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SCALING FOR SHOCK RESPONSE OF SUBMARINE EQUIPMENT

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PROBLEM STATEMENT

Previous analysis and studies at the University of Maryland and elsewhere have illustrated the difficulty in relating the equipment response at different charge weights for the same shock factor. A recent study [1] has shown a promising scaling law that appears valid over a wide range of charge sizes for the same hull. This report examines how far this range may be extended for both lower charge weights and higher charge weights; compares linear and parabolic least square fits of the data which are in the form of equipment peak acceleration response versus shock factor; introduces new scaling rules for equipment weight and equipment frequency for single-degree of freedom equipment; and points out the hazards of extrapolating over a wide range of shock factor using a limited range of data.

BACKGROUND

Two different model submarine hulls, each designed for approximately the same depth, were used in this study. Figure 1 shows the diameters, geometrical layout, and scantlings. Models B and F represent a 33-foot diameter hull and a 40.29-foot diameter hull, respectively. Each model shows a single-degree of freedom equipment as frame mounted. The earlier study [1] showed that a five frame model is adequate for the purpose of the investigation. These hulls were modeled as lumped parameter systems with a polygon of 36 sides to represent the cylindrical hull. The University of Maryland "HULL" code, which has been reported and described elsewhere [2], was the principal means used in the creation of the mathematical models.

The absolute acceleration of the equipment mass is the measure of response of the equipment as a function of the shock factor for a given charge weight. The variation of the equipment response is examined to establish trends that may affect equipment design. Figure 2 is a schematic of the shot geometry where the depth of the center line of the hull and the charge are always held at 60 feet so that the cavitation pressure remained the same in all cases. Neutral bouancy is always maintained.

The measure of the shock intensity used herein is the square root of the acoustic approximation of the energy flux density, or shock factor SF, where

$$SF = Q^{1/2}/R \qquad (1)$$

as shown by Cole [3]; Q is the charge weight in pounds of TNT, and R is the distance in feet between the hull and the charge.

Table 1 is a complete listing of all charge weights and geometries used to examine the effect of the charge size studies. Appendix 1 contains the data from which the scaling rules were derived.

The following presents two approaches to scaling: linear scaling and parabolic scaling, both of which employ the method of least squares.

LINEAR SCALING

A. Charge Weight Scaling

Figure 3 shows a plot of a typical linear least square fit for the equipment response as a function of the shock factor for a given charge weight. As observed in the earlier study [1], the acoustic pressure appears to be a key variable. Consequently, a scaling relationship is obtained by dividing the slope of the straight line for charge weight Q_a by the acoustic pressure:

$$\frac{\text{Slope of line a}}{\text{Acoustic pressure}} = s_a Q_a^{1/8}/(SF) \qquad (2)$$

where s_a is the slope of the line for charge weight Q_a. For equal shock factor, the slope of the line for charge weight b, s_b, is related to s_a as

$$s_b = s_a(Q_a/Q_b)^{1/8} \qquad (3)$$

This is called the one-eight charge weight scaling rule.

An initial attempt was made to examine the applicability of the one-eight scaling rule over the range of charge weights from 145-lb to 14,500-lb, and varying the shock factor out to 0.75 in increments of 0.15. Runs were made on models B and F, where the equipment weight was either 15 kips, 20 kips, or 25 kips, and the equipment frequency was either 20 Hz or 30 Hz. Figure 4 is a typical result, where for this case model B has a 20 kip, 20 Hz equipment mounted to a frame as shown in Fig.1. Figure 5 shows the linear least square fits through the data for each charge weight. Note that the symbols on this graph are used to identify each line more clearly and, consequently, are not data points. Figure 6 shows all of the slopes scaled to the 1740-lb charge by the one-eigth scaling rule.

An attempt was made to examine the response data for possible improvement of the 1/8 scaling rule. In particular, the standard deviation of $\Sigma(sQ^n)$ for different values of n are plotted in Fig.7 for model B, 20 kip, 20 Hz equipment response data. We observe that the minimum standard deviation for this plot occurs close to n = 0.1.

This analysis of the data suggests that we change the 1/8th power to 1/10 power in eq.(3). Figure 8 shows a comparison of the scaled slopes for the system described in Fig.4 using both scaling laws. Figure 9 shows the upper and lower bound data points from Fig.4 and the scaled slopes using the 1/10 scaling rule.

While the 1/10 scaling rule provides improvement in the scaled slopes for the charge weights shown in Fig.4, it nevertheless was obtained from an analysis of the response data as contrasted to the physical approach used to develop the 1/8 scaling rule. Further examination of the data in a more narrow charge weight range, as shown in Table 2 where the reference charge weight equals 1,740-lb, showed that the 1/8 scaling rule provided excellent results for charge weights ranging from 600-lb to 7,250-lb of TNT when compared with the 1/10 scaling rule. While the ratio of the mean scaled slope to the standard slope for each rule shown in Table 2 was very close to each other, there was considerable improvement in the standard deviation for the 1/8th scaling of the data in the case of model B, while there is little change in the standard deviation for model F. Consequently, while the 1/10th scaling rule yields improved results over a wide range of charge weights, the following discussion on equipment weight and frequency scaling will employ the narrower range of charge weights shown in Table 2, and eq.(3) will be used for charge weight scaling.

B. Equipment Weight Scaling

A rule for scaling the equipment weight was developed by observing that ordinarily two cubes of different weight scale by a factor of 1/3. Since we are plotting response data as a function of the shock factor which is a function of the square root of the charge weight as shown in eq.(1), the 1/3 scaling of the equipment weight was modified to 1/6. Consequently, given a reference slope s_a for equipment weight W_a, the scaled slope s_b for equipment weight W_b is given by

$$s_b = s_a (W_b/W_a)^{1/6} \qquad (4)$$

The application of the scaling rule is shown from the data in Table 3 for model B and in Table 4 for model F. The 20-kip equipment weight was used to provide the reference slope s_a. The slopes were scaled for the range of charge weights between 600-lb to 7,250-lb TNT, while the equipment frequency was 20 Hz in one case and 30 Hz in the other. The ratio of these scaled slopes to the reference slopes at 20 Hz show excellent agreement. For example, the ratios range from 0.9786 to 1.0131 for model B, 20 Hz equipment, and from 0.9838 to 1.0047 for the 30 Hz equipment as seen in Table 3. Similarly, Table 4 shows the ratios for model F, 20 Hz equipment ranging from 0.9875 to 1.0034, and 0.9899 to 1.0386 for the 30 Hz equipment.

C. Equipment Frequency Scaling

A scaling rule for the equipment frequency was developed from the data since no physical law could be found. This rule begins in the form

$$s_b = s_a (f_b/f_a)^n \qquad (5)$$

where

s_a = reference slope for a given charge weight, equipment weight, and equipment frequency f_a

s_b = slope for the same charge weight, equipment weight, but the equipment frequency is now f_b

The first set of data analysis is shown in Tables 5 and 6 for models B and F, respectively. Here the the charge weights range from 600-lb to 7,250-lb of TNT, the equipment weights are 15, 20, and 25 kips, respectively, and the frequency is either 20 Hz or 30 Hz. The average value of n equals 1.6029 for model B, and 1.6369 for model F.

The second set of data analysis is shown in Table 7 for model B and in Table 8 for model F, each carrying a 20-kip equipment subject to charge weights equal to 1,160-lb TNT and 1,740-lb TNT. The analysis sought the value of n in eq.(5) that fit the actual slopes over the frequency range from 15 Hz to 40 Hz in 5-Hz increments. All combinations of frequency ratios were examined. The average value of n and the standard deviation are shown for each case in Tables 7 and 8. Table 9 shows the results when both runs are averaged for models B and F, respectively. The overall results for both models are also shown in Table 9, where the average value for n equals 1.5598.

Figure 10 shows the data from Table 10 plotted as a scaled pseudo-velocity as a function of the frequency scaled to 40 Hz. The data points at a given frequency ratio tend to be grouped close to each other and the scaled pseudo-velocity coordinates increase with increasing values of frequency as one would expect.

The above analysis of the data suggests that a round-off value n = 1.6 seems reasonable for both models. Thus, eq.(5) reduces to

$$s_b = s_a(f_b/f_a)^{1.6} \qquad (6)$$

The results of the frequency rule were applied to both models B and F over a frequency range from 15 Hz to 40 Hz in increments of 5 Hz. The results are shown in Table 10. Charge weights of 1160-lb and 1740-lb were used for both models. The ratio of the scaled slopes using the frequency scaling rule to the actual scaled slopes show a maximum error of 5.36% for the ratio 0.9464 in the case of model B, and a maximum error of 4.44% for the ratio equal to 0.9556 Note that these maximum errors occur at the highest frequency, i.e., 40 Hz. This suggests that larger errors might occur at frequencies beyond 40 Hz for the two modelled vehicles studied herein.

D. Examples of Linear Scaling

The scaling rules expressed by eqs.(3), (4), and (5) are combined to form the general scaling rule:

$$s_b = s_a(Q_a/Q_b)^{1/8}(W_b/W_a)^{1/6}(f_b/f_a)^{1.6} \qquad (7)$$

This scaling rule was applied to the examples shown in Table 11. There are four examples, respectively, for models B and F. The percent differences between the slope found using eq.(7) and the actual slope are all under 5 % for these examples.

PARABOLIC SCALING

A. The Intercept Rule

Before analyzing the response data using a parabolic least squares fit, consider a typical plot of both a linear least square fit and a parabolic least square fit through the same data (x_i, y_i) as shown in Fig.11. Let the straight line be

$$y = Cx$$

where

$$C = \Sigma x_i y_i / \Sigma x_i^2 \qquad (8)$$

by least squares. Let the parabola be expressed as

$$y = Ax + Bx^2$$

where $A = \{(\Sigma x_i y_i)(\Sigma x_i^4) - (\Sigma x_i^3)(\Sigma x_i^2 y_i)\}/D$ (9)

$B = \{(\Sigma x_i^2)(\Sigma x_i^2 y_i) - (\Sigma x_i^3)(\Sigma x_i y_i)\}/D$ (10)

$D = (\Sigma x_i^2)(\Sigma x_i^4) - (\Sigma x_i^3)^2$

The point of intersection x_c of the two curves is

$$x_c = (C - A)/B \quad (11)$$

Substituting eqs.(8), (9), and (10) into eq.(11) yields,

$$x_c = \Sigma x_i^3/\Sigma x_i^2$$

Thus, x_c has the same value for each set of data provided the x_i values are identical. For most of the runs made in this study, the shock factors range from 0 to 0.75 in increments of 0.15. Consequently, $x_c = 0.61634$ for these cases. It is interesting to observe that x_c is independent of the y_i-values.

B. Generating a Parabolic Curve from Two Reference Data Points

A parabolic least squares fit is a better fit to the data. Suppose there exists a parabolic least squares fit through a given set of data as shown in Fig.12. First consider any two points on the curve: (x_1,y_1) and (x_2,y_2). The equation for the parabola that passes through these two points is

$$y = Ex + Fx^2$$

where

$E = [y_1 x_2^2 - y_2 x_1^2]/[x_1 x_2 (x_2 - x_1)]$ (12)

$F = [x_1 y_2 - x_2 y_1]/[x_1 x_2 (x_2 - x_1)]$ (13)

In the case of the response data, the x-coordinates are the shock factors. Choose 0.3 and 0.6 for the shock factors, i.e., $x_1 = 0.3$ and $x_2 = 0.6$. Equations (12) and (13) reduce to

$E = [4y_1 - y_2]/0.6$ (14)

$$F = [y_2 - 2y_1]/0.18 \quad (15)$$

where the y-coordinates represent the equipment acceleration.

Equations (14) and (15) and the scaling rules can be used to generated a parabola through two new values Y_1 and Y_2, where Y_1 and Y_2 are coordinates obtained by applying eq.(7). For example, suppose we have the least squares parabola through the data for a system composed of charge weight Q_a, equipment weight W_a and frequency f_a. Think of this curve as the parabola shown in Fig.12. Now consider a new system composed of charge weight Q_b, equipment weight W_b, and frequency f_b. Equation (7) predicts the slope for system b. Multiplying both sides of eq.(7) by x yields the following relationship between the ordinate values:

$$Y = y(Q_a/Q_b)^{1/8}(W_b/W_a)^{1/6}(f_b/f_a)^{1.6} = \alpha y \quad (16)$$

Y is the scaled ordinate through which a new parabola will be passed and y is the corresponding point on the reference parabola. Selecting the two coordinate values y_1 and y_2 from the reference parabola, we obtain the new ordinate values as

$$Y_1 = \alpha y_1$$

$$Y_2 = \alpha y_2$$

Y_1 and Y_2 are substituted into eqs.(14) and (15) to find the equation of the new parabola.

C. <u>Examples of Parabolic Scaling</u>

By way of illustration, consider the example of model B, 15 kip, 20 Hz equipment, 1,160-lb charge as providing the reference data. We wish to generate a parabola for model B, 25 kip, 30 Hz equipment, 3,625-lb charge. For this case eq.(16) reduces to

$$Y = 1.52375\ y \quad (17)$$

The parabola through the reference data is

$$y = 40.7703x + 21.6262x^2$$

Choosing $x_1 = 0.3$ and $x_2 = 0.6$, we obtain $y_1 = 14.1774$ and $y_2 = 32.2476$. Substituting these values into eq.(17) yields $Y_1 = 21.6029$ and $Y_2 = 49.1374$. These Y-coordinates are substituted into eqs.(14) and (15) to yield the scaled parabola

$$Y = 62.1239x + 32.9530x^2$$

which compares with the actual parabola

$$Y = 65.0874x + 28.9880x^2$$

Figure 13 shows the plot of each of these curves. There is a close fit between the scaled parabola (Y_{est}) and the parabola generated through the data (Y_{data}).

The second example examines the case of model B, 25 kip, 30 Hz equipment, 1,740-lb charge used as the reference data, to project a parabola for model B, 20 kip, 20 Hz equipment, 900-lb charge. The projected parabola is

$$Y = 47.8682x + 4.8035x^2$$

while the actual parabola is

$$Y = 43.0994x + 15.2513x^2$$

Figure 14 shows these two parabolas that yield reasonably close curves.

D. Extrapolation to Shock Factor of 1.0

Consider the straight line fit and the parabolic fit through the data for model B, 20 kip, 20 Hz equipment subject to a charge weight of 3625-lb of TNT as shown in Fig.15. Note that the shock factor extends to unity in this case. It is apparent that if one were to use the straight line for design purposes, the prescribed accelerations for lower shock factors would be conservative relative to the parabolic curve, while the opposite holds at the high end of the shock factors.

Consider the case where there is a limited amount of data collected at low shock factors and we wish to extrapolate to higher shock factors. For example, suppose the data points exist at shock factors of 0.15, 0.30, and 0.45, and we use least squares to generate the linear fit and the parabolic fit through the data as shown in Fig.16(a). The figure also contains the actual linear and parabolic curves through the data taken out to a shock factor of one. Clearly there is a noticible difference between the extrapolated curves and the actual curves. This difference decreases as one would expect as more data points are included in the generation of the extracted curves as shown in Figs.16(b) and (c). Similar results are shown in Fig.17 for the same equipment in model F subject to a 1,160-lb charge weight.

SUMMARY AND CONCLUSIONS

The results of this study have demonstrated that useful information may be obtained by using a computer as an initial surrogate for shock testing purposes. These results show the relative changes in shock design values for different boats and attack geometries. It is emphasized that the test sections were small in size and devoid of typical equipment present in a real compartment. Consequently, the results provide only trends in shock design values rather than absolute design numbers.

