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**CONSTITUTIVE MODELLING FOR GRANULAR MATERIAL
UNDER FINITE STRAINS WITH PARTICLE
SLIDINGS AND FABRIC CHANGES**

Final Report on Research
Sponsored by
United States Air Force
Office of Scientific Research
Bolling AFB, Washington, DC 20332-6448

by

Ching S. Chang

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CONSTITUTIVE MODELLING FOR GRANULAR MATERIAL
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Final Report on Research
under Grant No. AFOSR 89-0313

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by

Ching S. Chang

Department of Civil Engineering
University of Massachusetts
Amherst, MA 01003

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<p>The overall objective of this research is to develop a constitutive model for granular material under finite strains with explicit consideration of particle sliding and the consequent fabric change. The specific objective of this research is focused on the development of a stress-strain theory that accounts for the microstructure and the non-uniform strain field of the particle assembly. The developed theoretical model is evaluated by results obtained from computer simulation and experiments of granular material.</p>			
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ABSTRACT

Mechanical behavior of granular media is important in many fields of studies such as soil mechanics, powder mechanics, and ceramic mechanics. The mechanical behavior of granular media has been studied by borrowing the stress-strain models, such as elastic, elasto-plastic, or plastic models, developed for continuum materials. These continuum models consider neither the discrete nature nor the deformation mechanism of granular materials.

A more rational approach should be one that considers the granular system as an assemblage of particles. The stress-strain behavior for a granular material is defined for a representative-volume which consists of a sufficiently large number of particles to be representative of the material. When subjected to loading, the deformation of the granular system results from particle deformation as well as slip between particles.

Along this line, the aim of this research is to investigate the constitutive behavior of granular assemblies using a micro-mechanics approach, taking into account the non-uniform strain within the representative-volume. The nature of this investigation is focused on theoretical development. Formulation of macroscopic constitutive theory, firmly founded in micromechanics of particle interaction, provides an improved understanding and description of granular deformation behavior. The developed theory is evaluated by results obtained from computer simulation and experiments.

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Table 2.1 Summary of the constitutive equations.

CHAPTER 1**SUMMARY OF THE PROJECT****1.1 RESEARCH OBJECTIVES**

The overall objective of this research is to develop a constitutive model for granular materials under finite strains with consideration of particle sliding and the consequent fabric change. The specific objective of this research is focused on the development of a stress-strain theory that accounts for the microstructure and the non-uniform strain field of the particle assembly. The developed theoretical model is evaluated by results obtained from computer simulation and experimental tests.

1.2 ACCOMPLISHMENTS

A general stress-strain theory is developed for granular soil which explicitly accounts for the microstructure and the micro-deformation mechanisms of soil, such as sliding and separation of particles. The theory adopts a statistical mechanics approach in which the geometric structure of the particulate material is characteristically represented by the configurations of a set of micro-elements. Each micro-element is defined as a particle group consisting of a center particle and its neighbor particles. Following the concepts of statistical mechanics, the stress-strain behavior of the material can be interpreted as the average behavior, in a probabilistic sense, of the set of micro-elements.

Most of the research results have been published in Journals and Conference proceedings. A brief summary of the results is given here. The reference number in parenthesis noted in this summary is referring to the

sequential number in the publication list given in section 1.3.3 (i.e., Publications under this grant).

1.2.1 Theoretical development

- a) Continuum concepts of stress and strain were developed for a micro-element in terms of the discrete variables, such as contact forces and particle movements (2, 4, 9).
- b) A new definition of strain was derived from kinematics of particles. Unlike that in classic mechanics, the strain for granular material is asymmetric. The symmetric part of the strain is the usual Cauchy strain. The skew-symmetric part of the strain represents the spin of particles (7, 4, 9).
- c) Constitutive theory was developed considering the asymmetric stress tensor resulted from rolling resistance between particles. The granular material is derived to be a more generalized continuum in which couple stress can transmit. The effects of rolling resistance between particles on the deformation behavior of material are discussed (9, 12, 14).
- d) A general stress-strain theory for random packing of multi-sized granules has been developed. The stress-strain behavior is based on the properties at contacts between particles (3, 9, 16).
- e) The theory has been extended to large strain conditions including the mechanisms of sliding and separation of particles associated with non-uniform strain field (17, 18).

1.2.2 Computer Simulation

- a) The theory has been verified by computer simulation for assemblies of spheres and disks (5, 11, 14), and by experimental results from rod assemblies tested in a direct shear box (2, 3, 7).
- b) The theory has been applied to boundary value problems using an innovative numerical procedure similar to finite element formulation (12, 13, 14).

1.2.3 Experimental Evaluation

- a) The theory has been applied to study the behavior of sand (6, 15, 18) and cemented sand (10) on initial moduli, secant moduli and damping ratio, compared with experimental results for low amplitude loading conditions.
- b) The theory has been studied for the effects of fabric (packing structure) on the class of material symmetry (2, 8, 18).
- c) The extended theory for large strain conditions including the mechanisms of sliding and separation of particles (17, 18) is evaluated by comparing the predicted stress-strain behavior with that measured from experiments on different types of sands under various loading conditions (18).

In summary, the comparisons between the predicted and measured results for small and large strain conditions show that the model has the following capabilities:

- a) The model is capable of predicting macroscopic behavior such as moduli and failure of granular material based on the microscopic contact properties.
- b) The effect of microstructure, such as inherent anisotropy, can be explicitly accounted.

- c) The model accounts for the behavior associated with particle interaction and sliding at contact, thus capable of predicting behavior such as stress-induced anisotropy, path dependency, locked-in stress, plastic flow, dilatancy, friction loss (damping), and non-coaxial behavior under rotation of principal stress.
- d) The model is applicable to materials with cemented inter-particle properties.

1.3 SUMMARY OF THE PROJECT

1.3.1 Grant Information

Grant Number: AFOSR-89-0313
Amount: \$ 162,920
Period: May 1, 1989 - Sept. 30, 1991
Title: Constitutive Modelling for Granular Material under Finite Strains with Particle Slidings and Fabric Changes

1.3.2 Professional Personnel Associated with the Project

Principal Investigator : Ching S. Chang

Research Assistants :

Anil Misra (Ph.D. completed)
Mohammed Kabir (Ph.D. in progress)
Kofi Acheampong (Ph.D. in progress)
Yang Chang (Ph.D. in progress)
Manoj Sharma (Master's degree completed)
Sandeep Puri (Master's degree completed)
Sanjeev Joshi (Master's degree completed)

1.3.3 Publications under This Grant

Published Journal Papers

1. Chang, C. S., Misra, A., and Weeraratne, S. P., "Deformation Behavior of Sand in Cubical and Hollow Cylinder Devices," International Journal for Numerical and Analytical Methods in Geomechanics, John Wiley and Sons, Vol. 13, No. 5, 1989, pp. 493-510.
2. Chang, C. S. and Misra, A., "Theoretical and Experimental Study of Regular Packings of Granules," Journal of the Engineering Mechanics Division, ASCE, Vol. 115, No. 4, April 1989, pp. 704-720.
3. Chang, C. S. Misra, A., and Xue, J. H., "Incremental Stress-Strain Behavior of Packings Made of Multi-sized Particles," International Journal of Solids and Structures, Pergamon Press, Vol. 25, No. 6, 1989, pp. 665-681.
4. Chang, C. S., "Constitutive Modelling of Granular Materials as Generalized Continua with Non-Linear Kinematic Fields," Powders and Grains, Edited by J. Biarez and R. Gourves, A. A. Balkema Publishers, Rotterdam, The Netherlands, 1989, pp. 311-319.
5. Chang, C. S., and Misra, A., "Computer Simulation and Modelling of Mechanical Properties of Particulates," Journal of Computer and Geotechniques, Elsevier Science Publishers, Vol. 7, No. 4, 1989, pp.269-287.
6. Chang, C. S., Sundaram, S. S., and Misra, A., "Initial Moduli of Particulated Mass with Frictional Contacts," International Journal for Numerical and Analytical Methods in Geomechanics, John Wiley and Sons, Vol. 13, No. 6, 1989, pp. 626-641.
7. Chang, C. S., "Strain Tensor and Deformation for Granular Material," Journal of the Engineering Mechanics Division, ASCE, Vol. 116, No.4, 1990, pp. 790-804.
8. Chang, C. S., and Misra, A., "Packing Structure and Mechanical Properties of Granulates," Journal of the Engineering Mechanics Division, ASCE. Vol. 116, No. 5, 1990, pp. 1077-1093.
9. Chang, C. S., and Liao, C., "Constitutive Relations for Particulate Medium with the Effect of Particle Rotation," International Journal of Solids and Structures, Vol. 26, No. 4, 1990, pp. 437-453.
10. Chang, C. S., Misra, A., and Sundaram S. S., "Micromechanical Modelling for Behaviour of Cemented Sand Subjected to Low Amplitude Cyclic Loading," Geotechnique, Vol. 40, No. 2, 1990, pp. 251-263.
11. Chang, C. S. and Misra, A., "Application of Uniform Strain Theory to Heterogeneous Granular Solids," Journal of the Engineering Mechanics Division, ASCE. Vol. 116, No. 10, 1990, pp. 2310-2328.
12. Chang, C. S. and Ma Lun., "Modelling of Discrete Granulates as Micropolar Continuum," Journal of the Engineering Mechanics Division, ASCE. Vol. 116, No. 12, 1990, pp. 2703-2721.

13. Liao, C. L., and Chang, C. S., "Numerical Modelling for Large Deformation of Aggregates of Particles with Meta-Stable Structures," Mathematical and Computer Modelling, Vol. 14, 1990, pp. 921-926.
14. Liao, C. L., and Chang, C. S., "A Micro-Structural Finite Element Model for Granular Solids," Journal of Engineering Computations, 1991, (in print).
15. Chang, C. S., Misra, A., and Sundaram S. S., "Properties of Granular Packings under Low Amplitude Cyclic Loading," International Journal of Soil Dynamics and Earthquake Engineering, Vol. 10, No. 4, 1991, pp. 201-211.
16. Chang, C. S., and Ma, L., "A Micromechanical-Based Micro-polar Theory for Deformation of Granular Solids," International Journal of Solids and Structures, Vol. 28, No. 1, 1991, pp. 67-86.
17. Chang, C. S., Chang, Y., and Kabir, M., "Micromechanics Modelling for the Stress-Strain-Strength Behavior of Granular Material - I. Theory," Journal of the Geotechnical Engineering, ASCE, 1991 (accepted for publication).
18. Chang, C. S., Kabir, M., and Chang, Y., "Micromechanics Modelling for the Stress-Strain-Strength Behavior of Granular Material - II. Evaluation," Journal of the Geotechnical Engineering, ASCE, 1991 (accepted for publication).

Conferences Proceedings

1. Chang, C.S., "Micromechanical Modelling of Constitutive Equation for Granular Material," Micromechanics of Granular Materials, Edited by J. T. Jenkins and M. Satake, Elsevier Science Publishers, 1988, pp. 271-278.
2. Chang, C. S., "Constitutive Modelling of Granular Materials as Generalized Continua with Non-Linear Kinematic Fields," Powders and Grains, Edited by J. Biarez and R. Gourves, A. A. Balkema Publishers, Rotterdam, The Netherlands, 1989, pp. 311-319.
3. Chang, C. S., and Misra, A., "Stress-Strain Behavior of Sands - A Microstructural Approach," Proceedings of the Fourth International Symposium on Interaction of Non-Nuclear Munition with Structures, Panama City Beach, Florida, April 1989, pp. 354-459.
4. Liao, C.L., and Chang, C.S., "Numerical Modelling for Large Deformation of Aggregates of Particles with Meta-Stable Structure", Seventh International Conference on Mathematical and Computer Modelling, Chicago, Illinois, August, 1989, pp. 921-926.
5. Chang, C. S. and Misra, A., "Fabric and Effective Moduli of Frictional Granulates," Proceedings of the Twelfth Canadian

Congress of Applied Mechanics, Ottawa, Canada, May 1989, pp. 532-533

6. Liao, C. L., and Chang, C. S., "A Micro-Structural Finite Element Method for Granular Solids," First U.S. Conference on Discrete Element Methods, Golden, Colorado, October, 1989.
7. Chang, C. S., "Deformation of Granular Materials," Workshop on the Packing and Mechanics of Aggregates of Spheres, Mathematical Sciences Institute, Cornell University, N.Y., June 1989, pp. 22-24.
8. Liao, C. L., and Chang, C. S., "Numerical Modelling for Large Deformation of Aggregates of Particles with Meta-Stable Structures," Proceedings of the Seventh International Conference on Mathematical and Computer Modelling, Chicago, August 1989, pp. 471-477.
9. Chang, C. S., "Constitutive Modelling of Granular Materials as Generalized Continua," Proceedings of the International Conference on Micromechanics of Granular Media, Clermont-Ferrand, France, September 1989.
10. Chang, C. S., and Misra, A., "Stress-Strain Modeling of Heterogeneous Granular Solid Based on Micromechanics," Proceedings of the Third International Conference on Constitutive Law for Engineering Materials: Recent Advances and Industrial and Infrastructure Applications, Tucson, Arizona, Jan. 1991, pp.501-504.
11. Misra, A., and Chang, C. S., "Micro-mechanism of Induced Anisotropy in the Behavior of Granular Material," Proceedings of the Third International Conference on Constitutive Law for Engineering Materials: Recent Advances and Industrial and Infrastructure Applications, Tucson, Arizona, Jan. 1991, pp.531-534.
12. Misra, A., and Chang, C. S., "Mechanical Properties of Cemented Sands Based on Inter-particle Contact Behavior," Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics, March, 1991.

Thesis Completed under this Grant

1. Biswas, G.C., "Continuum Modelling for Discrete Geomaterials", submitted to the University of Massachusetts in partial fulfillment of the requirements of Ph.D. degree, Aug., 1989.
2. Misra, A., "Constitutive Relationships for Granular Solids with Particle slidings and Fabric Changes", submitted to the University of Massachusetts in partial fulfillment of the requirements of Ph.D. degree, Feb., 1991.

3. Sharma, M., "Discrete Element Analysis of Two and Three Dimensional Packings under various Stress Conditions", Master's project, University of Massachusetts, May, 1990.
4. Puri, S. "Engineering Properties of Discrete Granular Media using DEM", Master's project, University of Massachusetts, July, 1990.
5. Joshi, S. "Effects of Inter-Particle Contact on the Stress-Strain Behavior of Granular Soils", Master's project, University of Massachusetts, September, 1990.

CHAPTER 2

THEORETICAL DEVELOPMENT

The intrinsic feature of granular materials is their particulate structure. The micro-scale interaction between particles at contacts controls the macroscopic response. Mechanical behavior of granular material can be analyzed by two approaches which account for the particulate structure and the particle interactions, namely, the computer simulation approach and the micromechanics approach. The computer simulation approach solves for the deformation of an assembly based on the governing equations for the movement of each particle interacting with its surrounding particles. Along this line of approach, methodologies can be found in the work by Serrano and Rodriguez-Ortiz (1973) and Kishino (1988) for quasi-static condition and in the work by Cundall and Strack (1979) for dynamic condition. However, this approach is cumbersome for systems composed of large number of particles. For example, the number of particles is in the order of hundreds of thousands for a cubic inch of sand. Hence it is desirable to represent the discrete system with a more tractable continuum model.

In the micromechanics approach, the deformation behavior of the assembly is described by the continuum concepts of stress and strain. The constitutive relationship for a granular assembly is obtained by considering the material structure and particle interaction. A number of studies have been attempted along this line of approach. For

example, work can be found in Duffy (1959), Duffy and Mindlin (1957), Deresiewicz (1958), and Makhlouf and Stewart (1967) for regular packings, and Digby (1981), Walton (1987), Jenkins (1987), Chang (1988), and Bathurst and Rothenburg (1988) for random packings. However, all the aforementioned models are limited to the assumption that there is no sliding at contacts during deformation, thus only applicable to the conditions of small deformation.

In this study, we endeavor to extend the constitutive relationship for granular systems taking into account the effect of particle separation and sliding under large deformation. One of the major obstacles to account for particle sliding under large deformation is the non-uniform strain field (Chang 1989, Chang and Misra 1990b). In this chapter, we tackle this problem by introducing a distributive law which describes the heterogeneous field of strain.

In what follows, we first briefly review the previous work on granular mechanics. Then we describe the current approach of treating the material at three levels, namely, representative unit, micro-element and inter-particle contact. On this basis, the constitutive laws for each level are derived and the overall stress-strain relationship is described in terms of inter-particle contact behavior. The performance of this model compared with experimental results are discussed in chapters 3 and 4.

2.1 REVIEW OF PREVIOUS WORK ON GRANULAR MECHANICS

We categorize the work on constitutive mechanics of granular material into two areas, namely, 1) mobilized plane concept, and 2) micromechanical concept.

2.1.1 Mobilized plane concept

The concept of mobilized plane was first introduced by Coulomb (1773) to describe the resistance due to internal friction between particles. Based on this concept, Rankine (1857) developed theory of active and passive pressures and Caquot (1934) derived a solution of limiting equilibrium for an ideal granular wedge.

Taylor (1948) and Bishop (1950) differentiated the sliding within granular material from that of two solid blocks by addressing the mechanism of interlocking between particles of granular material. Interlocking of particles restricts the degree of mobilization thus attributes to a large portion of shear strength. As a result of the interlocking, dilatancy occurs with sliding.

Referring to the notion of dilatancy, Newland and Allely (1957) suggested that the relative direction of sliding between two blocks is in general not parallel to the mobilized plane, rather it is inclined at an angle. This mechanism leads to the phenomenon of dilatancy and, subsequently, to the difference between peak and residual stresses. Rowe (1962) postulated an energy ratio criterion to derive the relative direction of sliding between two blocks of particles in a random assembly subjected to a triaxial loading condition. Based on this derivation, the angle between sliding direction and mobilized plane is a

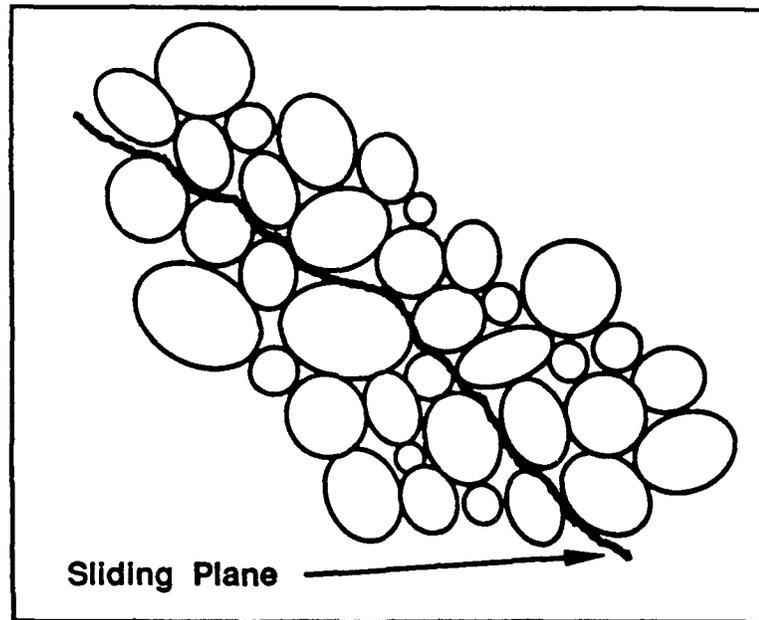
function of applied stress. Horne (1965, 1969) studied the Rowe's energy postulate in a more rigorous way by considering the sliding between pairs of particles in a random assembly. It is shown that the average of these sliding directions can be obtained by maximizing the energy transmission ratio.

Along this line, by accounting not only the directions but also the magnitudes of sliding over mobilized planes, general stress-strain relationship have been derived by many investigators, for example, Matsuoka (1974), de Josselin de Jong (1977), Prat and Bazant (1991) and Chang et al. (1989c).

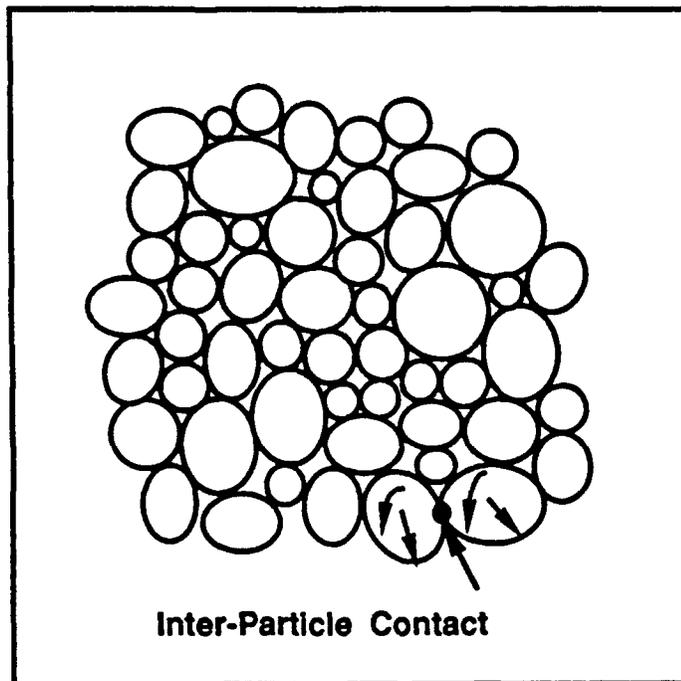
In the mobilized plane concept, it is necessary to describe phenomenologically the averaged behavior of particles on a mobilized plane. The microscopic structure and particle interaction are not directly considered. It is therefore desirable to chose an alternative approach based on micromechanical concept which, at a more fundamental level, considers explicitly the laws of mechanics governing the interaction of particles at contacts. The two concepts are schematically illustrated in Fig. 2.1.

2.1.2 Micromechanical concept

The earliest attempt based on this micromechanics approach was by Duffy and Mindlin (1957). They presented a stress-strain relation derived for a medium composed of a face-centered cubic array of elastic spheres in contact. Following their approach, stress-strain relationship have been developed by Deresiewicz (1958) for simple cubic



A. Mobilized Plane Concept



B. Micromechanical Concept

Fig. 2.1 Two concepts in constitutive mechanics of granular material: a) mobilized plane concept, and b) micromechanical concept.

array, and by Makhlof and Stewart (1967) for cubical-tetrahedral and tetragonal sphenoidal arrays.

