

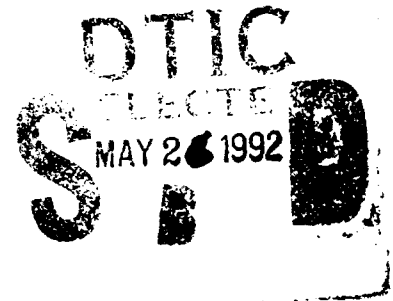
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EXPONENTIAL REPRESENTATION AND
CONSISTENCY CHECKING FOR M-LAYER

by

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ABSTRACT

M-Layer, the tropospheric propagation effect prediction program by NRaD (formerly NOSC), is revised for greater accuracy, speed and stability. This is achieved through converting the extended complex number representation into the representation by the complex exponent, improving the accuracy in Airy function computation, introducing a new mode locating algorithm and implementing a consistency checking procedure for determining the proper method to evaluate the height gain function. The revision has been documented and the new program source code has been delivered to NRaD. It is recommended that the mode search protocol, not just the mode locating algorithm introduced in this revision, be completely revised. Unlike the current approach of blanketing the whole possible region until exhaustion, modes should be searched according to their range attenuation rates one by one along a well defined path. This should result in a faster and even more stable program. The program size can also be reduced.

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I. INTRODUCTION

M-Layer is a FORTRAN program for computing the propagation factor of an electromagnetic (EM) wave in a stratified atmosphere. It is desirable to extend the capability of this program to include a layer of random medium representing the air-ocean interface where the thickness of this layer cannot be ignored, where the EM propagation and scattering are so strongly coupled that clutter and propagation effects within this layer cannot be dealt with separately, and where the grazing angle of the EM wave incident into this layer is so small that the curvature of the earth cannot be neglected. To achieve this goal, there are many basic theoretical problems which have to be answered. First of all, the effect of the earth curvature in this program is taken care of through the classical earth-flattening approximation [Ref. 1], but the result [Ref. 2] does not agree with the more recent diffraction theory of Fock [Ref. 3] near the surface of the earth. Then there is the question about the better method to model the atmospheric refractive index profile, either piecewise linear or quadratic, to be resolved by a new earth-flattening approximation under development at NPS. The new approximation will also determine the functions to be used for the representation of the EM fields in each layer through uniform asymptotic theories. Within some proper region, these new functions are expected to reduce to the Airy functions utilized by M-Layer. The evolutionary nature of this effort prompted this review to improve the inner workings of the M-Layer program.

In particular, the subroutines to search for the modes and those for evaluating the Airy functions will remain as an important part of a program investigating questions about EM wave propagation by solving the related boundary value problem.

It can never be overemphasized that a boundary value problem which includes a layer of random medium or some range dependent inhomogeneity, set up according to the Maxwell equations, will include backscattering in its solution. This is in sharp contrast to those numerical procedures based on the parabolic approximation to the wave equation for which the backscattering is completely ignored.

In what follows, the M-Layer program and the reasons for replacing the extended complex numbers with their complex exponent representations are discussed, together with some other problems encountered and resolved during this investigation.

A. M-LAYER

In M-Layer, the index of refraction of the atmosphere is assumed to be height dependent and is approximated with a continuous piecewise linear profile. The classical earth-flattening approximation is utilized to allow the use of the cylindrical coordinate system while retaining the effect of the curvature of the earth. This is done simply by substituting the index of refraction with the modified index of refraction, which also has a piecewise linear profile [Ref. 1].

The source of the EM radiation is assumed to be either a vertical electric dipole or a vertical "magnetic dipole", with the latter providing an approximation to

the radiation of a horizontal electric dipole. The dipole is located along the positive z-axis of the cylindrical coordinate system while the origin is sitting on the ground. The x-y plane is the "flattened" earth surface. After carrying out the Hankel transform along the radial direction, the resulting spectrum of the Hertzian dipole field within each layer of a linear segment of the modified refractive index profile is reduced to a linear combination of the Airy functions. Specifically, the layers are numbered to increase with height, with the first layer being the one right above the ground. The spectrum of the Hertzian dipole field is proportional to the product of the values, at the transmitter height and at the receiver height respectively, of the height-gain function. At a height within the i-th layer, the height-gain function is given by [Ref. 4]:

$$f_i(\rho, z) = B_i(\rho) [A_i(\rho) k_1(q_i) + k_2(q_i)] , \quad (1)$$

where ρ is the radial component of the propagation vector and is also the spectral variable of the Hankel transform; hence it is the same throughout all layers. It is a complex variable whose imaginary part represents the radial attenuation rate of the spectral component of the Hertzian dipole field. Under the classical earth-flattening approximation, the spectrum of the Hertzian dipole field contains a discrete portion and a branch cut. The discrete spectrum gives rise to the creeping wave modes diffracted by the earth surface and the dielectric waveguide modes supported by the layered atmosphere. The contribution from the branch cut is usually negligible, especially for the field in the shadow of the earth. The M-Layer program locates the

discrete spectrum for modes having a radial attenuation rate below a predetermined value. Contributions from these modes determine the propagation factor of the wave.

The variable q_i in the i -th layer is a dimensionless linear function of height z with the free space wavenumber k , the modified index of refraction m_i at the lower boundary $z = z_i$, the slope of the modified index of refraction $\alpha_i/2$ and ρ as parameters:

$$q_i = \sqrt[3]{\left(\frac{k}{\alpha_i}\right)^2 \left(m_i^2 + \alpha_i(z - z_i) - \frac{\rho^2}{k^2}\right)}. \quad (2)$$

The height dependence of the field is given in terms of the functions $k_1(q_i)$ and $k_2(q_i)$, which are proportional to the Airy functions $Ai(-q_i e^{j2\pi/3})$ and $Ai(-q_i)$ respectively. Of these two functions, at a height so large that q_i is large and positive, $k_1(q_i)$ represents a downward going wave and $e^{j4\pi/3}k_1(q_i) + k_2(q_i)$ represents an upward going wave. The coefficients A_i and B_i are determined by the conditions on the continuity of the Hertzian dipole field and its derivative across layer boundaries and by the normalization condition that the integral of the square of the height-gain function over all height equals unity.

To fulfill the radiation condition, the highest layer is given the same refractive index as the free space above it and only the outgoing wave is allowed within this layer. Below the "flattened" earth surface, the field is assumed to be a plane wave propagating downward. Hence only the normalization factors are required in the highest layer and in the ground. By assigning B_i to unity in the highest layer, all the

coefficients A_i and B_i can be determined, according to the boundary conditions, to within a multiplicative factor for B_i . This multiplicative factor is then deduced from the normalization condition. This procedure can also be carried out from the ground level up. That these coefficients can be computed either from the highest level down or from the lowest level up is a result of the fact that ρ belongs to the discrete spectrum of the Hertzian dipole field. Consequently, agreement between these two ways of evaluating the A_i and B_i coefficients confirms that a mode has been located accurately.

B. EXTENDED COMPLEX NUMBER REPRESENTATION

The discrete spectrum of the Hertzian dipole field corresponds to the zeroes of the modal function which is a determinant whose elements consist of $k_1(q_i)$ and $k_2(q_i)$ at the layer boundaries. Numerically, the magnitude of this modal function causes overflow and underflow problems as $k_1(q_i)$ or $k_2(q_i)$ becomes exponentially large or small for complex q_i values. In the M-Layer program, to overcome this problem, a complex number is written as a scaled number, which is complex, multiplied by a scaling factor which is an integer power of e , the base of natural logarithm. This integer is chosen so that the greater of the absolute values of the real part and the imaginary part of the scaled number lies within $e^{\pm 1}$. A complex number written in this form is called an extended complex number. Multiplication of two extended complex numbers requires summing the two integer exponents in addition to carrying out the regular complex multiplication of the scaled numbers. Addition

of two such numbers is achieved through the use of an addition subroutine: the larger scaling factor is factored out of both addends before they are combined. The scaling factor is adjusted after each addition and after a sequence of multiplications to make sure that the resulting scaled number is still within the desired range. Addition is troublesome when the two numbers to be added nearly cancel each other. Under this circumstance, the scaling factors of the two numbers are identical and both the real parts and the imaginary parts of the scaled numbers are almost equal with opposite signs. It is clear that the real part and the imaginary part of the sum lose their accuracies to different degrees; hence the phase angle may incur substantial error. To remedy this situation, interpolation procedures have to be devised.

As two complex numbers come close to cancel each other, they must be out of phase by almost 180 degrees. By factoring out the square root of their product instead of the scale factor, the resulting addends become reciprocal to each other, both lying within an identical small angle to, and on the same side of, the imaginary axis. They are close to the unit circle, but one is on the inside and the other is on the outside. Taking out further a phase factor of $\pi/2$ after writing the addends in their exponential forms, the exponents become small numbers for which Taylor series expansion of the exponential function converges rapidly and can be used for interpolating the sum to achieve higher accuracy. Note that after the extra phase factor of $\pi/2$ is removed from the addends, it is actually the difference of the resulting two reciprocals which is computed. This procedure effectively picks the direction on the complex plane along which the addends are almost opposing each

other to carry out their cancellation. The resulting sum has a phase angle nearly perpendicular to this chosen direction.

It is evident that the representation of a complex number by its complex exponent of base e provides better phase accuracy for addition. A one-to-one correspondence can be achieved by restricting the imaginary part of this exponent to within $-\pi$ and π . This will be called the exponential representation or the complex exponent representation henceforth. It is convenient for multiplication: adding the complex exponents of the two factors will suffice. Conversion of the M-Layer program from the extended complex number to the complex exponent representation has been carried out.

C. CONSISTENCY CHECKING

As better precision is achieved, problems with the mode search procedure and the evaluation of the A_i and B_i coefficients become severe. They are thoroughly investigated and resolved. For mode search, although the division of the region of interest into "contour rectangles" and further into square "meshes" and the search pattern to move around the sides of a "contour rectangle" to find and follow "phase lines" into it are kept, the basic assumption of Shellman and Morfitt [Ref. 5] that both the real and the imaginary parts of the modal function are linear along every edge of a mesh square is completely abandoned. For the evaluation of the A_i and B_i coefficients, the "test for evanescence" conditions have been removed. A consistency condition to determine whether to evaluate the coefficients from the ground level up

or from the top level down has been fomulated and incorporated into the program. This accomplishment leads to the relaxation of mode locating accuracy requirement which, combined with the improved precision of the revised program, makes the first order Newton-Raphson iteration unnecessary. The specific changes in the program and the resulting gains in speed, accuracy and execution stability are discussed in the following chapters. Recommendation to completely revise the mode search protocol to do without the "contour rectangles" is also provided.

II. PROGRAM REVISIONS

M-Layer is structured into three parts: setup, mode search and propagation factor evaluation. The main input is the modified refractive index values at specified heights so that a piecewise linear profile can be constructed. If the mode locations for the particular profile are available from a previous run of the program, they can also be included in the input and the mode search procedures will be bypassed. The various ranges and transmitter and receiver heights for which propagation factors are desired are also specified. The subroutine WVGSTDIN is called to input the information from an ASCII data file. The program then computes the constants to be used for mode search and propagation factor evaluation. The mode search is performed with the subroutine FNDMOD. The MODSUM subroutine is then invoked to first compute the A_i and B_i coefficients as explained in the Introduction, then compute the propagation factor and the propagation loss. The complete program structure is given in Figures 1 and 2. There are several other subroutines which are not included in these and other figures, such as DHORIZ for computing the horizon distance between a transmitter and a receiver for reference purpose; CHKMOD, a maintenance routine for removing zero from reported mode locations by older versions of the program; or AO2H2O, a routine to compute the atmospheric absorption coefficient due to oxygen and water vapor. They will not be discussed as

