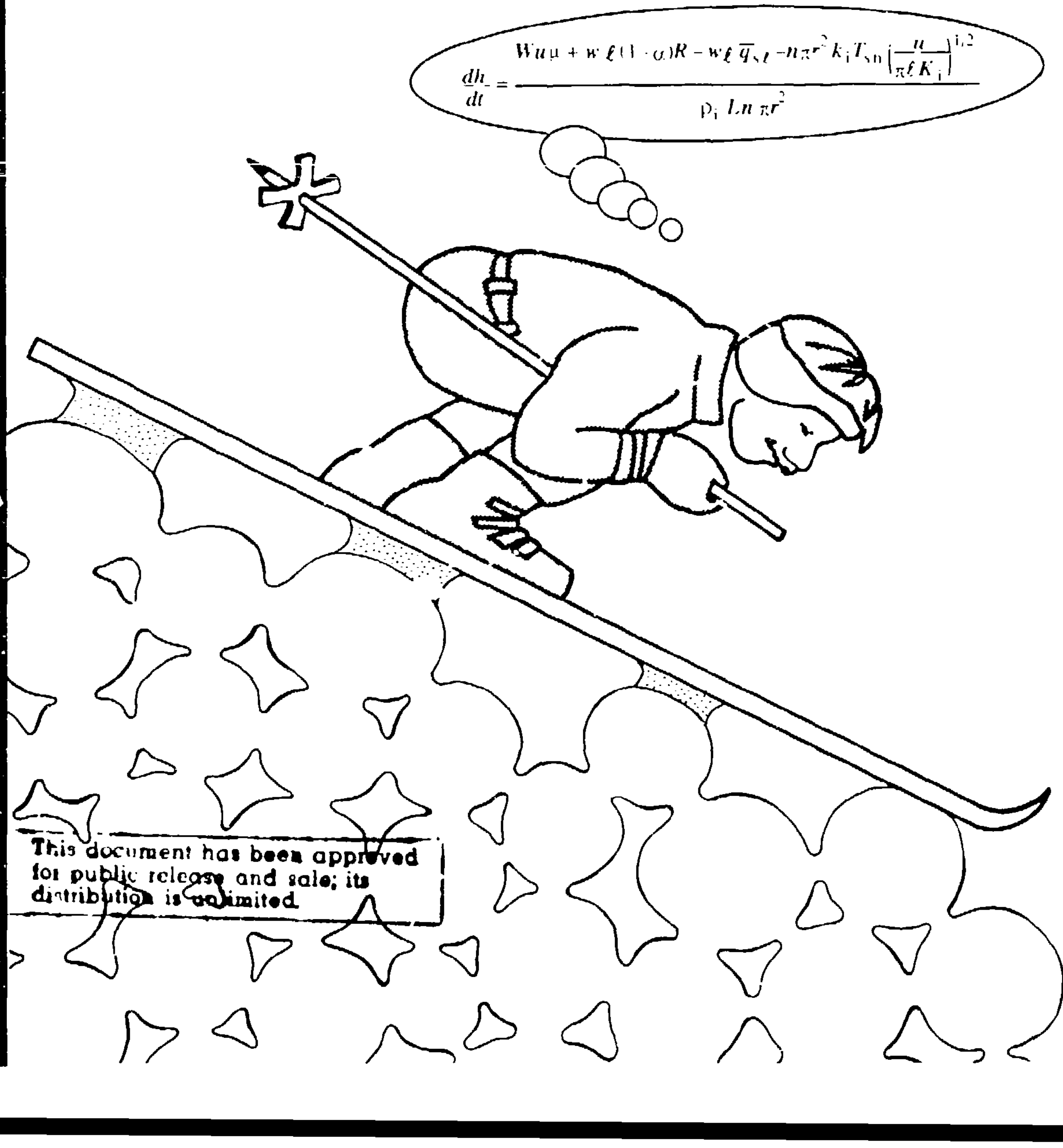
CRREL

A Review of the Processes That Control Snow Friction

Samuel C. Colbeck

April 1992



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PREFACE

This report was prepared by Dr. Samuel C. Colbeck, Geophysicist with the Snow and Ice Branch, Research Division, U.S. Army Cold Regions Research and Engineering Laboratory (CRREL). This project was funded by DA Project 4A762784AT42, *Design, Construction, and Operations Technology for Cold Regions*; Task FS, *Fire Support*, Work Unit 003, *Radiational Effects on Snow Signatures*.

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LIST OF SYMBOLS

A	actual contact area
A_0	contact area at rear
b	depth of compaction bulb
B	hardness
c	ratio of contact area to load
c_p	heat capacity
E	elastic tensial modulus
E_1	effective elastic modulus
F_{plow}	plowing force
h	film thickness
h_1	average asperity height
$\dot{h}$	rate of contact melting
H	slider thickness
k	thermal conductivity
l	slider length
L	latent heat
m	rate of meltwater production
n	number of contacts
q	heat flux
$\dot{q}$	rate of heat generation
r	contact radius
r_1	asperity tip radius
R	incident solar radiation per unit area
S	slippage factor
t	time
T	temperature
u	speed
w	width
W	weight
x	coordinate direction
α	albedo
β	constant
Δh	depression
ΔT	temperature rise
δT	temperature difference across a ski
$\tan \delta$	mechanical loss tangent
γ	shear strength
ε	coefficient
η	viscosity of water
κ	thermal diffusivity
μ	coefficient of friction
ν	Poisson's ratio
ξ	coefficient
ρ	density
σ	compressive strength
$\sigma(x)$	normal stress along x-axis
τ	shear strength
ϕ	fractional contact area

Subscripts

b	bulb
i	ice
sl	slider
sn	snow

A Review of the Processes That Control Snow Friction

SAMUEL C. COLBECK

I. INTRODUCTION

There is a long history of interest in snow friction, mostly because of the interest in recreational skiing. However, interest in snow friction also comes from a variety of subjects of practical importance including automobile tires, aircraft skis, ice breaker propulsion, and off-road vehicles. Kinetic friction has been studied the most and is discussed here because of the application of that information to skiing, but static friction is of more interest in applications such as automotive traction (Ahagon et al. 1988).

There has been considerable uncertainty about the mechanisms of snow and ice friction. In particular, there has been controversy over the melt-lubrication theory, but there is a considerable body of evidence to support the idea, and lubrication by melting is widely accepted in other areas of frictional studies, or tribology (e.g. Lim and Ashby 1987). Because this is the central issue to understanding friction over a wide range of natural conditions, much attention is devoted to it here. These issues are considered in the early stages of this review and then the pertinent observations on snow friction are summarized. This approach is taken because the purpose here is to understand snow friction, not just to summarize observations of it.

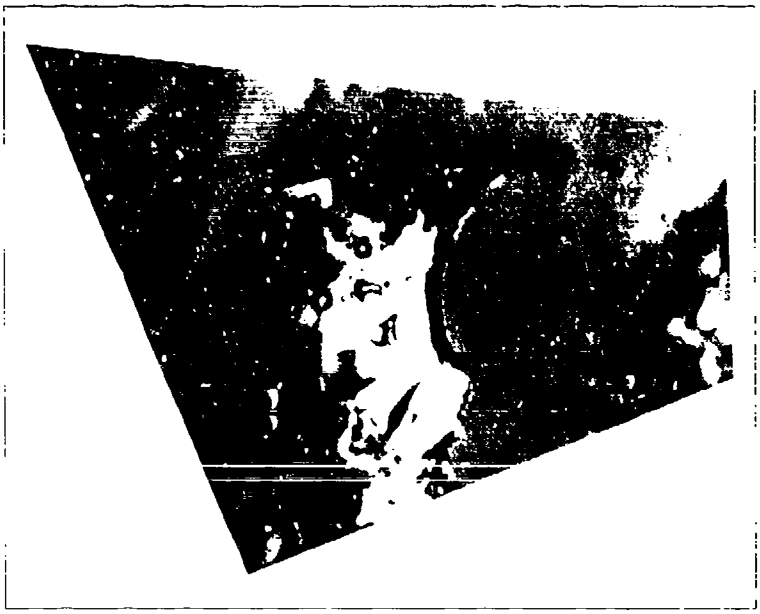
Work on this subject began some time ago (e.g. Gliddon 1923), but the most important period was when Bowden developed both the fundamental ideas behind the theory (e.g. Bowden and Hughes 1939) and introduced polytetrafluoroethylene (P.T.F.E.) as a better base for recreational skis (Bowden 195). There was also interest in aircraft skis in this period (e.g. Klein 1947), including a lot of interest in the USSR during the war years. Associated with the Winter Olympic Games in Sapporo, the Japanese published *Scientific Study of Skiing in Japan* (Society of Ski Science 1971), and the Scandinavians increased their efforts in the late 1970s. This high level of interest is likely to continue because of the widespread interest in recreational skiing and ski racing. As in many areas of technology, the subject has been developed to a very high level without a good physical understanding of the processes, but to continue the evolution, research will be necessary to improve our basic understanding of the mechanisms that account for the low friction of snow. The outstanding questions require increased knowledge of the contact area between snow and sliders, the role of meltwater lubrication including thickness of the water films, the occurrence of electrical charges, the possibility of capillary bonds, the action of dirt at the interface, and dry frictional processes. Knowledge of the effects of load, speed, temperature, snow type, and slider properties on all of these processes and parameters is critical to understanding even the simplest results from friction experiments or observations under natural conditions. Otherwise the results of laboratory experiments are often not transferable to other environments.

Beyond making a brief introduction to the snow surface, the purpose here is to review what is known about snow friction and to advance the field by testing some common assertions about snow friction. To this end the mechanisms responsible for sliding will be reviewed, especially dry sliding and meltwater lubrication. The descriptions of some of these mechanisms are expanded, but there is not enough information about them to combine them into a theory of snow friction for application. The measured values of friction will be summarized to identify the effects of load, speed, and temperature. Observations of interfacial heat and liquid generation will be reviewed and some new results introduced. When possible, the most general possible conclusions will be drawn from these tests, but the results are often specific to the particular test conditions. With some understanding of the processes and observations available, attempts to modify friction by adjusting the properties of ski bases are examined, because experience with ski waxing and structuring should help us understand the processes. This in turn should help identify the major remaining problems for research and the prospects for increasing our understanding of the subject.

II. SNOW SURFACE

Much information has been generated about the physical properties of snow, and some of that information is applicable to the snow surface. The snow temperature, density, strength, liquid-water content, and crystal types are of particular interest, but these snow parameters are difficult to measure on the surface and/or they assume rather different value, just because of the presence of the air/snow interface. Furthermore, the passage of a slider affects some of these parameters, especially temperature, density, liquid-water content, and crystal shape.

When snowfall occurs, the crystals can be classified using one of the standard classification systems such as Magano and Lee (1966). There is a wide range of possible crystal types depending on the prevailing temperature and humidity conditions during the for-



a. An ice particle polished by rubbing with a fiber block.

b. An ice particle polished by repeated passes of Nordic-type skis. The bumps on the polished surface appeared to have resulted from the ski drawing the water film as it moved past the ice grain.

Figure 1. Polished ice particles

mation of snow crystals in the atmosphere. Although there may not be any experimental observations showing the effects of different types of fresh snow crystals, skiers have learned that cold, fresh snow and man made snow are "aggressive" and require harder waxes. The lack of information about the compactive strength, porosity, and angularity of different types of snow surfaces limits the application of knowledge about snow to general statements about the prevailing conditions. A more detailed classification of snow surface conditions is needed to cover the complete range of conditions including fresh, windblown, wet, surface hoar, firnspiegel, glazed, rimmed, polished, and refrozen surfaces. A start on this classification can be found in Colbeck et al. (1990), but that classification was done without special attention to the surface conditions.

Snow friction is often of interest in situations where the surface has been polished by repeated crossings, and detailed microscopic observations of the snow surface following repeated passes are needed to document the effects that the slider has on the snow. The polished snow grains in Figure 1 show what appear to be meltwater caps formed on snow surfaces, and Figure 2 shows a surface that is highly polished from repeated ski passes. These polished grains can be generated by rubbing the snow surface with any flat object and indicate melting and refreezing on the ice grains that were contacted. Figure 1b shows a melted grain surface from the passage of Nordic skis. Definite evidence of melting and refreezing can be seen on this smooth, flat surface that is not oriented with the facets of the ice crystal. The small bumps on the surface suggest that the meltwater layer was drawn into liquid islands before it refroze. Figure 2 shows an oblique view of a highly polished snow surface where the smooth, dark surfaces appear to have resulted from melting and refreezing due to repeated passes of Nordic skis. This photograph shows contacts of about 100 to 300 μm in size and a contact area of nearly 50%, which is much higher than for surfaces that have not seen as many ski passes. These highly polished surfaces are well known to have lower friction than unpolished surfaces; this is not surprising since ice is slipperier than snow, possibly due to the dynamics of larger contacts (Colbeck 1988) or the lack of angular ice grains.

It is also important to look for evidence of recrystallization on the surface of ice asperities that may be deformed by dry processes as the slider passes. Both ice and other materials nucleate small crystallites at the surface that are oriented for easy glide. These would offer less resistance to deformation, so it is important to know if they exist on snow surfaces. Other signs of dry processes such as flake removal or gouging should be sought, especially at low speeds, small loads, and low temperatures where meltwater lubrication is limited. In my examinations of snow surfaces I have not seen such evidence but Huzioka (1962) saw ice chips at a low speed where less heat is generated. Unfortunately, since meltwater usually exists at the surfaces of the ice crystals during sliding, evidence of recrystallization or even gouging might be destroyed as the meltwater refreezes. Evidence of surface conditions during sliding is difficult to gather, and it is especially difficult to estimate the actual contact area during sliding (Ludema 1984). The ski temperature measurements described later give some information about the surface conditions during sliding.

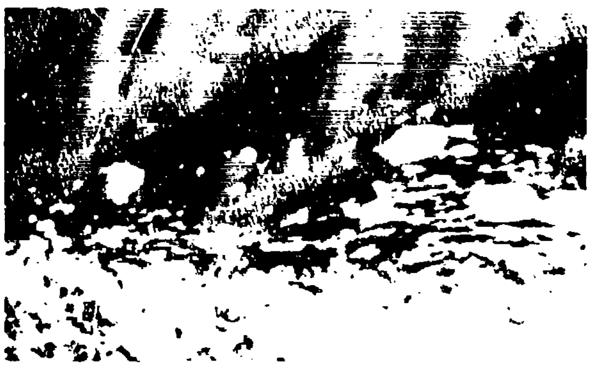


Figure 2. Oblique photograph of a highly polished area of snow after repeated passes of Nordic type skis. The black areas were the supporting surfaces that appeared at a distance to form a solid, icy surface with a mirror-like finish.

III. SLIDING MECHANISMS AND MODELS

The sliding processes are discussed before the measurements of friction are presented because it is necessary to have at least a general understanding of the processes before the experimental results can be put into perspective. Unfortunately this will lead to some repetition of the discussion of both theory and observations.

Various mechanisms contribute to the resistance to sliding over snow: plowing and compaction of snow in front of the slider, snow deformation below the slider, deformation or fracture of asperities, shearing of the water films that support the slider's weight, capillary attraction from other water attachments, and drag by surface dirt. Although adhesion is a very important part of friction when the sliding materials are similar in their molecular structure, are smooth, and have time to bond (Rabinowicz 1984), it is ignored here because ice grains usually slide on a very dissimilar material, such as polyethylene or ski wax. Furthermore, the surfaces are probably not molecularly smooth, the times for interaction are usually short, and polyethylene and P.T.F.E. surfaces are known to have low adhesion to other materials (Steijn 1967). Electrostatic forces may interact with some of these mechanisms, especially when solid-to-solid contacts cause electrostatic charges that attract dirt. Some information is given about each of these mechanisms here but there is not sufficient information available to give much detail for any of them. Furthermore, it is hard to generalize from most of the available measurements of friction, so the experimental results are not always helpful in testing the ideas presented about these mechanisms.

Although the mechanisms do not operate independently, different mechanisms do dominate under different conditions of load, speed, temperature, roughness, wetness, snow type, and slider characteristics. Later we will examine the individual mechanisms and describe some of their interactions. If the processes operated independently, the total friction (μ) could be expressed as the sum of a series of terms representing each mechanism, or

$$\mu = \mu_{plow} + \mu_{dry} + \mu_{lub} + \mu_{cap} + \mu_{dirt} \qquad (1)$$

where the subscripts *plow*, *dry*, *lub*, *cap*, and *dirt* represent the friction due to plowing, solid deformation, water lubrication, capillary attraction, and surface contamination, respectively. Other terms, such as one due to snow disaggregation, could also be included and might be important since snow grains seem to release by rebound after rapid ski passage. When two processes interact in parallel, however, the total friction must be expressed; for example, as

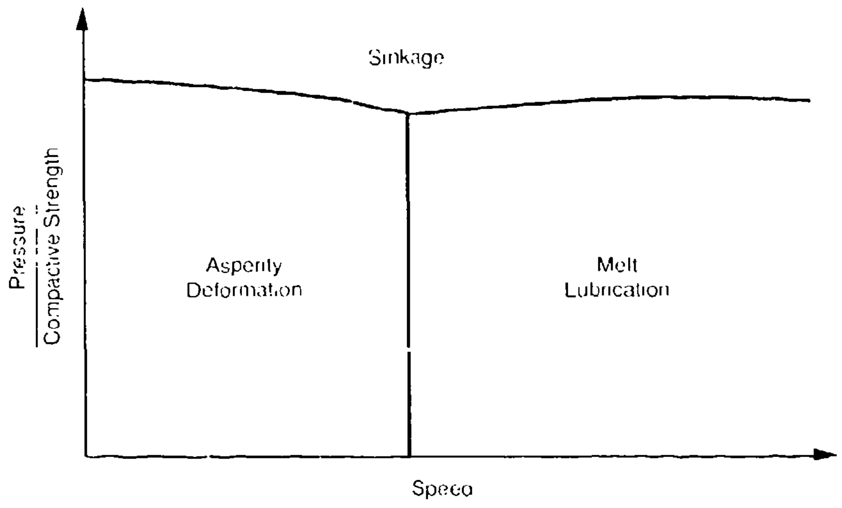
$$\frac{1}{\mu} = \frac{1}{\mu_{dry}} + \frac{1}{\mu_{lub}} \qquad (2)$$

Then it is much more difficult to describe the total friction because the processes must be understood independently and collectively.

