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DEPARTMENT OF DEFENCE

DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORY

MELBOURNE, VICTORIA

Aircraft Structures Technical Memorandum 549

COMPUTER RECOGNITION AND ANALYSIS OF PHOTOGRAMMETRIC TARGETS

by

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T.G. RYALL

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SUMMARY

This Technical Memorandum is a continuation of earlier work into the application of photogrammetric techniques for the modal analysis of vibrating structures. Methods are developed for the computer processing of photogrammetric negatives. The negatives capture the positions of retro-reflective targets located on the vibrating structure. With the relative distances between targets known, the distorted structure can be reconstructed. All targets are given a Global coordinate, *thus reducing the accumulation of errors over the negative. The techniques used to establish the* Global Coordinate System *are discussed as are the methods used to determine accurately the location of the retro-reflective targets.*

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CONTENTS

1 - INTRODUCTION

This Technical Memorandum is a continuation of earlier work into the application of photogrammetric techniques for the modal analysis of vibrating structures. Basically, photogrammetry is a method for the measurement of points on a body in a three dimensional coordinate system {1}.

In this instance, the structure is a steel plate (Refer - Figure 1). The plate has the dimensions 1022 x 902 x 3 mm and is clamped along one edge.

The experiment involved the placement of retro-reflective targets over the structure. Subjected to an excitation, the motion of the structure is frozen using a stroboscope and camera apparatus which is synchronized to the oscillation of the body {1}. Photographs of the vibrating system were taken at five stations around the plate {1}. At each station the phase between the stroboscope and exciter was increased from 0° to 360° in increments of 45° {1}. These increments ensure the photographs capture the plate in various stages of distortion.

The plate was excited at a frequency of 27.2 Hz, corresponding to the structure's fourth normal mode {1}.

Applying photogrammetric techniques, a three dimensional reconstruction of the plate distortion from the photographic negatives is possible; however the accurate determination of the target centres is essential. Whilst the peripheral dimensions of the targets may vary, the target centres are made to exacting standards.

It should be mentioned, the spacing between the retro-reflective targets must be sufficient to ensure the mode of vibration is discernible.

Previously, target centre coordinates were determined by manual measurement of the negatives therefore making the analysis extremely time consuming. This Technical Memorandum focuses on the processing of photogrammetric results in a more time efficient manner.

2 - COMPUTER ANALYSIS

If the processing of photographic data is to become more efficient computer interaction is essential. Whilst time consuming, an advantage with the manual processing of the negatives is the ability to magnify each target. The improved resolution ensures the target centres may be determined with considerable accuracy. Computer enhancement of the negative could yield similar results, the resolution only being restricted by the systems magnification capabilities. However, magnification and accuracy are proportional to data file size and computer processing time. A suitable compromise between accuracy and processing time must be maintained.

2.1 - IMAGES & PROCESSING

Computer processing of the results requires the images of the photographic negatives to be converted to light intensity data. For this application, a *Charged Couple Device* (CCD) camera is linked to a computer which operates a software package capable of isolating and capturing a single frame. An image of the photograph is captured and stored for processing.

The image quality is primarily a function of the CCD camera. The resolution is proportional to the number of sensors in the CCD. During this work two CCD camera systems were tested. The relative merits of each are briefly discussed.

2.2 - THE *VISUAL PRESENTER* SYSTEM

Compact, light weight and portable, the *Visual Presenter* system gives an image resolution of 768 x 512 pixels. No external light source is required, since two adjustable lights provide adequate illumination for the camera.

The *Visual Presenter* is primarily a multi-purpose conference, or seminar, projection system. Whilst the lens attachments supplied with the device are adequate for magnification of presentation slides and alike, the associated lens distortions are unacceptable for photogrammetry work. Similarly, the ability to move the camera over the target area is an essential feature of any system however, the construction of the *Visual Presenter* restricts the device in this capacity.

These deficiencies in the *Visual Presenter* could not be rectified and an alternative system was sought.

2.3 - THE *VIDEK* SYSTEM

The *Videk* camera system is not easily transported. The camera is mounted on an adjustable mast assembly which allows translation of the camera vertically (Z) and horizontally (Y). The system is restricted from movement in the X direction (Refer - Figure 2).

Unlike the *Visual Presenter*, this camera uses conventional lens attachments, permitting almost unrestricted magnification of the photographic negatives. With a resolution of 1340 x 1036 pixels, the *Videk* system clearly proved the most versatile of the two CCD systems.

All images were captured using the *Videk* camera.

3 - THE GLOBAL COORDINATE SYSTEM

The determination of the plate distortion from the photographic negative requires the position of each target on the plate to be accurately known. Whilst the magnification of individual target is essential, the relationship between targets is equally important. The targets must be related by a common *Global Coordinate System.*

A reference grid superimposed over the photographic negative provides this common coordinate system. The relative distances between target centres can be easily determined and the accumulation of errors over the target area is substantially reduced.

The reference grid must cover the entire target area.

3.1 - REFERENCE GRID

The reference grid used is computer generated and accurate to 1/1000 inch. In this first stage of the process only an image of the grid is required.

The grid, with overall dimensions 400 mm x 400 mm and 25 mm x 25 mm grid squares, is secured firmly to prevent any movement during the procedure. An image is taken for later analysis.

To ensure the grid lines are prominent, the captured images are primarily black and white where the background is black and the grid lines white.

The pixel intensity scale comprises 256 shades (Refer - Figure 3) between black and white. The computer must be able to distinguish between these contrasting light intensities. For this purpose an *Intensity Threshold Parameter* is nominated. Typically an intensity , $I(x,y) = 150$, is used where intensities below the threshold are deemed white and those above the threshold considered black.

With the Intensity Threshold Parameter known, the search for grid intersections can now commence. Firstly, the origin of the grid must be accurately located.

3.2 - COORDINATE AXES

The origin of the grid is arbitrarily nominated as the grid intersection point in the extreme upper left corner of the image and is given the *Global Coordinate* (1,1) (Refer - Figure 4A & 4B). Using the Intensity Threshold Parameter, line scans of this region identify the row and column in which the origin grid lines were detected.

These coordinates, (*Row,Column*), are refined using the principle of first moments where the pixel intensities are considered to be concentrated point masses. In computing the centre of gravity, intensities within a 50 pixel radius of the origin are considered. When the centre of gravity is determined, the origin is given these coordinates and the procedure repeated until the coordinates converge.

Similarly, line scans locate the grid point nearest the origin in the X direction. This point is given the Global Coordinate (2,1). First moment principles are again employed to determine accurately the coordinates of this intersection. Computing the gradient and displacement between (1,1) and (2,1), permits confident approximations for the coordinates of the remaining grid intersections to be made. Using first moments these approximations are refined and then recorded.

