TRANSFORMATION OF COMBAT DATA IN SUPPORT OF BATTLE TRACE

THESIS

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Preface

The purpose of this study, sponsored by TRAC-MTRY, was to find an appropriate way of characterizing battle data in such a manner as to remove the instability problems cited in their battle trace methodology. This research concerned itself only with an investigation of battle data and methods of displaying the results of combat.

Several Lanchester model based transformations were evaluated to suggest appropriate ways to measure relative combat effectiveness. Although the results of this analysis are purely demonstrative, the assessed models for battle trace will be valuable for further research.

I am in debt to a number of people who assisted me in completing this research. Major Hoffman of the TRAC-MTRY provided the data base and offered guidance in developing my research proposal and Colonel Baker provided assistance in accessing a Lanchester's Square Law transformation. Finally, I would like to express my thanks to Major Garrambone and Professor Reynolds, my thesis committee, for their invaluable assistance.

Hyoung-kyu Choi

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Abstract

The purpose of this study was to find an appropriate way of characterizing battle data as a function of time. This study was initially designed to remove the numerical instability problem cited in Barr's "battle trace" methodology as suggested by TRAC-MTRY. The research focused on the instability problem and identifying/recommending a technique that improved the efficiency of computation, and enhanced an analyst's ability to meaningfully interpret the battle trace. Two Lanchester's Square Law based methodologies are introduced. These methodologies were analyzed by observing battle trace plots and basic statistical changes associated with various battle segment interval sizes. The results of this analysis indicate that the battle trace of a constant value generated by Lanchester's Square Law integration seemed to be the best measure to characterize battle data as a function of time. This methodology has no computational problems, is numerically stable and efficient, and has a battle field interpretation with the proper interval size selection.

TRANSFORMATION OF COMBAT DATA IN SUPPORT OF BATTLE TRACE

I. Introduction

1.1 Background

Many military studies have sought to understand the essential combat factors that have led to success in battle. U.S. Army doctrine (Operations: Field Manual 100-5) identifies force strength, the effects of concentrated fire, and the use of maneuver as important factors in influencing the outcome of battles. Complementing these obvious parameters of battle, there exists a number of less tangible factors such as the level of training, troop motivation, leadership, determination, and audacity, which can also influence battle results (9:Ch 2, 4).

For the commander on the battlefield to perform astute decision making, the knowledge of such significant and intangible factors is paramount. Every commander must be aware of the status of friendly and enemy forces as well as the status of the combat environment. Centuries ago, Sun Tzu said in his work, "The Art of War," "Know the enemy, know yourself; your victory will never be endangered. Know the ground, know the weather; your victory will then be total" (17: 129).

Due to the complicated dynamic interactions within the combat environment, reasonable estimates of combat status are difficult to obtain. Since, military planners and commanders need to constantly improve their abilities to make combat decisions, methodologies have been developed to assist in learning about combat phenomena. Officers have improved their proficiency by studying historical battles, participating in field exercises, and by analyzing the output of computer simulations armed with an array of appropriate measures upon which to judge combat effectiveness.

Although many methods of estimating combat effectiveness have been developed, no single measure of combat effectiveness has been found to be suitable for all purposes. Of the existing measures, nearly all concentrate on understanding a battle based on end of battles results. Such summary analyses are only a first step in constructing explanatory models and fail to capture the dynamic combat factors which operate during the battle which effect battle outcomes (2:3, 4). Therefore, measures that can trace the battle process through time need to be generated (2:7). That is, there is a pressing need for the precise construction of a measure to trace the flow of the battle giving both an event history and perhaps a real time assessment of the status of the battle during the battle process.

This problem is currently being addressed by the U.S. Army Training and Doctrine Command (TRADOC) which is attempting to discover an effective way to measure combat effectiveness as a function of time. Such a measure might allow various combat factors to be identified once the trace of the event history of a battle is captured or reconstructed precisely in the form of an algebraic equation.

This requirement has led the TRADOC Analysis Command at Monterey (TRAC-MTRY) to investigate an approach suggested by Don Barr et al. to compute a measure of effectiveness in a "natural way." This approach is based on the concept of "Battle Trace" that was initially proposed by Dave Bitters at the U.S. Army Command and General Staff College (18:1).

