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The work reported on is part of a multidisciplinary effort aimed at the development of analytical and computational tools for the modeling and simulation of very flexible structures. This research is concerned with three main areas (i) Mathematical aspects, with special emphasis on nonlinear stability analysis; (ii) Modeling aspects, with emphasis in very flexible slender structures; and (iii) Numerical analysis aspects, with emphasis on methodologies suitable for large scale and accurate simulation. The present research effort has resulted in 36 publications, two conferences, and a strong collaboration (in a related research effort also sponsored by AFOSR) with groups at UC Berkeley and U. Maryland.

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Mathematical Modeling and Numerical Simulation of the Dynamics of Flexible Structures Undergoing Large Overall Motions

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Abstract

The present documment summarizes the main results obtained during the period 1988/1990 in areas related to the *mathematical modeling and numerical simulation of the dynamics of flexible structures undergoing large overall motions.* This is the final report of a two year research effort sponsored by AFOSR under current grant DJA/AFOSR 89-0294 with Stanford University, as a continuation of the effort initiated in 1986 under grant DJA/AFOSR 86-0292 with Stanford University.

The work reported on herein is part of a multidisciplinary effort aimed at the development of analytical and computational tools for the modeling and simulation of very flexible structures. This research is concerned with three main areas: **(i)** Mathematical aspects, with special emphasis on nonlinear stability analysis; **(ii)** Modeling aspects, with emphasis in very flexible slender structures; and **(iii)** Numerical analysis aspects, with emphasis on methodologies suitable for large scale and accurate simulation. The present research effort has resulted in 36 publications (see Appendix I), two conferences, and a strong collaboration (in a related research effort also sponsored by AFOSR) with groups UC Berkeley and U. Maryland.

Contents

1. Introduction

The present report gives an overview of the main results obtained during the past two years of research sponsored by AFOSR under current grant DJA/AFOSR 89-0294 with Stanford University. This effort is a continuation of a previous two–year research effort also sponsored by AFOSR under grant DJA/AFOSR 86-0292 with Stanford University. The goal of this work is the development of analytical and computational tools for design, analysis and simulation of the dynamics of very flexible structures undergoing large overall motions.

The organization of this report is as follows. Section **2** gives a brief outline of the scope of the proposed research along with a summary of the research goals set forth in our proposal to AFOSR. Section **3** gives a summary of the most important results obtained in the course of this investigation which fall within three main areas: (**i**) Mathematical aspects involved in the stability analysis of the mechanical systems of interest, (**ii**) Mathematical modeling of the physical systems of interest; in particular, very flexible structures, and (**iii**) Numerical analysis and computational aspects involved in the simulation of the dynamic response of these systems. Finally, Section **3** gives a brief outline of some lines of research to be pursued in a continuation of this effort. Further details on the scope of this proposed research are given in a follow–up proposal. Appendix I contains a complete relation of the 36 publication produced under the sponsorship of AFOSR since the initiation of this research.

2. Scope of the Proposed Research

As stated in our proposal to AFOSR, the central theme of the present investigation is the formulation, mathematical and numerical analysis of models for flexible multi–body systems, suitable for precision control and amenable to large–scale simulation. In particular, we are concerned with so–called *geometrically exact* structural models, a terminology introduced by S. ANTMAN and J. SIMO, for which no restriction is placed on the degree of allowable flexibility in the structure. From an engineering point of view, this class of models allow the correct modeling of important phenomena such as large deformations, nonlinear vibration, bifurcations and structural damage which may arise, for instance, as a result of control system breakdown. We remark that geometrically nonlinear effects play an important role in a number of engineering applications, particularly those involving rotating structures, and their prediction requires properly formulated nonlinear models. In fact, as demonstrated in SIMO & VU–QUOC [1988b], use of linearized theories can produce completely misleading and erroneous results.

Within the preceding context, the following three basic areas of investigation were proposed for the period 1988/1990 (see Section **5** of our renewal proposal):

i. *Formulation and analysis of geometrically exact shell models.* In particular two concrete objectives were proposed within this area of research. First, the formulation of geometrically exact shell models which account for thickness change. This class of models are of considerable interest in technological applications concerned, for instance, with composite shell structures where the prediction important effects such as *delamination* require the use of these enhanced models. Second, the formulation, algorithmic development and numerical analysis aspects involved in the implementation of inelasticity and damage constitutive models for geometrically exact shells.

ii. *Numerical analysis of the dynamics of shell models.* It was proposed to address in detail numerical analysis aspects involved in the actual implementation of geometrically exact shell models. In particular, the development of exact and singularity–free configuration update procedures, the investigation of appropriate finite element discretizations suitable for effective numerical simulations, and the use of covariant linearization techniques to derive exact tangent operators which plays a crucial role in incremental, iterative, solution strategies. The overall goal here is the analysis and development of computational capabilities suitable for large–scale simulation.

iii. *Stability Analysis. The Energy–Momentum method.* The two areas alluded to above are concerned primarily with the development of mathematical models, numerical analysis techniques and computational tools for the integrated design of flexible structures. In addition, it was proposed to continue the efforts on fundamental areas of hamiltonian and continuum mechanics, with emphasis on applications to stability analysis of relative equilibria. This problem is of particular importance in areas such as spacecraft dynamics where effects related to the flexibility of the appendages (antenae, solar panels, etc...,) play a critical role in the stability of the system.