Large amounts of computer generated data were collected for two submarine models each of which contained a single-degree of freedom frame-mounted equipment. One of the interesting observations made from the many plots of the least squares linear and parabolic curves is that the intersection of each pair of curves is independent of the ordinate values that were used to generate the curves, if the data were obtained at the same abscissa values.

The study attempted to develope some scaling rules for handeling field data that may exist for a given class of boat. The intent of these scaling rules is to allow greater useage of these data for different equipment subject to a variety of charge weights. Hence, a general scaling rule was developed that includes the charge weight, the equipment weight, and the equipment frequency. The general scaling rule was applied in two different ways. The first approach used eq.(7) assuming linear scaling only. The examples presented herein showed rather good results in projecting the slopes for a new model using the data from an existing model. The second approach to scaling used eq.(7) for generating a parabolic curve for a new system from the parabolic curve of an exiting system. These projected curves showed excellent corrolation with the existing parabolic curves.

A few words of caution are included with regard to using a linear curve to project shock design values for large shock factors. It is seen from the graphs that such a practice could lead to under-designing a new equipment. Likewise, it is shown that care must be exercised when one attempts to use a limited set of response data at low shock factors to generate straight lines or parabolas to the region of high shock factors.

Further efforts are needed to develope a rational scaling law for changes in the equipment frequency. While the data generated in this study suggests the 1.6 factor over a limited range in frequency, physical understanding would be useful for a larger frequency range and for other boat designs.

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APPENDIX

Summary of Data

MODEL B Raw Data

Table 1: 15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	6.3956	6.5686	6.5073	6.2041	6.0219	5.4830	6.0100
0.30	15.5715	14.5199	13.9880	13.5847	13.2670	12.6493	11.1395
0.45	24.8191	23.8275	23.0325	22.3908	22.2268	18.9597	18.2307
0.60	35.6442	33.3863	32.1668	31.0490	30.3143	27.3157	23.9631
0.75	42.7005	42.6560	42.7179	41.4267	39.4709	36.0714	30.8541

Table 2: 20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	6.3971	6.2860	6.2250	5.9436	5.7879	5.3364	5.8968
0.30	15.1429	14.0986	13.6857	13.2914	12.9770	12.0856	10.8218
0.45	23.6159	22.7440	22.0331	21.8526	21.3429	18.5255	17.7707
0.60	33.9585	31.7868	30.6274	29.6580	29.3813	26.1935	23.1815
0.75	40.7400	40.5836	40.8301	39.4717	37.5882	34.3608	29.8390

Table 3: 25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	6.2523	6.088888	6.0044	5.814763	5.6625	5.3142	5.7087
0.30	14.7866	14.07268	13.3875	13.00313	12.6960	11.5630	10.7310
0.45	22.4986	22.21325	21.9438	21.34868	20.8516	18.1265	17.1508
0.60	32.3408	30.30576	29.1889	28.28132	28.5723	25.5814	22.6494
0.75	38.8583	38.60455	38.9767	37.62002	35.8472	33.5140	29.1705

Table 4: 15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	13.3887	13.04272	12.8666	12.45935	12.1329	10.8451	11.1842
0.30	31.7052	30.17431	28.6857	27.8618	27.2055	24.4456	22.4415
0.45	46.7465	48.43918	46.9695	45.67122	44.5929	38.8641	34.1475
0.60	66.4412	62.73582	60.0099	58.72572	61.0938	54.7966	48.5730
0.75	79.9471	79.75784	79.7418	77.19367	73.4683	71.6140	62.5634

Table 5: 20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	12.7695	12.40776	12.2366	11.84915	11.5385	10.7133	10.8390
0.30	30.7301	28.71582	27.2771	26.49537	25.8732	23.2793	21.5012
0.45	44.3716	46.14995	44.7531	43.51862	42.4929	36.9790	32.4908
0.60	63.0184	59.4673	56.8628	55.85501	58.2238	52.1899	46.2318
0.75	75.6253	75.56751	75.5542	73.12894	69.6130	68.2240	59.5528

Table 6: 25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	12.1729	11.84982	11.6715	11.29958	11.0003	10.1368	10.2517
0.30	29.2534	27.31395	26.0330	25.27986	24.6671	22.1540	20.6460
0.45	42.1999	43.94956	42.6144	41.44941	40.4741	35.2021	30.9765
0.60	59.7403	56.41921	54.0933	53.16018	55.4650	49.6744	43.9811
0.75	71.5365	71.55432	71.6469	69.34348	66.1916	64.9949	56.6587

MODEL F Raw Data

Table 1: 15 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	7.8954	7.9567	7.6863	7.2106	7.1291	6.1514	6.0506
0.30	18.1235	16.7513	15.8125	15.9345	15.6288	15.1816	13.3299
0.45	28.8230	27.5148	26.9363	26.0286	26.5787	22.6295	21.9821
0.60	39.3150	39.1351	37.6583	36.6869	35.8768	32.4759	30.3252
0.75	48.0183	48.8537	49.2544	47.8262	46.6812	43.0371	36.9290

Table 2: 20 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	7.6006	7.6837	7.4226	6.9690	6.8157	6.0474	5.9362
0.30	17.4612	16.1299	15.2183	15.3387	15.0171	14.7941	12.8814
0.45	27.7596	26.5021	25.9723	25.0919	25.4454	21.7966	21.2668
0.60	37.7353	37.7127	36.2894	35.3497	34.4744	31.3103	29.2737
0.75	46.2129	46.9098	47.4665	46.0933	44.8990	41.5190	35.5876

Table 3: 25 kip, 20 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	7.3201	7.4187	7.1653	6.8329	6.6597	5.9444	5.7230
0.30	16.8233	15.6359	15.4732	15.0464	14.7190	14.3061	12.4578
0.45	26.7302	25.5608	25.0337	24.1837	24.5306	21.1075	20.5714
0.60	36.2431	36.3354	34.9587	34.0550	33.2822	30.1752	28.2534
0.75	44.4674	45.0318	45.7280	44.4152	43.2724	40.0407	34.2873

Table 4: 15 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	15.6560	15.6955	15.1687	14.6979	14.3238	12.7859	11.9343
0.30	36.1090	33.6002	33.2585	32.3418	31.6423	29.1711	26.0984
0.45	53.8523	51.8264	52.6217	51.1427	52.0236	45.4052	41.9457
0.60	77.1726	73.1771	70.3431	68.7074	67.3839	63.9906	57.3110
0.75	93.3833	95.0017	92.5846	89.6961	87.0836	83.9613	73.1756

Table 5: 20 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	15.0276	15.0960	14.0275	14.1371	13.7757	11.8305	10.9755
0.30	34.7248	32.2892	30.7098	31.0854	30.4166	26.9123	25.1295
0.45	51.7723	49.8015	48.6845	49.1839	50.0847	41.9781	40.1706
0.60	73.9152	69.7237	64.1160	65.1656	63.1655	59.2507	55.1499
0.75	90.3126	91.0161	84.9971	85.0584	81.8023	77.6631	70.3959

Table 6: 25 kip, 30 Hz

SF	600#	900#	1160#	1450#	1740#	3625#	7250#
0.15	14.4484	14.5159	14.0275	13.5944	13.2726	11.8306	10.7928
0.30	33.3854	31.0204	30.7098	29.8691	29.2302	26.9124	24.1915
0.45	49.7593	47.8747	48.6845	47.3078	48.2118	41.9781	38.6897
0.60	70.8399	66.8726	64.1160	62.6621	60.7416	59.2508	53.0729
0.75	86.5070	87.1686	84.9972	81.5211	78.6681	77.6632	67.7050

Model B: 20,000 lb Equipment

1160 lb Charge Weight

SF	15 Hz	20 Hz	25 Hz	30 Hz	35 Hz	40 Hz
0.15	3.819990	6.226134	9.087637	12.238800	15.473480	18.560340
0.30	8.032893	13.681590	20.251190	27.268910	34.495620	41.368350
0.45	13.584080	22.036340	33.151610	44.759670	56.390820	67.543670
0.60	18.987120	30.651740	42.471190	56.907090	71.583040	85.560710
0.75	25.162860	40.820010	56.671620	75.535850	94.524930	112.600300
Slope	31.806157	51.685496	73.004571	97.740999	122.738665	146.566350

1740 lb Charge Weight

SF	15 Hz	20 Hz	25 Hz	30 Hz	35 Hz	40 Hz
0.15	3.536816	5.785707	8.564591	11.534020	14.572360	17.477400
0.30	7.792947	12.972710	19.204730	25.864690	32.680320	39.197090
0.45	13.059040	21.342920	31.483390	42.492880	53.543880	64.089390
0.60	18.006650	29.613280	43.477870	58.673440	73.949280	88.507390
0.75	23.266290	37.572980	52.038400	69.585250	87.485980	104.474800
Slope	29.897925	48.736716	69.760933	93.740855	118.035353	141.156765

MODEL F: 20,000 lb Equipment

1160 lb Charge Weight

SF	15 Hz	20 Hz	25 Hz	30 Hz	35 Hz	40 Hz
0.15	4.5009	7.4226	10.7150	14.0275	18.6056	22.6535
0.30	9.2948	15.2183	23.4892	30.7098	40.7557	49.5455
0.45	15.8356	25.9723	37.2288	48.6845	64.4935	77.9303
0.60	22.1323	36.2894	51.0868	64.1160	84.9711	103.2119
0.75	28.9117	47.4665	66.7710	84.9971	112.2898	135.7298
Slope	36.8103	60.3959	85.7675	109.4485	144.8401	175.3978

1740 lb Charge Weight

SF	15 Hz	20 Hz	25 Hz	30 Hz	35 Hz	40 Hz
0.15	4.1204	6.8157	10.1127	13.7757	17.6195	21.4542
0.30	9.1132	15.0171	22.3452	30.4166	38.7149	47.0765
0.45	15.4792	25.4454	36.7982	50.0847	63.8880	77.3359
0.60	21.1495	34.4744	49.1826	63.1655	81.0514	98.7152
0.75	27.3474	44.8990	63.1937	81.8023	104.1352	126.3284
Slope	35.1660	57.6459	82.1693	107.4590	137.1630	166.5597

Table 1: Charge Weights and Standoff Distances

Standoff Distances (ft)						
Charge Weight (lbs TNT)	Shock Factor					
	0.15	0.30	0.45	0.60	0.75	1.00
145	80.3	40.1	26.8	20.1	16.1	12.0
364	127.2	63.6	42.4	31.8	25.4	19.1
600	163.3	81.6	54.4	40.8	32.7	24.5
900	200.0	100.0	66.7	50.0	40.0	30.0
1160	227.1	113.5	75.7	56.8	45.4	34.1
1450	253.9	126.9	84.6	63.5	50.8	38.1
1740	278.1	139.0	92.7	69.5	55.6	41.7
3625	401.4	200.7	133.8	100.3	80.3	60.2
7250	567.6	283.8	189.2	141.9	113.5	85.1
10875	695.2	347.6	231.7	173.8	139.0	104.3
14500	802.8	401.4	267.6	200.7	160.6	120.4

Table 2: Analysis of Charge Weight Scaling

Model B: 20 kip, 20 Hz Equipment			
Charge Weight (lbs TNT)	Slope	Scaled Slope 1/8 Rule	Scaled Slope 1/10 Rule
600	54.1896	47.4369	48.7165
900	52.4582	48.3087	49.1114
1160	51.6795	49.1255	49.6260
1450	50.1909	49.0600	49.2841
* 1740	48.6348	48.6348	48.6348
3625	43.8378	48.0500	47.1764
7250	39.1241	46.7647	45.1256
mean		48.1972	48.2393
standard deviation (σ)		0.8623	1.5803
mean/m_{1740}		0.9910	0.9919

Model F: 20 kip, 20 Hz Equipment			
Charge Weight (lbs TNT)	Slope	Scaled Slope 1/8 Rule	Scaled Slope 1/10 Rule
600	61.5524	53.8822	55.3357
900	61.1939	56.3533	57.2898
1160	60.3959	57.4111	57.9960
1450	58.7621	57.4381	57.7005
* 1740	57.6459	57.6459	57.6459
3625	52.5893	57.6424	56.5944
7250	47.3372	56.5818	54.5986
mean		56.7078	56.7373
standard deviation (σ)		1.2483	1.2072
mean/m_{1740}		0.9837	0.9842

* Reference Charge Weight

Table 3: Verification of Equipment Weight Scaling for Model B

Scaled Slopes for Equipment Frequency = 20 Hz							
Equipment Weight (kips)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
15	54.0802	52.4443	51.5109	49.8978	48.2771	43.5900	38.4860
* 20	54.1896	52.4582	51.6795	50.1909	48.6348	43.8378	39.1241
25	53.7547	52.2242	50.5736	49.9562	48.7033	44.3734	39.6378
Ratio of Slopes							
15	0.9980	0.9997	0.9967	0.9942	0.9926	0.9943	0.9837
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	0.9920	0.9955	0.9786	0.9953	1.0014	1.0122	1.0131
Scaled Slopes for Equipment Frequency = 30 Hz							
Equipment Weight (kips)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
15	101.9659	100.3369	98.1942	95.4414	93.8120	87.0673	76.9037
* 20	101.5200	99.8783	97.7300	95.0861	93.5423	87.0410	76.8491
25	99.8795	98.3512	96.3903	93.7959	92.4132	86.0157	75.9473
Ratio of Slopes							
15	1.0044	1.0046	1.0047	1.0037	1.0029	1.0003	1.0007
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	0.9838	0.9847	0.9863	0.9864	0.9878	0.9882	0.9883

* Reference slope

Table 4: Verification of Equipment Weight Scaling for Model F

Scaled Slopes for Equipment Frequency = 20 Hz							
Equipment Weight (kips)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
15	61.0000	60.6360	59.7358	58.1200	57.1953	51.9332	46.7648
* 20	61.5520	61.1940	60.3959	58.7620	57.6459	52.5893	47.3372
25	61.4510	61.1260	60.5987	58.8480	57.7671	52.6850	47.4037
Ratio of Slopes							
15	0.9910	0.9909	0.9891	0.9891	0.9922	0.9875	0.9875
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	0.9984	0.9989	1.0034	1.0015	1.0021	1.0018	1.0014

Scaled Slopes for Equipment Frequency = 30 Hz							
Equipment Weight (kips)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
15	118.4300	116.2420	113.6711	110.4670	108.4473	102.0326	90.6504
* 20	119.6390	116.7340	109.4485	110.2810	107.4590	99.0191	91.4334
25	119.0610	116.1830	113.5957	109.8920	107.2708	102.7712	91.3422
Ratio of Slopes							
15	0.9899	0.9958	1.0386	1.0017	1.0092	1.0304	0.9914
20	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
25	0.9952	0.9953	1.0379	0.9965	0.9982	1.0379	0.9990

* Reference slope

Table 5: Analysis of Equipment Frequency Scaling Rule on Model B Scaled Slopes

$$s_b = s_a \left[\frac{f_b}{f_a}\right]^n$$

Equipment Weight = 15 kips							
Equipment Frequency (Hz)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
20	56.7364	55.0201	54.0409	52.3485	50.6482	45.7309	40.3762
30	106.9740	105.2650	103.0170	100.1290	98.4290	91.3436	80.6810
n	1.5641	1.6001	1.5911	1.5995	1.6387	1.7063	1.7073

Equipment Weight = 20 kips							
Equipment Frequency (Hz)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
20	54.1896	52.4582	51.6795	50.1905	48.6348	43.8378	39.1241
30	101.5200	99.8783	97.7300	95.0861	93.5423	87.0410	76.8491
n	1.5483	1.5881	1.5714	1.5759	1.6131	1.6916	1.6650

Equipment Weight = 25 kips							
Equipment Frequency (Hz)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
20	51.7547	50.3176	49.7273	48.1324	46.9253	42.7534	38.1907
30	96.2331	94.7606	92.8713	90.3716	89.0394	82.8755	73.1747
n	1.5297	1.5612	1.5406	1.5537	1.5797	1.6324	1.6037

