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RESEARCH IN STOCHASTIC ANALYSIS AND ITS APPLICATIONS

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Co-Principal Investigators: Professor Stamatis Cambanis

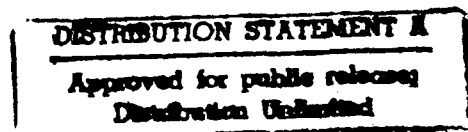
Professor Gopinath Kallianpur

Professor M. Ross Leadbetter

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Department of Statistics
University of North Carolina
Chapel Hill, NC 27599-3260

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13. ABSTRACT (Maximum 200 words) Research was conducted and directed in the area of stochastic analysis and its applications in engineering, neurophysiology and oceanography by the principal investigators, S. Cambanis, G. Kallianpur and M.R. Leadbetter and their associates. A list of the main areas of research activity is as follows: Infinite dimensional stochastic differential equations, stochastic partial differential equations, nonlinear filtering, random fields, point processes and random measures, exchangeable processes, nonstationary processes, Gaussian and non-Gaussian processes, wavelet approximation of random signals, sampling designs for time series, random measures associated with high levels, limit theorems for random measures, parameter estimation under dependence.					
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RESEARCH IN STOCHASTIC ANALYSIS AND ITS APPLICATIONS

SUMMARY OF RESEARCH ACTIVITY

Research was conducted and directed in the area of stochastic analysis and its applications in engineering, neurophysiology and oceanography by the principal investigators, S. Cambanis, G. Kallianpur and M.R. Leadbetter and their associates. A list of the main areas of research activity follows. More detailed descriptions of the work of all participants is given in the main body of the report.

Infinite dimensional stochastic differential equations

Stochastic partial differential equations

Nonlinear filtering

Random fields

Point processes and random measures

Exchangeable processes

Nonstationary processes

Gaussian and non-Gaussian processes

Wavelet approximation of random signals

Sampling designs for time series

Random measures associated with high levels

Limit theorems for random measures

Parameter estimation under dependence

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RESEARCH IN STOCHASTIC ANALYSIS AND ITS APPLICATIONS

STATISTICS CAMBANIS

The work briefly described here was developed in connection with problems arising from and related to the statistical communication theory and the analysis of stochastic signals and systems.

Part I considers questions raised by the observation of continuous time random signals at discrete sampling times, and the transmission or storage of analog random signals in digital form.

Part II considers non-Gaussian models frequently encountered in practical applications. The goal is to learn how Gaussian and linear signal processing methodologies should be adapted to deal with non-Gaussian regimes.

Part III initiates a study of wavelets and multiresolution analysis for random processes, and Part IV deals with random filtering and the harmonic analysis of nonstationary processes.

Item 8 is continuing joint work with E. Masry of the University of California, San Diego. Items 4,5,6,9 and 11 are in collaboration with visitors to the Center for Stochastic Processes: Houdré, Lawniczak, Leskow, Mandrekar, Rosinski, Surgailis, Taraporevala, Weron. Items 1, 2 and 3 are continuing work with current and former Ph.D. students Su and Benhenni.

I. DIGITAL PROCESSING OF ANALOG SIGNALS

Continuous time signals are typically sampled at discrete times and inferences are made on the basis of these samples, which may be further quantized (or rounded-off) for digital processing. Items 1 and 2 describe work completed on sampling designs for the estimation of regression coefficients and for the estimation of a random process, and Item 3 discusses work in progress on the degradation of the performance of sampling designs due to quantization.

1. Sampling designs for regression coefficient estimation with correlated errors.

[1]

The problem of estimating regression coefficients from observations at a

finite number of properly designed sampling points is considered when the error process has correlated values and no quadratic mean derivative. Sacks and Ylvisaker (1966) found an asymptotically optimal design for the best linear unbiased estimator (BLUE), which generally may lack numerical stability and requires the precise knowledge of the covariance function of the error process. Here, the goal is to find an asymptotically optimal design for a simpler estimator which is relatively nonparametric (with respect to the error covariance function). This is achieved by properly adjusting the median sampling design and the simpler estimator introduced by Schoenfelder (1978). Examples with Wiener and Gauss-Markov error processes are considered both analytically and numerically.

2. Sampling designs for estimation of a random process. [2]

A random process $X(t)$, $0 \leq t \leq 1$, is sampled at a finite number of appropriately designed points. On the basis of these observations, we estimate the values of the process at the unsampled points and we measure the performance by an integrated mean square error. We consider the cases where the process has a known, or partially or entirely unknown mean, i.e., when it can be modeled as $X(t) = m(t) + N(t)$, where $m(t)$ is the nonrandom large-scale mean structure and $N(t)$ is the small-scale random structure which models the temporal dependence and has zero mean and known covariance function. Specially, we consider (1) the case where $m(t)$ is known, (2) the semiparametric (regression) model where $m(t) = \beta_1 f_1(t) + \dots + \beta_q f_q(t)$, the β_i 's are unknown coefficients and the f_i 's are known regression functions, and (3) the nonparametric case where the macroscopic mean structure $m(t)$ is unknown. Here $f_i(t)$ and $m(t)$ are of comparable smoothness with the purely random part $N(t)$, and $N(t)$ has no quadratic mean derivative. Asymptotically optimal sampling designs are found for Cases (1), (2) and (3) when the best linear unbiased estimator (BLUE) of $X(t)$ is used (a nearly BLUE in Case (3)), as well as when the simple nonparametric linear interpolator of $X(t)$ is used. Also it is shown that the mean has no effect asymptotically on the overall

performance, and several examples are considered both analytically and numerically.

3. The effect of quantization on the performance of sampling designs. [3]

The most common form of quantization is rounding-off, which occurs in all digital systems. A general quantizer approximates an observed value by the nearest among a finite number of representative values. In estimating weighted integrals of time series with no quadratic mean derivatives, by means of samples at discrete times it is known that the rate of convergence of the mean square error is reduced from n^{-2} to $n^{-1.5}$ when the samples are quantized (Bucklew and Cambanis (1988)). For smoother time series, with $k = 1, 2, \dots$ quadratic mean derivatives, it is now shown that the rate of convergence is reduced from n^{-2k-2} to n^{-2} when the samples are quantized, which is a very significant reduction. The interplay between sampling and quantization is also studied, leading to (asymptotically) optimal allocation between the number of samples and the number of levels of quantization.

II. NONGAUSSIAN MODELS

In continuing the exploration of non-Gaussian models we have studied stable and more general infinitely divisible models. A new rich class of stationary stable processes generalizing moving averages is introduced and studied in Item 4; the study of infinitely divisible processes with stable marginal distributions is initiated in Item 5, and the ergodic properties of stationary infinitely divisible processes are studied in Item 6. Random processes with Eyraud-Farlie-Gumbel-Morgenstern finite dimensional distributions are studied in Item 7.

4. Generalized stable moving averages. [4]

Stationary stable processes have not yet been fully described and studied. The main two classes are the harmonizable ones (which are superpositions of harmonics with independent stable amplitudes) and the moving averages (which are

filtered white stable noise). These classes are disjoint and may not cover a vast collection of stationary stable processes. A new and very rich class of stationary stable processes is introduced and studied. This new class generalizes in a substantial way the moving averages, by means of an appropriate randomization of the filter in the filtered stable noise representation. First the characterization of the law of a generalized moving average (GMA) is established, in terms of the parameters in the generalized moving average representation. It is shown that GMA are mixing, so they have strong ergodic properties, and that they are not harmonizable. They lead to a wealth of new examples of self-similar processes, beyond the linear fractional stable motions, and also of processes which are reflection positive, which is a useful weakening of the Markov property.

5. Infinitely divisible processes with stable marginals. [5]

An interesting class of infinitely divisible processes consists of those whose marginal distributions are stable. Here their detailed study is initiated by finding for their multivariate distributions, conditions for mutual independence and finiteness of joint moments, and by determining the form of their regression function and of the conditional mean-square regression error.

6. Ergodicity and mixing of symmetric infinitely divisible processes. [6]

The hierarchy of ergodic properties of infinitely divisible stationary processes is studied in the language of the spectral representation of these processes. A characterization of ergodicity is given, it is shown that ergodicity and weak mixing are equivalent, and a new characterization of mixing is derived. Several examples are also discussed.