The battle trace concept suggests that an entire battle can be decomposed into a discrete series of smaller time segments and that a combat evaluation model can be used to characterize each time segment's combat by scoring battle results. The "battle trace" is a graphical plot of such characterized battle data associated with time (2:18). This methodology uses a proportional expression based on a simple Lanchester model of combat to compare two opposing units' relative combat effectiveness in each time segment(2:15-21, 17:1). The proportional expression is called "combat force elasticity" and is defined as a ratio of the two opposing combatants' percentage change of measured combat effectiveness at any particular time. This approach can provide the battle analyst with not only a way to capture battle results, but also offer an efficient way of demonstrating the effects of battlefield synergism (2:7). Unfortunately, TRAC-MTRY encountered multiple numerical instabilities when it tried to employ the battle trace concept in practice.

1.2 Study Problem

As mentioned above, the methodology suggested by Barr et al. required the computation of a combat force elasticity. The problem with this parameter is its numerical instabilities "at points where force sizes and attrition events are small, sparse, or nonexistent" (18:1).

The numerical instabilities cited by TRAC-MTRY can arise when the size of time segment (arbitrarily chosen by the analyst) is too small to contain enough interactive activities between opposing units. That is, sometimes, values of combat force elasticity can be zero or near zero. Since this is a ratio of battle data, some values are more likely to be infinite or near infinity thereby confounding the utility of the battle trace concept. The existence of this instability problem prevents efficient explanation of the battle state and undermines the analytical power of battle trace as a combat evaluation tool.

1.3 Study Objective

The objective of this research was to find an appropriate way of characterizing battle data in such a manner as to remove the data instability problem identified in Barr's battle trace methodology. In addition, this study was expected to provide analysts with a computationally sound way of capturing battle results and demonstrating the effects of various battlefield phenomena (2:8).

1.4 Sub-objectives

This study contained three sub-objectives-the accomplishment of which provided a solution to the entire research problem. Those sub-objectives were:

1) to investigate the sources and characteristics of the representative combat data;

2) to explore methodologies for addressing the instability problem;

3) to identify/recommend a technique that satisfies the instability problem which is computationally sound and provides combat effectiveness insights.

1.5 Scope, Assumptions, and Limitations

This study is limited in scope by the source and nature of the data provided by the sponsor TRAC-MTRY. The data was generated from the U.S. Army's combined arms combat model, Janus(A), and was taken from a single 151 minutes mechanizedinfantry/tank battle. This data is assumed to be representative of typical model output derived from a standard combat scenario. It is recognized that the battle trace process attempts to capture the essence of a stochastic process (combat) in an aggregated deterministic (Lanchester-type equation) mathematical model and that this connection is assumed appropriate (25:21). In addition, it is important to recognize that the findings of this thesis are limited in application to the data provided, although wider application would be suggested.

For convenience sake, this study of battle trace assumes the following: 1) two opposing forces are homogeneous, 2) attrition rate of coefficients are independent of time and space, and 3) the battle is an aimed fire attrition process.

1.6 Summary

This chapter provided a background to the understanding of the TRADOC problem of battle analysis and the difficulty of using battle trace with its current instabilities. In addition, the chapter discussed the thesis objective and sub-objectives, and identified the scope, assumptions and limitations inherent in the study. The next chapter will discuss the literature related to this study.

II. Literature Review

2.1 Introduction

The purpose of this review is to discuss the literature relevant to this research. This study will discuss the data sources of the study, concepts supporting the use of battle trace, and some methods for transforming this data. In sequence, the NTC operations, the Janus(A) computer simulation combat model, a comparison of the Janus(A) to NTC battle data, some combat evaluation measures, Lanchester-type models, the battle trace formula, and some forecasting methodologies will be discussed.

The NTC and Janus(A) review provides background on the data used in this study since the data was generated by Janus(A) based on a scenario adopted from an NTC battle. Two simple Lanchester-type models are discussed prior to describing the battle trace methodologies.

A discussion of battle trace introduces the model framework that was developed by Barr et al. The log trace concept is discussed to give additive sense to the battle trace concept rather than to employ reciprocal representations.

Finally, some data smoothing methods such as moving average and exponential smoothing are reviewed.

2.2 NTC Operations

2.2.1 General. The National Training Center (NTC) is located at Fort Irwin, California and covers about 640,000 acres of the Mojave Desert. NTC has provided the U.S. Army with the most realistic war experience next to actual combat(24:40). The NTC concentrates on a battle of Soviet equipped type forces against U.S. Army maneuver units, usually at the brigade level. U.S. Army units spend up to a year preparing for their fourteen days of training at NTC.