Table 6: Analysis of Equipment Frequency Scaling Rule on Model F Scaled Slopes

$$s_b = s_a \left[\frac{f_b}{f_a} \right]^n$$

Equipment Weight = 15 kips							
Equipment Frequency (Hz)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
20	63.9960	63.6140	62.6697	60.9750	60.0044	54.4839	49.0427
30	124.2470	121.9510	119.2541	115.8930	113.7737	107.0439	95.1027
n	1.6363	1.6050	1.5868	1.5839	1.5562	1.6656	1.6333

Equipment Weight = 20 kips							
Equipment Frequency (Hz)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
20	61.5520	61.1940	60.3959	58.7620	57.6459	52.5893	47.3372
30	119.6390	116.7340	109.4485	110.2810	107.4590	99.0191	91.4334
n	1.6391	1.5929	1.4663	1.5526	1.5360	1.5607	1.6236

Equipment Weight = 25 kips							
Equipment Frequency (Hz)	Charge Weight (lbs TNT)						
	600	900	1160	1450	1740	3625	7250
20	59.2080	58.8940	58.3864	56.7000	55.6582	50.7616	45.6731
30	114.7140	111.9410	109.4486	105.8800	103.3546	99.0193	88.0075
n	1.6312	1.5839	1.5498	1.5403	1.5265	1.6479	1.6177

Table 7: Analysis of Equipment Frequency Rule

Model B: 20 kip Equipment		
Frequency Ratio	n - Values for Scaling	
	Charge Weight (lbs TNT)	
	1160	1740
40/15	1.5576	1.5824
35/15	1.5938	1.6207
30/15	1.6197	1.6486
40/20	1.5036	1.5342
35/20	1.5455	1.5806
25/15	1.6265	1.6587
40/25	1.4827	1.4996
30/20	1.5714	1.6132
35/25	1.5441	1.5630
40/30	1.4081	1.4229
20/15	1.6877	1.6986
25/20	1.5476	1.6072
30/25	1.6005	1.6205
35/30	1.4774	1.4950
40/35	1.3282	1.3397
average n	1.5396	1.5621
standard deviation (σ)	0.0904	0.0905

Table 8: Analysis of Equipment Frequency Rule

Model F: 20 kip Equipment		
Frequency Ratio	n - Values for Scaling	
	Charge Weight (lbs TNT)	
	1160	1740
40/15	1.5918	1.5857
35/15	1.6167	1.6064
30/15	1.5721	1.6115
40/20	1.5381	1.5307
35/20	1.5631	1.5490
25/15	1.6559	1.6614
40/25	1.5222	1.5033
30/20	1.4663	1.5360
35/25	1.5573	1.5228
40/30	1.6393	1.5324
20/15	1.7211	1.7180
25/20	1.5717	1.5885
30/25	1.3373	1.4717
35/30	1.8175	1.5833
40/35	1.4336	1.4542
average n	1.5736	1.5631
standard deviation (σ)	0.1152	0.0699

Table 9: Combined Data Anlysis of Equipment Frequency Rule

	Model B	Model F	Models B & F
Average n	1.5526	1.5683	1.5596
Standard Deviation (σ)	0.0919	0.0938	0.0921

Table 10: Analysis of Equipment Frequency Rule

$$S_b = S_a \left[\frac{f_b}{f_a}\right]^{1.6}$$

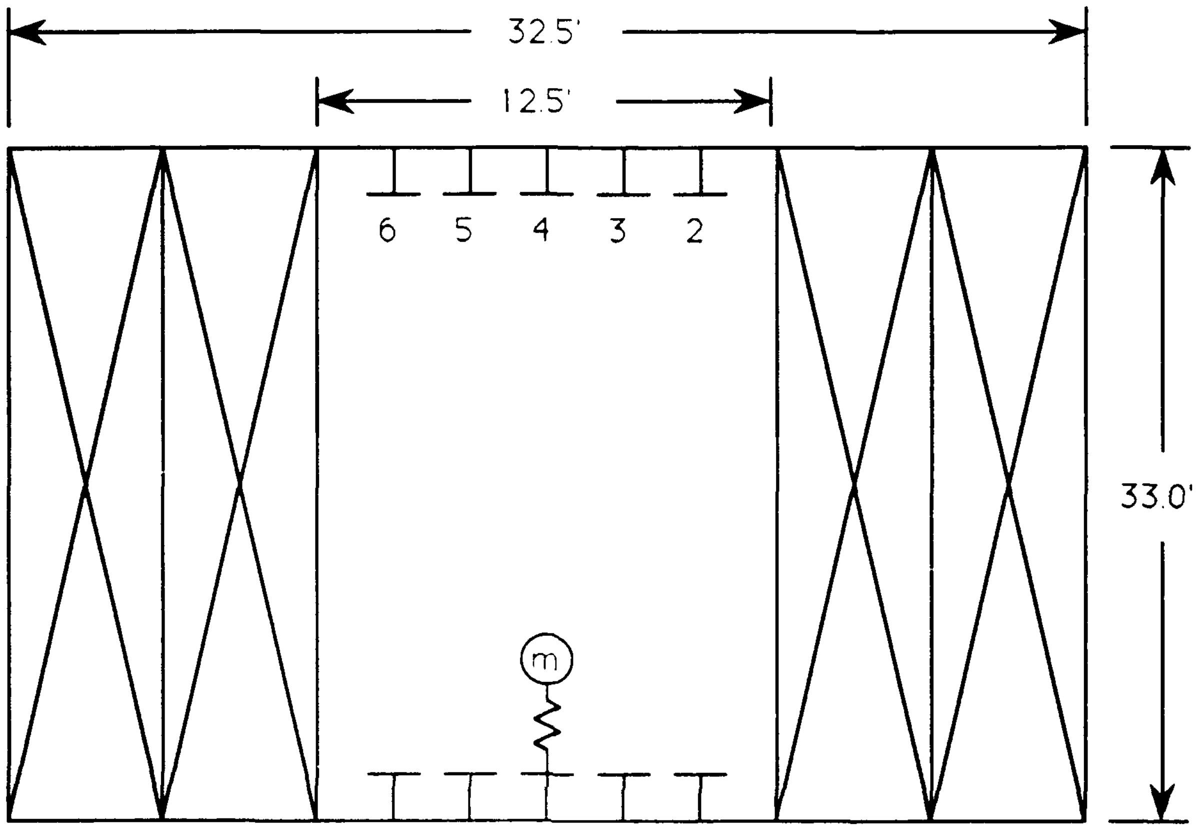
Model B: Charge Weight = 1160 lbs TNT			
Frequecy (Hz)	Actual Slope	Scaled Slope	Ratio
15	31.8062	72.0227	0.9866
20	51.6855	73.8626	1.0118
* 25	73.0046	73.0046	1.0000
30	97.7410	73.0107	1.0001
35	122.7387	71.6435	0.9814
40	146.5664	69.0943	0.9464
Model F: Charge Weight = 1740 lbs TNT			
Frequency (Hz)	Actual Slope	Scaled Slope	Ratio
15	35.1660	79.6308	0.9691
20	57.6459	82.3805	1.0026
* 25	82.1693	82.1693	1.0000
30	107.4590	80.2699	0.9769
35	137.1630	80.0630	0.9744
40	166.5597	78.5196	0.9556

* Reference slope

Table 11: Examples Using Equation (7)

s_a	Q_a (lb-TNT)	W_a (kips)	f_a (Hz)	Q_b (lb-TNT)	W_b (kips)	f_b (Hz)	s_b	s_{actual}	% diff
Model B									
97.7300	1160	20	30	1740	15	20	50.9441	50.6482	0.58
42.7534	3625	25	20	1160	20	35	125.2678	122.7387	2.06
42.7534	3625	25	20	1740	20	15	30.6948	29.8979	2.70
100.1290	1450	15	30	3625	25	20	42.8644	42.7534	0.25
Model F									
s_a	Q_a (lb-TNT)	W_a (kips)	f_a (Hz)	Q_b (lb-TNT)	W_b (kips)	f_b (Hz)	s_b	s_{actual}	% diff
109.4485	1160	20	30	1740	15	20	57.0526	60.0040	4.92
50.7620	3625	25	20	1160	20	35	148.7327	144.8401	2.70
50.7620	3625	25	20	1740	20	15	36.4496	35.1660	3.60
115.8930	1450	15	30	3625	25	20	49.6128	50.7620	2.26

Model B with internal SDOF equipmment



Section

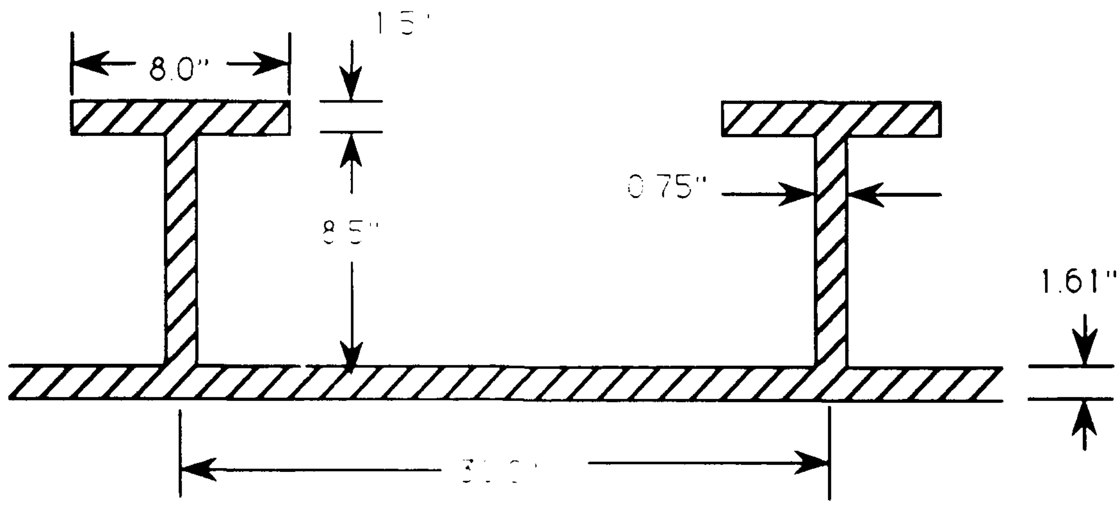
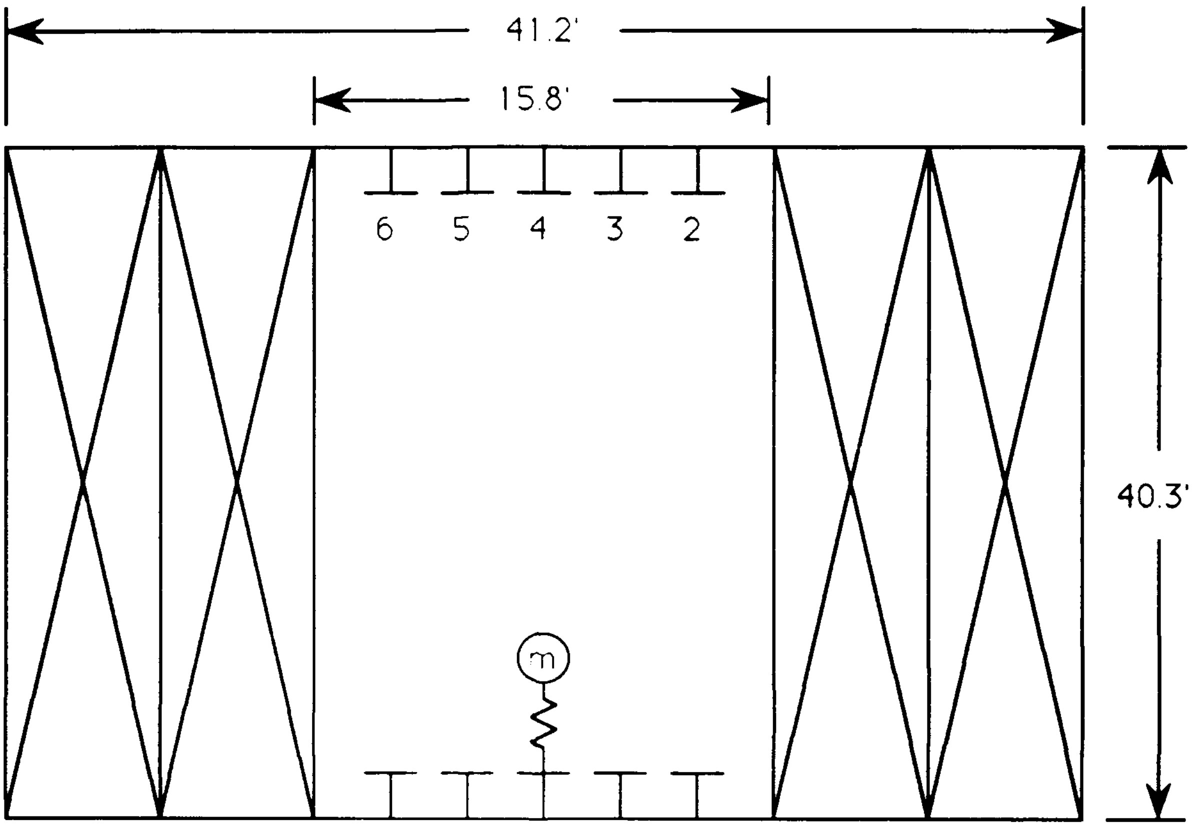


