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FEASIBILITY OF A KINEMATIC DIFFERENTIAL GLOBAL POSITIONING SYSTEM

by

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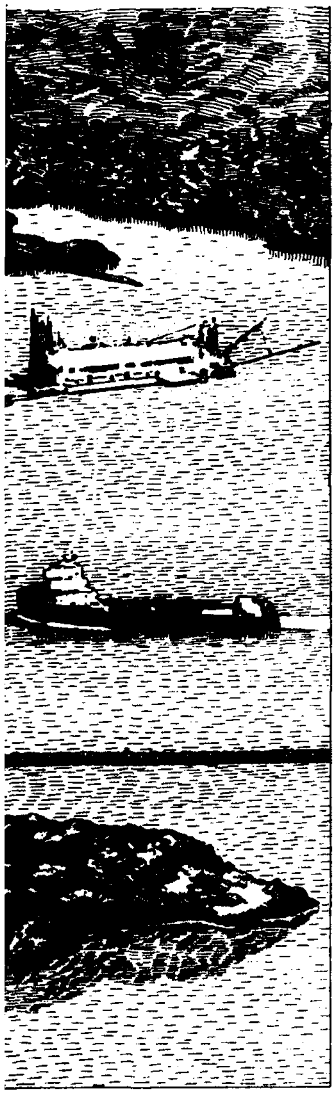

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This report considers the feasibility of using the Global Positioning System (GPS) to obtain three-dimensional positions, at decimeter (or better) accuracies, in real time for hydrographic survey vessels, dredges, and offshore tide gage buoys. The various technical constraints associated with using GPS to obtain high-accuracy positions of a moving platform are discussed. Real-time, decimeter-level kinematic differential GPS positioning can only be obtained if GPS carrier beat phase ambiguities can be resolved on the fly. Resolution of carrier beat phase ambiguities on the fly can only be achieved (if at all) with improved P-code pseudoranging. Dual-frequency receivers will be required. Codeless techniques for tracking the P-code may be almost as useful as code-correlation techniques in providing pseudorange measurements for carrier beat phase ambiguity resolution. Problems in four areas remain to be solved if decimeter-level systems are to be feasible: reducing pseudorange noise, avoiding multipath, overcoming possible adverse effects of selective availability, and overcoming the effects of vessel dynamics.

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PREFACE

This report was prepared under Work Unit 32479, Dynamic Positioning Systems, of the Dredging Research Program (DRP). The DRP is managed by the Coastal Engineering Research Center (CERC), US Army Engineer Waterways Experiment Station (WES). Technical Monitors for Headquarters, US Army Corps of Engineers, were Messrs. Robert H. Campbell, Glenn Drummond, Gerald Greener, Barry W. Holliday, David Mathis, and M. R. Miles. The US Army Topographic Engineering Center (TEC), formerly called the US Army Engineer Topographic Laboratories, was responsible for monitoring and technically reviewing actions associated with this work unit. The TEC Principal Investigator was Mr. Stephen R. DeLoach, Chief, Precise Survey Branch, Surveying Division, Topographic Developments Laboratory.

Mr. E. Clark McNair, Jr., CERC, is Manager of the DRP; Dr. Lyndell Z. Hales, CERC, is Assistant Manager. Dr. James R. Houston and Mr. Charles C. Calhoun, Jr., are Chief and Assistant Chief of CERC, respectively. This work was performed under DRP Technical Area 4, Vessel Positioning, Survey, Controls, and Dredge Monitoring Systems, which is managed by Mr. George P. (Pat) Bonner, Instrumentation Services Division, WES.

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COL David F. Maune, CE, was Commander and Director, and Mr. Walter E. Boge was Technical Director of the TEC during report preparation. Dr. Robert W. Whalin was Technical Director of WES. COL Leonard G. Hassell, EN, was Commander and Deputy Director.

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SUMMARY

This report considers the feasibility of using the Global Positioning System (GPS) to obtain three-dimensional positions, at decimeter (or better) accuracies, in real time for hydrographic survey vessels, dredges, and offshore tide gauge buoys. The various technical constraints associated with using GPS to obtain high accuracy positions of a moving platform are discussed. Real-time decimeter-level kinematic differential GPS positioning can only be obtained if GPS carrier beat phase ambiguities can be resolved on the fly. Resolution of carrier beat phase ambiguities on the fly can only be achieved (if at all) with improved P-code pseudoranging. Dual frequency receivers will be required. Codeless techniques for tracking the P-code may be almost as useful as code-correlation techniques in providing pseudorange measurements for carrier beat phase ambiguity resolution. Problems in four areas remain to be solved if decimeter-level systems are to be feasible: reducing pseudorange noise, avoiding multipath, overcoming possible adverse affects of Selective Availability, and the effects of vessel dynamics.

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1 INTRODUCTION

This report discusses the feasibility of using the Global Positioning System (GPS) to obtain three-dimensional positions in real time, at decimeter (or better) accuracies, for hydrographic survey vessels, dredges, and offshore tide gauge buoys.

The report is organized into five chapters:

• Chapter 2 sets the stage for Chapter 3 by reviewing each of the technical constraints which affect high accuracy kinematic differential GPS positioning. These include the various biases and errors in the GPS measurements themselves; and the siting and operation of GPS monitor stations.

• Chapter 3 is the heart of the report, discussing the performance which is expected from each of four possible methods for using GPS signals for kinematic differential positioning. These methods are:

Use of code pseudoranges only
Use of code pseudoranges and carrier beat phases
Use of carrier beat phase double differences, ambiguities resolved a priori
Use of carrier beat phase double differences, ambiguities resolved on the fly

• Chapter 4 deals with some other issues, respectively hardware and software requirements; the problem of transferring GPS antenna positions to other points of interest on the vessel; the requirements for achieving high accuracy kinematic differential GPS positioning in real time, including the requirement for a data communications link between monitor and user; and the problems of data synchronization.

• Chapter 5 summarizes the main points made in the report and draws some conclusions about the feasibility of production level kinematic GPS surveying systems.

The balance of this introductory chapter is a brief introduction to GPS. A description of GPS is followed by discussions of kinematic differential GPS positioning and differential GPS heights. Features relevant to this report are highlighted.

1.1 THE GLOBAL POSITIONING SYSTEM

The NAVSTAR Global Positioning System is a passive, satellite–based, navigational system operated by the United States Department of Defense. GPS is currently in the development stage with the early 1990's targeted for full operational capabilities. As shown in Figure 1, GPS consists of three segments: satellites, control systems, and users.

At the time of preparation of this document (March, 1989), six prototype (Block I) satellites are healthy and the first production (Block II) satellite has just been launched (on February 14). Further Block II launches are scheduled at approximately 60-day intervals, until 21 operational satellites and three in-orbit active spares have been launched into six orbital planes inclined 55° to the equator. The present schedule is for launches to be complete by early 1993, although global two-dimensional coverage may be available as early as mid-1990 (using both Block I and Block II satellites). The nominal satellite altitude is slightly higher than 20,000 kilometers, resulting in an orbital period of 12 sidereal hours. The satellite constellation will be oriented such that at least four satellites, viewed from any place on earth, will be visible above the horizon at virtually all times.

The control system consists of monitor stations on Diego Garcia, Ascension Island, Kwajalein, and Hawaii, and a master control station at the Consolidated Space Operations Center at Colorado Springs, Colorado. The purpose of the control system is to monitor the "health" of the satellites, determine their orbits and the behavior of their atomic clocks, and to transmit data to the satellites for re-broadcast to the users. These data contain ephemerides describing the satellite orbits, satellite clock synchronization parameters, and a satellite health (usability) flag.

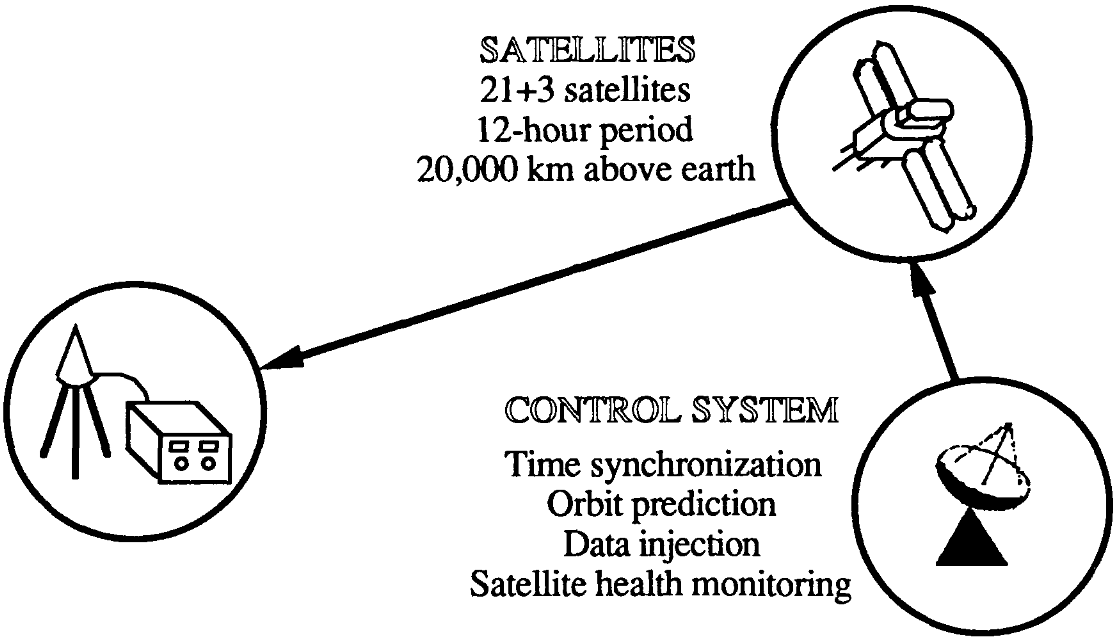

FIGURE 1: The Global Positioning System.

Each satellite transmits at two radio frequencies: 1575.42 MHz (L1) and 1227.6 MHz (L2). Three kinds of modulations are imposed upon these carriers: a 50 bit-per-second modulation which contains the "message" (information about orbits, clocks, health, etc.); a 10.23 MHz pseudo-random noise code, the P-code, associated with the GPS "Precise Positioning Service (PPS);" and a 1.023 MHz pseudo-random noise code, the C/A-code, associated with the GPS "Standard Positioning Service (SPS)." Each GPS satellite has a different C/A-code, and a different segment of the P-code. Thus the code identifies the satellite from which it came. GPS is designed so that the P-code (which is unclassified) can be replaced by a classified code, the Y-code, denying the PPS to non-qualified users (switching to the Y-code is referred to as "anti-spoofing"). In addition the capability of adding intentional degradation to the SPS is designed into GPS (this is referred to as "Selective Availability").

GPS receivers track these signal components from several GPS satellites simultaneously, in order to obtain enough information to determine their own position. The satellite orbital information in the message from each satellite is used to compute the position of that satellite. Either the C/A-code or the P-code can be used to measure the distances between the receiver and each satellite. Each distance measurement implies the receiver is somewhere on the surface of a sphere centered at the satellite, and with radius equal to the distance measurement. Simultaneous measurements to several different satellites yield several such spherical "Surfaces-of-position." In principal, the three-dimensional position of the receiver is found by finding the point at which these spheres intersect.

This simple explanation has ignored important practical details which will be discussed in detail in this report. One such "detail" is the problem of synchronizing receiver and satellite clocks. The standard method of solving this problem is to assume that the clocks in all GPS satellites are synchronized (by data from the control system), and to treat the offset of the receiver clock from these satellite clocks as a fourth unknown parameter to be solved for (together with three coordinates of position). Solving for these four parameters requires simultaneous tracking of at least four satellites.