7. On Eyraud-Farlie-Gumbel-Morgenstern (EFGM) random processes. [7]

A particularly simple class of multivariate distributions with given marginals are the EFGM distributions. Their consistent form makes them natural candidates for multivariate distributions of random process. The initial

interest of this study was to explore the properties of EFGM random processes. Instead it was discovered that there are fundamental problems inhibiting the definition of smooth continuous-parameter stationary EFGM processes. Their dependence structure is shown to be severely limited, which prevents them from enjoying some of the weakest regularity properties, such as continuity in probability and measurability. Discrete-time stationary EFGM processes should exist but nontrivial examples have not been constructed yet, and the most common types of dependence structure cannot be exhibited by them. The resulting limitations on nonstationary EFGM processes have not been explored. The conclusion of this investigation is a warning that these simple models of dependence may be inappropriate for sampled time or spatial processes. More complex and more realistic forms of bivariate dependence need to be used. The maximal and minimal correlation coefficient of a bivariate EFGM distribution is also considered and those with maximal correlation coefficient equal to one are characterized.

III. MULTIREOLUTION DECOMPOSITION AND WAVELET TRANSFORMS OF RANDOM SIGNALS

The wavelet approximation of deterministic and random signals at given resolution is considered in Item 8, along with the question of matching the wavelet to the signal for improved quality of approximation. The properties of the wavelet transform of random signals are considered in Item 9.

8. Wavelet approximation of deterministic and random signals: Convergence properties and rates. [8]

An n^{th} order asymptotic expansion is developed for the error in the wavelet approximation at resolution 2^{-k} of deterministic and random signals. The deterministic signals are assumed to have n continuous derivatives, while the random signals are only assumed to have a correlation function with continuous n^{th} order derivatives off the diagonal - a very mild assumption. For deterministic signals over the entire real line, for stationary random signals

over finite intervals, and for nonstationary random signals with finite mean energy over the entire real line, the moments of the scale function can be matched with the signal smoothness to improve substantially the quality of the approximation. In sharp contrast this is feasible only in special cases for nonstationary random signals over finite intervals.

9. Wavelet transforms of random processes. [9]

A study of the properties of wavelet transforms of random processes has been initiated. Their sampled values appear as coefficients in the wavelet approximation of the process at a given resolution. A natural question is what properties of the process are inherited to its wavelet transform, and, conversely, what properties of the process can be read-off properties of its wavelet transform. For random processes with finite second moment, properties of the random process such as periodically correlated, wide sense stationary, harmonizable, wide sense stationary increments and self-similarity, are characterized by means of analogous properties of their wavelet transforms at some scale. The case of random processes which do not have finite second moments, such as stable and other infinitely divisible processes, will also be considered.

IV. NON-STATIONARY PROCESSES

In pursuing the study of non-stationary processes, the random filters which preserve the normality of non-stationary random inputs are characterized and weak laws of large numbers are derived for periodically and for almost periodically correlated processes which are not stationary or harmonizable.

10. Random filters which preserve the normality of non-stationary random inputs. [10]

When a Gaussian signal goes through a (non-random) linear filter, its output is also Gaussian. We are interested in characterising and identifying

those random linear filters which are independently distributed of their random input and preserve its normality. When the input is a stationary Gaussian process, then the output is Gaussian only when the linear filter has non-random gain. Here we consider non-stationary random inputs, for which the situation is more delicate. When the input has stationary independent Gaussian increments, then the output is Gaussian only for linear filters with partly non-random gain and partly random sign. On the other hand when the Gaussian input has nonstationary independent increments, or is a non-stationary noise, or is harmonizable, then the output is Gaussian only for linear filters with random sign.

11. Laws of large numbers for periodically and almost periodically correlated processes. [11]

This paper gives results related to and including laws of large numbers for possibly non-harmonizable periodically and almost periodically correlated processes. In the case of periodically correlated processes, the conditions required for the weak law results are given in terms of the spectral distribution function associated with the average correlation function. The idea of a stationarizing random shift is used to obtain strong law results for such processes. In order to obtain similar results for almost periodically correlated processes, we have to impose the hypothesis $\sum \lambda_k^{-2} < \infty$ (excluding any $\lambda_k = 0$) on the countable set of frequencies $\{\lambda_k\}$ appearing in the Fourier series of the correlation function $R(t+\tau, t) \sim \sum_k a(\lambda_k, \tau) \exp(i\lambda_k t)$. The conditions required for the weak law results are again given in terms of the spectral distribution function associated with the average correlation function. A version of the random shift result is also obtained for these processes.

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G. KALLIANPUR

The research briefly described here falls in the following broad areas:

I. Infinite dimensional stochastic differential equations and stochastic partial differential equations. The general aim is to develop the theory necessary to treat stochastic models of infinite dimensional dynamical systems, in such areas as reaction-diffusions, neurophysiology and fluid mechanics. The work is a continuation of research done jointly with Professors T.S. Chiang and P. Sundar, and has been carried out with D. Baldwin, J. Xiong, R.L. Karandikar and his student, A. Bhatt of the Indian Statistical Institute. Baldwin and Xiong are Ph.D. students working with me.

I. Skeletal theory of filtering. This takes a new look at finitely additive nonlinear filtering theory and connects it with skeletons defined and used by Zakai in the study of Wiener functionals.

II. Feynman distributions. A definition of the Feynman "integral" as a distribution in the sense of Schwarz is given and its value is calculated.

IV. Time series. A time domain analysis of continuous time, periodically correlated (PC) processes is presented. This is joint work with H. Hurd.

V. Stationary random fields. Professor V. Mandrekar (on sabbatical leave at the Center) and I have started working on a monograph on the subject. In addition to presenting a comprehensive account of both the time domain analysis and the spectral theory of second order stationary random fields, some new results and new proofs of our earlier results have been obtained. Work is progressing on applications of the theory to ARMA models and texture analysis problems.

I. INFINITE DIMENSIONAL STOCHASTIC DIFFERENTIAL EQUATIONS (SDE) AND STOCHASTIC PARTIAL DIFFERENTIAL EQUATIONS. [1,2,3,4,5]

1. Nuclear space valued SDE's driven by Poisson random measures. [1,2]

Most of the work on stochastic partial differential equations (SPDE) or infinite dimensional stochastic differential equations (SDE) has been concerned with diffusions, i.e. where the randomness arises from space-time Gaussian white noise. However, there are important physical and physiological phenomena where it is a process with jumps such as a stream of Poisson impulses that is the cause of stochastic variability. An important class of examples is related to the fluctuation of voltage potentials in spatially extended neurons where the excitatory and inhibitory synaptic impulses are most naturally modeled as inputs of discontinuous processes. Furthermore, when more than one space dimension is involved, nuclear space valued (i.e. distribution valued) SDE's have to be considered.

The existence and uniqueness of solution has been established in joint work with J. Xiong. An outline of the proofs is given in [1], the details being provided in the first part of [2]. The SDE in question is of the following form

$$X_t = X_0 + \int_0^t A(s, X_s) ds + \int_0^t \int_U G(s, X_{s-}, u) \tilde{N}(duds)$$

where $A: \mathbb{R}_+ \times \Phi' \rightarrow \Phi'$, $G: \mathbb{R}_+ \times \Phi' \times U \rightarrow \Phi'$, (U, \mathcal{U}, μ) is a σ -finite measure space, $N(duds)$ is a Poisson random measure on $\mathbb{R}_+ \times U$ with intensity measure $\mu(du)ds$, and \tilde{N} is the compensated random measure.

2. Diffusion approximations. [2]

When the number of Poisson processes involved is large, as in the case with the fluctuation of neuron potentials, and the amplitude of each Poisson noise is small, one can expect it to be approximated by a Wiener process. This leads to a Φ' -valued diffusion equation of the form

$$dX_t = A(t, X_t)dt + B(t, X_t)dW_t$$

where W is a cylindrical Brownian motion on a suitable Hilbert space (or,

equivalently, a ϕ' -valued Wiener process). A and B are functions defined on $[0, \infty) \times \phi'$, A mapping into ϕ' and B a continuous linear operator on ϕ' . The precise conditions under which the diffusion approximation is derived together with the proof are given in [2].