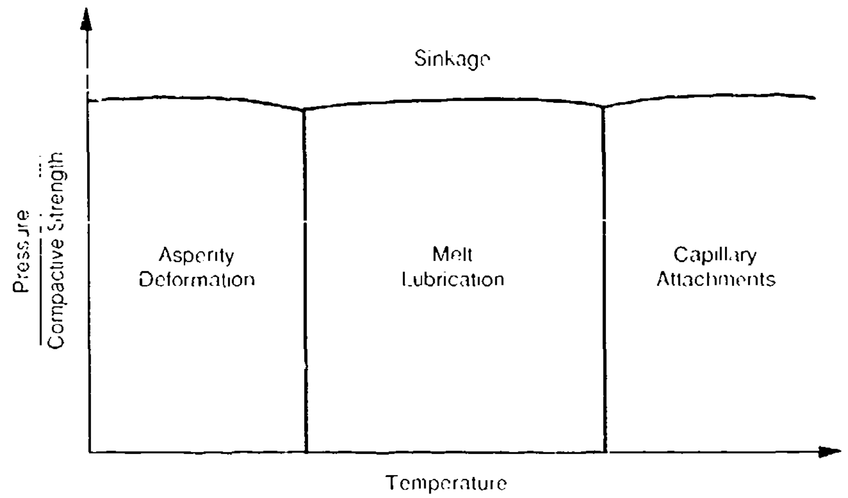
If enough information were available about the mechanisms, the contribution of each could be conveniently summarized in a friction mechanism map such as those developed for wear by Lim and Ashby (1987). Their maps were constructed from the available experimental data and then explained by theory. A map for snow is suggested in Figure 3a for two sliding mechanisms: asperity deformation and movement over films of meltwater. In reality the demarcations between these regimes are indistinct since there may be some sinkage occurring concurrently with either of the other two mechanisms, which themselves occur simultaneously when the melt films only partially separate the two solids.

Plowing, dirt, and capillary forces increase drag but cannot be added to this map without resorting to multiple axes that would include snow compressibility, dirt concentration, and liquid-water content or temperature. The effect of temperature is shown in Figure 3b, where capillary drag is included to account for liquid attachments that add drag but do not support the weight of the slider. The idea of a capillary attachment to a ski is shown in Figure 4, although these attachments have not been seen at a snow/slider interface itself. The processes shown in Figure 3b do not have clear boundaries either, and we expect neighboring processes to occur simultaneously. Since heat accumulates along the length of a slider, dry processes appear to dominate at the front of the slider (Klein 1947) but friction is reduced farther along the slider where there is enough accumulated heat to generate meltwater at the interface (Colbeck 1988). This idea is supported further by the temperature measurements of Colbeck and Warren (in press) who found that in softer snow, where the longitudinal contact between the two surfaces was more uniform, temperature increased along the entire length of the ski.

There are many other variables missing from this friction mechanism map, including slider characteristics such as roughness, thermal conductivity, flexibility, slider hardness, and water repellency as well as snow characteristics such as surface glazing, crystal angularity, and compressibility. The number of these important parameters or variables shows how difficult it is to describe snow friction in a general way and shows how much more complex the subject of snow friction is than that of many other subjects in tribology, such as two metals in contact. Nevertheless, it is worth looking



a. Speed and ratio of load to compactive strength of the snow surface.



b. Temperature and ratio of load to compactive strength of the snow surface.

Figure 3. Friction maps showing the dominant processes for different conditions vs the compactive strength of the snow surface.

at each of the mechanisms separately to show how they can be described.

Plowing and compaction

There is resistance to a slider's motion when the snow is compressed and/or pushed aside as the slider proceeds. Both of these processes dissipate energy and slow forward progress. Compaction was discussed by Nakaya et al. (1971), who showed examples of the compression bulb below a ski. This process depends greatly on the density of the snow and the pressure exerted, while temperature and speed are also important. From momentum considerations, Glenne (1987) suggested that the frontal impact resistance (I_{plow}) could be approximated by

$$F_{plow} = \rho_{sn} w u^2 \Delta h \tag{3}$$

where ρ_{sn} is snow density, w is slider width, u is its speed, and Δh is the sinkage depth. The equivalent coefficient

Figure 4. A capillary attachment. The water in this photograph bridges between the base of a ski and a glass bead. The hysteresis in the contact angle is visible due to the motion of the water over the dry ski.

of friction would be

$$\mu_{plow} = F_{plow}/W \tag{4}$$

where W is the total load. This approach is useful when Δh is known. But it seems likely that factors such as ρ_s, h, and W interact with the temperature and snow type to determine Δh and therefore μ_{plow}. Thus it is desirable to have both a more complete theoretical framework to understand plowing in snow as well as measurements of the resistive forces. In part this analysis might be based on measured values of the stresses required for the rapid compression of vented snow by Abele and Gow (1976). They showed that the compactive stress increases rapidly with increasing snow density so that when the snow is already highly compacted, further compaction is not necessary. When compaction does occur, data would have to be generated for each type of snow of interest at various temperatures.

If not solved, the problem of compactive stresses can at least be stated by equating the forward and vertical pressures. Thus

$$F_{plow} l = W \Delta h \tag{5}$$

where l is the slider length. The compactive pressure on the slider must correspond to the density of the compaction bulb below the slider and increase with the density of the snow. As the snow is compacted below the slider, the mass is conserved according to

$$(\Delta z + h)\rho_s = z\rho_b \tag{6}$$

where z is the depth of the compaction bulb and ρ_b is the density of the snow in that bulb. Accordingly, μ_{plow} is given by $\Delta h/l$, where Δh cannot be uniquely determined without knowing h and ρ_s. While ρ_b is the density that corresponds to the compactive stress W/wl, which can be obtained from tests such as Abele and Gow's (1976), b is still unknown so the problem is not solved. However, these relationships do suggest the possibility for some useful experiments based on the principles described above.

Dry rubbing and transition to meltwater lubrication

Dry rubbing

When meltwater lubrication is absent or insufficient, sliding must proceed totally or in part by elastic or plastic deformation and/or fracture of asperities on the surfaces (Kuroiwa 1977). Barnes et al. (1971) suggested that no meltwater was generated at −12°C when granite slid on ice at speeds of less than 1 mm/s and when brass slid on ice at speeds of less than 100 mm/s. Thus it is clear that meltwater is generated in all but possibly the coldest cases of skis or sleds travelling at normal speeds on snow. This also shows that metal runners are worse than wood runners at low temperatures.

Ice is more readily deformed at higher temperatures, especially above about −8°C. Below that temperature fracture of the asperities might be more likely, but this idea needs to be tested by microscopic observations and ultrasonic measurements. (Fracture of bonds at lower temperatures may explain the crunching sounds made by walking on cold snow.) The observation that hard waxes reduce friction at low temperatures (Shimbo 1971) can probably be explained by less gouging of the wax when it is harder and by the hard wax forming and retaining a smooth surface over which gliding can take place most easily.

Bowden and Tabor (1964) believed that asperities on the softer surfaces always yield plastically and stated that snow must behave in this way because the friction on a slider on snow is nearly proportional to load and is independent of the area of contact. Then movement is thought to be due to the deformation of ice grains and not to the deformation of the slider or its coatings. Thus it is generally thought that the slider base should be harder than ice at the ambient temperature. However, the bottom of the slider will be heated along its length so that deformation on the surface of the snow grains will actually take place at a temperature greater than the ambient temperature, especially over the parts of the slider that carry most of the load. In spite of the softer material being more easily deformed, the front of aluminum aircraft skis are degraded by dry friction (Klein 1947), perhaps because of dirt on the snow surface. In fact, one purpose of a ski wax should be to provide a sacrificial coating that is hard enough to cause the ice to deform nearly all of the time, but not so hard that it never fails. According to Barnes et al. (1971), the frictional effect of plowing of the softer material, usually the ice grains for a slider on snow, can be reduced if the slider is harder than the ice and the slider is smooth. This is an important statement about the use of hard and smooth waxes to reduce friction at low temperatures where meltwater lubrication is reduced, and it will be developed more quantitatively later.

The failure of ice depends on the rate of loading, which determines whether deformation is predominantly ductile or brittle. Although ice is stronger for faster rates of loading (Bowden and Tabor 1964), the yield stress at any temperature (Fig. 5) reaches a plateau at loading rates that are much slower than those of interest here, and the hardness does the same (Barnes et al. 1971). Because we are only concerned about high

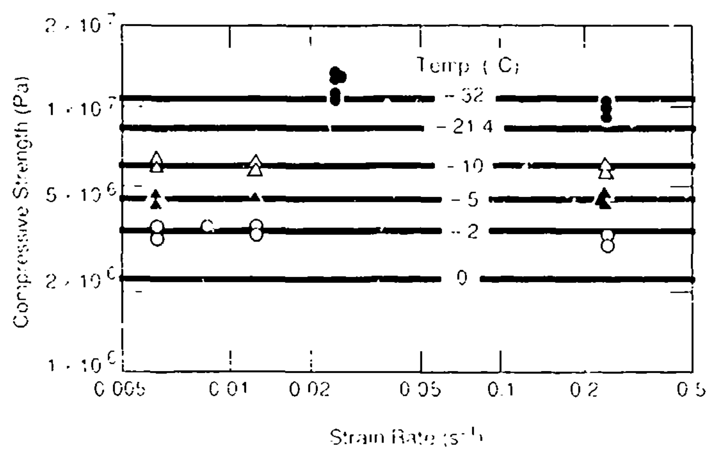


Figure 5. Compressive strength vs strain rate at various temperatures, from Carter's (1970) data.

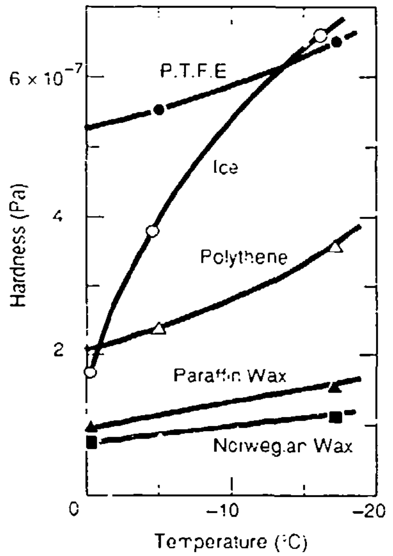


Figure 6. Hardness of various substances vs temperature (from Bowden and Tabor 1964). Only the hardness of ice changes rapidly over this temperature range.

homologous temperatures for ice, the hardness of ice increases rapidly as temperature decreases, whereas the hardness of waxes and plastics increases much more slowly over the same temperature range (Fig. 6). In addition, the hardness of ice is higher at high loading rates, whereas the hardness of waxes (at least paraffin wax) is independent of loading rate. For the materials tested, Bowden and Tabor (1964) showed that only P.T.F.E. is harder than ice but only at temperatures above about −15°C. Harder waxes have been marketed more recently, and Bowden and Tabor's tests do not account for the ductile-to-brittle transition that occurs in polymers at their transition temperature (Tobolsky 1960) and may occur in some of the hardest ski "waxes."

Temperature distribution

Regardless of the exact mechanism by which failure occurs, deformation of the asperities on the slider and/or the snow is necessary for movement of a slider when meltwater lubrication does not completely separate the sliding surfaces. The contact points in dry sliding are small asperities that heat rapidly because the energy is dissipated over a small area, i.e. "flash heating." This is an important phenomenon in friction because ice is much softer at higher temperatures and because the ice may eventually reach its melting temperature and produce lubricating meltwater. This subject has been explored many times for other materials, and the transition from dry to lubricated sliding has been observed both experimentally and theoretically in metals (e.g. Archard and Rowntree 1988).

The rate of heat generation by a slider ($\dot{q}$) is given by

$$\dot{q} = \mu u W \tag{7}$$

regardless of the mechanism by which the heat is produced. Although the fractional contact area has not been measured for dry sliding over snow, solid-to-solid contact areas are generally thought to be of the order of magnitude of 10^{-3} (Bowden and Hughes 1939) or smaller, so the stress concentration is large and the heat is dissipated through a small area. In addition, the coefficient of friction is larger for dry friction than for lubricated friction, so the heat generation is correspondingly greater. To analyze its thermal behavior we assume that the slider is perfectly smooth and that all of the asperities are ice particles in the snow. Then the ice particles are in contact along the entire length of the slider while any point on the slider is in contact intermittently with ice particles of contact radius r. Because of this intermittent contact of any point on the slider, Colbeck and Warren (in press) observed steady-state temperatures at the bases of skis that were just below, but not at, the melting temperature.

When contact is made, the temperature of an object rises above its initial value according to the widely used formula

$$\Delta T = 2q\,(t/\pi k \rho c_p)^{1/2} \tag{8}$$

where q is the heat flux at the surface, t is time after initial contact, k is thermal conductivity, ρ is density, and c_p is the heat capacity of the object. Accordingly, an ice grain travelling along the length of a slider would reach a higher temperature than any point on the slider because the ice grain would be in continuous contact whereas a point on the slider would only be in contact intermittently. The point on the slider would heat and cool in cycles as ice particles slide past that point. This suggests that any point on the slider would be heated in a stepwise fashion, as shown in Figure 7, where an ascending pattern of heating and cooling shows the individual heating and cooling cycles. As long as the motion continues, the cooling cycle is never long enough to return the point on the slider to the ambient temperature, and thus heat accumulates at that point and the temperature rises towards a quasi-equilibrium value. Once motion stops, the temperature decays slowly back toward the ambient value, as shown for a point on a slider in Figure 7. An example of the $t^{1/2}$ temperature rise and the slow decay measured on a coarse time scale at the

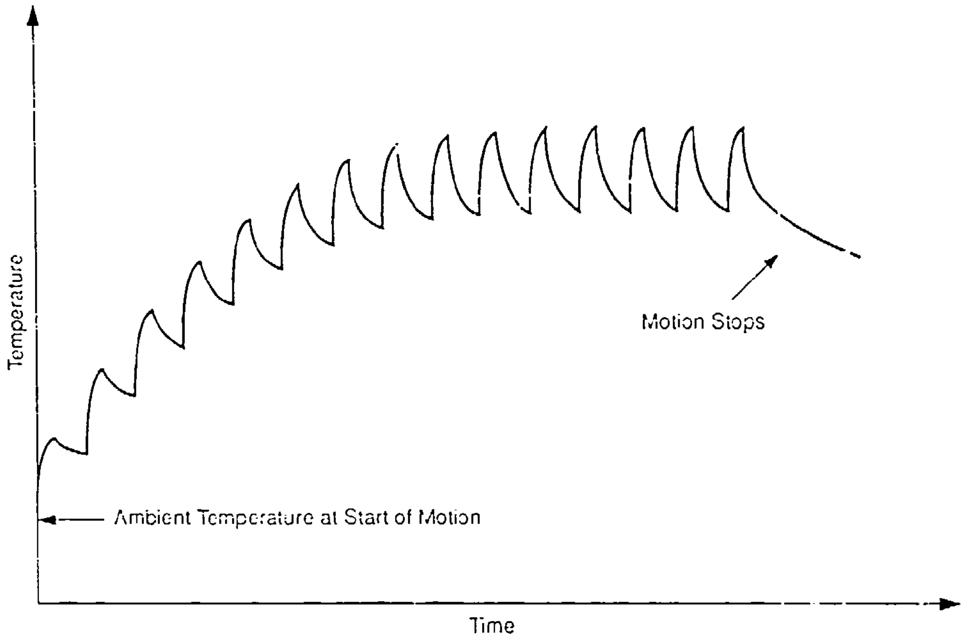


Figure 7. A hypothetical pattern of temperature rise at one point on a slider as it passes in and out of contact with water films.

base of a ski subjected to three cycles of stop–go motion is shown in Figure 8.

Assuming for convenience that the heat generated by friction flows equally into the ice contacts and the slider, the heat flux into one ice contact is $q/2n\pi r^2$, where n is the total number of contacts of contact radius r between the snow and the slider. Combining eqs 7 and 8 to show the dependence of the temperature rise of an ice particle on speed, load, and contact area gives

$$\Delta T = (\mu W u / n\pi r^2 k_i)(t\kappa_i/\pi)^{1/2} \qquad (9)$$

where κ is thermal diffusivity and the subscript i is for ice. As suggested in Figures 7 and 8, the effect for one point on a slider is cumulative until a plateau is reached. However, if the ice particle is in contact continuously, its temperature would rise steadily, which allows us to examine the conditions under which the temperature rise of an ice grain is sufficient to reach the melting point.

Reports of dog sleds operating at polar temperatures indicate that wooden sledge runners experience very high friction, presumably dry friction, when the temperature drops to about −40°C (e.g. Gould 1931) and that metal runners are even worse (Scott 1905). Many skiers have experienced this same increase in friction as temperature drops well below freezing. In eq 9, if we assume that t is the length of the slider (l) divided by the speed, the maximum possible temperature rise of an ice particle in contact with a slider is given by

$$\Delta T_{max} = (\mu W / n\pi r^2 k_i)(ul\kappa_i/\pi)^{1/2} \qquad (10)$$

With the common assumption that load-bearing area is proportional to weight (Bowden and Tabor 1956), eq 10 shows that the temperature rise increases as the square roots of slider speed and length. This suggests a greater possibility of reaching the melting temperature and experiencing meltwater lubrication in longer sliders, and it is observed that longer sliders have lower friction at low temperatures (Ericksson 1955), presumably because of the higher temperatures achieved at the interface. In addition, friction decreases as speed increases above the very low values where no meltwater is thought to exist and where dry rubbing is the only frictional process. For typical values for dry friction and downhill skis (μ = 0.3, W = 400 N, area = 0.16 m^2 with 0.1% in actual contact, u = 10 m/s, and l = 2 m), the maximum possible temperature rise of an ice grain during passage of the skis would be more than 10^3 K. Of course this temperature increase could not occur because the ice grains would start melting at some point along the length of the slider. Thus in most situations of interest, a small portion of the front of the slider is dry while the

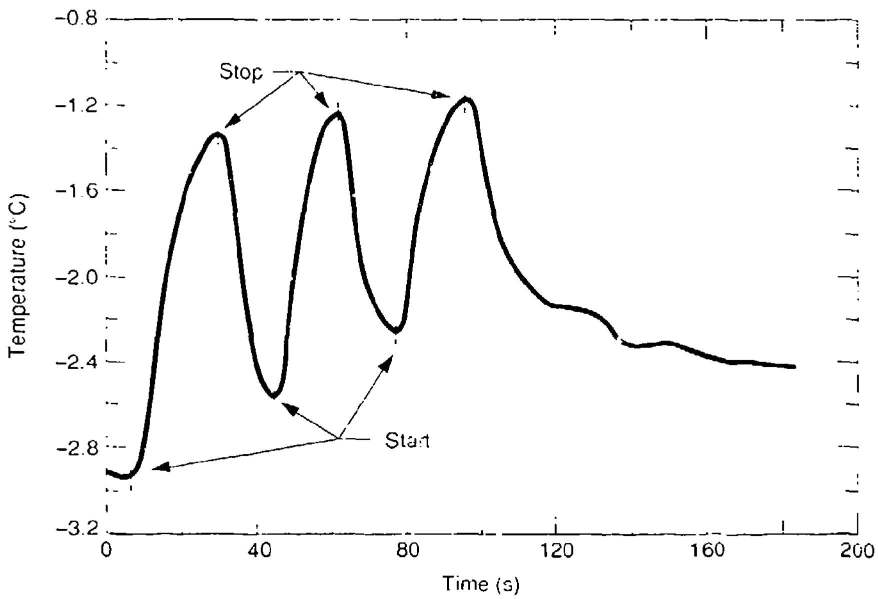


Figure 8. Temperature measured at the base of a ski during three cycles of start and stop (from Colbeck and Warren, in press).

remainder of the length is lubricated by meltwater.