The objectives of the NTC are defined in Army Regulation 350-50. Those objectives include the following: 1) to increase unit readiness for warfighting; 2) to provide tough and realistic training; 3) to provide feedback to army participants; 4) to provide a data source for training, doctrine, organization, and equipment improvements (6:3, 13:13). The realistic combat training environment of NTC ensures these objectives can be met.

2.2.2 Organizations. The units involved in NTC training can be divided into the following three groups: the visiting unit, the opposing force (OPFOR), and the Observer/Controller (O/C) group (22:3). Figure 1 shows these organizations.

2.2.2.1 The Visiting Unit. Fourteen U.S. Army brigades visit the NTC to conduct training every year. Usually, a brigade consists of a Brigade Headquarters, two

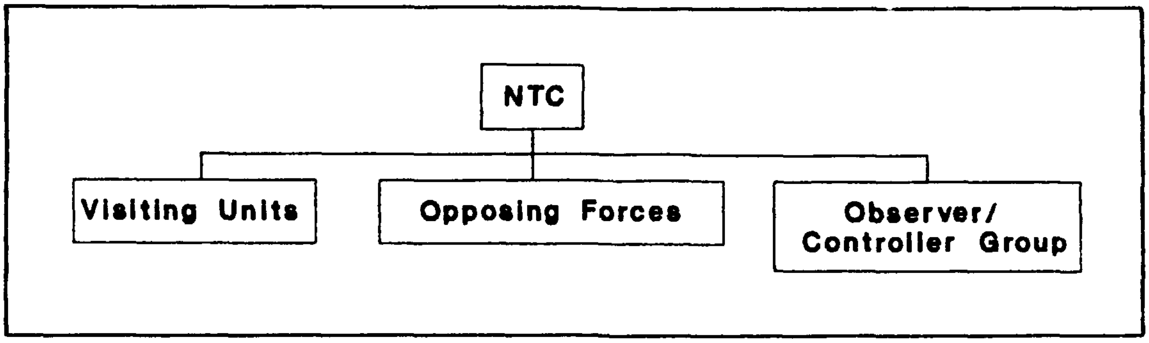


Figure 1. NTC Organization

heavy maneuver battalion/task forces, a field artillery battalion, a support battalion, two engineer companies, and other supporting units. A typical exercise involves a brigade conducting one week of preparation activities, ten days of force-on-force training, and four days of live fire training. During force-on-force training, each task force conducts five or six combat operations. Task forces participating in NTC combat training are equipped with the multiple integrated laser engagement system (MILES). MILES simulates weapon firing by generating a laser beam to the light receptors on an enemy target. This system of firing and recording provides realistic results of each direct fire combatant from the individual rifles to tanks and missiles (11:1-2). Results depending on accuracy are recorded as "either a near miss, a target hit, or a target kill" (22:4). Currently, this system is not available for aircraft or indirect fire.

2.2.2.2 The Opposing Force. The opposing force at the NTC is modeled after a Soviet Motorized Rifle

Regiment. This unit is permanently stationed at the NTC and support year round training. The OPFOR's mission is to provide to the training task force a tough, realistic enemy during their force-on-force battles (6:3). The OPFOR is equipped with simulated Soviet T-72 tanks, BMP armored amphibious infantry combat vehicles, and wheeled vehicles that resemble BRDM Armored amphibious reconnaissance vehicles (22:5; 10:Ch 5-15, Ch 5-21)

2.2.2.3 The Observer/Controller Group. The O/C observes all aspects of the training unit's operations, and helps conduct after action reviews. The O/C group works as "a battle arbitrator and safety agent," during the force-on-force battle (22:5).

The mission of the O/C group is particularly difficult, since the MILES system registers only firing results and cannot match who among the firer caused the kill. This loss of confirmed direct fire information is important for analysis, but a lack of actual indirect fire information makes even arbitration quite difficult. To do battle analysis, the results of every round fired must be known.

2.3 Janus(A)

2.3.1 General. Janus(A) is "a two-sided, interactive, closed, stochastic ground combat simulation." Janus(A) and its predecessor, Janus(T), was developed, maintained, and distributed by the U.S. Army TRADOC Analysis Command located at White Sands Missile Range, New Mexico (TRADOC-WSMR) as an analysis tools to support cost and operational effectiveness analysis (COEA), tactics and doctrine analysis, and other Army training studies (19:1). Additionally, Janus(A) has been used as "a high resolution scenario generator, a general defense plan (GDP) evaluator, and command post exercise (CPX) driver" (19:1). Recently, Janus(A) has supported additional types of training applications such as "company-level trainer, seminar or classroom trainer, and Operations Plan analyzer" (19:1). One of the major advantages of Janus(A) as a simulation model it is able to track and record all activities on the simulated battle field.