Our proposal contained a fourth area of research concerned with the formulation and numerical analysis of *conserving* (symplectic) *time stepping–algorithms* for nonlinear mechanics. Recently, we have obtained new and rather exciting results in this area, which will be reported on in the follow–up renewal proposal.

3. Summary of research objectives accomplished

During the last year we have made considerable progress in the three areas of research outlined above. The main developments are summarized below along with a comment on selected publications recorded in Appendix I.

i. *Formulation and analysis of geometrically exact shell models.* The two objectives set forth under this line of research have been completed. In SIMO, FOX & RIFAI [1990b], we have presented and developed in detail a fully nonlinear shell theory that incorporates finite thickness changes. A comprehensive modern treatment and summary of the current status of nonlinear shell theory is given in the thesis of FOX [1990]. In SIMO, MARSDEN & KRISHNAPRASAD [1988], we have completed our program on the systematic study of the hamiltonian structure of nonlinear elasticity, rods and shells. These results play a crucial role in the formulation and analysis of the algorithms described below. In SIMO & KENNEDY [1990], we have undertaken the formulation, numerical analysis and detailed implementation of inelastic constitutive models formulated in stress resultants. This paper also contains a number representative large scale numerical simulations; for instance, the the full simulation of the inelastic collapse of cylindrical shells in the plastic regime.

ii. *Numerical analysis of the dynamics of shell models.* The objectives and program of research set forth under this line of investigation have also been completed. In particular, in SIMO, FOX & RIFAI [1990a,b], we have provided a complete numerical analysis and algorithmic treatment of nonlinear (geometrically exact) shell theory incorporating thickness changes. This includes the construction of singularity–free shell parametrizations and the development of effective configuration update procedures which remain exact regardless the magnitude of the incremental displacements and/or rotations. Such a development exploits in a crucial manner the geometric framework developed in item **i** above, along with ideas of modern differential geometry. Related developments for nonlinear rods are addressed in SIMO & VU–QUOC [1988a,b,1990].

Simulations of the dynamics of flexible structures rely crucially on finite element discretizations of the continuum problem. For plates and shell structures, standard finite element methods often lead to discretizations that experience severe convergence and accuracy problems in the thin shell limit. In the past, ad–hoc methods have been developed, typically from an engineering perspective, to circumvent these well–known difficulties. In SIMO & RIFAI [1990] a general class of *stable* mixed finite element methods is developed for a number of *constrained problems*; in particular, very thin shell, which exhibit excellent accuracy characteristics for coarse meshes. These and related finite element techniques are employed and further developed in the specific context of shells in SIMO, FOX & RIFAI [1990a,b]. Further applications are considered in references [28-32] of Appendix I. An overview of the state of the art in mathematical and numerical analysis aspects related to rods, plates and shells is contained in the recent book quoted in reference [34].

iii. *Stability analysis. The Energy–Momentum method.* The progress made in this area of research over the past two years has led to what appears to be significant new results in methodologies for the stability analysis (and eventual bifurcation) of hamiltonian systems with symmetry. Initially, it was proposed to consider as a representative model problem the stability of coupled rigid bodies/flexible attachments by means of a methodology going back to ARNOLD [1966] and currently referred to as the energy–Casimir method. Remarkably, however, this well–established method fails for realistic models of the flexible appendage. The crucial difficulty lies precisely in the actual characterization of the Casimir functions associated with the mathematical model of the appendage.. We recall that Casimirs are conserved quantities in the reduced space; e.g., for rigid bodies the norm of the total angular momentum. For nonlinear elasticity, rods and shells no explicit Casimirs appear to be known.

In SIMO, POSBERGH & MARSDEN [1990a], the difficulties alluded to above are completely bypassed by exploiting directly the variational characterization of relative equilibria as critical points of the energy–momentum map, and working directly on the canonical phase space. Casimir functions, which as pointed out above may difficult to characterize or may not exist at all, play no role in this approach. The methodology is further refined, generalized and successfully applied to specific examples in LEWIS & SIMO [1990a]. Additional applications and mathematical aspects of the method are considered in references

[22-26] of Appendix I. The proceedings of the 1998 summer conference (see reference [35]) contain a number of related papers and give a recent overview of work in this area.

We conclude this brief overview by noting the positive effect on the research reported on above derived from our continued interaction with J.E. MARSDEN at Berkeley, and P.S. KRISHNAPRASAD at Maryland in a related effort also sponsored by AFOSR.