However, as far as the feasibility of decimeter-level positioning is concerned, the most important practical "detail" is the possible use of the carrier to provide distance measurements which are far more precise than the distances measured using the code. It is a rule of thumb that range measurements can be made with a precision of approximately 1% of the wavelength of the signal being used. The GPS L1 carrier (at about 1.6 GHz) has a wavelength of about 20 cm, implying a ranging precision of 2 mm. In contrast, the P-code (with chipping rate of about 10 MHz) has a chip length of 30 m, implying a ranging precision of about 30 cm, and the C/A-code (with chipping rate of about 1 MHz) has a chip length of about 300 m, implying a ranging precision of about 3 m.

However, the higher precision of carrier measurements does not come without cost. P-code ranges are unambiguous. C/A-code ranges are ambiguous with a modulus of about 300 km (that is, C/A-code ranges include a unknown multiple of 300 km), which are relatively easily resolved. However, L1 carrier ranges are ambiguous with a modulus of about 20 cm, which is much more difficult to resolve, and consequently it is important that the carrier be continuously tracked without cycle slips (or if they occur that they be correctable).

The feasibility of decimeter accuracies for GPS depends to a large extent on the feasibility of being able to realize the high precision of ranges derived from carrier beat phase measurements by reliably resolving the carrier ambiguity, and either preventing or reliably correcting carrier cycle slips. Discussion of how feasible this may be is the main topic of this report.

1.2 KINEMATIC DIFFERENTIAL GPS POSITIONING

GPS was designed to provide a point positioning capability. In this mode, the coordinates of a point are determined using signals received by a GPS receiver simultaneously from several GPS satellites. This is also called absolute positioning, and is the method designed to meet the primary GPS military missions, as well as many of the non-military applications. For most of these applications, the receiver will be mounted on a moving vehicle, and the kinematics of the receiver must be taken into account — hence the term kinematic positioning. Kinematic point positioning is possible since a GPS position can be determined from measurements obtained at a single epoch. Depending on the receiver kinematics and desired accuracy, it may be sufficient to compute independent single epoch GPS positions, or it may be necessary to combine GPS and other measurements observed along the receiver trajectory in a filter / smoother algorithm.

As shown in Figure 2, there are various GPS absolute positioning techniques, with differing accuracies. Civilians will have access to the Standard Positioning Service (SPS), using the C/A-code for range measurements. This is a little worse than half as accurate as the Precise Positioning Service (PPS), available to U.S. and NATO military users, as long as the intentional degradation known as selective availability has not been applied, in which case the SPS becomes about eight times less accurate than the PPS.

As is discussed in detail in Chapter 2, the ultimate limitation on GPS positioning accuracy is not the accuracy with which GPS range measurements can be made, but the effects of biases and errors on these measurements. Many of these biases have a high spatial correlation. In other words, two receivers reasonably near each other will experience almost the same effect from such biases. Examples of spatially correlated biases are those due to instability in the satellite and receiver clocks, due to inaccurate GPS ephemerides, and (to a lesser extent) due to atmospheric disturbances of the signal. Errors in position determination resulting from these biases will also have a high spatial correlation. Since the errors in point positioning at each of two receivers have almost the same magnitude and orientation, then the error in the position of one relative to the other will be much smaller. This is the basis of differential GPS positioning, also called relative GPS positioning. Biases in GPS measurements are discussed in detail in Chapter 2.

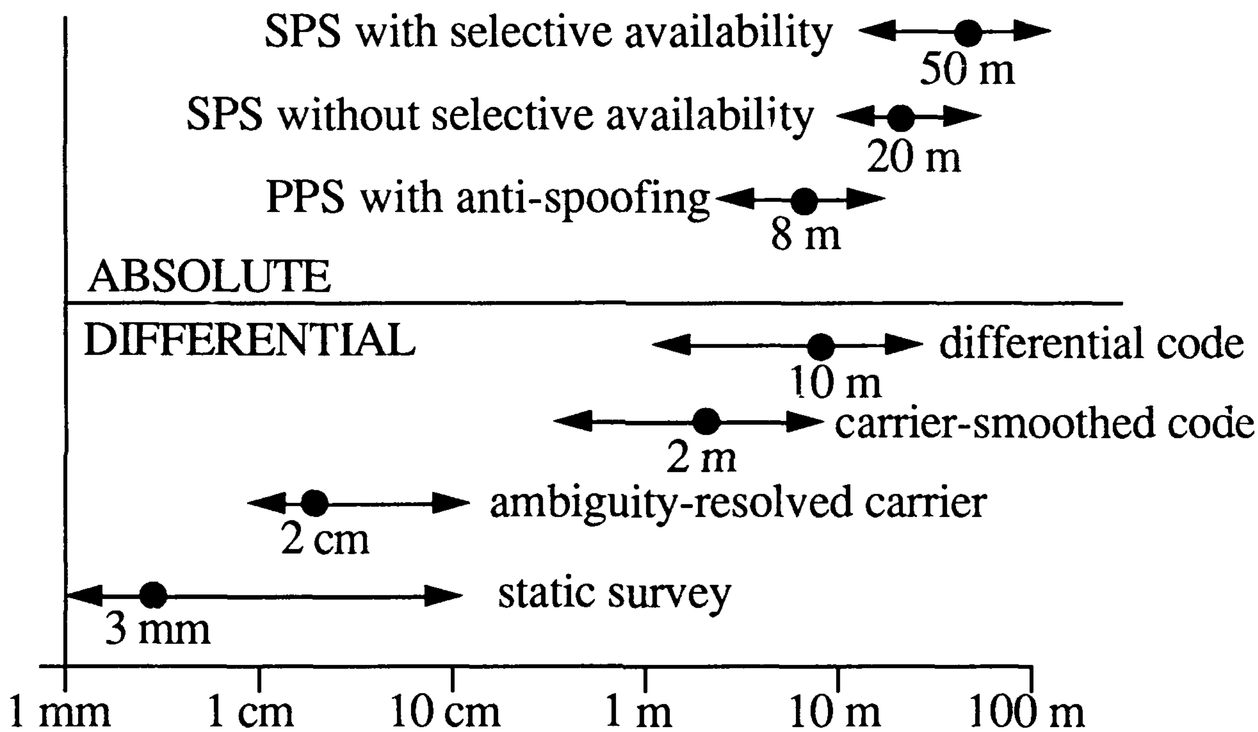

Figure 2: Methods of using GPS for positioning. (The arrows show the range of accuracies which can be expected. The dots and numbers represent 'typical' accuracies within that range.)

Figure 2 shows four differential GPS positioning methods. All require the operation of receivers at at least one station whose coordinates are known.

The method labelled *differential code* does not make any use of the carrier at all. It does permit PPS users to improve their accuracy slightly, and SPS users with selective availability to achieve accuracies nearly comparable to PPS users. This method is described in §3.1.

The method labelled *carrier-smoothed code* combines the differential positioning capabilities of the GPS code with the differential vehicle displacement-sensing capabilities of the carrier, and has been successfully demonstrated in several field trials. This method does not require that carrier ambiguities be resolved. The effect of carrier cycle slips is transitory. This method is described in detail in §3.2.

The method labelled *ambiguity-resolved carrier* realizes the full precision of carrier measurements by somehow resolving the carrier ambiguity, and preventing or reliably correcting cycle slips. How this might be accomplished is described in §3.3 and §3.4. These sections are the crux of this report.

The method labelled *static survey* is included for comparison only, since it does not have implications for kinematic applications. Static surveys benefit in comparison to kinematic surveys, from being able to use GPS range data collected over some period of time, which will reduce the effect of error sources which can be considered random over the observation period.

1.3 DIFFERENTIAL GPS HEIGHTS

GPS is a three-dimensional positioning system. However, determination of horizontal coordinates and determination of height have different characteristics. For vessels operating on the sea surface, horizontal coordinate determination is the primary tool for navigation and track control, while height determination monitors sea level. In this section, we discuss the characteristics of GPS height determination. We will show that differential GPS is well suited to monitoring sea level changes at the decimeter level, but that other factors limit how well differential GPS can determine sea level itself.

One characteristic of sea level (as contrasted with the horizontal coordinates of a vessel) is that it has a nearly constant value, that of mean sea level. This suggests the possibility of somehow using this fact to enhance the performance of GPS, either as a height determination tool, or in all three coordinates.

There are two special considerations in GPS height (or relative height) determination: satellite geometry, and the fact that GPS yields only heights above a mathematical reference surface, the ellipsoid.

For horizontal positioning, there is at least partial cancellation of biases in the measurements, since measurements can be made from satellites both north and south, and both east and west of the observer. Since there are no satellites below the observer, such bias cancellation is not possible for heights. Typically, vertical GPS positioning results are about 60% poorer than horizontal results.

This, however, is much less important than the fact illustrated in Figure 3: GPS-determined heights are above the ellipsoid (h), whereas we are usually interested in heights above some datum which is referenced to "sea level" (H). In this case there are four figures of the earth which we must take into account:

- The *terrain* is the actual surface of the earth, that is the seabed, or on land, the ground upon which we stand.
- *Sea level* is the actual surface of the sea, which is subject to both temporal and spatial variations. *Mean sea level* is the long term average (typically 20-year average) of sea level at some location.

* A *global geoid* is that equipotential surface of the earth's gravity field which most closely approximates mean sea level, averaged over the globe. (Local vertical datums have been established using various methods.)

* The geocentric reference *ellipsoid* is a mathematical figure which most closely approximates the global geoid, averaged over the globe. (Regional ellipsoids, which are not geocentric, have been determined somewhat differently, but are currently being abandoned in favour of geocentric ellipsoids.)

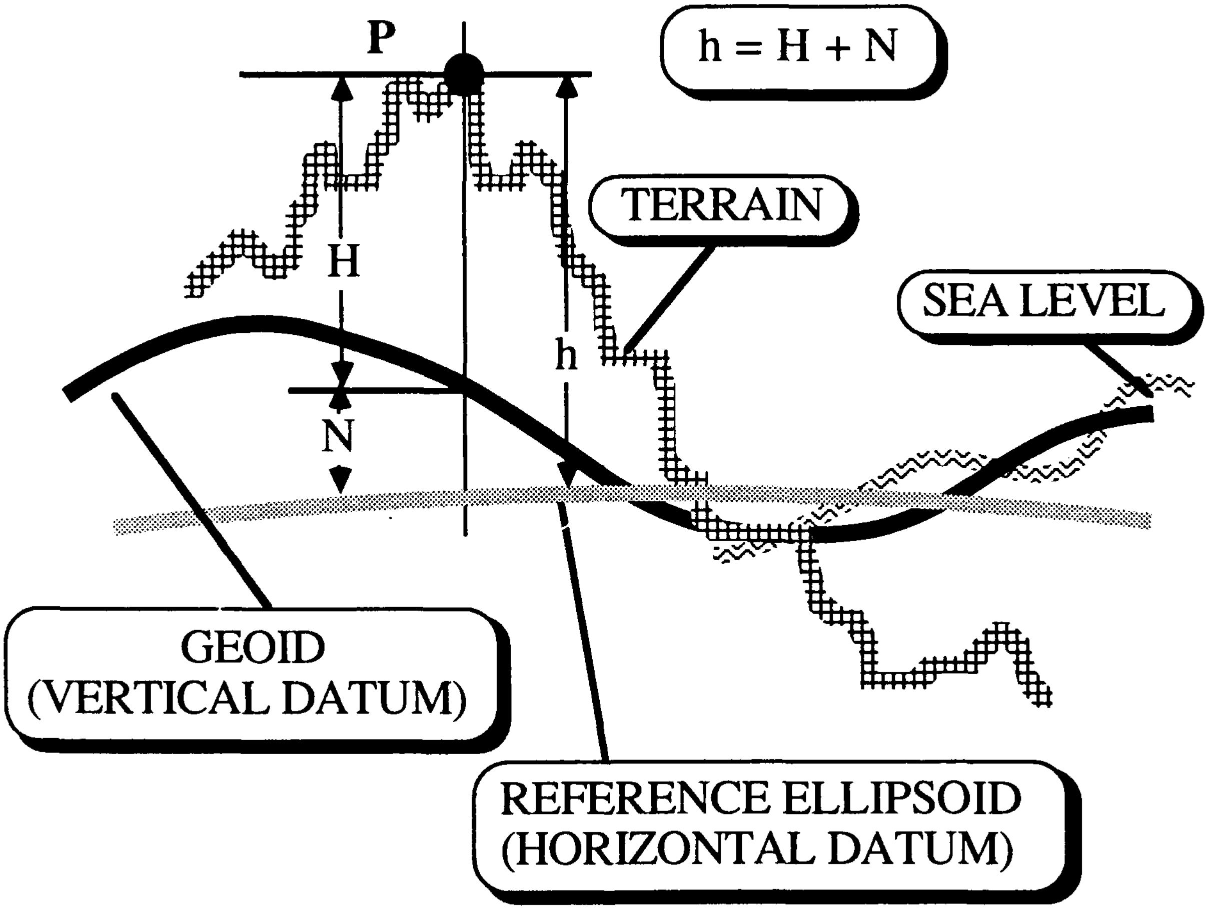

Figure 3: Four figures of the earth.

