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in Plasmas			AFOSR-87-0010
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# TECHNICAL REPORT AFOSR GRANT 87-0010

### STATISTICAL MECHANICS OF COLLECTIVE PHENOMENA IN PLASMAS

### NOVEMBER 1, 1990 - OCTOBER 31, 1991

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by Joel L. Lebowitz Professor of Mathematics and Physics Hill Center Rutgers University New Brunswick, NJ 08903 Principal Investigator





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#### ABSTRACT

We have carried out theoretical studies of equilibrium and non-equilibrium properties of strongly coupled particle systems, and of the phenomena which take place when neutral or partially ionized atoms interact strongly with external fields and with each other in a plasma. The main tools of our study are statistical mechanics and kinetic theory, including the transition from a microscopic to a hydrodynamic description. Quantum mechanics plays a central role in many of these problems, and is an important ingredient in our work.

## 1. ELECTRON VELOCITY DISTRIBUTION IN A WEAKLY IONIZED PLASMA WITH AN EXTERNAL ELECTRIC FIELD

We consider a weakly ionized gas in the presence of an externally imposed spatially homogeneous electric field E which may be either constant or vary in time. The density of the gas, the degree of ionization and the strength of the field are assumed to be such that: (i) the interactions between the electrons and of the electrons with the ions and neutrals can be described by Boltzmann type elastic collision integrals and, (ii) collisions between the electrons and the heavy components of the plasma, ions and neutrals, are adequately described by assuming the latter to be in a Maxwellian distribution with a priori given temperatures. Under these assumptions the time evolution of the spatially homogeneous electron velocity distribution function  $f(\underline{v}, t)$  will satify the Boltzmann equation

$$\frac{\partial f}{\partial t} - (e/m) \mathbf{E} \cdot \nabla f = (\mathbf{L}_{i} + \mathbf{L}_{n}) f + Q(f, f).$$
(1.1)

The right side of (1.1) consists of a linear part corresponding to elastic collisions

between the electrons and the heavy components (ions and neutrals) and a quadratic Landau type collision integrals for e-e interactions.

Eq. (1.1) differs from the frequently studied linear kinetic equation for an essentially zero density "electron swarm" by the omission of inelastic e-n collisions and the inclusion of elastic e-i and e-e collisions. It also differs from the strongly coupled plasma case in that we neglect collective plasma interactions.

We find that (in the approximation scheme used) this equation has for a stationary f (in a constant field), non-unique solutions for a certain range of E, in agreement with that found by us in earlier investigations. We interpret these to correspond to hysteresis effects when the field is changing very slowly indicating a very sharp changeover, possibly a discontinuous transition, in the stationary state of the electrons at a certain critical E. Possible applications of this effect are being investigated.

# 2. NEGATIVE TEMPERATURE STATES AND EQUIVALENCE OF ENSEMBLES FOR THE VORTEX MODEL OF TWO-DIMENSIONAL TURBULENCE

We studied Onsager's theory of large, coherent vortices in turbulent flows in the approximation of the point-vortex model for two-dimensional Euler hydrodynamics. In the limit of a large number of point vortices with the energy per pair of vortices held fixed, we prove the entropy defined from the microcanonical distribution as a function of the (pair-specific) energy has its maximum at a finite value and thereafter decreases, yielding the negative-temperature states predicted by Onsager. We furthermore show that the equilibrium vorticity distribution maximizes an appropriate entropy functional subject

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to the constraint of fixed energy, and, under regularity assumptions, obeys the Joyce-Montgomery mean-field equation. We also prove that, under appropriate conditions, the vorticity distribution is the same as that for the canonical distribution, a form of equivalence of ensembles. We also establish a large fluctuation theory for the microcanonical distributions, which is based on a level-3 large deviations theory for exchangeable distributions, We discuss some implications of that property for the ergodicity requirements to justify Onsager's theory, and also the theoretical foundations of a recent extension to continuous vorticity fields by R. Robert and J. Miller.

Although the theory of two-dimensional vortices is of primary interest, our proofs actually apply to a very general class of mean-field models with long-range interactions in arbitrary dimensions.

#### 3. QUANTUM PARTICLE IN AN ELECTRIC FIELD

The Einstein-Kubo relation between the diffusion constant D and the linear mobility  $\nu$  was investigated for a dissipative quantum particle in a potential  $V(x) = -Ex + V_0 \cos(k_0 x)$ . This system is directly related to a current biased Josephson junction. It is shown that  $D = kT\nu$  holds to all order in  $V_0$ provided the dissipation spectrum  $J(\omega)$  is linear near  $\omega \to 0$  and is not pathological. A generalized version of the Einstein relation is shown to hold also for sub-Ohmic dissipation,  $J(\omega) \sim \omega^{S}$ , 0 < s < 1, as  $\omega \to 0$  where the motion is sub-diffusive. The super-Ohmic case 1 < s < 2 was also studied.

We further obtained an analytic expression for the nonlinear mobility of a quantum particle moving down a cosine potential in the presence of a finite bias force in the small viscosity limit. This is used to explain some experimental

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observations on Josephson junctions in which the Coulomb blockade is important. The result is based on a resummation of a series expansion of the mobility in  $V_0$  (the strength of the cosine potential) and is applicable to a variety of regimes in the parameter space.

### 4. FLOQUET SPECTRUM FOR TWO-LEVEL SYSTEMS IN QUASIPERIODIC TIME DEPENDENT FIELDS

We studied the time evolution of N-level quantum systems under quasiperiodic time dependent perturbations. The problem is formulated in terms of the spectral properties of a quasienergy operator defined in an enlarged Hilbert space, or equivalently of a generalized Floquet operator. We discuss criteria for the appearance of pure point as well as continuous spectrum, corresponding respectively to stable quasiperiodic dynamics and to unstable chaotic behavior. We discuss two types of mechanisms that lead to instability. The first one is due to near resonances, while the second one is of topological nature and can be present for arbitrary ratios between the frequencies of the perturbation. We treat explicitly an example of this type. The stability of the pure point spectrum under small perturbations is proven using KAM techniques.

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Quantum Particle in a Washboard Potential – Part I: Linear Mobility and the Einstein Relation, with Y.C. Chen, preprint

Quantum Particle in a Washboard Potential – Part II: Nonlinear Mobility and the Josephson Junction, with Y.C. Chen, preprint

Atomic versus Ionized States in Many Particles Systems and the Spectra of Reduced Density Matrices: A Model Study, to appear in the <u>Journal of Statistical Physics</u>, (with N. Macris and P.A. Martin)

Spectral Properties of Quasiperiodically Driven Quantum Systems, to appear in <u>Proceedings of the Locarno Conference</u>, (with H. Jauslin)