The fraction of the length of the slider that is dry has a major influence on the overall friction experienced by the slider because the coefficient of friction for dry sliding is much greater than that for lubricated sliding. It is also important that the wear rate for dry sliding is much higher than for lubricated sliding, so the dry portion of a ski, for example, should be made from harder polymers and should be waxed for both lower temperatures and harder ice conditions. The length over which the front of a slider is dry (l_{dry}) can be found by rearranging eq 10 and then setting ΔT equal to the snow surface temperature. Colbeck (1988) derived a more complete expression including heat flow into the slider and found

$$l_{dry} = \frac{\pi u k_i^2}{\kappa_i}\left(-\frac{2u\mu_{dry}}{cT_{sn}} - \frac{k_{sl}}{\phi H}\right)^{-2} \qquad (11)$$

where the subscript sl refers to the slider, H is its thickness, and ϕ is its fractional contact area with the snow, or $n\pi r^2/wl$. Lehtovaara (198[illegible] [illegible] 13) derived a similar equation but assumed the [illegible] is a perfect insulator. Application of this equati[illegible] [illegible]s some very interesting results because it give[illegible] [illegible]ica tion of the degree to which dry versus lubr[illegible] occurs in many instances. It suggests that the many laboratory experiments cannot be applied to snow skis because the sliders used in the laboratory were too short and the speeds too low, thus giving a very different proportion of dry versus lubricated sliding between laboratory models and skis.

From eq 11 it is clear that l_{dry} is greater for highly conductive sliders. Accordingly, they have higher friction both because l_{dry} is greater and because once melting begins, it is not as intense since more of the heat generated by friction is conducted away from the interface. For a perfectly insulated slider with μ_{dry} equal to 0.2 and taking c as 3.05 MPa^{-1} from Huzioka's (1962) measurements, the length of the dry zone at the front of a slider is 7.9E-4 T_{sn}/u where T_{sn} is the temperature of the snow surface and u is in m/s. For a 2-m-long ski that is well insulated and moving at 10 m/s on snow of −10°C, l_{dry} is about 8 mm or about 0.4% of the total length. However, for friction tests such as those of Slotfeldt-Ellingsen and Torgersen (1983) with a speed of 0.3 m/s using a slider of total length 0.3 m, the dry length is 0.26 m or 88% of the total length. Thus the ski experiences predominantly lubricated friction along its length whereas the majority of sliders in laboratory tests experience mostly dry friction along their lengths. There is some uncertainty in these numerical results because Huzioka's data was taken when meltwater lubrication occurred, and the value of c would probably be smaller during dry rubbing. However, the conclusions are at

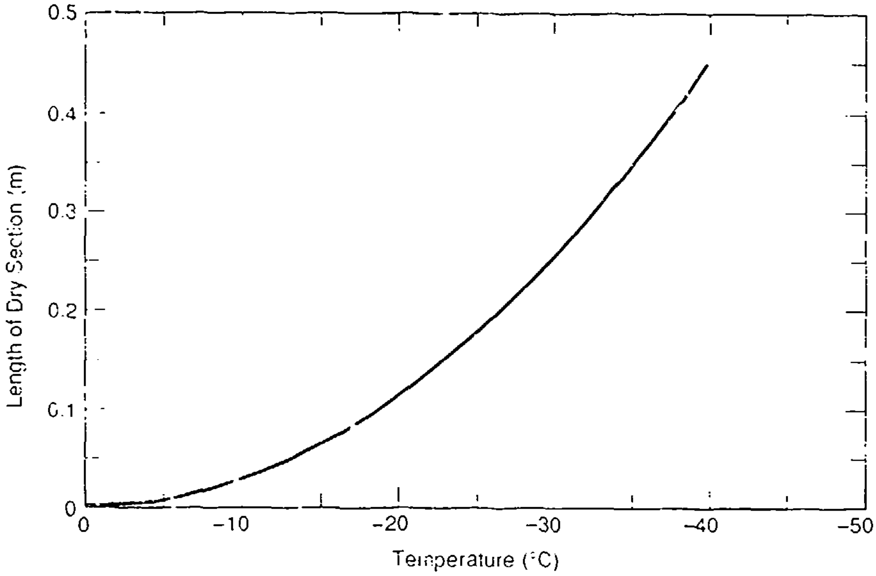


Figure 9. Length of the dry area at the front of a slider vs temperature calculated from eq 12 for a snow of compactive strength 10^5 Pa, speed of 30 m/s, and μ_{dry} of 0.2.

least qualitatively correct and suggest that considerable caution should be used in interpreting the results of friction experiments.

We can expand the approach taken to understanding the length of the dry section by making the common assumption that the pressure exerted by a slider is limited by the bearing capacity of the material beneath the slider or, in this case, by the pressure required to compress snow rapidly to the density in the pressure bulb. For a perfectly insulated slider, eq 11 shows that the dry length of the slider can be expressed as

$$l_{dry} = \frac{\pi k_i^2 T_m^2 A_q^2}{4\mu_{dry}^2 \kappa_i u W^2} \qquad (12)$$

where A_q is the total area of the slider and W/A_q is the pressure required for rapid compression of vented snow to the density achieved in the snow below the slider. Choosing a pressure of 10^5 Pa at a density of 400 kg/m^3 from the data of Abele and Gow (1976), eq 12 predicts a rapid increase in dry length with decreasing temperature, thus showing in part why friction decreases at higher temperatures (Fig. 9). Since the bearing strength increases rapidly with snow density, the dry length of the slider would decrease with increasing snow density, resulting in easier gliding on denser snow. Eq 12 also shows that the length of the dry section decreases with increasing speed, which partly explains why Kuroiwa (1977) observed decreasing friction as speed increased into the range where lubricated friction became important.

Ice deformation

Understanding dry sliding on snow would be much easier if the geometry of the ice asperities and the actual contact area were known and if the deformation rates of ice were known for complicated states of stress at different temperatures. The geometry of the asperities probably varies greatly with the snow type and, for fresh snow, changes greatly when a slider first passes. The contact area is thought to vary with load and the compactive strength of the snow, and the strength of ice varies with rate of loading, stress state, and temperature. Because of our incomplete knowledge of these important properties, it is more difficult to understand dry sliding than water-lubricated sliding over snow.

Normally we expect a mixture of dry and lubricated friction, with dry friction dominating at the front of the slider and meltwater lubrication dominating at the middle and/or rear of the slider, depending on how the load is distributed along the length of the slider. It is necessary to understand each separately and jointly since some contacts may not be completely separated by the meltwater film, but solid to solid interaction may occur in contacts that are partly lubricated by meltwater films. This area of tribology, known as elastohydrodynamics, is discussed later. First we look at solid to solid contact.

Archard and Rowntree (1988) described the temperature distribution across a single contact but, because the contacts are so small, we consider each contact to be completely dry until melting begins across the entire contact. It appears that the contacts at the front of the slider are completely dry with an average temperature that increases with distance behind the front of the slider until the melting temperature is reached at l_{dry}. Some possible mechanisms at these dry contacts in front of l_{dry} and in partially separated contacts behind l_{dry} are considered next.

The Plasticity Index was developed to distinguish between plastic and elastic responses when asperities on opposing faces interact and some form of deformation must take place. The index depends on the elastic constants and hardness of the interfacial materials as well as on the geometry of the asperities (Williamson 1984). It does not consider brittle failure although it could be important. Fein (1984) expressed it as

$$PI = \frac{0.83}{B} \frac{E_1 E_2}{E_1 + E_2} \left(\frac{r_1 + r_2}{r_1 r_2}\right)^{0.5} \left(h_1^2 + h_2^2\right)^{0.25} \quad (13)$$

where B is the hardness of the softer surface, E_j is an effective elastic modulus, r_j is asperity tip radius, h_j is average asperity height, and the subscripts refer to the two surfaces. The material properties of ice suggest that plastic yield is more likely than elastic deformation at a slider/snow interface when the asperities are as peaked as they would be for fresh snow. However, once the sharper asperities are removed and the ice contacts are flattened by rubbing, as shown in Figure 1, the geometry of the asperities suggests that the ice and/or the slider should respond elastically. Thus we suggest that virgin ice grains yield plastically (or fracture) whereas polished ice grains respond elastically because of their flat surfaces. Thus both modes of deformation must be considered.

Glenne (1987) reviewed the usual approach to dry friction where the actual contact area (A_{dry}) and coefficient of friction are given by

$$A_{dry} = \frac{W}{\sigma} \quad (14)$$

and

$$\mu_{dry} = \frac{\tau}{\sigma} \quad (15)$$

where σ is the compactive strength of the snow and τ is the shear strength of either the slider or the snow, whichever is weaker. It seems reasonable to assume that the contact area is determined by two processes. First, if the snow is of a low enough density, the ice grains are displaced from the surface such that the number of ice grains in contact increases until the compactive strength of the snow at that temperature and loading rate is reached. Second, if the ice is softer than the slider, the asperities on the slider may indent the ice grains and increase the contact area until the pressure is reduced to the hardness of ice for that temperature and rate of loading. The action of the first mechanism is one of the phenomena that separates the dry friction of snow from the friction of solid substances such as ice. However, this mechanism only occurs if the snow density is lower than the density required to support the load. In any case, a hard slider could indent the ice particles if the slider asperities were smaller than the grain size of the snow. Thus it is easy to understand that one objective of waxing at lower temperatures is to achieve a hard, smooth surface, although the prevailing idea seems to be that the wax should not be too much harder than the snow, possibly because the wax must yield to dirt on the snow surface.

If it is assumed that the snow grains fail in compression until the contact stress is reduced to the compressive strength of ice at that temperature, Carter's (1970) measured values for compressive strength can be used to determine the contact area. This suggests that c, the ratio of contact area to load, decreases as temperature decreases and equals 0.222 MPa at −5°C. Comparing this with Huzioka's (1962) value of 3.05 MPa, measured at −4°C in the presence of meltwater lubrication, gives the expected result that the contact area is smaller when only dry rubbing occurs. Huzioka found a frictional contact area of 1.4%, whereas a range of values of 0.2 to 0.04% are suggested here for Huzioka's load when no melting occurs. The value suggested by Bowden and Hughes (1939) for dry friction—0.1%—falls in this range.

To apply eq 15, Glenne (1987) suggested that for ice the ratio of τ to σ is about 0.06, although this is much less than the measured values of the coefficient of friction when only dry processes are likely. In their early work, Bowden and Tabor (1950, 1964) suggested that τ is about one-half the yield stress in tension and that σ is about three times that yield stress, or μ_{dry} is about one-sixth, which is close to Bowden's (1953) values of friction, at least for sliders at very low speeds where no melting or adhesion occurs. Taking values of the tensile and compressive failure stresses from Carter (1970) and assuming τ is one-half the tensile value, the calculated values of μ_{dry} are shown in Table 1. These values of the coefficient of friction have the wrong trend with temperature since low-speed and static friction are known to increase, not decrease, at lower temperatures (Bowden 1953).

This simple approach can be expanded by looking at

Table 1. Dry friction calculated from Carter's (1970) values of tensial and compressive strength.

T (°C)	$\tau \cdot 10^{6}$ (Pa)	$\sigma \cdot 10^{6}$ (Pa)	μ_{dry}
0	1.03	2.3	0.45
−2	1.07	3.5	0.31
−5	1.08	4.5	0.24
−10	1.1	6.5	0.17
−21.4	1.13	9	0.13
−32	1.17	11	0.11

Table 2. Dry friction calculated from eq 17 and Carter's (1970) values of tensial and compressive strength.

T (°C)	μ_{dry}
0	0.32
−2	0.27
−5	0.25
−10	0.22
−21.4	0.21
−32	0.13

the failure of an ice grain to see if ice grain failure and release from the surface might account for dry friction. This mechanism is similar to the process of flake production (e.g. Lim and Ashby 1987) and retains the assumption that all of the failure occurs in the ice, which is now subjected to a more complex state of stress. We assume that an ice asperity is subjected to a normal stress σ and a shear stress τ as shown in Figure 10. If μ_{dry} is large enough, tension may occur at the upstream corner at the base of the ice asperity, which could lead to failure and removal of the asperity by tensial failure starting at the upstream edge and propagating along the base. The normal stress on the asperity varies along its base as

$$\sigma(x) = \sigma - \frac{6\tau x}{r} \tag{16}$$

Using eq 15 and assuming that the asperity fails in tension on the base at its leading edge, the coefficient of dry friction can be expressed as

$$\mu_{dry} = 0.167 - \frac{\sigma(r)}{6\sigma} \tag{17}$$

where $\sigma(r)$ is taken as the tensile yield stress for ice. Using Carter's (1970) data for compressive strength for σ and for tensile yield for $\sigma(r)$, μ_{dry} is calculated at various temperatures and shown in Table 2 for the particular geometry assumed. This also shows higher friction at higher temperatures, but most of the values are about correct.

If the controlling processes operate on the scale of the size of the slider's asperities rather than on the scale of the size of the snow grains, the actual contact area is determined by the hardness of ice at high rates of loading. We would then expect much smaller values of contact area and much higher local stresses. The loading time of the asperities is approximately their size divided by slider speed, or about 10^{-7} s for a typical ski. The hardness of ice reaches a plateau as the loading rate increases, so taking Barnes et al.'s (1971) largest values for hardness, we would expect a pressure on the asperities of 50 MPa compared with Huzioka's (1962) value of 0.33 MPa and 4.1 MPa determined above. Thus, if the indentation of ice by the asperities on the slider determines the contact area, the fractional contact area for a ski would typically be less than 10^{-4} and the stresses exerted on the ice surfaces by the asperities of the slider would be very large. For solid ice surfaces, Barnes et al. (1971) suggested that these high stresses cause recrystallization in a 0.2-mm layer adjacent to the slider and

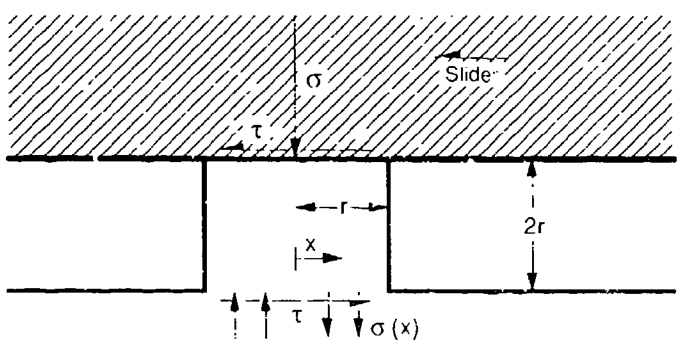


Figure 10. An ice asperity with a biaxial state of loading.

that this causes a reorientation of the ice crystals for favorable glide in shear tangential to the direction of motion. This is similar to oxidation-dominated friction for metals, and such observations must be made for snow to see if this is a common mechanism for dry sliding over snow grains. It is also important to observe the conditions under which the displacement or fracture of the ice grains occurs.

Assuming that the indentation hardness of ice determines the contact area and that the contacts fail in shear along the basal layers of ice crystals at the surface, we can calculate the coefficient of dry friction from measurements of shear in easy glide. However, there are two problems. First, there is a size effect on the strength of ice (Tusima and Fujii 1973), so that experimental results from samples of a normal size would suggest low values of shear strength. Second, few tests in shear have been done with such high rates of loading as are of interest here, and none have been done on thin crystallites oriented for easy glide.

The simple approach taken here to understanding a combined shear and normal load has produced the wrong trend with temperature because the strength in compression is a strong function of temperature, whereas the strength in tension or shear is not so temperature dependent. Thus the denominator in eq 15 increases rapidly as temperature decreases, and the calculated coefficient of friction decreases accordingly. It may be that ice asperities fail in tension or shear independently of the contact area and that only their tensile or shear strengths are important. Then the correct dependence on temperature would result, but without more information about the geometry of the surfaces this process cannot be analyzed. Moreover, it is likely that for most problems of interest virtually all ice deformation occurs at the melting temperature and that the effect of temperature is limited to changing the proportion of ice deformation versus meltlayer shear.

Although the values of μ_{dry} given in Table 2 are of about the right magnitude, there is much uncertainty about the application of these results because of the lack of direct observations of the contact area and failure mechanisms. To improve on this model of dry sliding it would be helpful to have profiles of slider bases such as waxed ski bases and microscopic examinations of snow surfaces after sliders have passed. This would provide information about the scale at which these processes operate and help answer questions about the role of grain displacement vs ice indentation, the contact area without meltwater present, and the role of asperities on the snow surface, including those due to fresh snow and those due to wind crust or man-made snow. It is clear from both field and laboratory observations that the snow surface is often smoothed by meltwater lubrication, probably because the asperities are melted to form smooth ice contacts, but similar observations are needed to identify the important mechanisms for dry sliding, in particular the mode of failure of the ice grains on the surface. Then more detailed models of the processes can be constructed, although even then the results would be uncertain because of the lack of information about the biaxial failure of ice.