With all of the grid intersection coordinates identified, the Global Coordinate System can now be established.

3.3 - GLOBAL COORDINATE SYSTEM LABELLING

The labelling sequence for the Global Coordinate System follows a standard (X,Y) format and the system is generated by rows, or Y coordinate.

With the origin as a reference, all points sharing a common gradient with the origin (1,1) and grid intersection (2,1) are segregated. These points are ordered, with the X coordinate increasing, and given Global Coordinates. This row of coordinates defines the X axis of the grid.

Incrementing to the next row of the grid, the intersection nearest the origin in the Y axis direction is identified and labelled (1,2). Using this point as a reference and knowing the slope of the grid, all points sharing a common gradient are identified. Similarly, these coordinates are ordered, with the X coordinate increasing, and labelled.

This procedure is repeated until all grid intersections are given Global Coordinates

With the Global Coordinate System established, the target negative can now be affixed firmly to the grid. Any movement of the grid or relative movement between the grid and photographic negative must not occur.

The photographic film on which the target images are captured is tinted. Superimposing the target negative onto the grid increases the intensity scatter. For target recognition, the reduced contrast implies a more sophisticated technique for the determination of the Intensity Threshold Parameter is required.

3.4 - INTENSITY THRESHOLD PARAMETER

For target identification, the Intensity Threshold Parameter must be calculated from line scans of the intensity data.

For an image known to contain at least one target, the distribution of intensities is expected to be bimodal, that is, we expect two distinct intensity peaks which correspond to the background and the targets. The Intensity Threshold Parameter should correspond to a minimum somewhere between these two intensity maxima.

The simplest polynomial which adequately describes this system is a 4th order polynomial. The roots of the polynomial's first derivative will give the turning points of the curve. A single real root in the solution of the polynomial implies the grid square contains no targets.

For target recognition, the Intensity Threshold Parameter is determined using this technique (Refer - Appendix 1).

4 - MAGNIFIED GRID IMAGES

In this phase of the procedure, image quality and the ability of the CCD camera to scan the photographic negative become essential features of a good system. Whilst the *Videk* camera has excellent image resolution, the restriction to translation in the Y - Z directions is a burden. For the purposes of this memorandum, the results are restricted to a corridor of grid squares located by the fixed X coordinate of the apparatus.

Moving from the global view, the camera is adjusted in the Z direction to isolate a single grid square. Ideally, each grid square over the target area should be systematically isolated and an image stored for processing.

Each image is processed separately and variations in lighting conditions make it necessary to compute an Intensity Threshold Parameter for each image.

Prior to the search for targets within the grid square, the boundaries of the square must be identified. Any targets located within the grid square are related to the Global Coordinate System via the *Reference Grid Intersection*. The Reference Grid Intersection is nominated as the intersection in the upper left quadrant of the image. The user is prompted by the program to supply the Global Coordinates of this reference point. Lines scans in both the vertical and horizontal directions locate this intersection and, using first moments, the coordinates are refined (Refer - Figure 5B).

Similarly, the procedure is repeated, locating the grid intersection in the upper right quadrant of the image. The displacement between these two points is computed.

Comparing this displacement, with the displacement for the non-magnified image will give the *Magnification Ratio*. The calculation of the Magnification Ratio permits all coordinates on the magnified images to be scaled to a common, non-magnified coordinate set and the boundaries of the grid square to be identified. Subsequent line scans for targets are constrained by these grid boundaries.

The search for targets within the grid square can now commence.

4.1 - TARGET DETECTION & CONFIRMATION

The determination of the target coordinates is a two phase process. Initially, targets are tentatively identified by a simple pattern recognition technique. The second, more rigorous test, is a comparison of the target's peripheral shape to that of an ideal ellipse. Both of these criteria are briefly discussed.

4.1.1 - THE PATTERN RECOGNITION CRITERION

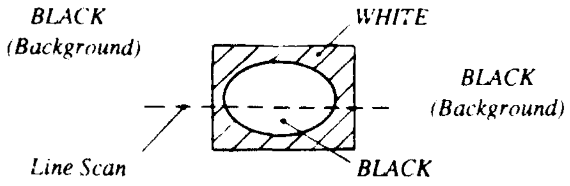
Using the Intensity Threshold Parameter, line scans of the image search for the intensity sequence -

BLACK - WHITE - BLACK - WHITE - BLACK
(Background) (Target) (Background)

The coordinates for all occurrences of this sequence are recorded for later processing.

The white pixels which form the target centre are not considered in this intensity sequence. Line scans which detect the target centre pixels are disgarded. These pixels are considered rogue white pixels.

The magnification of the target negative implies a single target will be identified many times. Therefore the redundant target identifications must be eliminated. In cases where multiple recognitions occur, only those sequences with a maximum number of black pixels bounded by white pixels are stored.



The second criterion must be satisfied prior to the targets confirmation.

4.1.2 - THE NON DIMENSIONAL RATIO CRITERION

The local area about the suspected target is considered in isolation and a binary image of this region is constructed. In the binary image, pixels are deemed to be either white (0) or black (1) (Refer - Figure 5A)

The peripheral dimensions of the target are primarily elliptical. The moments of intensity and area are combined to give a non dimensional ratio for the binary image (Refer - Appendix 2). The ratio is compared with that representative of an ideal ellipse, for which the ratio is 1.

$$R = \frac{ab}{4\sqrt{Det(A)}}$$

Where   *Det (A) - Determinant of the Intensity Matrix*
2a & 2b - Major & Minor axes of an Ellipse

(Refer - Appendix 2)

Only targets which have a ratio of approximately R=1 are accepted. As an initial estimate of the target centre coordinates, the centroid coordinates for the binary image are recorded.

4.2 - DETERMINATION OF TARGET CENTRE COORDINATES

The target centres, whilst actually circular, appear elliptical in shape. This phenomenon is as a result of the camera position when the photographs of the plate were taken. The camera stations were 2 m above the plate in a parallel plane and approximately equidistant from the plate's centre {1}.

Several techniques were employed in an endeavour to accurately locate the coordinates of the target centres. A description of each method is given.

4.2.1 - THE NON LINEAR LEAST SQUARES SOLUTION

The elliptical properties of the target's peripheral dimensions implies the target centres are similarly best approximated by an ellipse.

Using the preliminary estimate for the target centre, a local area encompassing the centre is isolated. For this region, calculation of the centre of gravity improves the initial estimate of the centre's coordinates. An ellipse fitting routine is employed to further refine the coordinates.