Failure of a face-centered packing has been investigated by Thurston and Deresiewicz (1959) and Rennie (1959). Rennie applied this results to the prediction of failure of random arrays, arguing that the random assembly consists of many face-centered miniature arrays in various orientations. Failure initiates when the most unfavorably oriented miniature arrays fail, causing the failure of the entire assembly of the particles. This concept of failure is however contradicted by experimental evidence.

More recently, with the aid of the knowledge gained in mechanics for composite material, the micromechanics approach for granular material has rapidly developed and shown fruitful results. Christoffersen et al. (1981) represented macroscopic stress of an assembly, on the basis of the principle of virtual work, in terms of microscopic contact forces. Stress-strain models for packings of equal sized spheres under axisymmetric loading conditions have been developed by Walton (1987) for isotropic packings and by Jenkins (1987) for idealized anisotropic packings. A general stress-strain theory for random packing of multi-sized granules has been developed by Chang (1988) and Chang et al. (1989a). The theory has been verified by computer simulation of disks (Chang and Misra 1989b) and by experimental results from rod assemblies (Chang and Misra 1989a). The theory has been applied to study the behavior of sand (Chang et al. 1989b) and

cemented sand (Chang et al. 1990), with discussions of the fabric effects (packing structure) on initial moduli (Chang and Misra 1990a).

It is noted that all the aforementioned stress-strain models for randomly packed granular material are limited to small strain conditions without the consideration of particle sliding. In the following, we aim to extend the previous model taking into the consideration of mechanisms of sliding and separation between particles.

2.2 THREE-LEVEL MICROMECHANICAL APPROACH

Granular material can be viewed at three levels, namely, 1) representative unit, 2) micro-element, and 3) contact. A representative unit is defined as an assembly which contains large number of particles to be representative of the granular materials. Each particle in the representative unit is in contact with several neighboring particles. These particles form a particle group, termed as micro-element. A micro-element, corresponding to each particle, is an elementary unit at microscopic level. There are several contacts between the center particle and its neighbors. A contact between a pair of particles is regarded as the basic unit of granular material. A schematic representation of the three levels of granular material is shown in Fig. 2.2.

Figure 2.3 shows the concept of micromechanics approach used in this chapter. In such approach, the constitutive behavior are defined for three levels:

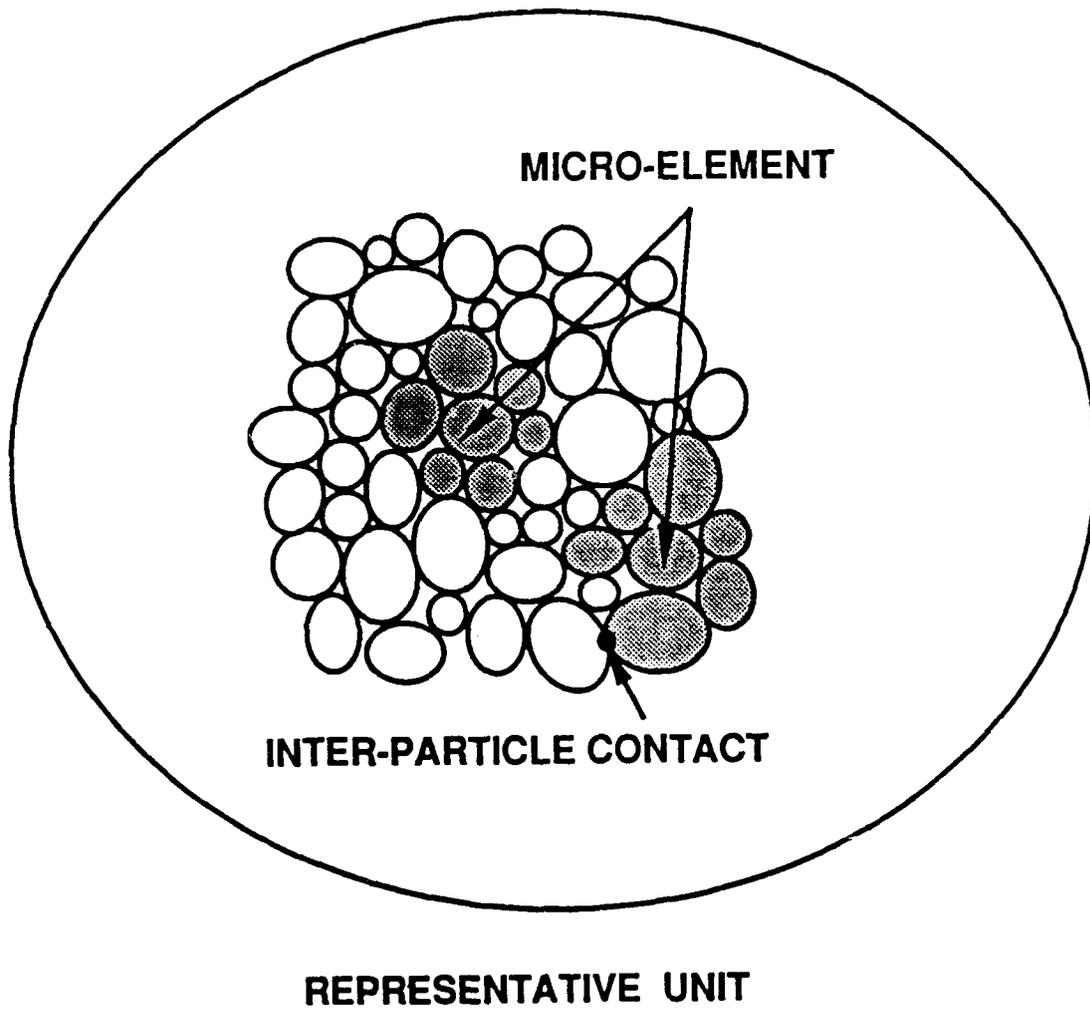


Fig. 2.2 Schematic representation of three levels of granular material.

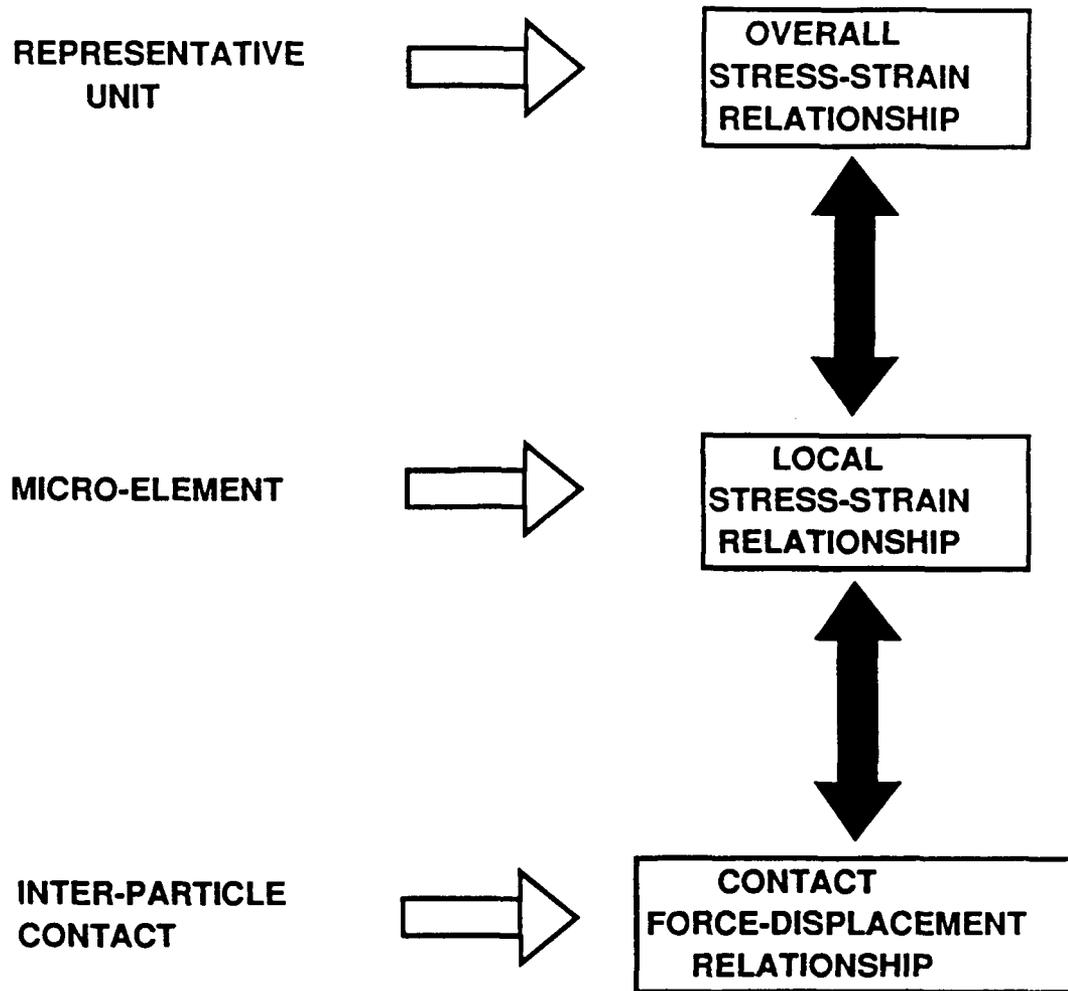


Fig. 2.3 Micromechanics approach for modelling the mechanical behavior of granular material.

1) At contact level, the constitutive law relates contact force and relative movement between two particles. At this level, continuum concept has not yet been introduced. We are dealing with the translation and rotation of discrete particles which result to the micro-scale discontinuity due to mechanisms such as sliding and separation.

2) At micro-element level, the constitutive law relates the stress and strain for the micro-element. The continuum concepts of stress and strain are now introduced. We seek to derive the stress-strain relationship for a micro-element based on contact behavior. To accomplish this objective, it is necessary to establish: a) the relationship between stress and contact forces, and b) the relationship between particle movement and strain.

3) At representative unit level, constitutive law relates the overall stress and overall strain. The behavior of the representative unit is obtained by averaging the behavior of micro-elements utilizing the principle of volume average. A concept of 'distributive law of strain' is introduced to facilitate the averaging process. Therefore, the macro-behavior of an assembly can ultimately be derived from the micro-behavior of a contact.

In what follows, the theoretical development of constitutive behavior are described for the three levels: 1) contact, 2) micro-element, and 3) representative unit. The objective is to derive the constitutive law of a representative unit based on the behavior of inter-particle contact.

2.3 BEHAVIOR AT CONTACT LEVEL

For materials such as granular soil, powder, and ceramic, granules are relatively rigid and the deformation of granules occurs mostly at contacts. Thus the granular material is envisioned to be rigid particles connected by imaginary elastic-plastic springs which allow sliding and separation of particles.

There are two modes of movement for a particle: translation, u_i , and rotation, ω_i . Based on the kinematics of two rigid particles of convex shape, the relative displacement $\Delta\delta_i^{nm}$ and relative rotation $\Delta\theta_i^{nm}$ between particle 'n' and particle 'm' at the contact point 'c', as shown schematically in Fig. 2.4, are given by

$$\Delta\delta_i^{nm} = \Delta u_i^m - \Delta u_i^n + e_{ijk} (\Delta\omega_j^m r_k^m - \Delta\omega_j^n r_k^n) \quad (2.1)$$

$$\Delta\theta_i^{nm} = \Delta\omega_i^m - \Delta\omega_i^n \quad (2.2)$$

where the quantity e_{ijk} = the permutation symbols used in tensor representation for cross product of vectors.

The relative angular rotation, $\Delta\theta_j^{nm}$, is related to the contact couple. The rolling and torsional resistance at contact are negligible for elastic particles of convex shape, but significant for angular particles. For simplicity, we limit our discussion to particles with convex shape and neglect the effect of contact couples.

The relative displacement representing spring-stretch at the contact point 'c', $\Delta\delta_j^{nm}$, is related to the contact force, Δf_i^{nm} , by a general expression as

$$\Delta f_i^{nm} = K_{ij}^{nm} \Delta\delta_j^{nm} - \hat{\Delta f}_i^{nm} \quad (2.3)$$

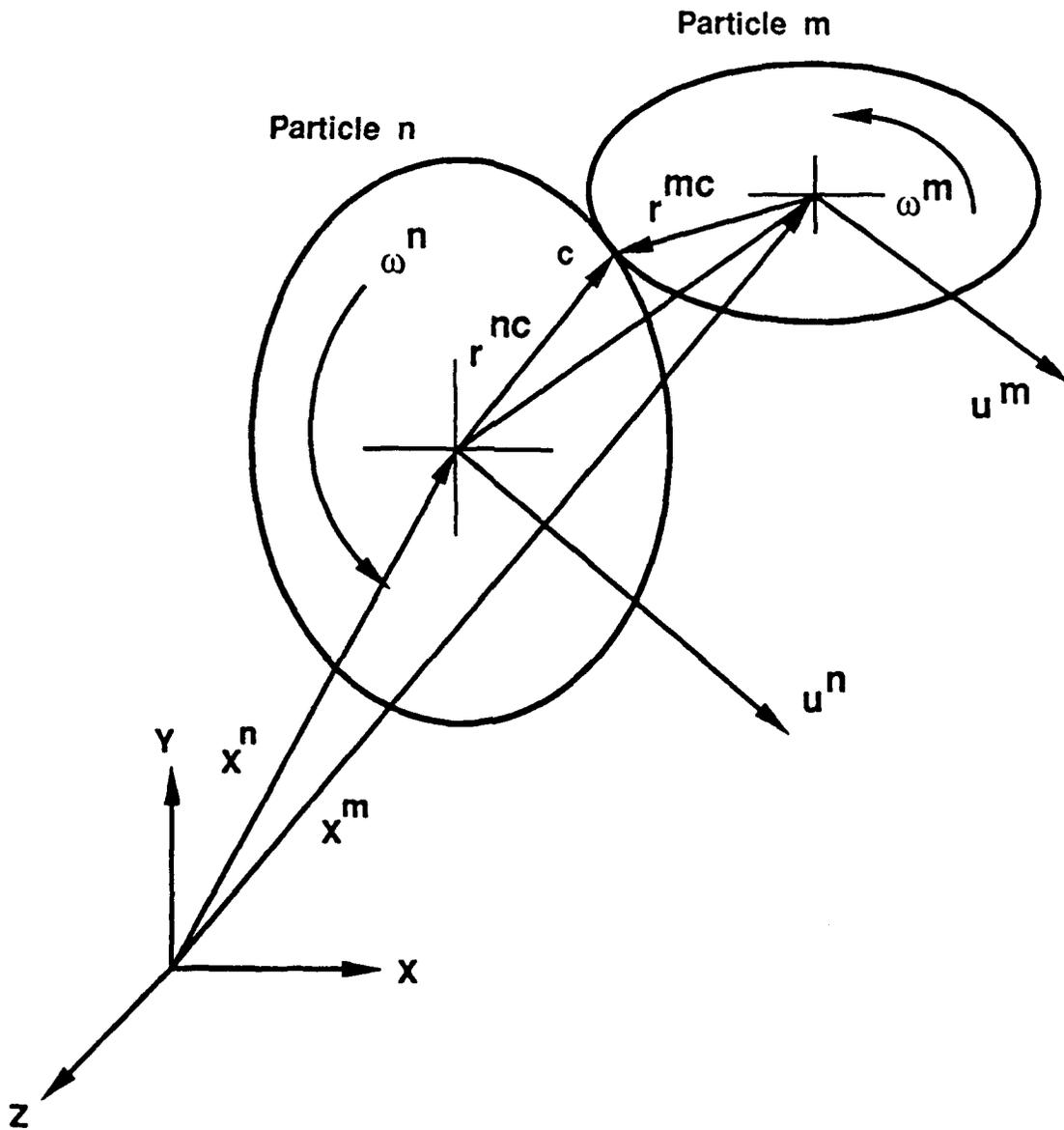


Fig. 2.4 Kinematics of two rigid particles of convex shape.

where the residual force $\Delta \hat{f}_i^{nm}$ and the contact stiffness tensor K_{ij}^{nm} are described below.

2.3.1 Residual Contact Force

During a small strain increment, the material is treated to be linear governed by the instantaneous stiffness tensor K_{ij}^{nm} . As a result of this linearization, the resultant contact force due to the strain increment, may exceed the contact strength. The effects are accounted by the residual force $\Delta \hat{f}_i$. In the case of sliding, the residual shear force, $\Delta \hat{f}_r$, is the amount of contact shear force in excess of the contact shear strength. In the case of particle separation, the residual normal force, $\Delta \hat{f}_n$, is the amount of contact normal force in excess of the contact tensile strength.

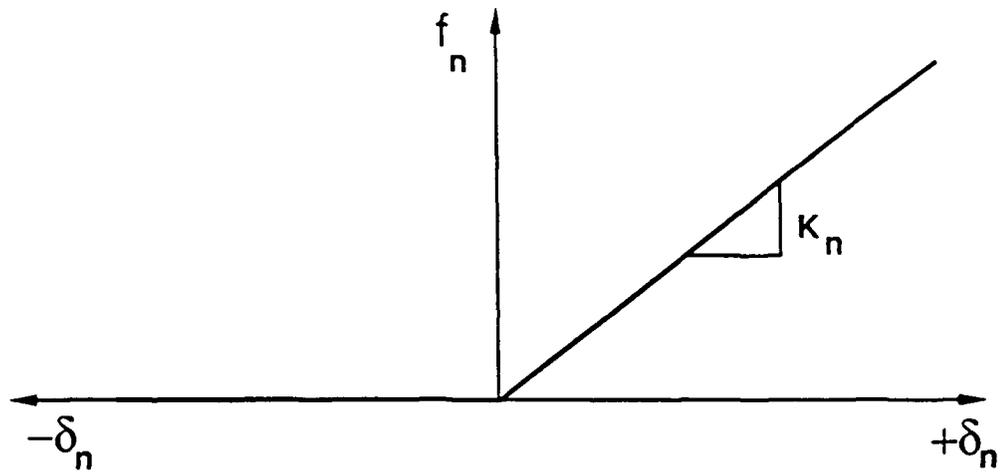
The force vector $\Delta \hat{f}_i$ for this contact is expressed as follows:

$$\Delta \hat{f}_i = \Delta \hat{f}_r \mathbf{r}_i + \Delta \hat{f}_n \mathbf{n}_i \quad (2.4)$$

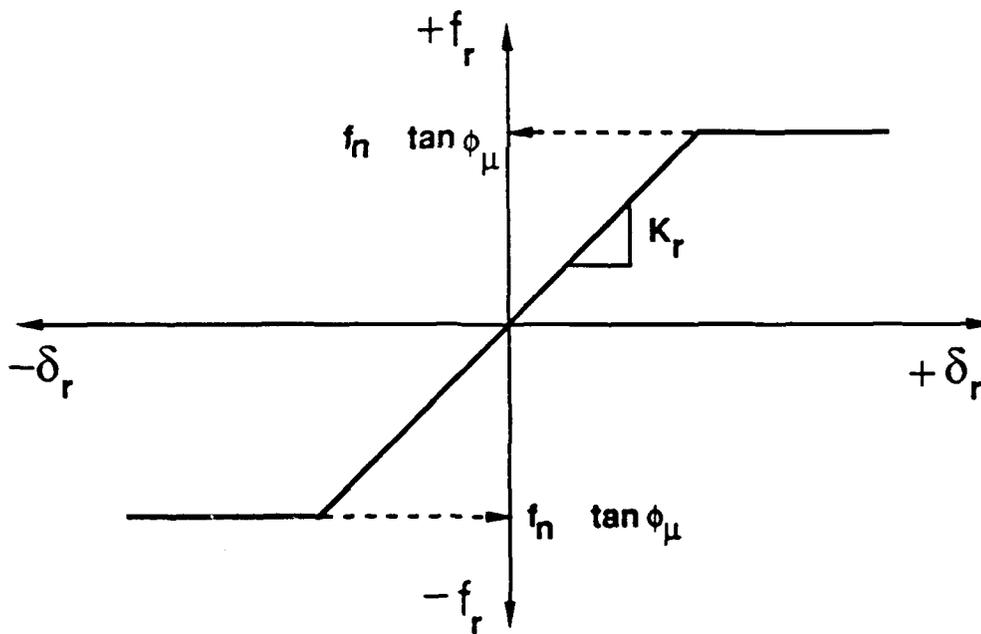
where \mathbf{r}_i is the unit vector in the direction of resultant shear on the contact plane. The vector \mathbf{n}_i is normal to the contact plane.

2.3.2 Contact Stiffness Tensor

It is assumed that the tangential and compressional stiffness are decoupled. The relationship between the resultant shear force f_r and the resultant shear displacement δ_r on the contact plane is $f_r = k_r \delta_r$. When the contact force exceeds the frictional strength, sliding occurs and k_r vanishes as illustrated in Fig. 2.5. The relationship between the contact force f_n and the relative displacement δ_n in the normal



A. Normal



B. Shear

Fig. 2.5 Force and displacement relationships in the directions of: a) normal, and b) shear on a plane of contact between two particles.

direction is $f_n = k_n \delta_n$. When the contact force tends to be in tension, particle separation occurs and k_n vanishes. The stiffness tensor at the contact between particles 'n' and 'm', K_{ij}^{nm} , takes the form

$$K_{ij}^{nm} = k_n n_i n_j + k_r (s_i s_j + t_i t_j) \quad (2.5)$$

where \mathbf{n} , \mathbf{s} and \mathbf{t} are the basic unit vectors of the local coordinate system at each contact as shown in Fig. 2.6. The vector \mathbf{n} is normal to the contact plane. The other two orthogonal vectors \mathbf{s} and \mathbf{t} are on the contact plane.

A frequently used formulation for contact stiffness between two elastic spheres is the Hertz theory (Johnson 1985). Considering the contact area to be circular with a parabolic pressure distribution, the deformation at the contact is obtained from elasticity solutions for pressure loads on semi-infinite media. This leads to the expression of normal stiffness as follows;

$$k_n = C_1 r E_\mu \left(\frac{f_n}{r^2 E_\mu} \right)^\alpha \quad (2.6)$$

where C_1 is a constant equal to $3^{1/3}$, r is the radius of particles, $\alpha = 1/3$, $E_\mu = G_\mu / (1 - \nu_\mu)$, G_μ and ν_μ are the particle shear modulus and Poisson's ratio respectively.

