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6. AUTHOR(S)
Dr. R. A. Nicolaides

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Carnegie-Mellon University
Department of Mathematics
Pittsburgh, PA 15213

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The overall goal of this research was the development and application of covolume methodology in CFD and related areas. The main framework of the covolume approach is now in place and its major characteristics are reasonably well understood. The research shows the algorithm to be a stable and accurate approach to computing viscous fluids on unstructured meshes. The covolume approach has several unique features, including an associated discrete vector field theory, which in turn permits covolume discretizations to exhibit important physical characteristics, for example being free of artificial vorticity creation. There is still need for work in compressible and three dimensional flows.

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Principle Investigator: R. A. Nicolaides, Carnegie Mellon University

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SUMMARY

The overall goal of the research was the development and application of covolume methodology in CFD and related areas. The main framework of the covolume approach is now in place and its major characteristics are reasonably well understood. The research shows the algorithm to be a stable and accurate approach to computing viscous fluids on unstructured meshes. The covolume approach has several unique features, including an associated discrete vector field theory, which in turn permits covolume discretizations to exhibit important physical characteristics, for example being free of artificial vorticity creation. There is still need for work in compressible and three dimensional flows.

REPORT

The major topic of the grant was to complete the development of a new methodology (the covolume approach) for the numerical solution of fluid dynamics problems on unstructured meshes. The same approach can be used to solve other problems and there is particular relevance to electromagnetics governed by Maxwell's equations. The major scientific achievements are as follows:

1. The covolume methodology was introduced for planar div-curl systems and rigorously analyzed for isotropic problems [1] and anisotropic problems [2]. The anisotropic case is particularly relevant in electromagnetics.



The isotropic problem is relevant to both incompressible fluid mechanics (for example in vorticity formulations) and electromagnetics.

2. The basic covolume algorithm for incompressible viscous flows governed by the Navier–Stokes equations was introduced, initially for the Stokes equations [3] and later for the nonlinear case, [4], [5].

3. It turned out that when specialized to uniform rectangular meshes in two dimensions, the covolume scheme reduces to the famous MAC discretization. This rather unexpected result has several consequences. First, it lends immediate credibility to the entire covolume approach, since the MAC technique is well established and is supported by an enormous body of empirical evidence about its convergence and accuracy. Second, it provides a whole new way of looking at the MAC scheme, a way which is based purely on physics and not on artifacts of coordinate systems, as in the usual approach. This new way of looking at the MAC scheme enabled us to provide the first rigorous analysis of its convergence. This work is reported in [6] for the linear case and in [7] for the full Navier–Stokes equations. These proofs settle long standing conjectures about the MAC scheme and are a major result from our research.

4. Another outcome of the covolume methodology for fluid mechanics is that it establishes an unusual equivalence between primitive variable and vorticity–velocity formulations at the discrete level. Hitherto, discretizing these formulations has been regarded as independent activities. This is clearly undesirable, bearing in mind that they are equivalent at the continuous level. In the covolume framework, parallel transformations can be made at the discrete

level which transform between the discrete systems. The covolume approach appears to be unique in having this property. It is elaborated in a survey article [8], scheduled to appear in mid 1992.

5. Underlying the existence of the discrete transformations mentioned in the previous paragraph is a discrete vector field theory which is reported in [9]. This theory contains natural analogs of results such as $\text{curl grad } \mathbf{u} = 0$ and $\text{div curl } \mathbf{u} = 0$. These results play an important part in obtaining good computational results to physical problems.

6. Towards the end of the grant period, some work was done on several additional topics. This includes

- (i) Extensions to compressible flows
- (ii) Further work on three dimensional extensions
- (iii) Construction of Voronoi–Delaunay mesh systems.

Reference [10] contains some work on (i) – (ii) which was partly carried out during the grant period. Topic (iii) refers to a new algorithm, which is optimally efficient and particularly suited to the requirements of partial differential equations. For example, it is easy to add or remove points from the mesh system and it is relatively easy to incorporate non convex boundaries. This work has not yet been published, and is continuing.

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STUDENTS SUPPORTED

William D. Rieder (Ph.D Carnegie Mellon, expected 1993)

M. Erik Reid (Dept. of Mathematics, Carnegie Mellon)

Xianon Wu (Ph.D Carnegie Mellon, 1991)