The terrain is up to 11,000 metres below (in the Mariana trench) and 8,800 metres above (on the top of Mount Everest) the other three surfaces. Separations between the geoid and ellipsoid may reach ±100 metres over the globe. Separations between mean sea level and the geoid are up to a meter at various locations around the world.

Differential GPS can only provide relative ellipsoidal heights, Δh. The height system used on land are heights above the geoid, or *orthometric heights*, H. In order to obtain relative orthometric heights, ΔH, from GPS-determined relative ellipsoidal heights, we require an independent knowledge of the relative *geoid-ellipsoid separation*, ΔN. The accuracy with which the relative geoid is determined depends upon the nature of the data which is used to compute it, as illustrated in Figure 4. Using only global geopotential coefficients, values of N accurate to about 1 m, and values of ΔN accurate to about 5-7 ppm of the horizontal separation between the points can be obtained (at the 1s level). Over a 100-km baseline, this implies ΔN can be modelled with a standard deviation of 70 cm. If gravity anomalies are used as well, these accuracies improve to about 0.5 m and 2-3 ppm respectively (or over a 100-km baseline, ΔN with standard deviation of 30 cm). In mountainous regions (not relevant here) the accuracies obtained are worse, unless terrain effects are also taken into account. Global geopotential coefficient sets are available and relatively easy to use (e.g.. Rapp and Cruz (1986)). Geoid models incorporating up-to-date gravity anomaly data are available only for certain areas (Vaníček et al., 1986; Rapp and Kadir, 1988), and are expensive to compute (but need only be computed once).

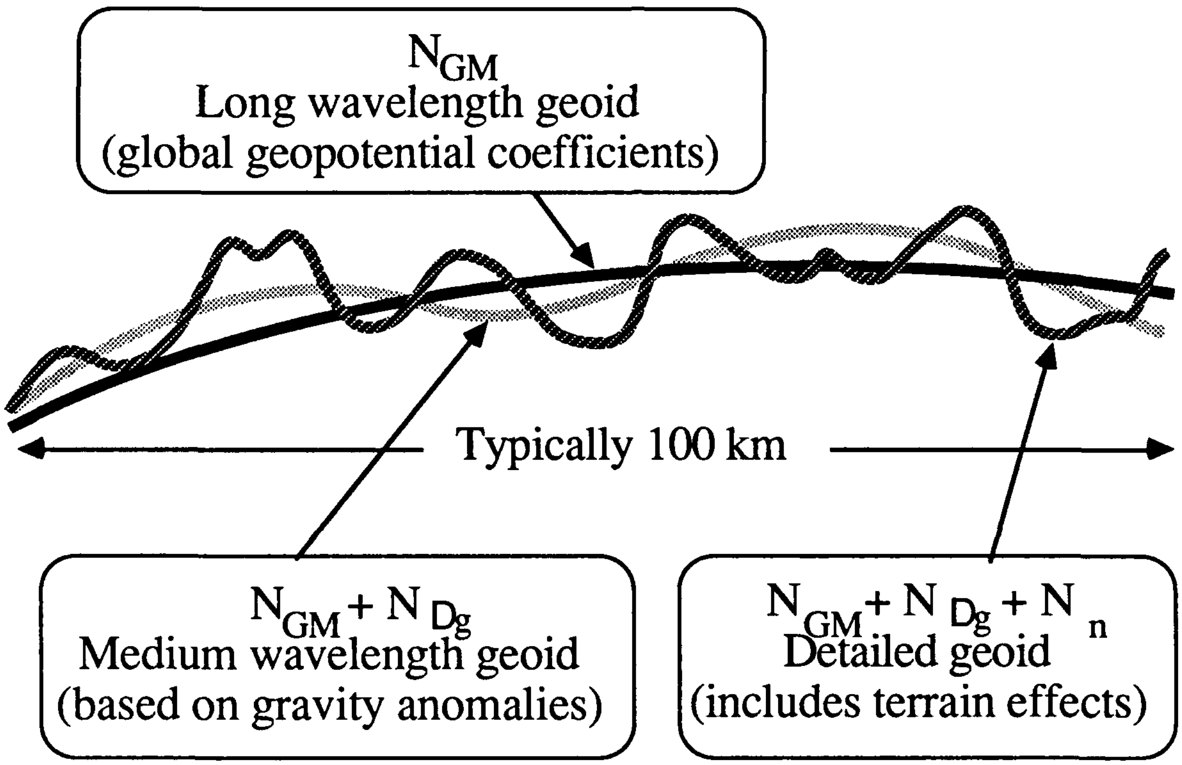

Figure 4: Geoid determination.

At present then, the accuracy with which ΔN can be determined limits decimeter-level determination of ΔH from GPS to baselines typically of only up to a few tens of kilometers in length. However, for the applications addressed in this report, ΔH is not the quantity we are most interested in obtaining.

Water depths on hydrographic charts are referred not to the geoid, but to another datum defined such that sea level seldom falls below it. Since 1980 for U.S. charts this datum is Mean Lower Low Water (MLLW), which is defined as the average of all the (daily) lower low waters over a specified 19-year period (the average of nearly 7000 numbers). MLLW is usually expressed as a distance below mean sea level. Since the range of tidal variations changes from place to place, so does the distance which MLLW is below mean sea level. Since MLLW is based on records of variations in instantaneous sea level, it is worth noting some of the causes of these variations:

- *waves*, with amplitudes from a few centimeters to many 10's of meters, and periods of seconds to minutes,

- *tides*, with amplitudes of up to 15 m, and 12- and 24-hour periods (as well as smaller amplitude constituents at other periods),

- *meteorological effects* (winds and barometric pressure variations) with amplitudes of up to a meter and periods of days,

- *ocean dynamic effects* (for example, the sea surface along the western boundary of the Gulf Stream is one meter higher than it would be if there were no Gulf Stream),

- *eustatic effects* (melting and freezing of polar ice caps) which produce detectable global changes in sea level over periods of decades.

Some of these sea level variations do not have a zero mean. One example is the Gulf Stream western boundary effect already mentioned. Another example is a prevailing onshore wind, which will tend to pile water up along the coast, leading to a mean sea level value which is higher than would be the case if there was no prevailing wind direction. This results in semi-permanent departures between mean sea level and the geoid called *sea surface topography* (SST). Little is known about SST for most locations, and it is a difficult quantity to measure.

A more serious problem is that even zero-mean variations in sea level such as waves and tides have an enormous range of periods over which they average to zero. Monthly means of hourly water level observations still exhibit typically 50-cm variations from month to month. Annual means still contain typically 10-cm variations from year to year.

How then is MLLW determined and how accurately? In principle there are three methods of determining MLLW for a location:

- At permanent tide gauges, when 19-year records are available, the rigorous calculation can be made. MLLW in this case is likely determined at better than a decimeter.
- At temporary tide gauges, when the available water level record is shorter than 19 years, an approximation of MLLW can be computed by augmenting the available water level record with a tidal model. Depending on the quality of the tidal model and the proximity of a permanent tide gauge, MLLW in this case may not be more accurate than a few decimeters.
- At locations never occupied by a tide gauge, tidal models alone must be used. MLLW in this case may not be more accurate than several decimeters.

The U.S. Army Corps of Engineers has applications which require determining variations in instantaneous sea level at the vessel. Figure 5 illustrates the scenario for such a determination using differential GPS. Assume that the GPS monitor is sited at or near a tide gauge, so that at the monitor we know the constant height relationships:

- H, the orthometric height of the GPS antenna above the geoid,
- MLLW, the distance the datum is below mean sea level,

and as well have available a continuous record of instantaneous sea level, ISL, from the tide gauge. Denoting the separation between instantaneous sea level and the antenna as ANT, we have the relationships

$$H + SST + MLLW = ISL + ANT \tag{1.1}$$

$$h = H + N \tag{1.2}$$

At the monitor we have the following situation: whether the monitor is close to a tide gauge or some distance away, the height difference between the GPS antenna and a fiducial mark near the tide gauge is known from spirit levelling, so that the quantity $[H_m + SST_m]$ is known). From the tide gauge ISL_m is measured. At the monitor ANT_m is varying and is a derived quantity.

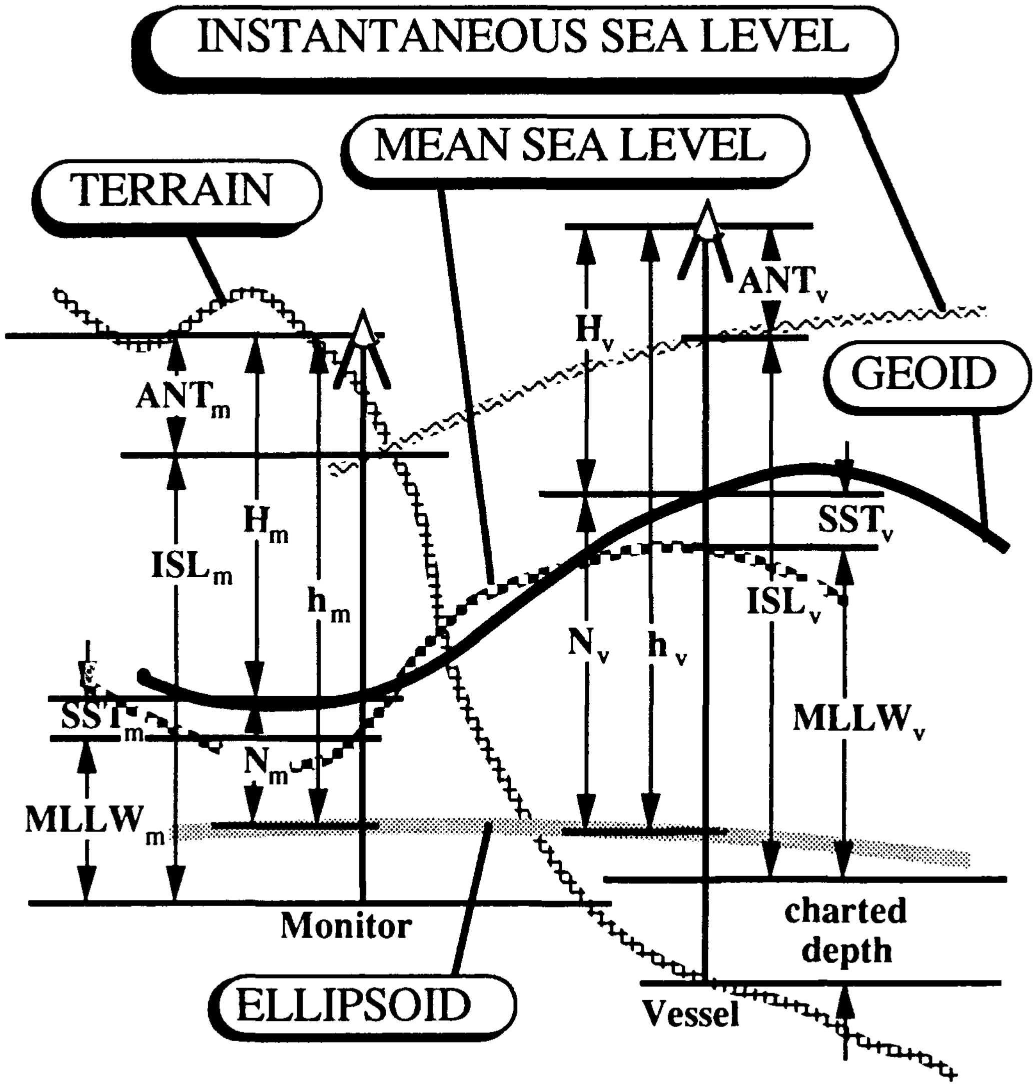

Figure 5: Instantaneous sea level determination.