For a contact of two topographically smooth spheres, the tangential stiffness under oscillating contact force was studied by Mindlin and Deresiewicz (1953). The tangential force at the contact results in deformation caused by the development of slip over a part of the contact surface. Clearly, when the tangential force exceeds the frictional strength at the contact, sliding takes place. A general expression for

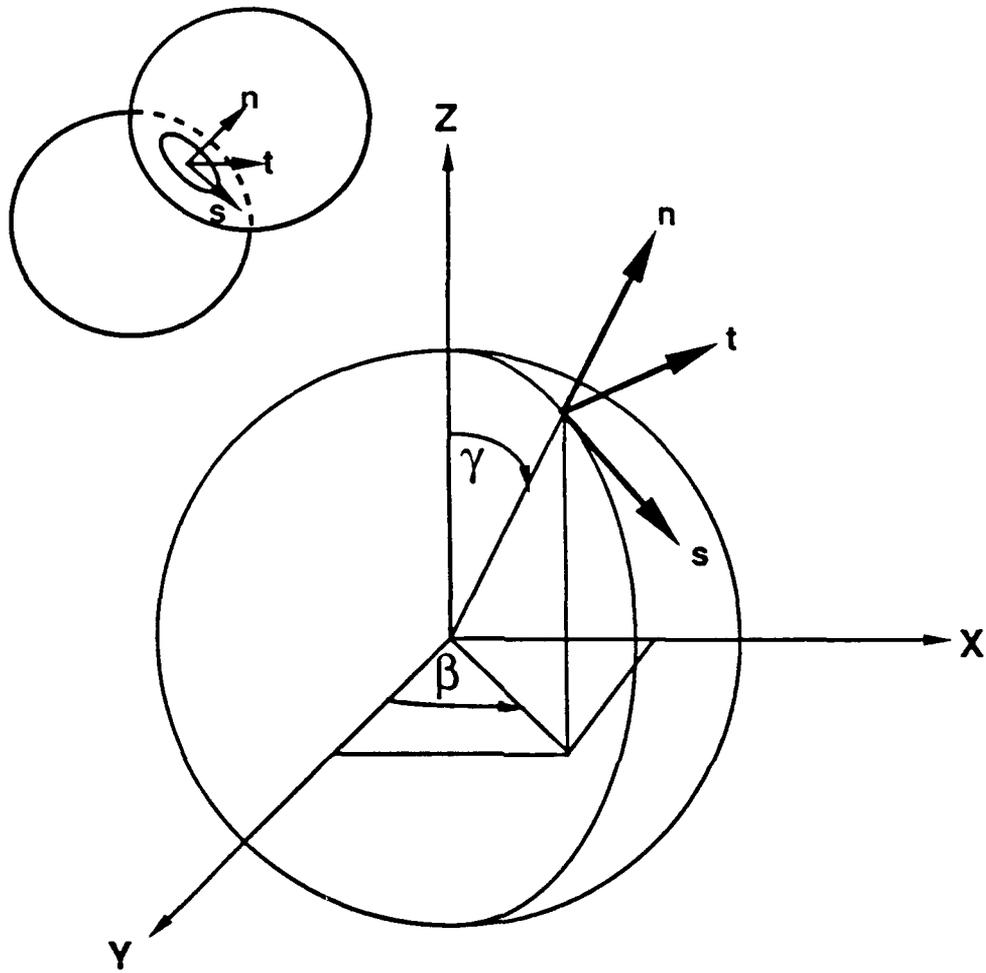


Fig. 2.6 Local co-ordinate system at an inter-particle contact.

the tangential stiffness of two particles can be written as a function of the contact force and the particle properties as follows:

$$K_r = C_2 K_n \left(1 - \frac{f_r}{f_n \tan \phi_\mu}\right)^{1/3} \quad (2.7)$$

where

$$C_2 = \frac{2(1-\nu_\mu)}{2-\nu_\mu} \quad (2.8)$$

ϕ_μ is the friction angle between two particles and f_r is the resultant shear force at the contact.

2.4 BEHAVIOR AT MICRO-ELEMENT LEVEL

Since the assembly consists of a large number of particles, it is expedient to treat translation and rotation of discrete particles as continuum fields. On this basis, the discrete system can be transformed into an equivalent continuum and its behavior can be described by the continuum concepts of stress and strain.

The strain and stress, for a micro-element, are defined in connection with the relative movement of the particles and the resulting contact forces, respectively. The stress-strain relationship is then derived based on contact behavior.

2.4.1 Strain

At the micro-element level, movement of particles can be determined by displacement gradient, $\Delta u_{i,j}^n$, with the usual affine (homogeneous deformation) assumption. Similarly, rotation of each particle can be determined by the gradient of particle rotation. However, with the

absence of couple stress of the micro-element, the gradient of particle rotation can be neglected. Therefore, particles within this group are assumed to have the same rotation, $\Delta\bar{\omega}_i^{-n}$.

It has been shown (Chang 1989, Chang and Ma 1990) that, for granular material, it is more precise to define an asymmetric deformation strain in terms of the continuum variables $u_{i,j}^n$ and $\Delta\bar{\omega}^{-n}$ in the following manner:

$$\Delta\epsilon_{ij}^n = \Delta u_{i,j}^n + e_{ijk} \Delta\bar{\omega}_k^{-n} \quad (2.9)$$

The symmetrical part of $\Delta\epsilon_{ij}^n$ is equal to the symmetrical part of displacement gradient, i.e.,

$$\Delta\epsilon_{(ij)}^n = \Delta u_{(i,j)}^n = \frac{1}{2} (\Delta u_{i,j}^n + \Delta u_{j,i}^n) \quad (2.10)$$

representing the usual symmetric Cauchy strain of the micro-element.

The skew symmetric part of $\Delta\epsilon_{ij}^n$ is given by

$$\Delta\epsilon_{[ij]}^n = \Delta u_{[i,j]}^n - e_{jik} \Delta\bar{\omega}_k^{-n} \quad (2.11)$$

where $\Delta u_{[i,j]}^n = (\Delta u_{i,j}^n - \Delta u_{j,i}^n)/2$ is the rigid body rotation tensor. The skew symmetric part of $\Delta\epsilon_{ij}^n$ thus represents the net spin of particles (i.e., the difference between rigid body rotation of the micro-element and the average rotation of particles).

The strain defined in this way not only is theoretically more generic but also furnishes a convenient kinematic relationship which relates strain to the spring stretch, $\Delta\delta_i^{nm}$, at the contact between the center particle 'n' and its neighbor particle 'm', given by

$$\Delta\delta_i^{nm} = \Delta\epsilon_{ij}^n L_j^{nm} \quad (2.12)$$

where, L_j^{nm} is the branch vector joining the centroids of particle 'n' and particle 'm'. Note that $L_j^{nm} = r_j^{nc} - r_j^{mc}$, and r_j^{nc} = the vector

measured from the centroid of the particle 'n' to the contact point 'c', as illustrated in Fig. 2.4.

It is noted that Eq. 2.12 is based on a kinematic assumption of affinity (i.e., linear displacement field). However, the displacement field is in general non-linear. By forcing particles to move in accordance with a linear displacement field, we impose a constraint to the system. As a result, this assumption leads to a stiffer system. The effect is particularly pronounced at large strain level when significant amount of sliding have taken place (Chang and Misra 1990b).

In this chapter, we derive a non-affine kinematic relationship so that it releases the restriction due to linear displacement field assumption. This is done by allowing the center particle to move freely to achieve equilibrium of contact forces while the surrounding particles are fixed. Thus, in addition to the movement in accordance with the linear displacement field, an extra movement, is expected for the center particle. This movement results in an additional stretch, $\Delta\delta_i^*$, at all the contacts of the center particle. Thus the total spring stretch between particles 'n' and 'm' becomes:

$$\Delta\delta_i^{nm} = \Delta\epsilon_{ij}^n L_j^{nm} + \Delta\delta_i^* \quad (2.13)$$

Since the center particle moves to satisfy the equilibrium of contact forces, the extra stretch $\Delta\delta_i^*$ can be obtained from the force equilibrium condition, given by

$$\Delta\delta_i^* = T_{ijm}^n \Delta\epsilon_{jm}^n ; \quad T_{ijm}^n = - \left[\sum_m K_{iq}^{nm} \right]^{-1} \left[\sum_m K_{qj}^{nm} L_m^{nm} \right] \quad (2.14)$$

Eq. 2.14 shows that the magnitude of $\Delta\delta_i^*$ is a function of particle arrangement and contact stiffness.

Substituting this equation into Eq. 2.13, the stretch of spring at contact relates to the strain of the micro-element by

$$\Delta \delta_i^{nm} = (\delta_{ij} L_m^{nm} - T_{ijm}^n) \Delta \epsilon_{jm}^n \quad (2.15)$$

When the displacement field is linear, T_{ijm}^n vanishes and Eq. 2.15 reduces to Eq. 2.12.

2.4.2 Stress

The relationship between the contact forces and the stress of the micro-element can be defined by employing the theorem of stress mean (Chang and Liao 1990). The local stress $\Delta \sigma_{ij}^n$ is expressed as the volume average of the dyadic product of contact force Δf_j^{nm} and branch vector L_i^{nm}

$$\Delta \sigma_{ij}^n = \frac{1}{2V^n} \sum_m L_i^{nm} \Delta f_j^{nm} \quad (2.16)$$

where V^n is the volume associated with the n-th micro-element.

Summation of the volume V^n over all micro-elements is equal to the total volume of the representative unit, such that $V = \sum_n V^n$.

2.4.3 Local constitutive law

Constitutive equation can thus be established to relate the local strain, $\Delta \epsilon_{ij}^n$, and the local stress, $\Delta \sigma_{ij}^n$, based on the following relationships: (1) stress versus contact forces (Eq. 2.16), (2) force versus spring-stretch at contact (Eq. 2.3), and (3) strain versus spring-stretch at contact (Eq. 2.15). The constitutive equation for the micro-element can thus be obtained in the following form:

$$\Delta \sigma_{ij}^n = C_{ijkl}^n \Delta \epsilon_{kl}^n + \hat{\Delta \sigma}_{ij}^n \quad (2.17)$$

where C_{ijkl}^n is the local stiffness tensor for the n-th micro-element given by

$$C_{ijkl}^n = \frac{1}{2V^n} \sum_m L_i^{nm} K_{jp}^{nm} (\delta_{pk} L_l^{nm} - T_{pkl}^n) \quad (2.18)$$

The instantaneous local stiffness tensor derived in Eq. 2.18 is a function of contact stiffness and the position of particles. The constitutive tensor has the following properties of symmetry:

$$C_{ijkl}^n = C_{jikl}^n = C_{jilk}^n = C_{ikjl}^n.$$

The stress $\Delta \hat{\sigma}_{ij}^n$ in Eq. 2.17 for the n-th micro-element, resulting from the residual forces due to sliding and separation, is given as follows;

$$\Delta \hat{\sigma}_{ij}^n = \frac{1}{2V^n} \sum_m L_i^{nm} \Delta \hat{f}_i^{nm} \quad (2.19)$$

2.5 BEHAVIOR AT REPRESENTATIVE UNIT LEVEL

In what follows, we consider a given volume to be representative of the granular solid such that the boundaries of the said volume are subject to displacements compatible with a uniform overall strain. Under such conditions, Hill (1967) has shown that, for heterogeneous material, the overall stress and strain can be expressed as the volume averages of their corresponding quantities at local level.

Thus the overall stress and strain, denoted by $\Delta \sigma_{ij}$ and $\Delta \epsilon_{ij}$, are regarded as volume averages of the local stress and local strain at the micro-element level, such that

$$\Delta \sigma_{ij} = \frac{1}{V} \sum_n V^n \Delta \sigma_{ij}^n \quad (2.20)$$

$$\Delta \epsilon_{ij} = \frac{1}{V} \sum_n V^n \Delta \epsilon_{ij}^n \quad (2.21)$$

Corresponding to the overall stress and strain, it is reasonable to define an overall stiffness tensor for the representative unit such that the overall stress-strain relationship can be expressed in the following form:

$$\Delta\sigma_{ij} = C_{ijkl} \Delta\epsilon_{kl} + \Delta\hat{\sigma}_{ij} \quad (2.22)$$

where the residual stress is the volume average of its local quantities, given by

$$\Delta\hat{\sigma}_{ij} = \frac{1}{V} \sum_n V^n \Delta\hat{\sigma}_{ij}^n \quad (2.23)$$

It is noted that the stiffness tensor C_{ijkl} is not the volume averaged stiffness tensor, \bar{C}_{ijkl} , defined by

$$\bar{C}_{ijkl} = \frac{1}{V} \sum_n V^n C_{ijkl}^n \quad (2.24)$$

To derive the overall stiffness tensor C_{ijkl} , it is essential to account for the nature of the randomly distributed local stiffness C_{ijkl}^n and local strain $\Delta\epsilon_{ij}^n$ of the micro-elements in a heterogeneous granular system. In order to delineate explicitly the distribution of local strains, we introduce a strain fraction by multiplying $\frac{V^n}{V}$ to the local strain, given by

$$\Delta e_{ij}^n = \Delta\epsilon_{ij}^n \frac{V^n}{V} \quad (2.25)$$

such that the volume average of strain, i.e., Eq. 2.21, can be rewritten as

$$\sum_n \Delta e_{ij}^n = \Delta\epsilon_{ij} \quad (2.26)$$

This means that the overall strain $\Delta\epsilon_{ij}$ is divided into fractions Δe_{ij}^n and distributed to each micro-element. The sum of all fractions is equal to the overall strain.

To facilitate a rational averaging process, we introduce a 'distributive law of strain' which relates the overall strain $\Delta \epsilon_{ij}$ to the strain fraction of a micro-element $\Delta \epsilon_{ij}^n$, given by,

$$\Delta \epsilon_{ij}^n = G_{ijkl}^n \Delta \epsilon_{kl} \quad (2.27)$$

where the dimensionless tensor G_{ijkl}^n , associated with each micro-element, describes indirectly the degree of heterogeneity of the material. From Eq. 2.27, the following condition must be satisfied:

$$\sum_n G_{ijkl}^n = I_{ijkl}; \quad (2.28)$$

where I_{ijkl} is a fourth rank identity tensor defined in terms of Kronecker delta δ_{ij} as

$$I_{ijkl} = \frac{1}{2} (\delta_{ik} \delta_{jl} + \delta_{jl} \delta_{ik}) \quad (2.29)$$

Thus, mathematically, G_{ijkl}^n is a probability density/distribution function in tensorial form.

For homogeneous material, the strain is equally distributed to each micro-element. Therefore the distribution tensor G_{ijkl}^n has the following form:

$$G_{ijkl}^n = I_{ijkl} \frac{V^n}{\bar{V}} \quad (2.30)$$

With the distributive law of strain (Eq. 2.27), from volume average of Eq. 2.17, and using Eqs. 2.20 and 2.25, the stiffness tensor C_{ijkl} can be written as a 'weighted' volume average of micro-element stiffness tensor C_{ijkl}^n , given by

$$C_{ijkl} = \sum_n C_{ijmn}^n G_{mnkl}^n \quad (2.31)$$

Since the representative unit consists of large number of randomly arranged particles, the heterogeneous system can be viewed as a

statistical homogeneous system. The strain distribution has no correlation with the geometric location and the distribution tensor G_{ijkl}^n should not be defined in geometric space. Instead, strain distribution is largely depending on the distribution of stiffness C_{ijkl}^n , indicating that the distribution tensor should be a function defined in stiffness space.

It is thus reasonable to establish the distribution tensor in terms of the variance of the micro-element stiffness C_{ijkl}^n with respect to the average stiffness of the packing \bar{C}_{ijkl} , defined as a dimensionless tensor v_{ijkl}^n given by

$$v_{ijkl}^n = C_{ijmn}^{-1} (C_{mnkl}^n - \bar{C}_{mnkl}) \quad (2.32)$$

The distribution tensor can be related to v_{ijkl}^n in many forms. It is found that a simple and suitable form for the distribution tensor is a generalized tensor form of Maxwellian type, i.e.,

$$G_{ijkl}^n = A_{ijmn} \text{Exp}(-v_{mnkl}^n) \frac{v^n}{V} \quad (2.33)$$

The constant tensor A_{ijkl} involved in the distribution function must satisfy the condition of identity (i.e., Eq. 2.28). Thus it can be obtained by

$$A_{ijkl}^{-1} = \frac{1}{V} \sum_n \text{Exp}(-v_{ijkl}^n) v^n \quad (2.34)$$

where the exponential term can be expanded into a series as follows:

$$\text{Exp}(-v_{mnkl}^n) = I_{mnkl} - v_{mnkl}^n + \frac{1}{2!} v_{mnpq}^n v_{pqkl}^n - \dots \quad (2.35)$$

The distributive tensor can also be obtained from a self-consistent method in which the micro-element is considered to be an inhomogeneity with constitutive tensor C_{ijkl}^n embedded in an infinite medium of constitutive tensor C_{ijkl} , similar to that used in the study of

polycrystalline behavior (Hutchingson, 1970, Nemat-Nasser and Mahrabadi, 1984). The usual self-consistent method can not be directly applied to granular materials because particles in granular media are not directly in analogy to the crystals in polycrystalline. In contrast to polycrystalline where deformation occurs primarily in crystals, the deformation of granular material principally occurs at contacts between particles. In addition, the fact that granular material has an asymmetric strain tensor adds complexity to the mathematical process in the self-consistent method. (Misra, 1990).

2.6 SUMMARY

Perceiving granular material as a collection of particles, a constitutive law for granular material is derived based on micromechanics approach, taking into account the mechanisms of sliding and separation of particles. For simplicity, the discussion of the present theory is restricted to the conditions of particles with convex shape.

Summary of the necessary equations for defining the constitutive tensor is given in Table. 2.1. Given also in Table 2.1 are the required parameters such as inter-particle contact properties and initial structure of micro-elements. In the present theory, structure of the granular material is intended to be represented statistically by a set of micro-elements where each micro-element is defined by a specific arrangement of particle group. This chapter is focused on the micromechanics frame work for the constitutive law of granular material.

CONSTITUTIVE LAW : $\Delta \sigma_{ij} = C_{ijkl} \Delta \varepsilon_{kl} + \Delta \hat{\sigma}_{ij}$

$$C_{ijkl} = \frac{1}{V} \sum_n C_{ijmn}^n H_{mnkl}^n V^n$$

$$\Delta \hat{\sigma}_{ij} = \frac{1}{V} \sum_n \Delta \hat{\sigma}_{ij}^n V^n$$

$$C_{ijmn}^n = \frac{1}{2V^n} \sum_c L_i^c K_{jp}^c (\delta_{pm} L_n^c - T_{pmn}^c)$$

$$H_{mnkl}^n = f(C_{ijmn}^n, \bar{C}_{ijmn})$$

$$\Delta \hat{\sigma}_{ij}^n = \frac{1}{2V^n} \sum_c L_i^c \Delta \hat{f}_j$$

$$\bar{C}_{ijmn} = \frac{1}{V} \sum_n C_{ijmn}^n V^n$$

$$T_{pmn}^c = \left[\sum_c K_{pq}^c \right]^{-1} [K_{qm}^c L_n^c]$$

MODEL REQUIREMENTS :

Representative Unit : No. of Micro-Elements, Volume of Representative Unit

Each Micro-Element : No. of Contacts, Volume of Micro-Element

Each Contact : Branch Vector, Contact Stiffness

Table 2.1 : Summary of the constitutive equations.

with the assumption that we already have the information regarding the representative set of micro-elements.

It is noted that the constitutive law is in an incremental form. At end of each stress increment, the initial structure of these micro-elements are updated. Thus, the non-linearity due to evolution of microstructure is incorporated.

The unique features of the present theory are listed as follows:

- 1) It explicitly accounts for the effects of micro-structure, thus is capable of modelling inherent anisotropy of material structure.
- 2) It accounts for mechanical properties at granular contact with the consideration of mechanism of sliding and separation between particles, thus is capable of modelling strength, residual strength, behavior of yielding, and behavior of plastic flow of the material.
- 3) It accounts for the evolution of micro-structure, thus is capable of modelling strain dependency, path dependency, stress-induced anisotropy, and behavior related to the memory of the material.

Therefore the present theory, in a more fundamental way, models many aspects of material behavior. This constitutive theory is potentially useful for comprehensive modelling of complex behavior for granular material.

CHAPTER 3

PREDICTIONS COMPARED WITH COMPUTER SIMULATION

The present micromechanics model is illustrated to predict the stress-strain behavior of a random packing of planar disks. To evaluate the applicability of the present model, the predicted results are compared with that obtained from computer simulation for the same random packing.

A periodic packing is used in this example. The representative unit of the packing shown in Fig. 3.1 is made up of 25 particles, 12 particles of radius 0.21 mm and 13 particles of radius 0.105 mm. As already noted, each particle in the representative unit is in contact with several neighboring particles. These particles form a particle group known as micro-element. A schematic representation of the 25 micro-elements corresponding to each particle is shown in Fig. 3.2. Each micro-element consists of a reference (or central) particle in contact with its neighbors. Microstructure of the random packing shown in Fig. 3.1 is represented by the 25 micro-elements.

In the stress-strain prediction, the contact normal stiffness is taken to be $k_n = 200$ kN/m. The range of the stiffness ratio, k_s/k_n , is from 0.3 to 1, and the range of inter-particle friction angle ϕ_u is 14 to 25 degrees. This range of inter-particle friction angles is comparable to the measured values reported for dry spheres manufactured from glass or steel materials (Skinner, 1969).