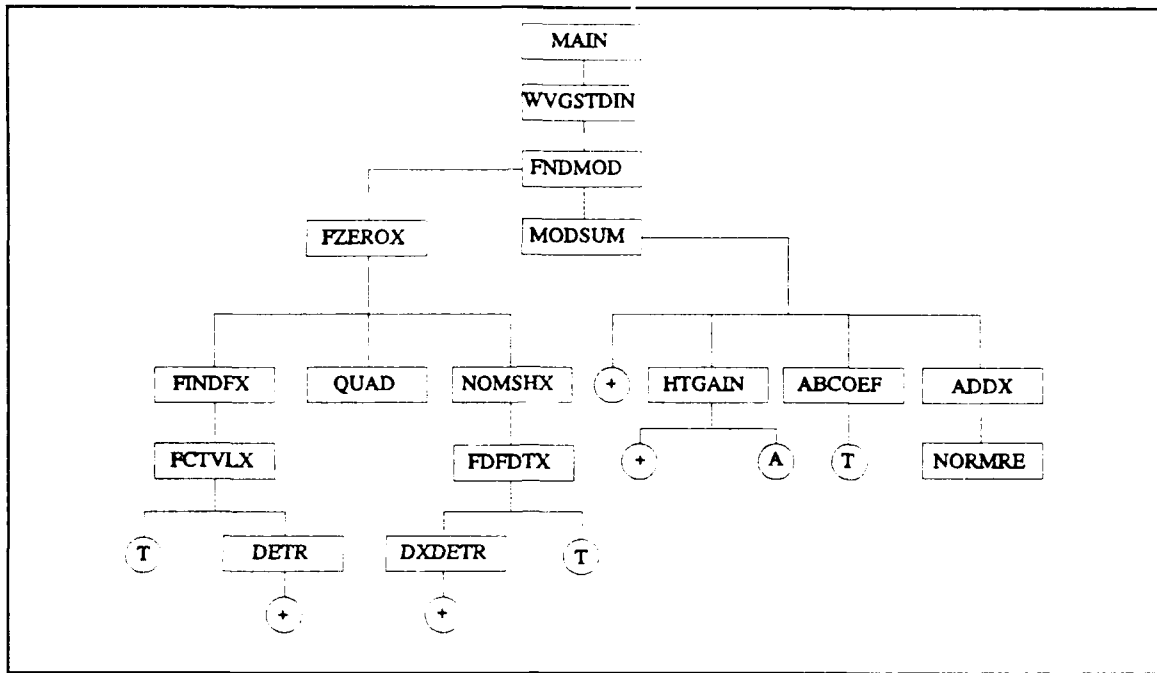


Figure 1 Original M-layer subroutines structure

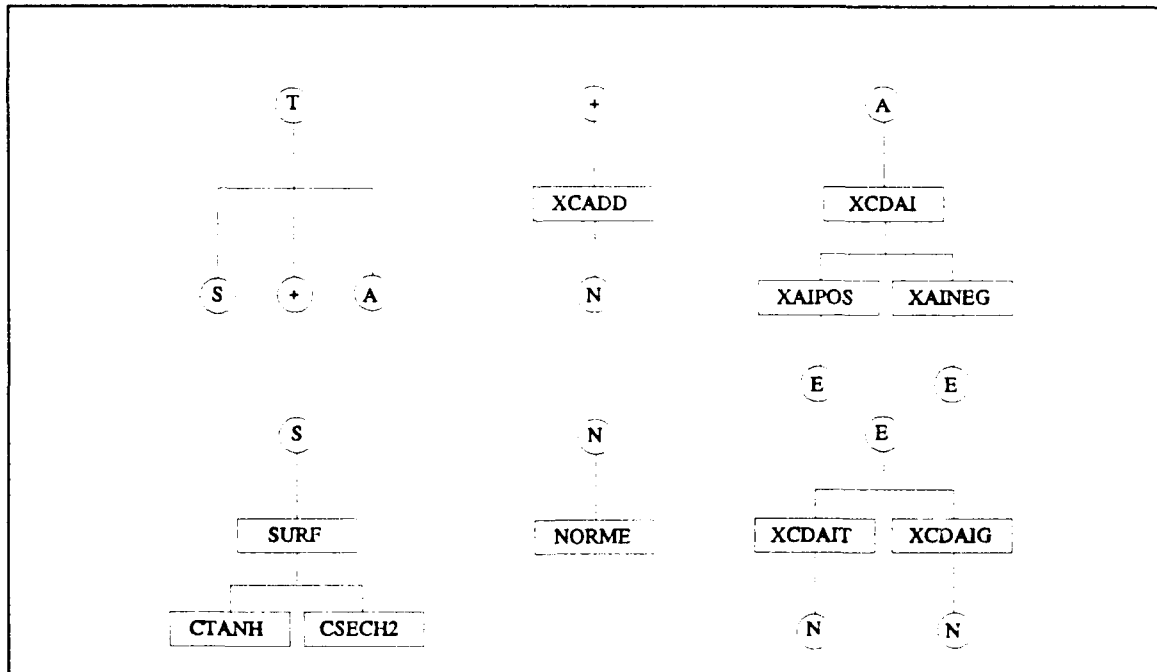


Figure 2 Original M-layer subroutines structure (continued)

they do not contribute directly to the main purpose of this program of locating the modes and computing the propagation factor.

The program structure has been altered as shown in Figures 3 and 4. Since the A_i and B_i coefficients have to be evaluated only once, they are now obtained through a call to the subroutine AECOEF directly from the main program right after the modes are located. Several subroutines are dropped in this revision for various reasons: The subroutines NORME and NORMRE are eliminated because they are no longer needed due to the change in complex number representation; The subroutines NOMSHX, FDFDTX and DXDETR are not used because the modes are now located with adequate precision without further iteration; The subroutine ADDX is not listed separately because it is called only once and has been reduced to only a few lines which are placed where the subroutine is called in the original program. On the other hand, changes in the mode search algorithm require the addition of two new subroutines: SURF0 is a modified and simpler version of SURF; ROOTS replaces QUAD. Due to the change in complex number representation, all subroutines listed below FNDMOD and MODSUM have been revised, including their input/output lists. But except for SURF0 and ROOTS, the utilities of these subroutines are the same as those of the original ones. Descriptions of these subroutines can be found in the report by Yeoh [Ref. 4].

The most significant changes have been made in XCADD, XCDAIT and XCDAIG for adopting the complex exponent representation and improving computation speed and accuracy; in FZEROX, FINDFX, ROOTS and SURF0 for

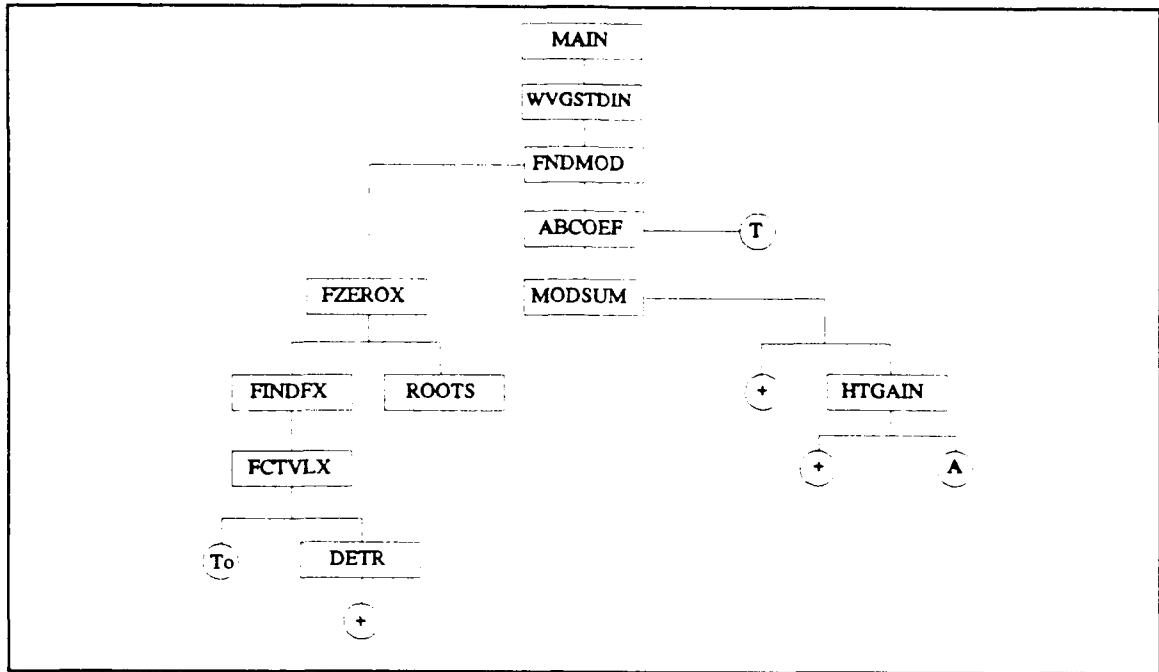


Figure 3 New M-layer subroutines structure

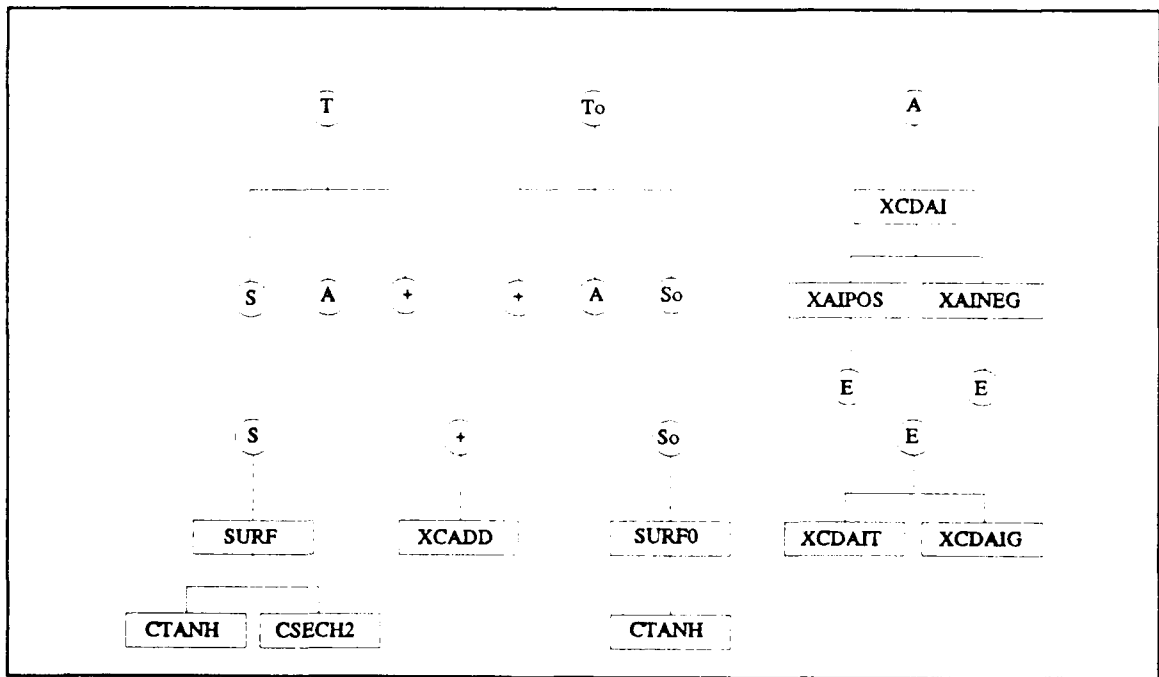


Figure 4 New M-layer subroutines structure (continued)

stabilizing and simplifying the mode search algorithm; and in ABCOEF for implementing the criteria to determine the reliable manner for evaluating the A_i and B_i coefficients. These changes are discussed in the sections below. The source code listings of the completely new subroutines XCADD and ROOTS and the significantly revised subroutines FZEROX and ABCOEF, which are compiled with Microsoft FORTRAN version 5.00, are attached as Appendices A through D. Validation of the revised program has been carried out at 9.6 GHz for all the 21 profiles listed in Yeoh [Ref. 4].

A. ADDITION SUBROUTINE

XCADD is the subroutine implementing the addition of complex numbers under the representation by their exponents. Given the double precision complex numbers z_1 and z_2 as the exponents of the addends, this subroutine returns the exponent of the sum. Since a double precision number has an accuracy of 53 bits, if the real parts of z_1 and z_2 differ by more than 53 bits, the exponent of their sum will simply be the one of the greater real part. When cancellation becomes serious, the square root of the addends is factored out first. Then the four-term Taylor series expansions of the resulting reciprocals are summed up. Since the leading term of the sum of the Taylor series is a good estimate of the sum of the reciprocals and the relative error of the four-term Taylor series sum is proportional to the fourth order of this leading term, the threshold for invoking this interpolation procedure is set at the highest possible value of 2^{-14} allowed under double precision. Experimenting

with this procedure shows that this interpolation improves accuracy as long as the threshold is set at a number between 2^{-24} and 2^{-14} .

B. AIRY FUNCTION EVALUATION

Similar to the original program, the evaluation of the Airy function adopted the algorithm prescribed by Schulten, et. al. [Ref. 6]. In the new program, changes are made to follow the advice of Schulten, et. al. concerning the region within which Taylor series expansion, instead of the faster Gaussian quadrature, has to be used to achieve double precision accuracy. Other changes in implementing the algorithm are described below.

1. XCDAIT

Due to the similarity in their Taylor series coefficients, the Airy function and its derivative are evaluated within a single loop. The relative accuracy of the derivative of the Airy function is set at the double precision limit of 2^{-54} .

2. XCDAIG

Six term Gaussian quadrature is used for evaluating the Airy function and its derivative outside the circle of radius 4.97 centered at (0.90, 2.80) on the complex plane. The use of four-term quadrature outside a radius of 15 from the origin suggested by Schulten, et. al. is not adopted. The six-term quadrature in this range retains a higher accuracy while overall speed improvement by using both the four-term and the six-term quadrature appears to be minimal.