A thorough explanation of this method is contained in the appendices (Refer - Appendix 3)

In brief, for each ellipse there exists a matrix A(i,j) where pixels are classified as either inside the boundaries of the ellipse, in which case A(i,j) = 1, or outside the ellipse where A(i,j) = 0. This implies a step change of intensity between the background pixels and those pixels forming the ellipse. The problem can be expressed in terms of an *Objective Function*. Minimising the Objective Function will yield the target centre coordinates. The target centre coordinates are referred to the *Reference Grid Intersection* and therefore the Global Coordinate System.

$$Objective\ Function = \frac{1}{N}\sum\{-O(i,j) + P + Q(A(i,j))\}^2$$

Where

N - Number of *Intensities* $I(x,y)$
$O(i,j)$ - Observed Intensity (x,y)
P - Background *Intensity*
Q - Foreground *Intensity*
(Refer - Appendix 3)

The primary deficiency in this method is the inherent assumption that the boundaries of the target centre behave as a step function; that is at the boundary between the background and the target centre there exists a step change in intensity. Examination of the intensity scatter around a typical target centre shows this not to be the case (Refer - Figure 6A & 6B). Whilst the Objective Function converges to a minimum, this should not be construed as the convergence of the function to the global minimum, or target centre, but instead implies the function has merely converged to a local minimum. This characteristic can be easily demonstrated by altering the initial estimates for the Objective Function.

An alternative technique is the determination of the target centre coordinates using *First Moments*. It should be noted this method is intended only to give a first estimate of the target coordinates and a function which better describes the properties of the system is essential in attaining more accurate results.

4.2.2 - THE FIRST MOMENTS SOLUTION

This method uses centre of gravity coordinates, X and Y, as an estimate for the target centre. Using initial estimates for X & Y, a circle of radius *R* radiates from the centre of gravity in pixel increments. The radius *R* has a fixed upper limit. For each increment in radius, the computed centre of gravity coordinates for the pixels bounded by the circle are differenced, in both X & Y, with those coordinates corresponding to best estimate for the target centre. The parameter *DXY*, defined as the summation of the X and Y differences squared, is a minimum at the centroid. The coordinates X & Y giving a lower *DXY* than the best estimate are stored whilst the radius is incremented. Those coordinates corresponding to a minimum *DXY* are deemed the target centre coordinates.

As with the *Non Linear Least Squares* method, the target centre coordinates are referred to the Reference Grid Intersection and therefore the Global Coordinate System.

The result obtained using both methods are compared.

5 - RESULTS

The anomaly in the Non Linear Least Squares solution namely the convergence of the Objective Function to the nearest local minimum and not the global minimum as intended, can be compensated. To induce convergence of the Objective Function to a global minimum, different initial estimates for the Objective Function were nominated. These results are presented (Refer - Table 1).

Method	Normalised Standard Deviation		Objective Function		
	(Minimum)	(Maximum)	Target 1	Target 2	Target 3
1	0.002377	0.001271	6665	207146	205805
2	0.002288	0.001223	4268	5321	4523

Table 1

Where
1 - Non Linear Least Squares Solution
2 - Non Linear Least Squares Solution (Using alternative initial estimates)

Notes -

For the standard deviation, the data presented are averaged over three scans of the same target area, where each standard deviation shown is normalised to either the minimum or maximum distance between targets captured in each scan.

The Objective Function data shown corresponds to a single scan of the target area.

Expectations of an improvement in the standard deviation using alternative initial estimates for the function are confirmed, the Objective Function having clearly converged to a different, lower minimum.

Table 2 shows the results obtained using the First Moments method.

Method	Normalised Standard Deviation	
	(Minimum)	(Maximum)
First Moments	0.002292	0.001226

Table 2

The effectiveness of the First Moments solution as an initial estimate is evident when comparing the Non Linear Least Squares solution, using nominated parameter starting values, with the First Moments result. Clearly, the implementation of an Objective Function has not significantly altered the First Moments estimate and the additional computational time required for the Objective Function solution is unwarranted.

6 - CONCLUSIONS

The primary intention at the commencement of this work was the development of more time-efficient processing techniques for the analysis of photogrammetric data whilst retaining a desirable degree of accuracy in the final result. To this end, this Technical Memorandum demonstrates that with continuing development and refinement the techniques presented will achieve these aims.

For the further development of the methods presented, the following recommendations are made:

Placement of the reference grid over the target area may, in some instances, cause targets to be inadvertently obscured by grid lines. An alternative reference system is a grid composed purely of intersection points. The elimination of grid lines reduces the likelihood of obscuring targets whilst, retaining the advantages of a Global Coordinate System.

The ability to freely move the camera in the X,Y & Z directions should be a feature of any system used for the scanning of photogrammetric negatives. Whilst the *Videk* apparatus permits Y and Z translations, the restriction in X direction is unacceptable. If this system is to prove useful in the analysis of photographic negatives, it must have three dimensional freedom.

In the calculation of the Magnification Ratio the displacements for both the magnified and non magnified images are derived from a single grid square (Refer - Figure 5B). A better estimate of the magnification between the two views could be achieved by considering several grid squares or, different sides of the same grid square.

The present Non Linear Least Squares method of solution is based on the assumption that there exists a step change in intensity between the background and target centre intensity levels. This assumption is invalid therefore prompting the need to find a function which better describes the system. Reference to Figures 6A & 6B suggest a biquadratic function, or a quadratic in both X and Y, would more adequately describe the unique properties of this system.

This Technical Memorandum has demonstrated the reliability of First Moments as an initial estimate for the target centre coordinates. With a view to time-efficient analysis, this simple method should be incorporated in future photogrammetric work as a preliminary coordinate estimation technique.