3. Propagation of chaos. [3]

This topic, investigated earlier for diffusions by T.S. Chiang, P. Sundar and myself in [3] is now studied for the Poisson random measure driven SDE's. For the discontinuous stochastic model it is shown that the asymptotic behavior of the sequence of empirical measures

$$\nu^n(\cdot) = \frac{1}{n} \sum_{j=1}^n \delta_{X_j^n(\cdot)}, \quad (\delta_x = \text{Dirac measure at } x)$$

is similar. In other words, ν^n converges weakly to the Dirac measure δ_{λ_0} where λ_0 is the unique solution of the corresponding McKean-Vlasov SDE. Hence the asymptotic behavior of ν^n is the same for the diffusion and the discontinuous cases but the McKean-Vlasov equations are, of course, different in the two cases.

4. Existence and uniqueness of nuclear space valued McKean-Vlasov SDE's. [4]

Jointly with D. Baldwin, the infinite-dimensional nuclear space-valued equations are considered (although similar techniques seem to work for the Hilbert space valued case). The McKean-Vlasov diffusion SDE is studied as an independent problem without linking it to the propagation of chaos of interacting systems. Existence and uniqueness of solution are established.

5. Propagation of chaos results for Hilbert space-valued diffusions. [5]

The research on this problem was begun more than a year ago during a brief visit to the Center by Professor Karandikar. However, the present work is a

substantial revision with new results obtained during further collaboration with Karandikar and Ajay Bhatt at the Indian Statistical Institute, Delhi. The asymptotics of the sequence

$$\Gamma^N := \frac{1}{N} \sum_{j=1}^N \delta_{X_j^N}$$

is investigated. The processes $X^{N,j}$ ($j=1, \dots, N$) satisfy an interacting system of Hilbert space (H -) valued SDE's of the following kind:

$$dX_t = -LX_t dt + Aa(t, X_t)dt + Bb(t, X_t)dW_t$$

where W is a cylindrical Brownian motion in H , a and b are suitable coefficient functions and L, A, B satisfy the following conditions:

- (i) $T_t = e^{-tL}$ is a contraction semigroup on H ;
- (ii) L^{-1} , A, B are mutually commuting bounded, self adjoint operators on H , and L^{-1} has a discrete spectrum;
- (iii) A, B and L^{-1} have common eigenfunctions;
- (iv) $A^2 L^{-1}$ and $B^2 L^{-\theta}$ ($0 < \theta < 1$) are nuclear.

The martingale problem for the McKean-Vlasov SDE in H is shown to have a unique solution. A propagation of chaos in $C([0, T], H)$ is obtained for Γ^N .

II. A SKELETAL THEORY OF NONLINEAR FILTERING. [6]

It is shown that the nonlinear filtering theory based on finitely additive white noise (developed earlier by R.L. Karandikar and myself) is a skeletal theory in the technical sense in which M. Zakai has introduced the term "skeleton" into the study of Wiener functionals [7].

Using the notation and terminology of [6] what is shown is the following. Let S be a Polish space and let F be an S -valued random variable such that $F = R_\alpha f$ for some accessible random variable f belonging to $\mathcal{L}^0(E, \mathcal{F}, \alpha; S)$, R_α being the lifting map. Then f is a skeleton of F . Suppose (H, B, μ) is an abstract

Wiener space and F a real valued random variable on $\Omega \times B$ such that $F = R_\alpha f$ for some $f \in \mathcal{L}^2(E, \mathcal{F}, \alpha)$. Then f is a φ -skeleton in the sense of Zakai for the orthonormal basis $\{\varphi_i\}$ appearing in the definition of the independent $N(0,1)$ random variables $L_0(\varphi_i)$ on B . The skeleton and F are related by

$$f(\omega, \sum_{i=1}^k L_0(\varphi_i)(x) \varphi_i) \rightarrow F(\omega, x)$$

in L^2 of the representation space.

Precisely this idea of a skeleton has also been used (with suitable modifications) by G.W. Johnson and myself in obtaining necessary and sufficient conditions for the existence of Stratonovich multiple Wiener integrals.

III. FEYNMAN DISTRIBUTIONS. [8]

The considerable literature on Feynman integrals (in the work of mathematicians at least) is devoted to finding a rigorous justification for R. Feynman's brilliant but heuristic derivation of the integral that bears his name. That such a justification is both necessary and difficult to obtain has been clear since R.H. Cameron in the U.S. and Y. Daletskii in the Soviet Union proved several years ago that the countably additive, complex-valued measure envisaged by Feynman cannot exist. Much of the effort in recent years has concentrated on establishing the Feynman integral via limiting procedures and analytic continuation techniques.

The present research is related to and an outgrowth of joint work with G.W. Johnson on homogeneous chaos expansions and with A.S. Ustunel on distributions on abstract Wiener spaces. The main idea is to think of the Feynman "integral" not as an integral but as a more general object such as a distribution in the sense of a Schwartz distribution in classical analysis. The space of test functionals is a subspace of Wiener functionals whose chaos expansions have natural extensions in the sense defined by Johnson and

Kallianpur in their work. A nuclear topology is introduced in this space Φ . The Feynman distribution is defined as a continuous linear functional on Φ .

It is not yet clear how general this definition is but it seems to contain the Albeverio-Hoegh-Krohn theory of Feynman-Fresnel integrals. A specific method for the evaluation of the distribution at any particular element of Φ is also given. The existence and properties of k -traces of Hilbert-space valued kernels plays an important part in the work.

IV. PERIODICALLY CORRELATED PROCESSES. [9]

A complex-valued second process $\{X(t), t \in \mathbb{R}\}$ is periodically correlated (PC) with period T if

$$m(t) := E(X(t)) = m(t+T) \quad \text{and} \\ R(s, t) := EX(s)\overline{X(t)} = R(s+T, t+T)$$

for every s, t and these conditions hold for no other T' , $0 < T' < T$. It is assumed without loss of generality that $m=0$. The results obtained fall into three categories:

1. Representation of PC processes: A quadratic mean continuous process X is PC with period T if and only if there exists a continuous group of unitary operators $\{U(\tau), \tau \in \mathbb{R}\}$ and a continuous, periodic function $P(t)$ taking values in $L^2(\Omega)$ such that

$$X(t) = U(t)P(t).$$

A stochastic Fourier integral representation for $X(t)$ in terms of a process $Z(\lambda, t)$ is derived where, for each t , $Z(\cdot, t)$ is orthogonally scattered and $Z(\lambda, t)$ is continuous in quadratic mean and periodic in t with period T .

A second result is a Karhunen-Loève type representation from which the

following expansion for the covariance is derived:

$$R(s, t) = \sum_{j=1}^{\infty} \sum_{k=1}^{\infty} r_{jk}([s/T] - [t/T]) \varphi_j(s) \overline{\varphi_j(t)}$$

where

(a) the $\varphi_j(t)$ are continuous periodic functions which are eigenfunctions of $R(s, t)$ ($0 \leq s, t \leq T$) in $L^2[0, T]$;

(b) $r_{jk}(m)$ is a cross correlation matrix of a stationary (possibly infinite dimensional) vector process with $r_{jk}(0) = \lambda_j \delta_{jk}$ and $\sum_{j=1}^{\infty} \lambda_j < \infty$, $\{\lambda_j\}$ being the eigenvalues of $R(s, t)$ in $L^2[0, T]$;

(c) the convergence is uniform.

2. Time domain analysis: Wold-like decomposition of PC processes. A PC process X with period T has the unique decomposition

$$X(t) = Y(t) + Z(t)$$

where the process Y, Z are mutually orthogonal, PC processes with the same period,

(a) Y is regular (purely non deterministic) and Z is singular (purely deterministic);

(b) U_Y and U_Z are, respectively, the restriction to the Hilbert spaces $H(Y)$ and $H(Z)$ of U_X , the generating unitary operator of X ;

(c) $Y(t)$ and $Z(t)$ are subordinated to $X(t)$.