C. MODE LOCATING

As explained in the Introduction, the modes are located at the zeroes of the modal function. These zeroes are located on the upper complex q_{11} plane. Here q_{11} is the value of q_1 on the earth surface, which, according to Eq.(2) of Chapter 1, is a linear function of ρ^2 . For a horizontally propagating mode, ρ/k is close to unity. The maximum range attenuation rate specified for the desired modes, which corresponds to a limit on the imaginary part of ρ , determines approximately the upper bound for the imaginary part of the q_{11} complex plane to be searched for modes. The Shellman and Morffit mode search procedure first divides the search region horizontally into "contour rectangles" each of which spans 160 meshes along the real q_{11} direction. A mesh is a square whose size is an adjustable parameter of the order 10^{-4} at 9.6 GHz for most of the cases considered herein. This parameter is determined by the frequency and the slope of the modified index of reflection in the lowest layer of the profile. The search commences at the top left corner of the "contour rectangle" whose left edge has a real coordinate value close to the difference of the real parts of the q_{11} values with the minimum modified index of refraction and the index near the surface substituted into Eq.(2) of Chapter 1. After the search over the initial rectangle is completed, the program moves to search the next rectangle until a specified maximum number of modes are found or a specified number of "contour rectangles" have been searched.

The search for zeroes makes use of the fact that a real function changes sign when it crosses a simple zero. Since a zero of a complex valued function $F(q)$ is

where both its real part and imaginary part vanish, a necessary condition for a point q_m to be a zero is that it is on the intersection of two curves defined by $\text{Im}\{F(q)\}=0$ and $\text{Re}\{F(q)\}=0$. The program searches around a "contour rectangle" for a sign change in $\text{Im}\{F(q)\}$ across an edge of a mesh bordering the side of the "contour rectangle" to determine that a line of $\text{Im}\{F(q)\}=0$ has been encountered. The search then follows this line into the meshes within the "contour rectangle", checking each mesh to see if a curve $\text{Re}\{F(q)\}=0$ enters the mesh under investigation. All these steps make use only of the assumption that the zeroes of the modal function are simple. Once both the curve $\text{Im}\{F(q)\}=0$ and the curve $\text{Re}\{F(q)\}=0$ are determined to be present within a mesh, the location of their possible interception is estimated. An algorithm for this estimate is required.

Shellman and Morffit [Ref. 5] introduced a further assumption that the functions $\text{Re}\{F(q)\}$ and $\text{Im}\{F(q)\}$ are both linear along the edges of a mesh. Based on this assumption, they try to estimate the locations where the curve $\text{Im}\{F(q)\}=0$ enters and leaves a mesh square and the location of q_m if a curve $\text{Re}\{F(q)\}=0$ also enters the same mesh. It is obvious that information about the locations where the curves enter and leave the mesh square is not essential. Furthermore, in the 18 m duct height case, the scheme causes the search path to loop around four contiguous meshes until the search is broken up by the limit on the number of meshes to be investigated. Replacing their technique requires major changes in the subroutines involved. A new subroutine ROOTS is provided to estimate the location of the

intersection of the curves $\text{Im}\{F(q)\}=0$ and $\text{Re}\{F(q)\}=0$. These changes eliminate the looping problem.

Another problem is encountered in the 40 m duct height case when a large number of zeroes are found in the lower half complex q_{11} plane. These zeroes appear to belong to the reflection coefficient on the wrong sheet of the branch cut and are not waveguide modes. This happens because the search region has been extended below the real q_{11} axis to avoid the singularity in SURF. The problem with this singularity should have been solved within SURF, especially because it occurs only when the derivative of the subroutine output variable **gamma** with respect to q_{11} is computed. Since this derivative is not needed during mode search, the extension of the search region to the negative q_{11} plane is unnecessary. A simplified routine, SURFO, is introduced which is exactly the same as SURF except that it does not evaluate the derivative of **gamma**. By using this subroutine instead of SURF, the search path in the revised program does not avoid the real and the imaginary axes.

1. FNDMOD

The search region is limited to the upper half q_{11} plane. All the modes found are ordered according to their range attenuation rates before those numbered beyond the maximum modes allowed are abandoned.

2. FZEROX

Since the curve $\text{Im}\{F(q)\}=0$ enters into a mesh square through an edge, the values of $\text{Im}\{F(q)\}$ must change sign over the end points of either one or all

three other edges. When there is only one other edge across which $\text{Im}\{F(q)\}$ changes sign at its end points, it is the edge across which the curve $\text{Im}\{F(q)\}=0$ exits the mesh square. Ambiguity arises when all edges indicate a change of sign at their end points. When this occurs, a "right turn rule" is adopted which assumes that the curve exits the edge to the right of the one along which it enters the mesh square. Such a rule avoids the retracing of the search path when the mesh square is revisited as entering this same mesh square from the left side of an edge after exiting from its right side requires a crossing of the $\text{Im}\{F(q)\}=0$ curve, which is prohibited under the simple zero assumption. On the other hand, the actual curve may have turned left and then returns to this mesh square, i.e., following a "left turn rule." Under such a scenario, this wrong choice would have left a segment of the curve not searched. This difficulty has not been observed during testing. In fact the ambiguous situation seldom occurs. Note also that, as remarked above, two lines of $\text{Im}\{F(q)\}=0$ do not cross each other unless a higher order zero is present. Hence only a right turn rule or a left turn rule for the curve to exit the mesh is allowed. Exiting the opposite edge demands a pair of crossing $\text{Im}\{F(q)\}=0$ curves within the mesh square. This violates the assumption that all zeroes are simple. Also note that, the possibility of vanishing $\text{Re}\{F(q)\}$ or $\text{Im}\{F(q)\}$ values at the corners of a mesh square is eliminated through a small adjustment in FINDFX.

3. FINDFX

Both the vertical shift away from the real q_{11} axis and the horizontal offset away from the imaginary axis are unnecessary and have been removed from

this routine. Furthermore, as a result of converting to the complex exponent representation, the sine and cosine of the argument of the modal function are examined for sign changes in FZEROX. This is implemented in FINDFX by including the cosine and sine values of the argument of the modal function in the output list. To avoid the indeterminate case when either the real or the imaginary part of the modal function becomes zero at any corner of a mesh square, the argument for computing the cosine and sine values is increased by 2^{-53} when this occurs. This is equivalent to a consistent small distortion of the particular corner of the mesh square. This will not cause any error in locating the zero because FINDFX still returns separately the unmodified exponent of the value of the modal function.

4. ROOTS

Assuming that the modal function is analytic within the mesh, this subroutine utilizes the values of the modal function at the four corners of the mesh square to determine the Taylor series expansion coefficients of the modal function to the third order. The roots of this cubic polynomial are then located using Cardan's solution by radicals. If the higher order coefficients fall below machine resolution for a root within the mesh square, these coefficients are regarded as zero and the order of the polynomial is reduced and can be solved more expediently. If the function is determined to be constant over the mesh square, the center of the square is taken as the root location.

D. EVALUATING A_i AND B_i

As discussed in the Introduction, the A_i and B_i coefficients can be evaluated either from the top level down or from the lowest level up. These two procedures are simply called "integration down" and "integration up" respectively in the original documentation [Ref. 4]. The location of a mode has been called an eigenvalue. That the results of integration down and integration up agree is a manifestation that the eigenvalue is located accurately.

The subroutine ABCOEF evaluates the coefficients A_i and B_i for each mode. If the range attenuation rate for a mode is greater than 0.1 dB/km, the coefficients are evaluated from the lowest layer up. Otherwise, it is evaluated from the top layer down. It is obvious that such a rule must be implemented because the results of integration up and integration down do not agree for many modes. Efforts are made to determine the cause of this discrepancy and to devise a means to resolve it.

Investigation reveals that inadequate precision in the location of the modes is one source of the problem. Since the B_i coefficients depend on the A_i coefficients while the A_i coefficients are obtained directly, only the A_i coefficients need to be examined. The A_i coefficients of the six modes of lowest range attenuation rates for all 21 profiles except the one without evaporation duct are computed using eigenvalues of different accuracy controlled by the first order Newton-Raphson iteration method. Table 1 shows the A_i coefficient computed with the new program. They are arranged from the top layer down. In the i -th layer, the A_i coefficient computed by integration downward depends only on A_{i+1} in the layer above while

TABLE 1. IMPROVING A_i ACCURACY WITH EIGENVALUE (18 M DUCT)

| mode 4 | | q-eigenvalue: .1888574325176803D+00 | | .1080678744810598D-01 | | eigenvalue difference: .00D+00 | | .00D+00 | | -.49D-13 | | -.66D-15 | | .12D-11 | | -.12D-10 | | .15D-06 | | .60D-07 | | |
|---------|---------|-------------------------------------|---------|-----------------------|---------|--------------------------------|---------|---------|---------|----------|---------|----------|---------|---------|---------|----------|---------|---------|---------|---------|---------|--------|
| layer # | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | Ai/down | |
| layer # | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | Ai/up | |
| 18 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 | .0261 | .6719 |
| 17 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 | -.0625 | .6368 |
| 16 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 | .0139 | .7440 |
| 15 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 | .1216 | .6353 |
| 14 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 | .0166 | .5471 |
| 13 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 | -.1565 | .5310 |
| 12 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 | -.3842 | .5659 |
| 11 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 | -2.2002 | -.8081 |
| 10 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 | -5.4648 | .2423 |
| 9 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 | -3.6974 | -.6979 |
| 8 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 | .3459 | -.7982 |
| 7 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 | .4098 | .8794 |
| 6 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 | .3480 | .8161 |
| 5 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 | .2923 | .8304 |
| 4 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 | .2359 | .8619 |
| 3 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 | .1831 | .8910 |
| 2 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 | .1300 | .9149 |
| 1 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 | .0586 | .9335 |

that computed by integration upward depends only on A_{i-1} in the layer below. Hence in each layer, the coefficient obtained by integration downward is listed above that obtained by integration upward. There are five sets of A_i values listed, with the magnitudes given in powers of 10 and the phase given as a multiple of π . They are obtained from eigenvalues of decreasing accuracy, the one used to compute the left most column being the most accurate. The first set is computed using an eigenvalue having a relative accuracy of 2^{-40} ; The second set uses an eigenvalue with a relative accuracy of 2^{-36} ; The relative accuracy of the eigenvalue for the third set is 2^{-36} ; For the fourth set, the first order Newton-Raphson iteration of the mode location is set at an absolute accuracy of 0.03 of the mesh size, same as that specified in the original program; The eigenvalue for the right most set is the mode location estimated by ROOTS without modification by the Newton-Raphson iteration. It is clear that, for this mode, the difference between these two methods of computing the coefficients becomes negligible as the accuracy in mode location increases. For example, in the 8-th layer, the magnitude of A_i computed by integrating downward changes from -1.9078 to 0.3482 to 0.3460 to 0.3459 , which agrees with the result computed by integrating upward. The phase follows the same trend to an agreement within 0.001π . Table 2 shows a similar set of output, but the coefficients fail to agree even when the relative accuracy is increased to 2^{-40} . Note that the actual difference in both the real part and the imaginary part of the two most accurate eigenvalues is about 2^{-48} . Double precision accuracy appears to be insufficient for the coefficients computed with these two methods to agree for all modes. Some interesting features

can be observed in both tables, which are present in all 120 sets of values computed. When disagreement is present in one set of A_i coefficients such as those in either Table 1 or Table 2, the change toward smaller differences with improving eigenvalue accuracy occurs mainly in one way of computation, but not both. For example, in Table 1, the values of integration downward improve with better eigenvalue accuracy, while those computed by integrating upward change little. In Table 2, the results of integration downward are the ones that are holding steady as the accuracy in eigenvalue improves. Furthermore, when disagreement occurs, the layer in which the A_i coefficient has the smallest magnitude, i.e., the one having the most negative power of 10, divides the table into two parts. The results of two different ways of computation agree in the layers above this one if they disagree in those below it, and vice versa. No explanation will be attempted. Instead, practical rules are drawn up to take advantage of these facts. In Table 1, the process of integration upward goes through the troublesome 10-th layer and produces results which agree with the results of downward integration before the downward process goes through the 10-th layer. On the other hand, the downward integration is tripped up going across the 10-th layer and produces results which fail to agree with the results from upward integration. It is clear that the results from upward integration are the correct ones. This conclusion is further supported by the fact that improving the accuracy of the eigenvalue does not change significantly the results of upward integration. Similar argument leads to the conclusion that in Table 2, the results of downward integration are the correct values.

It can be concluded from the above observations that one of the methods of computing the A_i coefficients converges to the correct value much faster than the other. It is also found that this method of faster convergence is always able to arrive at the correct values for A_i for all the cases under investigation.

Table 3 lists the statistics of the method of integration which yields the correct A_i coefficients for each of the 120 modes investigated. The differences in magnitudes and phases in the lowest layer and in the layer below the highest are also listed. Since for most of the cases when disagreement in A_i values occurs, the correct integration is upward, this is used as the default. To decide that downward integration should be utilized, the following steps are taken: The first A_i value of downward integration is computed and compared to the value from upward integration. If the magnitudes in dB disagree by less than 0.02 dB, their phases will be checked. If the phases differ by less than $10^{-3}\pi$, the agreement is deemed acceptable and the A_i and B_i coefficients computed from the lowest layer up are used. Otherwise, the coefficients are re-evaluated again from the highest layer down.

