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The goal of this project was to put the intuitive idea of gain-scheduling on a rigorous foundation for a class of nonlinear, distributed-parameter systems. This involved a study of the existence and characterization of the ideal, infinite-dimensional, feedback control. Since in most applications the feedback function cannot be computed in closed form it was necessary to study the convergence of approximate feedback functions, based on increasingly higher order finite-dimensional approximations of the system, to the ideal function. Finally, the results were applied to Burgers' Equation, which can be viewed as a low-order approximation to a wide variety of physical phenomena, including viscous compressible flow.

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Control of Nonlinear Distributed Parameter Systems with Application to Flow Control

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Control of Nonlinear Distributed Parameter Systems
with Application to Flow Control

The goal of this project was to put the intuitive idea of gain-scheduling on a rigorous foundation for a class of nonlinear, distributed-parameter systems. This involved a study of the existence and characterization of the ideal, infinite-dimensional, feedback control. Since in most applications the feedback function cannot be computed in closed form it was necessary to study the convergence of approximate feedback functions, based on increasingly higher order finite-dimensional approximations of the system, to the ideal function. Finally, our results were applied to Burgers' Equation, which can be viewed as a low-order approximation to a wide variety of physical phenomena, including viscous compressible flow.

The systems considered in this study were modeled by an abstract differential equation of the form

$$\dot{x} = F(x,u)$$

where $x \in \mathfrak{X}$, $u \in \mathfrak{U}$, the state space $\mathfrak{X}$ and the input space $\mathfrak{U}$ are Banach spaces with $\mathfrak{U}$ finite dimensional. In many cases of interest, F will be an unbounded, nonlinear, differential operator. For the purpose of computing equilibrium points and linearizing this operator we are led to restrict F to a dense subspace of $\mathfrak{X}$ with a norm that makes F differentiable. By considering the set of equilibrium points

$$\{(x^o(\alpha), u^o(\alpha)) \mid \alpha \in \mathcal{A}\}$$

we can compute the linearizations about these points to be

$$\dot{x} = D_1F(x^o(\alpha), u^o(\alpha))x + D_2F(x^o(\alpha), u^o(\alpha))u.$$

For each fixed α, we can design a linear feedback of the form

$$u = G(\alpha)x + H(\alpha)w$$

which yields the desired closed loop linear system

$$\dot{x} = [D_1F(x^o(\alpha), u^o(\alpha)) + D_2F(x^o(\alpha), u^o(\alpha))G(\alpha)]x + D_2F(x^o(\alpha), u^o(\alpha))H(\alpha)w.$$

The objective of gain scheduling is to derive a nonlinear feedback

$$u = S(x,w)$$

such that the linearizations of the closed-loop nonlinear system are equal to the desired closed-loop linear systems. This should produce desirable performance in a neighborhood of the equilibrium set. For scheduling to be possible when the state feedback G is arbitrary, the control transformation H must belong to a family of operators that depend on G. We have shown that a feedback of the form

$$S(x,w) = u^o(P_2(x,w)) + K(P_2(x,w)) \cdot ((x,w) - P(x,w)),$$

where $P = [P_1 \;\; P_2]: \mathfrak{H} \times \mathfrak{U} \to \mathfrak{H} \times \mathfrak{U}$ is a projection from the state-control space to the equilibrium set and $K = [G \;\; H]$, will produce the desired result.

Since we cannot generally compute $S(x,w)$, finite-dimensional approximations of order N of the original nonlinear system are considered on state spaces $\mathfrak{H}^N \subset \mathfrak{H}$, and finite-dimensional feedbacks $S^N(x^N,w)$ are computed. These feedback functions have the same structure as the infinite-dimensional functions and it is not hard to write down the conditions under which $S^N(x^N,w) \to S(x,w)$ as $N \to \infty$. It is not enough, however, that all of the individual functions in the feedback formula, e.g. K or u^o, converge. In many cases we must require that these functions converge in a locally uniform manner.