3. The relationship between $L^2[0, T]$ -PC processes and $L^2[0, T]$ -valued stationary sequences. $\{X_n\}$, $n \in \mathbb{Z}$, is an $L^2[0, T]$ -valued stationary sequence if each X_n is an $L^2[0, T]$ -measurable random variable and $E\langle X_n, h_1 \rangle \langle \overline{X_m}, h_2 \rangle = R(n-m; h_1, h_2)$, for every $h_1, h_2 \in L^2[0, T]$. $E\langle X_n, h \rangle$ is taken to be 0. After suitable and natural equivalence relations have been defined in \mathcal{S} , the class of

$L^2[0,T]$ -PC processes and \mathcal{S} , the class of all $L^2[0,T]$ -valued stationary sequences, a bijection is established between \mathcal{S} and \mathcal{S} . This fact makes it possible to relate the spectral theory of continuous time, $L^2[0,T]$ -PC processes to the spectral theory of $L^2[0,T]$ -valued stationary sequences and thus to solve the prediction problem for PC processes.

V. STATIONARY RANDOM FIELDS. [10]

Professor Mandrekar and I have started working on a monograph on second order, discrete time stationary random fields. Our aim is to combine our earlier work on time domain analysis with the spectral theory recently developed by various authors (e.g. Helson and Lowdenslager, Chiang Tse-Pei, Kallianpur, Miamee and Niemi, Korezlioglu and Loubaton). In the course of this work, we were able to improve several of the earlier results as well as discover new ones. This work is in progress and other problems (ARMA models) and applications (e.g. texture analysis) remain to be investigated.

References

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- [3] T.S. Chiang, G. Kallianpur and P. Sundar, Propagation of chaos and the McKean-Vlasov equation in duals of nuclear spaces, Appl. Math. Optim., 24, 55-83 (1991).
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- [8] G. Kallianpur, Traces, natural extensions and Feynman distributions, UNC Center for Stochastic Processes Technical Report No. 329, *Proceedings of the Conference on Gaussian Random Fields, Nagoya, Japan, August 1990*, (1991), to appear.
- [9] H. Hurd and G. Kallianpur, Periodically correlated and periodically unitary processes and their relationship to $L^2[0,T]$ -valued stationary sequences, UNC Center for Stochastic Processes Technical Report, in preparation.
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M.ROSS LEADBETTER

The research undertaken in this reporting period will be described in the following categories:

1. Random measures associated with high levels of a stationary stochastic process
2. Limit theorems for nonstationary random measures
3. Central limit theory for random additive functions of intervals
4. Estimation of tail properties of stationary stochastic sequences
5. Basic structure of random measures and point processes.

1. Random measures associated with high levels of a stationary stochastic process.

Major investigations were conducted on two types of random measure associated with high values of a (suitably mixing) stationary process $\{\xi_t, t \in \mathbb{R}\}$.

(a) Exceedance random measure for multiple levels.

If u_T is a level for each $T > 0$ (T being an "observation time") the corresponding exceedance random measure $\xi_T^{(u_T)}$ is defined for Borel sets B by

$$\xi_T^{(u_T)}(B) = \int_{T \cdot B} 1_{\{\xi(t) > u_T\}} dt .$$

That is $\xi_T^{(u_T)}(B)$ is the amount of time in the set B for which the process $\xi(t)$ exceeds u_T . The basic properties of ξ_T were developed in [1] where it was shown that any distributional limits for a normalized version $a_T \xi_T$ of ξ_T must necessarily be of a general compound Poisson type.

In the current period this work was generalized to apply to vector

exceedance random measures $\zeta_T^{(\underline{u}_T)} = (\zeta_{T,1}, \zeta_{T,2}, \dots, \zeta_{T,k})$ where $\zeta_{T,i}$ is the exceedance random measure corresponding to a level $u_T^{(i)}$, $\underline{u}_T = (u_T^{(1)}, \dots, u_T^{(k)})$.

This work required the development of new techniques for vector cases which were not needed for the scalar case.

An important class of processes considered are those we have called "deterministic at high levels". This notion generalizes the property of the stationary Gaussian process that excursions over a high level (i.e. "peaks") are approximately parabolic in shape, the parabola being determined by one random parameter (e.g. the height of the peak above the level). In particular this means that the peak height above a high level determines that above any higher level, which is essentially the defining property of the class considered.

The structure of such processes is discussed and the above general multilevel theory applied to the class. This work is nearing completion and will be reported in [2].

(b) The excursion random measure.

For a stationary (mixing) sequence of random variables $\{\xi_n\}$ the point N_n in the plane with points at $(j/n, u_n^{-1}(\xi_j))$, and a suitable family of levels $u_n(\tau)$, $\tau > 0$, has important bearing on extremal properties of $\{\xi_n\}$, and has been well studied (cf. [3]). For a stationary process $\{\xi(t)\}$ in continuous time and levels $u_T(\tau)$ a corresponding random measure ζ_T may be defined on (Borel sets of) the plane by

$$\zeta_T(E) = \int 1_{\{(t/T, u_T^{-1}(\xi_T)) \in E\}} dt.$$

ζ_T is intimately related to the exceedance random measures of (1) above by

$$\zeta_T(B \times [a,b)) = \zeta_T^{(u_T(a))}(B) - \zeta_T^{(u_T(b))}(B) .$$

The properties of the excursion random measure ζ_T have been investigated and reported in [4]. In particular, the possible distributional limits for ζ_T have been characterized as a subclass of infinitely divisible random measures, and the Laplace Transforms of the limits determined. A number of examples, including stable and Gaussian processes are discussed, illustrating the results.

2. Limit theorems for nonstationary random measures.

In this effort the convergence properties of non-stationary random measures on the real line were investigated along the same lines as the stationary case of [1]. Similar limit theorems hold, but less can be said about the specific form of the limit due to the nonstationarity. However sufficient conditions (in terms of convergence of "cluster size distributions") may be given, along with theorems characterizing the possible limit laws.

The results obtained have been generalized to apply to vector random measures, and specialized to the case of exceedance random measures by nonstationary processes (satisfying appropriate mixing conditions). The results of the work, which is joint with S. Nandagopalan, will be reported in [5].

3. Central limit theory for additive random interval functions.

A systematic treatment of central limit theory for additive functions of intervals was undertaken in the previous reporting period and described in [6]. We have further extended this work in collaboration with H. Rootzén in the present period. In particular central limit theorems for sums of a stochastic

sequence have been obtained, complementing the results previously included on array sums. This expanded work is reported in [6].

4. Estimation of tail properties of stationary sequences.

Further work has also been done on estimation of parameters of exponential and regularly varying tails of marginal distributions for a stationary sequence. The initial research was reported previously in [7] and submitted for publication to the Annals of Statistics. At the editor's suggestion further work was undertaken - to provide some additional theory, and applications to standard time series models.

This has been completed and the resulting paper is being reviewed by Annals of Statistics. Part of this work concerns estimation of the so-called extremal index of a stationary sequence and this is being continued in current research effort.

5. Basic structure of random measures and point processes.

Significant progress was made in this reporting period on continuing research into the development of basic foundational theory for point processes and random measures. Existing rigorous theory (cf. [8]) is typically based on a highly topological framework. However a principal use of the topology is to generate measure theoretic apparatus, which surely constitutes the natural "heart" of the subject.

This effort, then, is to develop the basic theory in such a way that as far as possible only necessary and minimal assumptions are used. For example the important Poisson processes and their relatives may be defined on a virtually arbitrary measurable space, whereas properties such as regularity, and representation in terms of atoms, require simple separation axioms.

The study has beneficial didactic consequences but its main objectives are

(i) to remove unnecessary assumptions that could burden applications and (ii) to provide the best framework for further theoretical development. In some areas - e.g. distributional convergence - at least implicit topological concepts are necessary to some extent and current effort is addressing these issues.