Once the correct method of evaluating the A_i and B_i coefficients is used, the accuracy of the mode location becomes less critical. For all the cases investigated, the A_i coefficients obtained from mode locations estimated with or without the Newton-Raphson first order iteration differ only by 0.06 dB in magnitude and 0.0013π in phase at most. In fact, few cases show differences more than 0.002 dB and 0.0001π . The Newton-Raphson iteration is not needed. Hence the subroutines NOMSHX, FDFDTX and DXDETR are removed.

TABLE 3. STATISTICS FOR EVALUATING A_i COEFFICIENT

| Duct height | Mode # | Evaluating Method | | $\Delta A_i $ (dB) | | $\Delta \arg(A_i)/\pi$ | |
|-------------|--------|-------------------|------|---------------------|-------|------------------------|--------|
| | | | | Layer | | Layer | |
| | | up | down | bottom | top-1 | bottom | top-1 |
| 02 | 1 | x | | | | | |
| | 2 | x | | | | | |
| | 3 | x | | | | | |
| | 4 | x | | 0.172 | | 0.093 | |
| | 5 | x | | | | | |
| | 6 | x | | 8.362 | | 1.3234 | |
| 04 | 1 | x | | | | | |
| | 2 | x | | | | | |
| | 3 | x | | 0.008 | | 0.0002 | |
| | 4 | x | | 1.030 | | 1.8717 | |
| | 5 | x | | 7.814 | | 1.2948 | |
| | 6 | x | | 0.002 | | 0.0001 | |
| 06 | 1 | x | | | | | |
| | 2 | x | | 0.002 | | 0.0004 | |
| | 3 | x | | 0.522 | | 0.0158 | |
| | 4 | x | | | | | |
| | 5 | x | | 13.278 | | 0.4377 | |
| | 6 | x | | 0.002 | | 0.0001 | |
| 08 | 1 | x | | | | | |
| | 2 | x | | 0.002 | | | |
| | 3 | x | | 0.002 | | 0.0001 | 0.0001 |
| | 4 | x | | 0.016 | | 0.0026 | |
| | 5 | x | | 4.066 | | 0.6355 | |
| | 6 | x | | 3.978 | | 0.6186 | |

TABLE 3. CONTINUED 1.

| Duct height | Mode # | Evaluating Method | | $\Delta A_i $ (dB) | | $\Delta \arg(A_i)/\pi$ | |
|-------------|--------|-------------------|------|---------------------|-------|------------------------|--------|
| | | | | Layer | | Layer | |
| | | up | down | bottom | top-1 | bottom | top-1 |
| 10 | 1 | x | | | | | |
| | 2 | x | | | | 0.0002 | |
| | 3 | x | | | | 0.0001 | |
| | 4 | x | | 0.04 | | 0.0008 | |
| | 5 | x | | 0.206 | | 0.0402 | 0.0001 |
| | 6 | x | | 0.002 | | 0.0001 | |
| 12 | 1 | x | | | | | |
| | 2 | x | | 0.006 | | 0.0003 | |
| | 3 | x | | 0.004 | | | |
| | 4 | x | | 1.808 | | 0.5661 | |
| | 5 | x | | 1.732 | | 0.5429 | |
| | 6 | x | | 1.472 | | 0.0414 | |
| 14 | 1 | x | | | | | |
| | 2 | x | | 0.002 | | 0.0001 | |
| | 3 | x | | 0.178 | | 0.0052 | |
| | 4 | x | | 0.024 | | 0.0005 | |
| | 5 | x | | 0.004 | | 0.0001 | |
| | 6 | x | | 0.85 | | 0.4711 | |
| 16 | 1 | x | | | | | |
| | 2 | x | | 0.006 | | 0.0002 | |
| | 3 | x | | 0.004 | | | |
| | 4 | x | | 0.006 | | 0.0001 | |
| | 5 | x | | 0.002 | | 0.0001 | |
| | 6 | x | | 0.004 | | 0.0077 | |

TABLE 3. CONTINUED 2.

| Duct height | Mode # | Evaluating Method | | $\Delta A_i $ (dB) | | $\Delta \arg(A_i)/\pi$ | |
|-------------|--------|-------------------|------|---------------------|-------|------------------------|--------|
| | | | | Layer | | Layer | |
| | | up | down | bottom | top-1 | bottom | top-1 |
| 18 | 1 | | x | | 0.008 | | 0.0001 |
| | 2 | x | | 0.002 | | 0.0001 | |
| | 3 | x | | | | 0.0001 | |
| | 4 | x | | | | | |
| | 5 | x | | 0.016 | | 0.0003 | |
| | 6 | x | | 0.002 | | | |
| 20 | 1 | | x | | 0.078 | | 0.0164 |
| | 2 | x | | | | | |
| | 3 | x | | 0.002 | | 0.0001 | |
| | 4 | x | | | | 0.0008 | |
| | 5 | x | | 0.16 | | 0.0195 | |
| | 6 | x | | 0.002 | | 0.0001 | |
| 22 | 1 | | x | | 8.708 | | 0.239 |
| | 2 | x | | | | | |
| | 3 | x | | 0.004 | | | |
| | 4 | x | | 0.016 | | | |
| | 5 | x | | 0.002 | | 0.0001 | |
| | 6 | x | | 0.31 | | 0.0117 | |
| 24 | 1 | x | | | | | |
| | 2 | | x | | 0.868 | | 0.2842 |
| | 3 | x | | 0.006 | | 0.0009 | |
| | 4 | x | | 0.002 | | 0.0001 | |
| | 5 | x | | 0.026 | | 0.0009 | |
| | 6 | x | | 0.008 | | 0.0001 | |

TABLE 3. CONTINUED 3.

| Duct height | Mode # | Evaluating Method | | $\Delta A_i $ (dB) | | $\Delta \arg(A_i)/\pi$ | |
|-------------|--------|-------------------|------|---------------------|-------|------------------------|--------|
| | | | | Layer | | Layer | |
| | | up | down | bottom | top-1 | bottom | top-1 |
| 26 | 1 | x | | 0.002 | 0.002 | 0.0001 | 0.0001 |
| | 2 | | x | | 4.308 | | 0.121 |
| | 3 | x | | 0.006 | | | |
| | 4 | x | | 0.002 | | 0.0001 | |
| | 5 | x | | | | 0.0001 | |
| | 6 | x | | 0.034 | | 0.0039 | |
| 28 | 1 | | x | | 0.028 | | 0.0014 |
| | 2 | | x | | 4.806 | | 0.0728 |
| | 3 | x | | | | | |
| | 4 | x | | | | | |
| | 5 | x | | 0.008 | 0.002 | 0.0002 | |
| | 6 | x | | 0.004 | | 0.0019 | |
| 30 | 1 | | x | | 1.562 | | 0.0165 |
| | 2 | x | | | | | |
| | 3 | | x | | 0.718 | | 0.2455 |
| | 4 | x | | | | | |
| | 5 | x | | 0.004 | | | |
| | 6 | x | | 0.724 | | 0.0522 | |
| 32 | 1 | | x | | 3.194 | | 0.1648 |
| | 2 | x | | 0.002 | | | |
| | 3 | | x | | 13.12 | | 0.1026 |
| | 4 | x | | 0.002 | | | |
| | 5 | x | | 0.382 | | 0.0099 | |
| | 6 | x | | 0.002 | | 0.0001 | |

TABLE 3. CONTINUED 4.

| Duct height | Mode # | Evaluating Method | | $\Delta A_i $ (dB) | | $\Delta \arg(A_i)/\pi$ | |
|-------------|--------|-------------------|------|---------------------|--------|------------------------|--------|
| | | | | Layer | | Layer | |
| | | up | down | bottom | top-1 | bottom | top-1 |
| 34 | 1 | x | | 0.002 | 0.002 | | |
| | 2 | | x | | 13.456 | | 0.0311 |
| | 3 | | x | | 1.014 | | 0.2347 |
| | 4 | x | | | | | |
| | 5 | x | | 0.03 | | 0.0006 | |
| | 6 | x | | 0.014 | | 0.0006 | |
| 36 | 1 | | x | | | 0.0001 | 0.0014 |
| | 2 | | x | | 1.686 | | 0.2224 |
| | 3 | | x | | 4.724 | | 0.0919 |
| | 4 | x | | | | | |
| | 5 | x | | 0.006 | | 0.0001 | |
| | 6 | x | | 0.02 | | 0.0001 | |
| 38 | 1 | | x | | 0.996 | | 0.0115 |
| | 2 | | x | | 4.974 | | 0.0152 |
| | 3 | x | | | | | |
| | 4 | | x | | 5.052 | | 0.0417 |
| | 5 | x | | | | 0.0001 | |
| | 6 | x | | 0.002 | | | |
| 40 | 1 | x | | 0.002 | 0.002 | | |
| | 2 | | x | | 3.85 | | 0.1226 |
| | 3 | | x | | 3.568 | | 0.1555 |
| | 4 | | x | | 3.448 | | 0.1678 |
| | 5 | x | | | | 0.0001 | |
| | 6 | x | | | | | |

III. CONCLUSION AND RECOMMENDATION

A. Performance

This revision of M-Layer converts the extended complex number representation of an exponentially large or small number into the direct representation by its complex exponent. The accuracy of the computation has been improved in two ways: First, an interpolation algorithm has been devised when severe cancellation of the addends is detected. Secondly, accuracy for the evaluation of the Airy function has been improved, not just by summing the Taylor series to double precision resolution and by adopting six-term Gaussian quadrature, but also by expanding the region within which the more expedient Gaussian quadrature is excluded in favor of the more accurate but time-consuming Taylor series summation. The improvement in accuracy is most easily seen from Table 1.

As discussed in the Introduction, evaluating the A_i and B_i coefficients either from the lowest layer up (integration up) or from the top layer down (integration down) must result in the same values. This property provides a consistency check for the accuracy of the computation. For the six modes of lowest range attenuation rates of the 20 profiles of different duct heights, Table 1 lists the maximum difference for each mode which shows a discrepancy between these two methods of evaluating the A_i coefficients. For each profile, the maximum value in magnitude difference in dB among all the layers is listed if it is greater than 2. If the phases of the coefficients

TABLE 1. MAXIMUM DIFFERENCE IN A_i COEFFICIENT BETWEEN INTEGRATION UP AND DOWN

| Duct height (m) | Mode # | Difference in A_i coefficient | | | |
|-----------------|--------|---------------------------------|---------|---------------------------------|---------|
| | | Magnitude difference in (dB) | | Phase difference over 0.1π | |
| | | original | revised | original | revised |
| 02 | 4 | 5.22 | | Yes | |
| | 6 | 61.16 | | Yes | |
| 04 | 4 | 22.46 | 2.3 | | |
| | 5 | 106.9 | | Yes | |
| 06 | 3 | 8.62 | | Yes | |
| | 5 | 32.36 | | | |
| 08 | 5 | 77.84 | | Yes | |
| | 6 | 44.9 | | Yes | |
| 10 | 5 | | | Yes | |
| 12 | 4 | 69.38 | | Yes | |
| | 5 | 46.32 | | Yes | |
| | 6 | 7.46 | | Yes | |
| 14 | 6 | 30.6 | | Yes | |
| 22 | 1 | 8.64 | | Yes | |
| 24 | 2 | 80.48 | | Yes | |
| 26 | 2 | 110.68 | | Yes | |
| 28 | 2 | 150.9 | 67.68 | Yes | Yes |
| 30 | 3 | 173.28 | 143.42 | Yes | Yes |
| 32 | 1 | 11.38 | | Yes | |
| | 3 | 525.04 | 188.04 | Yes | Yes |

TABLE 1. CONTINUED

| Duct height (m) | Mode # | Difference in A_i coefficient | | | |
|---|--------|---------------------------------|---------|---------------------------------|---------|
| | | Magnitude difference in (dB) | | Phase difference over 0.1π | |
| | | original | revised | original | revised |
| 34 | 2 | 37.98 | | Yes | |
| | 3 | 715.7 | 209.94 | Yes | Yes |
| 36 | 2 | 112.74 | | Yes | |
| | 3 | 957.92 | 231.68 | Yes | Yes |
| 38 | 2 | 107.44 | 52.26 | Yes | Yes |
| | 4 | 1249 | 255.8 | Yes | Yes |
| 40 | 3 | 167 | 112.72 | Yes | Yes |
| | 4 | 823.56 | 258.18 | Yes | Yes |
| Magnitude difference within 2dB are not listed. | | | | | |

deviate more than 0.1π in any layer, that particular mode is also singled out. The location of the mode of the revised program is within a relative accuracy of 2^{-40} achieved through first order Newton-Raphson iteration. Even though discrepancies still exist when the duct is 28 meters or higher, it is clear that the revised program computes more accurately than the original one.