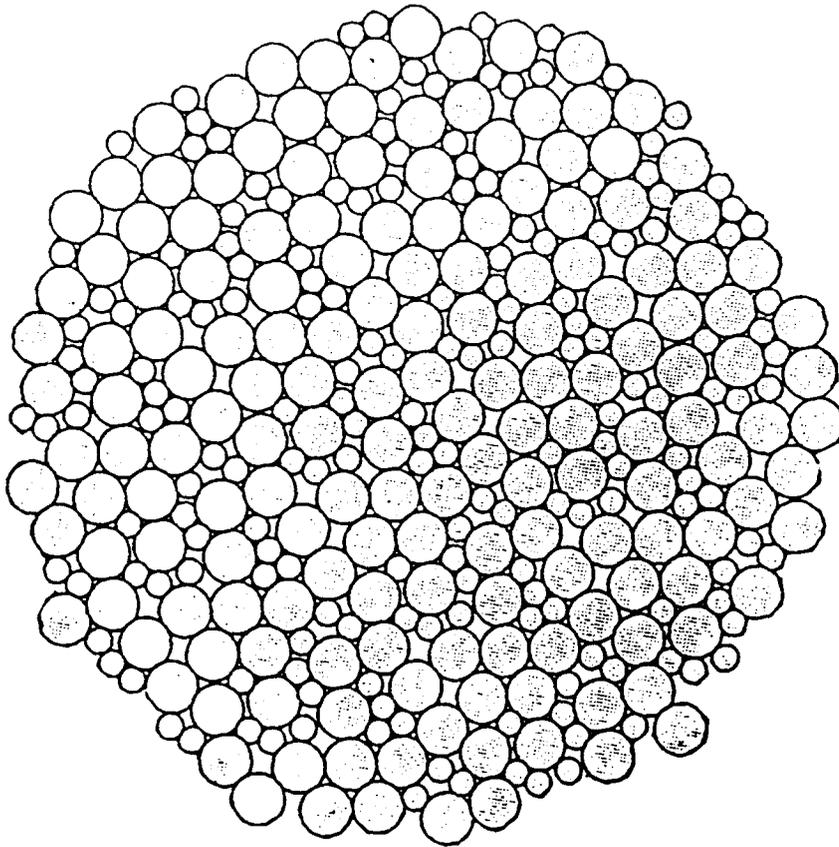


Fig. 3.1 Representative unit of periodic packing used in the examples.

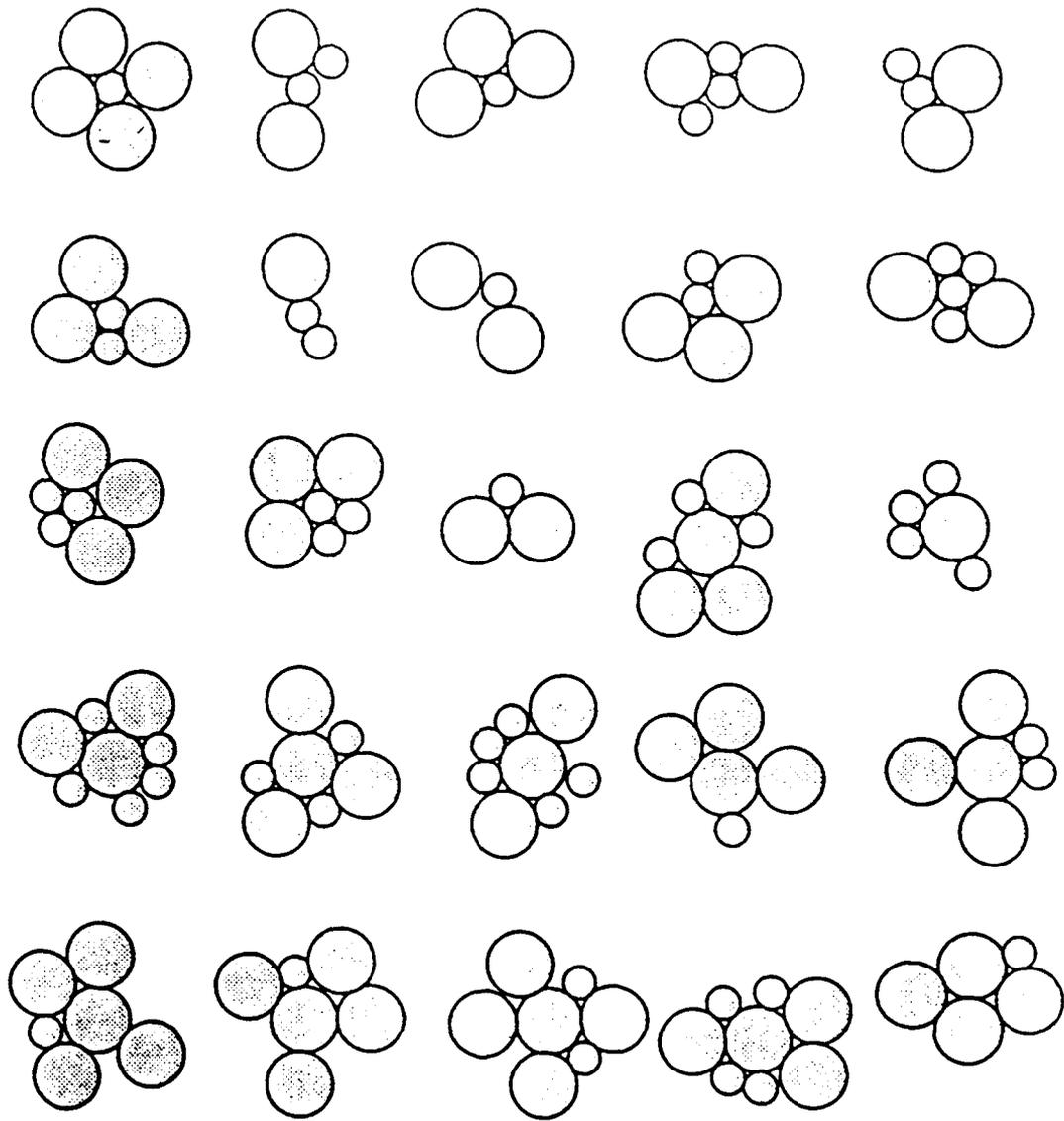


Fig. 3.2 Schematic representation of 25 micro-elements.

3.1 ONE DIMENSIONAL TEST

The stress-strain behavior of the packing is predicted under two loading conditions: one dimensional and biaxial compression. In the one-dimensional compression test, the packing was first isotropically compressed to $\sigma_0 = 203 \text{ kPa}$ (2.07 kg/cm^2). Then, the horizontal strain is held constant, i.e., $\Delta\varepsilon_{xx} = 0$, while the sample is strained in the vertical direction. The predicted results from the present self-consistent micromechanics model are compared with that obtained from the computer simulation method (TRUBALL) developed by Cundall and Strack (1979 a,b,c).

Fig. 3.3 shows the value of stress ratio K_0 , i.e. σ_h / σ_v , versus vertical strain for various values of ϕ_μ . From the state of isotropic compression, K_0 decreases from 1 sharply to a value and stays nearly constant with increasing vertical strain. For both self-consistent and computer simulation methods, the minimum K_0 value generally increases with decreasing ϕ_μ . In computer simulation, K_0 varies from 0.27 to 0.16 and for the micromechanics model, it varies from 0.22 to 0.14 for $\phi_\mu = 14^\circ$ and 25° respectively. Results of the two methods show reasonable agreement.

3.2 BIAXIAL COMPRESSION TEST

In the biaxial loading condition, the packing was first isotropically compressed to $\sigma_0 = 203 \text{ kPa}$ (2.07 kg/cm^2). Then, the horizontal stress is held constant while a compressive strain is applied in the vertical direction. The results of the prediction are plotted in

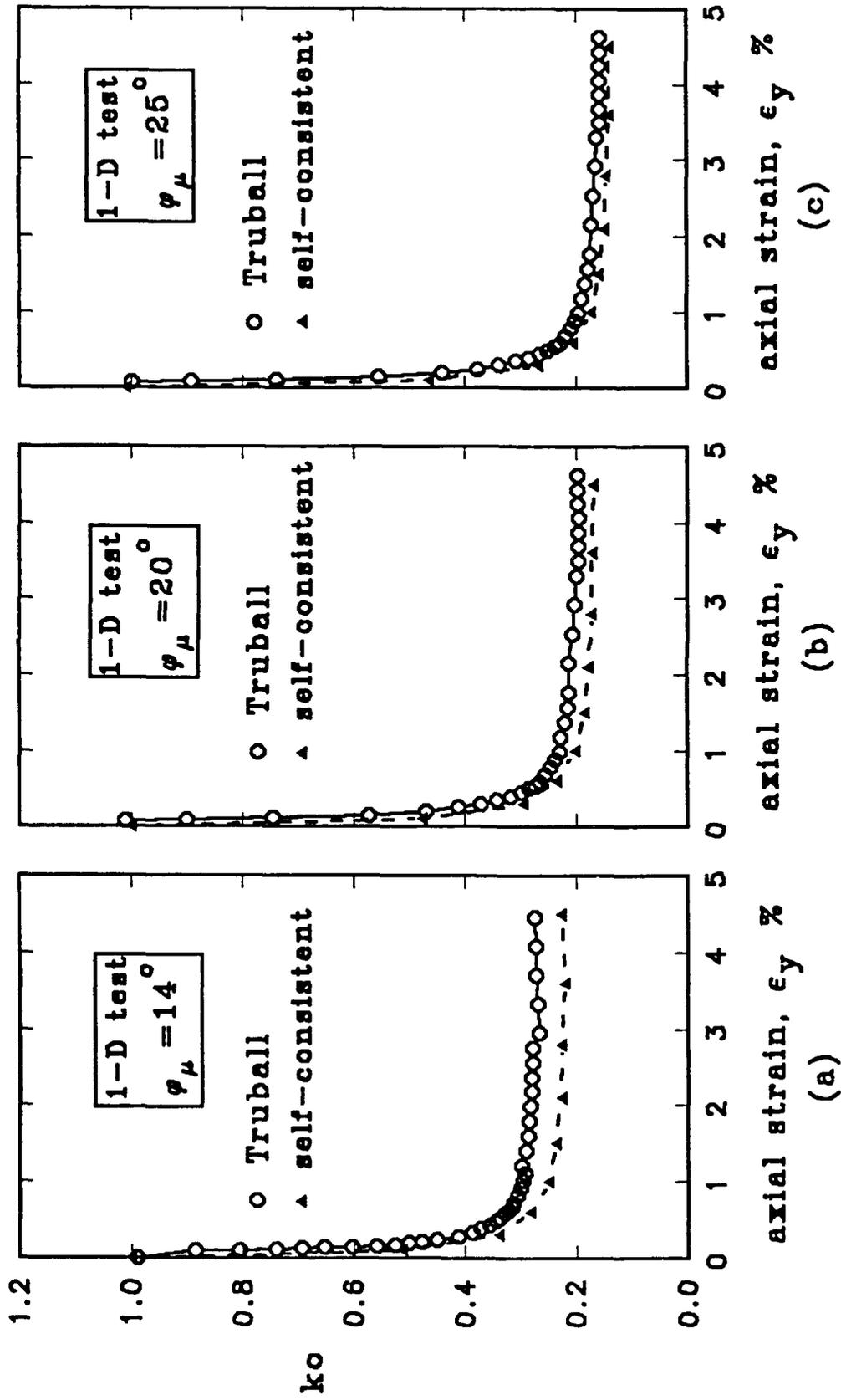


Fig. 3.3 K_0 versus vertical strain in one dimensional compression test predicted by computer simulation and present model.

term of stress ratio, q/p , defined as follows: $q=(\sigma_v - \sigma_h)/2$, and $p=(\sigma_v + \sigma_h)/2$. The curves of stress ratio q/p and volumetric strain versus vertical strain are shown in Fig. 3.4 for $\phi_\mu = 25^\circ$ and $k_s/k_n = 1$. Figure 3.5 plots the stress-strain curves for $k_s/k_n = 0.3$ and $\phi_\mu = 25^\circ$. Compared with Fig. 3.4, the effect of k_s on strength is insignificant.

The predicted stress-strain curves from the present micromechanics model show reasonable agreement with that obtained from computer simulation method. The discrepancies are caused by the assumption in the micromechanics model that the micro-element is embedded in an equivalent homogeneous media rather than in a heterogeneous media. The same degree of discrepancies was observed between the results of the micromechanics model and the computer simulation for material with larger number of particles. A typical example predicted using the micromechanics model for a packing with 250 particles is plotted in Fig. 3.6 for stress-strain curves at three confining pressures. Although the example is for idealized packings with circular disks, it is noted that the stress-strain response resembles very much the behavior of sand.

The predicted results from the present model have shown reasonable agreement with that obtained from computer simulation method. The discrepancies are due to the assumptions inherent in the self-consistent method which results in constraints imposed on the system. However, the general behavior remarkable resembles the observed behavior for sand.

The unique features of the present theory is that it explicitly accounts for the effects of micro-structure, and mechanism of sliding and separation between particles. Thus the model is capable of

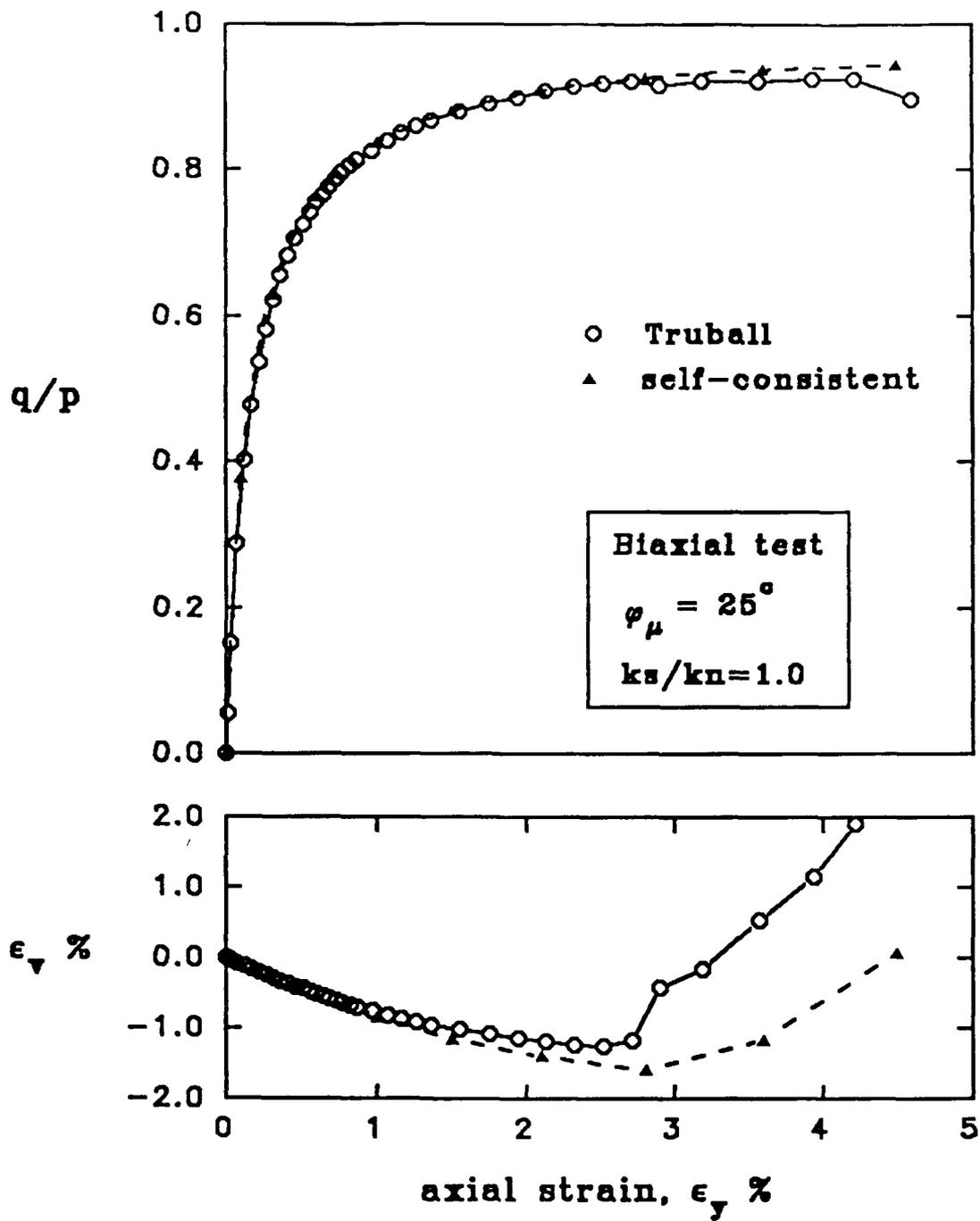


Fig. 3.4 Stress-strain curve of biaxial test for $k_s/k_n = 1$ predicted by computer simulation and present model.

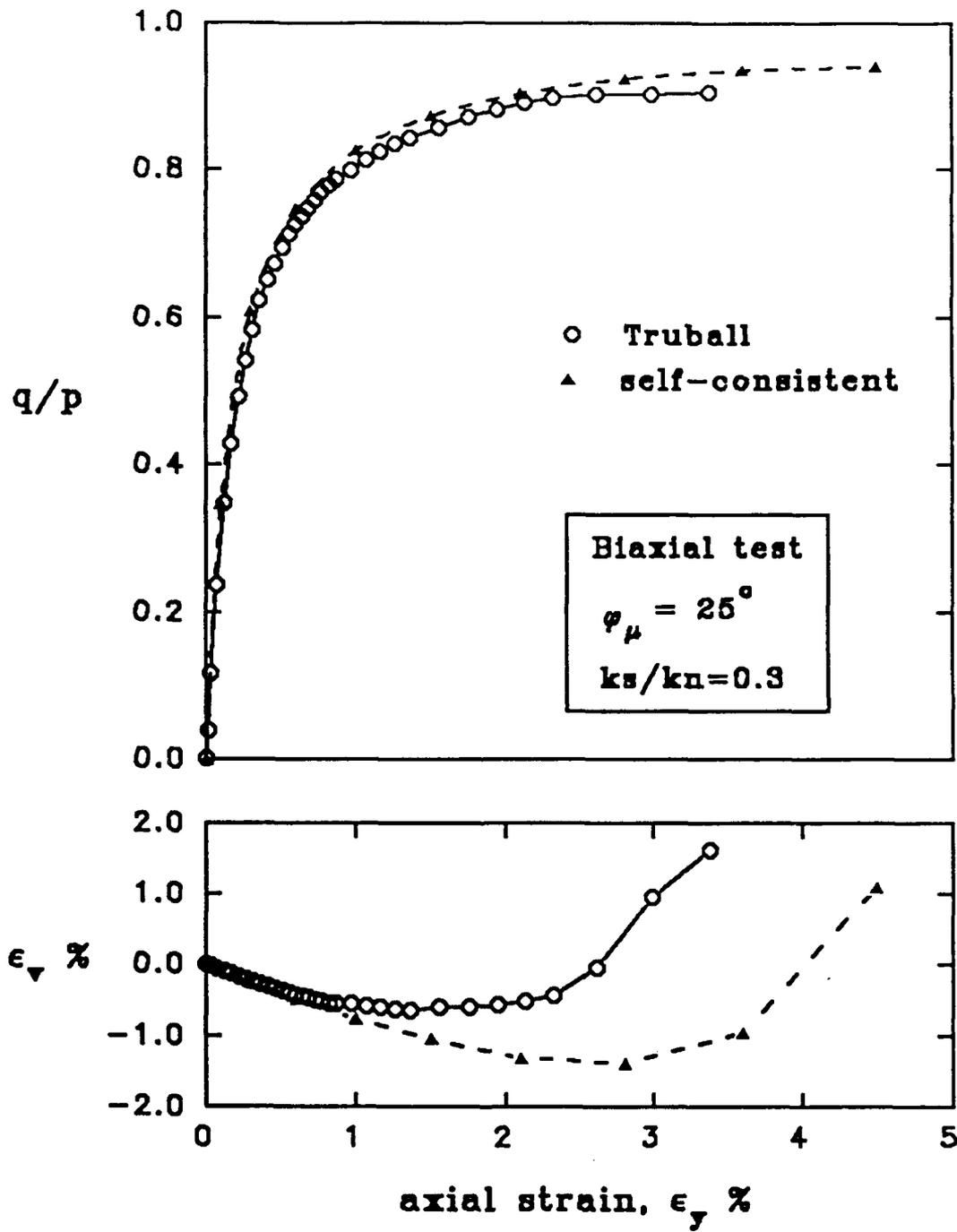


Fig. 3.5 Stress-strain curve of biaxial test for $k_s/k_n = 0.3$ predicted by computer simulation and present model.

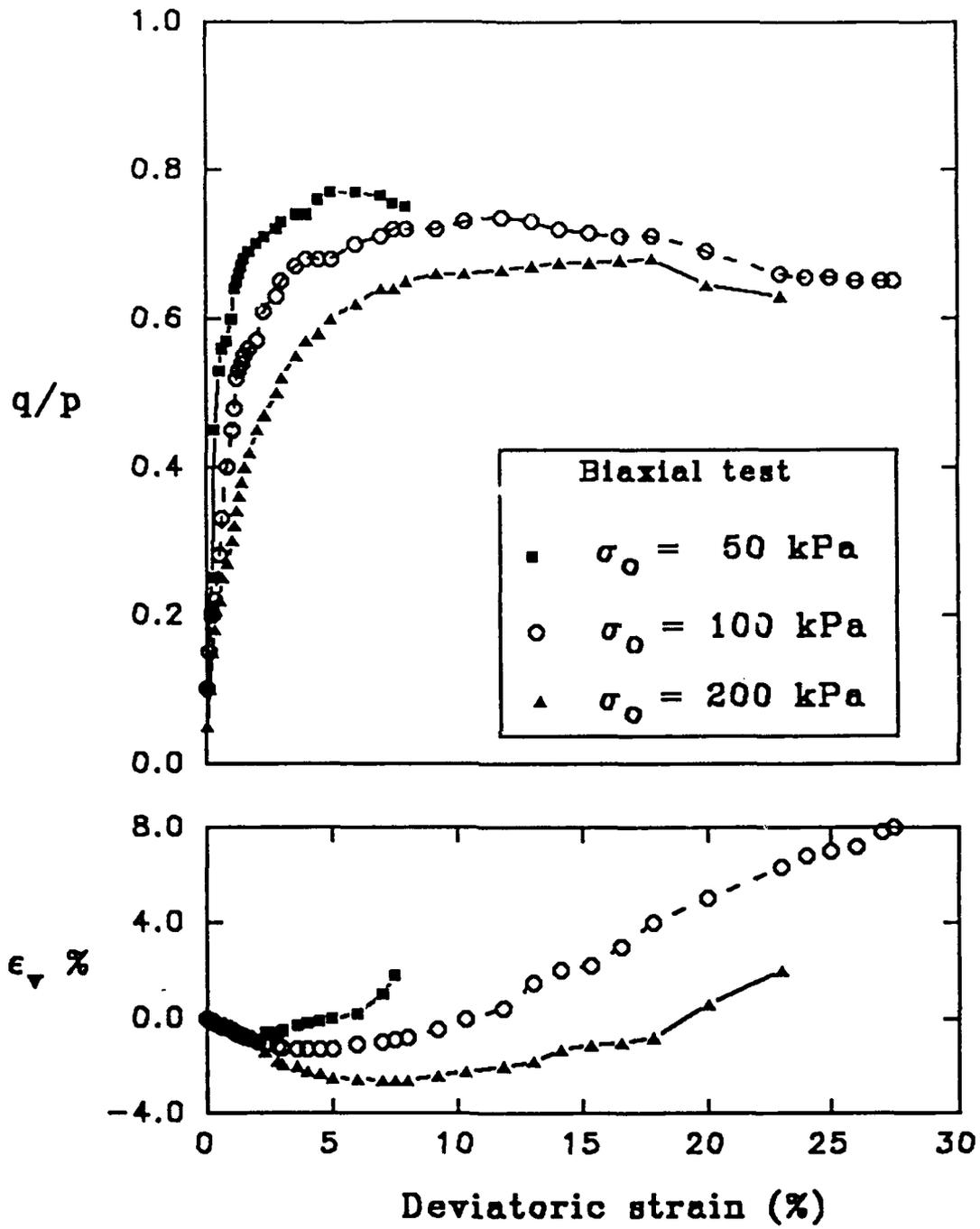


Fig. 3.6 Predicted stress-strain curve of biaxial test for various confining pressures (σ_0).

predicting failure strength and modelling plastic and path-dependent stress-strain behavior. The present constitutive theory is potentially useful for comprehensive modelling of complex behavior for granular material.