ACKNOWLEDGEMENTS

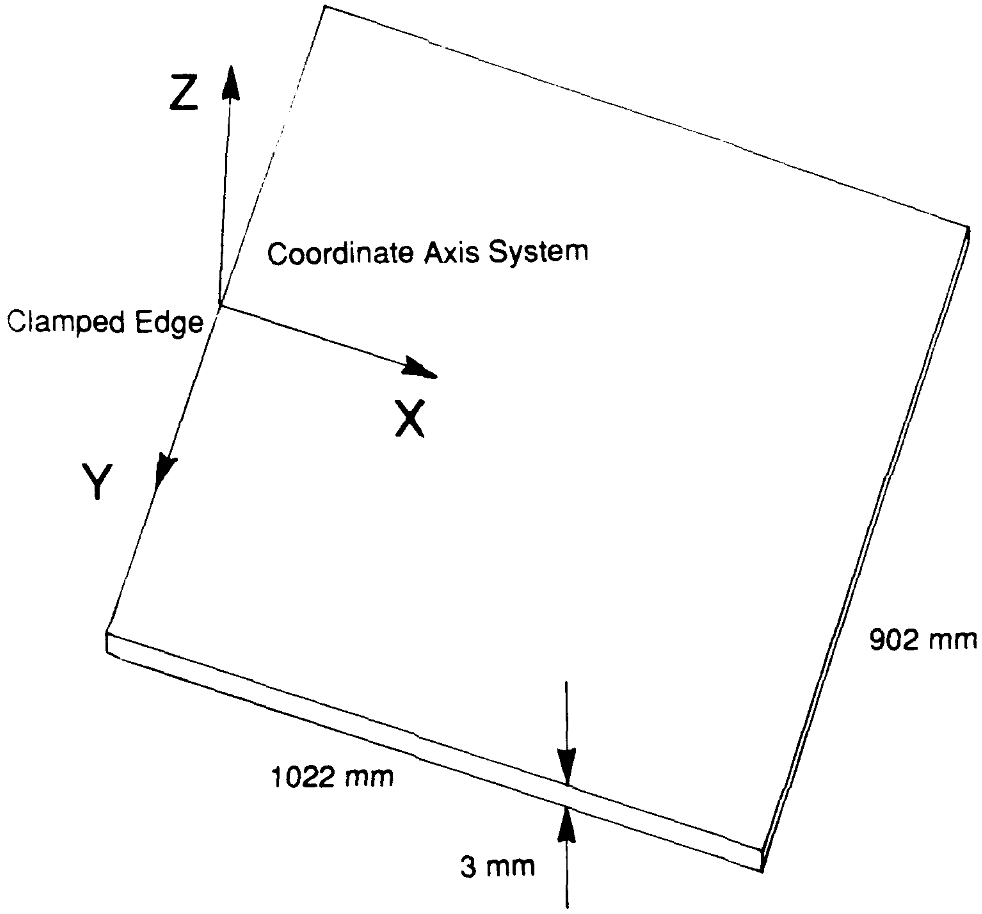
The authors wish to thank Ms. P. Cox for her invaluable assistance throughout this work.

REFERENCES

1. Farrell, P.A., Ryall, T.G. and Emslie, Betty
Use of Photogrammetry to Measure a Plate Vibration Mode
ARL-STRUC-TM 570, August 1990

Rectangular Cantilever Plate

Figure - 1



(**Source** - Aircraft Structures Technical Memorandum 570 {1})

VIDEK IMAGING SYSTEM SCHEMATIC

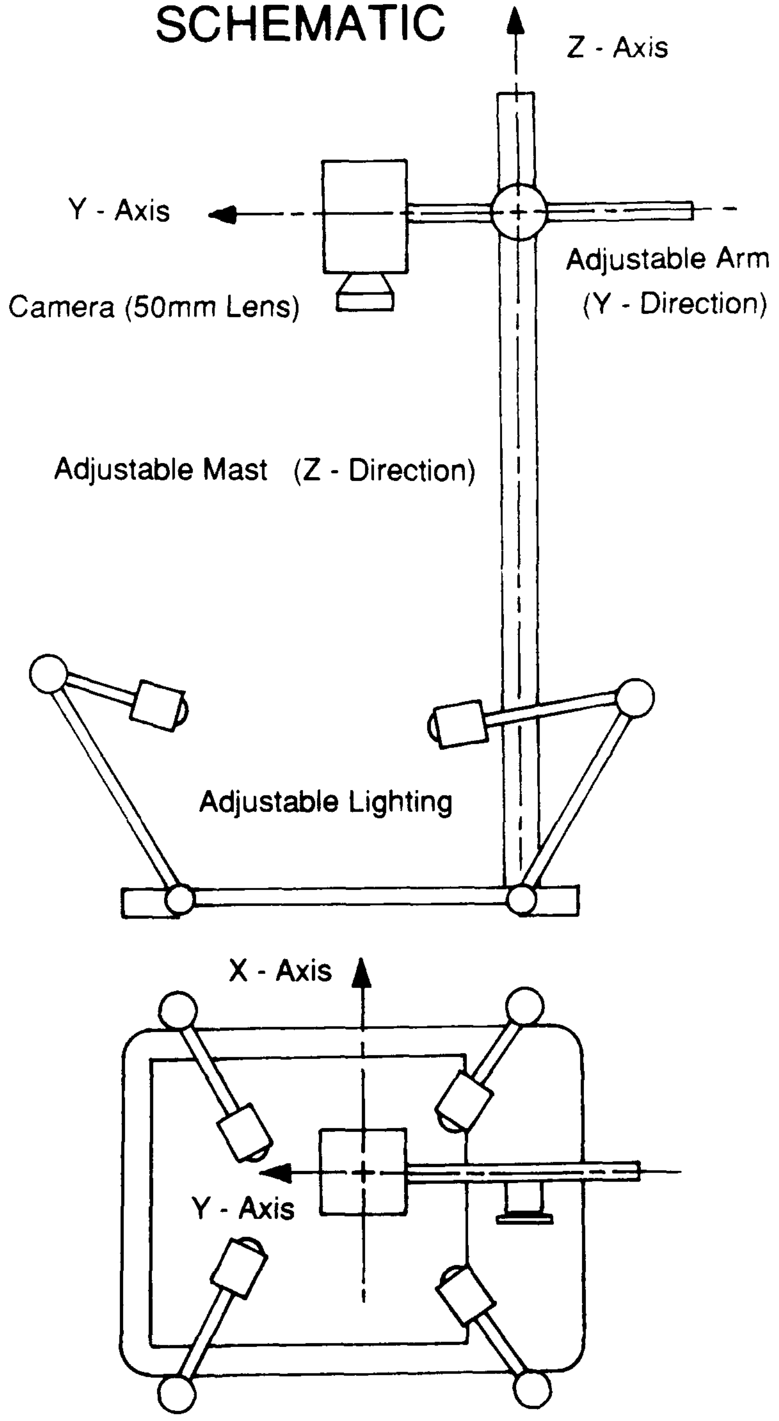
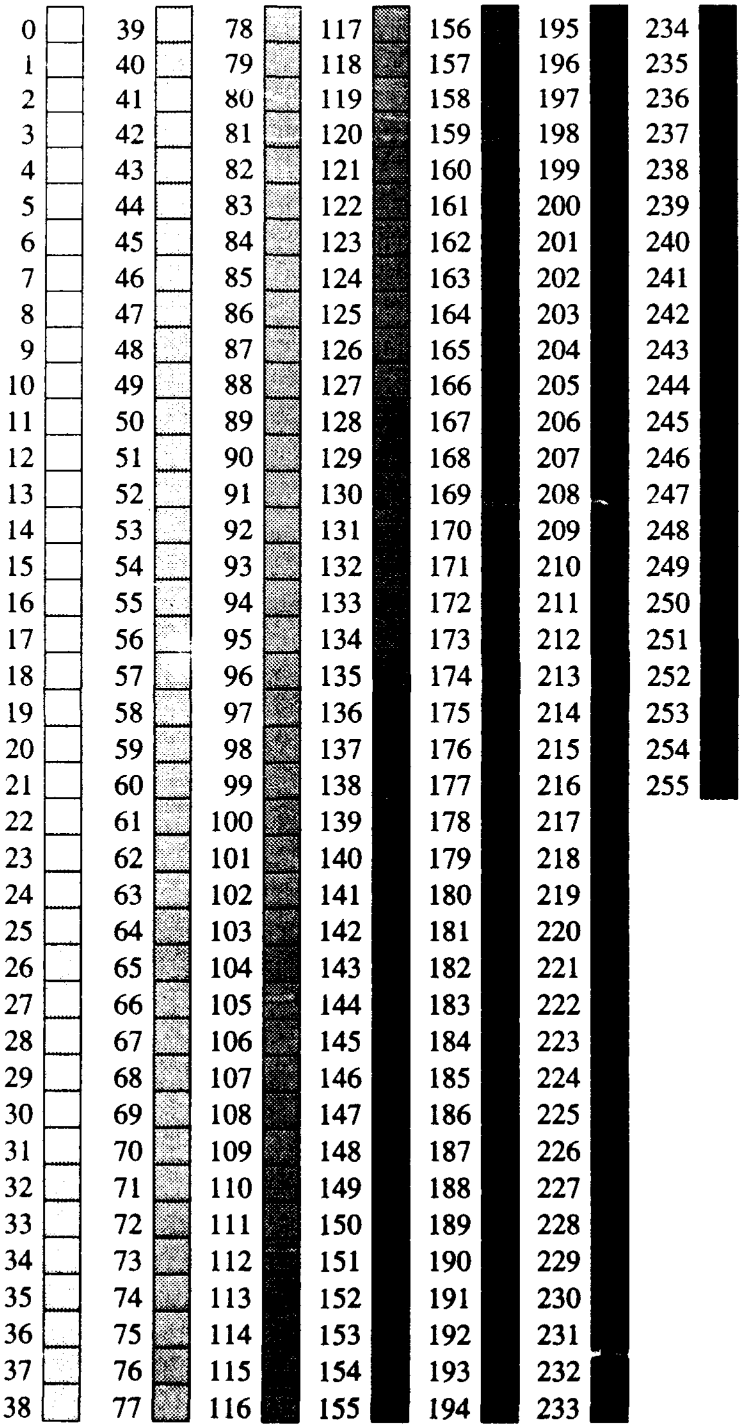