Figure 1A

Model F with internal SDOF equipmment



Section

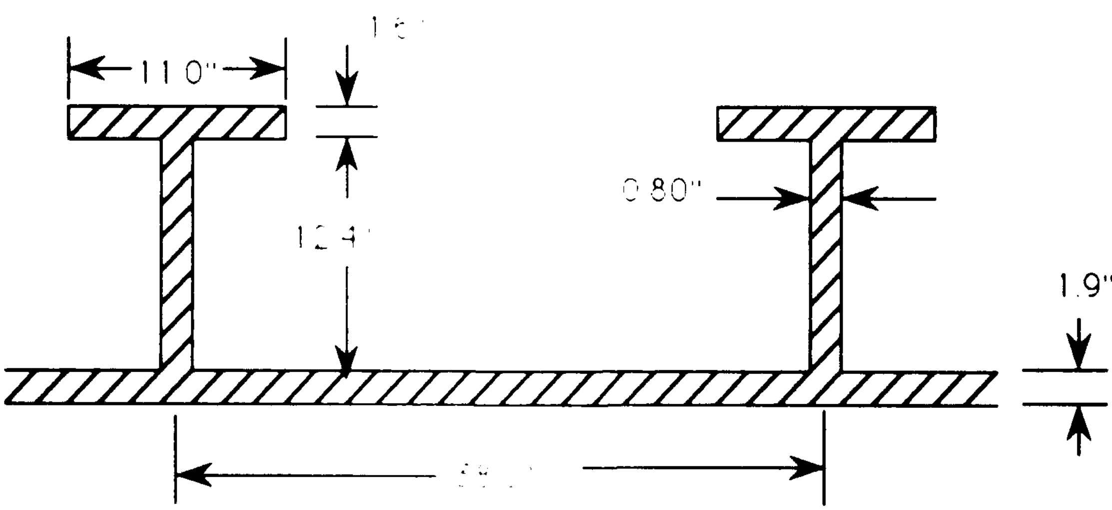


Figure 1B

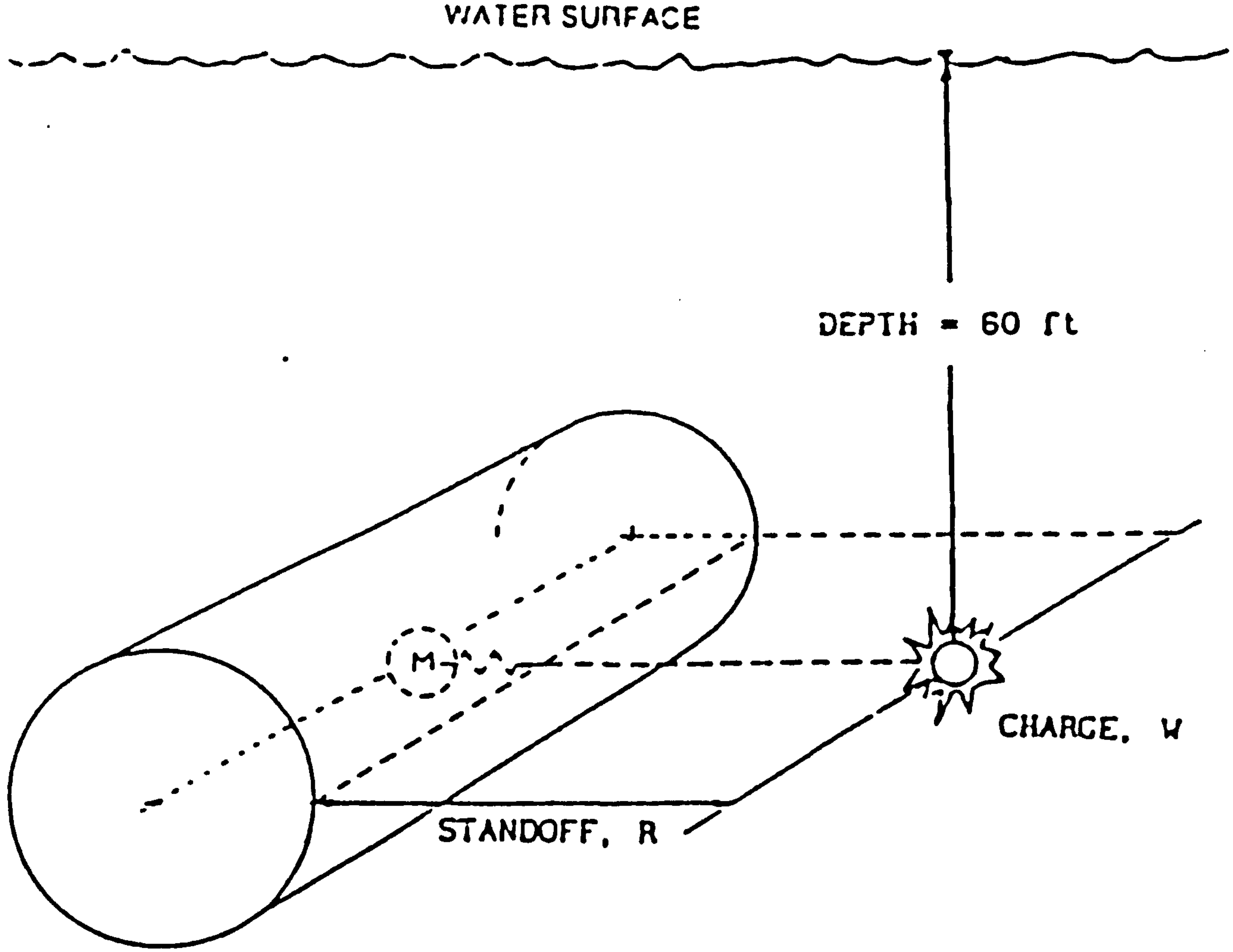
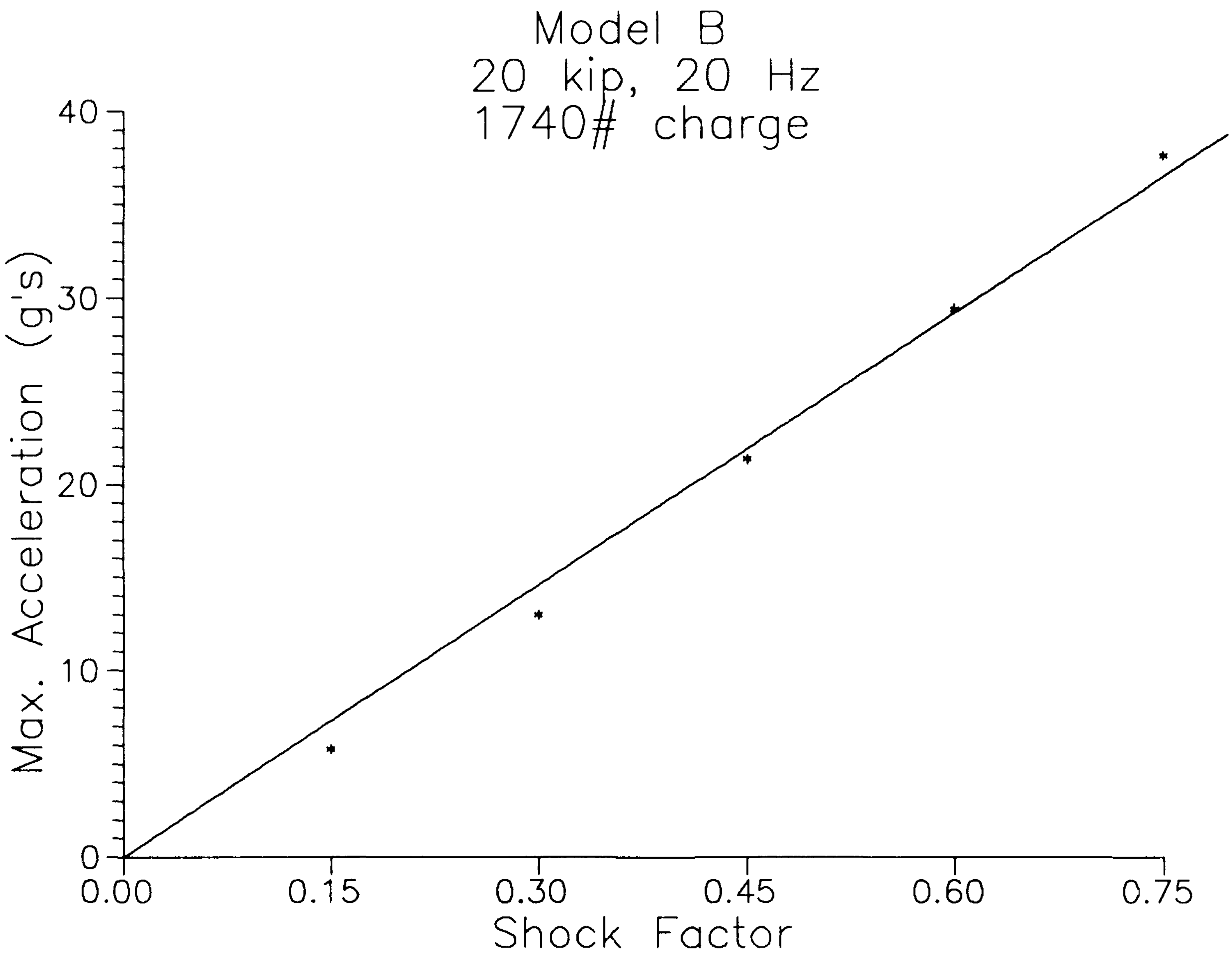


Figure 2



Model B
20 kip, 20 Hz
1740# charge
Max. Acceleration (g's)
0
10
20
30
40
0.00
0.15
0.30
0.45
0.60
0.75
Shock Factor

Figure 3

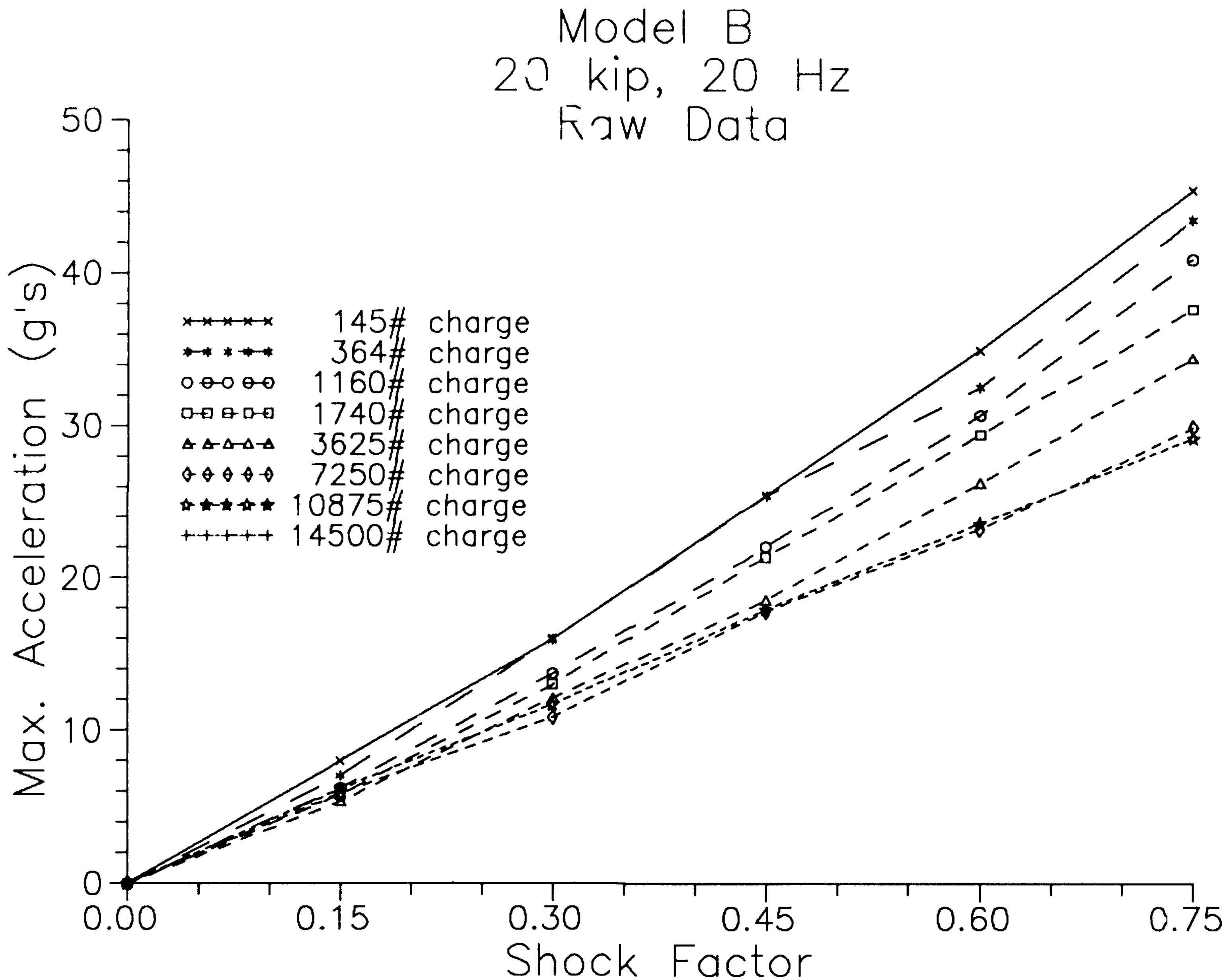

Model B
20 kip, 20 Hz
Raw Data
Max. Acceleration (g's)
Shock Factor
145# charge
364# charge
1160# charge
1740# charge
3625# charge
7250# charge
10875# charge
14500# charge
0
10
20
30
40
50
0.00
0.15
0.30
0.45
0.60
0.75