2.3.2 Simulation Type. "Janus(A) is interactive which means that model controllers and players make decisions during model execution (8:213). Janus(A) simulates two opposing forces, which are concurrently managed and controlled by two sets of players. The disposition of the opposing force is not completely known to the player in control of friendly force elements, unless reconnaissance finds these opponents (19:1). The major modeling focus in Janus(A) is upon the participating military systems in fire and maneuver on land (19:1).

Certain events simulated by "Janus(A) are not predetermined but occur according to laws of probability, and may or may not occur again if the same game is repeated"

(19:2). Thus, Janus(A) simulates the many combat stochastic processes.

2.3.3 Attrition Methodology. Janus(A) model contains tabled data of "probability of hit (PH) and probability of kill (PK)" for all possible direct fire engagements. These data sets are indexed on functions of "weapon and target movement, target orientation, target posture, and target range" (19:2).

For indirect fire engagements, the Janus(A) model specifies "probability distributions for aim errors and round-to-round dispersion errors" (19:2). These probability distributions are functions of range from firer-to-target for each type of indirect firer weapon system (19:2).

2.3.4 Resolution. As a high resolution combat model, Janus(A) normally simulates battalion to brigade size operations. The model can simulate movement routes, various speeds of ground vehicles, dismounted infantry operations, aircraft, logistics and resupply operations. Search and detection operations specified by sensors and radars performances are affected by weather (19:2).

Other functional areas portrayed by Janus(A) are direct and indirect fire, obscuration, maneuver through obstacles/barriers, minefields, minefield breaching, helicopters, and chemicals effects (8:347).

2.3.5 Model Output. Janus(A) records all significant events during the simulation (8:5). Through its post processor, raw and integrated reports can be created at the request of the model user. Those reports can include the following: 1) artillery summary report; 2) direct fire summary report; 3) killer-victim scoreboard; 4) force loss analysis; 5) system exchange ratio; 6) system contribution; 7) detections scoreboard; 8) range analysis (19:5, 8:326-346).

2.4 Comparison of the Janus(A) to NTC Battle Data

2.4.1 General. In June 1990, a study of the "Comparison of the Janus(A) Combat Model to National Training Center (NTC) Battle Data" was reported by Capt. David A. Dryer of TRAC-MTRY. A summary of this report describes the rationale for employing the Janus(A) combat model with NTC battle data and provides background on the set of data that was used in this research.

2.4.2 Purpose. The purpose of this comparison study was to qualify NTC laser battles as scenarios for use in Janus(A). The objectives of the study include the following: 1) to document the best NTC data qualification methodology for Janus(A); 2) to modify the Janus(A) database to replicate the NTC environment; 3) to conduct Janus(A) runs with a qualified NTC scenario; 4) to conduct comparisons between the NTC laser battle and Janus(A) scenario runs; 5) to note limitations in NTC data and suggest improvements in the Janus(A) combat model; 6) to highlight modeling inconsistencies between the NTC and Janus(A) (14:1).

2.4.3 Scenario Qualification. All available data sources are used to get the most accurate description of the actual NTC combat scenario. A graphical representation of the qualification methodology and data sources used is shown in Figure 2.

2.4.3.1 NTC Scenario Qualification. NTC scenarios provide the initial mission, task organization, concept of operation, and critical mission events. The NTC battle being used in this study is a modernized armored task force defense-in-sector mission which took place in the Siberia training area of NTC during FY 1988 (14:8).

Time period selected for the NTC scenario is limited to about four hours. Janus(A) can simulate a maneuver with at most forty nine path segments and the smallest interval size is five minutes. Thus, the scenario time period for the most accurate movement routes will be at most 245 minutes (14:14).

2.4.3.2 Janus(A) Data Base Modification. The Janus(A) standard combat scenario was altered to more closely replicate the performance of NTC combat systems, so modifications were made to the probability of kill tables, rates of movement, and weather conditions in the model (14:20).