Figure - 2

PIXEL INTENSITY SCALE

Figure - 3



GLOBAL COORDINATE SYSTEM

Figure - 4 A

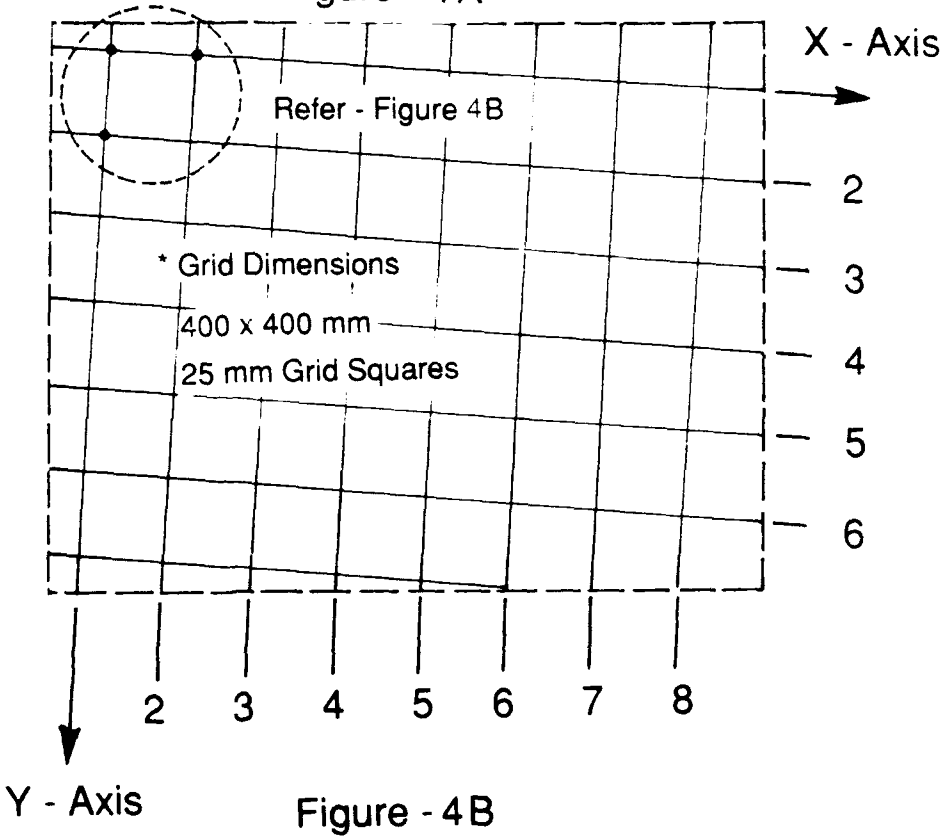
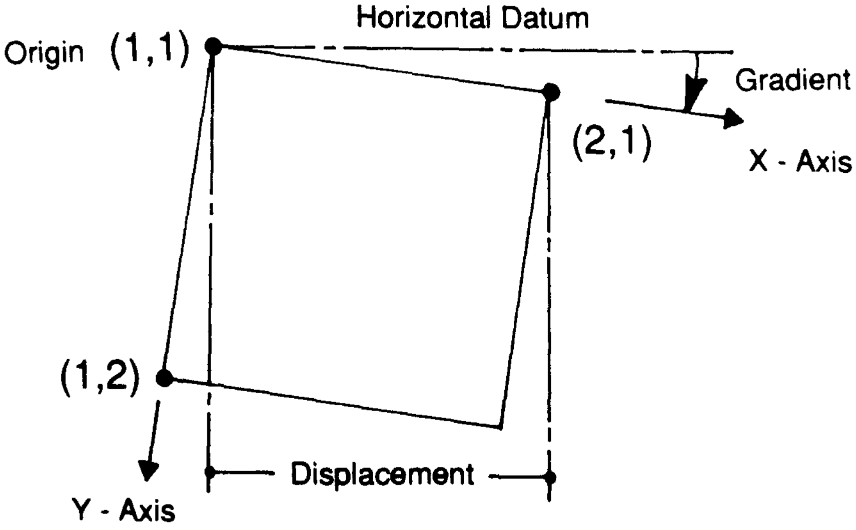
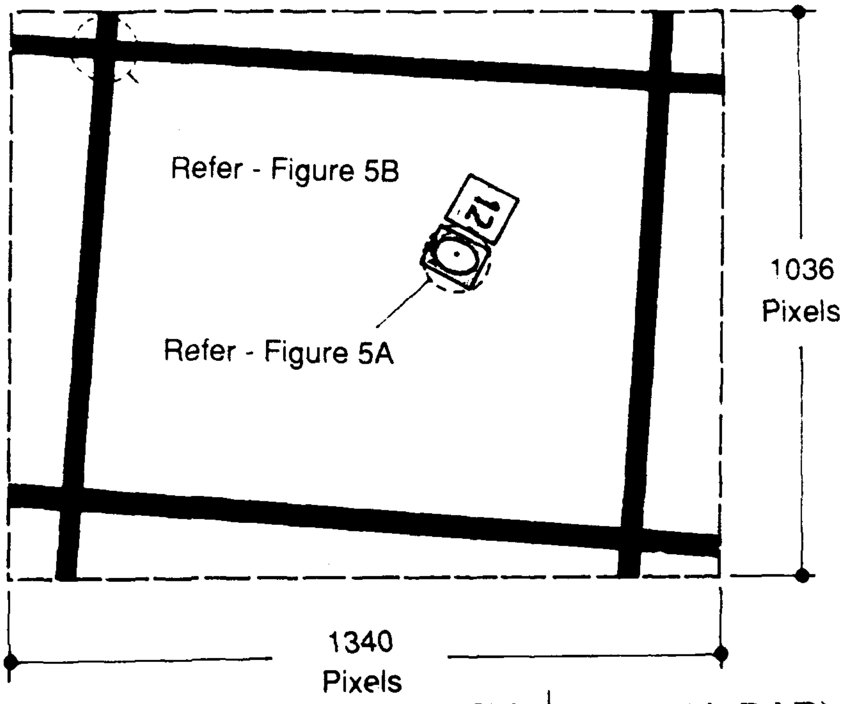