On the vessel we have the same two equations. Now, however, we assume that ANT_v, the height of the antenna above the waterline of the vessel, is known, and ISL_v is the quantity to be determined (we examine the assumption that ANT_v is known in §4.2.)

Then using

$$h_m = H_m + N_m \tag{1.3}$$

$$h_v = H_v + N_v = ISL_v + ANT_v + N_v - SST_v - MLLW_v \tag{1.4}$$

the differential GPS height difference equation becomes

$$\Delta h = h_v - h_m = ISL_v + ANT_v - H_m + \Delta N - SST_v - MLLW_v . \tag{1.5}$$

In this equation Δh is measured using differential GPS. If we assume all but the first term on the right hand side may be considered to have constant values, then changes in ISL_v will be determined with the same accuracy as Δh. This is the major result which we seek. Variations in sea level at the vessel can be monitored by differential GPS with the full accuracy with which GPS determines relative ellipsoid heights.

However the actual value of ISL_v will be determined with an accuracy which depends on the accuracy with which all terms on the right hand side can be determined. We have assumed that we can directly measure ANT_v and H_m with an accuracies consistent with decimeter-level positioning. However the determination of SST_v and $MLLW_v$ will be neither as simple nor as accurate, and may only be consistent with positioning in height to several decimeters.

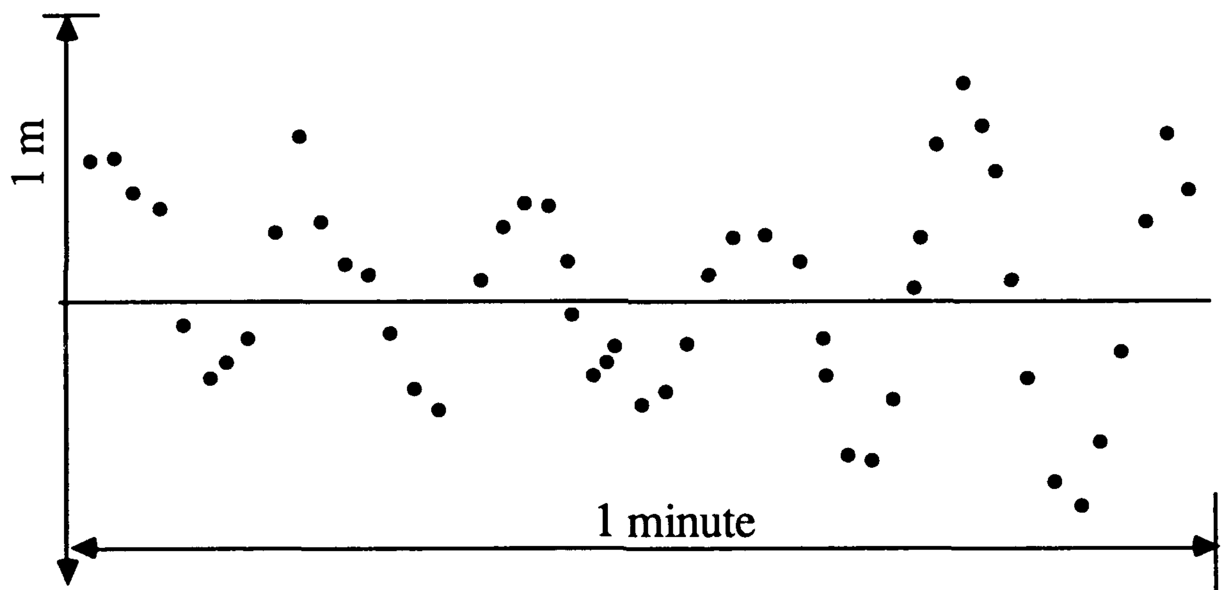

Figure 6: GPS-derived ship's heave
(from Lachapelle et al. (1988)).

Experimental confirmation is shown in Figure 6 that variations in sea level can indeed be measured at the decimeter level. Recall the potentially advantageous characteristic of sea level, that its variations are about a reasonably constant mean sea level. Figure 6 illustrates that differential GPS ellipsoid heights determined by carrier-smoothed code measurements (see §3.2) contain height information which may well be able to resolve variations in height at the decimeter level, whereas the horizontal position recovery from carrier-smoothed code measurements is only at the meter-level. This plot was obtained by comparing each GPS determined height with a several minute average of these heights. It is believed to represent heave of the hydrographic survey vessel, although no independent heave measurements were made to confirm this.

2 TECHNICAL CONSTRAINTS ON HIGH ACCURACY KINEMATIC DIFFERENTIAL GPS POSITIONING

This section highlights the technical constraints imposed on a real-time differential GPS kinematic positioning system by inherent properties of GPS measurements (see §2.1) and by the monitor station (see §2.2).

2.1 BIASES AND ERRORS IN GPS MEASUREMENTS

Global Positioning System measurements can be represented schematically by one of the two following observation equations. The first equation holds for pseudorange measurements P, the second for carrier beat phase measurements F (in units of length). Both equations are equally valid for code-correlation and so-called codeless receivers.

$$P = r + c\,(dt - dT) + d_{ion} + d_{trop} + aM + e_P \qquad (2.1)$$

$$F = r + c\,(dt - dT) - d_{ion} + d_{trop} + lN + e_F \qquad (2.2)$$

The following abbreviations have been used:

r: geometric range between satellite and receiver
c: speed of light in vacuum
dt: satellite clock error
dT: receiver clock error
d_{ion}: signal path lengthening due to ionospheric refraction
d_{trop}: signal path lengthening due to tropospheric refraction
a: code chip length
l: carrier signal wavelength (half wavelength for codeless receivers)
M: code ambiguity parameter (constant integer number)
N: carrier cycle ambiguity parameter (constant integer number)
e_P: pseudorange measurement noise and unmodelled errors
e_F: carrier beat phase measurement noise and unmodelled errors.

The code ambiguity parameter, M, is zero for code-correlation receivers. The primary differences between pseudorange and carrier beat phase measurements as described by equations (2.1) and (2.2) are the opposite sign in the ionospheric delay, different level of measurement noise, and different size for some of the unmodelled errors.

A note about code-derived pseudoranges. The conventional use of the code is based on tracking the code by correlating it with a replica generated within the receiver. This requires knowledge of the code, and the official U.S. Department of Defense (DoD) policy is that the the P-code will be replaced by the Y-code, knowledge of which will not be available to non-qualified users. However, "codeless" techniques for tracking the phase of the code (P-code, Y-code, or C/A-code) have been developed, as described in §2.1.5, which may well prove to be almost as useful as code-correlation in providing pseudorange measurements. In this report we refer to such measurements as "codeless pseudoranges." When we refer merely to pseudoranges, we mean either code-correlation pseudoranges or codeless pseudoranges.

2.1.1 SATELLITE EPHEMERIS ERRORS

Satellite ephemeris errors enter the observation equations described above if the geometric distance between receiver and satellite is replaced by a function of receiver and satellite coordinates. The size of the errors in the broadcast satellite ephemeris is presently believed to be about 20 m, mainly in along-track direction. If not corrected or eliminated, the satellite ephemeris error will result in position errors of the same magnitude. In differential positioning, the ephemeris error leads to worst-case baseline errors which can be characterized by the following rule of thumb (e.g., Beutler et al. (1984))

$$\frac{db}{b} = \frac{dr}{r} \tag{2.3}$$

where db is the baseline error, b is the baseline length, and dr is the satellite ephemeris error. For ephemeris errors of 20 m and average distances between the satellite and the receiver of 20,000 km we obtain from equation (2.3) baseline errors of the order of 1 part per million (ppm) of the baseline length amounting to 10 cm for baselines of 100 km.

GPS satellite ephemeris of higher accuracy are presently available, with some time delay, from the National Geodetic Survey and from private industry.

2.1.2 CLOCK ERRORS

The size and behavior of the satellite clock errors depends on the type of clock used in a particular satellite at a particular time (cesium, rubidium, quartz oscillator). The major portion of the clock error is monitored by the GPS control segment. The observed clock errors are subsequently extrapolated and broadcast in the GPS satellite message in form of polynomial coefficients. Typically, independent single epoch error estimation is performed (sometimes referred to as white noise modelling), or the clock error is eliminated through observation differencing between the stations observing a baseline.

The receiver clock error is similar to the satellite clock error described above. Since on average the clocks in GPS receivers will be of poorer quality than those in GPS satellites, parametric modelling will be even more difficult. The usual approach is the estimation of independent single epoch clock errors, or the elimination through differencing between simultaneously observed signals of different satellites.

2.1.3 IONOSPHERIC DISPERSIVE REFRACTION

The ionosphere extends roughly from 50 km to 1000 km in altitude. In this region, the incident solar radiation leads to the separation of free electrons which in turn interact with the electromagnetic GPS signal. The important result for GPS ranging of this interaction is the ionospheric delay of the signal, lengthening the signal path as measured by pseudoranges (group delay) and shortening the signal path as measured by carrier beat phases (phase delay). The ionospheric delay can be shown to be proportional (in first-order approximation) to the Total Electron Content (TEC) along the signal path, and inversely proportional to the square of the signal carrier frequency f (e.g., Hartmann and Leitinger (1984)) according to:

$$d_{ion} = 40.3 \frac{TEC}{f^2} . \quad (2.4)$$

The vertical TEC value can vary between $1 \cdot 10^{17}$ m^{-2} (night time, solar activity minimum) and $2 \cdot 10^{18}$ m^{-2} (day time, solar activity maximum), resulting in ionospheric delays between 1.5 m and 30 m. Additional spatial variations exist on a global scale, resulting in larger delays for equatorial and polar regions. The delay is increased for non-vertical propagation of the signal through the ionosphere.

Presently, reliable models to predict the ionospheric activity do not exist. The frequency dependence of the ionospheric delay can be exploited in dual frequency systems to eliminate the delay through a particular linear combination of the L1 and L2 observations. This elimination goes hand in hand with a threefold increase in measurement noise. For single frequency measurements, the effect of the ionosphere on differential positioning has to be neglected. For mid-latitudes, the ionospheric delay is expected to show a large degree of spatial coherence resulting in systematic errors in scale for single frequency differential positioning (Georgiadou and Kleusberg, 1988). However, strong spatial and temporal variations in ionospheric activity have been shown to exist in northern latitudes (Beutler et al., 1988). In these regions, dual frequency equipment may be required for 1 ppm results on baselines as short as 5 km (Beutler et al., 1988).

2.1.4 TROPOSPHERIC EFFECTS

The troposphere is the lower portion of the atmosphere extending up to several tens of kilometers. The tropospheric propagation delay of GPS signals is not frequency dependent and is identical for pseudorange and carrier beat phase observations. The total effect can be split in so-called dry and wet delay component (e.g., Hopfield (1971)), which can be determined approximately from surface meteorological measurements. The dry component of the delay depends on atmospheric pressure and temperature, and amounts to approximately 2.5 m in zenith direction for moderate climates. The wet component depends on water vapor pressure and temperature, and amounts to a few decimeters in zenith direction. Typical errors when modelled from surface measurements of atmospheric pressure, temperature, and humidity are of the order of several centimeters, mainly resulting from poor modelling of the wet component. The tropospheric delay and its modelling error increase rapidly for signals observed near the horizon. To reduce these effects, satellites in low elevations (less than 10 degrees) should not be used for positioning.

