Research in Non-Linear Water Waves

Final Technical Report

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The work carried out under this grant was a study of the effect of shear in air and water on the stability and structure of non-linear surface waves and the generation of waves by wind. The results have been or will be published in 4 papers. Titles and abstracts follow.

Calculations are carried out of the shape of gravity and gravity-capillary waves on deep water in the presence of a thin sheet of uniform vorticity which models the effect of a wind drift layer. The dependence of the fluid speed at the wave crest is determined and compared for gravity waves with the theory of Banner and Phillips (1974). It is found that this theory underestimates the retardation due to drift and tendency to break. The retardation disappears when capillary forces are significant, but in this case it is found that there can be a significant alteration of the wave shape.


A simple water-wave instability induced by a shear flow is re-examined, using a cubic equation first derived by Stern and Adam (1973) for a piecewise constant vorticity model. The instability criteria and the growth rate are computed. It is found that this mechanism is effective only if the surface drift velocity exceeds the minimum wave speed for capillary-gravity waves, and only if the drift-layer thickness lies within a band which depends on the wavelength and the drift velocity.


Stern and Adam and subsequent workers have considered the linear stability of two-dimensional, parallel, ideal fluid flow with shear in the presence of a free surface. In these studies a fluid current is modelled as a finite layer of constant vorticity above a simi-infinite stagnant region, corresponding to a piecewise-linear velocity profile. Here, an investigation of the stability of currents for several smooth velocity profiles is presented. With surface tension present it is found that the fluid surface velocity must still exceed the minimum wavespeed of stagnant fluid for instability to occur; a result highlighted by Caponi, Yuen, Milinazzo and Saffman (see above) for piecewise-linear profiles. Instability growth rates are found to be significantly smaller than those associated with a piecewise-linear profile. There are also qualitative differences in the stability characteristics; in particular, transition is associated with an exchange of stability for smooth profiles, but not for the piecewise-linear profile.

A linear stability analysis of the inviscid, parallel flow of air over water leads to an eigenvalue problem for the wavespeed, which is solved numerically for two air profiles. Agreement of growth rates with those predicted from the Miles mechanism is poor. Both methods of calculating growth rates exhibit high sensitivity to changes in the wind profile. This sensitivity may help to explain discrepancies between the results of experimental studies of wave growth rates. In the limit of a highly sheared wind profile the numerical computations retrieve the Kelvin-Helmholtz instability.