Definition: *Let (f_n) be a sequence of functions defined on an open set A. We say that (f_n) converges locally uniformly (l.u.) to a function f if every $a \in A$ has a neighborhood $\mathcal{N}(a)$ such that $f_n(x) \to f(x)$ uniformly for x in $\mathcal{N}(a)$.*

The condition of most significance for controller design is that we must require that for every $\xi \in \mathfrak{H}$, $G^N(\alpha)\Pi^N \cdot \xi \to G(\alpha) \cdot \xi$ in $\mathfrak{U}$, l.u. in α, where $\Pi^N: \mathfrak{H}^N \to \mathfrak{H}$ is a projection operator.

To proceed further, we must specify how the state feedback gains will be computed and show that the desired convergence does indeed occur. In this work, the feedback gains were computed by solving a linear quadratic regulator (LQR) problem. Relying heavily on the previous work of Gibson, Banks, and Kunisch, the following result was proved.

Let $\mathfrak{H}^N$, $N = 1,2,\ldots$ be a sequence of finite-dimensional linear subspaces of $\mathfrak{H}$ and $\Pi^N: \mathfrak{H} \to \mathfrak{H}^N$ be the canonical orthogonal projections. Assume that for every $\alpha \in \mathcal{A}$, $T^N(t;\alpha)$ is a sequence of strongly continuous semigroups on $\mathfrak{H}^N$ with infinitesimal generators $A^N(\alpha) \in \mathcal{L}(\mathfrak{H}^N)$. Given operators $B^N(\alpha) \in \mathcal{L}(\mathfrak{U}, \mathfrak{H}^N)$ and $D^N(\alpha) \in \mathcal{L}(\mathfrak{H}^N)$, we consider the family of regulator problems:

$(\mathfrak{R}^N, \alpha)$ Given $\alpha \in \mathcal{A}$. minimize $J^N(x^N(0), u; \alpha)$ over $u \in L^2(0,\infty;\mathfrak{U})$, where

$$\begin{aligned} \dot{x}^N(t) &= A^N(\alpha)x^N(t) + B^N(\alpha)u(t), \qquad t > 0, \qquad \alpha \in \mathcal{A}, \\ x^N(0) &= x_0^N \equiv \Pi^N x_0, \end{aligned} \tag{1}$$

and

$$J^N(x_0^N, u; \alpha) = \int_0^\infty \{\langle D^N(\alpha)x^N(t), x^N(t)\rangle + \langle Q(\alpha)u(t), u(t)\rangle\} dt, \tag{2}$$

Assumptions:

H1 For each $x_0^N \in \mathfrak{H}^N$ and for each $\alpha \in \mathcal{A}$, there exists an admissible control $u^N \in L^2(0,\infty;\mathfrak{U})$ for $(\mathfrak{R}^N,\alpha)$ and any admissible control for (1), (2) drives the state of (1) to zero asymptotically.

H2 (i) Every $\alpha \in \mathcal{A}$ has a neighborhood $\mathcal{N}(\alpha)$ such that for each $z \in \mathfrak{H}$, we have $T^N(t;\alpha)\Pi^N z \to T(t;\alpha)z$ with the convergence uniform in (t,α) on $I \times \mathcal{N}(\mathcal{A})$, where I is an arbitrary bounded subset of $[0,\infty)$.

(ii) Every $\alpha \in \mathcal{A}$ has a neighborhood $\mathcal{N}(\alpha)$ such that for each $z \in \mathfrak{H}$, we have $T^N(t;\alpha)^*\Pi^N z \to T(t;\alpha)^* z$ with the convergence uniform in (t,α) on $I \times \mathcal{N}(\mathcal{A})$, where I is an arbitrary bounded subset of $[0,\infty)$.

(iii) For each $v \in \mathfrak{U}$, $B^N(\alpha)v \to B(\alpha)v$ locally uniformly in α.

(iv) For each $z \in \mathfrak{H}$, $B^N(\alpha)^*\Pi^N z \to B(\alpha)^* z$ locally uniformly in α.