Since the spatial coherence of tropospheric conditions is very limited, an elimination of tropospheric delay through observation differencing in differential positioning is not advisable, not even over very short distances.

2.1.5 CODE AMBIGUITY

The code ambiguity, aM, appears in the pseudorange observation equation (2.1) for codeless receivers only (e.g., MacDoran et al. (1985)). It accounts for an unknown integer number M of of code chips of length a in the measurement. The code chip length for codeless measured P-code pseudoranges is 29.305 m. Codeless measured C/A-code pseudoranges are not included here since the C/A-code is expected to be available to every user. The ambiguities of codeless derived P-code pseudoranges are usually resolved using C/A-code derived unambiguous pseudoranges.

Actually, there exists an additional different type of C/A-code ambiguity (not shown in equation (2.1)) of about 300 km resulting from the code length of 1 msec. This ambiguity is easily determined and of no further importance in the present context.

2.1.6 CARRIER CYCLE AMBIGUITY

The carrier ambiguity, lN, appears in the carrier beat phase observation equation (2.2) for code correlating and for codeless receivers. It accounts for an unknown integer number N cycles of length l in the measurement. For code correlating receivers, l is the L1 or L2 carrier wavelength, whichever is tracked. For codeless receivers, l is half the L1 or L2 carrier wavelength. The ambiguity parameter is in general an arbitrary unknown number which changes if the receiver loses lock to the tracked signal. The ambiguity parameters and the differential position cannot be simultaneously and instantaneously determined. However, in static differential positioning, both sets of parameters become simultaneously determinable through the accumulation of information over extended periods of time. If the ambiguity parameters are determined either through static positioning or by some other means (see §3), the only real difference between pseudorange and carrier beat phase measurements is the much lower noise level in the latter. Using carrier beat phases with pre-determined ambiguities for positioning has been referred to as carrier ranging (Melbourne, 1985).

2.1.7 CARRIER CYCLE SLIPS

Carrier cycle slips result if the receiver loses lock to the tracked signal. After regaining the phase lock to the signal, the ambiguity parameter N in equation (2.2) has settled on a different integer number than before. The change in carrier beat phase ambiguity, by definition an integer number of carrier wavelengths (sometimes with intermediate steps over half cycles), is called a cycle slip. For codeless receivers, the cycle slip is an integer multiple of carrier half-wavelengths.

Reasons for phase lock loss may be receiver power failure, signal shading, carrier signal multipath interference (see below), or incompatibility between high receiver dynamics and narrow tracking loops.

2.1.8 MULTIPATH AND ANTENNA PHASE VARIATION

Under this heading all errors originating in the antenna and its immediate environment are summarized. Multipath interference has been shown to be a major portion of the unmodelled pseudorange measurement error (Evans, 1986). Carrier beat phase measurements are less affected by multipath (Georgiadou and Kleusberg, 1988). In both cases, the effect can be identified through its repeated occurrence for identical satellite-to-receiver geometry after one sidereal day. The periods of multipath induced measurement errors range from a few minutes to several tens of minutes. The size of P-code pseudorange multipath errors ranges from up to 10 m for rather old receiving equipment technology (Evans, 1986) to less than a meter for more recently developed equipment (Counselman and Ladd, 1987). The reduction of multipath effects, especially in pseudorange measurements for both code correlation and codeless receivers, is presently actively pursued (Young et al., 1987; Ladd, personal communication, 1989), and focuses primarily on antenna design. Equally important is the selection of the antenna site to avoid any conductive material in the immediate vicinity.

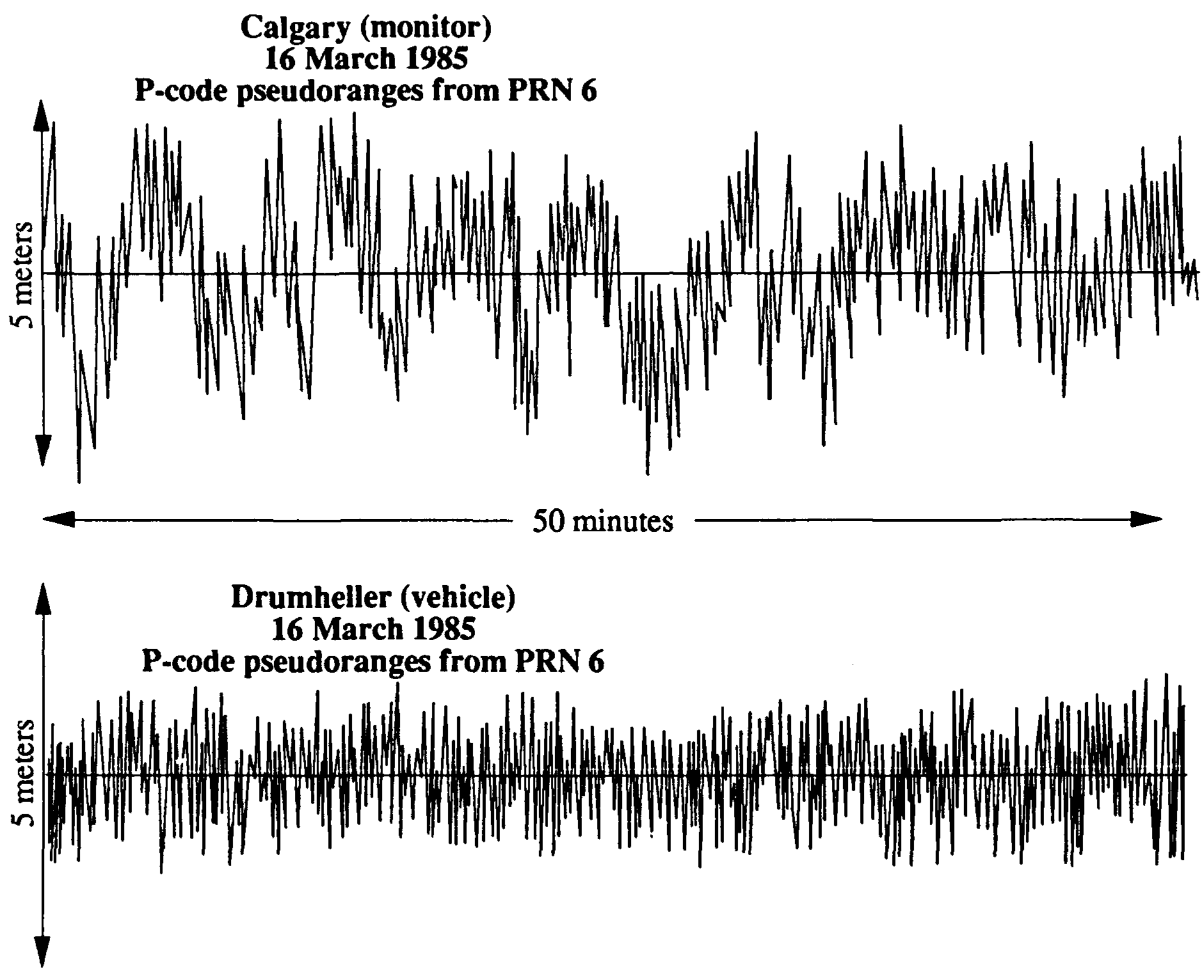

Figure 7: Multipath effects on static and moving GPS antennas

As described above, multipath errors depend on the geometry between satellite, receiver antenna, and electrically conducting material in the environment of the receiver. For a static receiver, this geometry changes only due to the orbital motion of the satellite. Since this orbital motion is rather slow, the multipath errors change equally slowly. For kinematic applications, the change of geometry will also depend on the motion of the receiver. For rapid receiver motion, the net effect may be a randomization of the multipath errors as shown in Figure 7. This figure shows multipath errors observed on P-code pseudoranges measured simultaneously from a static receiver and a moving receiver (antenna mounted on a truck roof top). The technique used for deriving these error time series is described by Evans (1986). For the static receiver (station Calgary, top plot), the cyclic behavior of the multipath error is quite obvious. Typical error periods are of the order of 5 to 15 minutes. (The noise-like variation on top of the cyclic variations are primarily due to receiver noise.) For the moving receiver (station Drumheller, lower plot), this cyclic pattern is strongly reduced and replaced by an increased noise level compared to the static receiver.

2.1.9 MEASUREMENT NOISE

By measurement noise, we understand those errors that remain after all propagation errors, clock errors, and errors related to the physical properties of the antenna have been taken into account. This noise level depends on the measurement type (pseudorange, carrier beat phase), the way of measuring (code correlating, codeless), the mode of operation (low dynamics and narrow tracking bandwidth, high dynamics and wide bandwidth), and the sophistication and smartness of the receiver circuits.

The analysis of measurements of a code correlating receiver in low dynamics setting (Evans et al., 1985) yielded receiver noise levels of 20-65 cm for P-code pseudoranges and 1-3 mm for carrier beat phases. For a P-code chip length of about 30 m and a carrier wavelength of about 20 cm, these noise levels can be expressed as 1%-2% of chip length/wavelength. Accordingly, the noise in C/A-code pseudoranges would be at the several meter level.

For codeless technology, the noise level will be somewhat higher, since the signal-to-noise ratio is lower in this case. For both code correlation and codeless technology, averaging or filtering reduces the impact of the measurement noise on positioning, if the measurement noise is white.

2.1.10 SELECTIVE AVAILABILITY

Selective Availability (S/A) is the intentional degradation of the position-related information contained in the GPS signals to restrict non-authorized users to 100 m absolute position accuracy. The procedures for implementation of S/A are classified and unknown to the authors. Possible ways of achieving the S/A goal include degradation of the accuracy of the broadcast orbital parameters, and dithering the satellite clock which directly transforms into pseudorange and carrier beat phase measurement errors.

Degrading the broadcast orbit information to an accuracy of 100 m would result in differential position errors of about 5 ppm, or 0.5 m over distances of 100 km. However, since GPS orbits can be observed and predicted with high accuracy by organizations outside the GPS control segment, they may produce real-time orbits of higher quality and more or less offset the effect of S/A.

The other way of S/A implementation is the satellite clock dithering. It results in random errors in observed pseudoranges and carrier beat phases. If these are observed simultaneously at two stations and differenced for the purpose of differential positioning, the effect of satellite clock dithering will disappear, at least at the accuracy level discussed in this report. If, however, observations are not differenced directly, but corrections valid for certain time intervals are computed at the monitor station for transmission to the remote station, the degree of error removal will very much depend on the temporal coherence of the clock dithering. Only limited tests have been done to assess the implications of S/A on data transmission rates (Kalafus, 1982), primarily because nobody knows much about the spectral distribution of clock dither errors. Further research seems to be required here, as S/A is introduced in the Block II satellite constellation. The satellite clock dithering effects of Selective Availability affect P-code pseudoranges, C/A-code pseudoranges, and carrier beat phases in the same manner.

2.2 MONITOR STATION SITING AND OPERATION

The primary task of the differential GPS monitor station is to provide the remote user with data to be used for correcting GPS observation errors and biases. GPS monitor stations can be set up on a permanent basis, or may be established temporarily for a particular application. For differential positioning purposes, one GPS monitor station is sufficient. If countrywide networks of monitor stations are set up (e.g., Steeves et al. (1986)), additional positioning redundancy is provided if corrections can be received from more than one monitor station.

2.2.1 SITE SELECTION CONSIDERATIONS

The site of the monitor station(s) has to be selected to enable uninterrupted and undistorted reception of the GPS signals, and to provide data transmission coverage for the area of application. Uninterrupted and undistorted signal reception means that the station has to be set up away from any signal shading objects like buildings, trees, or power poles. Continuous power supply must be available. No electrically conducting material should be in the immediate vicinity of the receiving antenna to avoid any GPS signal distortion through multipath effects. If data transmission to the remote stations is done through line of sight communication, the monitor site has to be selected to guarantee intervisibility.