There are other existing models of dry friction that could be applied to ice. If the slider is harder and plows the ice grains, Briscoe and Tabor's (1978) model of the viscoelastic plowing of solids could be used to describe the friction. The contact area would be determined in part by the hardness of ice, but the deformation would be controlled by its elastic modulus. They proposed

$$\mu = 0.5\pi \left(A_0 B\right)^{1/3} r_a^{-2/3} \left(1 - \nu^2\right)^{1/3} E^{-1/3} \tan\delta \qquad (18)$$

where A_0 = contact area at rest,
B = hardness,
ν = Poisson's ratio,
E = elastic modulus in tension,
$\tan\delta$ = mechanical loss tangent.

μ is very sensitive to the curvature (r_a) of the asperities on the slider and, if the slopes of these asperities are gentle enough, there is probably no gouging but only elastic depression as the solids come into contact. This is described later.

Meltwater lubrication

The prevailing belief is that the coefficients of friction of snow and ice are low because of lubrication provided by meltwater. At low temperatures where meltwater lubrication disappears, the friction of snow is similar to that of sand (Bowden 1953), which is a comparison often made by early polar travellers when dog sledding at temperatures of −40°C. The idea of meltwater lubrication was promoted by Bowden and Hughes (1939), who showed that pressure melting is not likely because the depression of the melting temperature is too weak to lower the melting temperature by a significant amount. In fact, using the value of contact pressure given above, the depression of the melting temperature is only about 0.25°C. However, the actual solid-to-solid contact may be considerably less, and the process of pressure melting-regelation is re-examined later.

The idea of meltwater lubrication by frictional heating has been supported by much of the past research and was reviewed by Glenne (1987). Meltwater and its ejecta have been seen directly when objects slide on ice (Tusima and Yosida 1969). Probably because of meltwater lubrication, the addition of heat has been shown to be beneficial at low temperatures (Pfalzner 1947).

although this procedure must be done cautiously since too much meltwater clearly increases friction, and the introduction of heat above the interface is a very inefficient process. The basic idea applies to other materials as well (Bowden and Persson 1961, Archard and Rowntree 1988, Lim and Ashby 1987) and can be observed most easily in materials where the phase transformation leaves clear evidence (Bowden and Persson 1961), as shown in Figure 1b.

As a test of the meltwater lubrication theory, we assume that no solid-to-solid contact occurs and ignore the energy conducted away from the interface. Then an upper limit for the thickness of the meltwater film can be calculated from the energy used to push an object over snow by assuming that the power required to propel the object equals the power consumed by phase change, or

$$\mu_{lub} W u = \dot{m} L \rho_i \tag{19}$$

where $\dot{m}$ is the meltwater mass production rate. Taking $\dot{m}$ as the rate of surface melting ($\dot{h}$) over the actual contact area,

$$\dot{h} = \mu_{lub} u / c L \rho_i \tag{20}$$

where c is the ratio of actual contact area to load on the slider. According to one of Amonton's laws for friction (Bowden and Tabor 1950), this ratio is constant. Using Colbeck's (1988) equation for the melt component of friction (Lehtovaara [1989] derived a similar relation),

$$\mu = c u \eta / h , \tag{21}$$

the melt rate is found as

$$\dot{h} = \eta u^2 / h L \rho_i \tag{22}$$

where η is water's viscosity. If the water were removed from the contacts by the squeeze mechanism described by Colbeck (1988), the thickness of the film would be in balance when

$$h^4 = 3 c r^2 \eta^2 u^2 / 2 L \rho_i \tag{23}$$

Taking c as 3.05 MPa^{-1} from Huzioka's (1962) data, r as 1 mm, and u as 10 m/s, the film thickness would be 1.5 µm if removal occurred by squeeze only. This value is less than the values deduced by Ambach and Mayr (1981) from dielectric measurements, but is greater than that calculated by Evans et al. (1976) or Colbeck (1988). It suggests a coefficient of friction of 0.036, which is about correct. A much smaller value for the film thickness is calculated using the shear removal mechanism suggested by Colbeck (1988) whereby the thickness is

$$h^2 = \eta u \pi r / L \rho_i \tag{24}$$

For a speed of 10 m/s the film is then less than 0.43 µm and μ_{lub} is 0.13. A larger thickness could occur if the water film slipped along the slider, as could happen for smooth, hydrophobic sliders. However, Huzioka's photographs suggest that the shear removal mechanism is operative and thus the calculation using the shear mechanism cannot be ignored. Accordingly, the calculated value for film thickness is probably too small to separate the solids under most conditions of interest, even when only squeeze occurs, and this suggests that there is solid-to-solid interaction as well as meltwater lubrication.

With these thin films, Evans et al. (1976) suggested that rather than ice deforming, the solids may be separated by a few molecular layers in which most of the frictional force is generated. They suggested that the meltwater layer would not be thick enough to prevent all solid-to-solid contact and that there would be a combination of dry and lubricated friction occurring simultaneously where only the highest asperities would contact the other surface. These concepts can best be tested for snow by using the experimental results of Huzioka (1962). Even if only squeeze removal occurred and no heat were conducted away from the interface, for those experiments the meltwater film would have been only 0.17 µm thick, clearly not enough to separate the solids. Thus it is likely in Huzioka's experiment that both meltwater lubrication and solid-to-solid interaction occurred and that both of these processes probably occur in most cases of sliders on snow unless meltwater and/or heat are available from other sources.

Another way of testing the idea of sliding without solid-to-solid interaction is to assume that all of the heat generated by friction is generated by shearing the meltwater films; this leads to the derivation of eq 21. Then, using the experimental results of Huzioka (1962), the water film in his experiment should have been only 0.0011 µm thick. This is clearly much too thin to separate the asperities, and thus it is easy to understand why Huzioka concluded that solid-to-solid contact occurred in his experiment and extrapolated this conclusion to higher temperatures and values of heat production.

The theory of meltwater lubrication has been examined for ice by Evans et al. (1976), Oksanen and Keinonen (1982), and Akkok et al. (1987) and for snow by Colbeck (1988) and Lehtovaara (1989). The two modelling efforts for snow were rather different, with the former concentrating on the heat and mass flows while the latter was more concerned with the interaction of a ski with the snow through load distribution. Both considered dry friction and calculated a coefficient of friction that depended on both wet and dry processes. While Lehtovaara assumed that meltwater was removed only by the squeeze mechanism, Colbeck considered both squeeze and shear removal of the meltwater from the films but suggested that shear would be a more efficient mechanism. This led to a simple expression for film thickness (h), given in eq 24.

It is possible that the shear mechanism described by Colbeck (1988) is too efficient because it does not allow for slippage of the water films along the slider or for meltwater to be retained by rough surfaces, and it will be modified accordingly. If the slider base is smooth and hydrophobic, melt films can be observed to slide along the base of the slider, as shown in Figure 4 for a capillary attachment. Then the squeeze mechanism would be relatively more important, a process that has been frequently used in tribology (e.g. Moore 1965) and was used exclusively by Lehtovaara (1989). In addition, if the weight of the slider is carried by solid-to-solid contacts or very large hydrodynamic forces, as described in elastohydrodynamics (e.g. Fowles 1969), then the liquid could be retained around the moving asperities and would increase the thickness of the meltwater film and decrease the friction accordingly. Thus, at low temperatures where meltwater lubrication is essential, a smooth gliding surface would be desirable to allow water slippage, whereas at high temperatures, where water attachments increase drag, a rough slider surface would be useful to disrupt the water attachments. In addition, since water slides more readily along hydrophobic surfaces, the shear removal mechanism may be less effective if the contact surface of the slider is hydrophobic; thus a hydrophobic surface would appear to be advantageous at all temperatures.

While Colbeck (1988) assumed that contact area increased proportionately to load and looked at the effect of contact size, Lehtovaara (1989) used the number of contacts and the hardness of the snow as parameters. Both concluded that friction would decrease to a minimum before increasing with speed and that the friction would be greater for a larger number of contacts because of the dynamics of the water film. Both predicted that the minimum friction would occur at lower values of speed as temperature increased. However, Colbeck predicted that friction would be lower at higher temperatures until capillary attachments became important, whereas Lehtovaara predicted that friction would increase with temperature due to the thicker meltwater films. This demonstrates a fundamental difference between the two approaches, both of which contain assumptions that remain to be tested.

Colbeck's (1988) eq 4 describes the balance among melt, removal, and film thickness. It is modified to account for the slippage of the water films along the surface of the slider by introducing a factor, S, that fractionally reduces the shear mechanism. The result is

$$\frac{\pi\eta}{L\rho_i} = \frac{2\pi}{3\eta c}\left(\frac{h^2}{ur}\right)^2 + S\,\frac{h^2}{ur} \,. \tag{25}$$

where the terms represent meltwater production rate, removal by squeeze, and removal by shear, respectively. This can be solved for the ratio

$$\frac{h^2}{ur} = \frac{-S + \left(S^2 + \dfrac{8\pi^2}{3cL\rho_i}\right)^{1/2}}{4\pi \,/\, 3c\eta} \,. \tag{26}$$

The values of friction computed from eqs 21 and 26 are shown in Figure 11, where it varies with the effectiveness of the shear removal process as characterized by S. It appears from the high values of the coefficient of friction for large values of S that Colbeck's (1988) assumed shear mechanism is too efficient, that the value of c obtained from Huzioka's (1962) data is too large, and/or that other processes operate as well. If we conclude that shear is effective at low speeds and it looses its effectiveness at higher speeds, then the calculated values of μ_{melt} are in the range of values reported by Kuroiwa (1977). However, as was shown above, even if no removal occurs by shear, the assumption that all of the contact is through meltwater films leads to the conclusion that the meltwater films are too thin to separate the solids for the surface roughnesses that exist on most sliders. Thus there must be some solid-to-solid interaction, which greatly complicates models of snow friction.

Elastohydrodynamics and thin films

Elastohydrodynamics was developed to study frictional resistance when there is solid-to-solid interaction due to partially developed lubricating films. Some of the weight of the slider is borne either by direct contact or by inertial forces in the meltwater films due to the very rapid passage of the asperities. This is a very complicated subject requiring numerical solutions to coupled differential equations describing the mixture of physical processes that occur when the solids interact in this way. While no attempt is made here to obtain numerical solutions, the subject will be described, the information needed will be discussed, and some possible benefits mentioned. Other thoughts about thin films are also presented.

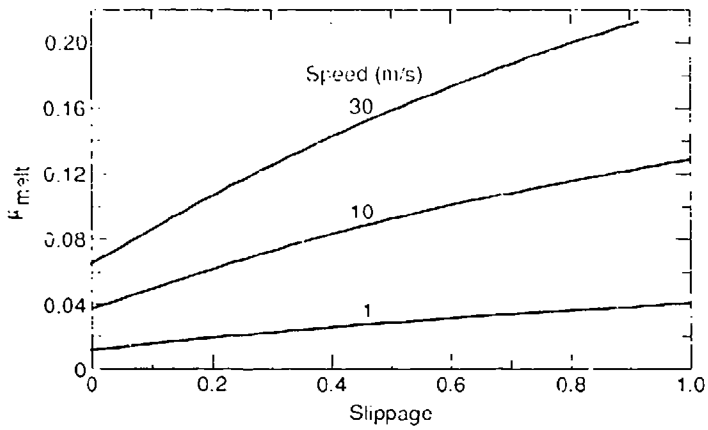


Figure 11. μ_{melt} vs slippage for various speeds for a contact size of 1 mm. When S is 0 no shear removal occurs, and when S is 1.0 no slippage occurs.

Polyethylene is commonly used at lubricated surfaces because of its favorable properties. Chemically, it forms a weak bond with ice, which reduces adhesion, and, except at very low temperatures, it is harder than ice so it is not gouged by ice. It has a high strength but is more elastic than ice and thus asperities on the surface of the slider can deform elastically to allow passage of the solids. From the Plasticity Index given in eq 13, an ice/polyethylene interface should respond elastically when asperities interact as long as their surface slopes are less than about 1%. Asperities with steeper slopes would be subject to plastic deformation and, if ice, removal by melting as well. Thus to minimize friction, the sliding surface should be hard but elastic with very gently sloping surface relief. Skis that are structured by indentation may have the additional advantage of providing a smooth running surface while the indentations allow increased flexure of the polyethylene. With increased flexibility, the polyethylene base could act like a series of shock absorbers that flex individually as the ice passes, as suggested in Figure 12. This figure also suggests that the water film increases in thickness along the length of the contact and could be shed at the downstream side of the ice grain. The individual asperities on the slider deform elastically to allow the gently sloping roughness elements on the ice grain to pass. It is important to note that only the peaks of the ice grain are in contact so that only they are melted, thus smoothing the ice grain as the slider passes and providing for continued smoothing with repeated passes. This scenario suggests that the ice grains are smoothed to accommodate the scale of roughness on the slider, so the roughness of the slider would determine the roughness of the ice grains after the slider had passed a short distance. Lehtovaara (1989) found that ice could also smooth the slider, although, when the ice is very cold, it can be observed to shred polymer ski bases.

The thickness of the water film away from the high points of the asperities could be considerably greater than those portions of the films trapped between the solids, which may account for the difference between the thick films reported by Ambach and Mayr (1981) and the thin films calculated above. The scenario depicted in Figure 12 reopens the old issue of pressure melting, since the upstream portions of the ice asperities would see much higher stresses than the downstream portions. If the upstream pressure on the asperity is assumed to be limited by the indentation hardness of ice

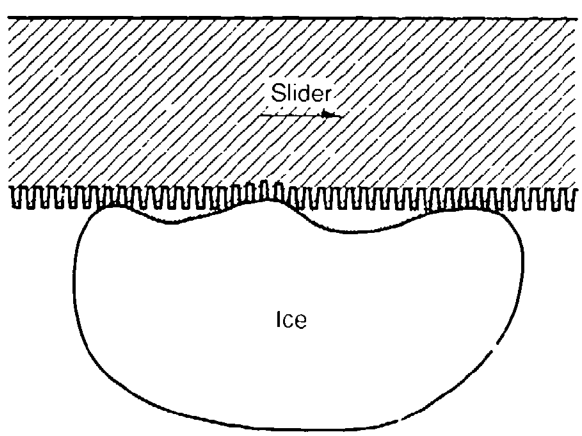


Figure 12. Hypothesized action of a highly flexible polyethylene base passing over an ice grain. The slider base deforms elastically while the ice grain is smoothed by melting on the summits and refreezing in the valleys.

at 0°C at high loading rates while the downstream pressure is atmospheric, the difference in melting temperature across an asperity could be as high as 1.1°C. Even if the heat flow path through the ice asperity is only 1 µm, the melt rate along the direction of movement would be less than 10 mm/s, which shows that, at most speeds of interest, the asperity would have to be removed or avoided by some other process. Thus, while the temperature depression caused by pressure melting is large, the process is still too slow to be of much interest.

Elastohydrodynamics can be used when only a limited amount of meltwater is available and some solid-to-solid interaction takes place. The role of meltwater is still important, but other processes must also occur. Often some polishing of the sharp asperities must take place before the surfaces are sufficiently smooth to allow only elastic deformation of the asperities. As the asperities approach there is a very rapid compression of the meltwater film, which can be very thin. These high-pressure films shear rapidly and allow the asperities to pass while reducing or eliminating the solid-to-solid contact. Even asperities that are not aligned for contact, but would come close to touching, may interact through the liquid pressure field and produce drag (Fowles 1969).

The meltwater films may change viscosity due to pressure effects, although solid-like structures are not likely to appear in these thin films in the time available. Furthermore, if the pressure is limited by the hardness of ice at high loading rates at 0°C, the viscosity would only be reduced by about 3% (Dorsey 1940).

Patir and Cheng (1978) have shown that the thickness of a water film can change with the orientation of the roughness elements. When the elements are oriented longitudinally with the slider, the meltwater films tend to be thinner. This suggests a positive feedback since, if longitudinal gouging occurs because the films are thin, the water film thickness would be reduced even more. This is a strong argument for the frequent resurfacing of sliders when optimum performance is needed. When the elements are oriented transversely across the slider, the water film thickness is increased because of fluid pressure increase on the upstream sides of the asperities. Thus, when the water film is too thin or too thick, its thickness and the entire dynamics of the interaction of the asperities can be modified to a substantial degree by changing the orientation of the surface structure of the slider. The orientation is especially important when the surface roughness elements on the slider are less than the thickness the water film would have if the surface were smooth. This appears to be the case for most situations with sliders on snow and thus a transverse structure should be beneficial at low temperatures while a longitudinal structure should be better at high temperatures. If the viscosity of water were greater and its surface energy less, perfectly flat surfaces would probably work best.

Without better knowledge of the geometries of the sliding surfaces, it is difficult to get quantitative results from elastohydrodynamics, but its qualitative application is useful anyway. For example, as a rule-of-thumb, lubrication works well when the lubricant separates the solids by a distance of at least two to four times the root mean-square value of the surface roughness (Fein 1984). Since this quantity of lubricant is not usually available for sliders moving over snow at subfreezing temperatures, it is clear that some consideration has to be given to ways of making the sliding process as efficient as possible.

Fowles (1969, 1971) showed that even melting can occur on upstream sides of the asperities because the high rate of liquid shearing in the thin films dissipates much energy locally. Thus the rougher elements on an ice surface can be removed preferentially by yield or melting, while the asperities on a polyethylene surface deform elastically. When large stresses develop locally on an asperity, the plastic region may be quite small, but the thermal effects can be felt over most of the asperity. These effects depend critically on the degree of overlap of the asperities and on the speed (Fowles 1971). Below a critical speed or overlap, the process appears to be isothermal, so that intense melting only occurs under conditions of high speed and direct overlap of the opposing asperities. The magnitude of the temperature rise calculated by Fowles suggests very rapid rates of local melting, and thus rapid elimination of the larger asperities on the snow grains is possible.

It has been observed that kinetic friction is higher at lower speeds where less heat is generated (Bowden and Tabor 1964, Kuroiwa 1977). This occurs not just because more meltwater is generated at higher speeds but because the meltwater is more effective. At higher speeds, asperities with significant overlaps can pass while maintaining a finite film thickness between them and thus avoiding solid-to-solid contact or solid yielding. Although such conclusions are based on qualitative reasoning from elastohydrodynamics, they do indicate that the theory can be a powerful tool in understanding the friction of sliders on snow. It is mostly the surface geometry of the sliders that must be known before more quantitative use could be made of the existing theory.