References

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- [2] M.R. Leadbetter and T. Hsing, On multiple-level excursions by stationary processes with deterministic peaks, in preparation.
- [3] M.R. Leadbetter, G. Lindgren and H. Rootzén, *Extremes and Related Properties of Random Sequences and Processes*, Springer, NY, 1983.
- [4] T. Hsing and M.R. Leadbetter, On the excursion random measures for stationary processes, UNC Center for Stochastic Processes Technical Report No. 350, Oct. 91. Submitted to *Ann. Prob.*
- [5] S. Nandagopalan, M.R. Leadbetter and J. Hüsler, Limit theorems for multi-dimensional random measures, in preparation.
- [6] M.R. Leadbetter and H. Rootzén, On central limit theory for families of strongly mixing additive functions of intervals, UNC Center for Stochastic Processes Technical Report No. 295, May 90, extended version to appear in *Festschrift for G. Kallianpur, P.K. Sen, et al. eds.*, Springer, NY.
- [7] H. Rootzén, M.R. Leadbetter and L. de Haan, Tail and quantile estimation for strongly mixing stationary sequences UNC Center for Stochastic Processes Technical Report No. 292, Apr. 90. Revised for *Ann. Statist.*
- [8] O. Kallenberg, *Random Measures*, Academic Press, 1983.

RICHARD C. BRADLEY

Professor Bradley of the department of Mathematics at Indiana University visited the Center for five months and conducted research on random fields, including their dependence structure and central limit theory, [1,2,3] and on Markovian representations of time series [4].

1. On the spectral density and asymptotic normality of weakly dependent random fields. [1]

For weakly stationary random fields, conditions on coefficients of "linear dependence" are given which are, respectively, sufficient for the existence of a continuous spectral density, and necessary and sufficient for the existence of a continuous positive spectral density. For strictly stationary random fields, central limit theorems are proved under the corresponding "unrestricted ρ -mixing" condition and just finite or "barely infinite" second moments. No mixing rate is assumed.

2. Equivalent mixing conditions for random fields. [2]

For strictly stationary random fields indexed by \mathbb{R}^d or \mathbb{Z}^d , certain versions of the "strong mixing" condition are equivalent to corresponding versions of the " ρ -mixing" condition.

3. Some examples of mixing random fields. [3]

Several classes of strictly stationary random fields are constructed, with various combinations of "strong mixing" properties. The purpose is to "separate" various mixing assumptions that are used in the literature on limit theory for random fields.

4. A limitation of Markov representation for stationary processes. [4]

Strictly stationary random sequences are constructed which satisfy ϕ -mixing (and even a slightly stronger mixing condition) with an arbitrarily fast mixing rate (but not m -dependence), but which cannot be represented as an instantaneous function of a strictly stationary real Harris recurrent Markov chain. The examples here are a modification of similar ones constructed earlier by Berbee and the author which had an exponential mixing rate.

References

- [1] R.C. Bradley, On the spectral density and asymptotic normality of weakly dependent random fields, UNC Center for Stochastic Processes Technical Report No. 332, March 91.
- [2] R.C. Bradley, Equivalent mixing conditions for random fields, UNC Center for Stochastic Processes Technical Report No. 336, March 91.
- [3] R.C. Bradley, Some examples of mixing random fields, UNC Center for Stochastic Processes Technical Report No. 342, March 91.
- [4] R.C. Bradley, An addendum to "A limitation of Markov representation for stationary processes", UNC Center for Stochastic Processes Technical Report No. 335, March 91.

XAVIER FERNIQUE

During his four month stay at the Center, Professor Fernique of the University of Strasbourg continued his pioneering work on sample path properties of stationary Gaussian processes taking values in general topological spaces [1,2]. His main results are summarized below.

Let E be a Lusin and quasi-complete topological vector space, and E' its topological dual. Let X be a stationary Gaussian random function on \mathbb{R}^d and taking values in E . It is shown that if for all $y \in E'$, the real random variable $\langle X, y \rangle$ has a modification with locally bounded paths, then X has the same property.

Denote by c_0 the space of infinite sequences $\{a_n\}$ such that $a_n \rightarrow 0$. It is then shown that X has a modification with continuous paths if E does not contain c_0 (in the sense made precise in [1]). In the opposite direction, if E contains c_0 , a Gaussian stationary E -valued random function can be constructed with bounded paths which has no modification with continuous paths.

In a further study of Fréchet spaces not containing c_0 , Fernique has obtained the following result [2]. Let (X_n) be a symmetric sequence of E -valued random variables. Suppose the random Fourier series $\sum X_n \exp(i\langle \lambda_n, t \rangle)$, $t \in \mathbb{R}^d$, has partial sums which are a.s. locally uniformly bounded in E . Then it converges a.s. uniformly on every compact in \mathbb{R}^d to an E -valued random function with continuous paths.

References

- [1] X. Fernique, Analyse de fonctions aléatoires gaussiennes stationnaires à valeurs vectorielles, UNC Center for Stochastic Processes Technical Report No. 331, Mar. 91.
- [2] X. Fernique, Sur les espaces de Fréchet ne contenant pas c_0 , UNC Center for Stochastic Processes Technical Report No. 333, Mar. 91.

TAILEN HSING

Professor T. Hsing of the Statistics Department of Texas A&M University visited for the 1990-91 academic year. During this period he conducted research in point processes and random measures associated with high values of a stochastic process, parameter estimation for extreme order statistics in a weakly dependent sequence, and on a theory for "sliced inverse regression". This work was described in the following technical reports:

1. Point process convergence and asymptotic behavior of sums of weakly dependent random variables with heavy tails. [1]

Let $\{\xi_j\}$ be a strictly stationary sequence of random variables such that $P\{|\xi_1| > x\}$ is regularly varying at infinity with index $-\alpha < 0$. Let a_n be such that $n P\{|\xi_1| > a_n\} \rightarrow 1$ as $n \rightarrow \infty$ and let the point process N_n (on $\mathbb{R} - \{0\}$) have atoms at ξ_j/a_n . The possible limits in distribution for N_n are characterized under a weak mixing condition and a necessary and sufficient condition is given for convergence. This yields the limiting distribution for $S_n = \sum_{j=1}^n \xi_j$ and illuminates the influence of the extreme order statistics on S_n .

2. On some estimates based on sample behavior near high level excursions. [2]

Let $\{\xi_j\}$ be a stationary sequence of weakly dependent random variables with marginal distribution F , and let $M_n^{(k)}$ be the k -th largest value of ξ_j , $1 \leq j \leq n$. The estimation of the parameters of the asymptotic distribution of $M_m^{(k)}$ is considered using a procedure motivated by a limit theorem pertaining to the point process having points at $(j/n, n(1-F(\xi_j)))$, $j=1,2,\dots$. A number of statistical issues concerning the procedure are addressed, including the means for selecting the tuning parameters. Estimation of the filter of the stable moving average process, for which similar principles apply, is also considered.

3. On the excursion random measure of stationary processes. [3]

The excursion random measure of a stationary process is a random measure on $(-\infty, \infty) \times (0, \infty)$, which records the extent of excursions of high levels by the process. (See report for M.R. Leadbetter where a precise definition is given). The excursion random measure is asymptotically infinitely divisible and satisfies certain conditions of stability and independent increments, under very general conditions. A number of examples, including stable and Gaussian processes, are considered, illustrating the results.

4. An asymptotic theory for sliced inverse regression. [4]

Sliced inverse regression is a nonparametric method for achieving dimension reduction in regression problems. It is widely applicable, extremely easy to implement on a computer, and requires no nonparametric smoothing devices such as kernel regression. If Y is the response and $X \in \mathbb{R}^D$ the predictor, in order to implement sliced inverse regression, one requires an estimate of $\Lambda = E\{\text{cov}(X|Y)\} = \text{cov}(X) - \text{cov}\{E(X|Y)\}$. The inverse regression of X on Y is clearly seen in Λ . One such estimate is Li's (1991) 2-slice estimate, defined as follows: the data are sorted on Y , then grouped into sets of size 2, the covariance of X is estimated within each group, and these estimates are averaged. In this paper, we consider the asymptotic properties of the 2-slice method, obtaining simple conditions for $n^{1/2}$ -convergence and asymptotic normality. A key step in the proof of asymptotic normality is a central limit theorem for sums of conditionally independent random variables. We also study the asymptotic distribution of Greenwood's statistics in non-uniform cases.