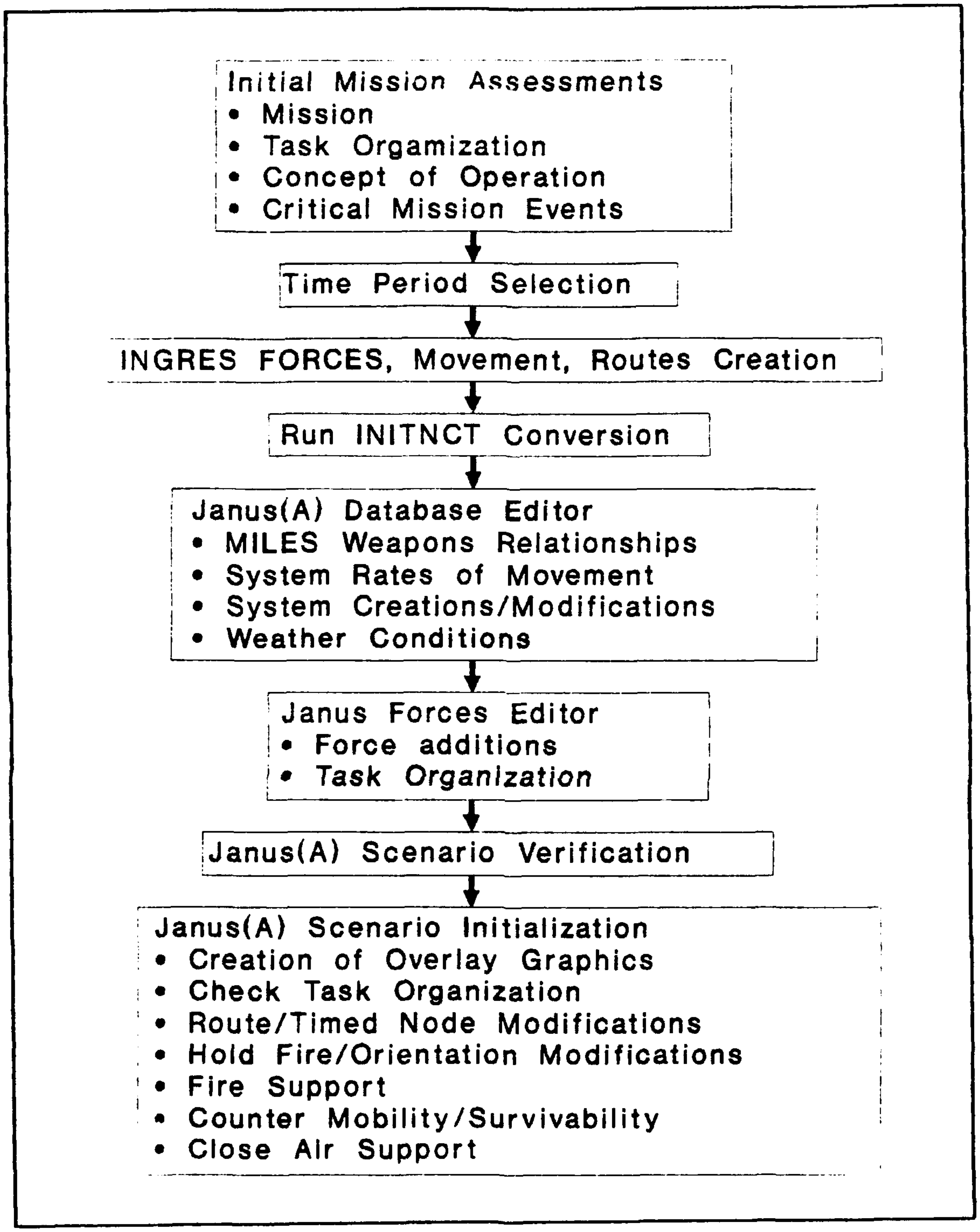


Figure 2. Janus-NTC Qualification Methodology (14:9)

2.4.4 Results of Comparison. Direct fire and attrition data were compared between the NTC mission and the

Janus(A) simulation with data observations and some statistical testing. The direct fire comparison shows statistically significant differences with Janus(A) firings being slightly lower than the actual NTC mission (14:43-44).

The attrition comparison shows that some kill rates in NTC are much lower than Janus(A). This means that the probability of hit (PH) values in Janus(A) are much higher than the average NTC performance. Moreover, Janus(A) does not model target overkill and weapon engagements beyond the weapon's maximum effective range (14:57-58).