Figure 4

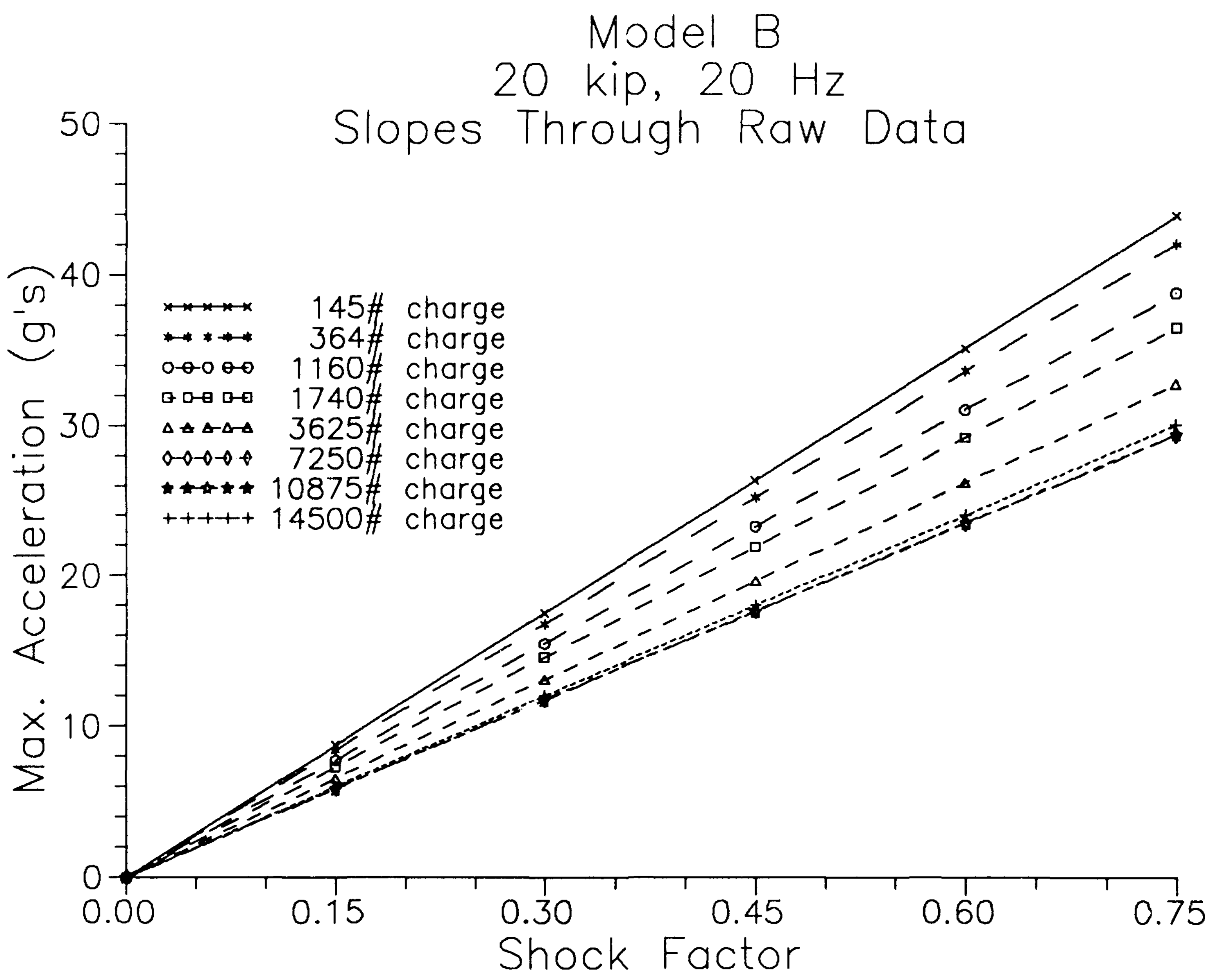
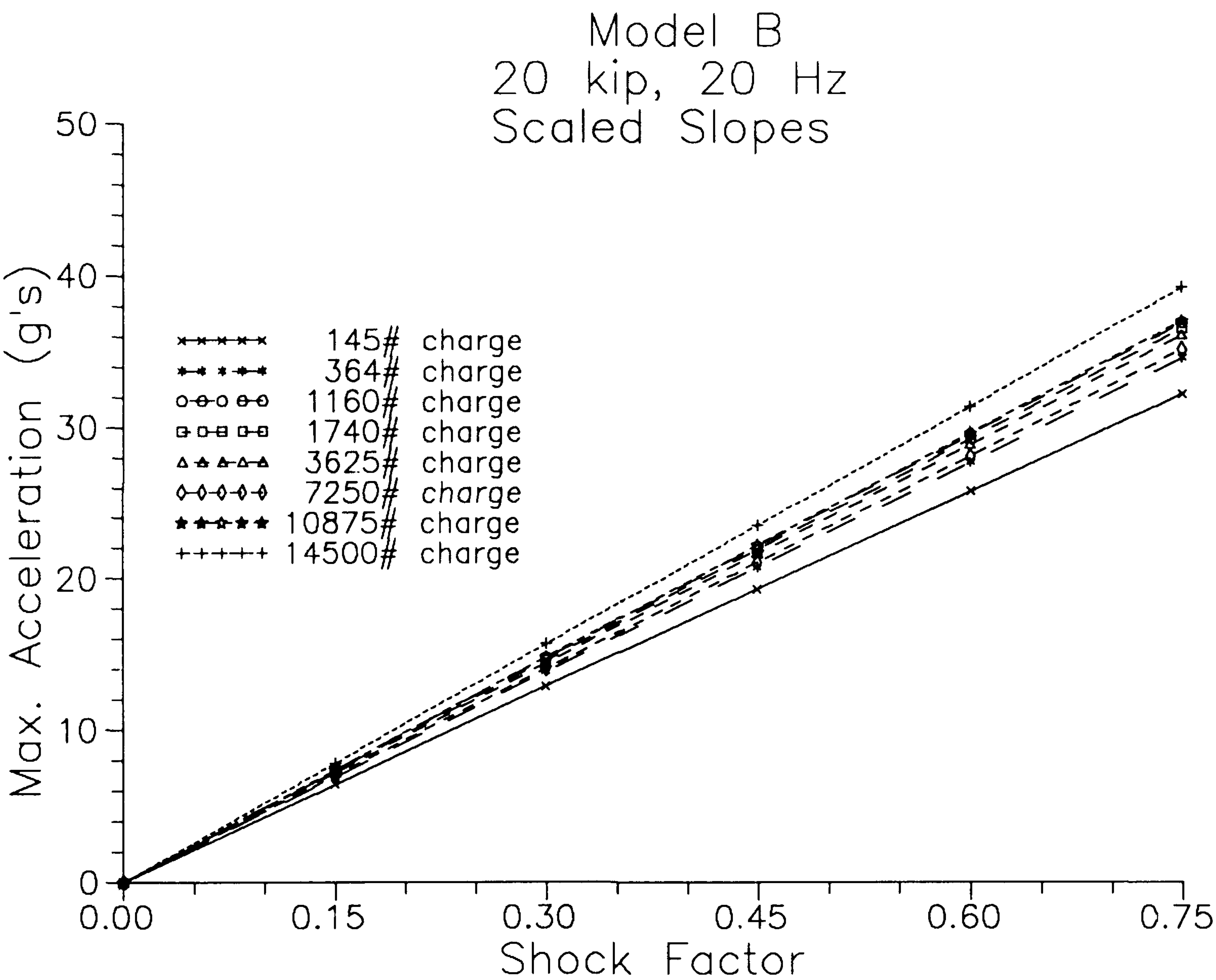
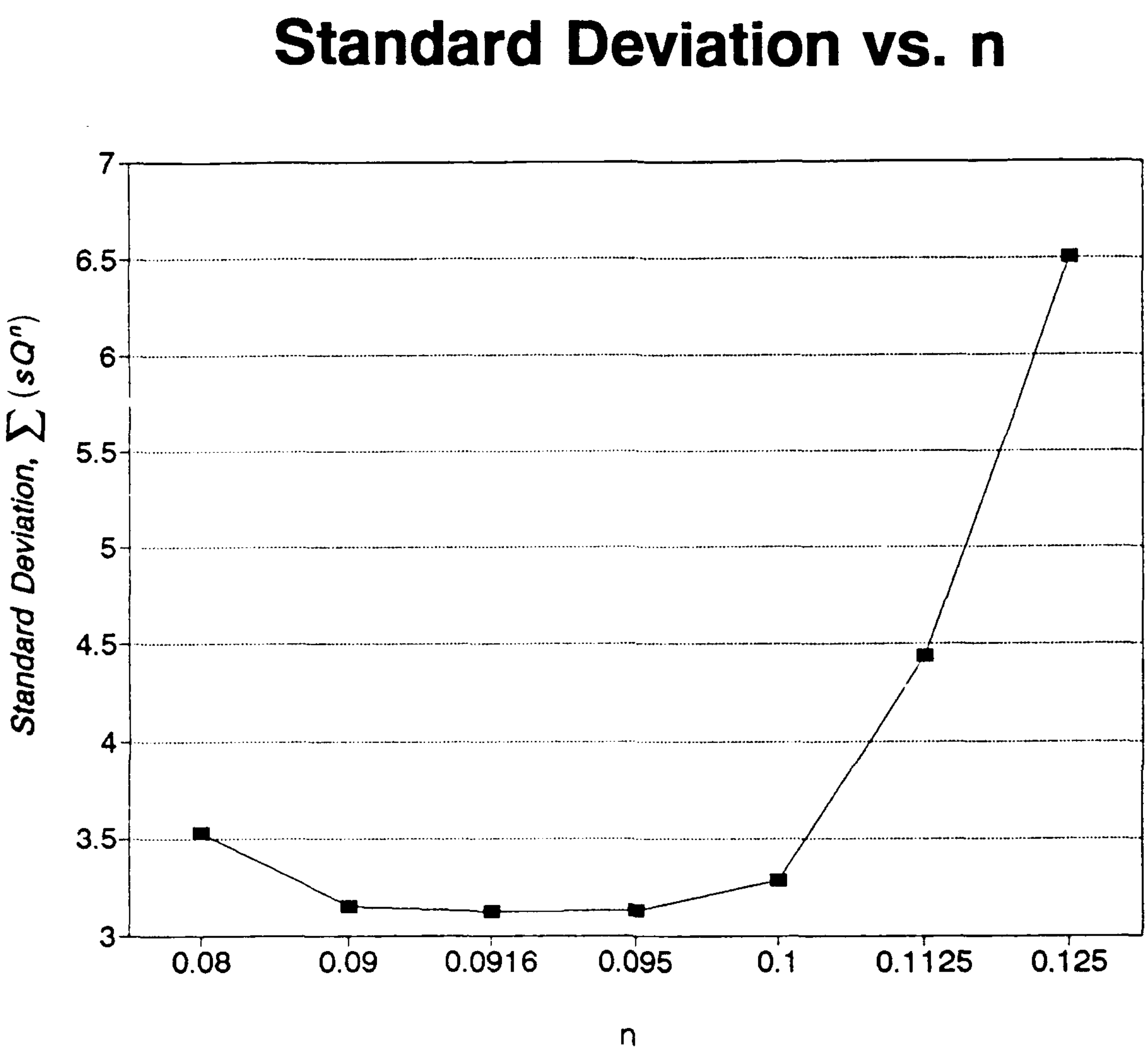

Model B
20 kip, 20 Hz
Slopes Through Raw Data
145# charge
364# charge
1160# charge
1740# charge
3625# charge
7250# charge
10875# charge
14500# charge
Max. Acceleration (g's)
Shock Factor
0
10
20
30
40
50
0.00
0.15
0.30
0.45
0.60
0.75

Figure 5



Model B
20 kip, 20 Hz
Scaled Slopes
145# charge
364# charge
1160# charge
1740# charge
3625# charge
7250# charge
10875# charge
14500# charge
Max. Acceleration (g's)
Shock Factor
0
10
20
30
40
50
0.00
0.15
0.30
0.45
0.60
0.75

Figure 6



Standard Deviation vs. n
Standard Deviation, $\sum (sQ^n)$
7
6.5
6
5.5
5
4.5
4
3.5
3
0.08
0.09
0.0916
0.095
0.1
0.1125
0.125
n