2.2.2 FUNCTIONAL REQUIREMENTS

The monitor station in GPS differential positioning consists of three functional components: a GPS receiver, a data processing and logging unit, and a data transmitter unit. The receiver tracks all visible GPS satellites and measures pseudoranges and carrier beat phases, preferably in dual frequency P-code mode (see §3.4.1). The data processor checks the data quality and computes from valid measurements monitor station corrections for all satellites at a predefined rate and format. All raw measurement data records are logged to be retrievable for post processing. The data transmitter unit broadcasts the monitor station correction and status information to the remote user. Additional raw carrier beat phase data transmission is required, if carrier beat phase ambiguity resolution is part of the differential positioning process (see §3.3 and §3.4).

2.2.3 OPERATIONAL REQUIREMENTS

All operations of the monitor station are required to be fully automatic. In case of ar unforeseen interruption, the monitor station must have a restart capability. The station must be capable of providing and transmitting warning messages if anything goes wrong.

3 KINEMATIC DIFFERENTIAL GPS TECHNIQUES

Kinematic differential GPS techniques can be classified according to the type of measurement and the type of processing utilized. GPS measurement types are pseudoranges and carrier beat phases which can be used either alone or combined with each other. Carrier beat phase measurements alone are processed in kinematic positioning as double differences or as triple differences. Each of these cases is treated in more detail below. The techniques involve significantly different levels of processing and modelling complexity. The achievable relative positioning accuracies differ by several orders of magnitude.

In general, the accuracy of differential kinematic positioning will depend on the observation accuracy and a factor describing the propagation of the observation errors into position errors. This factor can be approximated by the Geometric Dilution of Precision (GDOP) (e.g., Wells et al. (1987)). For the full GPS satellite constellation, typical GDOP values are below 4. The observation accuracy mentioned above depends on the measurement noise, any unmodelled biases and errors (see §2.1), and inaccuracies introduced through the differential correction determined at the monitor station.

3.1 USING CODE PSEUDORANGES ONLY

Differential pseudorange positioning is the simplest and least accurate technique for differential kinematic positioning. A minimum of four pseudoranges observed simultaneously with the roving receiver, and pseudorange corrections determined at the monitor station are used to solve for instantaneous position and clock error of the roving receiver. Data structures for the transmission of the correction message from the monitor station have been specified (Kalafus et al., 1986). It remains to be seen, whether these structures remain appropriate after Selective Availability is introduced (see §4.3).

3.1.1 P-CODE PSEUDORANGES

P-code pseudoranges can be measured using either code-correlation or codeless techniques. Pseudoranges obtained with a code-correlation receiver are unambiguous. The code ambiguity of codeless observed P-code pseudoranges can be resolved using C/A-code pseudoranges. The main error sources in pseudorange differential positioning are signal multipath and pseudorange measurement noise. Dual frequency measurements can be used to correct the differential ionospheric refraction effects over long distances, if necessary. For short baselines with no appreciable differential ionospheric delays (at the measurement noise level), the pseudorange observation on both frequencies can be averaged to reduce the impact of measurement noise (e.g., Lachapelle et al. (1984)).

The United States DoD reserves the right to switch from P-code to the classified Y-code which is unknown to non-military users. In this event, non-authorized users would not be able to utilize P-code correlation receivers.

3.1.2 C/A-CODE PSEUDORANGES

Since C/A-code is available on one frequency only, a correction for ionospheric delay is not possible. However, for rather short distances (below 200 km) between monitor and remote station, the C/A-code pseudorange errors due to measurement noise and signal multipath by far exceed the ionospheric delay errors. C/A-code measurements are inherently ambiguous by a multiple of 1 msec (see §2.1).

3.1.3 RESULTS REPORTED IN THE LITERATURE

Typical results of differential pseudorange positioning based on TI 4100 measurements have been reported by Lachapelle et al. (1984). In a test conducted over distances of about 100 km, they used averages of L_1 and L_2 P-code pseudoranges as observations. For horizontal positions, an accuracy of 5 m was achieved. Vertical differential position accuracy was 10 m. They also quote an accuracy figure of 15 m for differential C/A-code pseudorange positioning.

3.2 USING BOTH CODE PSEUDORANGES AND CARRIER BEAT PHASES

The following is the basic idea behind the combination of pseudoranges and carrier beat phase observations: pseudoranges have high noise level and strong geometric information on position, whereas carrier beat phase changes over time have low noise level and weak information on position, but strong information on position change. Combining these two observables gives low noise level and strong geometry.

3.2.1 COMBINING CODE AND CARRIER OBSERVATIONS

Combining pseudoranges and carrier beat phase measurements is based on Ron Hatch's idea (Hatch, 1982). Pseudoranges contain information on the receiver-to-satellite range at a high noise level. The carrier beat phase changes in time (Doppler) contain information on change of the receiver-to-satellite range with a low noise level. These properties are used to filter/smooth the high pseudorange noise with the carrier beat phase observations. The filtering is done independently for each satellite being observed, leading to a relatively simple and fast processing. The filtering process produces low noise, slowly changing pseudorange corrections at the monitor station, to be transmitted to the remote station. These corrections are applied to the filtered pseudoranges at the remote (moving) station. The computation of positions from filtered pseudoranges is identical to using raw pseudoranges.

3.2.2 COMBINING CODE AND CARRIER IN THE POSITION SOLUTION

In this processing technique, pseudoranges and carrier beat phase changes (Doppler) observed at the monitor station and at the remote station are combined in one least-squares adjustment giving the relative position of the remote station with respect to the monitor station (Kleusberg, 1986). This method requires all raw data observed at the monitor station to be transmitted to the remote station, and the actual processing is more complex than in the method described in the previous section. Theoretically, the relative positioning results are the same for both methods. The more complex technique described in this section may be advantageous if measurements from additional sensors are to be incorporated in the adjustment.

3.2.3 P-CODE VS C/A-CODE, EACH COMBINED WITH CARRIER

The primary differences are higher measurement noise and larger multipath errors in C/A-code pseudoranges, and the ability to correct the ionospheric delay in P-code pseudoranges through dual frequency observations. The higher noise and multipath leads to a slower convergence of the filtering process described above. The uncorrected ionospheric delay can introduce systematic errors in the relative positioning results obtained from C/A-code measurements.

3.2.4 EFFECT OF CYCLE SLIPS

The treatment of cycle slips involves two issues: firstly, they have to be detected and, secondly, they have to be corrected if possible. Cycle slips can easily be detected in dual frequency receivers by tracing the ionospheric delay (Goad, 1986). A reliable method of cycle slip detection in single frequency receivers requires more than four simultaneously tracked satellite signals and is based on incompatibilities between carrier beat phase observations contaminated by cycle slips (e.g., Goad (1988)).

If the conditions as described in §3.3.4 below are met, the cycle slips can be directly corrected. If this is not possible, the following measures can be applied to the procedures above. For the method described in §3.2.1, the filtering procedure for a particular signal is reset to its initial status, if a cycle slip is detected. For the method described in §3.2.2, the carrier beat phase change contaminated by the cycle slip is rejected and not used in the adjustment. In both cases, the result is a temporary accuracy degradation of the relative positioning.

3.2.5 RESULTS REPORTED IN THE LITERATURE

Results published for relative positioning using any of the above described methods of combining pseudoranges and carrier beat phases range from slightly less than one meter to 3-4 m in relative positioning accuracy. This variation in accuracy can be attributed to differences between P-code and C/A-code measurements, number of cycle slips, and poor reference positions with which to compare.

3.3 USING CARRIER BEAT PHASE: AMBIGUITIES RESOLVED A PRIORI

If the carrier beat phase ambiguities are resolved a priori, the carrier beat phase observations can be interpreted as very precise pseudorange measurements (see §2.1). In this case, carrier beat phase instantaneous kinematic positioning is possible as long as simultaneous carrier beat phase measurements (with resolved ambiguities) to at least four satellites are available. The accuracy of the determined positions depends on the satellite geometry and the carrier beat phase noise level or unmodelled errors, whichever is larger. The following sub-sections discuss ways for resolving a priori the carrier beat phase ambiguity, and the impact of phase lock loss resulting in carrier beat phase cycle slips.

3.3.1 USING INITIAL UNKNOWN STATIC BASELINE

The basic idea is to perform initially a static survey of an arbitrary baseline for the sole purpose of determining the carrier beat phase ambiguities of the satellite signals observed (or their double differences). This is usually done by subjecting the carrier beat phase double differences to a least-squares adjustment. The primary unknowns of the adjustment are the baseline components and the carrier beat phase ambiguities. If the real number estimates for the carrier beat phase ambiguities unambiguously identify the corresponding integer numbers, the ambiguities are resolved. The simultaneously determined precise baseline vector may be of secondary importance only. The observation time required for the determination of the initial baseline will vary between about 30 minutes and several hours (Goad, 1988), depending on satellite geometry, baseline length, and use or single or dual frequency measurements. Both GPS receivers, the monitor and the rover, have to remain at rest during the measurements. For the following kinematic survey, the ambiguities are assumed to remain constant and equal to their determined value.

3.3.2 USING INITIAL KNOWN STATIC BASELINE

If the initial baseline is known with sub-half-cycle accuracy, the carrier beat phase ambiguities can be determined instantaneously (one or a few measurement epochs). Simultaneously, the initial baseline accuracy can be improved. Everything else is identical to the description in §3.3.1.

The initial knowledge of the baseline components is not required, if the baseline is determined simultaneously by other means, e.g., by electronic distance measurements and theodolite measurements.

3.3.3 USING ANTENNA SWAPPING

Antenna swapping as first described by Remondi (1986) involves the exchange of the antennas between the endpoints of an unknown baseline. Obviously, this technique is limited to very short baselines. Linear combinations of the carrier beat phase measurements before and after the antenna exchange can be used to either eliminate the carrier beat phase ambiguities and determine the baseline vector, or to determine both the carrier beat phase ambiguities and the baseline vector. As in the case described in §3.3.2, only a few measurements in each antenna position are required.

3.3.4 EFFECT OF CYCLE SLIPS

As pointed out by Remondi (1986), Mader (1986), Goad (1988), and many other investigators involved in kinematic or semi-kinematic surveying, maintaining phase lock to the observed satellite signals is of utmost importance. Since at every epoch four unknowns have to be determined (3 coordinates and the receiver clock offset), a minimum of four carrier beat phase measurements with predetermined ambiguities must be available.

As long as multichannel receivers are used, enough GPS satellites are visible, and cycle slips occur only occasionally, leaving at least four channels uninterrupted at any time, the cycle slips will require only some additional processing. Dual frequency receivers provide additional redundancy since it is not required that both channels keep continuous phase lock. If less than four channels keep phase lock at any instant in time, the cycle slips cannot be recovered from the phase observations alone. In this case only two options exist:

- Restart the kinematic survey by re-determining the carrier beat phase ambiguities with one of the techniques described in §3.3.1 through §3.3.3.

- Bridge the period of phase lock loss by means of other measurements of the motion of the receiver . These measurements in turn can be used to determine the cycle slips. For short periods of phase lock loss (less than 1 to 2 minutes), a high quality inertial survey system would be capable of providing position change with the required accuracy. For longer periods, no useful instrumentation is known to the authors. For very low dynamic motion of the receiver (no accelerations) it may be possible to bridge short data gaps by extrapolation of the previous receiver motion.

3.3.5 RESULTS REPORTED IN THE LITERATURE

No real-time applications of differential positioning with carrier beat phases have been reported in the literature or are known to the authors. Post-mission results have been reported by Mader (1986) and Landau (1988). Mader (1986) analyzed dual frequency carrier beat phase data collected simultaneously in a fixed-wing aircraft and a ground based monitor station. After elaborate processing and data editing, decimeter-level agreement was achieved for the vertical component in comparison to altimeter measurements. Similar observations were used, and similar accuracies were achieved by Landau (1988) for land vehicle positioning.

3.4 USING CARRIER BEAT PHASE: AMBIGUITIES RESOLVED ON THE FLY

The main difference in comparison to the positioning technique described in §3.3 is the requirement of resolving carrier beat phase ambiguities (or double differences thereof) while the remote receiver is moving. The obvious advantage is that none of the rather cumbersome procedures for determining the initial ambiguity, described in §3.3, have to be followed. Of even more importance is that this technique can recover from possible cycle slips even if they occur simultaneously in all channels.