References

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- [2] T. Hsing, On some estimates based on sample behavior near high level excursions, UNC Center for Stochastic Processes Technical Report No. 337, Apr. 91.
- [3] T. Hsing and M.R. Leadbetter, On the excursion random measure of stationary processes, UNC Center for Stochastic Processes Technical Report No. 350, Oct. 91.
- [4] T. Hsing and R.J. Carroll, An asymptotic theory for sliced inverse regression, UNC Center for Stochastic Processes Technical Report No. 327, Jan. 91.

OLAV KALLENBERG

Professor Kallenberg of the department of Mathematics at Auburn University visited the Center for two months, and conducted research on exchangeable stochastic processes

Symmetries on random arrays and set-indexed processes. [1]

A process X on the set \tilde{N} of all finite subsets J of N is said to be spreadable, if $(X_{pJ}) \stackrel{d}{=} (X_J)$ for all subsequences $p = (p_1, p_2, \dots)$ of N , where $pJ = \{p_j : j \in J\}$. Spreadable processes are characterized in this paper by a representation formula, similar to those obtained by Aldous and Hoover for exchangeable arrays of random variables. Our representation is equivalent to the statement that a process on \tilde{N} is spreadable, if and only if it can be extended to an exchangeable process indexed by all finite sequences of distinct elements from N . The latter result may be regarded as a multivariate extension of a theorem by Ryll-Nardzewski, stating that, for infinite sequences of random variables, the notions of exchangeability and spreadability are equivalent.

Reference

- [1] O. Kallenberg, Symmetries on random arrays and set-indexed processes, UNC Center for Stochastic Processes Technical Report No. 345, Sept. 91.

TORGNY LINDVALL

Dr. Torgny Lindvall, of the Mathematics Department of Chalmers University Sweden, visited the Center for a two month period (Nov.-Dec. 1990). Dr. Lindvall is a foremost expert on the method of "coupling" in probability theory. During his visit he worked in inequalities for stochastic processes, Poisson approximation and problems in coupling. The results obtained will be reported in [1].

Reference

- [1] T. Lindvall, "The Coupling Method", Wiley, New York, to appear.

V. MANDREKAR

Professor V. Mandrekar of the Department of Statistics and Probability at Michigan State University visited the Center for 7 months. He worked on the spectral analysis of stochastic sequences with finite moments of order $p \geq 1$, including stable sequences. He also worked with S. Cambanis, J. Rosinski and D. Surgailis on the study of a new rich class of stationary stable processes, and with G. Kallianpur on stationary random fields.

1. On the spectral representation of a sequence in Banach space. [1]

The paper gives the spectral representation of absolutely continuous random sequences with finite moments of order $p \geq 1$ (or more generally with values in a Banach space). It is shown that this class includes both the stable harmonizable and the moving average sequences, for which explicit representations are computed. This work also provides a spectral representation for non-stationary processes in the general set-up. The special case when $p = 2$ (or when the Banach space is a Hilbert space) is presently under investigation.

2. Spectral theory of periodically and quasi-periodically stationary stable sequences. [2]

This paper develops a spectral theory for (periodically) T-stationary sequences and quasi-periodically stationary stable sequences. We use the notion of the unitary operator of a periodically correlated (PC) sequence to extend the spectral theory and representations introduced by Gladyshev for PC sequences to the stable sequences. The extended results are obtained by using the spectral representation for an isometric operator on a space possessing the unconditionality property for martingale differences. The quasi-periodically stationary stable sequences are then seen to be a natural extension of the

T-stationary sequences. Finally, we show that the purely non-deterministic PC sequences may be given as moving averages of white sequences with respect to a periodic family of coefficients.

3. Generalized stable moving averages. [3]

Stationary stable processes have not yet been fully described and studied. The main two classes are the harmonizable ones (which are superpositions of harmonics with independent stable amplitudes) and the moving averages (which are filtered white stable noise). These classes are disjoint and may not cover a vast collection of stationary stable processes. A new and very rich class of stationary stable processes is introduced and studied. This new class generalizes in a substantial way the moving averages, by means of an appropriate randomization of the filter in the filtered stable noise representation. First the characterization of the law of a generalized moving average (GMA) is established, in terms of the parameters in the generalized moving average representation. It is shown that GMA are mixing, so they have strong ergodic properties, and that they are not harmonizable. They lead to a wealth of new examples of self-similar processes, beyond the linear fractional stable motions, and also of processes which are reflection positive, which is a useful weakening of the Markov property.

4. Stationary random fields. [4]

Professors Mandrekar and Kallianpur have started working on a monograph on second order, discrete time stationary random fields. The aim is to combine our earlier work on time domain analysis with the spectral theory recently developed by various authors (e.g. Helson and Lowdenslager, Chiang Tse-Pei, Kallianpur, Miamee and Niemi, Korezlioglu and Loubaton). In the course of the work, they were able to improve several of the earlier results as well as

discover new ones. This work is in progress and other problems (ARMA models) and applications (e.g. texture analysis) remain to be investigated.

References

- [1] A. Makagon and V. Mandrekar, On the spectral representation of a sequence in Banach space, UNC Center for stochastic Processes Technical Report No. 328, Feb. 91.
- [2] H.L. Hurd and V. Mandrekar, Spectral theory of periodically and quasi-periodically stationary SoS sequences, UNC Center for Stochastic Processes Technical Report No. 349, Sept. 91.
- [3] S. Cambanis, V. Mandrekar, J. Rosinski and D. Surgailis, Generalized stable moving averages, in preparation.
- [4] G. Kallianpur and V. Mandrekar, Stationary Random Fields, in preparation.

JAN ROSINSKI

Professor Rosinski of the Department of Mathematics at the University of Tennessee in Knoxville visited the Center for 4 months. He conducted research primarily on classes of non-Gaussian processes, studying a new and rich class of stationary stable processes jointly with S. Cambanis, V. Mandrekar and D. Surgailis, and also the distribution of certain important functionals of infinitely divisible processes.

1. Generalized stable moving averages. [1]

Stationary stable processes have not yet been fully described and studied. The main two classes are the harmonizable ones (which are superpositions of harmonics with independent stable amplitudes) and the moving averages (which are filtered white stable noise). These classes are disjoint and may not cover a vast collection of stationary stable processes. A new and very rich class of stationary stable processes is introduced and studied. This new class generalizes in a substantial way the moving averages, by means of an appropriate randomization of the filter in the filtered stable noise representation. First the characterization of the law of a generalized moving average (GMA) is established, in terms of the parameters in the generalized moving average representation. It is shown that GMA are mixing, so they have strong ergodic properties, and that they are not harmonizable. They lead to a wealth of new examples of self-similar processes, beyond the linear fractional stable motions, and also of processes which are reflection positive, which is a useful weakening of the Markov property.

2. Distributions of subadditive functionals of sample paths of infinitely divisible processes. [2]

Subadditive functionals on the space of sample paths include suprema,

integrals of paths, oscillation on sets, and many others. In this paper we find an optimal condition which ensures that the distribution of a subadditive functional of sample paths of an infinitely divisible process belongs to the subexponential class of distributions. Further, we give the exact tail behaviour of the distributions of such functionals, thus improving many recent results obtained for particular subadditive functionals and for particular infinitely divisible processes, including stable processes.

3. Remarks on strong exponential integrability of vector valued random series and triangular arrays. [3]

A sharp result on the strong exponential integrability of the norm of sums of independent uniformly bounded Banach space-valued random vectors is established based on Lévy's decoupling method.

References

- [1] S. Cambanis, V. Mandrekar, J. Rosinski and D. Surgailis, Generalized stable moving averages, in preparation.
- [2] J. Rosinski and G. Samorodnitsky, Distributions of subadditive functions of sample paths of infinitely divisible processes, UNC Center for Stochastic Processes Technical Report No. 338, Apr. 91.
- [3] J. Rosinski, Remarks on strong exponential integrability of vector valued random series and triangular arrays, UNC Center for Stochastic Processes Technical Report No. 339, May 91.