Figure - 4 B



MAGNIFIED GRID SQUARE



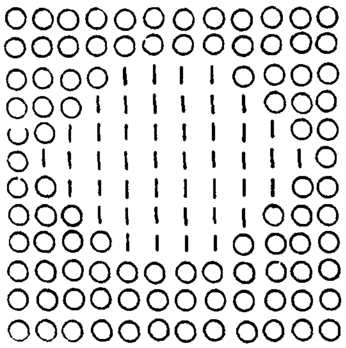


Figure - 5A

* Targets are confirmed using a binary image. Area and moments of intensity are determined for the shape and combined to give a non-dimensional ratio. This ratio is compared with the value for an ellipse.

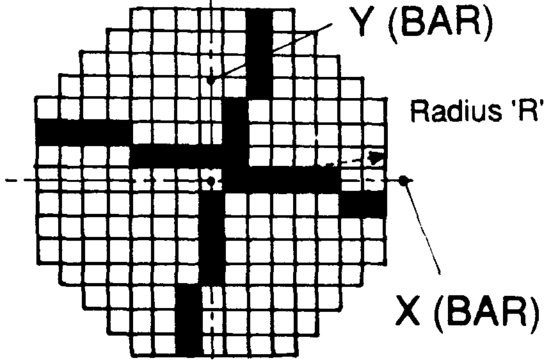


Figure - 5B

* Using an initial estimate for the grid intersection coordinates, pixels within a radius 'R' are considered in the calculation of X(bar) & Y(bar). Taking the coordinates X(bar) & Y(bar) as a new estimate for the grid intersection, the procedure is repeated until the coordinates converge.

TARGET CENTRE SURFACE PLOT

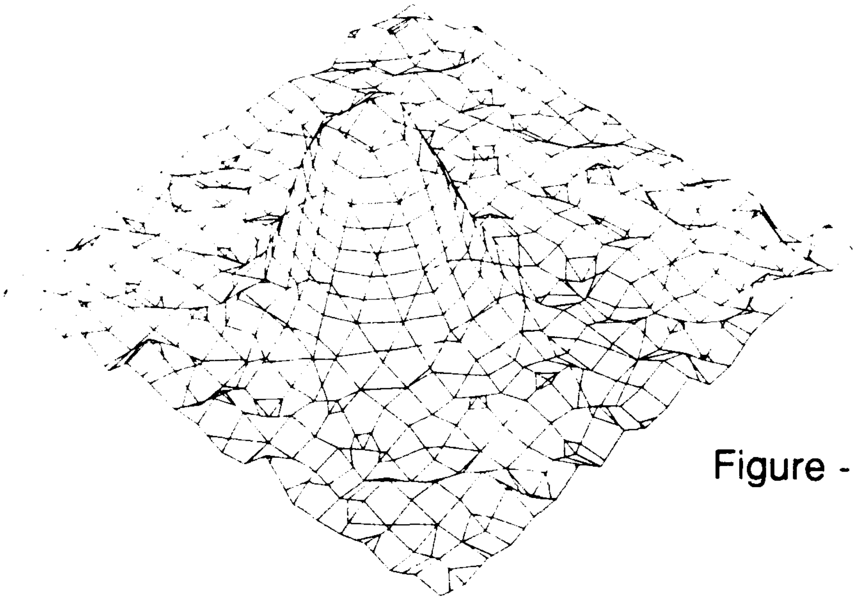


Figure - 6A

Note - This is an inverted 'NEGATIVE' image

TARGET CENTRE CONTOUR PLOT

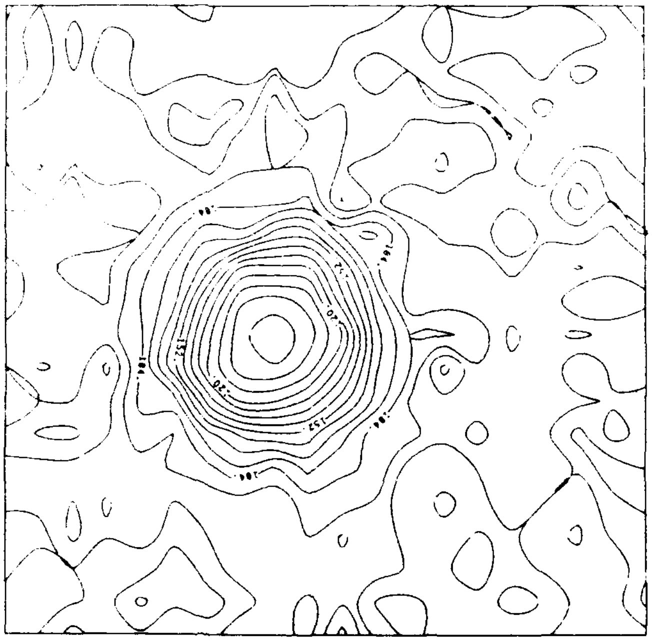
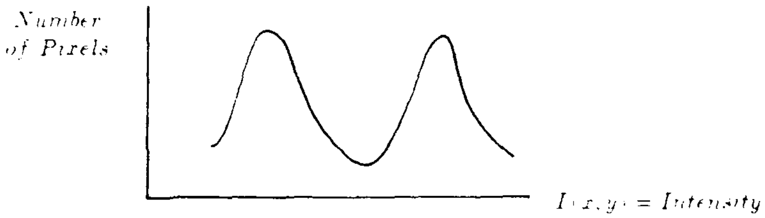


Figure - 6B

Appendix 1
A Technique for the Calculation of the Intensity Threshold Parameter

The distribution of intensities for any captured image is expected to be bimodal.