3.4.1 COMBINING CODE AND CARRIER OBSERVATIONS, REVISITED

From the observation equations listed in §2.1 it can be seen that if the code ambiguity is known, the primary differences between carrier beat phase measurements and pseudoranges are the carrier beat phase ambiguity, and the different measurement noise level. Thus, subtracting pseudoranges from simultaneous carrier beat phase measurements yields a noisy observation of the carrier beat phase ambiguity. If this ambiguity remains constant for a long enough time (i.e., no cycle slips occur), it can be determined from the noisy measurements. The procedure followed in practice is not so simple (e.g., Wübbena (1989)) and involves several linear combinations of simultaneous observables (for example, "wide lane" and "narrow lane" combinations). For this purpose it is of critical importance that dual frequency P-code pseudoranges and carrier beat phase measurements be available.

The cycle-slip-free observation sequence must be long enough to enable the reduction of the pseudorange noise to significantly below the half wavelength of the carrier signal. Problem areas in this regard are unmodelled pseudorange multipath, and unmodelled delays in the receiver (Wübbena, personal communication, 1989).

3.4.2 USE OF CARRIER BEAT PHASE OBSERVATIONS ONLY

If carrier beat phases are measured for more than four satellite signals simultaneously, these measurements contain information that can be used to resolve the carrier beat phase ambiguities (Loomis, 1989). However, a long and uninterrupted observation time span is required (more than one hour for the simultaneous observation of seven satellites). No cycle slips are permitted during this time span.

3.4.3 EFFECT OF CYCLE SLIPS

Since the receiver can recover from phase lock losses in all channels, the importance of cycle slips is greatly reduced. Their remaining primary effect is the requirement for continuous data checking, and, if necessary, for cycle slip correction.

3.4.4 RESULTS REPORTED IN THE LITERATURE

Successful post-mission applications of the technique described in §3.4.1 have been reported by Seeber and Wübbena (1989) and by Purcell et al. (1989). The first reference includes ambiguity resolution with dual frequency TI 4100 observations for two examples: differential kinematic aircraft positioning and ships attitude determination from three widely spaced GPS receivers. The primary problem with this rather outdated type of equipment is its high sensitivity to multipath.

The second reference deals with dual frequency P-code correlation observations from two Rogue receivers for aircraft heading determination. This recently developed GPS receiver type has been designed to be less sensitive to multipath errors. The Rogue receiver can also be operated in codeless mode. For codeless operation, ambiguity resolution has not yet been achieved.

Similar developments using dual frequency codeless pseudoranging and carrier beat phase observations are under way at other institutions (Counselman and Ladd, 1987; Ladd, personal communication, 1989).

No results using real data have been reported for the differential kinematic positioning method described in §3.4.2.

4 OTHER ISSUES

This chapter discusses three issues other than those surrounding the GPS observations themselves, which are pertinent if real-time decimeter-level differential kinematic GPS positioning is to be feasible. The features required for differential kinematic GPS hardware and software are outlined in §4.1. The problems involved, and methods possible for transferring positions determined for GPS antennas to other points of interest on a vessel are discussed in §4.2. The real-time requirements for data communications and data synchronization are addressed in §4.3.

4.1 DIFFERENTIAL KINEMATIC GPS POSITIONING HARDWARE AND SOFTWARE

GPS receiver design features which are essential if real-time decimeter-level kinematic differential GPS positioning is to be achieved have been identified elsewhere in this report. In this section they are summarized.

The GPS **antenna** design should minimize sensitivity to both code and carrier multipath signals (see §2.1.8).

Dual frequency receivers are required to provide the additional information which can assist with carrier beat phase ambiguity resolution (see §3.4.1), cycle slip detection and correction (see §3.2.4, §3.3.4, and §3.4.3), and for longer baselines, to eliminate ionospheric effects (see §2.1.3).

Tracking of the **P-code** rather than simply the C/A-code is required in order to take advantage of the lower measurement noise and smaller *multipath* errors provided by the P-code (see §3.4.1). P-code tracking can use either code correlation techniques or (at least in principle) codeless techniques (see §2.1.5).

Multichannel receivers with as many channels as the maximum number of simultaneously visible GPS satellites (10- to 12-satellite capability is emerging as the standard) make real-time cycle slip detection and correction feasible (see §3.2.4, §3.3.4, and §3.4.3), and allow ambiguity resolution from carrier measurements alone over a period of an hour or so (see §3.4.2).

A **communication interface** to permit real-time reception and use of a correction message from the monitor station(s) is required for any real-time differential GPS system. For decimeter-level performance, the correction message data rate required may be much higher than the standard of 50 bits per second which has been proposed (see §4.3).

Software must provide the GPS-related features of differential corrections, real-time cycle slip detection and correction, and automatic carrier ambiguity resolution, as well as computing antenna positions based on the observation equations (2.1) and (2.2). Careful attention should be paid to incorporating all possible accuracy validation indicators in the software (checking ephemeris message, differential correction message, and observed data against predicted values, estimating observation variance and propagating that into position variance estimates). Additional requirements for incorporation of data from additional sensors in order to provide positioning of other points of interest on the vessels are discussed in §4.2.

Receiver **output data rates** can be whatever the user requires (typically 1 Hz, or one position / second), as long as the internal processing is smart enough to implement the software features just listed. If the receiver is merely a sensor, and these features are implemented in an external processor, then output data rates of many Hz (possibly as high as 10 to 20 Hz) may be required.

4.2 TRANSFERRING GPS ANTENNA POSITIONS TO OTHER POINTS OF INTEREST ON THE VESSEL

GPS is a means to an end — attaching coordinates to data collected by other sensors (such as an echo sounder, or a tide gauge buoy). Positions determined for a GPS antenna must be transferred to the other points on the vessel where these sensors are located. To minimize shadowing of satellites and multipath, the GPS antenna should usually be placed at or near the highest point on the vessel — for example at the top of the mast. The offsets between the antenna location and the other points of interest should be surveyed before the vessel leaves port to better than a decimeter in a ship-fixed coordinate system (intervening bulkheads and decks sometimes make this difficult). The real-time positioning problem then is to relate this ship-fixed coordinate system to the earth-fixed coordinate system in which the GPS positions are determined, and to apply the offsets in the earth-fixed system. It is commonly assumed that the relationship between ship-fixed and earth-fixed systems can be modelled by roll angle, pitch angle, and ship's heading. Since the applications requiring decimeter-level positioning will likely demand calm seas as well, this assumption is probably valid. However, roll and pitch motion will in general be different and vary depending on the aspect of the sea waves causing the motion, and the actual direction of travel of a ship will not in general coincide with its heading. More sophisticated models of the relationship between ship-fixed and earth-fixed coordinate systems should be investigated. For example, the roll/pitch motion of a dredging vessel due to movement of the boom, and the heave of a vessel due to surge may well have significant effects at the decimeter level.

Let us consider three possible methods for determining the orientation of the ship-fixed coordinate system (roll, pitch, and heading). The first method is to use only the fine structure in the trajectory of the GPS antenna. The advantage of this method is that it does not require any additional sensors. Some studies of this approach have been done. For example, Figure 8 illustrates that the roll experienced by the GPS antenna on a 30-m hydrographic survey vessel (in the outer harbour of Halifax, Nova Scotia) is certainly above the decimeter level. This figure shows the capability of a single GPS antenna to detect roll. The long period dip in the GPS-derived "roll" is due to actual deviation of the ship's track from a straight line. Unless very straight lines are being run, separating the roll from deviations in the line being run will be very difficult from an unaided sequence of GPS positions. The higher peaks on the TRIM derived roll may be due to overshoot by the TRIM sensor. Although it may be possible to determine roll from the trajectory of a single GPS antenna, pitch and particularly heading are not well determined.

The second method of orienting the vessel is to use an array of GPS antennas to determine orientation as well as position. A minimal array of two antennas in the fore-aft direction will sense pitch and heading. An array of three will sense roll, pitch, and heading. It may be possible to use "short-baseline" arrays in which the antennas are mounted within a meter or so of each other on a rigid manufacturer-supplied platform. There may be some advantages in considering the alternative of "long-baseline" arrays, where the antennas are mounted at the extremities of the ship. While some manufacturers have looked at the possibility of GPS antenna arrays for orientation, to our knowledge there have been no field experiments reported upon to date.

The third method of orienting the vessel is to use a single GPS antenna together with independent orientation sensors, for example, a gyrocompass to provide heading, and inclinometers (such as the TRIM unit the output of which is illustrated in Figure 8) to provide roll and pitch. Using independent orientation sensors introduces the problem of data synchronization between the GPS and addition sensor data (see §4.3).

Making vessel orientation measurements which are compatible with decimeter-level positioning, whether from GPS antenna arrays or from additional sensors or from a combination of both, is one of the areas requiring research.

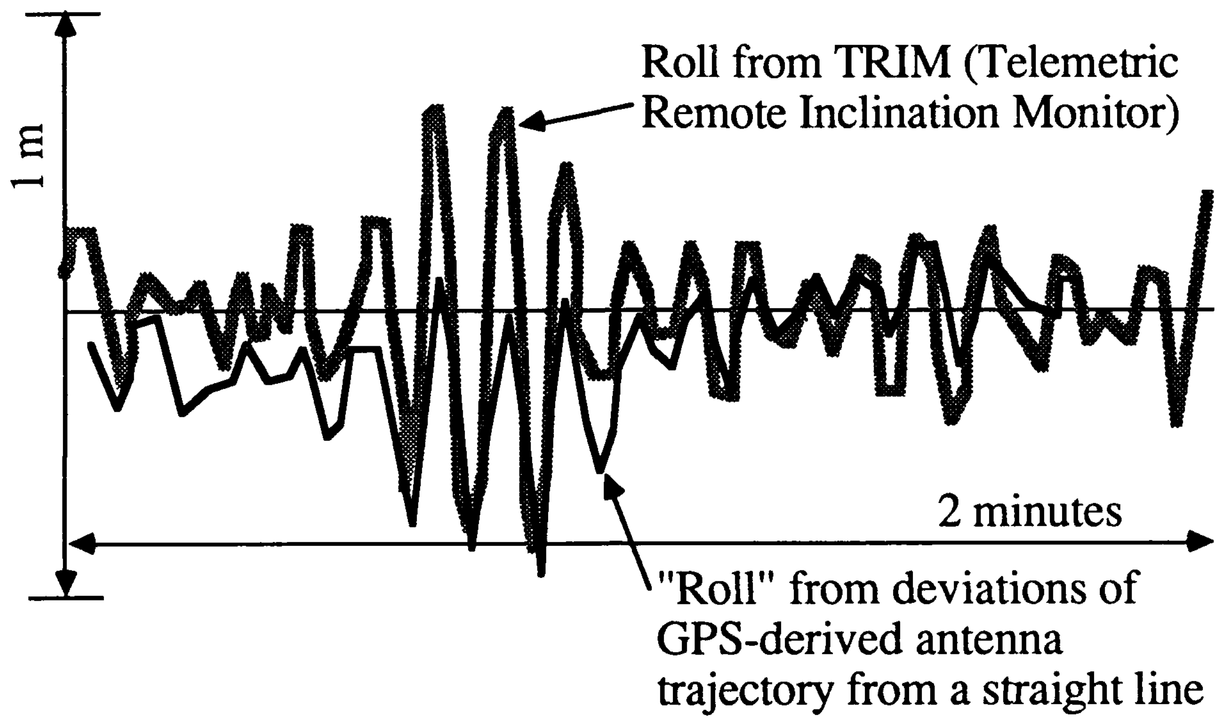

Figure 8: Comparison between TRIM and GPS-derived ship's roll (from Lachapelle et al. (1988)).