Figure 7

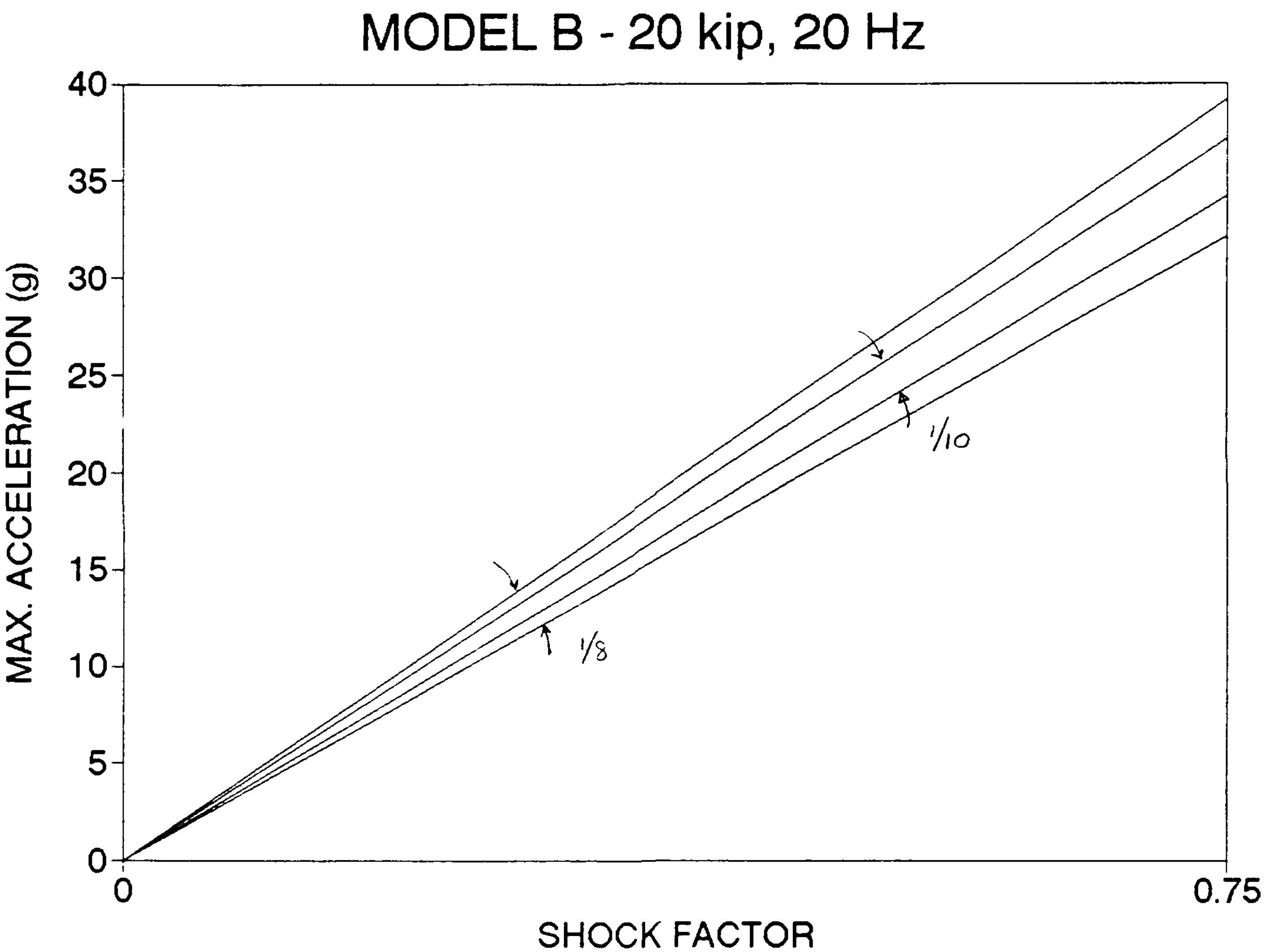

MODEL B - 20 kip, 20 Hz
MAX. ACCELERATION (g)
40
35
30
25
20
15
10
5
0
0
0.75
SHOCK FACTOR
1/10
1/8

Figure 8

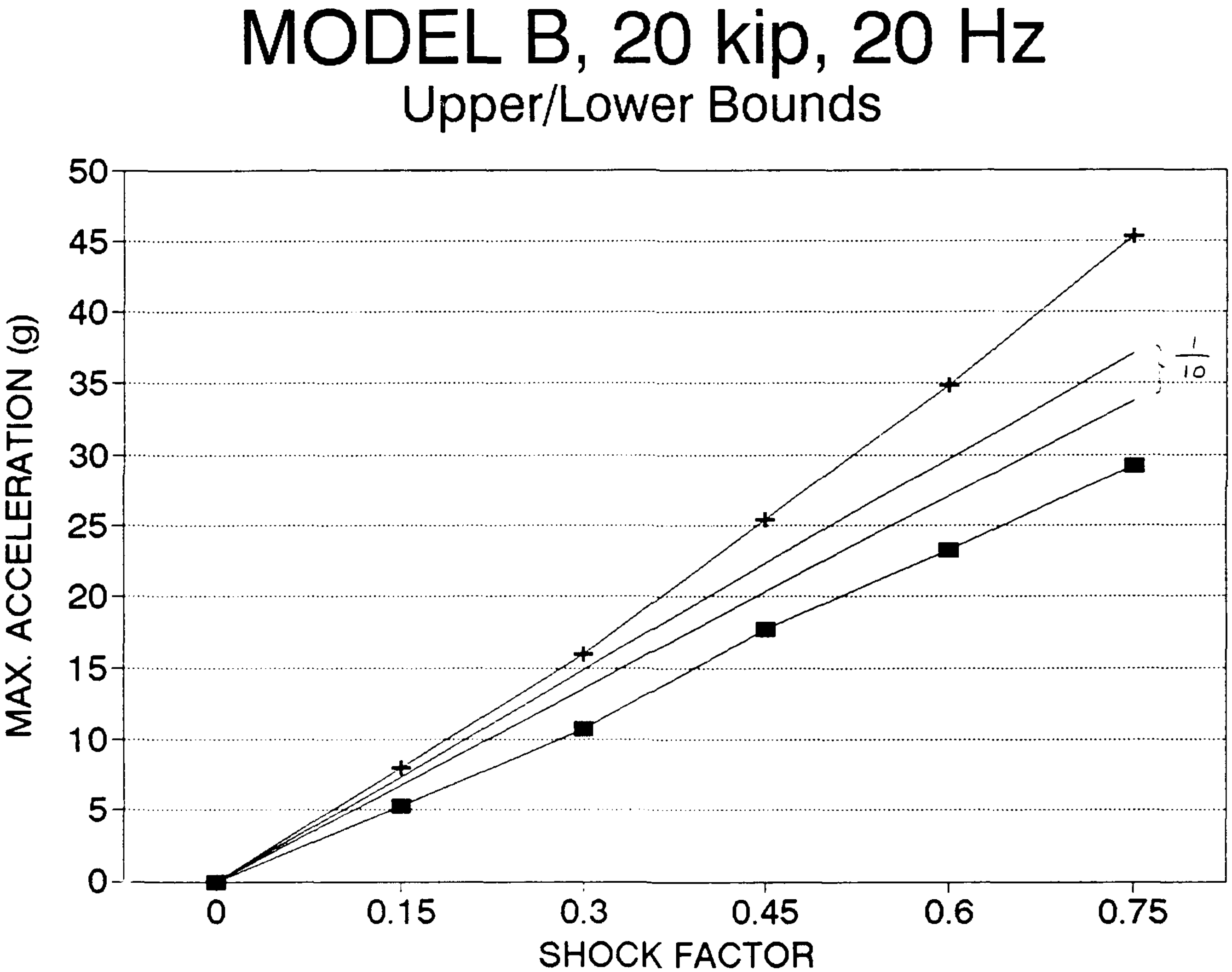
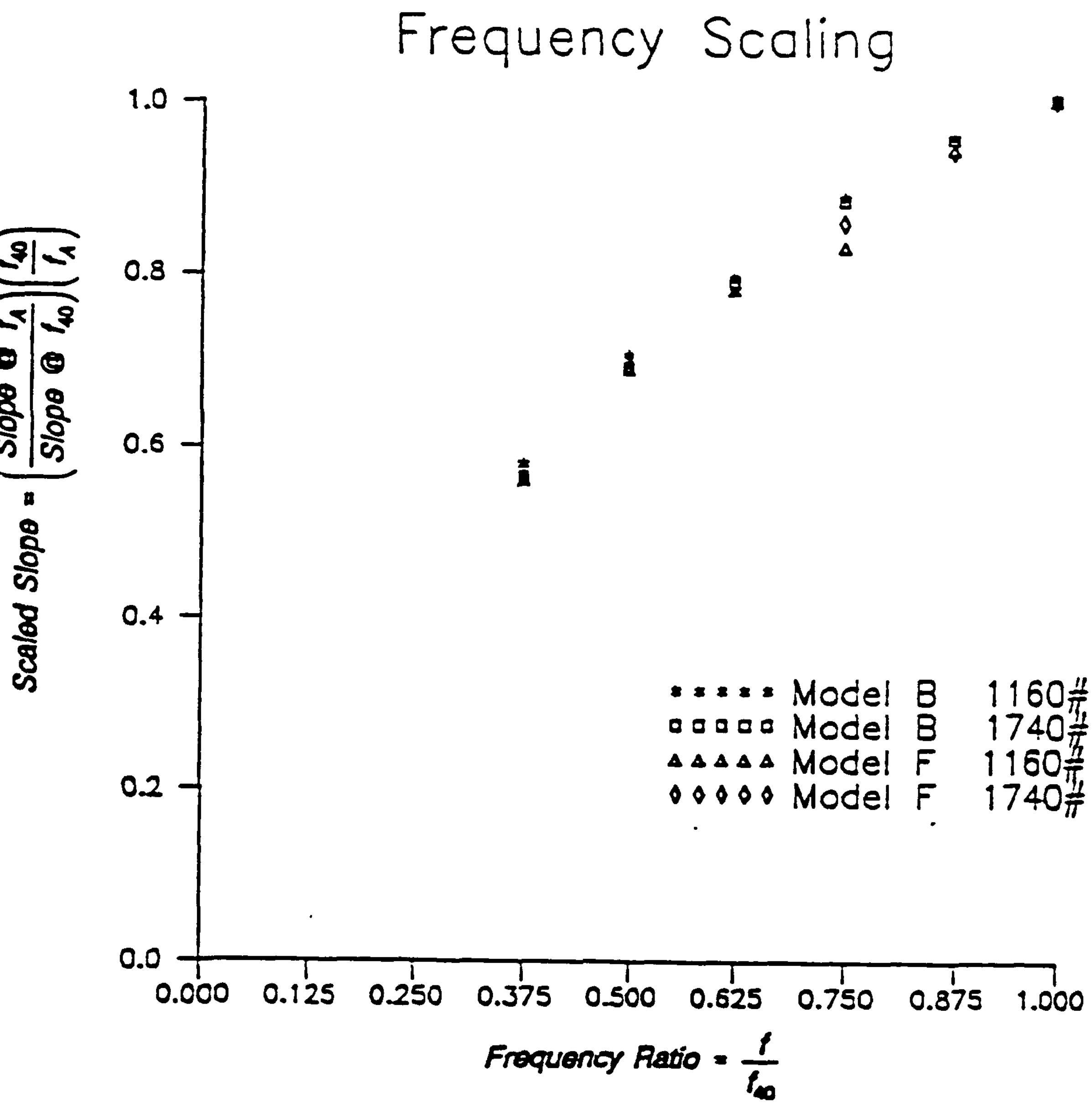

MODEL B, 20 kip, 20 Hz
Upper/Lower Bounds
MAX. ACCELERATION (g)
50
45
40
35
30
25
20
15
10
5
0
0
0.15
0.3
0.45
0.6
0.75
SHOCK FACTOR
1/10

Figure 9



Frequency Scaling
Scaled Slope = (Slope @ f_A / Slope @ f_40)(f_40 / f_A)
1.0
0.8
0.6
0.4
0.2
0.0
0.000
0.125
0.250
0.375
0.500
0.625
0.750
0.875
1.000
Frequency Ratio = f / f_40
Model B 1160#
Model B 1740#
Model F 1160#
Model F 1740#

Figure 10

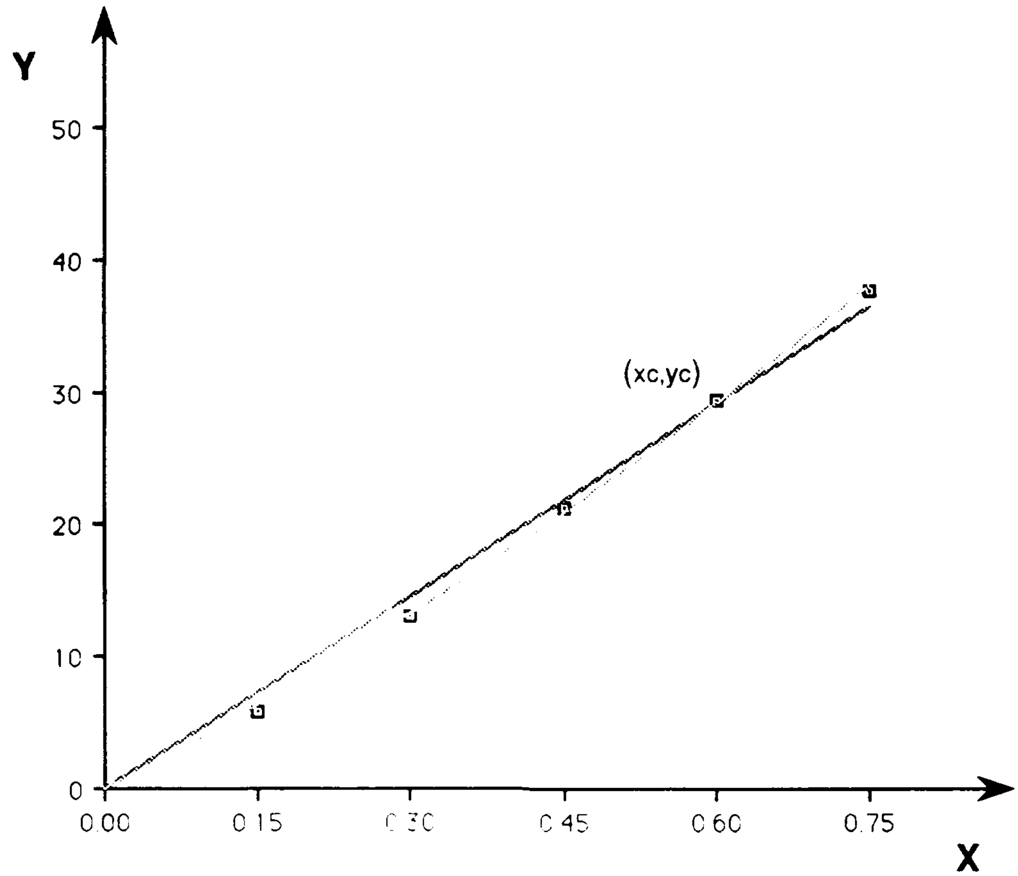


Figure 11

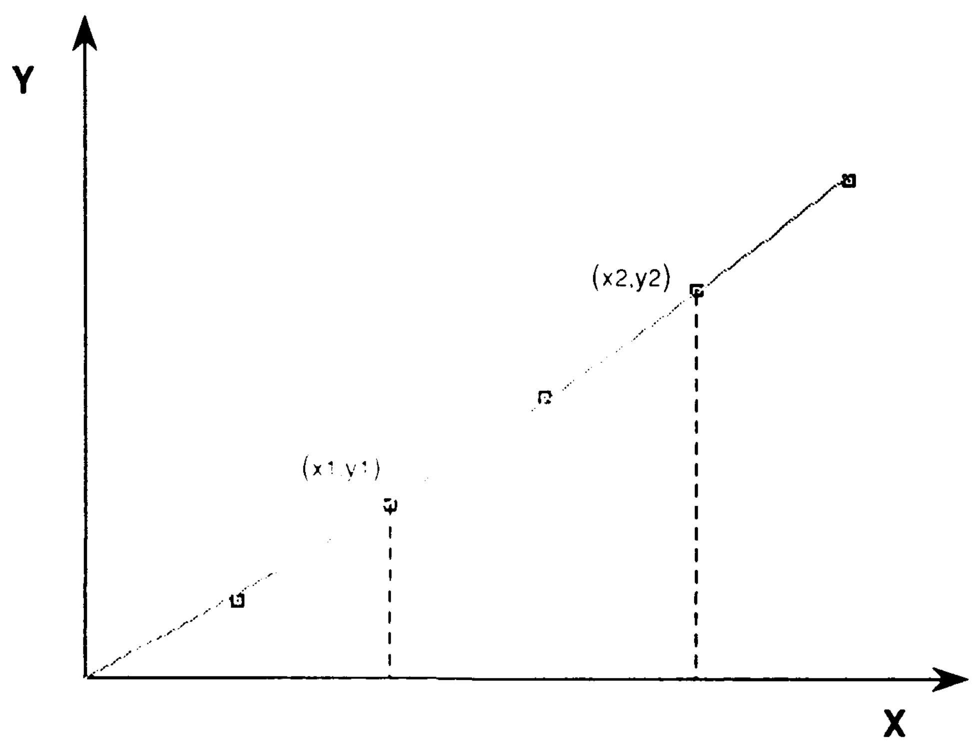
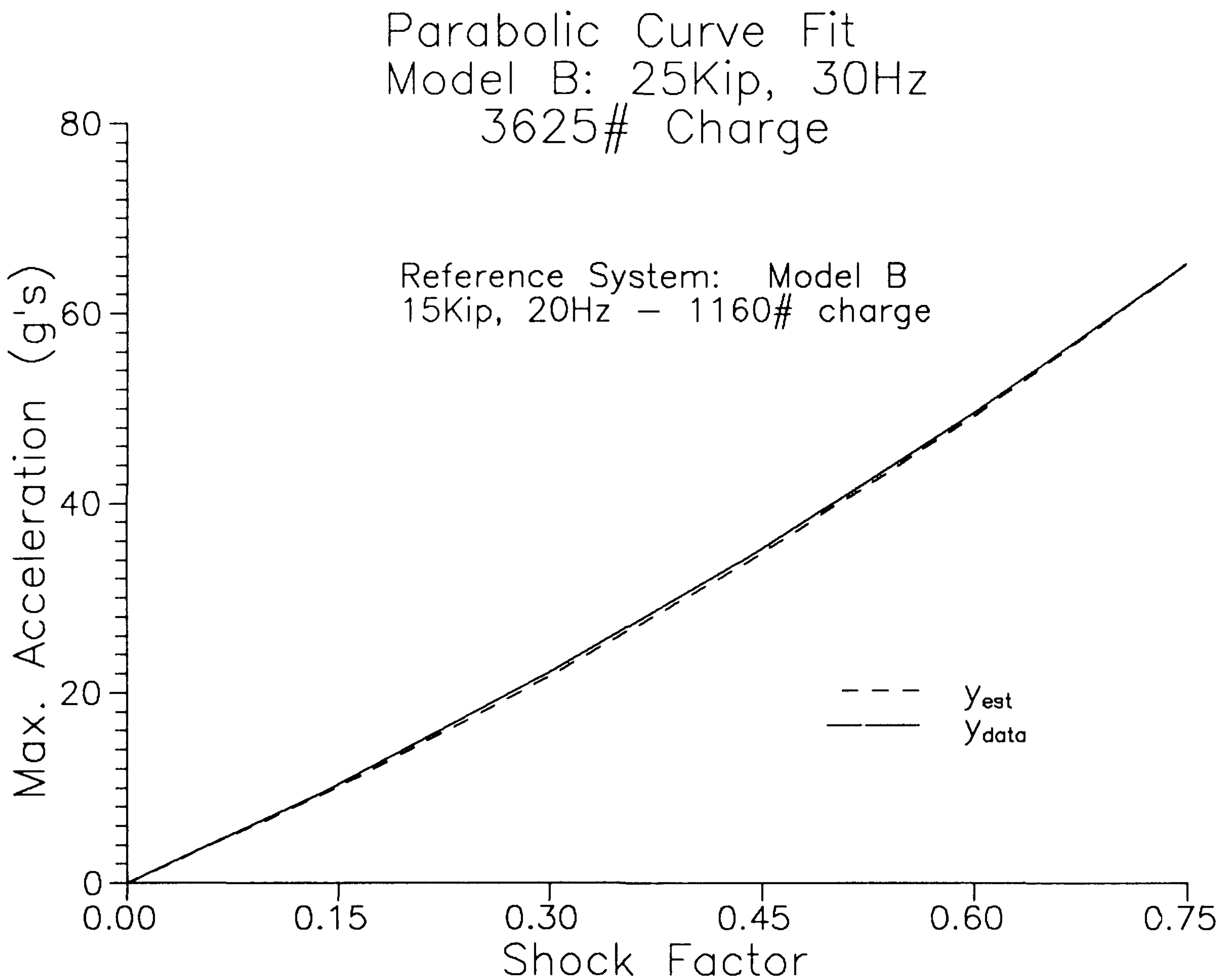


Figure 12



Parabolic Curve Fit
Model B: 25Kip, 30Hz
3625# Charge
Reference System: Model B
15Kip, 20Hz – 1160# charge
Max. Acceleration (g's)
80
60
40
20
0
0.00
0.15
0.30
0.45
0.60
0.75
Shock Factor
y_{est}
y_{data}