CHAPTER 4

EVALUATION WITH EXPERIMENTAL RESULTS

Chapter 2 described the constitutive model for granular materials based on micro-scale interaction between particles including the effects of sliding and separation between particles. In this chapter, the theory is preliminarily evaluated for two conditions, namely, small strain and large strain.

For small strain conditions, we evaluate the capability of the constitutive model by comparing the predicted mechanical properties, such as initial moduli, secant moduli and damping ratio, with those measured from experiments on sands under low amplitude cyclic loading. The focus is on the model capability in simulating:

- 1) the macroscopic stress-strain behavior based on microscopic contact property,
- 2) the stress-induced and inherent anisotropy,
- 3) the response and energy dissipation of material under cyclic loading, and
- 4) the effect of cemented inter-particle properties.

For large strain conditions, the predicted stress-strain behavior is compared with experimental results on different types of sands under various loading conditions, such as:

- 1) one dimensional compression,
- 2) cubical tests,
- 3) circular stress paths on octahedral plane, and

4) direct simple shear.

The comparison of salient features of constitutive behavior is presented to evaluate the performance of the developed constitutive model for granular material.

4.1 MODEL EVALUATION FOR SMALL STRAIN CONDITIONS

The assumption of uniform strain is substantiated with the results of computer simulation for packings under small strain conditions (Chang and Misra 1989). With this assumption, the present constitutive model reduces to the same form as that of the previously developed model (Chang 1988, 1989) which has been applied to the studies of moduli and damping under low amplitude cyclic load. A few examples are provided here to illustrate the model capabilities.

For the problems of small strain, the structure of granular material is statistically represented by a set of binary-units in various orientations. The binary-unit, defined as two particles in contact, is the basic constituent unit of the system. For convenience, in all examples the granular material are envisioned to be collections of elastic spheres. Although the proposed theory is not limited to spherical particles, it is difficult to implement arbitrary shaped particles because of the invalidity of the usual Hertzian contact theory and the manifold complexity involved in the description of packing geometry. For packings with equal-sized spherical particles, the structure can be characterized by the distribution of contact orientations and the average coordination number.

4.1.1 Macro-behavior in terms of micro-property

For isotropic packing of equal-sized elastic spheres, the shear modulus, G , and the Poisson's ratio, ν , were derived in terms of contact properties (Chang et al. 1990), given by

$$\nu = \frac{\nu_{\mu}}{10 - 6\nu_{\mu}} \quad (4.1)$$

$$G = F(z) P(E_{\mu}) \sigma_c^{\alpha} \quad (4.2)$$

where ν_{μ} is the Poisson's ratio of the granule, and σ_c is the confining stress. In Eq. 4.2, $F(z)$ is a function representing packing structure, given by

$$F(z) = \frac{1}{10} z^{1-\alpha}, \quad (4.3)$$

where z is a dimensionless variable, defined by

$$z = \frac{M}{3V} r^3 = \frac{\bar{n}}{4\pi(1+e)} \quad (4.4)$$

where r is the value of radii of spheres, M/V is the number of contacts (double count) per unit volume of the representative unit. The parameter z is also a function of the void ratio, e , and the average coordination number, \bar{n} . The void ratio e is defined as the ratio of the volume of voids to the volume of solids, and the co-ordination number \bar{n} is defined as the average number of contacts per particle.

In Eq. 4.2, $P(E_{\mu})$ is a function of granule properties, given by

$$P(E_{\mu}) = C_1 (2+C_2) E_{\mu}^{1-\alpha} \quad (4.5)$$

In general, the value of constant C_2 is between 0.5 to 1 and the value of α is between 0.3 to 0.5. For Hertz-Mindlin theory, α is 1/3, $C_1 = 1.44$, $E_{\mu} = G_{\mu}/(1-\nu_{\mu})$, G_{μ} and ν_{μ} are the shear modulus and Poisson's ratio of the granule, respectively. C_2 is given by

$$C_2 = \frac{2(1-\nu_\mu)}{2-\nu_\mu} \quad (4.6)$$

Chang et al. (1990) have shown that Eq. 4.2 becomes identical with Hardin and Black's (1966) expression, using $\alpha = 0.5$ and an appropriate relationship of $e-\bar{n}$ curve. It has also been shown that, for $\alpha = 0.33$ with selected values of modulus and Poisson's ratio for quartz, the predicted results, as in Fig. 4.1, are comparable with experimentally measured shear modulus (Chung et al. 1984) on Monterey No. 0 sand in resonant column device under various confining pressures. In Fig. 4.1, the predicted results are also compared with two empirical equations proposed by Hardin and Black (1966), and Chung et al. (1984).

4.1.2 Stress-induced and inherent anisotropy

Chang et al. (1990) have shown that, for an geometric isotropic packing under isotropic stress, the contact forces and the corresponding force-dependent contact stiffness are same for all contacts. As a result, the constitutive matrix computed from the contact stiffness resembles the general Hooke's law of the elasticity theory for isotropic material.

However, when the stress becomes anisotropic, the contact forces and the corresponding force-dependent contact stiffness are no longer same for all contacts. This results to a anisotropic constitutive matrix even though the packing geometry is isotropic. The change of moduli with applied stress is generally referred as stress-induced anisotropy. The moduli predicted by the model has shown good agreement

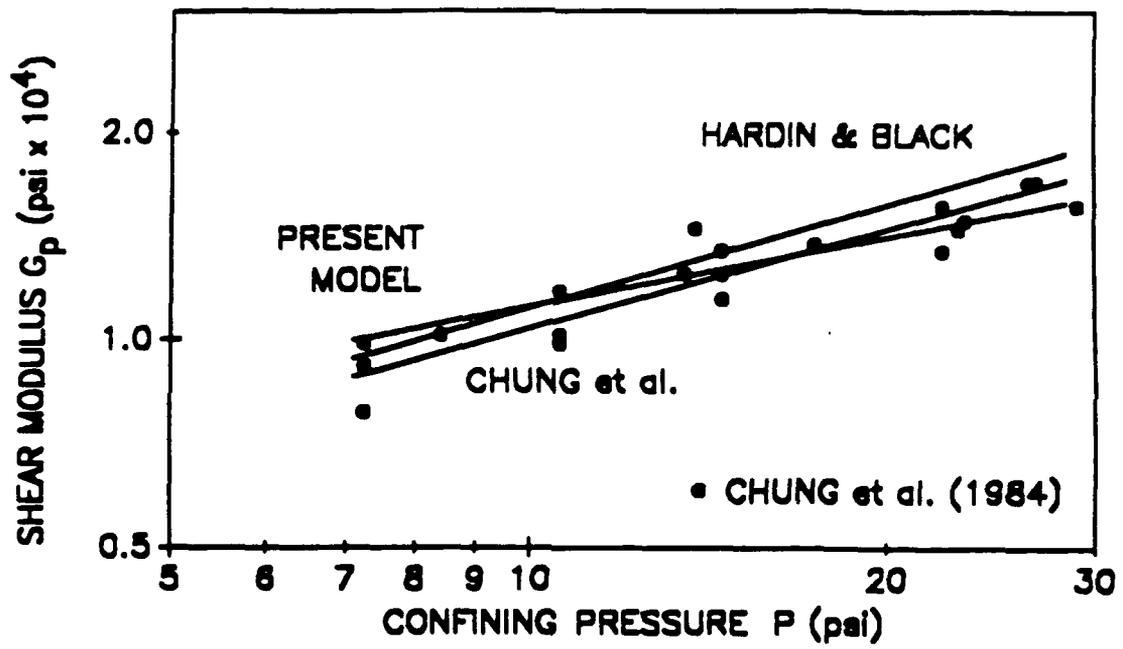


Fig. 4.1 Comparison of measured and predicted results of shear moduli for sand (Chang et al. 1991a).

with those measured from experiments conducted by Yanagisawa (Chang et al. 1990).

Another source of anisotropy is due to the packing anisotropy, termed as inherent anisotropy. Characterization of packing structure by fabric tensor was discussed by Chang and Misra (1990) to relate material symmetries. To illustrate the model capability of simulating inherent and induced anisotropy, predicted moduli are compared with that obtained experimentally by Stokoe et al. (1985) from wave velocities measured in various directions of a large cubic sample of sand.

The measured initial moduli are plotted in Fig. 4.2a by symbols, for various initial confining stress. The measured moduli are different in vertical and horizontal directions, showing that the soil, prepared by air pluviation method, has an inherent transverse isotropic packing structure.

After the sample is isotropically stressed, two loading conditions were followed to show the stress-induced anisotropy. Figure 4.2b shows the results of increasing vertical stress with constant horizontal stress. Figure 4.2c shows the results of decreasing horizontal stress with constant vertical stress.

The predictions in all cases were performed (Chang et al. 1991a) by using the material properties, G_{μ} , ν_{μ} and ϕ_{μ} , and a parameter a_{20} describing the distribution of contact orientations. The comparison shows that the model can effectively simulate both inherent and induced anisotropy.

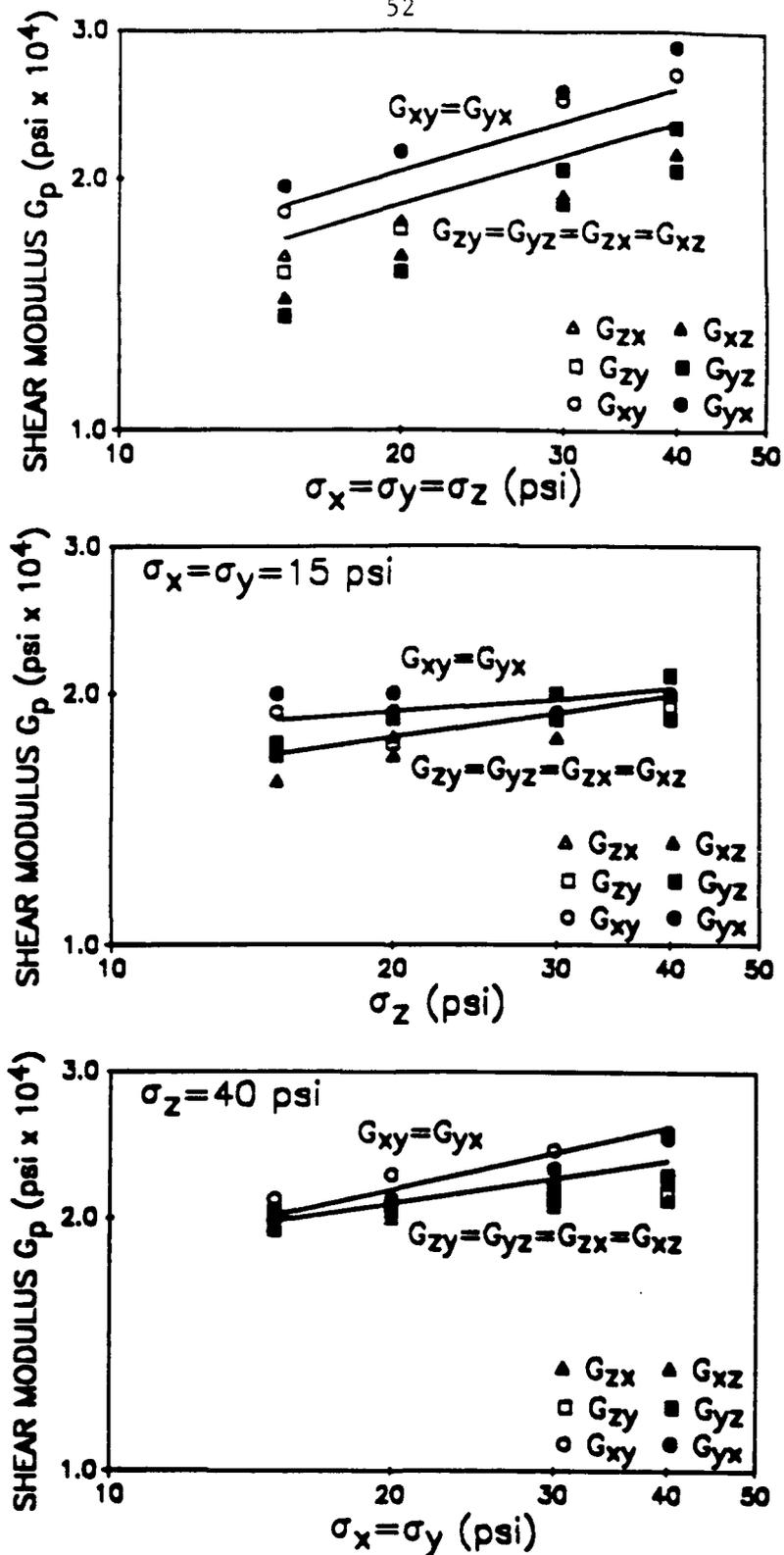


Fig. 4.2 Comparison of measured and predicted shear moduli for sand under three loading conditions: a) isotropic compression, b) σ_z increases, $\sigma_x = \sigma_y = \text{constant}$, and c) $\sigma_z = \text{constant}$, $\sigma_x = \sigma_y$ decreases (Chang et al. 1991a).

4.1.3 Cyclic loading and related energy dissipation

The constitutive model is capable of predicting behavior such as, initial and secant moduli, and damping ratios, under low amplitude cyclic loading. The predicted (Chang et al. 1990) secant moduli and damping ratio are compared with that measured by Acar and El-Tahir (1986) for Monterey No. 0 sand and shown in Figs. 4.3 and 4.4.

In the Hertzian contact theory, partial sliding can occur at a contact even though the two particles are still intact under low amplitude cyclic loading. The partial sliding causes degradation of the material and affects the value of secant shear modulus G_s . The damping results from energy dissipation due to the partial sliding between particles. The present constitutive theory, with microscopic contact consideration, is particularly useful and realistic in predicting this type of behavior.

4.1.4 Inter-particle properties - cemented sand

The model is capable of simulating the behavior of cemented sand by incorporating, other than friction, the mechanism of adhesion at inter-particle contact. A comparison between predicted and measured results for various degrees of cementation can be found in the reference of Chang et al. (1990). Typical examples are shown in Figs. 4.3 and 4.4 for the effects of cementation. The model realistically captures the effects of cementation.

4.2 MODEL EVALUATION FOR FINITE STRAIN CONDITIONS

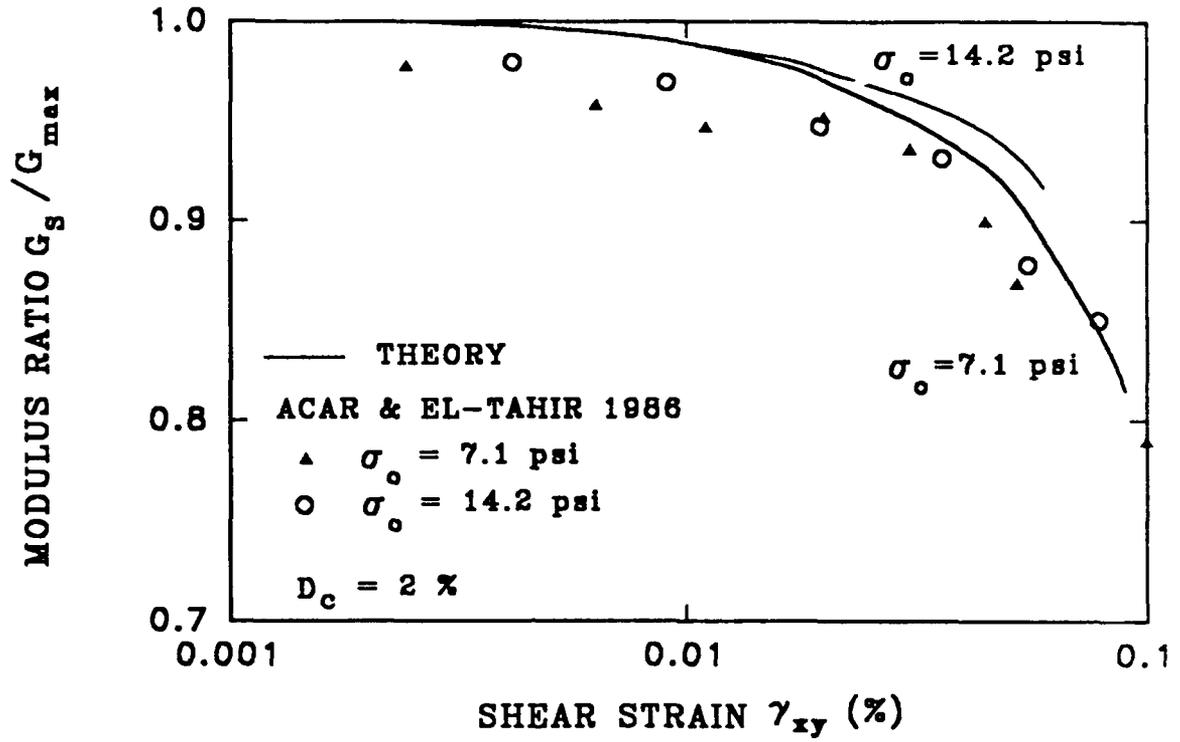


Fig. 4.3 Comparison of predicted and measured secant moduli for sand
(Chang et al. 1990).

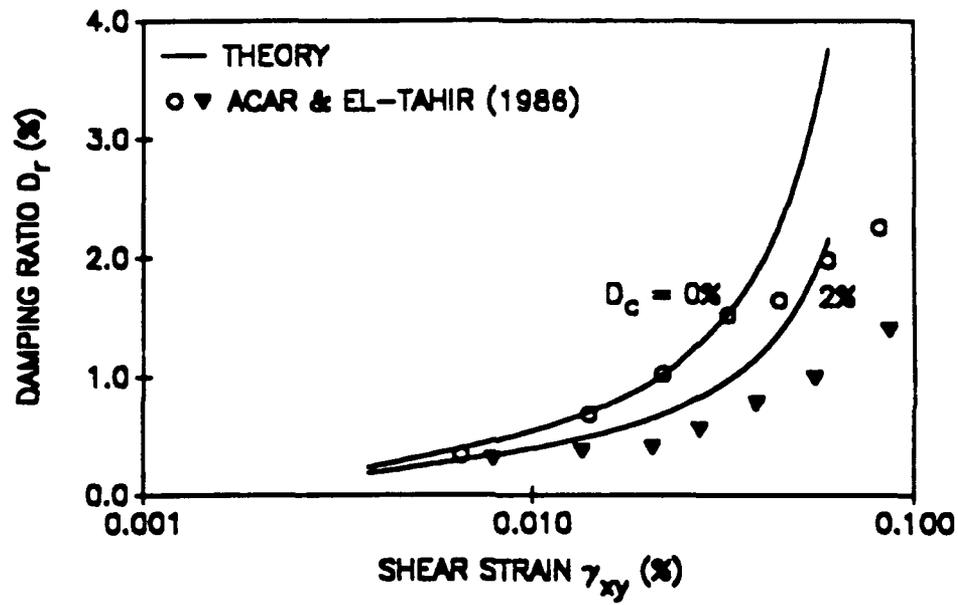


Fig. 4.4 Comparison of predicted and measured damping ratio for sand (Chang et al. 1990).

For the problems of small strain, the structure of granular material is represented statistically by a set of binary-units which is not sufficient for large strain problems. Therefore, the structure of granular material is represented statistically by a set of micro-elements, where each micro-element is defined by a specific arrangement of particle group.

To illustrate the current model, the geometry of five micro-elements were randomly generated; each consists of eight spherical particles with 0.2mm radius. Each micro-element was rotated on the axes 6 times to create six micro-elements as shown in Fig. 4.5, resulting total of 30 micro-elements. The idealized material, represented by these thirty micro-elements, has material symmetry along the directions of three axes. The idealized material has an average co-ordination number of 7 and void ratio 0.7, representing an uniformly graded medium dense sand with rounded particles.

To evaluate the capability of the constitutive model at large strain conditions, the stress-strain behavior of this idealized material is predicted for the following loading conditions:

- 1) one dimensional compression,
- 2) cubical tests,
- 3) circular stress paths on octahedral plane, and
- 4) direct simple shear.

