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13. ABSTRACT *(Maximum 200 words)*

A series of numerical experiments which were carried out with Michael Henderson at IBM (Yorktown) uncovered a very puzzling phenomenon of phase repulsion. This effect consists of the following: when two (identical) oscillators interact, their phases actually repel: one oscillator wants to be "slightly ahead or slightly behind the other; this happens despite the apparently synchronizing effect of the coupling. This phase-replusion phenomenon was totally unexpected and a full explanation remains to be given. The PI did provide an explanation in a simple case, based on estimates (he would like to find a more universal explanation of this apparently basic effect.) The PI expects phase repulsion to play an interesting role in large networks of coupled oscillators.

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1. Dynamics of coupled relaxation oscillators.

One of the aims of this research is to discover fundamental phenomena in systems of interacting oscillators of relaxation type. The developments of the last decade or so provided a good opportunity to make significant progress in this direction. On the one hand, systems of (simpler) phase oscillators have been studied extensively ([ADO], [BH], [EK], [MMS], [TMS]); on the other hand geometric understanding of relaxation oscillators with forcing has been achieved ([L1]).

As the first step in the direction of understanding coupled relaxation oscillators, I concentrated on the simplest but already nontrivial case of two oscillators. This case is already harder than the classical problem of periodically forced van der Pol equations considered by Cartwright, Littlewood and Levinson ([LI] and references therein) and later in [L1].

1. Numerical experiments.

A series of numerical experiments which we carried out with Michael Henderson at IBM (Yorktown) uncovered a very puzzling phenomenon of phase repulsion. This effect consists in the following: when two (identical) oscillators interact, their phases actually repel: one oscillator wants to be "slightly" ahead or slightly behind the other; this happens despite the apparently synchronizing effect of the coupling. This phase-repulsion phenomenon was totally unexpected and a full explanation remains to be given. I did provide an explanation in a simple case, based on estimates (but I would like to find a a more universal explanation of this apparently basic effect.) I expect phase repulsion to play an interesting role in large networks of coupled oscillators.

Using graphics software written by Michael Henderson I also observed an even more puzzling fact: a certain circle mapping associated with a different system, a single forced relaxation oscillator, seems to be nearly piecewise linear for small values of the relaxation parameter. If true, this linearity would be a major surprise – mathematicians such as Littlewood and Levinson put major efforts into this problem and did not come to this conclusion. (Littlewood considered his work on this problem to be technically the most difficult, and his paper [LI], one in a series he wrote on this problem, is 110 pages long.) The (hypothetical) linearity hidden in a nonlinear system could be viewed as a new kind of normal form or universality phenomenon for a class of singularly perturbed problems.

2. Theoretical analysis.

The study that I carried out so far consists of two parts: first, the reduction of dimension of the system, and second, analysis of the reduced system.

The main result of this study is the development of an approach for treating many similar problems involving different types of coupling.

As a case study I analyzed a large part of the bifurcation diagram on a prototypical example. As it turned out, the parameter range of the system was broken up into two different regions, one "chaotic" and the other "regular". In paricular, for the parameters chosen from the "chaotic" part of the bifurcation diagram there coexist two stable periodic regimes – depending on the initial conditions, the ratio of frequencies of the two oscillators can take on one of two possible values. In addition, there are many unstable regimes when the ratio of the frequencies wanders randomly. These solutions are not observed numerically – they occupy the set of measure zero, but they are of interest because their stable manifolds serve as the "watershed" separating the basins of attraction of the two stable regimes. These basins interlace in an interesting way which is described completely. A related result describes the attractor which resembles a "crumpled" 2–torus embedded in the phase space. The physical significance of this result lies in the fact that it illuminates the way the initial conditions approaching two different stable modes interlace.

The behavior in the "non–chaotic" range was analyzed as well; the most notable observation there was the discovery of an additional stable regime corresponding to the out–of–phase oscillation; I am looking forward to exploring the implications of this for networks of oscillators.

The results of this research will be published in several journal articles, in particular, in a tutorial paper in Int. J. Bifurcation and Chaos.

Modeling, geometry and nonlinear dynamics of mechanical systems.

In a different direction, I continued my earlier work ([L2, 4, 5]) on geometric phases in mechanical problems. A key aspect of this work is the derivation and analysis of prototypical models of freely rotating elastic bodies with the goal of understanding the effects of vibrations upon spatial orientation. One of the problems is to describe a relationship between the geometric phase of the system on one hand and a certain phase of the internal vibration on the other. More precisely, I am exploring a relationship between two angles, one of which describes the orientation of the body in space while the other gives the diffence in phase between the vibrational modes of the same frequency.

I am planning to publish an announcement of the results of this research in the proceedings of the upcoming workshop "Dynamics and Control of Freely Rotating Systems of Rigid Bodies" (to be held at the Fields Institute in Waterloo, Canada, March 8–10), with more details in a later publication.

Another project that I started but which is still not completed deals with exploring the possibility that the well–known effect of stabilization by high–frequency vibration ([L3]) may be interpreted as a manifestation of the geometric phase.

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