(v) For each $z \in \mathfrak{H}$, $D^N(\alpha)\Pi^N z \to D(\alpha)z$ locally uniformly in α.

Theorem. *Suppose (H1), (H2) hold, $Q(\alpha) > 0$, $D(\alpha) \geq 0$ and $D^N(\alpha) \geq 0$ and let $R^N(\alpha)$ denote the unique nonnegative selfadjoint Riccati operators on $\mathfrak{H}^N$ for the problems $(\mathfrak{R}^N;\alpha)$. Further assume that for each $\alpha \in \mathcal{A}$, a unique nonnegative selfadjoint Riccati operator on $\mathfrak{H}$ for the problem $(\mathfrak{R};\alpha)$ exists and is a strongly continuous function of α. Let $S(t;\alpha)$ and $S^N(t;\alpha)$ be the semigroups generated by $A(\alpha) - B(\alpha)Q^{-1}(\alpha)B^*(\alpha)R(\alpha)$ and $A^N(\alpha) - B^N(\alpha)Q^{-1}(\alpha)B^{N*}(\alpha)R^N(\alpha)$ on $\mathfrak{H}$ and $\mathfrak{H}^N$, respectively and suppose $\| S(t;\alpha)z \| \to 0$ as $t \to \infty$, for all $z \in \mathfrak{H}$ and $\alpha \in \mathcal{A}$. If for every $\widehat{\alpha} \in \mathcal{A}$ there is a neighborhood $\mathcal{N}(\widehat{\alpha})$ and positive constants M_1, M_2 and ω independent of N and t such that*

$$\| S^N(t;\alpha) \|_{\mathfrak{H}^N} \leq M_1 e^{-\omega t} \quad \text{for } t \geq 0, \quad N = 1,2,\ldots, \quad \alpha \in \mathcal{N}(\widehat{\alpha})$$

and

$$\| R^N(\alpha) \|_{\mathfrak{H}^N} \leq M_2 \qquad \text{for } N = 1,2,\ldots, \quad \alpha \in \mathcal{N}(\widehat{\alpha})$$

then

$$R^N(\alpha)\Pi^N z \to R(\alpha)z \quad \text{for every } z \in \mathfrak{H}, \quad \text{locally uniformly in } \alpha.$$

This result replaces the conditions on the LQR feedback gains with conditions on the open-loop system. To verify that these conditions can indeed be satisfied, we restrict attention to systems whose families of linearizations consist of parabolic differential equations of the form

$$\dot{x} = a(s,\alpha)\frac{\partial^2 x}{\partial s^2} + b(s,\alpha)\frac{\partial x}{\partial s} + c(s,\alpha)x + f(s,\alpha)u \quad t>0,\ 0\leq s\leq 1,\ \alpha\in\mathcal{A},$$

with initial condition $x(0,s)=x_o(s)$, boundary conditions $x(t,0)=x(t,1)=0$, and solutions in the space $\mathfrak{H}=L^2(0,1)$. Define the operator $A_k(\alpha)$ through the sesquilinear form

$$\sigma(y,z;\alpha) = \int_0^1 [a(s,\alpha)\frac{\partial y}{\partial s}\frac{\partial \overline{z}}{\partial s} - (b(s,\alpha)\frac{\partial y}{\partial s} + (c(s,\alpha)-k)y)\overline{z}\ ]ds = \langle -A_k(\alpha)y, z\rangle$$

where k is a constant chosen to make the form coercive. Defining

$$w(\alpha) = (I - A_k(\alpha))^{-1}z,$$
$$w^*(\alpha) = (I - A_k^*(\alpha))^{-1}z,$$

it can be shown that **H2** (i-ii) is satisfied for the systems being considered if the following condition is met:

(C1): *For every $z\in L^2(0,1)$, there are functions $\widetilde{w}^N{:}\,\mathcal{A}\to\mathfrak{H}^N$ and $\widetilde{w}^{*N}{:}\,\mathcal{A}\to\mathfrak{H}^N$, $N=1,2,\ldots$, such that $\| w(\alpha)-\widetilde{w}^N(\alpha)\|_1\to 0$ and $\| w^*(\alpha)-\widetilde{w}^{*N}(\alpha)\|_1\to 0$ locally uniformly in α.*