Figure 13

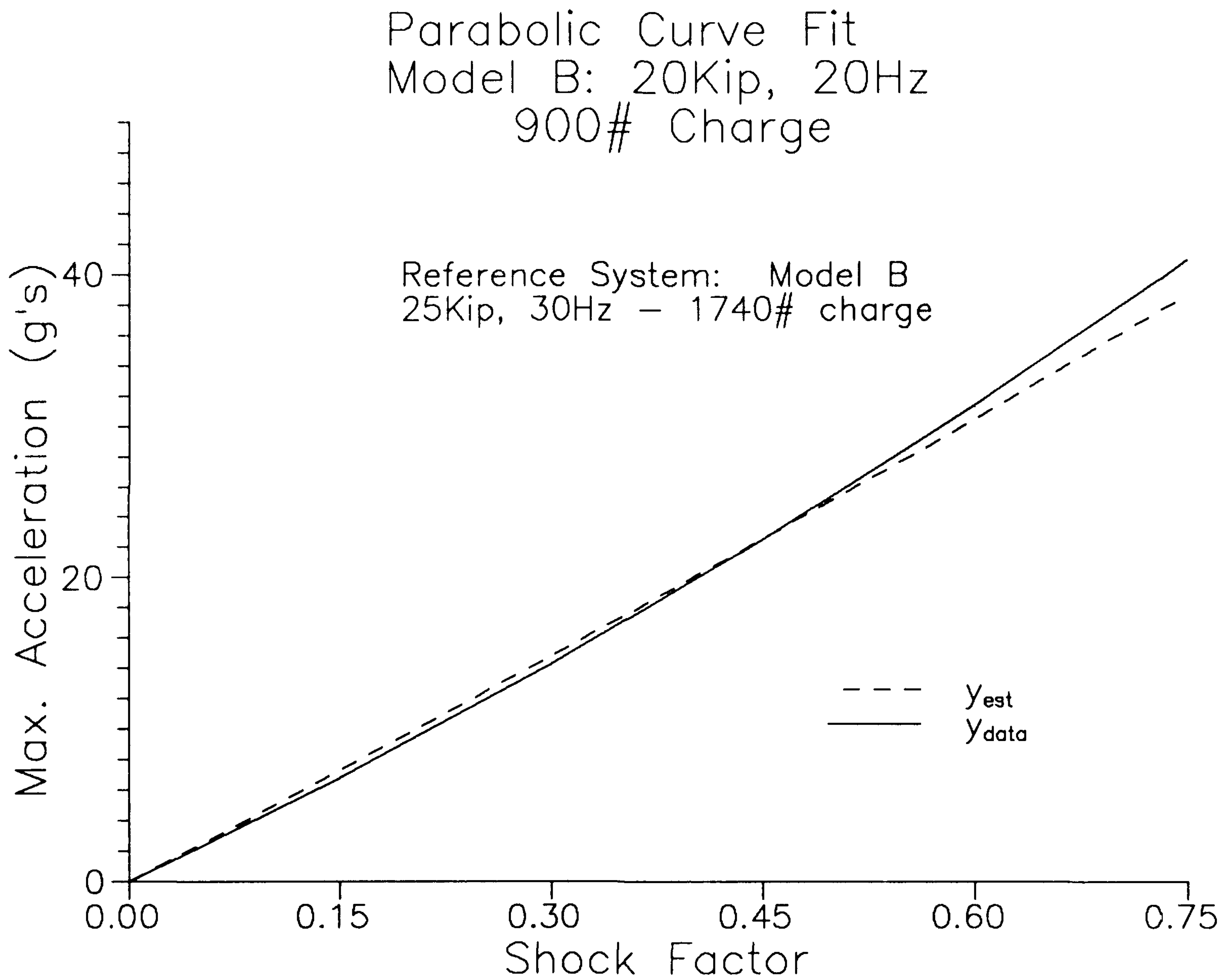

Parabolic Curve Fit
Model B: 20Kip, 20Hz
900# Charge
Reference System: Model B
25Kip, 30Hz – 1740# charge
Max. Acceleration (g's)
40
20
0
0.00
0.15
0.30
0.45
0.60
0.75
Shock Factor
y_{est}
y_{data}

Figure 14

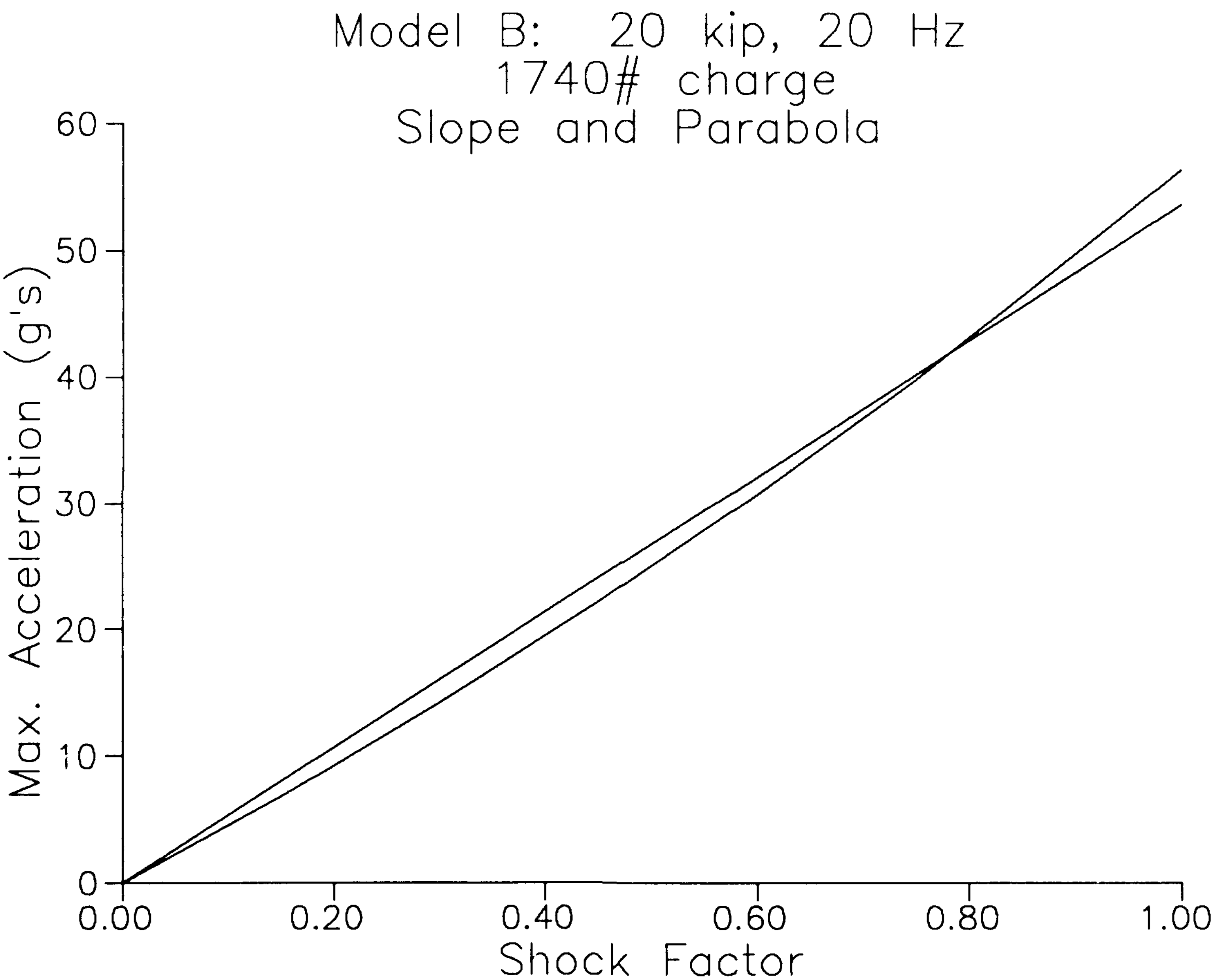

Model B: 20 kip, 20 Hz
1740# charge
Slope and Parabola
Max. Acceleration (g's)
0
10
20
30
40
50
60
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor

Figure 15

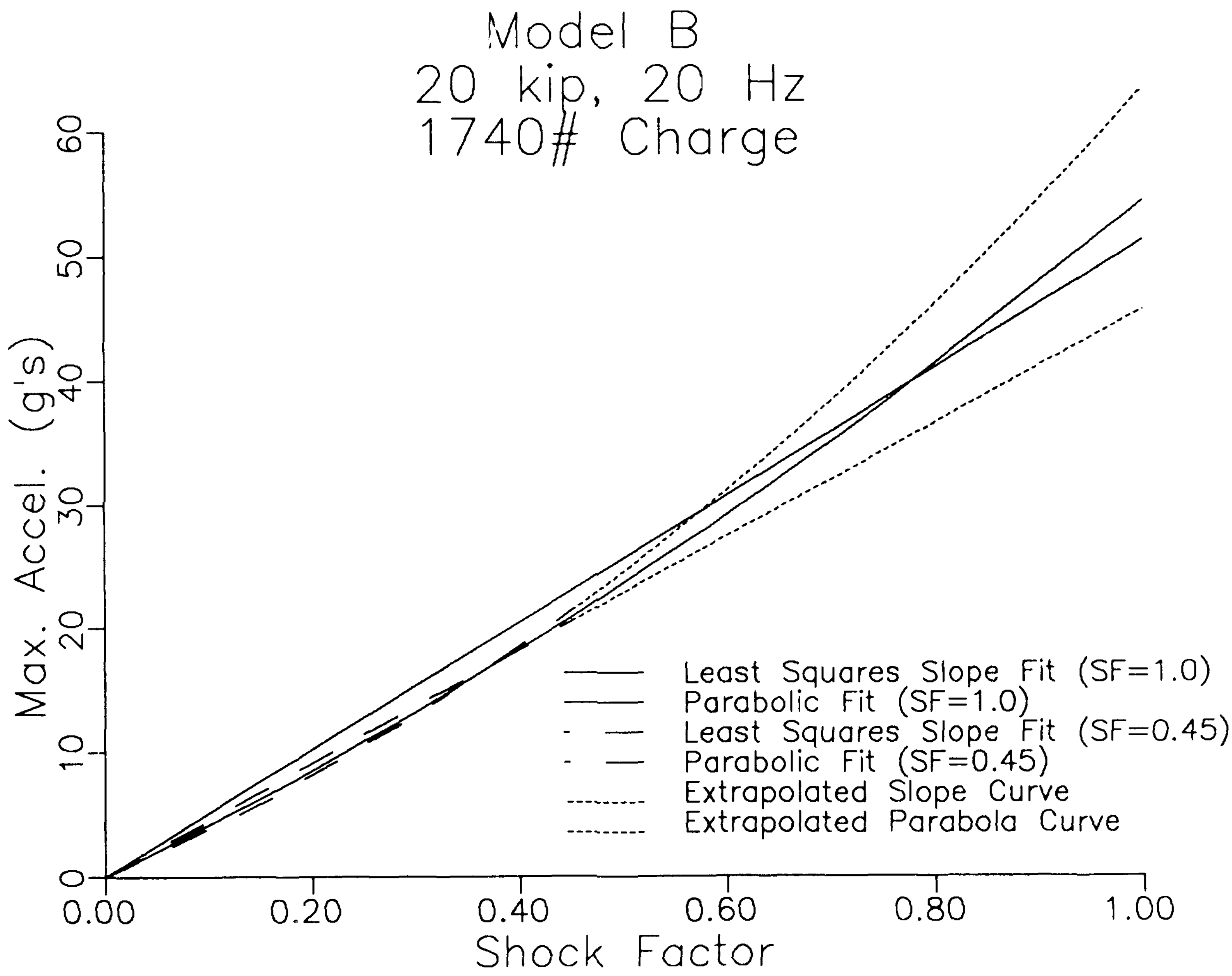
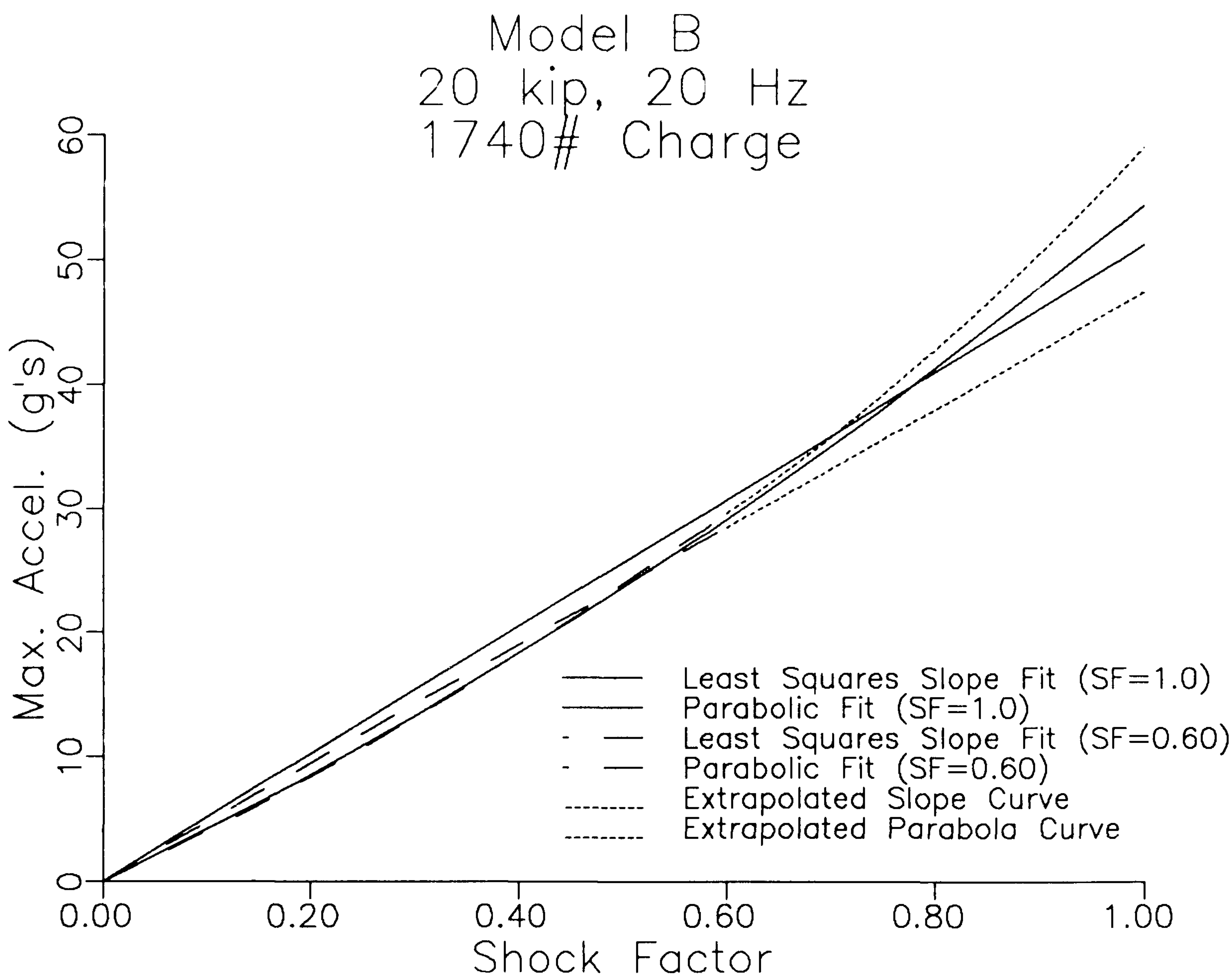

Model B
20 kip, 20 Hz
1740# Charge
Max. Accel. (g's)
0
10
20
30
40
50
60
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor
Least Squares Slope Fit (SF=1.0)
Parabolic Fit (SF=1.0)
Least Squares Slope Fit (SF=0.45)
Parabolic Fit (SF=0.45)
Extrapolated Slope Curve
Extrapolated Parabola Curve

Figure 16(a)



Model B
20 kip, 20 Hz
1740# Charge
Max. Accel. (g's)
0
10
20
30
40
50
60
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor
Least Squares Slope Fit (SF=1.0)
Parabolic Fit (SF=1.0)
Least Squares Slope Fit (SF=0.60)
Parabolic Fit (SF=0.60)
Extrapolated Slope Curve
Extrapolated Parabola Curve

Figure 16(b)