For the cases where the two methods of evaluating the A_i and B_i coefficients disagree, it has been observed that one of the methods always leads to A_i values which are little changed when the accuracy in mode location is varied, while the other method produces A_i values which shift toward the results of the other method as the accuracy of mode location improves. Based on this observation, a consistency

check is implemented into the program to identify the method which converges better. For the 120 cases investigated, when this method of faster convergence is used, the A_i coefficients obtained from mode locations estimated with or without the Newton-Raphson first order iteration differ only by 0.06 dB in magnitude and 0.0013π in phase at most. In fact, few cases show differences more than 0.002 dB and 0.0001π . This allowed the Newton-Raphson iteration to be removed in this revision.

Table 2 compares the performance between the original and the revised programs. The time spent to find the modes has been reduced by an average of 22.58%. The revised program can always produce the modes found by the original program. Moreover, the mode search is stable for the new program: the time it requires to search for the modes is about the same for similar profiles. The sudden jumps in mode search time for the 24 m and the 40 m cases, which indicate troubles during the search, no longer happen.

With the proper method of evaluating the A_i and B_i coefficients determined by the consistency check, the output of the revised program differs from the original program in some cases. The most serious deviation has been observed for the 38 m duct height case as shown in Tables 3 and 4. For example, at a range of 36.5 km with the transmitter at a height of 25 m and the receiver at 10 m, the coherent path loss is 175.93 dB from the original program, and is 167.90 dB from the revised program.

TABLE 2. OVERALL MODE SEARCH PERFORMANCE COMPARISON

| DUCT HEIGHT (meters) | ORIGINAL PROGRAM | | REVISED PROGRAM | | Time Improvement |
|-------------------------|------------------|-------|-----------------|-------|------------------|
| | Time | Modes | Time | Modes | |
| 00 | 0:00:37 | 3 | 0:00:35 | 3 | 5.40% |
| 02 | 0:32:14 | 9 | 0:31:55 | 9 | 0.98% |
| 04 | 1:14:12 | 25 | 1:05:04 | 25 | 12.31% |
| 06 | 2:10:18 | 53 | 1:56:50 | 53 | 10.33% |
| 08 | 0:35:58 | 39 | 0:29:25 | 39 | 18.21% |
| 10 | 0:53:24 | 59 | 0:48:32 | 61 | 9.11% |
| 12 | 1:09:40 | 86 | 1:01:44 | 89 | 11.39% |
| 14 | 1:20:42 | 94 | 1:11:13 | 97 | 11.75% |
| 16 | 1:54:35 | 95 | 1:18:07 | 97 | 31.82% |
| 18 | 1:45:09 | 100 | 1:27:15 | 104 | 17.02% |
| 20 | 1:46:19 | 103 | 1:34:20 | 105 | 11.27% |
| 22 | 1:52:54 | 105 | 1:35:18 | 100 | 15.59% |
| 24 | 3:42:59 | 106 | 1:46:47 | 107 | 52.11% |
| 26 | 2:07:42 | 106 | 1:43:55 | 108 | 18.62% |
| 28 | 2:00:05 | 107 | 1:44:59 | 109 | 12.57% |
| 30 | 1:59:59 | 107 | 1:46:19 | 108 | 11.39% |
| 32 | 1:55:29 | 108 | 1:42:58 | 110 | 10.84% |
| 34 | 2:29:57 | 109 | 2:15:58 | 111 | 9.32% |
| 36 | 2:31:40 | 109 | 2:17:20 | 112 | 9.45% |
| 38 | 2:38:44 | 110 | 2:18:09 | 111 | 12.97% |
| 40 | 5:41:17 | 95 | 2:39:39 | 111 | 53.22% |
| Total | 40:23:54 | | 31:16:22 | | 22.58% |

TABLE 3. ORIGINAL PROGRAM OUTPUT: 38 M DUCT

| frequency = 9600.0000 mhz | | | | | | | |
|---------------------------|-----------|-----------|------------------------------|--------------------------------|-------------------------------|---------------------------------|-----------------|
| range (km) | zt (m) | zr (m) | coherent mode sum (db) | incoherent mode sum (db) | coherent path loss (db) | incoherent path loss (db) | horizon (km) |
| 27.3 | 25.0 | 4.0 | -15.30 | -15.62 | 156.10 | 156.43 | 28.9 |
| 27.3 | 25.0 | 6.0 | .62 | -2.35 | 140.18 | 143.16 | 30.7 |
| 27.3 | 25.0 | 8.0 | -1.11 | -4.21 | 141.92 | 145.01 | 32.3 |
| 27.3 | 25.0 | 10.0 | -27.26 | -12.66 | 168.06 | 153.46 | 33.6 |
| 36.5 | 25.0 | 4.0 | -16.94 | -16.62 | 160.28 | 159.96 | 28.9 |
| 36.5 | 25.0 | 6.0 | -.73 | -2.05 | 144.07 | 145.39 | 30.7 |
| 36.5 | 25.0 | 8.0 | -2.21 | -3.72 | 145.55 | 147.06 | 32.3 |
| 36.5 | 25.0 | 10.0 | -32.59 | -14.29 | 175.93 | 157.64 | 33.6 |
| 45.8 | 25.0 | 4.0 | -19.89 | -16.96 | 165.20 | 162.26 | 28.9 |
| 45.8 | 25.0 | 6.0 | -2.81 | -1.89 | 148.11 | 147.19 | 30.7 |
| 45.8 | 25.0 | 8.0 | -4.11 | -3.43 | 149.41 | 148.74 | 32.3 |
| 45.8 | 25.0 | 10.0 | -28.57 | -15.22 | 173.88 | 160.52 | 33.6 |

TABLE 4. REVISED PROGRAM OUTPUT: 38 M DUCT

| frequency = 9600.0000 mhz | | | | | | | |
|---------------------------|-----------|-----------|------------------------------|--------------------------------|-------------------------------|---------------------------------|-----------------|
| range (km) | zt (m) | zr (m) | coherent mode sum (db) | incoherent mode sum (db) | coherent path loss (db) | incoherent path loss (db) | horizon (km) |
| 27.3 | 25.0 | 4.0 | -14.38 | -15.66 | 155.18 | 156.47 | 28.9 |
| 27.3 | 25.0 | 6.0 | .42 | -2.37 | 140.39 | 143.18 | 30.7 |
| 27.3 | 25.0 | 8.0 | -1.52 | -4.21 | 142.33 | 145.02 | 32.3 |
| 27.3 | 25.0 | 10.0 | -21.20 | -12.51 | 162.01 | 153.31 | 33.6 |
| 36.5 | 25.0 | 4.0 | -17.32 | -16.60 | 160.66 | 159.94 | 28.9 |
| 36.5 | 25.0 | 6.0 | -.48 | -2.08 | 143.82 | 145.42 | 30.7 |
| 36.5 | 25.0 | 8.0 | -1.62 | -3.73 | 144.96 | 147.07 | 32.3 |
| 36.5 | 25.0 | 10.0 | -24.56 | -14.04 | 167.90 | 157.38 | 33.6 |
| 45.8 | 25.0 | 4.0 | -20.26 | -16.93 | 165.57 | 162.23 | 28.9 |
| 45.8 | 25.0 | 6.0 | -3.14 | -1.93 | 148.44 | 147.23 | 30.7 |
| 45.8 | 25.0 | 8.0 | -4.62 | -3.46 | 149.92 | 148.76 | 32.3 |
| 45.8 | 25.0 | 10.0 | -25.40 | -14.90 | 170.71 | 160.21 | 33.6 |

B. Recommendation

The mode search protocol of this program needs to be revised. Since the search is limited by the maximum range attenuation rate accepted, it is logical to begin with locating the mode of the lowest or the highest attenuation, then proceed to look for the next one in the order of increasing or decreasing attenuation rate. Furthermore, under the assumption of analyticity over the search region, there should be only one connected "phase line" of vanishing real part of the modal function on which all the modes are located. The partition of the search region into rectangles as has been done in this program tends to cut the "phase line" into segments before the program starts to search for the end points of these segments and then follow the segments in different directions. It is clear that a better way is to search for one end of the "phase line" along a line of a constant attenuation rate in the search region, either at the maximum accepted or the minimum possible attenuation, then follow this "phase line" all the way to the other end. This technique works even if the "phase line" branches off into several directions at a Stokes' point.

APPENDIX A: SUBROUTINE XCADD

This Appendix lists the addition subroutine XCADD which returns the complex exponent of the sum when the complex exponents of the addends are given. This is a complete re-write of the original subroutine of the same name.

```

1      subroutine xcadd(zx,z1x,z2x)
2      c
3      c      Given z1x and z2x, this subroutine adds the two complex numbers
4      c      z1=exp(z1x) and z2=exp(z2x) for z=exp(zx) and returns zx.
5      c
6      c      inputs...
7      c          z1x=complex exponent of the complex number z1
8      c          z2x=complex exponent of the complex number z2
9      c
10     c      outputs...
11     c          zx=complex exponent of the complex number z
12     c
13     c      subroutines called...
14     c
15     c*****
16     implicit real*8 (a-h,o-z)
17     complex*16 zx,z1x,z2x,zt1x,zt2x,clogzh,dsum,czero,cerrx,cone,chpi
18     parameter(pi=3.141592653589793238462643d0,twopi=2.d0*pi,
19     +   hpi=0.5d0*pi,zero=0.d0,c16=1.d0/6.d0,
20     +   bit14=1.d0/16384.d0,bit24=bit14/1024.d0,ctol=bit14,
21     +   dpi=2259.d0/4294967296.d0/4294967296.d0,hdpi=dpi/2.d0,
22     +   e2m54=-3.742994775023704819d1,e2p27=-0.5d0*e2m54,
23     +   chpi=(0.d0,1.57079632679489661923132d0),cone=(1.d0,0.d0),
24     +   czero=(0.d0,0.d0),cerrx=(-3.742994775023704819d1,0.d0))
25     c      cerrx=e2m54=-54*log(2)=exponent below machine accuracy
26     dimension ztmp(2),stmp(2)
27     equivalence (ztmp,clogzh),(stmp,dsum)
28     c*****
29     c      Replace the input variables with a local variable so that
30     c      equations in the form of y=x+y will not lead to confusion.
31     c
32     zt1x=z1x
33     zt2x=z2x
34     c
35     clogzh=0.5d0*(zt1x-zt2x)
36     dxh=ztmp(1)
37     if(dxh .lt. zero) then
38         zx=zt2x
39         dxh=-dxh
40     else
41         zx=zt1x
42     end if
43     c*****
44     c      machine accuracy = 2**(-53)
45     c      2**(27)=e**e2p27
46     c
47     if (dxh .ge. e2p27) then

```

```

48         return
49     else
50         zx=0.5d0*(zt1x+zt2x)
51         dsum=cexp(clogzh)
52         dsum=1.d0/dsum+dsum
53         if (cdabs(dsum) .gt. ctol) then
54             zx=cdlog(dsum)+zx
55         else
56 c      Cancellation is serious. Im[clogzh] is close to pi/2 or -pi/2.
57             yi=dnint(ztmp(2)/twopi)*2.d0
58             ztmp(2)=ztmp(2)-pi*yi
59             dyi=dpi*yi
60             if (ztmp(2) .lt. zero) then
61                 clogzh=-clogzh
62                 dyi=-dyi
63             end if
64             ztmp(2)=(ztmp(2)-hpi)-hdpi-dyi
65             dsum=2.d0*clogzh*(cone+c16*clogzh*clogzh)
66             if (dsum .eq. czero) then
67 c      Note that a complete cancellation of two nonzero numbers of
68 c      order one is considered to be as accurate as what is allowed
69 c      by the machine and the algorithm.
70                 zx=cerrx+chpi+zx
71             else
72                 dsum=cdlog(dsum)
73                 if (stmp(1) .lt. e2m54) stmp(1)=e2m54
74                 zx=dsum+chpi+zx
75             end if
76         end if
77         return
78     end if
79 c
80     end