The predicted results are compared with the measured behavior from experiments with various types of sands. Since the predictions for all cases are based on the idealized material, the predictions are not

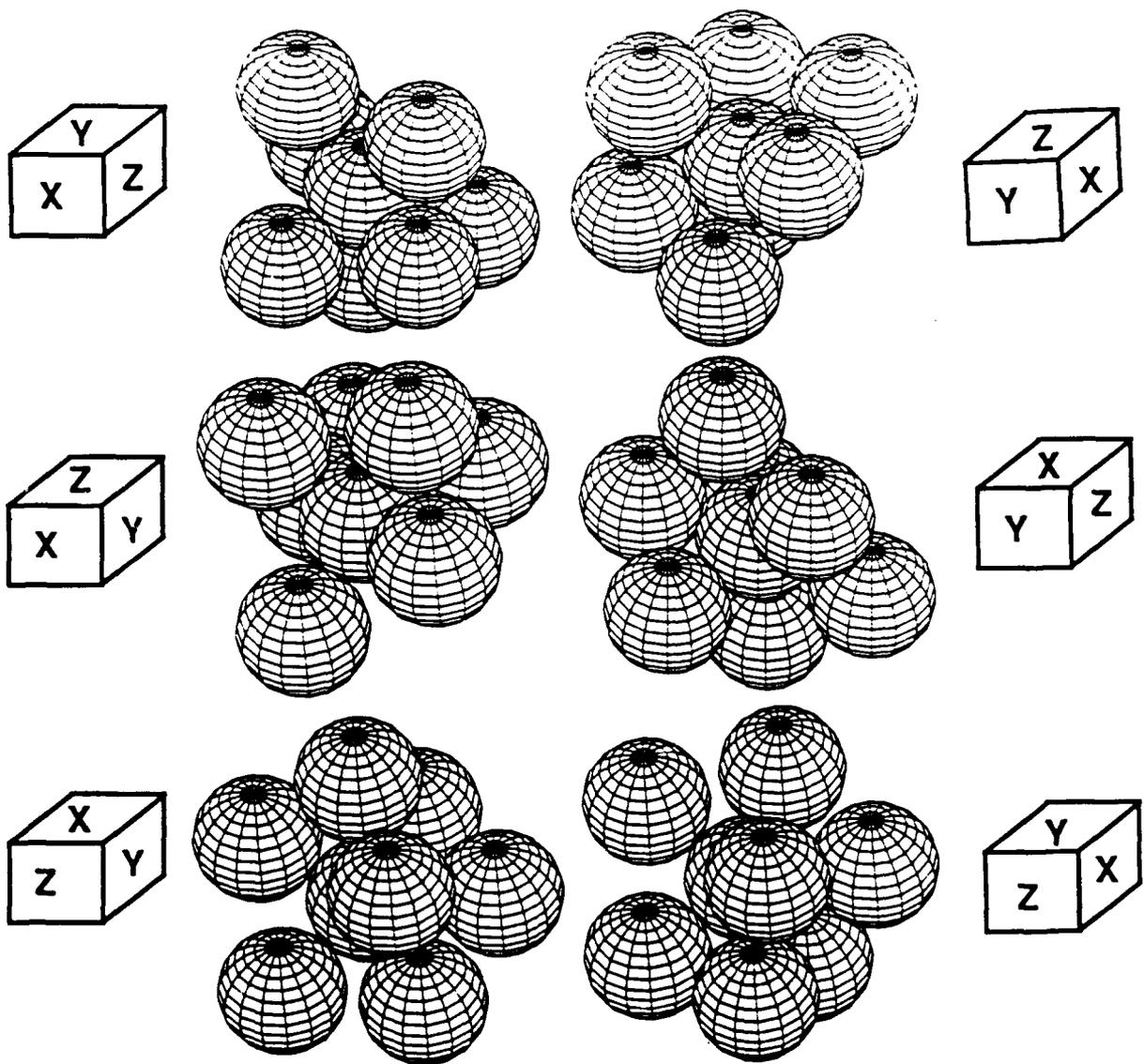


Fig. 4.5 Configurations of six micro-elements.

intended to fit any particular experiment curve. The comparison is merely for the purpose of evaluating, in a qualitative sense, the general model performance under various loading conditions.

Only three basic material parameters are used for this model, namely, the normal contact stiffness k_n , the shear contact stiffness k_s , and the inter-particle friction angle ϕ_μ . For the predictions of all cases listed above, the normal and shear contact stiffness, $k_n = k_s = 105$ N/m, are used. Although the non-linear Hertzian type of contact can be readily used for these predictions, constant contact stiffness are purposely used here to demonstrate that the nonlinear stress-strain behavior is predominantly caused by the mechanism of particle sliding.

4.2.1 One dimensional compression

The typical behavior of granular material in one dimensional compression is shown in Fig. 4.6. The value of K_o is nearly constant during loading but gradually increases during unloading. At microscopic level, a substantial amount of sliding occurs during loading which results locked-in stress during unloading, thus reflects by a higher K_o . The present theory, accounting for the mechanism of sliding at contact, is capable of predicting the familiar phenomenon of K_o increase with the over-consolidation ratio (OCR).

The idealized material is also used to study the one-dimensional compression behavior of granular material preceded by isotropic compression. For this purpose, $\phi_\mu = 17^\circ$ is selected and the prediction is compared with the experimental results on medium dense Napa Basalt

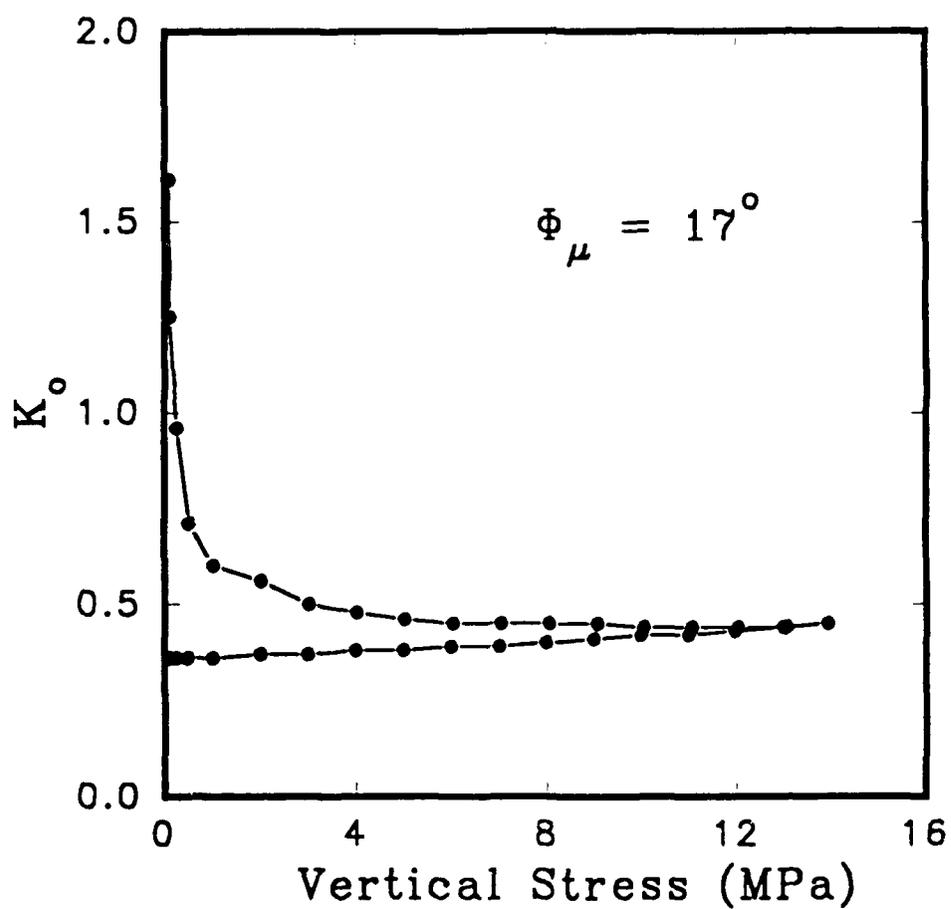


Fig. 4.6 Predicted loading-unloading behavior in one dimensional compression for idealized material.

(Lade 1975) shown in Fig. 4.7. The prediction gives excellent agreement with the experimental behavior even though the soil structure is hypothetical.

The behavior of granular material under one-dimensional compression preceded by anisotropic consolidation is also studied. For convenience, $\phi_{\mu} = 15^{\circ}$ is selected for the predictions of three initial anisotropic stress ratio, 0.5, 1.0 and 1.5. The results are in good agreement with the experimental results on Toyoura sand (Okochi and Tatsuoka 1984), as shown in Fig. 4.8. It may be seen from Fig. 4.8 that the K_0 values measured under different initial stress conditions converge to an identical value at large axial stress.

4.2.2 Cubical tests

The constitutive model is used to predict stress-strain and volume change behavior of the idealized granular material under cubical triaxial loading conditions with different values of b , where b is defined as

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} \quad (4.7)$$

Figure 4.9 shows the predicted stress-strain and volume change behavior using ϕ_{μ} equal to 25° . The predicted behavior is compared with the experimental results obtained from cubical triaxial tests on dense Monterey No. 0 sand (Lade and Duncan 1973), shown in Fig. 4.10. It is noted that the nonlinear stress-strain behavior is predominantly caused by sliding of particles since the contact stiffness are constants.

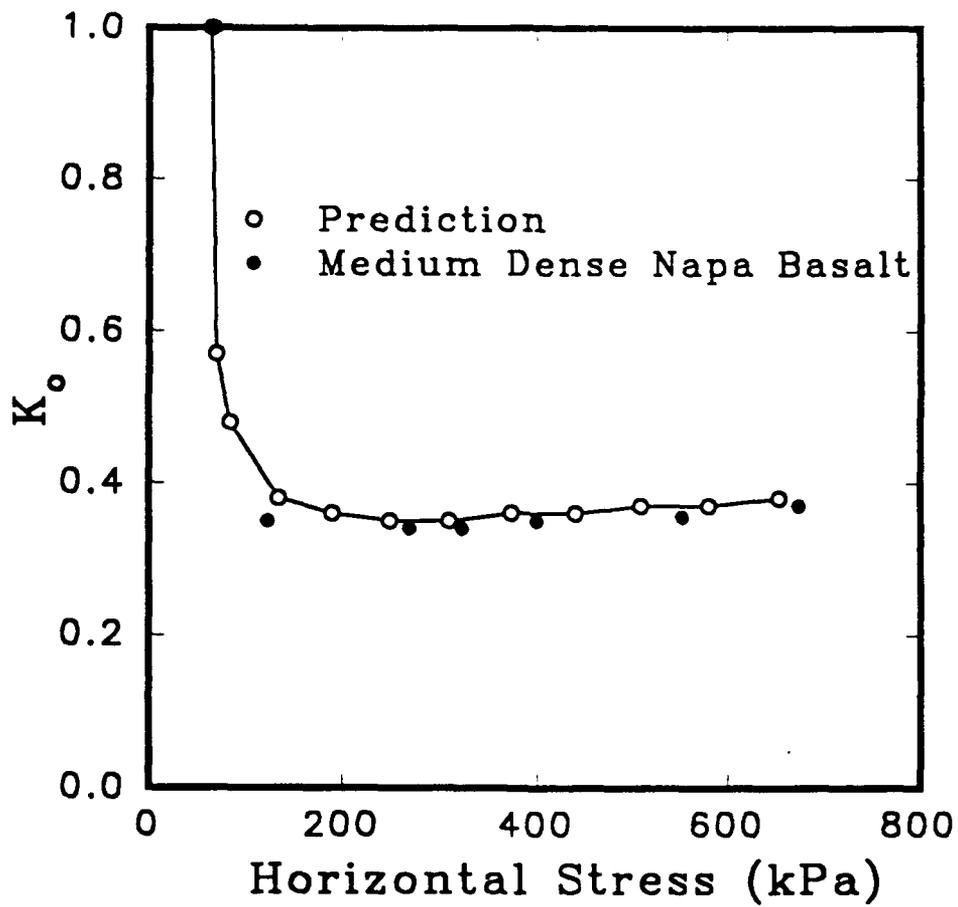


Fig. 4.7 Comparison of predicted K_0 behavior with the experimental results on medium dense Napa Basalt performed by Lade (1975).

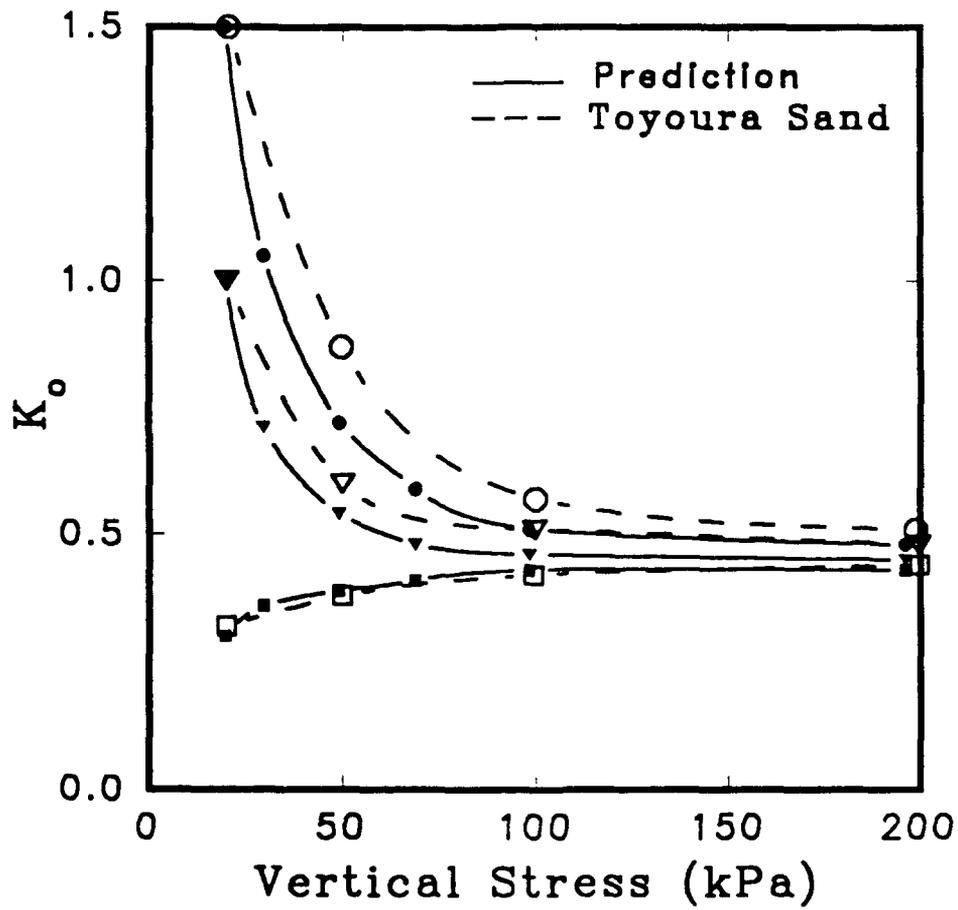


Fig. 4.8 Comparison of predicted K_0 behavior with the experimental results on Toyoura sand for different initial anisotropic stress conditions performed by Okochi and Tatsuoka (1984).

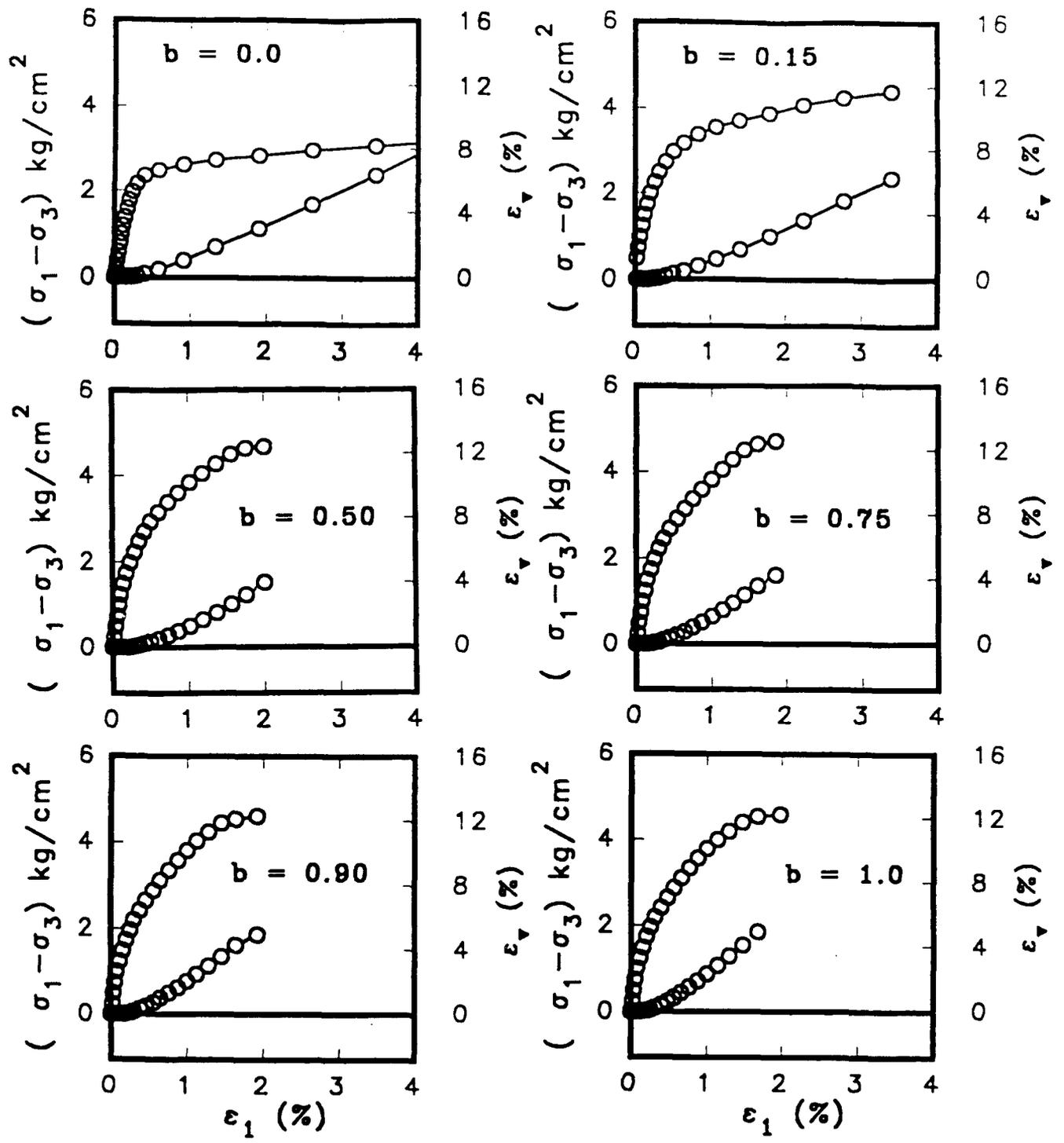


Fig. 4.9 Predicted stress-strain and volume change behavior for the idealized material under cubical triaxial loading conditions with various values of b .

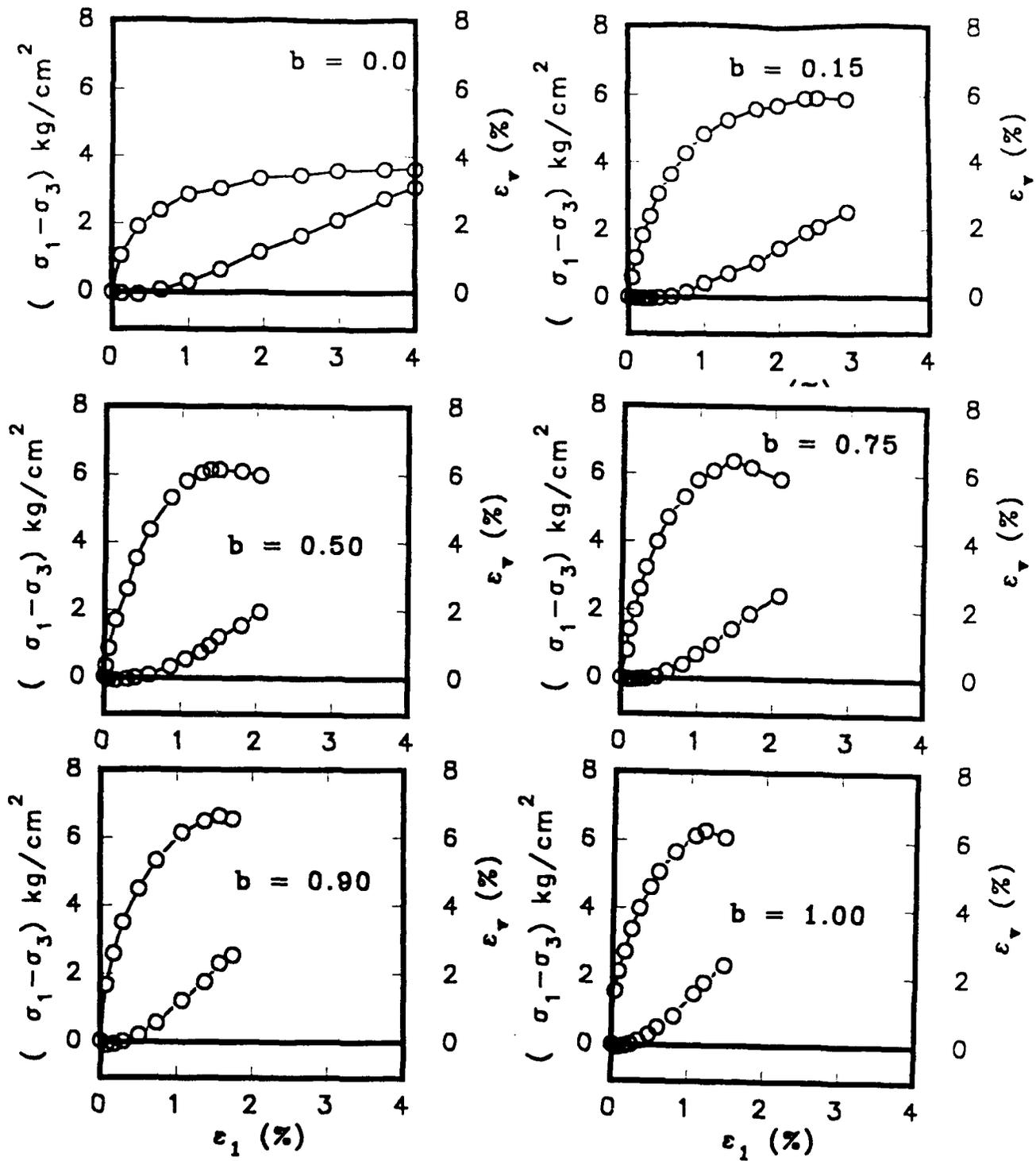


Fig. 4.10 Measured stress-strain and volume change behavior for dense Monterey No. 0 sand under cubical triaxial loading condition with various values of b (Lade and Duncan, 1973).

Although the prediction is based on the idealized material represented by 30 micro-elements, the predicted trend of stress-strain and volumetric strain characteristics are remarkably similar to that of experimental values for Monterey sand.