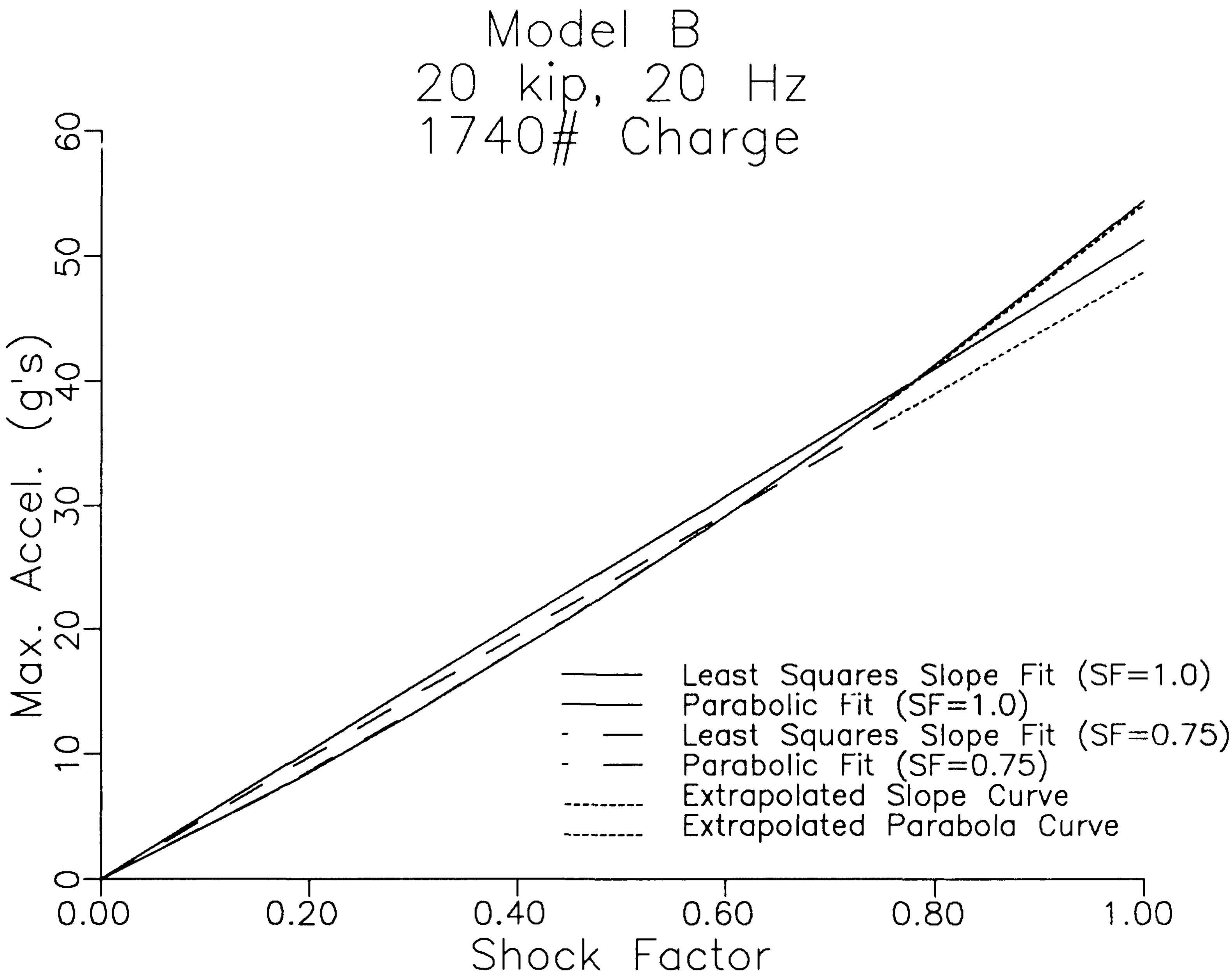

Model B
20 kip, 20 Hz
1740# Charge
Max. Accel. (g's)
0
10
20
30
40
50
60
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor
Least Squares Slope Fit (SF=1.0)
Parabolic Fit (SF=1.0)
Least Squares Slope Fit (SF=0.75)
Parabolic Fit (SF=0.75)
Extrapolated Slope Curve
Extrapolated Parabola Curve

Figure 16(c)

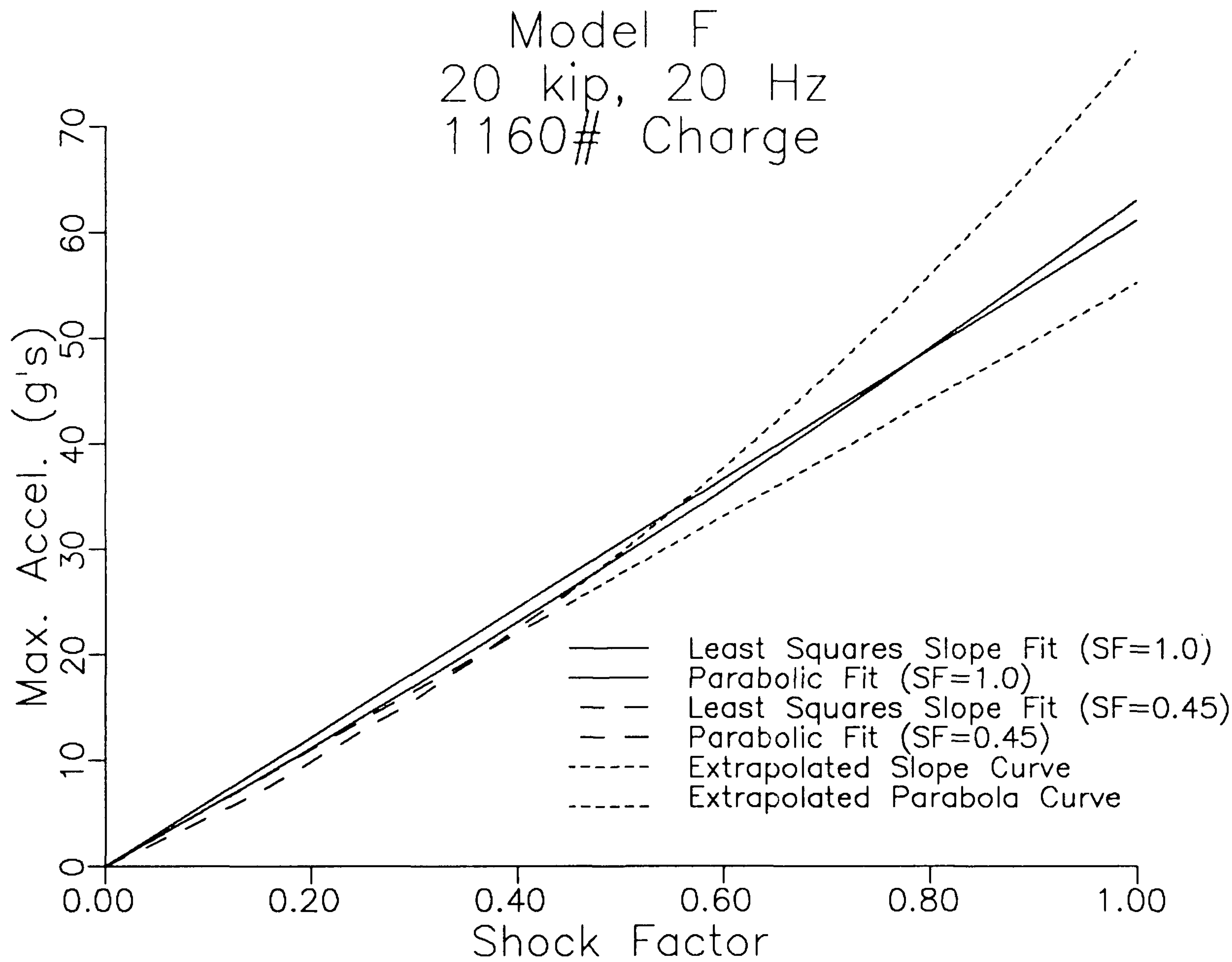

Model F
20 kip, 20 Hz
1160# Charge
Max. Accel. (g's)
0
10
20
30
40
50
60
70
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor
Least Squares Slope Fit (SF=1.0)
Parabolic Fit (SF=1.0)
Least Squares Slope Fit (SF=0.45)
Parabolic Fit (SF=0.45)
Extrapolated Slope Curve
Extrapolated Parabola Curve

Figure 17(a)

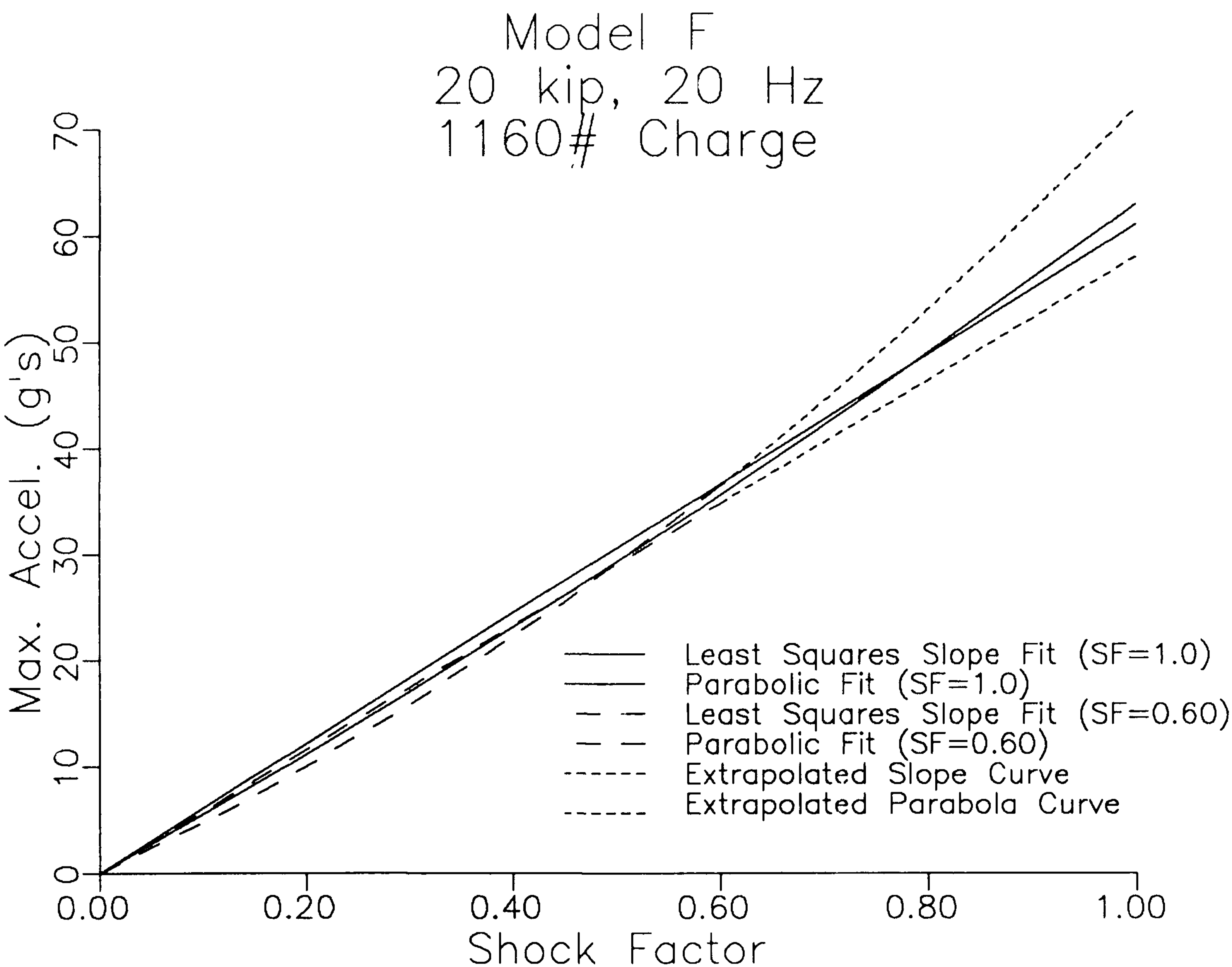

Model F
20 kip, 20 Hz
1160# Charge
Max. Accel. (g's)
0
10
20
30
40
50
60
70
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor
Least Squares Slope Fit (SF=1.0)
Parabolic Fit (SF=1.0)
Least Squares Slope Fit (SF=0.60)
Parabolic Fit (SF=0.60)
Extrapolated Slope Curve
Extrapolated Parabola Curve

Figure 17(b)

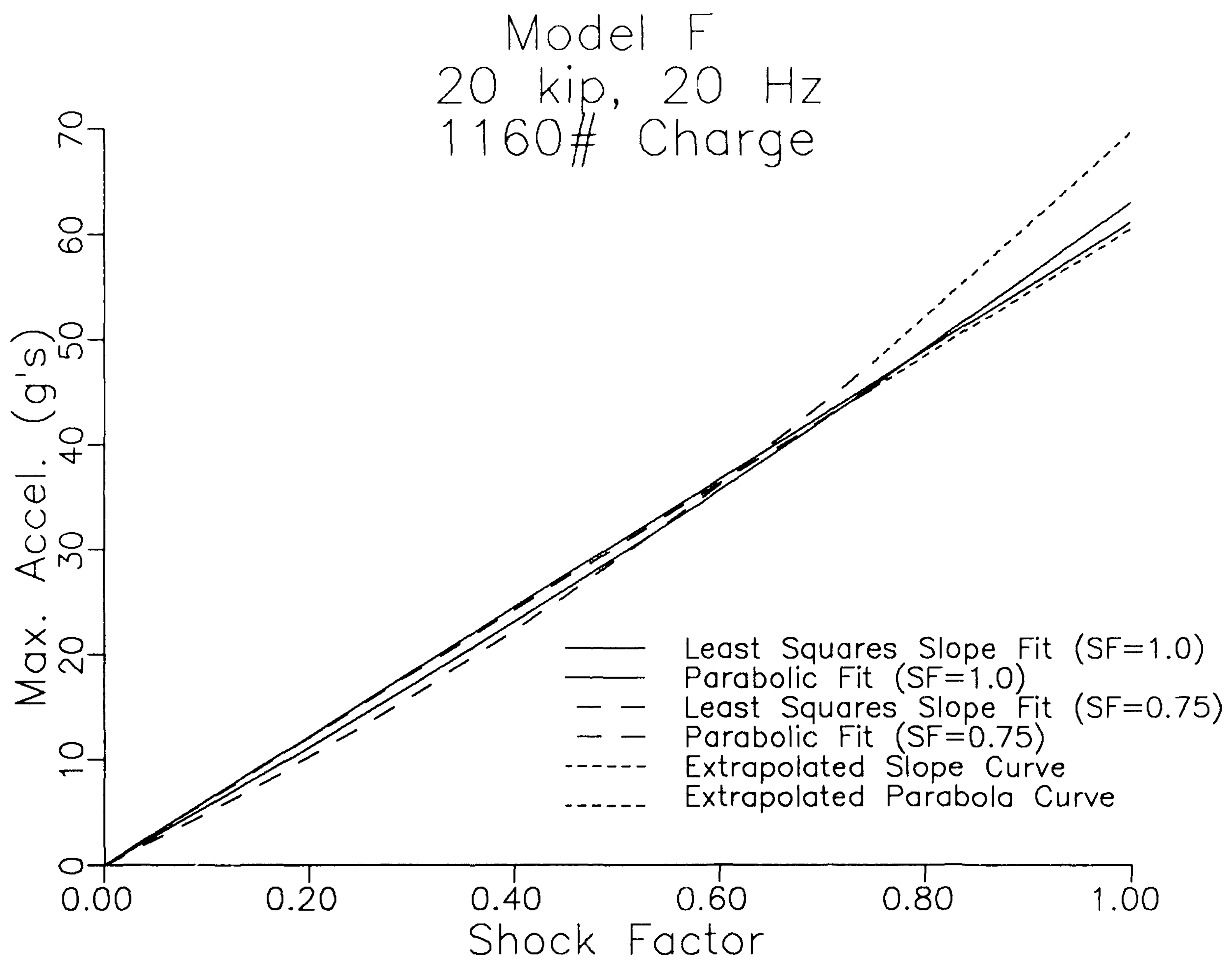

Model F
20 kip, 20 Hz
1160# Charge
Max. Accel. (g's)
0
10
20
30
40
50
60
70
0.00
0.20
0.40
0.60
0.80
1.00
Shock Factor
Least Squares Slope Fit (SF=1.0)
Parabolic Fit (SF=1.0)
Least Squares Slope Fit (SF=0.75)
Parabolic Fit (SF=0.75)
Extrapolated Slope Curve
Extrapolated Parabola Curve

Figure 17(c)