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# Theory of System Reliability Demonstration, Burn-In Design, and Record Statistics

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### RESEARCH OF SAUL BLUMENTHAL ON AFOSR GRANT 89-0357

#### IRELIABILITY DEMONSTRATION.

1. Reliability Acceptance Testing for New Series Systems, to appear Naval Research Logistics, 1992.
2. Two-Stage Reliability Tests For New Series Systems. to appear J. Statist.Planning and Inference, 1992( with X.H.Zhou).
3. New Approximations for the Event Count Distribution for Superimposed Renewal Processes at the Time Origin with Application to the Reliability of New Series Systems. to appear Operations Research, 1992.

Series systems exhibit wear out if the components tend to wear out as they age. Acceptance sampling criteria are usually stated in terms of Mean Time Between Failures (MTBF) for an aged (or equilibrium) system and tests based on MILSTD781D are applied. However, the acceptance test is often administered to a newly produced system. In [1], we examine the consequences of using the MILSTD781D tests on new systems and show that the chances of accepting poor systems are dramatically greater than for the aged systems. In [1], we have developed tests that take this aging phenomenon into account, and are intended to be applied to new systems. Detailed properties of these plans, and their design are presented.

A major limitation of the test plans developed in [1] is that a great deal of information about the component failure distributions must be known in order to use

them. For instance, if all components have the same gamma failure distribution, the shape parameter of that distribution must be known. In [2], my graduate student, X.H. Zhou, and I have shown that the operating characteristics of the plans of [1] are very sensitive to departures from the assumed shape parameter. This problem can be overcome by using a two-stage adaptive procedure that would use first stage data to estimate the shape parameter and then use that estimate in the second stage to run the demonstration test. In [2], we have investigated this and found that such a plan does accomplish its desired goal. The analytic results are asymptotic as the number of first stage test systems becomes infinite, but simulation results verify that even when the number of systems being tested at the first stage is moderate, the goal is attainable. Technical complications arise because of the counting nature of the data so that the shape can be estimated only if a value of the scale parameter is assumed. However, in spite of this, estimates of the shape that give good operating characteristic values at the second stage for any value of the scale parameter are obtained.

The plans in both [1] and [2] depend on a Poisson approximation for the number of observed failures during the test period. This approximation was derived by Grigelionis (1964) as a limit when the number of components on test (number of components per system times the number of systems on test) becomes infinite. In [3], painstaking calculations of the exact distribution of the number of failures reveal that this approximation is not very close even when there are several hundred components on test, but may still give an almost correct test plan. For small numbers like 25, it is a very poor approximation and leads to incorrect test plan specification. Exact calculations are extremely tedious, can only be carried out for relatively small numbers of failures and for certain distributional forms so that designing test plans based on these does not seem practical. However, a binomial approximation has been found which gives a very good fit to the true distribution for small numbers of components and which depends in a fairly simple way on the distribution of times between component failures. For gamma distributions, where the exact computations are feasible, test plans have been designed using both the exact computations and the binomial approximation. They are very close even when there are only 25 components, provided that the shape parameter is not too small or the hypothesized parameter values are not too close together.

## II. ESTIMATION WITH TRUNCATED DATA

1. Estimation of Sample Size with Grouped Data, under revision for J. Statist.Planning and Inference (with R. C. Dahiya).
2. Population Size Estimation with Truncated Data:A Survey, to appear Bayesian Analysis in Statistics and Econometrics. (Prem K. Goel and N.S. Iyengar, eds). 1992. pp.361-367. Lecture Notes in Statistics, #75, New York:Springer-Verlag.

We are given  $N$  random variables  $X_1, \dots, X_N$  with common density  $f(x; \theta)$  where both  $N$  and  $\theta$  are unknown. The values of the  $X$ 's can be observed only if they lie in the interval  $(0, T)$ . In addition, in [1], we assume that the interval  $(0, T)$  is subdivided and only the number of  $X$ 's falling in each subinterval is reported, with all other details about actual values being lost. The problem addressed is estimation of  $N$  and  $\theta$ . The motivation is provided by some papers in the literature in which the number of bugs in a computer program was being estimated and data were obtained in this form.

We show that the usual practice of spreading the observations uniformly over the interval leads to stochastically inconsistent estimates. Restrictions on the estimation procedures are derived to assure the stochastic consistency of the estimator of  $N$ , and alternative methods of ungrouping are developed which can be used to compute optimal estimators that were proposed by Blumenthal in previous papers. Further, iterative computing schemes are examined, and Monte Carlo comparison of several estimators is carried out. Some detailed examples that illustrate the extent of the possible error in using the older schemes are now being computed for the revision.

In [2], the truncated sampling literature related to estimation of  $N$  is surveyed, with an emphasis on papers that apply these results to estimating the number of errors in computer programs. Some new avenues for future research are indicated in the paper.

### III. BURN-IN.

1. Burn-in with Mixed Populations. Ohio State University, Statistics Department Technical Report No. 372, August 1987 (with U.Q. Pan).
2. Burn-in using a sequential screening procedure. Technical Report 1987.14, Dept of Mathematics and Statistics, Wright State Univ., December 1987 ( with U.Q. Pan).
3. A Reliability Model for Burn-in. Technical Report 1987.12, Dept of Mathematics and Statistics, Wright State Univ., November 1987 ( with U.Q. Pan).

Closely related to the problem described above is the burn-in problem. In its simplest form, a batch of  $M$  items contains  $N$  (unknown) defectives with known failure distribution. A sequential stopping rule is desired for removing enough of the defectives to assure that with a specified high probability the reliability of the remaining items in the lot after burn-in exceeds some given lower bound. Various stopping rules have been studied for both exact and approximate (large lot size) properties. The technical report [1] is based on the dissertation of Dr. Pan. In [2], a procedure for burn-in based on a sequential estimation scheme published previously by Blumenthal is extracted from [1], additional computations of its small lot size properties are made, and some additional analytic properties are developed. In [3], the characterization of the fixed burn-in time procedure that achieves the after burn-in reliability goal is extracted from [1]. This fixed time depends on knowing more than is generally available, but serves as a benchmark to which to compare the sequential schemes to see how much extra time they take because the extra information is not available.



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