PADMANABHAN SUNDAR

Dr. Sundar of the Mathematics Department at Louisiana State University continued his research on stochastic partial differential equations during his one month visit to the Center in the summer. He has been interested in generalizations of Girsanov's theorem for laws induced by solutions of SPDE's.

He does not have a general result yet and has been working on special examples that might suggest the lines of attack on the problem. One result he has obtained is the following: If $u(t,x,w)$ is the unique solution of the SPDE

$$\frac{\partial u}{\partial t} = \Delta u + \dot{W}_{tx}, \quad 0 < x < \pi,$$

with 0 initial condition and Neumann boundary conditions, then the (Gaussian) measure induced by u is singular with respect to the law of the Brownian sheet.

A technical report containing examples and general results will be prepared after further work on these problems.

DONATAS SURGAILIS

Dr. Surgailis of the Department of Mathematical Statistics of the Lithuanian Academy of Sciences in Vilnius visited the Center for 2 months. He introduced a rich new class of stationary stable processes generalizing the moving averages of stable noise and initiated an indepth study of this class jointly with S. Cambanis, J. Rosinski and V. Mandrekar.

1. Generalized stable moving averages. [1]

Stationary stable processes have not yet been fully described and studied. The main two classes are the harmonizable ones (which are superpositions of harmonics with independent stable amplitudes) and the moving averages (which are filtered white stable noise). These classes are disjoint and may not cover a vast collection of stationary stable processes. A new and very rich class of stationary stable processes is introduced and studied. This new class generalizes in a substantial way the moving averages, by means of an appropriate randomization of the filter in the filtered stable noise representation. First the characterization of the law of a generalized moving average (GMA) is established, in terms of the parameters in the generalized moving average representation. It is shown that GMA are mixing, so they have strong ergodic properties, and that they are not harmonizable. They lead to a wealth of new examples of self-similar processes, beyond the linear fractional stable motions, and also of processes which are reflection positive, which is a useful weakening of the Markov property.

Reference

- [1] S. Cambanis, V. Mandrekar, J. Rosinski and D. Surgailis, Generalized stable moving averages, in preparation.

PH.D STUDENTS

PH.D. DEGREES AWARDED

Y.C. SU, SAMPLING DESIGNS FOR ESTIMATION OF REGRESSION COEFFICIENTS AND OF A RANDOM PROCESS. [1]

Mr. Su completed his Ph.D. research under the direction of S. Cambanis. His thesis research addressed two problems of sampling designs for stochastic processes discussed in items 1 and 2 under S. Cambanis and abstracted below.

The problem of estimating regression coefficients with correlated errors and of estimating a time series, both from observations of the continuous-time series at a finite number of appropriately designed sampling points, are considered when the time series has correlated values but no quadratic mean derivative. For the estimation of regression coefficients we find an asymptotically optimal sampling design for an estimator which is simpler than the best linear unbiased estimator and relatively nonparametric, i.e. less dependent on the covariance function. For the estimation of a random process we consider the cases where the mean of the process is known, partially known, or entirely unknown. Asymptotically optimal sampling designs are found when the best linear unbiased estimator is used, as well as when the simple nonparametric linear interpolator is used. It is shown that the mean has no effect asymptotically on the overall performance, and examples are considered both analytically and numerically.

Reference

- [1] Y.C. Su, Sampling designs for estimation of regression coefficients and of a random process, UNC Center for Stochastic Processes Technical Report No. 356, Dec. 91.

DISSERTATIONS IN PREPARATION

D. BALDWIN, NUCLEAR SPACE VALUED MCKEAN-VLASOV STOCHASTIC DIFFERENTIAL EQUATIONS

Mr. Baldwin is completing his Ph.D. research under the direction of G. Kallianpur. He is studying infinite dimensional nuclear space-valued equations, (although similar techniques seem to work for the Hilbert space valued case). The McKean-Vlasov diffusion SDE is studied as an independent problem without linking it to the propagation of chaos of interacting systems. Existence and uniqueness of solution are established.

J. XIONG, STOCHASTIC DIFFERENTIAL EQUATIONS IN DUALS OF NUCLEAR SPACES

Mr. Xiong is in the final stages of completing his Ph.D. research under the direction of G. Kallianpur. His work is described in I.1 and 2 under G. Kallianpur and is abstracted below.

The following stochastic differential equation is considered on Φ'

$$dX_t = A(t, X_t)dt + \int_U G(t, X_{t-}, u) \tilde{N}(du, dt)$$

where $A: R_+ \times \Phi' \rightarrow \Phi'$ and $G: R_+ \times \Phi' \times U \rightarrow \Phi'$ are measurable, Φ is a countably Hilbert nuclear space with strong dual Φ' , U is a Blackwell space and N is a Poisson random measure on $U \times R_+$. The existence and uniqueness of solutions are established by using a Galerkin-type approximation and a modification of the Yamada-Watanabe argument about the uniqueness of strong solutions. A limit theorem is also proven for approximating a nuclear space valued diffusion process by a sequence of processes which are solutions of the SDE's considered above. These results are applied to certain neurophysiological problems. Further results on propagation of chaos and solutions of the corresponding McKean-Vlasov SDE have also been obtained.

TECHNICAL REPORTS
CENTER FOR STOCHASTIC PROCESSES

- 308. H.L. Hurd and J. Leskow, Strongly consistent and asymptotically normal estimation of covariance for almost periodically correlated processes, Sept. 90.
- 309. O. Kallenberg, From optional skipping to random time change - on some recent advances in exchangeability theory, Sept. 90. *Proc. of the Second World Congress of the Bernoulli Society*, to appear.
- 310. S. Cambanis and W. Wu, Multiple regression on stable vectors, Sept. 90. *J. Multivariate Anal.*, to appear.
- 311. W. Wu, Heavy tailed models: bootstrapping the sample mean and stable dependence structure, Sept. 90. (Dissertation).
- 312. T.S. Chiang, G. Kallianpur and P. Sundar, Propagation of chaos for systems of interacting neurons, Sept. 90. *Proc. Trento Conf. on Stochastic Partial Differential Equations*, Springer-Verlag, 1991, to appear.
- 313. T. Koski and S. Cambanis, On the statistics of the error in predictive coding for stationary Ornstein-Uhlenbeck-processes, Sept. 90. *IEEE Trans. Information Theor.*, to appear.
- 314. J.M.P. Albin, On the general law of iterated logarithm with application to Gaussian processes in \mathbb{R}^n and Hilbert space and to stable processes, Sept. 90. *Stochastic Proc. Appl.*, to appear.
- 315. S. Nandagopalan, Multivariate extremes and estimation of the extremal index, Sept. 90. (Dissertation).
- 316. G.W. Johnson and G. Kallianpur, Multiple Wiener integrals on abstract Wiener spaces and liftings of p-linear forms, Oct. 90. *Proceedings of the Conference on White Noise Analysis*, T. Hida et al., eds., World Scientific, 1990, 208-219.
- 317. V.V. Sazonov and V.V. Ulyanov, An improved estimate of the accuracy of Gaussian approximation in Hilbert space, Oct. 90.
- 318. Y.C. Su and S. Cambanis, Sampling designs for regression coefficient estimation with correlated errors, Feb. 91.
- 319. G. Kallianpur and A.S. Ustunel, Distributions, Feynman integrals and measures on abstract Wiener spaces, Oct. 90.
- 320. G. Kallianpur and V. Perez-Abreu, The Skorohod integral and the derivative operator of functionals of a cylindrical Brownian motion, Oct. 90. *Appl. Math. Optimization*, to appear.
- 321. G.W. Johnson and G. Kallianpur, The analytic Feynman integral of the natural extension of pth homogeneous chaos, Oct. 90. *Circolo Rendiconti di Palermo. Proceedings of the Conference on Measure Theory, Oberwolfach, 1990, (1991)*, to appear.
- 322. D. Daley and T. Rolski, Finiteness of waiting-time moments in general stationary single-server queues, Nov. 90. *Ann. Appl. Probab.* 1992, to appear.