The predicted variation of peak frictional angle with b is shown in Fig. 4.11, along with other experimental measurements obtained from literature for various types of granular materials (Reades and Green 1974, Sutherland and Mesdary 1969, Lade and Duncan 1973). The present theory, even with the idealized material, predicts reasonable results which, both qualitatively and quantitatively, fall in the range of that observed from experiments.

The predicted failure surface on octahedral plane is compared with the failure surfaces empirically hypothesized by Mohr-Coulomb, Matsuoka (1974) and Lade and Duncan (1973), as shown in Fig. 4.12. It is the unique capability of the present theory which predicts the failure surface of granular material in a stress space solely on the basis of inter-particle friction property.

The predicted directions of the strain increments form acute angles with the failure surface on triaxial plane, and nearly perpendicular to the failure surface on octahedral plane, as shown in Fig. 4.13. This behavior is also in agreement with that observed experimentally for sand (Ko and Scott 1967, Lade and Duncan 1973).

4.2.3 Circular stress paths

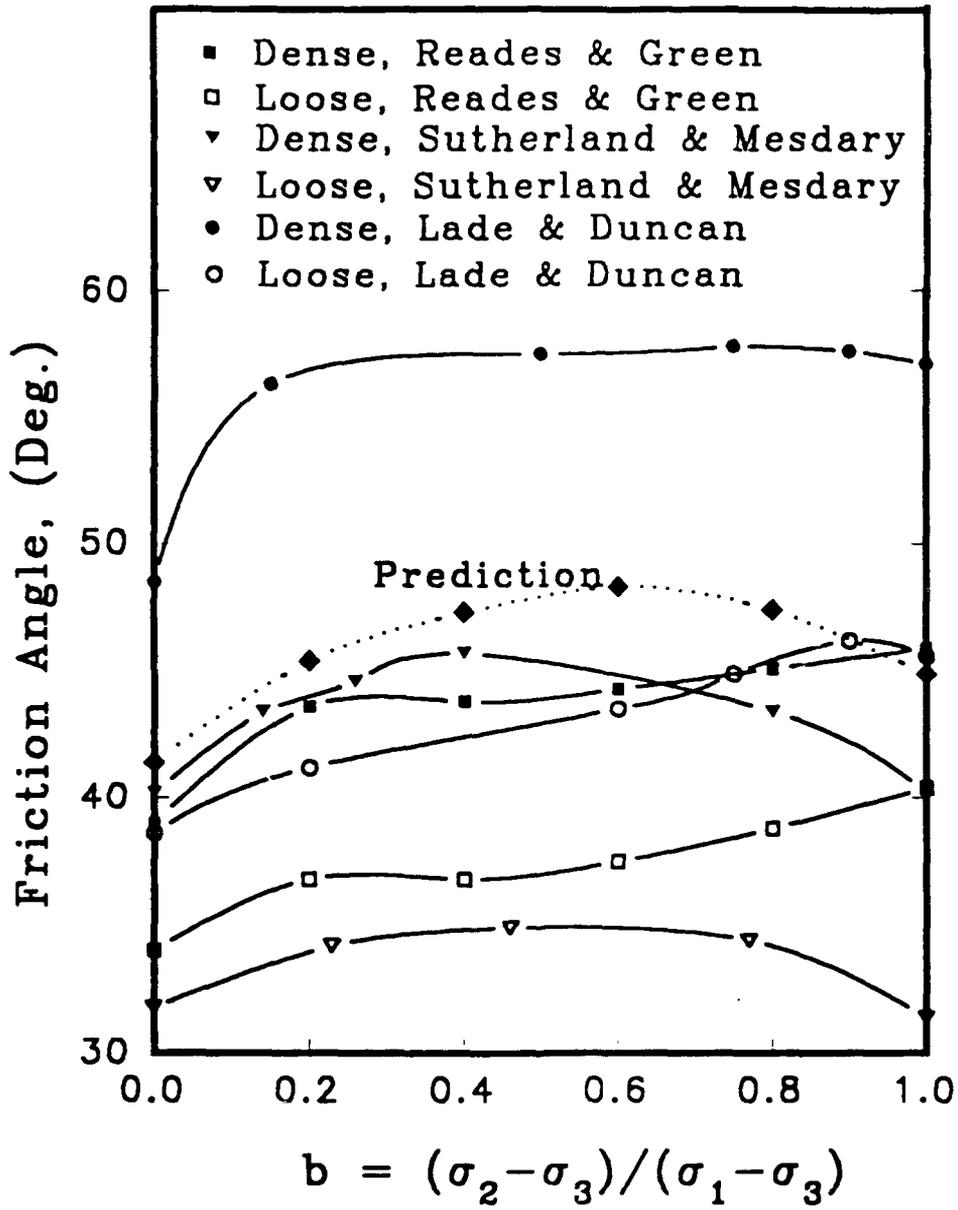


Fig. 4.11 Predicted variation of peak friction angle with b for the idealized material, along with other experimental measurements.

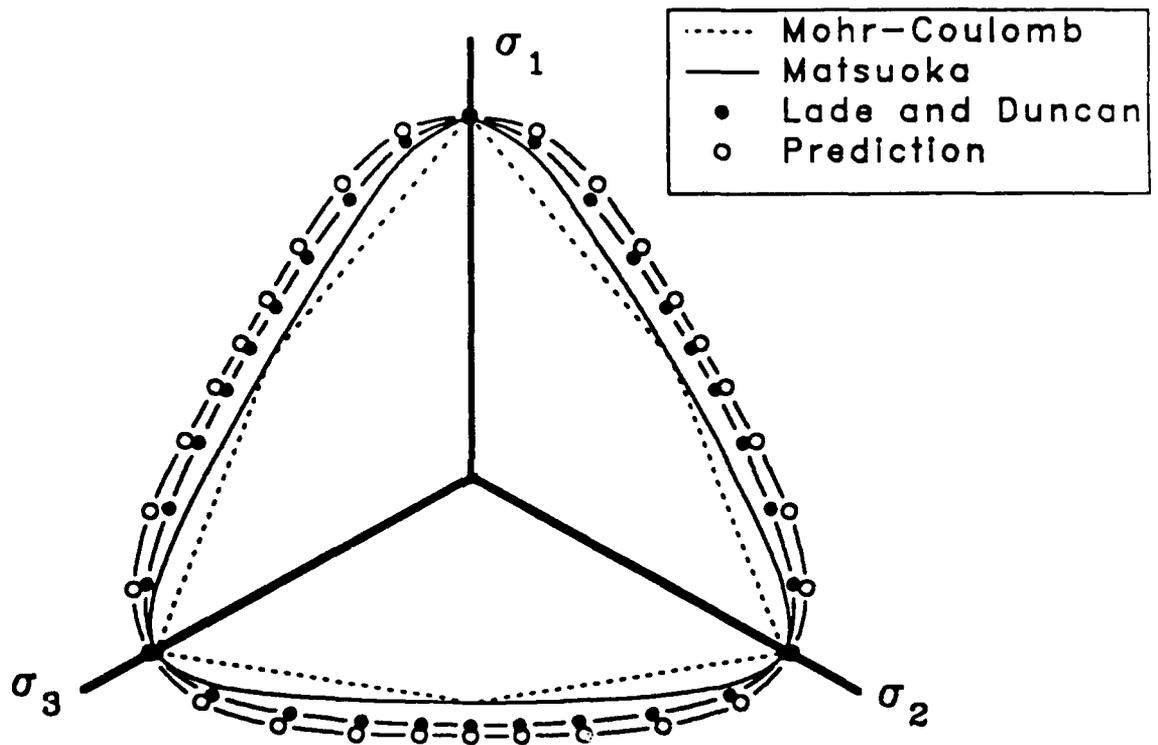


Fig. 4.12 Predicted failure surface on octahedral plane compared with the failure surfaces empirically hypothesized by Mohr-Coulomb, Matsuoka (1974) and Lade and Duncan (1973).

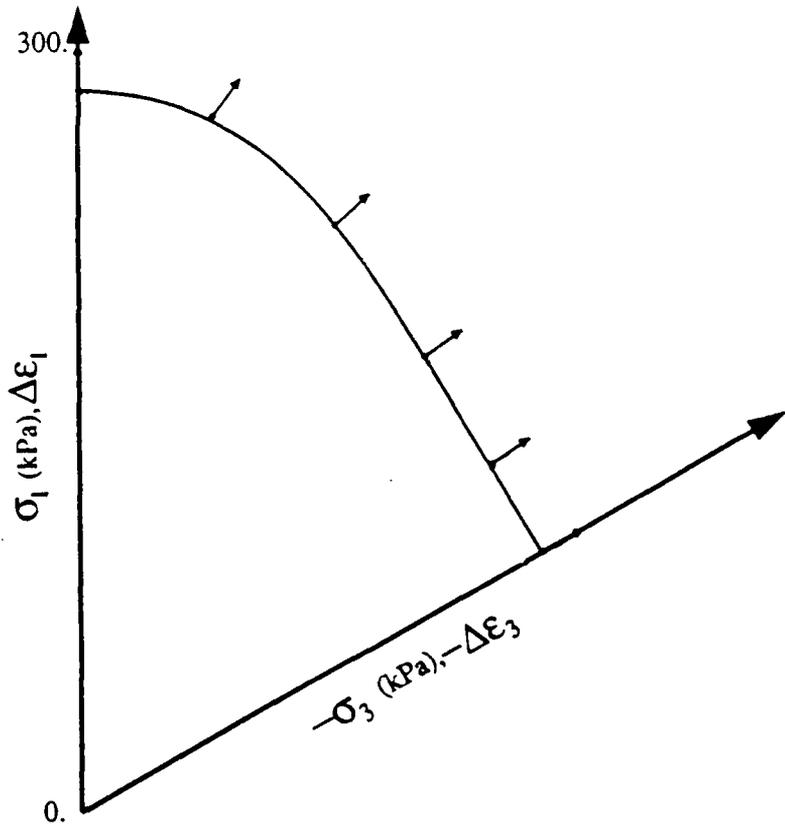
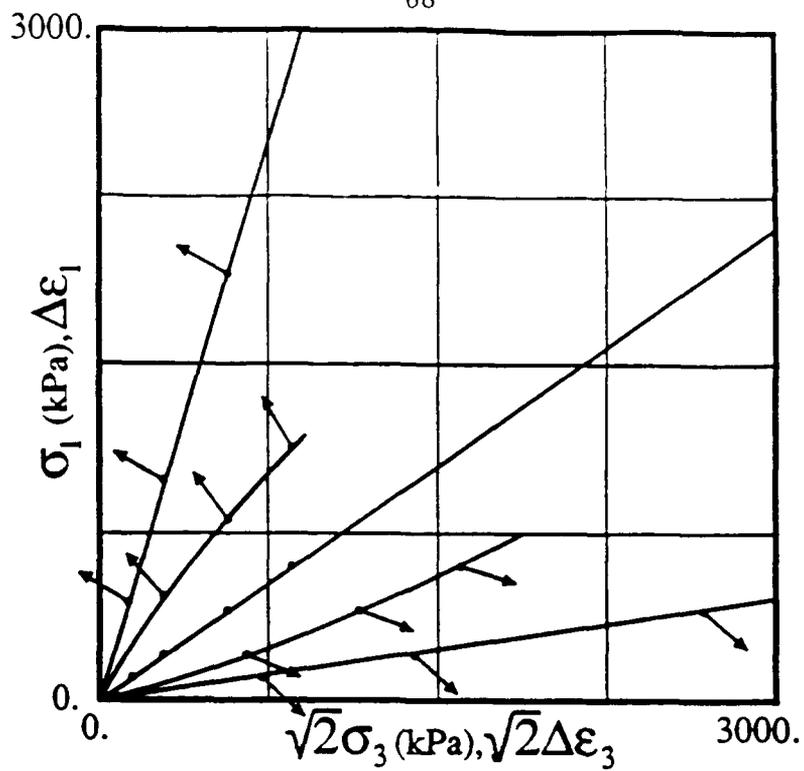


Fig. 4.13 Predicted directions of the strain increment vectors at failure on a) triaxial plane and b) octahedral plane for the idealized material.

To further evaluate the capability of the model, the idealized material subjected to circular stress path on octahedral plane is predicted. A classical model, such as an isotropic hardening plasticity model, would predict elastic response for the circular stress path within the yield surface. This is in contradiction to the observed experimental results. The present theory models the plastic sliding at contacts instead of using a priori empirically hypothesized yielding surface. Thus the present model is capable of predicting material response under complex loading conditions such as the circular stress path. Figure 4.14 shows the comparison of predicted strain path with that measured from experiments on Hostun sand performed by Lanier and Zitouni (1988).

4.2.4 Direct simple shear

Predictions of strain-controlled direct simple shear are performed on the idealized material using ϕ_{μ} equal to 25° . The packing is initially consolidated one-dimensionally to a vertical stress of 134.4 kPa, followed by a shear strain. Figure 4.15a shows the predicted stress-strain curves. The predicted stress ratio at failure is 0.78.

Figure 4.15b shows that the predicted intermediate principal stress is not equal to the mean of the other two principal stresses, opposed to Hill's (1950) proposition for ideal plastic material. However the predicted trends are in agreement with that measured from simple shear tests on sand reported by Roscoe et al. (1967).

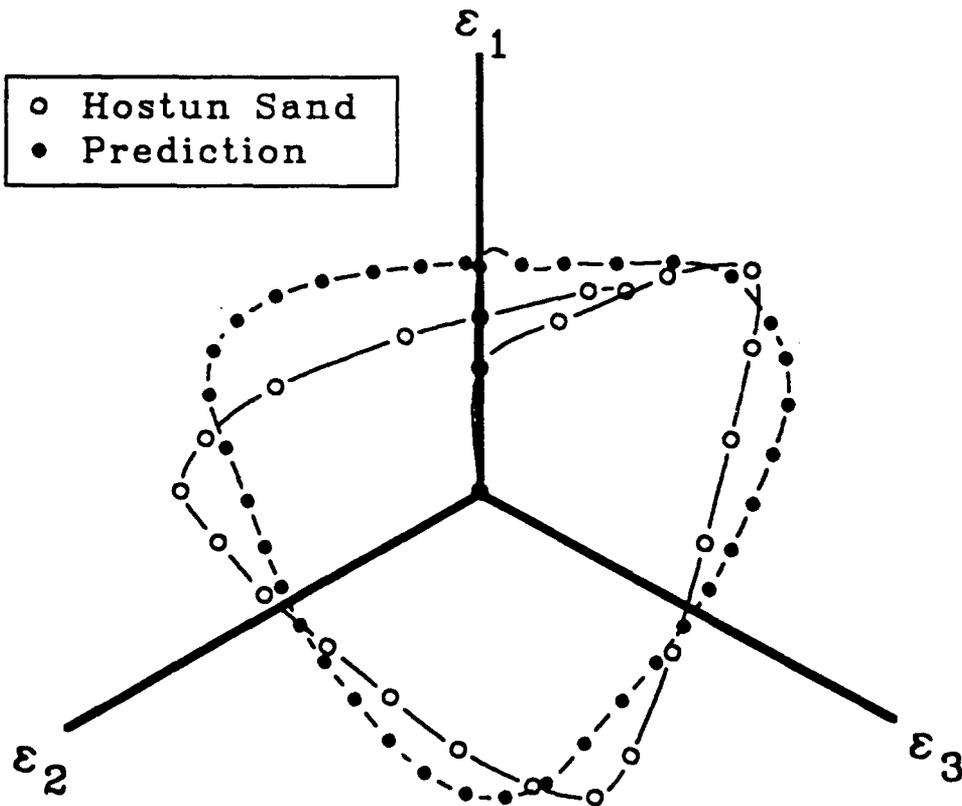


Fig. 4.14 Comparison of predicted strain path with that measured from experiment on Hostun sand performed by Lanier and Zitouni (1988).

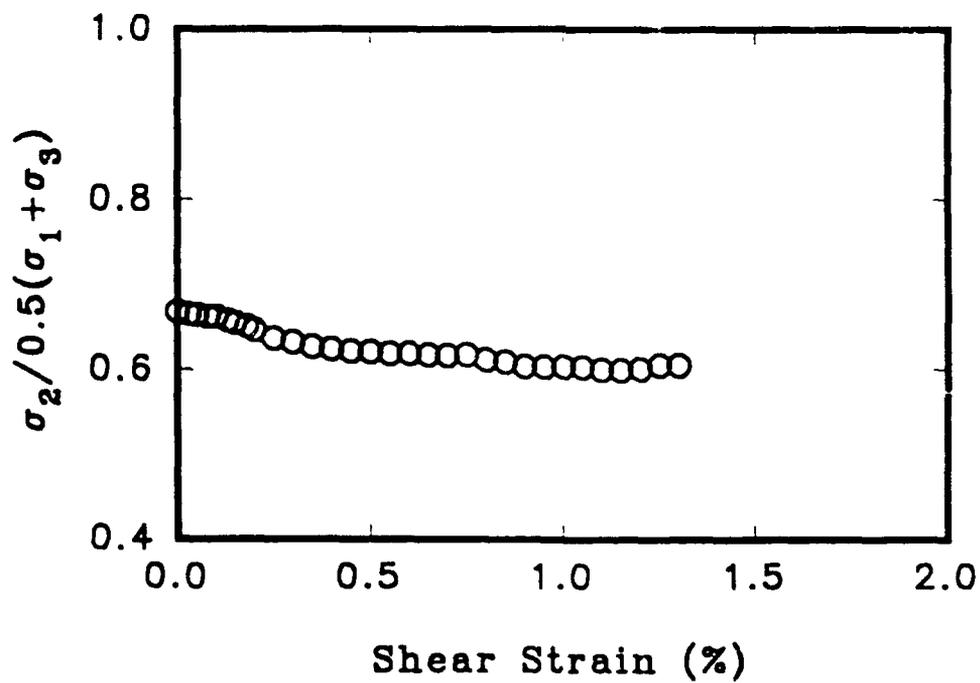
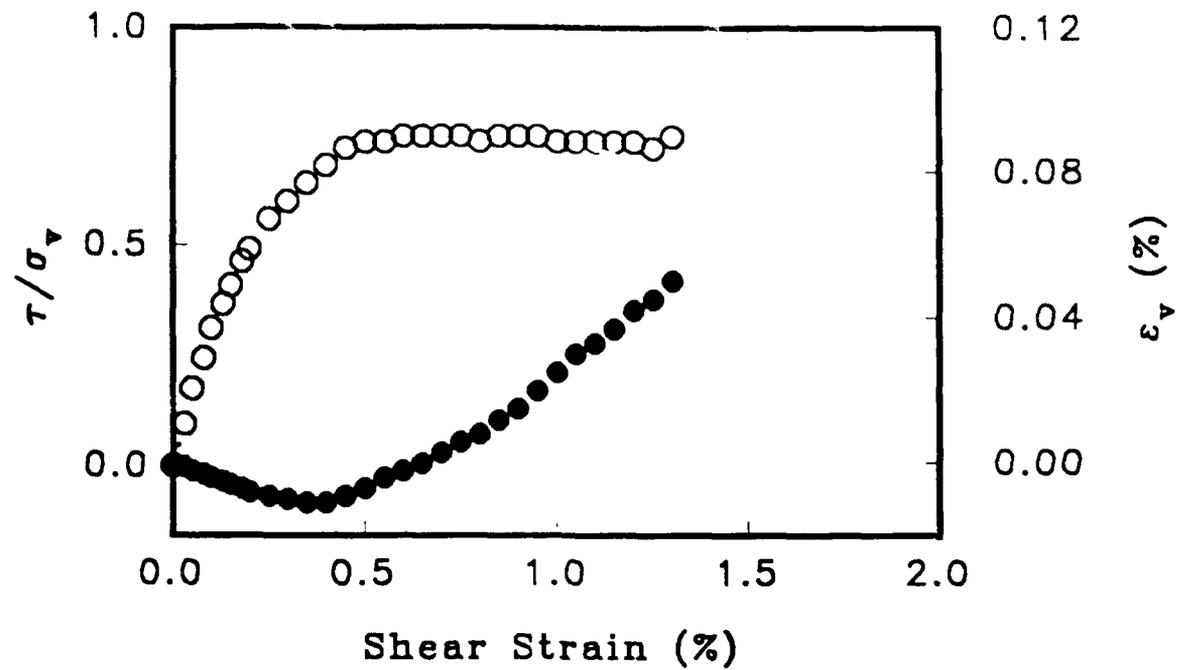


Fig. 4.15 a) Predicted stress-strain curve for the idealized material in direct simple shear test; b) Variation of intermediate principal stress with other two principal stresses.

To evaluate the behavior of rotation of principal stress, the predicted results are compared with an empirical equation by Oda and Konishi (1974). The empirical equation relates the inclination angle ψ , i.e. angle between the horizontal plane and the plane on which the major principal stress acts, to the stress ratio in simple shear test on the granular material, given by

$$\tau/\sigma_v = \kappa \tan \psi \quad (4.8)$$

This equation has been validated for Leighton Buzzard sand in monotonic and cyclic simple shear tests. The measured values of κ for Leighton Buzzard sand varies from 0.58 to 0.67 (Cole 1967, Budhu 1979). Figure 4.16a shows the predicted relationship between the stress ratio and $\tan \psi$, which gives a value of $\kappa = 0.64$.

In addition to the angle ψ , the following angles β , ω , χ , and ξ are defined respectively as the angles between the horizontal plane and the plane of maximum shear stress, the plane of maximum stress ratio, the plane of major principal stress increment, and the plane of the major principal strain increment.