Other mechanisms

Snow surfaces tend to accumulate dirt and, if the slider is electrostatically charged, it might retain dirt. Even microscopic rock dust would affect the sliding mechanisms since the separations are so small. A hard particle trapped between two softer surfaces would indent or gouge one or both surfaces. When the particles are well rounded, the Plasticity Index shows that plastic yield of the ice is likely to occur unless the dirt particles are soft organic materials that might be shredded between the sliding surfaces. To minimize drag by foreign material, graphite waxes are used on skis to allow electrostatic charges to drain away. Unfortunately, graphite is soft and a good thermal conductor and it can also drain heat. Since the electrostatic charges are most likely to develop at low temperatures where the heat is most needed at the interface, the coincidence of good electrical and thermal conduction in graphite is unfortunate.

Electrostatic charges are common on rubbing bodies, and coatings to induce electrical conductivity to minimize charge accumulation on surfaces are widely used. It is known that electrical charges can arise from different temperature gradients in the rubbing solids (Bowden and Tabor 1956), different surface temperatures, stresses in the solids, or phase boundaries. Since meltwater is shed from ice grains, it is likely that the displaced water preferentially removes accumulated charge, thus leaving the ice grain with the opposite charge. Citing earlier work, Dorsey (1940) reported that merely dipping ice into water is sufficient to give ice a positive charge, which in turn might induce a negative charge on the slider.

When dry rubbing occurs, the charge separation has been observed to depend on temperature and speed and therefore on the duration of the contact (Latham and Mason 1961). Takahashi (1969a) found that rubbed ice surfaces, including ice surfaces covered by a liquid layer (Takahashi 1969b), developed a large negative potential. This might be explained by shearing of the electrical double layer at the ice/water interface, which could preferentially remove positive charges and leave the ice surface negatively charged (Shewchuk and Iribarne 1974). For 100 µm droplets of pure water at −10°C, the charge separation increased rapidly above a threshold speed of 10 m/s, thus suggesting that this effect is only important at higher speeds. However, the results for pure water were erratic, suggesting that both the magnitude and the sign of this effect were very sensitive to small amounts of impurities in the water. Sign reversals have been observed in other experiments as well, and the theory is not sufficiently well developed to allow us to identify the important processes with confidence (e.g., Baker and Dash 1989). Pruppacher and Klett (1978) reviewed the processes that might account for charge separation on ice, but the information does not allow us to decide which mechanisms are operative on sliders, let alone how much charge separation there might be. It is very likely that charge separation occurs on gliding skis, and it seems certain that charge separation would attract impurities of a size that would interfere with gliding. It also seems possible, although not important, that charge separation could change the pressure of the slider on the snow. For example, electrical double layers might support a pressure of about 10^5 Pa (Tabor 1974) which is much smaller than the yield stress of ice. The occurrence of charges might be most important when dry sliding occurs at low humidities and/or when the most meltwater is being shed from the snow grains. Measurements of electrostatic charges on skis should be made to identify the conditions under which this phenomenon occurs.

Measurements of the water pressure at the base of a gliding ski are also necessary to identify the conditions where capillary action may add drag to the ski. Colbeck (1988) suggested that there are two types of snow grains that interact with sliders: those that support the weight of the slider with either solid-to-solid contact or through high-pressure water films, and nonsupporting grains that are in contact with the slider through a liquid bond, such as the one shown in Figure 4. These exert a drag on the slider, because of the hysteresis in the contact angle, and a downward force, because of the reduced pressure in any water attachment with this shape. While their existence is unproven, capillary islands or electrical charges are the only proposed mechanisms for the increase in drag that is associated with wet conditions.

Capillary forces should be high when sliding over fresh, wet snow because of the smaller and more numerous pores under those conditions. The suction in a pore, shown in Figure 13, increases as the inverse of the pore size, and fresh or fine-grained snow has smaller pores. For coarse-grained snow the geometry shown in Figure 4 suggests that the forces would be greater when less water was available because the force on each particle would be greater for smaller liquid islands. However, more islands would be active if more water were avail

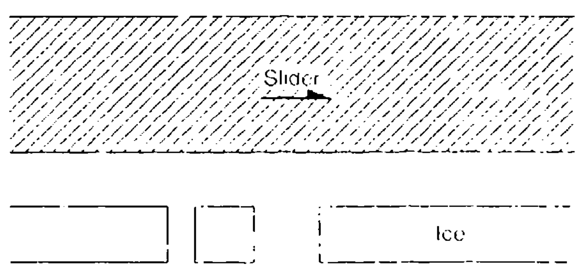


Figure 13. Water drawn out of small pores. The water bridges develop suction that increases as the inverse of the pore size. The same idea applies to water held in crevices and at junctions.

Table 3. Contact angles of water on various substrates from Bowden and Tabor (1964).

Substrate	*Contact angle (deg)*	*Comment*
Ski lacquer	0	
Paraffin wax–graphite	98.5	Decreases with time
Paraffin wax	118	Decreases with time
Aluminum stearate	122	38° when rubbed under water
Nylon	0	
Perspex	0	
P.T.F.E.	126	Remains constant

able, so the effect for coarse-grained snow may be greatest at intermediate values of water supply. While the geometry of the pore or ice particle may determine the water pressure in the connection, the surface area on the slider is largely controlled by the contact angle of water on the slider base. As shown in Figure 4, there is a hysteresis in that angle as the water slides over a dry base. On the leading edge the angle increases as the water advances onto the dry surface and, on the trailing edge, the angle decreases as the water recedes from a wetted surface. This change in angle greatly increases the drag on the slider.

The downward pull on the slider exerted by an island is the pressure in the island times the contact area. Both pressure and contact area are determined by the shape of the island, which is affected by the geometry of the ice surface and the contact angle on the slider. Bowden and Tabor (1964) reported contact angles for various substances of interest and also reported that, for some of these surfaces, the angle decreases with time as the molecules in the substance reorient. Some of these angles are listed in Table 3, where the disadvantage of the old ski lacquers and the great advantage of P.T.F.E. is clear. While the time constant was not reported for the decrease in contact angle, Bowden and Tabor did report that the effect of wetting of newly waxed skis could be observed after a "short distance of sliding." It is not clear if all modern waxes have this same disadvantage but if they do, that would explain why skis are roughened to help overcome capillary bonds. It also points out one of the great differences in polymers since some, such as nylon, will allow water to penetrate (Cohen and Tabor 1966), which reduces the strength, increases the real contact area, and thus increases friction (Lancaster 1972). Water penetration should be reduced by proper waxing.

Combined models of the processes

From what is known about the different processes, it is likely that various ones dominate under different snow conditions such as temperature, crystal type, and wetness and different slider characteristics such as length, speed, load, and thermal conductivity. From the available evidence, it seems that plastic deformation or fracture of the asperities dominates under conditions of fresh dry snow, low speeds, and low temperatures. At higher speeds and higher temperatures on smoother snow, the processes are predominantly elastic deformation of the solids with hydrodynamic lubrication between the solids. For still higher temperatures with wet snow, capillary attachments increase drag on the slider base in a manner that is sensitive to the contact angle of water on the slider, the roughness of the slider, and the geometry of the snow grains. Given this complexity, it is clear why Lehtovaara (1989) said that "one particular ski can operate optimally only in a small range of track and air conditions." For this reason we will construct a simple model that describes snow friction in terms of the amount of water available for lubrication at the interface. The idea behind this model is shown in Figure 14: the dominant process depends on the film thickness, which in turn depends on both the amount of meltwater being produced and the efficiency with which it is used.

Colbeck (1988) described the overall friction of a slider on snow as consisting of three components whose relative significance depended primarily on the ambient temperature but also on the heat loss into both the slider and the supporting ice grains. In dry, cold snow much of the heat produced at the interface is lost to conduction; thus the friction is determined by a combination of the forces required to deform the solids and the hydrodynamic drag in thin films. When the snow is wet and heat generation at the interface is a disadvantage, the friction is controlled by drag due to capillary forces and the hydrodynamic drag in thicker films. Unfortunately, the only one of these three processes that has been described quantitatively at this time is the hydrodynamic drag, and that has only been described for snow if the interfaces are smooth and if the heat and mass balances are known at the interface. Lehtovaara (1989) modelled friction as the sum of wet and dry components that operated over different portions of the base. The wet friction was described as in eq 21 and the dry friction as in eq 15. When the appropriate areas over which these processes operate are known, a generally applicable model of friction can be constructed. Unfortunately, these areas are not only unknown but elastohydrodynamics suggests that the contact areas are even ill-defined since the solids can interact through fluid pressure without coming in direct contact. Thus a simple

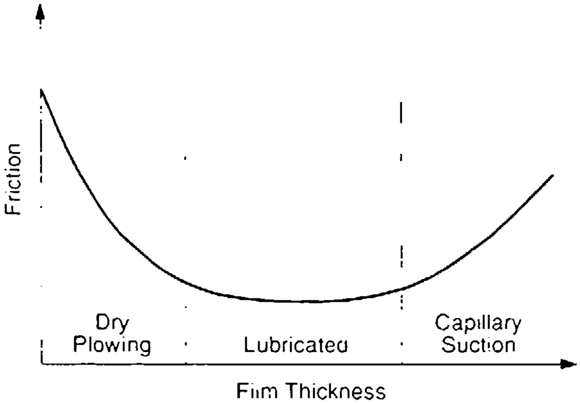


Figure 14. Friction vs water film thickness. The friction is greater when the films are thinner, because of solid-to-solid interactions, and when there is too much water present.

empirical approach will be used here that is based on the three main processes used in Figure 14. Since the relative importance of these three processes is controlled by the thickness of the meltwater film, which is in turn at least partly controlled by the heat balance at the sliding interface, we begin by examining that heat balance.

Interfacial energy balance

The heat available for melting at the interface is the heat generated by friction and radiation absorption minus the net energy balance due to conduction. The total energy balance is

$$W\mu u + wl\left(1-\alpha\right)R \;=\; n\pi r^2 \rho_i L\dot{h} + q_i n\pi r^2 + wlq_{sl} \tag{27}$$

where $\dot{h}$ is the rate of melting at the contacts, q_i is the heat flux into the ice, and q_{sl} is the heat flux into the slider. α is the albedo of the base of the slider and R is the solar radiation impinging directly at the interface. Later we show that this can greatly affect the interfacial temperature and therefore the friction. The heat flow into the slider and the heat flow into the ice must be treated separately because the slider accumulates heat as it moves but the ice is constantly renewed. Assuming the snow grains instantly reach 0°C when the slider arrives and remain at that temperature during the passage of the slider, Colbeck (1988) suggested that the heat flux into an ice grain is

$$q = -\frac{k_i T_{sn}}{\left(\pi \kappa_i t\right)^{1/2}} \tag{28}$$

where T_{sn} is the snow surface temperature in °C and t is the time since the onset of contact for that snow grain. If the grain is in contact with the ski along its entire length l, the average heat flow into the ice grain is given by

$$\bar{q} = -2\left(\frac{u}{\pi l \kappa_i}\right)^{1/2} k_i T_{sn} \tag{29}$$

The total heat loss into all ice grains during passage is $n\pi r^2$ times this value.

To analyze the heat flow into the slider, Colbeck (1988) assumed that any point on the slider was at 0°C when in contact with an ice particle and at T_{sn} otherwise. It was also assumed that the top of the slider was at the same temperature as the snow surface and that the heat flow was only vertical. These assumptions are shown to be rather poor by the temperature measurements at the base of skis reported by Colbeck and Warren (in press), as well as some new measurements discussed later. Accordingly, we take the heat loss into the slider as a parameter that can be greatly affected by such simple things as the color of the slider and its metal content. A more complete energy balance of a slider than that done by Colbeck and Warren (in press) is needed to get the necessary information about the thermal response of skis to heat production by speed, radiation balance, and ambient temperature. When eqs 27 and 29 are combined and the ratio of area to weight is taken as c, the rate of melting over contacts of area $n\pi r^2$ is

$$\dot{h} = \frac{wl}{Wc\rho_i L}\left(\frac{W\mu u}{wl} + R - q_{sl}\right) + 2\left(\frac{u}{\pi l \kappa_i}\right)^{1/2} \frac{k_i T_{sn}}{\rho_i L} \tag{30}$$

where T_{sn} is in °C. This is an expanded version of Lehtovaara's (1989) eq 19 and is similar to the equations derived by Colbeck (1988). It gives the meltwater production rate, which, along with the shear removal rate, controls the thickness of the water films. The significance of the different terms is described later.

Empirical model

The experimental evidence shows that there is a range from dry sliding, where too little meltwater is available, to very wet sliding, where too much meltwater is available. Because of this, Colbeck's (1988) empirical model was based on water film thickness. While this model captures the basic ideas about the physics of the processes as explained in this monograph, the model's use is limited by the lack of convenient ways to measure the film's thickness. Conceptually it is merely an attempt to fit equations to the existing ideas about how friction works, but it does not increase our understanding of the processes.

Taking wet and dry friction as parallel processes that operate according to the ideas of elastohydrodynamics, the total friction is given by the sum of the interaction of these parallel processes and the friction due to capillary attraction, or

$$\mu = \mu_{cap} + \frac{\mu_{dry}\,\mu_{lub}}{\mu_{dry} + \mu_{lub}} \qquad (31)$$

The experimental evidence suggests that dry friction decays exponentially as the thickness of the meltwater film increases, so we assume that

$$\mu_{dry} = \varepsilon \exp(-\xi h) \qquad (32)$$

where ε and ξ are constants that can be determined experimentally but that would vary with the prevailing conditions. While μ_{lub} can be determined from the physical theory outlined above for the dynamics of the water film, there is no physical or experimental basis for quantifying the frictional force due to capillary attachments. Since the importance of these attachments must increase with the availability of water, and removal by both shear and squeeze increases as powers of h, we assume that the drag increases as a power of h. Accordingly,

$$\mu_{cap} = \beta h^3 \qquad (33)$$

where β is a constant for given conditions. An example of how the total friction and its components change with water film thickness is shown in Figure 15 for a typical set of conditions. Overall the friction is a balance among its different components with the result that it reaches a minimum at an intermediate value of film thickness.

Summary of models and processes

It should be clear from the preceding discussions that some of the fundamental information about snow friction is missing. There will be much speculation about the sliding processes until we know the scales at which the dominant mechanisms operate. Studies of the surface topography of sliders and snow must be done in conjunction with measurements of friction, electrical charges, surface temperatures, and water film thicknesses. Then the theory of elastohydrodynamics can be applied to test ideas about possible mechanisms. Until then, discussions of the mechanisms will be speculative and models of the processes will be qualitative at best. Empirical models such as the one presented above may be of some practical value but cannot possibly include all of the parameters of interest. Given this situation, the experimental observations discussed next are particularly valuable but, as was explained above, the applicability of an experimental result is limited to the range of conditions over which the experiment was conducted, because the processes that control the friction change with parameters such as speed and slider length. Nevertheless, much has been learned from both laboratory tests and skier experience.

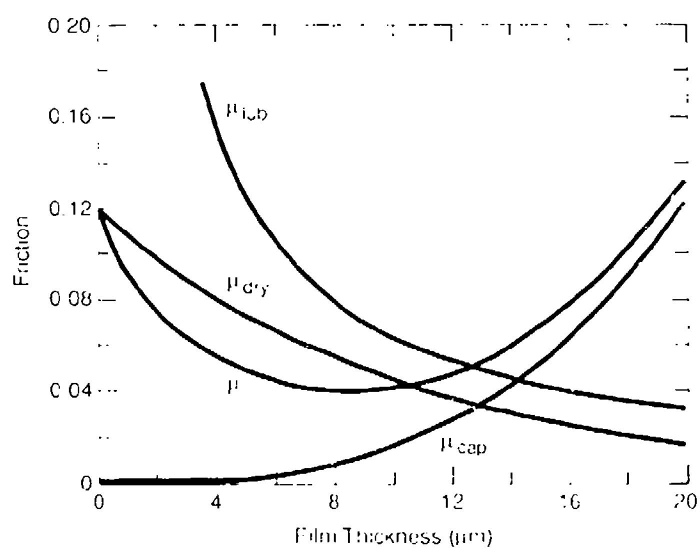


Figure 15. Example of total friction vs film thickness from the empirical model of Colbeck (1988). The total friction is the sum of three components due to dry friction, melt film shearing, and capillary attraction.

IV. SNOW FRICTION MEASUREMENTS

There is a long history of measurements of snow friction but, because of the large variety of important parameters in these tests, it will still be a long time before there is enough good quality data to provide all of the needed information. Certainly much of the problem is due to the lack of measurements under the conditions of primary interest: long sliders moving at high speeds under a variety of natural snow conditions. Accordingly, the available experimental results are useful for investigating the important processes but do not provide information about the coefficient of friction under most conditions of interest. The existing experimental evidence does give us information about how friction varies with parameters such as speed, load, and temperature under laboratory conditions.

Effect of speed

At low speeds, Bowden and Tabor (1964) found that the friction of miniature skis dropped slowly as speed increased gradually, so, at a speed of 0.03 m/s, the friction was only slightly less than its static value. The same pattern was observed for a variety of materials but with less friction for waxed wood than for lacquered wood, Perspex, or aluminum. However, the friction for all of these materials dropped greatly when the speed was increased to 5 m/s, presumably because of the greater heat and meltwater production at these higher speeds.

Shimbo (1961) observed similar behavior for P.T.F.E., with the friction dropping very rapidly as speed first increased, as shown in Figure 16. For ice the friction continues to drop with increasing speed but with snow it is usually found that friction increases at speeds above 5 to 10 m/s. As shown in Figure 16, Kuroiwa (1977) observed this behavior for both waxed and unwaxed polyethylene in both wet and dry snow. Spring (1988) also observed this increase, especially for wet snow, while the friction of dry snow stayed low at least up to 10 m/s, as shown in Figure 16. Colbeck (1988) explained both the decrease at low speeds and the increase at higher speeds in terms of the dynamics of the water films. However, it seems likely that the reduction of solid-to-solid interaction as speed first increases and the greater heat loss at high speeds are at least partly responsible.

From the data it is clear the friction decreases below the static value once speeds are great enough to introduce some lubricating meltwater and that friction increases again once the speed exceeds 2 m/s, according to Spring (1988), or 10 m/s, according to Kuroiwa (1977). It is not likely that the trend seen in Spring's results can continue since aircraft landings on snow and ski racing would be impossible. For the same reason, it is unlikely that the steepening trend shown in Kuroiwa's data could be correct. However, it does appear to be correct that the friction increases at higher speeds, possibly because of both the dynamics of the water film and because, as shown in eq 29, the average heat flow into the ice grains increases with speed as would the heat loss from the slider due to energy exchange with the air.