Where, the *Threshold Intensity Parameter* is given by the local minimum.

A *4th Order* polynomial describes the curve.

$$N(x) = Ax^4 + Bx^3 + Cx^2 + Dx + E \qquad (1)$$

We restrict the intensity variations to the range $-1 < X < 1$

Therefore we define -

$$X = \frac{I(x,y) - 127.5}{127.5} \qquad (2)$$

We use *least squares* to solve for the coefficients A - E

Separating the *Odd* & *Even* powers

For the *Odd* powers

$$D\sum x^2 + B\sum x^4 = \sum xN(x) \qquad (3A)$$

$$D\sum x^4 + B\sum x^6 = \sum x^3N(x) \qquad (3B)$$

Using matricies, we solve for B and D

$$\begin{pmatrix} \sum x^2 & \sum x^4 \\ \sum x^4 & \sum x^6 \end{pmatrix} \begin{pmatrix} D \\ B \end{pmatrix} = \begin{pmatrix} \sum xN(x) \\ \sum x^3N(x) \end{pmatrix}$$

Where

$\sum X^n$ - Summation of all intensities I(x,y) to the power n
N(x) - Number of pixels having intensity 'x'

Similarly, solving for all *Even* powers

$$E\sum 1 + C\sum x^2 + A\sum x^4 = \sum N(x) \qquad (4A)$$

$$E\sum x^2 + C\sum x^4 + A\sum x^6 = \sum x^2 N(x) \qquad (4B)$$

$$E\sum x^4 + C\sum x^6 + A\sum x^8 = \sum x^4 N(x) \qquad (4C)$$

Using matricies, we solve for A, C and E

$$\begin{pmatrix} \sum 1 & \sum x^2 & \sum x^4 \\ \sum x^2 & \sum x^4 & \sum x^6 \\ \sum x^4 & \sum x^6 & \sum x^8 \end{pmatrix} \begin{pmatrix} E \\ C \\ A \end{pmatrix} = \begin{pmatrix} \sum N(x) \\ \sum x^2 N(x) \\ \sum x^4 N(x) \end{pmatrix}$$

Taking the *First Derivative* of the polynomial gives :

$$4Ax^3 + 3Bx^2 + 2Cx + D = 0 \qquad (5)$$

Solving for the root of the polynomial gives the turning points for the curve.

If there is only one real root between $-1 < X < 1$ the intensity is *unimodal* that is, there were no targets detected.

Appendix 2
Criterion for the Confirmation of Targets

The [illegible] properties I_{xx}, I_{yy}, I_{xy} and area are combined to give the [illegible] ratio R. The ratio is computed for the target and compared with an Ideal Ellipse for which $R = 1$.

For the target, $\bar{X}$ and $\bar{Y}$ are the centre of gravity coordinates and are given by

$$\bar{X} = \frac{\sum I(x,y) X}{\sum I(x,y)} \quad (1)$$

$$\bar{Y} = \frac{\sum I(x,y) Y}{\sum I(x,y)} \quad (2)$$

where

$$I(x,y) = [illegible]$$

The moments of area are given by

$$I_{xx} = \sum I(x,y)(X - \bar{X})^2 \quad [illegible]$$

$$I_{yy} = \sum I(x,y)(Y - \bar{Y})^2 \quad [illegible]$$

$$I_{xy} = \sum I(x,y)(X - \bar{X})(Y - \bar{Y})$$

For the ellipse we calculate the principal moments of inertia [illegible] which are invariant under translation and rotation.

Firstly

$$I_{xx} = \int_0^{2\pi}\int_0^{1} [illegible]\, dr\, d\theta$$

$$= \int_0^{2\pi} [illegible]\, d\theta \quad [illegible]$$

$$= \left(\frac{ab^3}{4}\right)[illegible]$$

$$= \frac{ab^3\pi}{4}$$

and similarly,

$$\begin{aligned}
I_{yy} &= \int_0^{2\pi}\int_0^1 (a^2 r^2 cos^2\theta)(ab)\,r\,dr\,d\theta \\
&= \int_0^{2\pi}\left[\frac{a^3 r^4 b}{4} cos^2\theta\right]_0^1 d\theta \\
&= \left(\frac{a^3 b}{4}\right)\left[\frac{\theta}{2}\right]_0^{2\pi} \\
&= \frac{a^3 b\pi}{4}
\end{aligned} \tag{3B}$$

thus

$$I_{xx} = \frac{ab^3\pi}{4} \qquad I_{yy} = \frac{a^3 b\pi}{4}$$

Normalising the expressions for I_{xx} and I_{yy} by the area of an ellipse gives I^*_{xx} & I^*_{yy} -

$$I^*_{xx} = \frac{I_{xx}}{\pi ab} = \frac{b^2}{4} \qquad I^*_{yy} = \frac{I_{yy}}{\pi ab} = \frac{a^2}{4} \tag{4A \& 4B}$$

The normalised Intensity matrix is given by -

$$A = \begin{pmatrix} I^*_{xx} & 0 \\ 0 & I^*_{yy} \end{pmatrix}$$

The determinant for this matrix can be written -

$$\begin{aligned}
Determinant &= (I^*_{xx})(I^*_{yy}) - (I_{xy})^2 \\
&= \left(\frac{b^2}{4}\right)\left(\frac{a^2}{4}\right) \\
&= \frac{a^4 b^4}{16}
\end{aligned} \tag{5}$$

Using the expression for the area of an ellipse, we can derive the non-dimensional ratio R, thus

$$\begin{aligned} Area &= 4\pi\sqrt{Det(A)} \\ \pi ab &= 4\pi\sqrt{Det(A)} \\ ab &= 4\sqrt{Det(A)} \end{aligned} \tag{6}$$

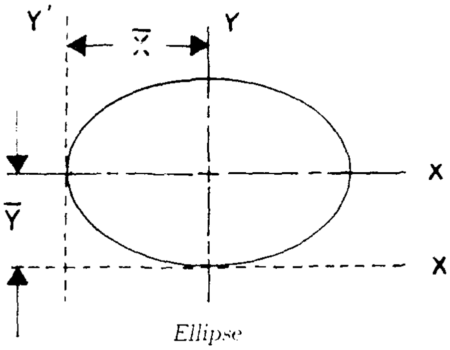
Where, R is defined as -

$$R = \frac{ab}{4\sqrt{Det(A)}} \tag{7}$$

For an *Ideal Ellipse*, R=1

The ratio R is computed for suspect targets and compared with the value for an *Ideal Ellipse*.