```

APPENDIX B: SUBROUTINE FZEROX

This Appendix includes the listing of the subroutine FZEROX which identifies the meshes which may contain modes within a contour rectangle. The Shellman-Morffit mode locating algorithm has been completely replaced.

```

1      subroutine fzerox(tleft,tright,tbot,ttop,tmesh,zeros,ni,nf)
2  c*****
3  c  fzerox is a routine for finding the zeroes of a complex function, f,
4  c  which lie within a specified rectangular region of the
5  c  complex q11 plane, assuming that the function has only
6  c  simple zeroes over this rectangle.
7  c
8  c  parameters specifying the search rectangle:
9  c  tleft - value of the real part of q11 at the left edge.
10 c  tright- value of the real part of q11 at the right edge.
11 c  tbot  - value of the imaginary part of q11 at the bottom edge.
12 c      (this is set to 0.)
13 c  ttop  - value of the imaginary part of q11 at the top edge.
14 c  tmesh - set equal to about half the average spacing between
15 c      zeroes within the rectangle. A smaller value may be used
16 c      as a safety measure, but too small a value will result
17 c      in excessively long run time.
18 c  zeros - output list of (complex) values of q11 at which
19 c      zeroes are found.
20 c  nf-ni - the number of zeroes found
21 c
22 c  subroutines called--
23 c      findfx
24 c      roots
25 c      nomshx
26 c*****
27      implicit double precision (a-h,o-z)
28      complex*16 f10,f01,f11,fxnew,fxold,fx00,fx10,fx01,fx11,
29      +          czero,one,ci,sol,zeros
30      parameter(czero=(0.d0,0.d0),one=(1.d0,0.d0),ci=(0.d0,1.d0))
31 $include: 'mlaparm.inc'
      ***** Begin listing of: mlaparm.inc
1  c
2  c      include file to define the
3  c          maximum # of layers (mxlayr)
4  c          maximum # of modes (mxmode)
5  c
6      parameter (mxlayr=35 )
7      parameter (mxmode=127)
      ***** End listing of: mlaparm.inc
32      dimension kedge1(100),kedge2(100),kedge3(100),kedge4(100),
33 c  +  loc12r(mxmode),loc12i(mxmode),loc23r(mxmode),loc23i(mxmode),
34 c  +  loc34r(mxmode),loc34i(mxmode),loc41r(mxmode),loc41i(mxmode),
35 c  +  sol(3),theta(2),zeros(2*mxmode+1)
36 c
37 c
38      common /tmccom/tmesh

```



```

39 c*****
40 c   maxnsq - maximum number of mesh squares allowed on any one
41 c     phase line
42 c   maxnt - maximum number of times fzero will reduce tmesh
43 c
44     maxnsq=3*max0(int((ttop-tbot)/tmsh0),int((tright-tleft)/tmsh0))
45     maxnt=2
46 c*****
47     tmesh = tmsh0
48     ntime = 0
49     go to 7
50 c
51 5   tmesh=tmesh/2.0d0
52     ntime = ntime+1
53     if(ntime .gt. maxnt) go to 97
54 c
55 7   continue
56
57 c*****
58 c   calculate coordinates of rectangle edges in tmesh units
59 c
60     jlt = idnint(tleft/tmesh-0.5d0)
61     jrt = idnint(tright/tmesh+0.5d0)
62     jtop = idnint(ttop/tmesh+1.5d0)
63     jbot = 0
64 c
65 c   initialize parameters for starting search at upper left
66 c   corner of search rectangle
67 c
68     ki = jtop
69     kr = jlt
70     kedge = 1
71     call findfx(kr,ki,fxnew,xnew,ynew)
72     nre1=0
73     nre2=0
74     nre3=0
75     nre4=0
76     knot12=0
77     knot23=0
78     knot34=0
79     knot41=0
80     nf=ni
81     ni1=ni+1
82     go to 15
83 c*****
84 10  continue
85     if(nrz1 .lt. 2) go to 15

```

```

86 c   write(16,2000) nrzl
87     go to 5
88 15  nrzl=0
89     nrsqu = 0
90 20  fxold=fxnew
91     xold=xnew
92     yold=ynew
93     go to (21,26,31,36),kedge
94 c*****
95 c   search along left edge of rectangle for changes in the
96 c   sign of imag(f)
97 c
98 21  continue
99     if(ki.eq.jbot) then
100       kedge=2
101       go to 26
102     end if
103     ki = ki-1
104     call findfx(kr,ki,fxnew,xnew,ynew)
105     if (yold*ynew .gt. 0.d0) go to 20
106     if(nre1.eq.0) go to 23
107 c
108 c   check if crossing point has been previously found
109 c
110     do 22 k=1,nre1
111       if(ki.eq.kedge1(k)) go to 20
112 22  continue
113 c
114 c   follow phase line through rectangular region
115 c
116 23  fx01=fxold
117     fx01r=xold
118     fx01i=yold
119     fx00=fxnew
120     fx00r=xnew
121     fx00i=ynew
122     li = ki
123     lr = jlt
124     go to 43
125 c*****
126 c   search along bottom edge of rectangle for changes in the
127 c   sign of imag(f)
128 c
129 26  continue
130     if(kr.eq.jrt) then
131       kedge=3
132       go to 31

```

```

133     end if
134     kr = kr+1
135     call findfx(kr,ki,fxnew,xnew,ynew)
136     if (yold*ynew .gt. 0.d0) go to 20
137     if(nre2.eq.0) go to 28
138 c
139 c   check if crossing point has been previously found
140 c
141     do 27 k=1,nre2
142     if(kr.eq.kedge2(k)) go to 20
143 27  continue
144 c
145 c   follow phase line through rectangular region
146 c
147 28  fx00=fxold
148     fx00r=xold
149     fx00i=yold
150     fx10=fxnew
151     fx10r=xnew
152     fx10i=ynew
153     li = jbot
154     lr = kr-1
155     go to 48
156 c*****
157 c search along right edge of rectangle for sign changes in imag(f).
158 c
159 31  continue
160     if(ki.eq.jtop) then
161         kedge=4
162         go to 36
163     end if
164     ki = ki+1
165     call findfx(kr,ki,fxnew,xnew,ynew)
166     if (yold*ynew .gt. 0.d0) go to 20
167     if(nre3.eq.0) go to 33
168 c
169 c   check if crossing point has been previously found
170 c
171     do 32 k=1,nre3
172     if(ki.eq.kedge3(k)) go to 20
173 32  continue
174 c
175 c   follow phase line through rectangular region
176 c
177 33  fx10=fxold
178     fx10r=xold
179     fx10i=yold

```

```

180     fx11=fxnew
181     fx11r=xnew
182     fx11i=ynew
183     li = ki-1
184     lr = jrt-1
185     go to 53
186 c*****
187 c search along top edge of rectangle for sign changes in imag(f).
188 c
189 36 continue
190     if(kr.eq.jlt) go to 80
191     kr = kr-1
192     call findfx(kr,ki,fxnew,xnew,ynew)
193     if (yold*ynew .gt. 0.d0) go to 20
194     if(nre4.eq.0) go to 38
195 c
196 c check if crossing point has been previously found
197 c
198     do 37 k=1,nre4
199     if(kr.eq.kedge4(k)) go to 20
200 37 continue
201 c
202 c follow phase line through rectangular region
203 c
204 38 fx11=fxold
205     fx11r=xold
206     fx11i=yold
207     fx01=fxnew
208     fx01r=xnew
209     fx01i=ynew
210     li = jtop-1
211     lr = kr
212     go to 58
213 c*****
214 c enter mesh square from left side or exit rectangle at right edge.
215
216 41 lr=lr+1
217     if (lr .le. jrt-1) go to 42
218     nre3=nre3+1
219     kedge3(nre3)=li+1
220     go to 10
221 42 fx01=fx11
222     fx01r=fx11r
223     fx01i=fx11i
224     fx00=fx10
225     fx00r=fx10r
226     fx00i=fx10i

```

```

227 43   continue
228       call findfx(lr+1,li+1,fx11,fx11r,fx11i)
229       call findfx(lr+1,li,fx10,fx10r,fx10i)
230 c*****
231 c     Determine the edge of exit of im(f)=0 from current mesh.
232       edgeit=fx01i*fx11i
233       edgeib=fx00i*fx10i
234       if (edgeib .gt. 0.d0) then
235 c         Im(f)=0 goes through the 01 to 10 line.
236         if (edgeit .gt. 0.d0) then
237 c           Im(f)=0 goes through the 10 to 11 edge (edge 1).
238           lout=1
239         else
240 c           Im(f)=0 goes through the 01 to 11 edge (edge 2)
241           lout=2
242         end if
243       else
244 c         Im(f)=0 goes through the 00 to 10 edge (edge 4)
245         lout=4
246         if (edgeit .lt. 0.d0) then
247 c           Im(f)=0 also runs through 01 to 11 and 10 to 11 edges.
248 c           Store crossing location and in/out information.
249           knot34=knot34+1
250 c           loc34r(knot34)=lr
251 c           loc34i(knot34)=li
252         end if
253       end if
254 c*****
255       go to 60
256 c*****
257 c     enter mesh square from bottom side or exit rectangle at top edge.
258 46   li=li+1
259       if (li .le. jtop-1) go to 47
260       nre4=nre4+1
261       kedge4(nre4)=lr
262       go to 10
263 47   fx00=fx01
264       fx00r=fx01r
265       fx00i=fx01i
266       fx10=fx11
267       fx10r=fx11r
268       fx10i=fx11i
269 48   continue
270       call findfx(lr,li+1,fx01,fx01r,fx01i)
271       call findfx(lr+1,li+1,fx11,fx11r,fx11i)
272 c*****
273 c     Determine the edge of exit of im(f)=0 from current mesh.

```

```

274     edgeil=fx00i*fx01i
275     edgeir=fx10i*fx11i
276     if (edgeir .gt. 0.d0) then
277 c       Im(f)=0 goes through the 00 to 11 line.
278         if (edgeil .gt. 0.d0) then
279 c           Im(f)=0 goes through the 01 to 11 edge (edge 2)
280             lout=2
281         else
282 c           Im(f)=0 goes through the 00 to 01 edge (edge 3).
283             lout=3
284         end if
285     else
286 c       Im(f)=0 goes through the 10 to 11 edge (edge 1)
287         lout=1
288         if (edgeil .lt. 0.d0) then
289 c           Im(f)=0 also runs through 00 to 01 and 01 to 11 edges.
290 c           Store crossing location and in/out information.
291             knot41=knot41+1
292 c           loc41r(knot41)=lr
293 c           loc41i(knot41)=li
294         end if
295     end if
296 c*****
297     go to 60
298 c*****
299 c enter mesh square from right side or exit rectangle at left edge.
300
301 51     lr=lr-1
302         if (lr .ge. jlt) go to 52
303         nre1=nre1+1
304         kedge1(nre1)=li
305         go to 10
306 52     fx1i=fx01
307         fx11r=fx01r
308         fx11i=fx01i
309         fx10=fx00
310         fx10r=fx00r
311         fx10i=fx00i
312 53     continue
313         call findfx(lr,li+1,fx01,fx01r,fx01i)
314         call findfx(lr,li,fx00,fx00r,fx00i)
315 c*****
316 c       Determine the edge of exit of im(f)=0 from current mesh.
317         edgeit=fx01i*fx11i
318         edgeib=fx00i*fx10i
319         if (edgeit .gt. 0.d0) then
320 c           Im(f)=0 goes through the 01 to 10 line.

```

```

321         if (edgeib .gt. 0.d0) then
322 c         Im(f)=0 goes through the 00 to 01 edge (edge 3).
323             lout=3
324         else
325 c         Im(f)=0 goes through the 00 to 10 edge (edge 4)
326             lout=4
327         end if
328     else
329 c         Im(f)=0 goes through the 01 to 11 edge (edge 2)
330             lout=2
331         if (edgeib .lt. 0.d0) then
332 c         Im(f)=0 also runs through 00 to 10 and 00 to 01 edges.
333 c         Store crossing location and in/out information.
334             knot12=knot12+1
335 c             loc12r(knot12)=lr
336 c             loc12i(knot12)=li
337         end if
338     end if
339 c*****
340         go to 60
341 c*****
342 c enter mesh square from top side or exit rectangle at bottom edge.
343 56     li=li-1
344         i4 (li .ge. jbot) go to 57
345         nre2=nre2+1
346         kedge2(nre2)=lr+1
347         go to 10
348 57     fx01=fx00
349         fx01r=fx00r
350         fx01i=fx00i
351         fx11=fx10
352         fx11r=fx10r
353         fx11i=fx10i
354 58     continue
355         call findfx(lr,li,fx00,fx00r,fx00i)
356         call findfx(lr+1,li,fx10,fx10r,fx10i)
357 c*****
358 c         Determine the edge of exit of im(f)=0 from current mesh.
359         edgeil=fx00i*fx01i
360         edgeir=fx10i*fx11i
361         if (edgeil .gt. 0.d0) then
362 c             Im(f)=0 goes through the 00 to 11 line.
363             if (edgeir .gt. 0.d0) then
364 c                 Im(f)=0 goes through the 00 to 10 edge (edge 4)
365                 lout=4
366             else
367 c                 Im(f)=0 goes through the 10 to 11 edge (edge 1).

