tag{7}$$

which defines the contour of the yield function in the $\tau_1$, $\tau_2$-plane.

As noted by Hill (1950), it is only necessary to compute coordinates for a 30° segment of this plane, the remainder of the locus being determined by the symmetry constraints for an isotropic material. This locus for c = 2.64 is displayed in Figure 3 as a solid line. It is seen that the $(J_2, J_3)$ surface is a fluted cylinder with the von Mises cylinder inscribed. For Material 10 which provides only for isotropic hardening, these surfaces would expand uniformly as plastic deformation progresses. It should be mentioned that DYNA3D presently uses only the Krieg-Key radial return algorithm (Krieg and Key, 1976) to return the stress state to the updated yield surface each cycle. This algorithm is clearly more appropriate for the $J_2$ surface than for the $(J_2, J_3)$ function. However, it is still attractive due to its simplicity and the assurance that the yield surface will be intersected, although for the latter it corresponds to a nonassociated flow rule.* A "normal return" algorithm would seem preferable or perhaps the recently published Nemat-Nasser algorithm (Nemat-Nasser 1991), but either of these would require more extensive calculations per cycle. These matters may be academic in view of the following discussion.

## 7. MATERIAL STABILITY, CONVEXITY REQUIREMENTS

It is clear from inspection that the c = 2.64 loading function in Figure 3 violates the requirement that the surface be convex, as deduced by Drucker (1951, 1959) from his postulates for material stability. Although the author has made a considerable number of computer runs using this value of c with no evidence of instability, these cases were not designed to test all types of loading and unloading. Thus, it is appropriate to ask: Can any $(J_2, J_3)$ load function satisfy the convexity requirement? The answer is yes and the limitation on c is determined by evaluating the curvature of the load function at its point of tangency to the von Mises circle. Thus, at $\tau_1 = 0$,

$$\frac{d^2\tau_2}{d\tau_1^2} = \frac{1 - \frac{4}{3}c}{\sqrt{2}\,k} \tag{8}$$

This happens to be the exact curvature since $\frac{d\tau_2}{d\tau_1} = 0$ at this location.

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* This consideration does not invalidate the results displayed in this report since, for both pure shear and uniaxial loading, radial and normal return coincide.

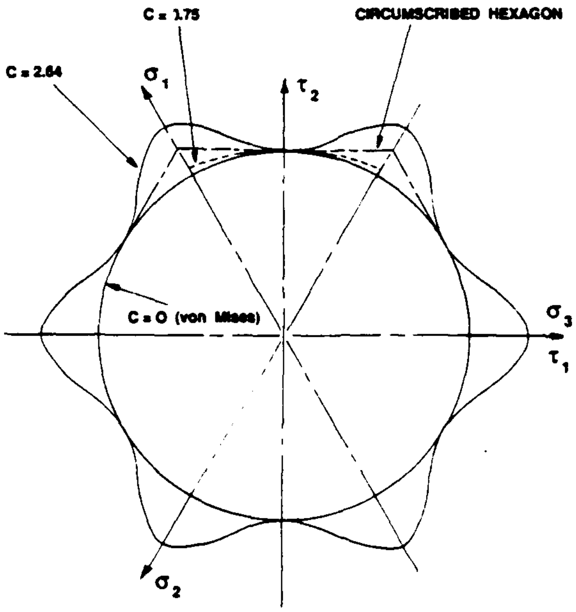

Figure 3. Axial View of Yield Loci.

Therefore, the transition in sign of the curvature occurs at c = 0.75 and the load function is everywhere convex for this or lesser values of c. The form of the load function for c = 0.75 is plotted in the upper sextant of Figure 3. Unfortunately, for this value of c, the percentage difference between the maximum and minimum radii of the load function is only about 6%. It can now be seen that the greatest percentage difference in radii consistent with convexity requires a load function in the form of a hexagon circumscribed about the von Mises circle, as shown in the upper portion of Figure 3. Even for this form of yield function (with its attendant analytical complexities), the percentage difference in radii is only 15.5%, about half of what is needed to reconcile the discrepancies associated with experimental data.

## 8. CONCLUDING REMARKS

It is realized that other investigators (Shrivastava, Jonas, and Canova 1982; Asaro and Needleman 1985; Weerasooriya and Swanson 1991) have treated the subject matter of this paper by micromechanical modeling and have attributed the cited discrepancies to formation of texture. However, the end objective of such research does not appear to be the identification of load functions appropriate to classical plasticity.

This author is convinced that material behavior must be modeled within the framework of continuum mechanics if efficient, large-scale computations are to yield valid results for engineering purposes. While the study reported herein may seem inconclusive, it is believed that use of a nonconvex loading function may be permissible for certain classes of problems, especially if the computer program is modified to test on the sign of the plastic work increment in each element and to terminate the calculations if a negative work increment is predicted. It is very desirable to resolve this matter before proceeding to studies of strain-rate effects, stress rate models, etc., where uncertainties regarding the loading function may obscure interpretation of other types of experimental results.

It should be mentioned that the author has added a new subroutine to a research version of DYNA3D which includes many effects previously not present in a single material model, specifically:

(1) Finite elastoplastic straining.

(2) Mixed kinematic/isotropic hardening, as recommended by Hodge (1957).

(3) Arbitrary shape of uniaxial stress-strain function through input of tabular data.

(4) Choice of stress rate formulation (Jaumann [1905] or Green and Naghdi [Green and Naghdi 1965; Green and McInnis 1967]).

(5) Choice of several equation-of-state models.

It is planned to use this tool to study finite plasticity for various proportional and nonproportional loading paths.

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