Appendix 3
The Determination of Target Centres using Non-Linear Least Squares



Ellipse

The target centres are best approximated by an ellipse. The method by which the ellipse is estimated is shown below.
The general equation for an ellipse is given by -

$$\frac{(X - X_0)^2}{2\sigma_1^2(1-\rho^2)} - \frac{\rho(X - X_0)(Y - Y_0)}{(1-\rho^2)\sigma_1\sigma_2} + \frac{(Y - Y_0)^2}{2\sigma_2^2(1-\rho^2)} = 1 \qquad (1)$$

Where estimates of, X_0 and Y_0, are given by -

$$X_0 = \frac{\sum I(x,y)X}{\sum I(x,y)} \qquad Y_0 = \frac{\sum I(x,y)Y}{\sum I(x,y)} \qquad (2A \;\&\; 2B)$$

and the parameters σ_1^2, σ_2^2 and ρ are given by -

$$\sigma_1^2 = \frac{\sum I(x,y)(X - X_0)^2}{\sum I(x,y)} \qquad \sigma_2^2 = \frac{\sum I(x,y)(Y - Y_0)^2}{\sum I(x,y)} \qquad (3A \;\&\; 3B)$$

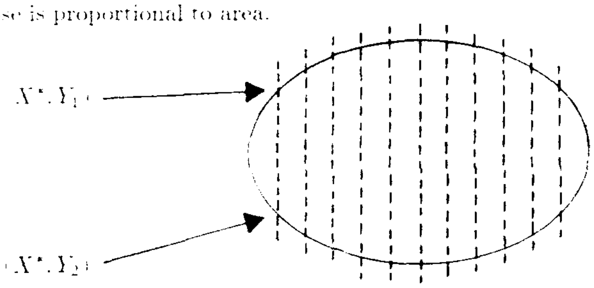
and

$$\rho = \frac{\sum I(x,y)(X - X_0)(Y - Y_0)}{\sum I(x,y)\sigma_1\sigma_2} \qquad (3C)$$

and

$$I(x,y) = Intensity$$

The ellipse can be divided in into line segments. Integrating the line segments over the ellipse is proportional to area.



The points of intersection are (X^*, Y_1), and (X^*, Y_2)

The line segments intersect the ellipse only if -

For X intercept

$$\left\{ \frac{\rho (X^* - X_0)}{(1 - \rho^2) \sigma_1 \sigma_2} \right\}^2 - \frac{2}{\sigma_2^2 (1 - \rho^2)} \left\{ \frac{(X^* - X_0)}{2\sigma_1^2 (1 - \rho^2)} - 1 \right\} > 0 \tag{4}$$

or

$$\frac{2}{\sigma_2^2 (1 - \rho^2)} - \frac{(X^* - X_0)^2}{(1 - \rho^2) \sigma_1^2 \sigma_2^2} > 0 \tag{4A}$$

thus, intersection occurs if -

$$X_0 - \sigma_1 \sqrt{2} \leq X^* \leq X_0 + \sigma_1 \sqrt{2}$$

Similarly, the 'Y' intersection points are determined -

$$Y = Y_0 + \sigma_2^2 (1 - \rho^2) \left\{ \frac{\rho (X^* - X_0)}{(1 - \rho^2) \sigma_1 \sigma_2} + \sqrt{\frac{2}{\sigma_2^2 (1 - \rho^2)} - \frac{(X^* - X_0)^2}{(1 - \rho^2) \sigma_1^2 \sigma_2^2}} \right\} \tag{5}$$

Therefore, we can write

$$Y_1 = Y_0 + \rho \left(\frac{\sigma_2}{\sigma 1} \right) (X^* - X_0) + \sigma_2 \sqrt{1 - \rho^2} \left\{ 2 - \frac{(X^* - X_0)^2}{\sigma_1^2} \right\}^{1/2} \tag{5A}$$

$$Y_2 = Y_0 + \rho \left(\frac{\sigma_2}{\sigma 1} \right) (X^* - X_0) - \sigma_2 \sqrt{1 - \rho^2} \left\{ 2 - \frac{(X^* - X_0)^2}{\sigma_1^2} \right\}^{1/2} \tag{5B}$$

thus

$$Y_0 - \sigma_2 \sqrt{2} \leq Y^* \leq Y_0 + \sigma_2 \sqrt{2}$$

For particular values of X_0, Y_0, σ_1, σ_2 and ρ there exists a matrix $A(i,j)$, where $A(i,j) = 1$ for pixels within the ellipse and $A(i,j) = 0$ for pixels outside the ellipse.

We can express the problem as the minimisation of the *Objective Function* given by -

$$Objective\ Function = \frac{1}{N}\sum\{-O(i,j) + P + Q(A(i,j))\}^2 \qquad (6)$$

Where

N - Number of *Intensities* $I(x,y)$
O(i,j) - Observed Intensity (x,y)
P - Background *Intensity*
Q - Foreground *Intensity*

and

$$\overline{X} = \frac{\sum X(i,j)}{N} \qquad (7)$$

Thus

$$P + Q\overline{A} = \overline{O} \qquad (8A)$$

$$P\overline{A} + Q\overline{A^2} = \overline{OA} \qquad (8B)$$

therefore

$$P = \frac{\overline{O}\left(\overline{A^2}\right) - \overline{A}(\overline{OA})}{\overline{A^2} - (\overline{A})^2} \qquad Q = \frac{\overline{OA} - (\overline{O})\,(\overline{A})}{\overline{A^2} - (\overline{A})^2} \qquad (9A\ \&\ 9B)$$

The parameters are incremented until the *Objective Function* converges to a minimum.

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16. ABSTRACT

This Technical Memorandum is a continuation of earlier work into the application of photogrammetric techniques for the modal analysis of vibrating structures. Methods are developed for the computer processing of photogrammetric negatives. The negatives capture the positions of retro-reflective targets located on the vibrating structure. With the relative distances between targets known, the distorted structure can be reconstructed. All targets are given a Global coordinate, *thus reducing the accumulation of errors over the negative. The techniques used to establish the* Global Coordinate System *are discussed as are the methods used to determine accurately the location of the retro-reflective targets.*

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