The predicted results of these angles are plotted in Fig. 4.16b which shows that :

- a) The angle ξ is not equal to the angle ψ indicating that the principal axes of the strain-increment do not coincide with the principal axes of stress. Thus behavior of sand is in contrary to the postulate in plasticity theory (Hill 1950).
- b) The angle χ is not equal to the angle ξ indicating that the axes of stress-increment (χ) and strain-increment (ξ) do not coincide,

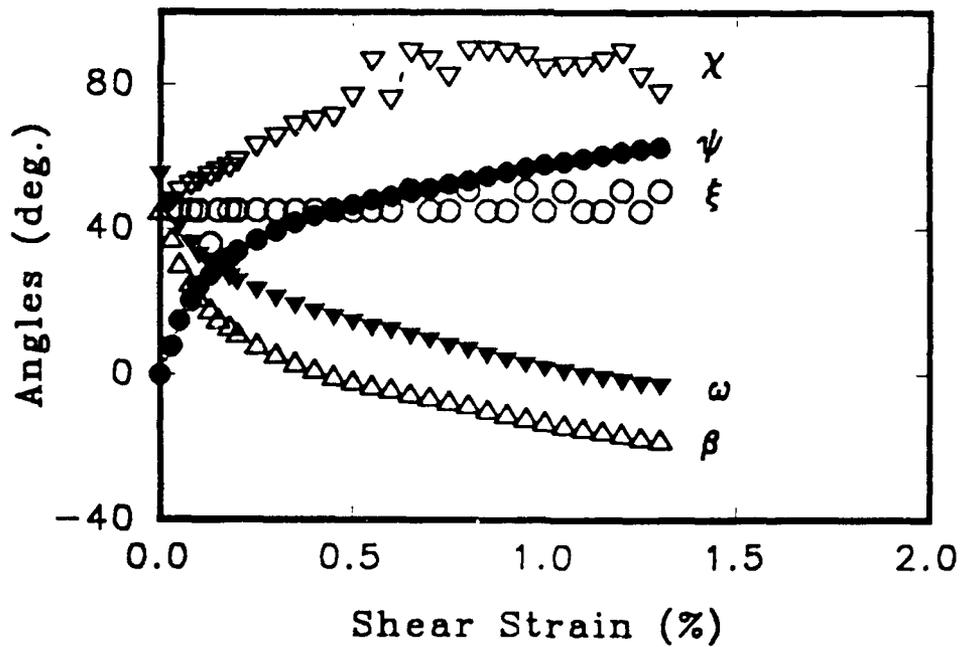
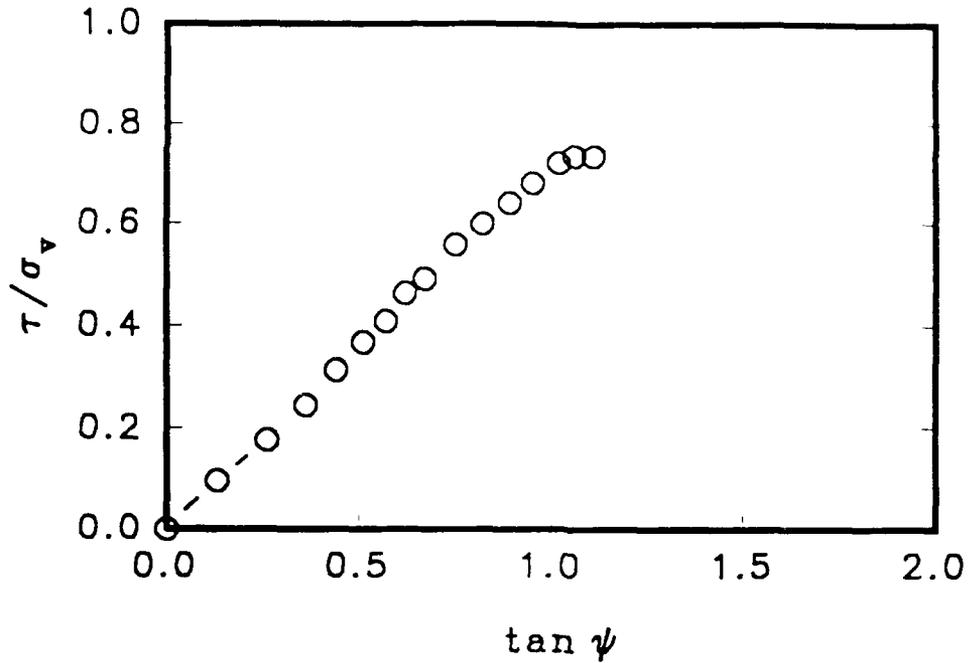


Fig. 4.16 a) Predicted relationship between the stress ratio and $\tan \psi$; b) Predicted results of angles defined as the inclinations of the planes of: 1) major principal stress, ψ , 2) maximum shear stress, β , 3) maximum stress ratio, ω , 4) major principal strain increment, ξ , and 5) major principal stress increment, χ .

which is the characteristics for elastic material (Hill, 1950).

Thus sand is not an elastic material, even in the early stage of the test.

- c) The angle ω decreases as shear strain increases and approaches to zero at large shear strain. This indicates that the plane of maximum stress ratio does not occur on the horizontal plane except at a later stage of the test.
- d) The angle β is approximately zero at the peak shear stress, indicating that the horizontal plane is the plane of maximum shear stress. On this basis the peak friction angle of the idealized material in direct simple shear is 48.6° . On the other hand, by treating the horizontal plane as the maximum stress ratio, the angle of friction becomes 37° . In cubical tests the maximum value of peak friction angle is 48° at $b = 0.6$, whereas for triaxial compression and extension cases the friction angles are 41° and 45° respectively.

The predicted trends of these angles are in agreement with that measured from simple shear tests on sand reported by Roscoe et al. (1967).

4.3 SUMMARY

The developed constitutive model has been evaluated by comparing the predicted behavior of the idealized material with that obtained from experiments on different sands under various loading conditions. For small strain conditions, the model is evaluated for its applicability in predicting initial moduli, secant moduli and damping ratio under low

amplitude loading. For large strain conditions, the model is evaluated for its capability in predicting stress-strain-strength behavior under various stress paths.

The results show that the model has the following capabilities:

- 1) The macroscopic stress-strain behavior, such as moduli and failure strength, can be predicted in terms of microscopic contact properties.
- 2) The effect of microstructure, such as inherent anisotropy, can be explicitly accounted.
- 3) The model can account for the behavior associated with particle interaction and sliding at contact, such as stress-induced anisotropy, path dependency, locked-in stress, plastic flow, dilatancy, friction loss (damping), and non-coaxial behavior under rotation of principal stress.
- 4) The model can account for materials with cemented inter-particle properties.

It is noted that in all the predictions, only three basic material parameters are used to represent the stiffness and inter-particle friction of the contact.

The theory used in this chapter focuses on the development of mechanics of granular material. It does not address issues on representation of geometric structure of granular material. However, the predicted behavior for an idealized material represented by 30 randomly generated micro-elements, has shown remarkable similarity to the stress-strain behavior observed from experiments under various

loading conditions. This agreement in the predicted and measured behavior indicates the potential applicability of this model. Further study is under progress on statistical representation of microstructure of granular soil.

The purpose of this chapter is to illustrate, aside from the prevailing models, an alternative way of modelling stress-strain behavior for granular material. The proposed theory is conceptually simple and is useful in discerning the essence of all entities comprising the behavior of granular material.

CHAPTER 5
SUMMARY AND CONCLUSION

Perceiving granular material as a collection of particles, a constitutive law for granular material is derived based on micromechanics approach, taking into account the mechanisms of sliding and separation of particles. In the present theory, structure of the granular material is intended to be represented statistically by a set of micro-elements where each micro-element is defined by a specific arrangement of particle group.

The constitutive law is in an incremental form. At the end of each stress increment, the initial structure of these micro-elements are updated. Thus, the non-linearity due to evolution of microstructure is incorporated.

The unique features of the present theory are listed as follows:

- 1) It explicitly accounts for the effects of micro-structure, thus is capable of modelling inherent anisotropy of material structure.
- 2) It accounts for mechanical properties at granular contact with the consideration of mechanism of sliding and separation between particles, thus is capable of modelling strength, residual strength, behavior of yielding, and behavior of plastic flow of the material.

- 3) It accounts for the evolution of micro-structure, thus is capable of modelling strain dependency, path dependency, stress-induced anisotropy, and behavior related to the memory of the material.

Therefore the present theory, in a more fundamental way, models many aspects of material behavior.

The developed constitutive model has been evaluated by comparing the predicted behavior of the idealized material with that obtained from experiments on different sands under various loading conditions. For small strain conditions, the model is evaluated for its applicability in predicting initial moduli, secant moduli and damping ratio under low amplitude loading. For large strain conditions, the model is evaluated for its capability in predicting stress-strain-strength behavior under various stress paths.

The results show that the model has the following capabilities:

- 1) The macroscopic stress-strain behavior, such as moduli and failure strength, can be predicted in terms of microscopic contact properties.
- 2) The effect of microstructure, such as inherent anisotropy, can be explicitly accounted.
- 3) The model can account for the behavior associated with particle interaction and sliding at contact, such as stress-induced anisotropy, path dependency, locked-in stress, plastic flow, dilatancy, friction loss (damping), and non-coaxial behavior under rotation of principal stress.

- 4) The model can account for materials with cemented inter-particle properties.

The predicted behavior for an idealized material has shown remarkable similarity to the stress-strain behavior observed from experiments under various loading conditions. This agreement in the predicted and measured behavior indicates the potential applicability of this model. Further study is under progress on statistical representation of microstructure of granular soil.

The results illustrate that the micromechanics approach of modelling stress-strain behavior for granular material is conceptually simple and is useful in discerning the essence of all entities comprising the behavior of granular material.

REFERENCES

Acar, Y.B. and El-Tahir, E.A. (1986), "Low Strain Dynamic Properties of Artificially Cemented Sand", *Journal of Geotechnical Engineering, ASCE*, Vol. 112, 1001-1005.

Bathurst, R.J. and Rothenberg, L. (1988), "Micromechanical Aspects of Isotropic Granular Assemblies with Linear Contact Interactions", *Journal of Applied Mechanics, ASME*, Vol. 55, 17-23.

Bishop, A.W. (1950), discussion of "The Measurement of the Shear Strength of Soils" by Skempton and Bishop, *Geotechnique*, Vol. II, No. 2, 113-116.

Budhu, M. (1979), "Simple Shear Deformation of Sands", Ph.D. dissertation, Univ. of Cambridge, London.

Caquot (1934), "Equilibre des Massifs a Frottement Interne", *Stabilite des Terres Pulverents et Coherentes*, Gauthier Villars, Paris.

Chang, C.S. (1988), "Micromechanical Modelling of Constitutive Equation for Granular Material," *Micromechanics of Granular Materials*, Edited by J. T. Jenkins and M. Satake, Elsevier Science Publishers, 271-278.

Chang, C.S. (1989), "Constitutive Modelling of Granular Materials as Generalized Continua with Non-Linear Kinematic Fields," *Powders and Grains*, Edited by J. Biarez and R. Gourves, A. A. Balkema Publishers, Rotterdam, The Netherlands, 311-319.

Chang, C.S. and Misra, A. (1989a), "Theoretical and Experimental Study of Regular Packings of Granules," *Journal of the Engineering Mechanics Division, ASCE*, Vol. 115, No. 4, 704-720.

Chang, C.S. and Misra, A. (1989b), "Computer Simulation and Modelling of Mechanical Properties of Particulates," *Journal of Computer and Geotechniques*, Elsevier Science Publishers, Vol. 7, No. 4, 269-287.

Chang, C.S., Misra, A. and Xue, J.H. (1989a), "Incremental Stress-Strain Behavior of Packings Made of Multi-sized Particles," *International Journal of Solids and Structures*, Pergamon Press, Vol. 25, No. 6, 665-681.

Chang, C.S., Sundaram, S.S. and Misra, A. (1989b), "Initial Moduli of Particulated Mass with Frictional Contacts," *International Journal for Numerical and Analytical Methods in Geomechanics*, John Wiley and Sons, Vol. 13, No. 6, 626-641.

- Chang, C.S., Weeraratne, S.P. and Misra, A. (1989c), "Slip Mechanism-Based Constitutive Model for Granular Soils", *Journal of Engineering Mechanics*, ASCE, Vol. 115, No. 4, 790-807.
- Chang, C.S. and Liao, C. (1990), "Constitutive Relations for Particulate Medium with the Effect of Particle Rotation," *International Journal of Solids and Structures*, Vol. 26, No. 4, 437-453.
- Chang, C.S. and Ma Lun (1990), "Modelling of Discrete Granulates as Micropolar Continuum," *Journal of the Engineering Mechanics Division*, ASCE. Vol. 116, No. 12, 2703-2721.
- Chang, C.S. and Misra, A. (1990a), "Packing Structure and Mechanical Properties of Granulates," *Journal of the Engineering Mechanics Division*, ASCE. Vol. 116, No. 5, 1077-1093.
- Chang, C.S. and Misra, A. (1990b), "Application of Uniform Strain Theory to Heterogeneous Granular Solids," *Journal of the Engineering Mechanics Division*, ASCE. Vol. 116, No. 10, 2310-2328.
- Chang, C.S., Misra, A. and Sundaram S.S. (1990), "Micromechanical Modelling for Behaviour of Cemented Sand Subjected to Low Amplitude Cyclic Loading," *Geotechnique*, Vol. 40, No. 2, 251-263.
- Chang, C.S., Misra, A. and Sundaram S.S. (1991a), "Properties of Granular Packings under Low Amplitude Cyclic Loading", *International Journal of Soil Dynamics and Earthquake Engineering*,
- Chang, C.S., Chang, Y. and Kabir, M.G. (1991b), "Micromechanics Modelling for the Stress-Strain-Strength Behavior of Granular Materials - I. Theory", submitted to the *Journal of Geotechnical Engineering*, ASCE.
- Chang, C.S., Kabir, M.G. and Chang, Y. (1991), "Micromechanics Modelling for the Stress-Strain-Strength Behavior of Granular Materials - II. Evaluation", Submitted to ASCE, *Journal of Geotechnical Engineering Division*.
- Christoffersen, J., Mehrabadi, M.M. and Nemat-Nasser, S. (1981), "A Micromechanical Description of Granular Material Behavior", *Journal of Applied Mechanics*, Vol. 48, No. 2, 339-344.
- Chung, R.M., Yokel, F.Y. and Drnevich, V.P. (1984), "Evaluation of Dynamic Properties of Sands by Resonant Column Testing", *Geotechnical Testing Journal*, ASTM, 60-69.
- Cole, E.R.L. (1967), "Soils in the Simple Shear Apparatus", Ph.D. dissertation, Univ. of Cambridge, London.

- Coulomb, C.A. (1773), "Essai sur une application des regles de maximis et minimis a quelques problemes de statique relatifs a l'architecture", Memoires de Mathematique and de Physique, presentes a l'Academie Royale des sciences par divers Savans, and lus dans ses Assemblees, Vol. 7, 343-382, Paris.
- Cundall, P. A. and Strack, O. D. L. (1979a), A discrete numerical model for granular assemblies, *Geotechnique*, 29, No. 1, 47-65.
- Cundall, P. A. and Strack, O. D. L. (1979b), The Development of Constitutive Laws for Soil using the Distinct Element Method, Third Int. Conf. Numerical Methods in Geomechanics, Aachen, 289-298.
- Cundall, P. A. and Strack, O. D. L. (1979c), The distinct element method as a tool for research in granular material, Part II, report to NSF, Dept. of Civil and Mineral Engineering, Univ. of Minnesota.
- de Josselin de Jong, G. (1977), "Constitutive Relations for the Flow of a Granular Assembly in the Limit State of Stress", Proc. 9th Int. Conf. on Soil Mech. and Found. Engg., Specialty Session 9, 87-95.
- Deresiewicz, H. (1958), "Stress-Strain Relations for a Simple Model of a Granular Medium," *Journal of Applied Mechanics*, ASME, 402-406.
- Digby, P.J. (1981), "The Effective Elastic Moduli of Porous Granular Rock", *Journal of Applied Mechanics*, ASME, Vol. 48, No. 4, 803-808.
- Duffy, J. (1959), "A Differential Stress-Strain Relation for the Hexagonal Close Packed Array," *Journal of Applied Mechanics*, Trans ASME, 88-94.
- Duffy, J. and Mindlin, R.D. (1957), "Stress-Strain Relations and Vibrations of Granular Media," *Journal of Applied Mechanics*, ASME, 585-593.
- Eshelby, J. D. (1957), "The Determination of the Elastic Field of an Ellipsoidal Inclusion and Related Problems," *Proceedings of Royal Society of London*, Vol. A241, 376-396.
- Hardin, B.O. and Black, W.L. (1966), "Sand Stiffness under Various Triaxial Stresses", *Journal of Soil Mechanics and Foundation Engineering*, ASCE, Vol. 92, 27-42.
- Hill, R. (1950), *The Mathematical Theory of Plasticity*, Clarendon Press, Oxford.
- Hill, R. (1967), "The Essential Structure of Constitutive Laws for Metal Composites and Polycrystals," *Journal of Mechanics and Physics of Solids*, Vol. 15, No. 2, 79-95.

Horne, M.R. (1965), "The Behaviour of an Assembly of Rotund, Rigid, Cohesionless Particles I and II", Proc. Roy. Soc. A, Vol. 286, 62-97.

Horne, M.R. (1969), "The Behaviour of an Assembly of Rotund, Rigid, Cohesionless Particles III", Proc. Roy. Soc. A, Vol. 310, 21-34.

Hutchinson, J. W. (1970), "Elastic-Plastic Behavior of Polycrystalline Metals and Composites," Proc. of Royal Soc. of London, Vol. A319, pp. 247-272.

Jenkins, J.T. (1988), "Volume Change in Small Strain Axisymmetric Deformations of a Granular Material", Micromechanics of Granular Materials, Eds. M. Satake, and J.T. Jenkins, Elsevier, Amsterdam, The Netherlands, 143-152.

Johnson, K.L. (1985), Contact Mechanics, Cambridge University Press, London.

Kishino, Y. (1988), "Disc Model Analysis of Granular Media", Micromechanics of Granular Materials, M. Satake and J. T. Jenkins eds., Amsterdam, 143-152.

Ko, H.-Y. and Scott, R.F. (1967), "Deformation of Sand in Shear", Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 93, No. SM5, 283-310.

Lade, P.V. (1975), "Elasto-Plastic Stress-Strain Theory for Cohesionless Soil with Curved Yield Surfaces," Report No. UCLA-ENG-7594, Soil Mechanics Laboratory, University of California at Los Angeles, 97.

Lade, P.V., and Duncan, J.M. (1973), "Cubical Triaxial Tests on Cohesionless Soil," Journal of the Soil Mechanics and Foundations Division, ASCE, Vol. 99, No. SM10, 793-812.

Lanier, J., and Zitouni, Z. (1988), "Development of a Data Base Using the Grenoble True Triaxial Apparatus," Proceedings of the Constitutive Equations for Granular Non-Cohesive Soils, Cleveland, 47-58.

Makhlouf, H. and Stewart, J.J. (1967), "Elastic Constants of Cubical-Tetrahedral and Tetragonal Sphenoidal Arrays of Uniform Spheres", Proceeding, International Symposium of Wave Propagation and Dynamic Properties of Earth Materials, Albuquerque, NM, 825-837.

Matsuoka, H. (1974), "Stress-Strain Relationships of Sands Based on the Mobilized Plane," Soils and Foundations, Vol. 14, No. 2, 47-61.

- Mindlin, R.D. and Deresiewicz, H. (1953), "Elastic Spheres in Contact Under Varying Oblique Forces", *Journal of Applied Mechanics*, Vol. 20, No. 3, 327-344.
- Misra, A. (1990), "Constitutive relationships for granular solids with particle slidings and fabric changes," Ph.D dissertation, Univ. of Massachusetts, Amherst.
- Mura, S. (1985), "Micromechanics of Defects in Solids," Martinus Nijhoff, The Hague, Netherlands.
- Newland, P.L. and Allely, B.H. (1957), "Volume Changes in Drained Triaxial Tests on Granular Materials", *Geotechnique*, Vol. VII, No. 1, 17-34.
- Oda, M. and Konishi, J. (1974), "Rotation of Principal Stresses in Granular Material in Simple Shear", *Soils and Foundations*, Vol. 14, No. 4, 39-53.
- Okochi, Y., and Tatsuoka, F. (1984), "Some Factors Affecting K_c Values of Sand Measured in Triaxial Cell," *Soils and Foundations*, Vol. 24, No. 3, 52-68.
- Prat, P. C. and Bazant, Z. P. (1991), "Microplane Model for Triaxial Deformation of Saturated Cohesive Soils", *Journal of Geotechnical Engineering*, ASCE. (in press)
- Rankine, W.J.M. (1857), "On the Stability of Loose Earth", *Phil. Trans. Royal Soc., London*.
- Reades, D.W., and Green, G.E. (1974), discussion of "Cubical Triaxial Tests on Cohesionless Soil", by Lade, P.V. and Duncan J.M., *Journal of the Soil Mechanics and Foundations Division*, ASCE, Vol. 100, No. GT9, 1065-1067.
- Rennie, B.C. (1959), "On the Strength of Sand", *J. of Aust. Math. Soc.*, Vol. 1, 71.
- Roscoe, K.H., Bassett, R.H. and Cole, E.R.L. (1967), "Principal Axes Observed during Simple Shear of Sand", *Proc. Oslo Geotechnical Conference*, 231-237.
- Rowe, P.W. (1962), "The Stress-Dilatancy Relation for Static Equilibrium of an Assembly of Particles in Contact", *Proc. Roy. Soc. A*, Vol. 269, 500-527.
- Serrano, A.A. and Rodriguez-Ortiz, J.M. (1973), "A Contribution to the Mechanics of Heterogeneous Granular Media", *Proc. of the Symp. on the Role of Plasticity in Soil Mechanics*, Cambridge, 215-228.

Skinner, A. E. (1969), "A Note on the Influence of Interparticle Friction on the Shearing Strength of a Random Assembly of Spherical Particles," *Geotechnique*, Vol. 19, No. 1, 150-157.

Stokoe, K.H., Lee, S.H.H. and Knox, D.P. (1985), "Shear Moduli Measurements under True Triaxial Stresses", *Advances in the Art of Testing Soils under Cyclic Conditions*, Ed. V. Khosla, 166-185.

Sutherland, H.B., and Mesdary, M.S. (1969), "The Influence of the Intermediate Principal Stress on the Strength of Sand," *Proceedings of the Seventh International Conference on Soil Mechanics and Foundation Engineering, Mexico*, Vol. 1, 391-399.

Taylor, D.W. (1948), *Fundamentals of Soil Mechanics*, J. Wiley and Sons, New York.

Thurston, C.W. and Deresiewicz, H. (1959), "Analysis of a Compression Test of a Model of a Granular Medium", *J. Appl. Mech.*, Vol. 26, 251.

Walton, K. (1977), "The Effective Elastic Moduli of a Random Packing of Spheres", *Journal of Mechanics and Physics of Solids*, Vol. 35, No. 3, 213-226.