```

```

368         lout=1
369     end if
370 else
371 c     Im(f)=0 goes through the 00 to 01 edge (edge 3)
372     lout=3
373     if (edgeir .lt. 0.d0) then
374 c     Im(f)=0 also runs through 00 to 10 and 10 to 11 edges.
375 c     Store crossing location and in/out information.
376         knot23=knot23+1
377 c     loc23r(knot23)=lr
378 c     loc23i(knot23)=li
379     end if
380 end if
381 c
382 c*****
383 60 continue
384     nrsqu=nrsqu+1
385     if(nrsqu .gt. maxnsq) go to 95
386 c*****
387 c Test for there being at least one re(f)=0 line entering and
388 c     leaving the mesh square.
389 c
390     if ((fx00r*fx10r .gt. 0.d0) .and. (fx01r*fx11r .gt. 0.d0)
391 + .and. (fx00r*fx01r .gt. 0.d0)) go to (41,46,51,56) lout
392 c
393 c Compute the values of the modal function at the corners of a
394 c     mesh square to determine its Taylor series to the 3rd order
395 c     for estimating its root locations.
396 c
397 c     f00=one
398     f10=cdexp(fx10-fx00)-one
399     f01=cdexp(fx01-fx00)-one
400     f11=cdexp(fx11-fx00)-one
401 c
402 c*****
403 c     write (16,3001) ni,nf,lr,li,knot12,knot23,knot34,knot41
404 c 3001 format('/ ni, nf, lr, li and knot12, 23, 34 and 43 before ROOTS
405 c + :', 2i6,2x,2i6,2x,4i6)
406 c
407 c***** estimate locations of zeroes by radicals *****
408 c
409     call roots(f10,f01,f11,sol,nrsol)
410 c
411     do 63 n=1,nrsol
412         ureal = dreal(sol(n))
413         uimag = dimag(sol(n))
414         if (ureal .lt. 0.d0 .or. ureal .gt. 1.0d0) go to 63

```



```

415         if (uimag .lt. 0.d0 .or. uimag .gt. 1.0d0) go to 63
416 62      theta(1)=(lr+ureal)*tmesh
417         theta(2)=(li+uimag)*tmesh
418         nf = nf+1
419         zeros(nf)=dcmplx(theta(1),theta(2))
420         nrzl=nrzl+1
421 63      continue
422 c*****
423 c      write (16,3002) ni,nf,nrsol
424 c 3002 format(/' out of ROOTS at 63, ni, nf and # of roots ',3i4)
425 c*****
426 c      continue following the phase line
427         go to (41,46,51,56) lout
428 c*****
429 cc
430 80      continue
431 c
432         return
433 c*****
434 95      continue
435         write(16,9500)
436         write(16,4001)lr,li,ni,nf,tmesh
437         write(* ,9500)
438 4001 format('go to 5 from 95 at lr, li =',i6,',',i6,' ni, nf =',i6,
439         +',',i6,', mesh size =',d14.6)
440         go to 5
441 c*****
442 97      continue
443         write(16,9700)
444         write(16,4002)lr,li,ni,nf,tmesh
445         write(* ,9700)
446 4002 format('go to 5 from 97 at lr, li =',i6,',',i6,' ni, nf =',i6,
447         +',',i6,', mesh size =',d14.6,/'zeroes found are kept.')
448 c      nf=ni
449 c
450         return
451 c
452 c**** format statements
453 9500 format(/5x,'too many squares on same phase line -- ',
454         $      'reduce tmesh and start over')
455 9700 format(/5x,'tmesh has been reduced but problems remain in',
456         $      ' executing fzerox')
457 c
458         end

```

APPENDIX C: SUBROUTINE ROOTS

This Appendix contains the listing of the subroutine ROOTS. This subroutine replaces the portion of the subroutine FZEROX where the coefficients of a quadratic equation are determined, and the subroutine QUAD for locating the zeroes of a quadratic polynomial. In the revised subroutine FZEROX, the roots of a cubic polynomial has to be found. This subroutine determines these zeroes by radicals.

```

1      subroutine roots (f1,f2,f3,sol,nrsol)
2  c*****
3  c This subroutine finds the roots of a third order polynomial by
4  c radicals when the values of this polynomial at z=0, z=1, z=i and
5  c z=1+i are given as f0=1, f1+f0, f2+f0 and f3+f0 respectively.
6  c Note that this algorithm takes cubic roots of two complex numbers
7  c (hence the name 'solution by radicals') and use their linear
8  c combinations as the roots of a third order polynomial.
9  c*****
10     implicit real*8 (a-h, o-z)
11     complex*16 f1,f2,f3,zero,one,ci,ep14,em14,ep23,em23,
12     +      fa,fb,fc,fd,fa1,fa2,fa3,fa1s,p,q,delt,z,zm,u,v,sol
13     parameter (xbit52=52.d0*0.69314718055994531d0,thrd=1.d0/3.d0,
14     +      bit50=1.d0/33554432.d0/33554432.d0,bit51=bit50/2.d0,
15     +      bit52=bit51/2.d0,tol=0.001d0,
16     +      zero=(0.d0,0.d0),one=(1.d0,0.d0),ci=(0.d0,1.d0),
17     +      ep14=(0.5d0,0.5d0),em14=(0.5d0,-0.5d0),
18     +      ep23=(-0.5d0,0.86602540378443864675d0),
19     +      em23=(-0.5d0,-0.86602540378443864675d0))
20     dimension sol(*)
21     fa=one
22     fb=(f2-ci*f1+em14*f3)
23     fc=((ep14+one)*f1-(em14+one)*f2+ci*f3)
24     fd=(em14*(f2-f1)-ep14*f3)
25     if (cdabs(fb) .le. bit50) fb=zero
26     if (cdabs(fc) .le. bit51) fc=zero
27     if (cdabs(fd) .le. bit52) fd=zero
28     if (fd .ne. zero) then
29         fa1=(-thrd)*fc/fd
30         fa2=fb/fd
31         fa3=fa/fd
32         fa1s=fa1*fa1
33         p=thrd*fa2-fa1s
34         q=0.5d0*(fa3+fa1*fa2)-fa1*fa1s
35         if (p .eq. zero) then
36             if (q .eq. zero) then
37                 nrsol=1
38                 sol(1)=fa1
39                 return
40             else
41                 nrsol=3
42                 u=((-2.d0)*q)**thrd
43                 sol(1)=u+fa1
44                 sol(2)=ep23*u+fa1
45                 sol(3)=em23*u+fa1
46                 return
47             end if

```

```

48     else
49         if (q. eq. zero) then
50             nrsol=3
51             sol(1)=fa1
52             u=cdsqrt((-3.d0)*p)
53             sol(2)=fa1+u
54             sol(3)=fa1-u
55             return
56         else
57             v=p/q
58             z=p*v*v
59             absz=cdabs(z)
60             if (absz .lt. tol) then
61                 zm=-z
62                 fn=dint(1.d0-xbit52/dlog(absz))
63                 lastn=idint(fn)-1
64                 dnn=fn-0.5d0
65                 dnd=fn+1.0d0
66                 delt=one
67                 do 100 nt=1,lastn
68                     dnn=dnn-1.d0
69                     dnd=dnd-1.d0
70                     delt=(dnn/dnd)*delt*zm+one
71 100             continue
72                 delt=(0.5d0*delt/q)**thrd
73                 u=p*delt
74                 v=-1.d0/delt
75             else
76                 delt=cdsqrt(one+z)-one
77                 u=(q*delt)**thrd
78                 v=-p/u
79             end if
80             nrsol=3
81             sol(1)=u+v+fa1
82             sol(2)=ep23*u+em23*v+fa1
83             sol(3)=em23*u+ep23*v+fa1
84             return
85         end if
86     end if
87     else if (fc .ne. zero) then
88         if (fb .eq. zero) then
89             if (fa .eq. zero) then
90                 nrsol=1
91                 sol(1)=zero
92                 return
93             else
94                 nrsol=2

```

```

95         z=cdsqrt(-fa/fc)
96         sol(1)=z
97         sol(2)=-z
98         return
99     end if
100    else
101        fa1=0.5d0*fb/fc
102        fa2=fa/fc
103        z=fa2/fa1/fa1
104        absz=cdabs(z)
105        if (absz .lt. tol) then
106            fn=dint(1.d0-xbit52/dlog(absz))
107            lastn=idint(fn)-1
108            dnn=fn-0.5d0
109            dnd=fn+1.0d0
110            deltt=one
111            do 200 nt=1,lastn
112                dnn=dnn-1.d0
113                dnd=dnd-1.d0
114                deltt=(dnn/dnd)*deltt*z+one
115 200        continue
116            deltt=-0.5d0*deltt/fa1
117            nrsol=2
118            sol(1)=fa2*deltt
119            sol(2)=1.d0/deltt
120            return
121        else
122            deltt=cdsqrt(one-z)
123            nrsol=2
124            sol(1)=-fa1*(one-delt)
125            sol(2)=-fa1*(one+delt)
126            return
127        end if
128    end if
129    else if (fb .ne. zero) then
130        nrsol=1
131        sol(1)=-fa/fb
132        return
133    else
134        nrsol=1
135        sol(1)=ep14
136        return
137    end if
138    end

```

APPENDIX D: SUBROUTINE ABCOEF

This Appendix contains the listing of the subroutine ABCOEF. The consistency self-checking procedure has been implemented to determine the correct method to evaluate the A_i and B_i coefficients.

```

1      subroutine abcoef(zero,m)
2  c*****
3  c      For each mode m, this subroutine calculates A-B coefficients in
4  c      all layers for combining two linearly independent solutions of
5  c      Stokes' equation to form the height gain function:
6  c
7  c      height gain=exp(bcoefx(l,m))*(k1*exp(acoefx(l,m))+k2)
8  c
9  c      where k1 and k2 are two independent solutions to Stokes'
10 c      equation. In the top layer (i.e. nzlayr) the height gain is:
11 c
12 c      height gain=exp(bcoefx(l,m))*h2
13
14 c      where h2 is a solution to the Stokes' equation associated
15 c      with outgoing energy flow. Here k1 and k2 are proportional
16 c      to the k1 and k2 used by Marcus and the h2 is proportional
17 c      to a modified Hankle function of order 1/3.
18
19 c      inputs...
20 c      zero-an eigenvalue in q11 space
21
22 c      outputs...
23 c      acoefx-two dimensional array of complex exponents
24 c      coefficients used to combine two linearly
25 c      independent solutions of stokes' equation
26 c      bcoefx-two dimensional array of complex exponents
27 c      coefficients used for normalizing the height gains
28
29 c      note: acoefx and bcoefx are passed by the
30 c      common block /pap2/
31
32 c      subroutines called...
33 c      xcdei
34 c      xcadd
35
36 c      common block areas...
37 c      com1
38 c      com2
39 c      pap1
40 c      pap2
41 c*****
42
43      implicit real*8(a-h,o-z)
44      complex*16 acoefx,bcoefx,cqij,h2xq1,dh2xq1,h2xq2,dh2xq2,k1xq1,
45      $ dk1xq1,k1xq2,dk1xq2,k2xq1,dk2xq1,k2xq2,dk2xq2,h2dk1x,
46      $ dh2k1x,h2dk2x,dh2k2x,numax,denax,numbx,denbx,int1x,int2x,
47      $ hyx,dhyx,k1dhyx,dk1hyx,dk2hyx,k2dhyx,gamma,dgamdq,i,

```

```

48 $ koa123,rtsumx,zero,q1,q2,sumx,surfno,dqij,dqijdz,sqng,
49 $ dnumbx,dhux,dh1x,e13x,cneg,cldqzl,cldqzm,cigama,koawav,tthd,
50 + taccoef,dacoef
51
52 parameter(downi=1.d-3,downr=1.d-3/0.4342944819032518d0,
53 + pi=3.141592653589793238462643d0,
54 + i=(0.0d0,1.0d0),tthd=(2.d0/3.d0)*i,
55 + cneg=(0.0d0,3.141592653589793238462643d0),e13x=cneg/3.d0)
56 c*****
57 c mxlayr=maximum number of layers allowed
58 c mxmode=maximum number of modes allowed
59
60 c
61 c use include file for parameters of
62 c use include file for parameters of
63 c mxlayr max # layers
64 c mxmode max # modes
65 c
66 $include: 'mlaparm.inc'
***** Begin listing of: mlaparm.inc
1 c
2 c include file to define the
3 c maximum # of layers (mxlayr)
4 c maximum # of modes (mxmode)
5 c
6 parameter (mxlayr=35 )
7 parameter (mxmode=127)
***** End listing of: mlaparm.inc
67 c
68 c
69 c*****
70 c acoefx-two dimensional complex array used for combining two
71 c independent solutions to stokes' equation
72 c bcoefx-two dimensional complex array used for normalizing height
73 c gain
74 c cqij-two dimensional array containing coefficients for evaluating
75 c qij in terms of q11
76 c dqij-array containing coefficients for evaluating qij in terms of
77 c q11
78 c dqijdz-array containing derivatives of qi(z) in the different
79 c layers
80 c zi-array containing input hesights for the modified refractivity
81
82 dimension acoefx(mxlayr,mxmode),
83 $ bcoefx(mxlayr,mxmode),
84 $ dqij(mxlayr),cqij(mxlayr,2),dqijdz(mxlayr),zi(mxlayr+1)
85 c*****