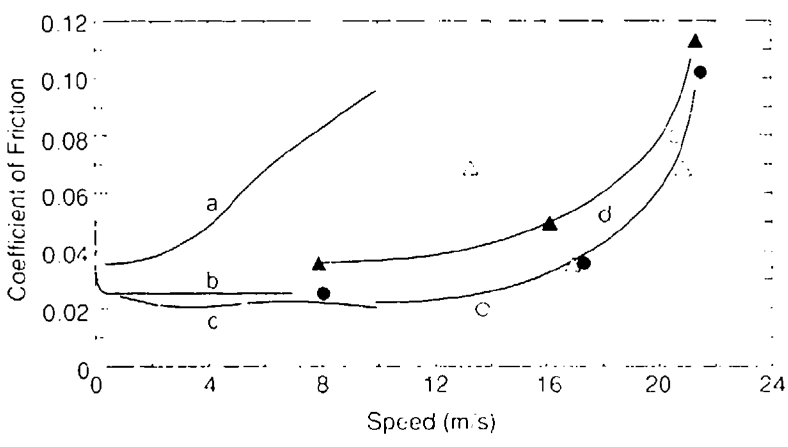


Figure 16. Measured values of coefficient of friction vs speed: a is for dense, wet snow (from Spring 1988), b is for P.T.F.E. on wet snow (from Shimbo 1961), c is for dense, dry snow at −7.5°C (from Spring 1988), and d is for waxed (circles) and unwaxed (triangles) polyethylene on dry (solid symbols) snow at −2.5 to −1.6°C and on wet (open symbols) snow (from Kuroiwa 1977).

Casassa et al. (1991) measured the friction of snow on snow and snow on ice. While their work was designed to provide information about flowing snow and not about sliders on snow, the sliding of ice on snow or snow on ice might provide some insight into the processes of snow friction at low values of load. At these loads, snow particle disaggragation and movement can add drag, and ice-to-ice adhesion must also be considered. Thus their results are not used here, but they do show interesting effects of speed at different temperatures.

Effect of load

When the coefficient of friction is independent of the load on the slider, the processes of friction can be described more easily, since it is then assumed that the contact area is proportional to the load (Bowden and Tabor 1956). Kuroda (1942) found that the coefficient of static friction remained constant with increasing pressure for a variety of snow conditions and materials, and Bowden (1953) found a similar result. He also showed that the coefficient depended on the type of surface, being lowest for P.T.F.E. Shimbo (1961), Keinonen (1978), and Spring et al. (1985) found similar results for both static and kinetic friction. At a speed of 0.1 m/s, Bowden and Hughes (1939) found that the coefficient of friction of a ski on snow decreased when the load exceeded about 50 kg, and Ericksson (1955) found that it decreased for increased loads at 2.5 m/s. With these speeds and ranges of loads, the frictional processes would have changed from dry processes to lubricated processes as the load increased, and thus the experimental results may simply indicate that the mode of sliding moved from the left-hand side to the middle of Figure 14.

Effect of temperature

There has been a lot of interest in the effect of ambient temperature on sliding friction, because the effect is discernable to both ice skaters and skiers and it is relatively easy to observe experimentally. Eq 30 shows that the rate of meltwater production decreases as temperature drops, and it is clear that the thickness of the layer of meltwater decreases with decreasing temperature. Ambach and Mayr (1981) found that the effect of temperature was most pronounced in the first one-third of a ski run before the speed increased to the highest values. Thus it is the combination of weight, friction, speed, and the energy balance at the interface that controls the generation of meltwater: the ambient air temperature, snow surface temperature, and radiation balance all contribute to this energy balance. Since snow surface temperature is very difficult to measure because of radiational effects on sensors, use of the air temperature as a parameter is common, although the slider/snow interfacial temperature, not the air temperature, is of primary interest.

Bowden and Hughes (1939), Klein (1947), and Ermakov (1984) measured steady increases in friction with decreasing ambient temperature, and Colbeck and Warren (in press) found that ski bases were colder at lower ambient temperatures. Bowden (1953) found that static friction increased at lower temperatures but that it also increased at air temperatures above freezing and that both subfreezing and suprafreezing effects depended

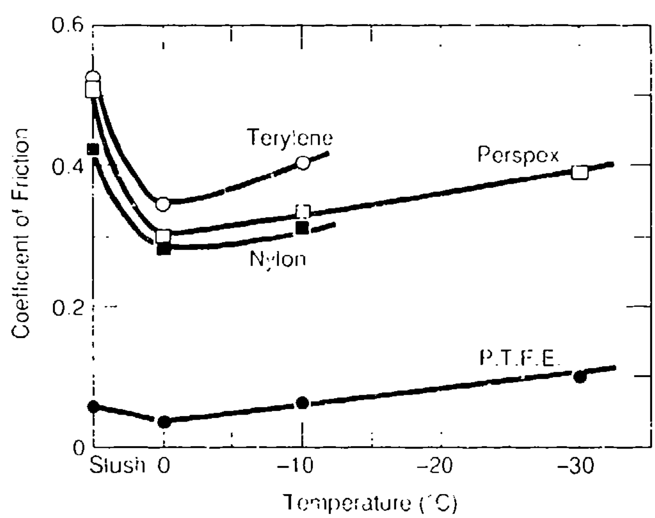


Figure 17. Coefficient of friction vs temperature for different materials sliding on snow (from Bowden and Tabor 1964).

greatly on the type of material applied to the base of a ski. Bowden and Tabor (1964) found a similar response at low speeds (Fig. 17) but found a great difference between P.T.F.E. and other materials. At a speed of 2.5 m/s, Ericksson (1955) observed a more rapid rate of increase in friction for steel runners than for wooden runners as the temperature dropped, presumably because of the greater heat loss through the metal. However, Ericksson found that the friction on the wood runner increased above about −1°C, not above 0°C as Bowden had observed. This suggests that for highly polished, hydrophilic runners, capillary drag may become important at temperatures just below the melting temperature. Outwater (1970) suggested that this minimum in friction occurs at different subfreezing temperatures for different ski surfaces.

Effect of snow type

It is generally agreed that fresh, cold, and man-made snow are aggressive because they erode the base of a ski and increase friction. Accordingly, harder waxes are used under these conditions. Conversely, old, warm, and dense snows exhibit low friction. This is partly due to the decrease in friction with increasing grain size (Fig. 18), which can be explained by the dynamics of the water film (Colbeck 1988) and by the greater elastic response when the grains are bigger or smoother. Klein (1947) stated further that the resistance to sliding was high with fresh snow until it lost its dendritic structure, and he suggested that finer snow structures had higher sliding resistances.

Hämäläinen and Spring (1986) found that the kinetic friction of skis tended to decrease for harder snow surfaces, but the effect was not very large. Thus several investigators found the highest friction for fresh snow and the second highest for wet snow, while older, higher density, frozen snows had lower friction (e.g. Kuroda 1942). It appears that crushed ice has rather different properties than most snow (Shimbo 1961, Lehtovaara 1989) and should be avoided as a substitute for snow in laboratory experiments.

Effects of slider characteristics

The effect of length observed by Ericksson (1955) and shown in Figure 19 is well known to skiers; it is one of the reasons why longer skis are used for racing events that emphasize speed rather than turning. This can be at least partly explained by the thickening of the water film along the length of the ski, as discussed earlier. The effect of the thermal conductivity of a slider on ice was observed by Bowden and Hughes (1939), who first suggested use of low-conductivity metals for ski edges. This idea was extended by the theory of Colbeck (1988) and the thermal model of a ski by Colbeck and Warren (in press) who suggested that highly conductive materials such as aluminum should be avoided, especially close to the base of the ski.

Snow skis are routinely imprinted with different roughnesses and coated with waxes of different hardnesses. In an important series of tests, Shimbo (1961, 1971) showed the effect of hardness and roughness on kinetic friction at a speed of 2.4 m/s. At an ambient temperature of 3°C, the kinetic friction is less for rougher surfaces (Fig. 20) whereas at −2°C, the ki-

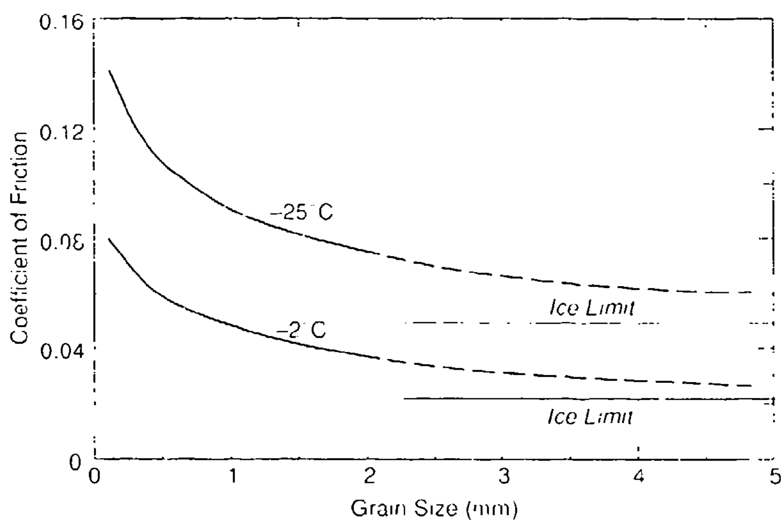


Figure 18. Coefficient of friction vs snow grain size at two temperatures (after Ericksson 1955).

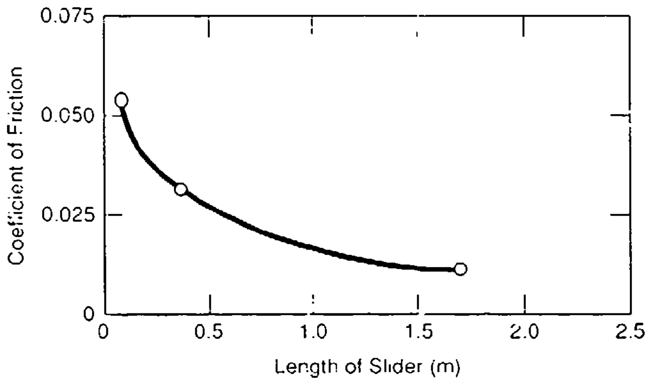


Figure 19. Coefficient of friction vs runner length at -4 °C (after Ericksson 1955).

netic friction increased slightly with roughness. These tests should be more detailed in the finer scale of roughnesses, especially in the range of 0 to 10 µm where ski roughnesses generally occur.

The theory of elastohydrodynamics described above should apply to snow when the slider is harder than snow but is still highly elastic. When the slider is softer and begins to erode and roughen, the increased roughness elements of the slider probably cause further erosion of the slider because of increased solid-to-solid interaction. Figure 21 shows Shimbo's results for wax hardness (a lower value of penetration indicates a harder surface). The results of these tests explain what is common knowledge in skiing: harder waxes work better at lower temperatures, and softer waxes work better at higher temperatures. The effect is especially pronounced at lower temperatures where the coefficient of friction decreases rapidly as wax hardness increases, possibly because the harder ice surfaces are capable of penetrating the polymer bases of skis at these temperatures. As shown in Figure 6, ice increases in hardness with falling temperature much more rapidly than P.T.F.E., and ice is the harder substance below about −15°C. Since ice is capable of eroding ski bases at lower temperatures, it is very important to protect them from roughening at lower temperatures since, as shown in Figure 20, friction increases with roughness at subfreezing temperatures. The effect of roughness on friction at lower temperatures is probably much greater than suggested by the results in Figure 20, but tests such as these remain to be done at lower temperatures.

There are other important effects on sliding friction

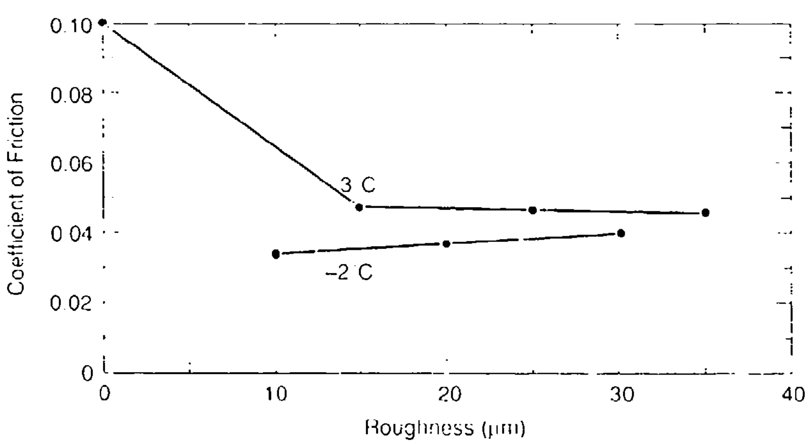


Figure 20. Coefficient of friction vs roughness at two temperatures (after Shimbo 1971).

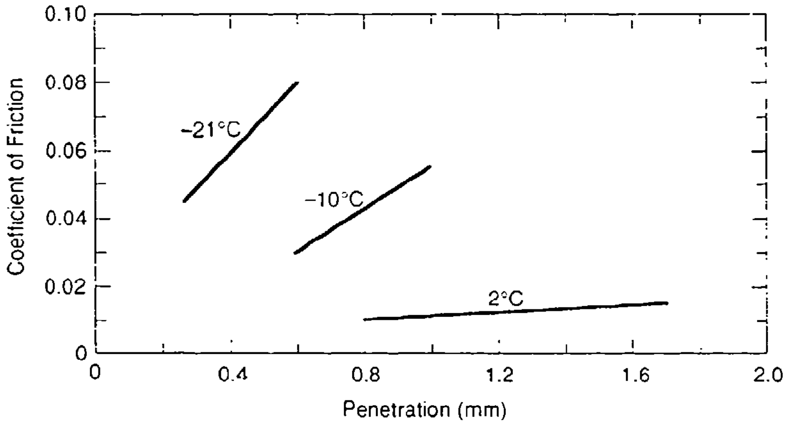


Figure 21. Coefficient of friction vs penetration into wax at different temperatures (after Shimbo 1971). Penetration is inversely related to hardness.

but there are no observations of these effects to report. Electrical charges have been observed to accumulate when snow is rubbed against itself (Chalmers 1952), and these charges may account for some dirt accumulation by the slider. Measurements of both the charges and the dirt are lacking and, while the charges should not be too difficult to measure, the dirt may be hard to observe because it is likely to be in the size range of micrometers. Micrometer-size particles would be easily attracted by electrical charges and, if harder than ice, would be

V. OTHER SLIDER MEASUREMENTS AND SIMULATIONS

The most useful information that could be obtained at this time would come directly from the sliding interface: the distribution of thicknesses and areas of the meltwater films, the surface roughness of sliders, the surface roughness of the supporting ice grains, the contact areas of the solids, the deformation of both solid surfaces, their temperatures, and the nature of capillary bonds. Here we review the small amount of information that is available about these subjects: information on the contact area beneath the slider and the thickness of the meltwater films.

Temperature measurements at the base of the slider are an indirect measure of the processes and properties at the interface, but they are important in their own right. They are much more easily made than any of the other measurements, and it is very important to know the temperature at which the interfacial processes take place. While ambient air temperature was reported as the most important parameter for the friction measurements given in the last section, it would have been much more informative to report the interface temperature of the slider, which is not determined by air temperature alone. In fact, in some circumstances solar radiation may be more important than air temperature. Temperature is also a direct measure of the generation of the interfacial heat, which generates the meltwater that controls the friction

Film thickness

Bowden and Hughes (1939) made the first attempt to measure the thickness of a meltwater film. However, they used a crude method based on the electrical conductivity of salt water, and their estimate was almost certainly too high. Ambach and Mayr (1981) used a 100-mm^2 capacitance probe placed at the base of a ski to determine the thickness of the film and thus obtained the first values that are at least close to being correct. Because a calibration procedure was necessary to convert the voltage signal into a film thickness, the reported values of thickness are not necessarily exact, but they are almost certainly of the right order of magnitude and show trends with varying conditions that are very helpful in thinking about the processes. Ambach and Mayr found that the water film thickness varied with snow temperature, speed, ski base preparation, and snow surface conditions. The film thickness (Fig. 22) decreased with both snow and air temperatures. While it would be speculative to extrapolate these values to lower temperatures, it seems reasonable that the film thickness would continue to increase with air temperature, snow wetness, or solar radiation absorption. Similar measurements in a much lower temperature range would pro

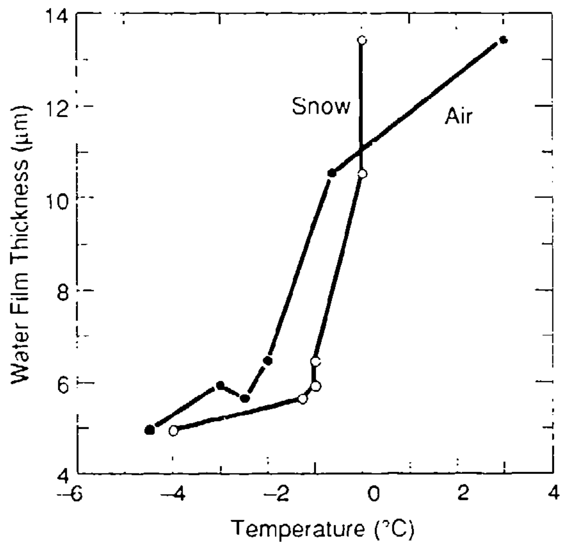


Figure 22. Water film thickness vs temperatures of the snow or air (after data of Ambach and Mayr 1981).

vide a great deal of information about the transition to dry sliding.

Ambach and Mayr (1981) found that use of the recommended TOKO wax for their range of snow temperatures produced thicker water films and presumably lower friction. However, their tests failed to show any effect of ski roughness. While the results of these tests are from a limited range of conditions, they add a great deal of credibility to the meltwater lubrication theory and confirm some conclusions drawn from theory, experiment, or experience. For example, the observation that less meltwater is generated when the snow is colder supports the use of eq 30. The observation of the effect of waxes on the thickness of meltwater films helps quantify thinking about the use of waxes. It suggests, for example, that the difference in film thickness between TOKO yellow and TOKO green wax at a snow temperature of −11°C would be about 1.5 μm. Using eq 21, this suggests that the yellow wax would increase friction about 30% over the green wax. This difference is great enough to be observed by skiers, which explains why wax technology has evolved to a high level even without detailed measurements of the dominant processes. The green wax is recommended for these conditions, and the yellow wax is recommended for much warmer and wetter conditions.