323. T. Hsing, Point process convergence and asymptotic behavior of sums of weakly dependent random variables with heavy tails, Nov. 90.
324. S. Cambanis, On Eyraud-Farlie-Gumbel-Morgenstern random processes, Nov. 90. *Advances in Probability Distributions with Given Marginals*, G. Dall'Aglio et al., eds., Kluwer Academic, 1991, 207-222.
325. G. Kallianpur, A skeletal theory of filtering, Dec. 90. *Liber Amicorum for Moshe Zakai*, Academic Press, 1991, 213-243.
326. V.V. Sazonov and V.V. Ulyanov, Speed of convergence in the central limit theorem in Hilbert space under weakened moment conditions, Jan. 91.
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328. A. Makagon and V. Mandrekar, On the spectral representation of a sequence in Banach space, Feb. 91.
329. G. Kallianpur, Traces, natural extensions and Feynman distributions, Feb. 91. *Proceedings of the Conference on Gaussian Random Fields, Nagoya, Japan, August 1990*, (1991), to appear.
330. H.L. Hurd and J. Leskow, Estimation of the Fourier coefficient functions and their spectral densities for ϕ -mixing almost periodically correlated processes, Mar. 91.
331. X. Fernique, Analyse de fonctions aléatoires gaussiennes stationnaires à valeurs vectorielles, Mar. 91.
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344. G. Kallianpur and J. Xiong, A nuclear-space-valued stochastic differential equation driven by Poisson random measures, Sept. 91. *Proc. International Conference on SPDE's*, Springer, to appear.
345. O. Kallenberg, Symmetries on random arrays and set-indexed processes, Sept. 91.
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355. J. Olsson and Holger Rootzén, An image model for quantal response analysis in perimetry, Nov. 91.
356. Y.-C. Su, Sampling designs for estimation of regression coefficients and of a random process, Dec. 91.

IN PREPARATION

- D. Baldwin and G. Kallianpur, Existence and uniqueness of solution of nuclear space valued McKean-Vlasov equation.
- K. Benhenni and S. Cambanis, The effect of quantization on the performance of sampling designs.
- S. Cambanis and C. Houdré, Wavelet transforms of random processes.
- S. Cambanis, V. Mandrekar, J. Rosinski and D. Surgailis, Generalized stable moving averages.
- S. Cambanis and A. Taraporevala, Infinitely divisible processes with stable marginal distributions.

H. Hurd and G. Kallianpur, Periodically correlated and periodically unitary processes and their relationship to $L^2[0,T]$ -valued stationary sequences.

G. Kallianpur, R.L. Karandikar and A. Bhatt, Propagation of chaos for interacting Hilbert space valued diffusions.

G. Kallianpur and V. Mandrekar, Stationary Random Fields.

M.R. Leadbetter and T. Hsing, On multiple-level excursions by stationary processes with deterministic peaks, in preparation.

S. Nandagopalan, M.R. Leadbetter and J. Hüsler, Limit theorems for multi-dimensional random measures, in preparation.

P. Sundar, Generalizations of Girsanov's theorem for laws induced by solutions of SPDE's.

STOCHASTIC PROCESSES SEMINARS

- Sept. 7 Inference for almost periodically correlated processes, J. Leskow, University of California, Santa Barbara
- Sept. 12 Some random time change representations of stable integrals, O. Kallenberg, Auburn University and University of North Carolina
- Sept. 21 Stratonovitch calculus for non adapted two parameter processes, Michelle Thieullen, Universite Paris VI and University of North Carolina
- Oct. 3 On the central limit theorem in Hilbert space, V.V. Sazonov, Steklov Mathematical Institute, Moscow, USSR
- Oct. 19 Canonical correlations between past and future of stochastic processes, A.M. Yaglom, Institute of Atmospheric Physics, Moscow, USSR
- Oct. 25 On a nonlinear autoregressive process in the theory of signal compression, T. Koski, Lulea University of Technology and University of North Carolina
- Nov. 8 Explosion times for Doleans-Dade stochastic differential equations, M. Thieullen, Universite Paris VI and University of North Carolina
- Nov. 19 Monotonicity properties of birth and death processes, T. Lindvall, University of Goteborg and University of North Carolina
- Dec. 13 Stratonovich multiple Wiener integrals and anticipative integration, G. Kallianpur, University of North Carolina
- Jan. 23 Markov random graphs and polygonal fields on the plane, D. Surgailis, Institute of Mathematics and Cybernetics, Vilnius, USSR
- Jan. 30 Long memory shot noises and limit theorems with application to Burgers' equation, D. Surgailis, Institute of Mathematics and Cybernetics, Vilnius, USSR
- Feb. 6 The synthesis of random field representations and multiobjective optimization, G. Christakos, University of North Carolina
- Feb. 13 Zero-one laws for multilinear forms in Gaussian and other infinitely divisible random variables, J. Rosinski, University of Tennessee, Knoxville
- Feb. 20 Travelling waves in branching diffusions, S.P. Lalley, Purdue University
- Feb. 27 Strong consistent model class selection for a stationary ergodic process based on log-likelihood, J.C. Kieffer, University of Minnesota
- Mar. 6 Compactness of the fluctuations associated with some generalized nonlinear Boltzmann equations, G. Giroux, University of Sherbrooke

- Mar. 20 Regularity of Gaussian stationary vector valued random functions, X. Fernique, University of Strasbourg and University of North Carolina
- Apr. 1 On the spectral density and asymptotic normality of weakly dependent random fields, R.C. Bradley, Indiana University and University of North Carolina
- Apr. 9 Semigroup methods in the theory of infinitely divisible distributions on groups and vector spaces, T. Byczkowski, Wroclaw Technical University and University of Tennessee, Knoxville
- Apr. 11 An overview of some one-dimensional stochastic neuronal models, Petr Lansky, Czechoslovak Academy of Sciences and North Carolina State University
- Apr. 12 Contagious processes in finite populations, D. Daley, Australian National University
- Apr. 17 Regularity of Gaussian stationary vector valued random functions I, X. Fernique, Université de Strasbourg and University of North Carolina
- Apr. 25 Regularity of Gaussian stationary vector valued random functions II, X. Fernique, Université de Strasbourg and University of North Carolina
- May 13 Recent results on SPDE's of Ginsburg-Landau type, T. Funaki, Nagoya University
- May 20 System identification of the dynamical characteristics of catchment response to rainfall: Limitations and applications to assessing the effects of environmental changes, A.J. Jakeman, Australian National University
- May 21 Limit theorems for sums of independent random variables and applications, Boris V. Gnedenko, Moscow State University
- May 23 Limit theorems for order statistics and applications, Boris V. Gnedenko, Moscow State University
- Aug. 26 Almost sure expansions for sums of independent random variables, V.M. Zolotarev, Steklov Mathematical Institute, Moscow
- Aug. 28 Local time and Palm distributions for exchangeable interval partitions, O. Kallenberg, Auburn University and University of North Carolina

PROFESSIONAL PERSONNEL

Faculty Investigators

S. Cambanis
G. Kallianpur
M.R. Leadbetter

Visitors

R.C. Bradley	Indiana University	Feb. - June 91
X. Fernique	University of Strasbourg	mid Feb. - mid May 91
T. Hsing	Texas A&M University	Sept. 90 - May 91
O. Kallenberg	Auburn University	July - Aug 91
T. Lindvall	Chalmers University	Nov. - Dec. 90
V. Mandrekar	Michigan State University	Jan. - July 91
J. Rosinski	University of Tennessee	Jan - Apr. 91
P. Sundar	Louisiana State University	June 91
D. Surgailis	Lithuanian Academy of Science	Sept. - Oct. 91

Graduate Students

D. Baldwin
Y.C. Su
J. Xiong

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Faculty Investigators: S. Cambanis, G. Kallianpur and M.R. Leadbetter
for two summer months

Visitors: R.C. Bradley, X. Fernique, T. Hsing, T. Lindvall, V. Mandrekar
and J. Rosinski

Graduate Students: Y.C. Su and J. Xiong

ARMY RESEARCH OFFICE SUPPORT

Faculty Investigators: S. Cambanis, G. Kallianpur and M.R. Leadbetter
for one summer month

Visitors: O. Kallenberg, P. Sundar and D. Surgailis