3. Proposed areas of research in a continuation effort

The objective of this Section is to advance a number of lines of research to be pursued in a continuation of this investigation. A detailed discussion of these research objectives is deferred to a follow–up proposal to be submitted to AFOSR shortly after this report. In keeping with the structure of this report, the proposed research is organized along the same lines considered in Section **2.**

i. *Mathematical models of flexible structure.* Realistic models of spacecrafts, space platforms and related devices invariably involve *non–smooth* intersections of structural elements such as plates, rods and shells. The appropriate mathematical and numerical modeling of these intersection problems has been a subject of considerable interest and some controversy in recent years. In particular, shell models with ***drill rotations*** have been proposed as a rather practical means of resolving shell intersection problems. We plan to investigate the mathematical foundations and numerical implications of this class of widely used structural models.

ii. *Numerical analysis.* As noted in our proposal, a large body of computational experience indicates that standard time–stepping algorithms are not suitable for the long–term simulation on nonlinear dynamical systems. In fact, our recent results suggest that non of the existing and widely used time integrators in structural dynamics conserve fundamental constants of motion such as angular momentum and energy. Recently, we have been successful in developing algorithms for rigid body dynamics which conserve *exactly* these constants of motion without resorting to Lagrange multiplier and related (awkward) devices. Aside from its practical implications, conservation properties lead naturally to nonlinear notions of ***algorithmic stability.*** Our objective is to further analyze these methods and extend them to more sophisticated and mechanically more interesting models such as rods and shells.

Standard iterative solution procedures for the numerical simulation of nonlinear problems break down in the presence of nonlinear effects such as transfer of stability and bifurcation as a result of symmetry breaking. In such situations which arise, in particular, in numerical simulations of nonlinear shells, it is necessary to use continuation methods. Our preliminary work in WRIGGERS & SIMO [1990] suggests that it is possible to develop algorithms of this type which allow the *direct computation* of critical points by iterative methods that exhibit *quadratic* rate of convergence.

iii. *Geometric methods in stability and bifurcation analysis.* As pointed out above, the energy–momentum method developed in SIMO, POSBERGH & MARSDEN [1990] offers a general methodology for the systematic stability analysis of relative equilibria in hamiltonian systems. The crucial tool in this approach is a block–diagonalization result which decouples, as far as the stability analysis is concerned, rotational from internal or vibrational modes. Note that these modes are dynamically coupled. Nevertheless, we show that a change of variables can be explicitly constructed such that the second variation of the energy–momentum map block–diagonalizes. As a result, the stability conditions associated with the rigid modes become explicit.

The preceding block diagonalization result has important implications in stability and bifurcation analyses of rotating flexible structures that we plan to explore in detail in the future. In particular, our preliminary work in references [1] and [2] suggests a further simplification of the energy–momentum method crucial in concrete applications. Furthermore, the coordinate change involved in the block–diagonalization theorem has the remarkable property of providing an explicit *normal form* for the linearized dynamics at a critical point. Preliminary results indicate that this property plays a key role in bifurcation analyses at a relative equilibrium.

Appendix I: Publications under AFOSR Support

I.1. *Publications in Referee Journals*

[1] Simo, J.C., D. Lewis, and J.E. Marsden, [1990] "Stability of relative equilibria. Part I: The reduced energy–momentum method," SUDAM Report No. 89-3. Submitted to *Archive for Rational Mechanics and Analysis.*

[2] Simo, J.C., T. Posbergh, and J.E. Marsden, [1990a] "Stability of relative equilibria. Part II: Application to nonlinear elasticity," SUDAM Report No. 89-4. Submitted to *Archive for Rational Mechanics and Analysis.*

[3] Simo, J.C., T. Posbergh, and J.E. Marsden, [1990b], "Stability analysis of coupled rigid bodies and geometrically exact rods: Block diagonalization and the energy–momentum method," *Physics Reports,* to appear.

[4] Lewis, D., and J.C. Simo, [1990], "Nonlinear stability of pseudo–rigid bodies," *Proceedings of the Royal Society of London, A,"* **427**, *281-319.*

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[9] Simo, J.C., and J.G. Kennedy, [1990], "On a Stress Resultant Geometrically Exact Shell Model. Part V: Inelastic constitutive models in stress resultants; Formulation and numerical algorithms," *Computer Methods in Applied Mechanics and Engineering,* to appear

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[14] Simo, J.C., J.E. Marsden and P.S. Krishnaprasad, [1988], "The Hamiltonian structure of elasticity. The convective representation of solids, rods and plates," *Archive for Rational Mechanics and Analysis*, **104**, 125-183.

[15] Simo, J.C. and L. Vu-Quoc, [1988a], "On the dynamics in Space of rods undergoing large motions -A geometrically exact approach," *Computer Methods in Applied Mechanics and Engineering,* **66**, 125-161.

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II: Geometric and Computational Aspects," *Computer Methods in Applied Mechanics and Engineering,* **58**, 79-116.

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[22] Marsden, J.E., J.C. Simo, D. Lewis and T.A. Posbergh [1989], "A block diagonalization theorem in the energy-momentum method," *Contemporary Mathematics,* AMS, **97**, 297-313.

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I.3. *Co-Edited Books and Conference Proceedings*

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I.4. *PhD Dissertations*

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