Contact area

The concept of contact area in snow tribology must be expanded to include solid-to-solid contact and solid-to-solid interaction through a liquid bridge. It may not be possible to separate these two forms of interaction since the liquid bridge is sometimes a very thin liquid film under high pressure squeezed between two asperities, even asperities that would pass without any contact. The total contact area as seen through a glass plate mounted in a ski is an approximation of the true interaction between the solids since it gives the total of the contact area but does not provide information about direct contact versus interaction through a liquid film. In addition, it is very unfortunate that these observations have not been made over a range of conditions to find, for example, how the contact area changes with load.

Based on the hardness of ice, Bowden and Tabor (1964) estimated that the fractional contact area would be between 10^{-4} and $3 \cdot 10^{-5}$ for temperatures from just below the melting temperature to −20°C. This assumes that the slider is harder than ice and that none of the load is carried by pressure in the water films. These are upper limits for the direct solid-to-solid contact and would be reduced significantly if the dynamic pressure in the water films separated the solids. Huzioka (1962) observed the contacts directly through a glass plate and found a much larger fractional contact area of $1.4 \cdot 10^{-2}$. Much of that was due to the supporting water films, which were sheared away from the contacts along with some ice filings. Presumably at higher speeds, where more heat and meltwater are generated, the ice filings would disappear as the water film thickened. It is not clear what would happen then to the contact area, and continued measurements such as Huzioka's are essential to understanding snow friction.

Torgensen (as reported by Perla and Glenne 1981) found a contact area for new snow of less than 1% that increased to nearly 80% for new, melting snow. More recently Pihkala and Spring (1986) measured the thermal conductivity across the sliding interface as an indirect measure of the contact area and found much higher values than had been previously reported. For dense snow at or below −5°C, they reported contact areas of 5 to 15%, and for dense, wet snow they observed contact areas of 45 to 100%, depending on the liquid water content of the snow. Apparently the liquid water content of the snow has a large effect on contact area, but this area may have to be apportioned among solid-to-solid, load-bearing water films, and capillary bridges.

Slider temperatures and heat flow

Although the temperature at the base of the slider is an indirect indication of the processes, it is easily measured under a variety of conditions. Furthermore, by measuring the temperature profile in a slider, the heat flow into the slider can be calculated, which provides information about the removal of heat from the area of meltwater production. The technique for making these

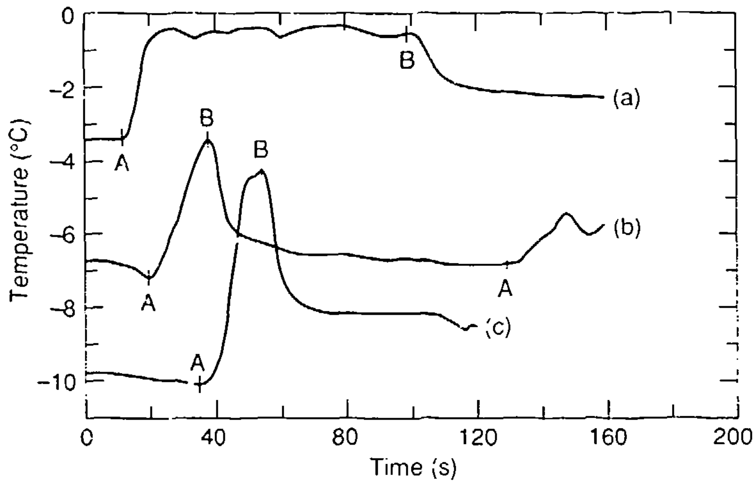


Figure 23. Ski base temperature vs time for three runs at different ambient temperatures (after Colbeck and Warren, in press).

measurements was described in Warren et al. (1989), and the results were summarized in Colbeck and Warren (in press). Some more recent results are added here. Similar measurements are frequently made by tribologists.

Typical results of temperature measurements at the base of a downhill ski are shown in Figure 23; it can be seen that there is a sudden increase in temperature at the onset of motion and that the temperature reaches a plateau if given sufficient time. This steady-state temperature is determined by the balance of heat production, heat flow, melting, and removal of meltwater from the interface. The temperature rise decreases with increasing ambient temperature but increases with increasing load and speed. The temperatures in a transverse profile across the base of the ski respond quickly to pressure changes while turning, and temperatures are a good measure of the weight distribution both across and along a ski. They show, for example that skier pressure increases from the inside to the outside of a ski even when the skier perceives the weight to be evenly distributed. On a hard snow surface most of the weight is carried under the skier, but when skiing in soft snow the weight is more evenly distributed, as indicated by a continuous increase of temperature from the front to the back of the ski.

Heat production by friction is load × speed × the coefficient of friction. Colbeck and Warren (in press) showed that the temperature rise at the base of a ski increases with speed and load, and thus it is reasonable to assume that the temperature rise increases with the coefficient of friction as well. Accordingly, I interpret the temperature rise at the base of the ski to be a good indicator of the friction on the ski, a result that is clearly shown in Figure 24. Wood skis are known to have a higher level of friction than P.T.F.E. (Bowden 1955) but, as is shown in Figure 24, well-waxed wood runs at a much lower temperature than unwaxed wood. Presumably the unwaxed side generates more meltwater through frictional heat dissipation, but the waxed side uses meltwater more efficiently. We will return to this issue in the next chapter when we look at the total energy balance.

Skis are generally constructed of different materials with markedly different thermal responses. The rate of heat flow into a ski is an important aspect of snow friction since, as has been shown earlier (e.g. Bowden and Hughes 1939), friction increases when the slider is more conductive and can remove more heat from the interface. Some idea of the heat flow patterns can be derived from the temperature response at three levels in a Rossignol DH ski (Fig. 25). This type of thermal response was simulated in a numerical model by Colbeck and Warren (in press) for various positions along the bottom of the DH ski, and some results are shown in Figure 26. For four transverse positions from the steel edge in Figure 26a to the centerline of the ski in Figure 26d, the heat flux for four different material compositions can be seen. Figure 26a shows that the steel edge (cases 1 and 4) dominates the heat flux 3 mm from the edge and that the heat flux is significantly reduced when the steel is replaced by a ceramic edge (cases 2 and 3). Figures 26b and 26c show that the heat fluxes at 10 mm and 25 mm from the edge are reduced if the steel edge is replaced by a ceramic edge and/or if the aluminum plate across the bottom of the ski is replaced by a polymer (cases 2, 3, and 4). Figure 26d shows that, at the

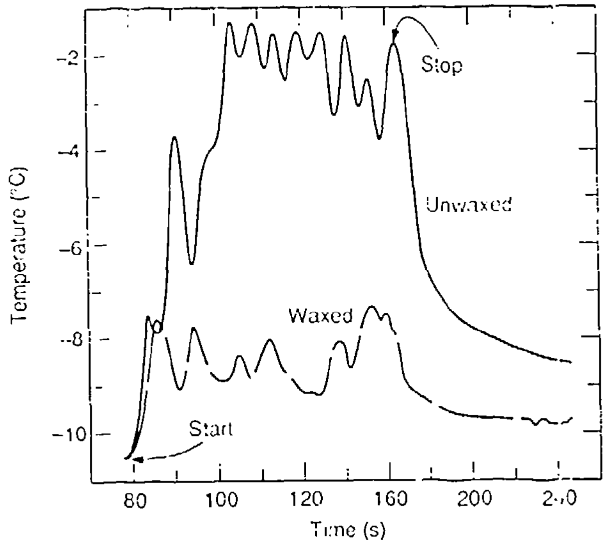


Figure 24. Ski base temperature vs time for a wood ski that was one-half waxed and one-half bare wood along its length.

centerline of the ski, the aluminum plate dominates the heat flux (cases 1 and 2) and that the steel edge has a significant effect too when it is combined with the aluminum plate (case 1). With a large aluminum plate running across the base of a ski with steel edges, the heat available for melting at the base in the center section of the ski is reduced by about 50%.

Although incomplete, the variety of friction and other measurements reviewed here shows both the scope of the information that is available and the nature of the information that should be generated about snow friction. When combined with current ideas about the processes, some useful approaches to achieving low friction are apparent.

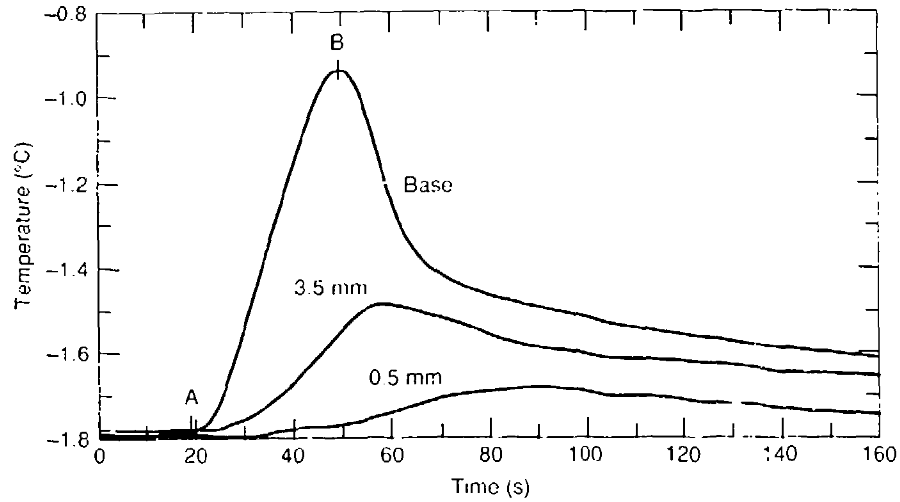


Figure 25. Temperature at three heights in a Rossignol DH ski vs time (after Colbeck and Warren in press). The temperature traces show the propagation of a wave of heat into the ski.

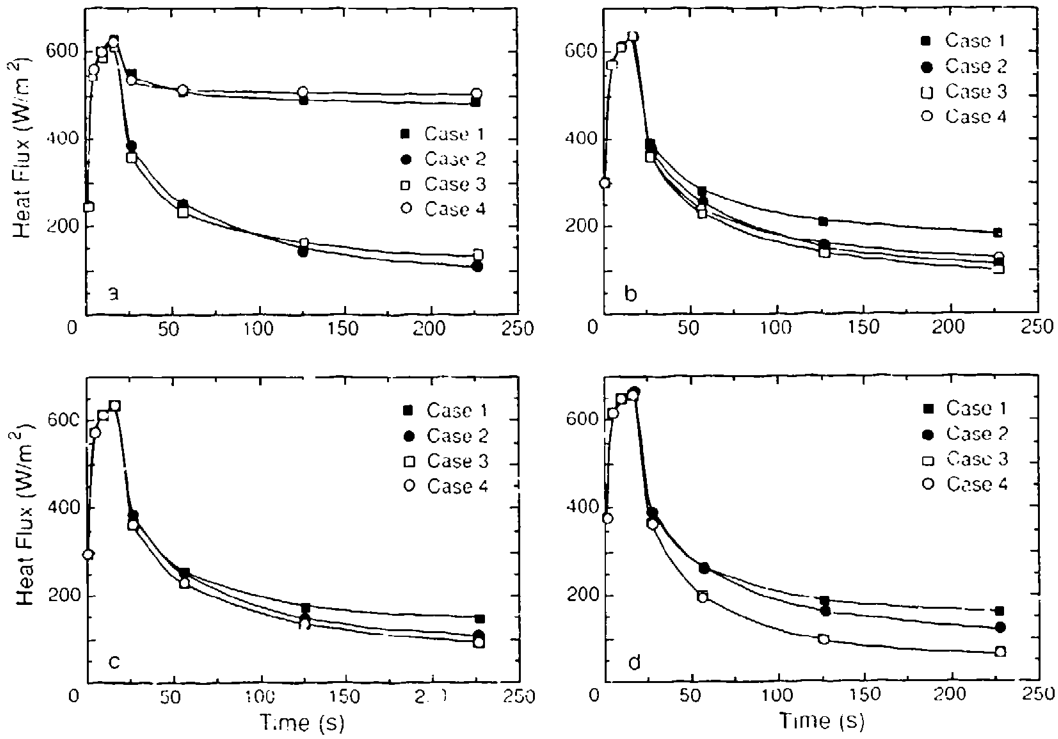


Figure 26. Computed heat flux vs time for four transverse positions and four different material configurations in a simulated DH ski for an ambient temperature of –5°C and speed of 18 m/s. a) 3 mm from the edge, b) 10 mm from the edge, c) 25 mm from the edge, and d) on the centerline, 37 mm from the edge.

VI. FRICTION ADJUSTMENTS

To design a slider with the optimum friction for a given set of conditions many things must be known. The processes of greatest interest occur at the sliding interface, but there are many factors that affect those processes. These slider properties are important:

- Load distribution
- Surface roughness
- Surface hydrophobicity
- Elastic and plastic characteristics of the surface layer
- Adhesiveness to ice
- Surface contamination
- Electrical and thermal conductivity
- Color

These snow properties are important:

- Temperature
- Hardness
- Bearing strength
- Wetness
- Grain size
- Polish
- Surface contamination
- Depth

The following processes are also important:

- Ice melting
- Heat flow into the slider
- Heat flow into ice grains
- Shear of meltwater films
- Elastic and/or plastic deformation of solids
- Capillary bridging
- Solar radiation absorption
- Cooling of the slider
- Drag by dirt
- Electrical charging
- Solid-to-solid adhesion

Given the difficulty of controlling or even measuring all of these properties, it is hard to design well-controlled experiments to find the effect of any given parameter. Shimbo's (1971) results are probably the best example of informative experimental results, but not all of the experimental conditions were reported. In this chapter existing experimental and conceptual results are discussed with the aim of achieving the lowest friction allowed by the conditions.

Interfacial temperature

While waxes are usually applied according to air temperature and snow conditions, the wax actually works at the sliding interface, whose temperature is affected by a balance among heat production, heat loss, and latent heat use. The interfacial temperature can be considerably greater than the air temperature and thus some knowledge of the actual sliding temperature would help choose the proper wax and/or surface structure for the prevailing conditions. The differences between ambient and sliding temperatures are especially important since the temperature, on the absolute scale, is usually close to the melting temperature of ice, and the properties of ice change rapidly in that temperature range even if the properties of the slider do not.

Assuming that conditions are the same all over the slider's base, the total energy balance from eqs 27 and 29 is

$$W u \mu + w l (1-\alpha) R = \rho_i L n \pi r^2 \dot{h} + w l \dot{q}_{sl} + n \pi r^2 k_i T_{si} \left(\frac{u}{\pi l \kappa_i} \right)^{1/2} \qquad (34)$$

The first term in this equation represents heat production by friction, the second term represents heat production by solar radiation absorption, the third term represents heat loss due to melting, the fourth term represents heat loss by heat flow into the slider, and the fifth term represents heat loss by heat flow into the ice. Except for the third term, each is evaluated individually.

Frictional heat

If the slider carries a weight of 400 N at a speed of 10 m/s with a coefficient of friction of 0.03, the friction produces heat at the rate of 120 watts. Thus, as a rule of thumb, a downhill racer typically generates as much heat under each ski as a 200 or 300 W light bulb.

Solar heat

Taking a slider area of 0.16 m^2 with a solar flux of one half the solar constant reaching the base of the slider by diffusion from the underlying snow, the heat production rate is 112 W at the base if the albedo is 0. For a white base this value would be an order of magnitude less than for a black base, so the color of a slider is clearly very important. When the slider is black and is receiving intense solar radiation, it can absorb nearly as much heat at the base as it produces by friction and, when the heat absorption at the sides and top are considered, the total solar radiation effect could control the heat balance of the slider. This can clearly be seen in Figure 27, where the ski cools rather than warms at the start of a downhill run because it had been heated by the sun. Then when motion stops, the ski heats rather than cools, as was found in situations where sunshine was not a factor (e.g. Figs. 23 and 24). In fact, the base of the ski was heated to above the melting temperature although the air temperature was -9°C. In this situation it is clear that the entire ski would be heated sufficiently to conduct a significant amount of heat to the base when it started to move again. An all-wood ski of area 0.16 m^2 and thickness 20 mm would store enough energy to melt about 5400 mm^3 of water per degree of temperature rise above 0°C if the heat could be completely recovered. Although heat is used very inefficiently, as shown by Pfalzner (1947), little meltwater is needed to lubricate the ski. Furthermore, a warm ski would greatly affect friction in the early part of a ski run, as is suggested by the high ski temperatures shown in Figure 27. After the start of motion, the ski base cools for about 100 s before reaching a plateau that itself is considerably higher than would be expected in the absence of radiation absorption. In Figure 24 the waxed temperature plateau is nearly 5°C cooler than the hot wax plateau in Figure 27, although the conditions of the two tests were similar except for the intense solar input occurring during the later test.

Conduction into the slider

Heat flow through the slider is a very important consideration since it can vary over a wide range. For the wood ski described above, the steady-state heat flow through the ski would be about $1.34\delta T$ W, where δT is the temperature difference driving the heat loss from the ski. In this case the heat flow would be small compared with the heat generated by friction; except in its transient phase, the heat flow into or out of a wood ski is negligible. For an all-aluminum ski of the same dimensions, however, the steady-state heat flow would be $1600\delta T$ W, which is very significant since it would consume most or all of the heat production by friction for common values of δT. Since skis are normally a complicated mixture of materials, Colbeck and Warren (in press) studied the heat flow patterns with a numerical model; some of the results are shown in Figure 26.