H2 (iii-v) are easily satisfied by appropriate selection of the approximating problems. The remaining hypotheses of the theorem, save the strong continuity of the Riccati operator, can be shown to follow from **C1** and the condition:

(C2): *The pair $(A(\alpha),B(\alpha))$ is exponentially stabilizable, locally uniformly in α. This means that for every $\widehat{\alpha}\in\mathcal{A}$, there is a neighborhood $\mathcal{N}(\widehat{\alpha})$ and a bounded linear operator $K_s{:}\,H^0(0,1)\to\mathfrak{U}$, independent of α, such that for every $\alpha\in\mathcal{N}(\widehat{\alpha})$, the semigroup $T_s(t;\alpha)$, generated by $A(\alpha)+B(\alpha)K_s$, satisfies $\| T_s(t;\alpha)\| \leq M_1 e^{-\omega_1 t}$ for some positive constants M_1 and ω_1.*

To show that the above theory can be applied to a relevant example, we consider a modified form of Burgers' equation given by

$$x_t = \epsilon x_{ss} - x x_s + cx + uf$$

and subject to the conditions

$$x(s,0) = x_o(s), \quad x(0,t) = x(1,t) = 0, \quad 0 \le s \le 1, \quad t \ge 0.$$

The constant c is used to make the equilibrium point at the origin unstable, u is the control input and f is the control influence function. The linearization of this system is given by

$$x_t = \epsilon x_{ss} - x^o(\alpha)x_s + (c - x_s^o(\alpha))x + \alpha f,$$

where $x^o(\alpha)$ is the equilibrium state. In the state space formalism, we identify the state operator as

$$A(\alpha)x = \epsilon x_{ss} - x^0(\alpha)x_s + (c - x_s^0(\alpha))x.$$

The linearizations are parabolic partial differential equations, as required by our results. Moreover, $\{A(\alpha) : \alpha \in \mathcal{A}\}$ constitute an analytic family of operators and therefore generates a family of bounded operators $T(t;\alpha)$ that are analytic in t and α.

The finite-element technique is used to obtain finite-order approximations to the original system, and $x(s)$ is approximated as a piecewise-linear continuous function. Relying heavily on the analyticity of $T(t;\alpha)$ and using elementary facts concerning finite-element approximations we have shown that conditions C1 and C2 are satisfied by our example. In addition, using some recent results of Duncan on continuity of Riccati operators depending on a parameter we are able to show that $R(\alpha)$ is indeed strongly continuous. Thus, we can be assured of convergence of the approximate controllers to the ideal controller. A number of computations were performed to illustrate the convergence numerically.

In summary, we have placed the intuitive idea of gain-scheduling on a rigorous mathematical foundation for a class of nonlinear distributed-parameter systems. We have considered the problem of approximating the ideal controller by finite-dimensional controllers and have given conditions for the appropriate convergence to take place. By specializing to the case of parabolic linearizations, these conditions can be replaced by a much simpler set of conditions which we have shown to be satisfied by a modified form of Burgers' equation.

We have also explored several interesting problems for finite-dimensional nonlinear systems. Several schemes for incorporating global information into a gain-scheduled controller were investigated. As discussed above, our current controllers can not be guaranteed to produce desired performance outside a neighborhood of the equilibrium manifold. To modify undesirable performance far from equilibrium, it is necessary to incorporate the desired control action into the local nonlinear controller. It was found that our gain-scheduled controller provides a useful structure for incorporating this information. Unstructured, spline-based methods did not perform as well. We have also explored the efficacy of a nonlinear PID controller applied to a highly nonlinear model of a chemical process. The performance was comparable to that achieved by much more sophisticated nonlinear controllers that required more information for design and more complex on-line computation.

PAPERS WHICH ACKNOWLEDGE THE GRANT

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