We have assumed in this section that the vessel behaves like a rigid body. If vessel deformations are significant at the accuracy level discussed here, more sophisticated methods are required for transferring the position from the antenna to other points of interest. In this case, the variations of the inter-antenna baselines on the ship may give some insight into the structural rigidity of the vessel.

4.3 DATA COMMUNICATIONS AND DATA SYNCHRONIZATION

In this section, we consider two additional requirements for real-time kinematic differential GPS positioning. The first requirement is that differential data from the GPS monitor be communicated to the vessel. The second requirement is that all forms of data be synchronized. This include GPS observations, differential GPS corrections, additional positioning sensor data, and the primary data for which positioning is required (e.g., bathymetry).

The data content, format, and rate of differential correction messages for single-monitor differential GPS applications have been extensively studied by Special Committee 104 (SC104) on Differential NAVSTAR/GPS Service, established by the Radio Technical Commission for Marine Service (RTCM), as described in Kalafus et al. (1986). The standard proposed by this committee is based on as close commonality as possible between the format and rate of the differential correction messages and the format and rate of the GPS satellite broadcast (ephemeris) message. This implies a data rate of 50 bits per second. The main difference between the two is that while the satellite broadcast message is fixed in length, the differential correction messages are variable in length, depending on the number of satellites visible at the monitor station.

The final report of this committee is nearing completion (Kalafus, personal communication, 1989), and will propose 64 message types, of which six will have a final format specification, six will have a tentative format specification, nine will be reserved for particular uses with formats yet unspecified, and the remaining 43 types will be as yet undefined as to either purpose or format.

RTCM SC-104 Differential GPS Correction Message Types

Message Type	Purpose	Status
1	Differential GPS corrections	Final
2	Delta differential GPS corrections (ephemeris change)	Final
3	Monitor station parameters	Final
4	Surveying	Tentative
5	Constellation health	Tentative
6	Null frame	Final
7	Beacon almanacs	Tentative
8	Pseudolite almanacs	Tentative
9	High rate differential GPS corrections	Final
10	P-code differential corrections	Reserved
11	C/A-code L1, L2 delta corrections	Reserved
12	Pseudolite station parameters	Reserved
13	Ground transmitter parameters	Tentative
14	Surveying auxiliary message	Reserved
15	Ionosphere (troposphere) message	Reserved
16	Special message	Final
17	Ephemeris almanac	Tentative
18-59		Undefined
60-63	Differential Loran-C messages	Reserved

The messages in final format have been tested by members of the committee and others and found to be satisfactory. The tentatively specified messages have not yet been tested.

Message Type 1, the primary message for differential pseudorange users, specifies 50 bits for each satellite which is visible from the monitor station. This message contains predicted parameters to be used in a pseudorange correction equation (among other parameters). Assuming that this message is transmitted as often as possible at 50 bits per second, that other messages are transmitted infrequently, and that eleven satellites are visible from the monitor (nearly the maximum number of satellites possible), then the repetition interval of the Type 1 message, averaged over 10 minutes, would be one complete message every 18 seconds or so.

Message Type 4, the primary message for differential carrier beat phase users, tentatively specifies 90 bits for each satellite. This message contains instantaneous (unsmoothed) carrier beat phase measurements (among other parameters). The repetition interval for this message (under the same conditions as before) would be more than 30 seconds. This may be compatible with static surveying applications, where 30-second observation intervals are commonly used. For kinematic surveying, observation intervals of one second or shorter are necessary. In this case the two options are to attempt to interpolate, or to shorten the repetition interval.

Interpolation is not likely to be successful. The parameters in both Message Type 1 and Message Type 4 will age over the repetition interval. The main agent of this aging will be Selective Availability. Assuming that the effect of Selective Availability on the second derivative of the pseudorange measurement errors is 3.7 mm /sec^2 (1σ), then the error in the Message Type 1 differential correction will be 2 m after 20 seconds, and 5 m after 30 seconds (Kalafus et al., 1983). There is no reason to assume that the Message Type 4 error would be different.

The repetition interval can be shortened by transmitting at a higher rate. Transmitting at 2400 bits per second, for example, would permit the complete Message Type 4 for 11 satellites to be transmitted in less than a half-second. There is no barrier to transmitting at this rate, in terms of off-the-shelf communications equipment. The main disadvantage is that this would be incompatible with the SC104 proposed standard.

Research is required to determine the actual impact of Selective Availability upon aging of differential corrections, and hence upon the transmission rate of differential corrections. Message Type 4 remains to be tested. The SC104 restricted their attention to single-monitor applications of differential GPS. While multiple-monitor networks have been proposed, detailed attention has not yet been paid to the format and content of appropriate differential correction messages from multiple-monitor networks.

When data from two or more sensors (typically a positioning sensor like GPS and environmental sensors such as an echo sounder and a tide gauge) are to be used together, two data synchronization issues arise. The first is that the time tags associated with each sensor's data must be referred to the same time base. The second is that the data samples themselves must somehow be synchronized (so that a particular GPS-determined position can be associated with a particular depth measurement, for example).

If data collected on a vessel moving at 5 m$\cdot$s^{-1} is to be synchronized in position to the decimeter level, then the data must be synchronized in time to better than $\frac{1}{50}$ second. A GPS antenna on top of a 10-m mast on a vessel which is rolling 30° will undergo a maximum velocity of 5 m.s^{-1} due to the rolling motion alone. To achieve synchronization to $\frac{1}{50}$ second requires that on board each vessel, one clock be used as a master, and all other local sensor clocks (which provide the data time tags) be automatically referenced to the master clock (for example, by recording local time for each master clock one-second pulse).

True synchronization of the data sampling is more difficult, since it requires simultaneous triggering of all sensors. One approach would be to have the master clock control the sampling in all of the sensors. However, different command processing delays, and different data filtering delays in each sensor mean that a single simultaneous master clock trigger pulse must be supplemented by calibration delays in each sensor, in order to achieve true data synchronization. Not all sensors are likely to permit either the external triggering or the calibration delays. An alternative is not to strive for true data synchronization at the decimeter level, but to use precisely synchronized time tags to interpolate the data.

5 SUMMARY AND CONCLUSIONS

This chapter summarizes the essential constraints on differential kinematic GPS positioning explained in Chapter 2, and discusses the feasibility of development of production quality systems based on the methods described in Chapter 3.

A note about use of the P-code: The conventional use of the code is based on tracking the code by correlating it with a replica generated within the receiver. This requires knowledge of the code, and the official DoD policy is that the the P-code will be replaced by the Y-code, knowledge of which will not be available to non-qualified users. However, "codeless" techniques for tracking the phase of the code (P-code, Y-code, or C/A-code) have been developed, as described in §2.1.5, which may well prove to be almost as useful as code-correlation in providing pseudorange measurements. In this chapter we refer to such measurements as "codeless pseudoranges." When we refer merely to pseudoranges, we mean either code-correlation pseudoranges or codeless pseudoranges.

5.1 SUMMARY OF MAIN POINTS

Three possible methods for precise differential kinematic positioning have been described in Chapter 3. The first method, consisting of smoothing the pseudoranges with carrier beat phases yields meter-level accuracies. The integer nature of carrier beat phase ambiguities is not exploited. The method is applicable to C/A-code receiver measurements. The processing is straightforward and yields reliable results as long as a minimum of four satellite signals are tracked and cycle slips are rather infrequent. At the meter-level accuracy, this approach is the most promising one since it relies on C/A-code only and does not require carrier ambiguity resolution.

The second approach is based on carrier beat phase measurements only. It requires the resolution of the initial integer carrier beat phase ambiguities, and also requires continuous phase lock to at least four GPS satellite signals. If L1 carrier beat phases are utilized only, the accuracy of such a system will be limited by differential ionospheric delays, approaching several decimeters over distances of hundreds of kilometers and high ionospheric activity. If both L1 and L2 phase measurements are used, the primary limiting factor on accuracy will be residual orbital errors. Two problem areas exist in developing such a system. Firstly, resolving the initial ambiguity requires either that remote and monitor station are brought close together, or that both stations remain static for an extended period of time. Neither of these options may be available in a real world hydrographic environment. Secondly, the method requires continuous phase lock to at least four satellites. Though theoretically possible, this may not be feasible, if the remote station passes under low bridges, or if some of the satellite signals are temporarily shaded. The use of inertial navigation units, integrated with the GPS system, will bridge loss of GPS signals for short periods of time. For longer gaps, however, such inertial aiding will be of no benefit. Therefore, a system based on carrier beat phase measurements only will exhibit low reliability and is not considered.

The third approach combines the positive aspects of the two previously described methods. It consists of an improvement of the "code smoothing with carrier beat phase" method to an accuracy level that allows the resolution of the carrier beat phase ambiguity on the fly. Only dual frequency P-code pseudoranges are believed to provide the accuracy required for this method. Both code correlation and codeless P-code pseudoranges are candidates for such a system, if pseudorange multipath can be controlled to an acceptable level.

5.2 FEASIBILITY OF METER-LEVEL DIFFERENTIAL KINEMATIC POSITIONING

Systems capable of producing differential positioning with meter-level accuracy in a marine environment are available. It has been demonstrated that positioning results at this accuracy level can be obtained using single frequency C/A-code equipment (e.g., Lachapelle et al. (1988); Nard (1986)). The latter reference describes one of the most advanced systems presently available. The system consists of a transportable GPS monitor station, a dedicated data communications link, and a remote GPS receiver/processor. The GPS data processing is based on smoothing C/A-code pseudoranges with carrier beat phase as described in §3.2.1. Its successful marine operation in real-time over distances exceeding 100 km has been demonstrated by Nard and Gounon (1988).

No major technological development is required for this type of system. Therefore, most likely more systems of this type will be available when GPS becomes fully operational. Over distances of a few hundred kilometers, these single frequency systems will not be seriously affected by uncorrected ionospheric refraction, at least not at the few meter-level accuracy. Presently unknown is the impact of Selective Availability on differential positioning. Depending on the frequency distribution of Selective Availability errors, the format and rate of differential data correction transmissions may be affected. However, none of the sources queried voiced concerns about differential GPS errors caused by Selective Availability, at the accuracy level discussed here.

5.3 FEASIBILITY OF DECIMETER-LEVEL DIFFERENTIAL KINEMATIC POSITIONING

Presently, systems capable of producing decimeter-level differential positioning in a marine environment are not available. A production level system must be capable of either pseudoranging with an accuracy of several centimeters, or resolving the carrier beat phase ambiguities "on the fly." Instantaneous pseudoranging at the centimeter accuracy level seems not to be feasible. If at all, resolution of carrier beat phase ambiguities can be achieved only with improved P-code pseudoranging, not with C/A-code pseudoranges. Differential ionospheric delays cannot be neglected over distances of a hundred kilometers. Therefore, dual frequency systems will be required. The feasibility of a decimeter system will depend on solving problems in the following areas: Pseudorange noise, Multipath, Selective Availability.

The noise level in the pseudoranges needs to be low enough to enable the unambiguous determination of carrier beat phase ambiguities. This determinability will also depend on the frequency of cycle slips. Because of the higher signal-to-noise ratio, code correlation receivers have a definite advantage here over codeless receivers.

Antenna sensitivity to pseudorange multipath has to be reduced by improving antenna design, so that long periodic pseudorange multipath induced errors remains below the half carrier cycle level. In addition to antenna design, the multipath errors may be reducible by properly selecting the antenna environment.

Recent developments indicate that receiver noise and multipath errors can be reduced to enable the determination of carrier ambiguities with P-code correlation pseudoranges (Melbourne, personal communication, 1989). For codeless P-code pseudoranging receivers under development, confidence was expressed to be able to resolve carrier ambiguities (Ladd, personal communication, 1989).

For real-time applications, Selective Availability errors have to be recovered in real-time at the few centimeter level. Depending on the (unknown) short periodic spectral components of S/A, this real-time error recovery may require much higher data rates for the communications link than those needed for meter-level differential kinematic positioning. To recover the full accuracy potential of differential carrier beat phase positioning, the original carrier beat phase observations at the monitor must be transmitted, rather than smoothed parameters derived from these observations.

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