While the steady-state response of a wood or plastic ski is not very important in the overall energy balance, their transient responses can be. In Figure 28 the temperature rise vs height is shown for three times during the transient phase of heating in the DH ski; similar profiles were obtained from an all plastic ski (Colbeck and Warren, in press). The temperature gradients observed in these skis show that the heat flow during the initial period of motion would be similar to the rate of

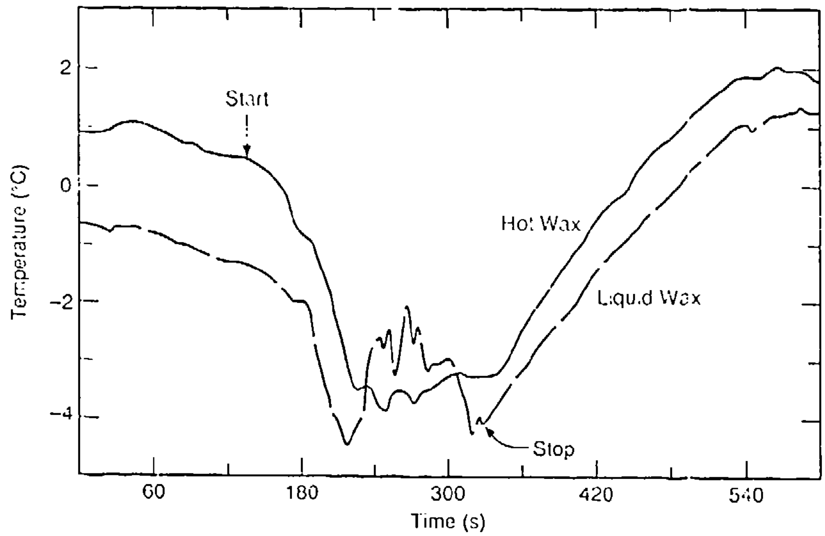


Figure 27. Ski base temperature vs time with an air temperature of −9°C but strong solar radiation absorption. The diffuse heating to the ski base by the sun causes a very large rise in the ski temperature so that the ski cools rather than warms at the start of motion. One side of the length of the ski was waxed with a liquid wax and the other side with a hot wax. The side with the hot wax reached a lower temperature plateau even though that side of the ski had a positive bias, probably because that side of the ski was facing the sun.

heat production by friction, thus greatly reducing the energy available to generate meltwater. For a step change in temperature on the base of a slider, the heat flow into the ski at the base would decrease with time according to

$$q_{sl}(0,t) = \frac{k_{sl}\,\delta T}{(\pi \kappa_{sl} t)^{1/2}} \qquad (35)$$

For a polyethylene slider of 0.16 m^2 area, this shows that heat flow would equal $75\delta T$ W after 1 s of heating and would drop rapidly thereafter. This accounts for the rapid rises in temperature shown in Figures 8, 23, 24, 26, and 28, since the heat loss into the ski would fall rapidly with time. For an all-wood ski the heat flow would be about one-half of that value, depending on the type of wood and the orientation of the wood grain. In either case, the heat flow into these sliders is only important during the transient phase of heating.

Conduction into ice

Only transient heat flows into the ice because the slider passes over an ice particle in a time period of l/u. If the ice particles immediately rise to the melting temperature and stay at that temperature until the slider passes, integrating eq 28 over the time period l/u gives the average heat flow into the ice grains during the passage of the slider as given by eq 29. While the heat flux is high because the ice grains are only in a rapid transient mode for a short period of time, the contact area of the ice particles is a small percentage of the total

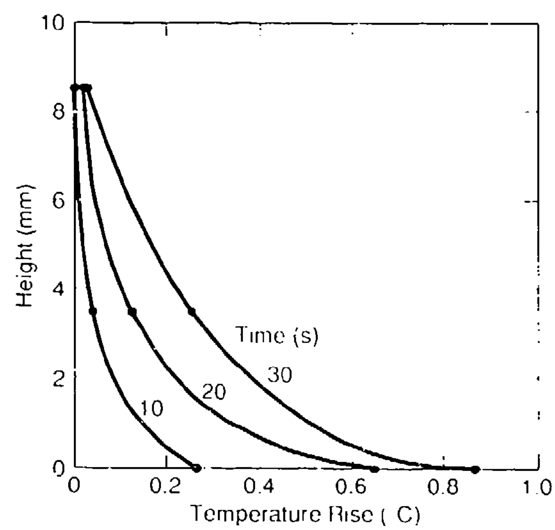


Figure 28. Temperature rise vs height at different times from the profiles shown in Figure 25.

area of the slider and this must be taken into consideration. For a slider of 2 m length moving at 10 m/s with a fractional contact area of 1% with the ice particles or their liquid films, the average heat flow into the ice would be $8.4\delta T$ W. This is greater than the steady-state heat loss into a wood slider but is still a small fraction of the heat that would be generated by the slider under those conditions. Thus while heat flow into the ice cannot be ignored, it is not a major term in the energy balance as long as the contact area is not much more than 1%.

Summary of interfacial temperature

It is clear from the discussion above that there are certain things in the design of a slider that should be done to make it better suited to its intended use. For example, highly conductive sliders are not well-suited for use at low temperatures because they remove heat from the interface, which reduces meltwater production and, in the extreme, reduces the temperature at which the solids must deform. Because ice rapidly becomes less deformable as the temperature drops, any deformation necessary to accommodate the passage of asperities would be more difficult if the interface temperature decreased. In addition, it is very clear from the slider temperature measurements shown in Figure 27 that a slider base can be directly heated by incoming solar radiation that penetrates the snow and diffuses up to the base of the slider. Direct radiation absorption on the top and sides is also important since the general warming of the slider would affect friction once the slider started to move.

The thermal properties, not just the mechanical properties, of the slider are important. Even its color can affect its performance, especially when there is either too much or too little meltwater available. Although these ideas remain to be tested in well-controlled experiments, the arguments and temperature measurements given here strongly suggest that sliders should be nonconductive and dark at low temperatures where it is important to retain as much heat and produce as much meltwater as possible. However, at high temperatures, where too much water appears to slow the slider, they should be white to minimize solar radiation absorption. Unfortunately, the black bases in skis being produced today are usually softer than the white bases because of the use of carbon or graphite. Thus the darker bases in the current generation of skis are not necessarily suited for use at low temperatures.

Surface roughness and waxing

Some time ago it was suggested that the sliding surface should be rough when too much water produced drag and smooth when little meltwater was available. A variety of patterns can be imprinted on or ground into the bottom of a slider, and they should have several important characteristics. The simplest concept is of roughness, but their geometry can vary longitudinally along and across the slider to suit different purposes. "Binders" are commonly used to coat surfaces subjected to frictional wear, and these generally adhere better if the surfaces are slightly roughened; in the case of metals, a roughness of about 0.5 µm is best (Clauss 1972). Oleamide and stearamide are known to lubricate polyethylene and can be incorporated in the bulk of the polymer, where they diffuse to the surface to provide an effective lubricating layer (Cohen and Tabor 1966).

The theory of elastohydrodynamics shows that plastic deformation of the slider can be avoided if the roughness elements are well-rounded, so at low temperatures the slider should be prepared with as smooth a running surface as possible. In this situation, roughness with a transverse orientation tends to increase the thickness of the water film, and roughness elements with a longitudinal orientation should be avoided. More quantitative statements could be made after use of the elastohydrodynamic models, but for now, Shimbo's (1971) results shown in Figure 20 are the only quantitative results available, and they do not include roughness orientation or even roughness results from the range of most interest. Clearly much research can be done to provide information about slider roughness.

Binders are commonly used to reduce friction and wear, and ski waxes are just one example of them. In general, they are applied in thicknesses of 1 to 4 µm and consist of organic resins such as phenolics and epoxies or sometimes silicones or alkyds (Clauss 1972). For application on metals, they are baked onto the surface at temperatures up to 200°C and take from 15 min to several hours to cure. They produce a more uniform coat when the substrate is slowly heated up to temperature, although air-drying versions of binders are also available. However, as with ski waxes, these versions are less durable and do not produce the wide range of desired effects. The higher temperature plateau of the liquid wax as compared with the wax applied at a high temperature (Fig. 27) is an example of this. Presumably, the ski with the hot wax reaches a cooler plateau (Fig. 27) because of lower friction, just as was shown in Figure 24 for an unwaxed vs a waxed wood ski.

Waxes are applied to skis for several reasons, and different waxes have much different effects on snow friction (Bowden and Tabor 1964). Waxes are more highly developed today so more subtle variations are likely to occur compared with those shown by Bowden and Tabor. To be most effective, the wax must not interfere with the desired surface roughness of the slider, which suggests that wax applications should be very thin—too thin to see. If well-bonded to the base of the slider, the wax should be effective in altering the hydrophobicity and hardness of the slider without changing the roughness. Shimbo (1971) showed that, for most waxes tested, the thickness of the wax did not affect the friction, although the roughness of the slider was not reported. As reproduced in Figure 21, Shimbo did show that the hardness of the wax has a large effect on friction, especially at lower temperatures where ice hardens much more rapidly than waxes. At these temperatures, the wax must be hard enough to withstand plastic deformation as indicated by the Plasticity Index. The wax should always be harder than the ice, which may be the main advantage of modern waxes.

Temperature and pressure are known to vary over the length and width of the sliders, so optimum use of surface roughening and waxing should take this into consideration. At the speed of snow skis the length of the dry area at the front of the ski is likely to be quite small if the load is evenly distributed, but would be somewhat longer in most actual situations. For example, with a Rossignol DH ski on a hard surface, Colbeck and Warren (in press) found that most of the heating was on the inside underneath the center section of the ski. The temperatures were less at the front, rear, and outside of the ski, so those areas should be waxed and perhaps structured differently than the inside edge beneath the skier. Although Pfalzner (1947) showed that the process is very inefficient, if heat is added to a slider it should be done in the coldest areas where it would be most effective.

Slider surface material

The surface material of a slider need only be a fraction of its total thickness but it can have a large effect on how the slider performs. The surface layer of most skis is about a 1-mm-thick layer of polyethylene, which has several distinct advantages over most other materials. Most plastics tend to be self lubricating and therefore have low friction. All have low thermal conductivity, which is a disadvantage in most frictional applications but an advantage with snow. Unfortunately, their electrical conductivity is also rather low, so they do not dissipate electrical charges readily and thus may attract dirt particles. Most plastics have the important advantage of absorbing vibrations well because of their high elasticity, and some have high impact resistance as well (Clauss 1972). While fluorocarbons have low friction, they do not have a high resistance to plastic deformation, and thus Teflon has not been widely used on sliders on snow. High molecular weight (2 to 5 million) polyethylene, however, does have outstanding abrasional resistance and favorable elastic properties and is widely used for skis. Some manufacturers vary the molecular weight to achieve better characteristics in different temperature ranges where different processes control the friction, although this effect of molecular weight is controversial. Bases are available in sintered form and thus waxes can be retained in the pores. In general, the higher molecular weight polyethylenes are used at lower temperatures where higher impact resistance and better wear characteristics are important (Clauss 1972). With the long-chain polymers used for skis, the molecules can be drawn out in the direction of motion so that the sliding actually occurs on oriented surfaces on the molecular scale (Tabor 1974).

VII. SUMMARY

Although there has been a long history of interest in and study of snow friction, as in other areas of tribology, the basic processes by which sliders move over snow are subjects of conjecture. Snow surfaces should be observed to characterize the roughness elements over which a slider must move. This will help in understanding how the basic processes differ with the type of surface. Observations of snow grains, such as those in Figures 1 and 2, are a start in gathering this kind of information, but other evidence is also needed, especially observations of the changes that occur in fresh snow grains during a sequence of passes. Information about the development of roughness profiles on both the snow and slider surfaces would help us understand the scales at which the prevailing processes occur. While some observations of the contact area have been made, many more are needed over a wide range of conditions. It is also necessary to refine this measurement to distinguish between solid-to-solid contacts and contacts with meltwater films. If possible, it would also be very useful to know if capillary bonds, such as the one shown in Figure 4, exist on snow surfaces and, if so, what their contribution is to the drag.

It has long been assumed that the drag on a slider on snow consists of a mixture of components that arise from different mechanisms operating simultaneously. If the snow density is too low to support the rapid application of stress by the slider, the snow is plowed and compacted. At low speeds, temperatures, and loads on hard snow surfaces, the dominant process is usually deformation of the asperities on one or both surfaces. However, when meltwater is generated by phase change at the interface, combined solid-to-solid interaction and meltwater lubrication are the dominant processes. There does not appear to be sufficient heat available to generate enough meltwater to separate the surfaces completely and so, under most circumstances of subfreezing conditions, it will be necessary to use elastohydro dynamics to understand how the asperities interact to cause drag. When greater quantities of meltwater are present, complete separation of the asperities is possible, but the surface area for drag increases considerably. In addition, capillary attachments may then add drag.

Several other mechanisms may be important too, including electrostatic charging and contamination by dirt particles attracted by the charges. Of all of these mechanisms, the one that is easiest to understand is the shear of meltwater films. Their generation is controlled by heat production at the interface and heat flow into both the slider and the ice grains. While heat flow into the ice grains is never large compared with heat production, either heat flow into highly conductive sliders or solar radiation absorbtion by dark sliders can control the interfacial temperature. Thus the choice of both slider material and color is critical and depends on the conditions for which the slider will be used.

The front of a slider moving at subfreezing temperatures tends to be dry and it can be easily abraded. The material and coating chosen for the front should consider the higher friction and abrasion as well as the lower temperatures there. Depending on the weight distribution, the same may be true for the rear. Because the fraction of the slider that is subjected to dry sliding depends on the thermal properties of the slider and the rate of heat generation, extreme caution should be used in applying the results of laboratory tests to other conditions. In particular, the short and slow sliders generally used in the laboratory may experience a much larger proportion of dry sliding than the prototypes they are intended to represent.

Polyethylene surfaces are known to have low friction and are better than most other polymers, although water is an ineffective lubricant for them (Cohen and Tabor 1966). This appears to result from a combination of properties that distinguish this particular material:

- It is hydrophobic and remains so under humid conditions.
- It is hard and is not easily damaged by ice particles over most of the range of temperatures of interest.
- It is highly elastic and so asperities can deform to allow relative movement of the surfaces by elastic, rather than permanent, deformation.
- It can be smoothed and imprinted with different patterns that allow more favorable interactions with ice grains in the presence of meltwater.
- It can be made porous.
- It can be readily coated with waxes and other polymers.
- Ice does not readily adhere to it.
- Its friction is not greatly affected by surface contamination (Lancaster 1972).

Some of these characteristics could be used to greater advantage by using elastohydrodynamics to predict how the meltwater films and the ice grains interact with different patterns on the slider surface.

Experimental observations provide much-needed guidance as to how friction varies with temperature and grain size and with slider roughness, hardness, speed, load, and length. Since friction is generally low, it is difficult to conduct tests to show how it varies, and these tests are even more difficult under the conditions of most interest. The results clearly show that friction is high when too much water is present and that it is lowest at or just below 0°C. It then decreases as temperature drops, probably because more heat is conducted away from the interface rather than because ice is harder at lower temperatures. Most of the frictional processes may occur at the melting temperature because of flash heating at the contacts. Some roughness is desirable when too much water is present, probably because it helps break up the water films, and longitudinal patterns should help to remove water. Friction decreases as speed first increases because of the onset of lubricated melting, but then increases at higher speeds because of more heat loss into the ice grains. Friction is reasonably independent of load, perhaps because the contact area increases proportionately. Friction decreases as slider length increases, probably because a greater proportion of its length is lubricated by meltwater, and friction decreases as grain size increases because of the dynamics of the water film.

The thicknesses of the meltwater films have been deduced from two types of electrical measurements. The capacitance results of Ambach and Mayr (1981) suggested film thicknesses of 5 to 10 µm under the conditions of most interest. They found decreasing film thicknesses at lower temperatures, as would be expected from the energy balance, and thicker films when the proper ski wax was used. However, films of these thicknesses cannot be explained in terms of the energy that is available to create meltwater even when fairly extreme assumptions are made about the processes. It is very important to resolve this issue because the energy balance method suggests that the films are too thin to completely separate the surfaces, while the capacitance results suggest that they are. Neither method has to be accepted without question, so the issue must be resolved to guide future thinking about the very processes that account for friction.

The contact area has been observed to increase with temperature, possibly just because of the amount of increased meltwater. If it were determined by the hardness of ice, it should have a value of less than 10^{-4}, but the observed values are greater. Contact area appears to exceed 10^{-2} for most conditions of interest but we do not know how much of that contact is solid-to-solid, through high-pressure liquid films or through low-pressure capillary attachments.

Slider temperatures can be easily measured and they provide indirect evidence about the frictional processes. Furthermore, since wax applications work at the interface, their selection should be guided by knowledge of the temperature at the slider interface, not by air temperature. Interface temperatures increase rapidly following the onset of motion unless the slider has been preheated by direct solar radiation absorption. The effectiveness of waxing can be seen by the different temperatures at which waxed and unwaxed interfaces run, and heat conduction through the slider can be observed. The heat flow through sliders of complex structure has been computed with a numerical model, and the results show how the use of highly conductive materials can cause a significant reduction in the heat available for providing meltwater.

Given the wide variety of conditions that occur, it is not possible to construct a slider that would have optimal performance for all conditions. Slider characteristics can be modified by restructuring their base and recoating them with the appropriate wax. However, the thermal and mechanical characteristics are very important, and even their color can affect their performance under many conditions. An average skier generates more than 200 W of heat by frictional heating, and solar input can increase that substantially. However, some metal sliders are highly conductive and can greatly reduce the amount of heat available. Coatings are frequently applied to polymers to reduce friction, and only very thin coatings are necessary to have an effect. The technology to reduce friction on snow has developed to a high level through the use of polyethylene and appropriate coatings without a detailed understanding of the processes described here. However, further evolution is desirable, and that appears to require improved descriptions of the processes.

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13. ABSTRACT *(Maximum 200 words)*

There is a long history of interest in snow friction, but it is still necessary to speculate about the details of the processes. Roughness elements and contact areas must be characterized before the basic processes can be well understood. These parameters change with movement over snow and, in fresh snow, probably change along the length of the slider. Friction results from a mixture of processes: dry, lubricated, and possibly capillary. Dry rubbing occurs at low speeds, loads, and/or temperatures and is characterized by solid-to-solid interactions requiring solid deformation. With small quantitites of meltwater present, elastohydrodynamics must be used to account for processes at partially separated surfaces and, when too much water is present, the contact area increases and there may be capillary attachments. Static charging probably occurs and may attract dirt that, even in the size range of micrometers, could complicate the processes. Slider thermal conductivity and even color are very important. Heat is generated by friction and solar radiation absorbtion but some is conducted away by the slider and ice particles. The remaining heat is available to generate meltwater, which acts as a lubricant. Polyethylene bases offer many advantages including low ice adhesion, high hydrophobicity, high hardness and elasticity, good machinability, and good absorption of waxes. While sliders must be designed for use over a narrow range of snow and weather conditions, polyethylene bases can be structured and waxed to broaden that range. The important processes operate, not at the air temperature, but at the ski base temperature, which is highly dependent on such things as snow surface temperature, load, and speed.

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