```



```

86
87     common /com1/freq,waveno,sqng
88     common /com2/cqij,dqij,dqijdz,nzlayr
89     common /pap1/nrmode,koa123,surfno,zi
90     common /pap2/acoefx,bcoefx
91
92 c*****
93 c     check for single layer
94 c
95 c     set a complex variable koawav=-i*koa123/(waveno*waveno) to
96 c     avoid repeating computations
97
98     koawav=-i*koa123/(waveno*waveno)
99
100    if(nzlayr .eq. 1)then
101        q1=cqij(1,1)+zero*dqij(1)
102        call surf(q1,gamma,dgamdq)
103        call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
104        dh2xq1=dh2xq1+e13x
105        int1x=cdlog(koawav*dgamdq-q1/dqijdz(1))+2.0d0*h2xq1
106        int2x=2.0d0*dh2xq1-cdlog(-dqijdz(1))
107        call xcadd(sumx,int1x,int2x)
108        rtsumx=0.5d0*sumx
109        bcoefx(1,m)=-rtsumx
110        return
111    end if
112
113        cldqzl=cdlog(-dqijdz(1))
114
115 c     if l equals one then initialize cumulants and caculate a's and
116 c     b's in bottom layer using ground boundary conditions.
117
118        q1=cqij(1,1)+zero*dqij(1)
119        call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
120        dk2xq1=dk2xq1+cneg
121        dk1xq1=dk1xq1-e13x
122        call surf(q1,gamma,dgamdq)
123        cigama=cdlog(i*gamma)
124        call xcadd(numax,cldqzl-cneg+dk2xq1,cigama+cneg+k2xq1)
125        call xcadd(denax,cigama+k1xq1,cldqzl+dk1xq1)
126        acoefx(1,m)=numax-denax
127        call xcadd(denbx,k2xq1,acoefx(1,m)+k1xq1)
128        bcoefx(1,m)=-denbx
129
130 c     calculate contributions to normalizing integrals.
131
132        call xcadd(hyx,k2xq1,acoefx(1,m)+k1xq1)

```

```

133      hyx=bcoefx(1,m)+hyx
134      call xcadd(dhyx,dk2xq1,acofex(1,m)+dk1xq1)
135      dhyx=bcoefx(1,m)+dhyx
136      int1x=cdlog(koawav*dgamdq-q1/dqijdz(1))+2.0d0*hyx
137      int2x=2.0d0*dhyx-cldqzl
138      call xcadd(sumx,int1x,int2x)
139
140      do 9010 l=2,nzlayer-1
141          lm1=l-1
142          cldqzl=cdlog(-dqijdz(l))
143          cldqzm=cdlog(dqijdz(lm1))
144          q1=cqij(l,1)+zero*dqij(l)
145          call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
146          dk2xq1=dk2xq1+cneg
147          dk1xq1=dk1xq1-e13x
148          q2=cqij(lm1,2)+zero*dqij(lm1)
149          call xcdai(-q2,k2xq2,dk2xq2,k1xq2,dk1xq2,h2xq2,dh2xq2)
150          dk2xq2=dk2xq2+cneg
151          dk1xq2=dk1xq2-e13x
152          call xcadd(hyx,k2xq2,acofex(lm1,m)+k1xq2)
153          call xcadd(dhyx,dk2xq2,acofex(lm1,m)+dk1xq2)
154          k1dhyx=k1xq1+dhyx
155          dk1hyx=dk1xq1+hyx
156          dk2hyx=dk2xq1+hyx
157          k2dhyx=k2xq1+dhyx
158          call xcadd(denax,cldqzm+k1dhyx,cldqzl+dk1hyx)
159          call xcadd(numax,cldqzl+cneg+dk2hyx,cldqzm+cneg+k2dhyx)
160          acoefx(l,m)=numax-denax
161          call xcadd(denbx,k2xq1,acofex(l,m)+k1xq1)
162          numbx=bcoefx(lm1,m)+hyx
163          dnumbx=bcoefx(lm1,m)+dhyx
164          bcoefx(l,m)=numbx-denbx
165
166      c      calculate contribution to normalizing integrals.
167
168          int1x=cdlog(-q1/dqijdz(l)+q2/dqijdz(lm1))+2.0d0*numbx
169          call xcadd(sumx,sumx,int1x)
170          call xcadd(dhux,dk2xq1,acofex(l,m)+dk1xq1)
171          dhux=bcoefx(l,m)+dhux
172          int1x=2.0d0*dnumbx-cldqzm
173          int2x=2.0d0*dhux-cldqzl
174          call xcadd(sumx,sumx,int1x)
175          c      xcadd(sumx,sumx,int2x)
176      9010      continue
177
178
179      c      if l equals nzlayer, calculate a's and b's using outgoing

```

```

180 c   wave in top layer.
181
182       nzm1=nzlayr-1
183       q1=cqij(nzlayr,1)+zero*dqij(nzlayr)
184       call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
185       dh2xq1=dh2xq1+e13x
186       q2=cqij(nzm1,2)+zero*dqij(nzm1)
187       call xcdai(-q2,k2xq2,dk2xq2,k1xq2,dk1xq2,h2xq2,dh2xq2)
188       dk2xq2=dk2xq2+cneg
189       dk1xq2=dk1xq2-e13x
190       call xcadd(hyx,k2xq2,acoeffx(nzm1,m)+k1xq2)
191       numbx=bcoeffx(nzlayr-1,m)+hyx
192       bcoeffx(nzlayr,m)=numbx-h2xq1
193
194 c   calculate contribution to cumulants.
195
196       int1x=cdlog(-q1/dqijdz(nzlayr)+q2/dqijdz(nzm1))+
197 $     2.0d0*numbx
198       call xcadd(sumx,sumx,int1x)
199       call xcadd(dhyx,dk2xq2,acoeffx(nzm1,m)+dk1xq2)
200       dnumbx=bcoeffx(nzm1,m)+dhyx
201       int1x=2.0d0*dnumbx-cdlog(dqijdz(nzm1))
202       call xcadd(sumx,sumx,int1x)
203       dhux=bcoeffx(nzlayr,m)+dh2xq1
204       int2x=2.0d0*dhux-cdlog(-dqijdz(nzlayr))
205       call xcadd(sumx,sumx,int2x)
206
207 c   renormalize b's so that height gain integral equals unity.
208
209       rtsumx=.5d0*sumx
210       do 9000 ll=1,nzlayr
211           bcoeffx(ll,m)=bcoeffx(ll,m)-rtsumx
212 9000     continue
213
214 c*****
215
216       l=nzlayr
217       lm1=l-1
218       cldqzm=cdlog(dqijdz(lm1))
219       cldqzl=cdlog(-dqijdz(l))
220
221
222 c   calculate q and associated quantities at bottom of layer l
223
224       q1=cqij(l,1)+zero*dqij(l)
225       call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
226       dh2xq1=dh2xq1+e13x

```

```

227
228      q2=cqij(lm1,2)+zero*dqij(lm1)
229      call xcdai(-q2,k2xq2,dk2xq2,k1xq2,dk1xq2,h2xq2,dh2xq2)
230      dk2xq2=dk2xq2+cneg
231      dk1xq2=dk1xq2-e13x
232
233 c*****
234 c      Caculate acoefx(lm1,m),bcoefx(lm1,m)
235 c      and cumulants using outgoing wave in nzlayr
236 c*****
237      dh2k1x=dh2xq1+k1xq2
238      h2dk1x=h2xq1+dk1xq2
239      h2dk2x=h2xq1+dk2xq2
240      dh2k2x=dh2xq1+k2xq2
241
242      call xcadd(denax,cldqzl-cneg+dh2k1x,cldqzm+cneg+h2dk1x)
243      call xcadd(numax,cldqzm+h2dk2x,cldqzl+dh2k2x)
244
245 c  If in the nzlayr-1 layer the magnitudes of A coefficients from
246 c  integration up and down differ by less than 0.02 dB and their
247 c  phases differ by less than 0.001pi, the A and B coefficients
248 c  obtained from integration up will be accepted.
249
250      tcoef=numax-denax
251      dcoef=tcoef-acoefx(lm1,m)
252      difr=dabs(dreal(dcoef))
253      if (difr .lt. downr) then
254          difi=dimag(dcoef)/pi
255          difi=dabs(difi-dnint(difi/2.d0))*2.d0)
256          if (difi .lt. downi) return
257      end if
258
259      acoefx(lm1,m)=tcoef
260      call xcadd(denbx,k2xq2,acoefx(lm1,m)+k1xq2)
261      bcoefx(lm1,m)=h2xq1-denbx
262
263 c      calculate contributions to cumulants
264
265      sumx=cdlog(-q1/dqijdz(l)+q2/dqijdz(lm1))+2.0d0*h2xq1
266      call xcadd(dhlx,dk2xq2,acoefx(lm1,m)+dk1xq2)
267      dhlx=bcoefx(lm1,m)+dhlx
268      int1x=2.0d0*dh2xq1-cldqzl
269      call xcadd(int1x,sumx,int1x)
270      int2x=2.0d0*dhlx-cldqzm
271      call xcadd(sumx,int1x,int2x)
272
273      do 9030 l=nzlayr-1,2,-1

```

```

274      lm1=l-1
275      cldqzl=cdlog(-dqijdz(l))
276      cldqzm=cdlog(dqijdz(lm1))
277
278 c      calculate q and associated quantities at bottom of layer l
279
280      q1=cqij(l,1)+zero*dqij(l)
281      call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
282      dk2xq1=dk2xq1+cneg
283      dk1xq1=dk1xq1-e13x
284
285      q2=cqij(lm1,2)+zero*dqij(lm1)
286      call xcdai(-q2,k2xq2,dk2xq2,k1xq2,dk1xq2,h2xq2,dh2xq2)
287      dk2xq2=dk2xq2+cneg
288      dk1xq2=dk1xq2-e13x
289      dh2xq2=dh2xq2+e13x
290
291 c*****
292 c      Calculate acoefx(lm1,m),bcoefx(lm1,m) and cumulants
293 c      using continuity relations in terms of the linearly
294 c      independent functions k1 and k2
295
296      call xcadd(hyx,k2xq1,acoefx(l,m)+k1xq1)
297      call xcadd(dhyx,dk2xq1,acoefx(l,m)+dk1xq1)
298      k1dhyx=k1xq2+dhyx
299      dk1hyx=dk1xq2+hyx
300      dk2hyx=dk2xq2+hyx
301      k2dhyx=k2xq2+dhyx
302
303      call xcadd(denax,cldqzl-cneg+k1dhyx,cldqzm+cneg+dk1hyx)
304      call xcadd(numax,cldqzm+dk2hyx,cldqzl+k2dhyx)
305      acoefx(lm1,m)=numax-denax
306      call xcadd(denbx,k2xq2,acoefx(lm1,m)+k1xq2)
307      numbx=bcoefx(l,m)+hyx
308      dnumbx=bcoefx(l,m)+dhyx
309      bcoefx(lm1,m)=numbx-denbx
310
311 c      calculate contributions to cumulants.
312
313      int1x=cdlog(-q1/dqijdz(l)+q2/dqijdz(lm1))+2.0d0*numbx
314      call xcadd(sumx,sumx,int1x)
315      call xcadd(dhlx,dk2xq2,acoefx(lm1,m)+dk1xq2)
316      dhlx=bcoefx(lm1,m)+dhlx
317      int1x=2.0d0*dnumbx-cldqzl
318      int2x=2.0d0*dhlx-cldqzm
319      call xcadd(sumx,sumx,int1x)
320      call xcadd(sumx,sumx,int2x)

```

```

321
322 9030    continue
323
324 c*****
325 c      if l equal to one calculate ground
326 c      contribution to cumulants and renormalize bcoefx's
327
328         l=1
329         q1=cqij(l,1)+zero*dqij(l)
330         call xcdai(-q1,k2xq1,dk2xq1,k1xq1,dk1xq1,h2xq1,dh2xq1)
331         dk2xq1=dk2xq1+cneg
332         dk1xq1=dk1xq1-e13x
333
334         call xcadd(hyx,k2xq1,acoefx(l,m)+k1xq1)
335         call xcadd(dhyx,dk2xq1,acoefx(l,m)+dk1xq1)
336         call surf(q1,gamma,dgamdq)
337         numbx=bcoefx(l,m)+hyx
338         dnumbx=bcoefx(l,m)+dhyx
339         int1x=cdlog(koawav*dgamdq-q1/dqijdz(l))+2.0d0*numbx
340         int2x=2.0d0*dnumbx-cdlog(-dqijdz(1))
341         call xcadd(sumx,sumx,int1x)
342         call xcadd(sumx,sumx,int2x)
343
344 c      renormalize b's so that height gain integrals equal unity.
345
346         rtsumx=.5d0*sumx
347
348         do 9020 ll=1,nzlayr-1
349             bcoefx(ll,m)=bcoefx(ll,m)-rtsumx
350 9020    continue
351
352         bcoefx(nzlayr,m)=-rtsumx
353
354
355     return
356     end

```

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