2.4.5 Summary. While Dryer's report addresses the shortfalls between exercise performance and model performance, it also recognizes the inherent ability of combat models to better capture and record all the significant events occurring on the battle field a task beyond the current instrumented range capability. In addition, the reader is appraised of the model's capabilities to provide for the examination of all the cause and effect relationships required to perform true combat analysis.

2.5 Combat Evaluation (Measures of Effectiveness)

2.5.1 General. A large number of measures relating to battle processes are in use today. Those measures are used for predicting combat results, comparing combat systems, and assisting commanders in combat decision making.

Unfortunately, no one measure can satisfy all purposes or adequately accommodate any particular point of view on combat performance. Generally, any way of measuring combat effectiveness believed suitable for a certain purpose may be wholly unsuited for another (2:3). Moreover, various points of view on combat may require totally different ways of measuring combat effectiveness. Therefore, selecting a good measure of effectiveness is very difficult and may be the key element of analyzing combat effectiveness (2:3).

2.5.2 Attrition Rate of Coefficients. The "attrition rate" of a combatant, called the kill rate, is the basic concept employed in mathematical modeling of combat engagements (23:65).

> The attrition rates are expressed as numbers of individual targets (combatants, weapons, etc.) destroyed in a unit of time or as an area (target) effectively destroyed in a unit of time. (23:65)

The attrition rate may be linked to the total unit time required to destroy one combatant or weapon. The simplest relationship for this time T is represented as (23:66):

$$T = t + \frac{1}{\alpha P_s} \qquad (1)$$

where

T = the required total unit time

t = the acquisition time

α = the firing rate

P_S = the single shot probability

Therefore, the attrition rate coefficient κ can now be represented as:

$$k = \frac{1}{T} = \frac{1}{t + \frac{1}{\alpha P_s}} \qquad (2)$$

This equation is adopted from J. S. Przemieniecki's notation of "effective firing rate" and has a similar sense with F. W. Lanchester's attrition rate coefficient defined by Seth Bonder as "the average or expected rate at which a weapon system can destroy targets" (23:66; 3:222).

Finding the values for these attrition rates could be based on estimations of the time required to perform the activities, but for the most part these coefficients were drawn from statistical averages of combat losses from actual or exercise battles for select intervals of time. The raw data forms used to display the system-on-system deaths are known as killer-victim scoreboards.

2.5.3 Killer-Victim Scoreboards. A "killer-victim scoreboard" shows the numbers of each type of weapon killed by each type of weapon on the opposing side in a matrix format (7:Ch 30, 20). The killer-victim scoreboard can be created as a summary of the results of an entire battle or for any particular time interval of a battle. Table 1 shows a typical killer-victim scoreboard. Three different types of Blue weapons and two types of Red weapons are listed in Table 1. Each element, a_{ij}, on this killer-victim matrix

Table 1. Killer-Victim Scoreboard (7:Ch 30, 21)

Killer Weapons		Victim Weapons				
		Blue			Red	
		1	2	3	4	5
Blue	1	0	0	0	a_{14}	a_{15}
	2	0	0	0	a_{24}	a_{25}
	3	0	0	0	a_{34}	a_{35}
Red	4	a_{41}	a_{42}	a_{43}	0	0
	5	a_{51}	a_{52}	a_{53}	0	0

represents the number of j type weapons eliminated by weapons of type i. Note, there are no deaths here due to fratricide. Essentially, any element a_{ij} in this killer-victim matrix can be converted to a kill rate of the i^{th} weapons against the j^{th} target by simply dividing the average number of kills per weapon by the interval of battle time. DARCOM-P 706-102 cites this method of converting the killer-victim scoreboard to a matrix of kill rates as a method suitable for weapon system and battle analysis, especially, useful for large scale problems (7:Ch 30, 21-25).

2.5.4 Force Effectiveness Indicator. One measure of the effectiveness of a force is the "force effectiveness indicator (FEI)" which is defined in DARCOM-P 706-102 as the ratio of the total "value" of the Blue force to the total "value" of the Red force (7:Ch 26, 7). The following equation represents this FEI measurement:

$$FEI = \frac{TVB}{TVR} \tag{3}$$

where

TVB = the total force value for Blue which is computed as the sum of the products of the number of each type Red weapon destroyed multiplied by the value of that type Red weapon

TVR = the total force value for Red is computed similarly.

This method requires that the value of every weapon on the battle field must be known, either simply assigned based on expert opinion or derived through a mathematically attractive algorithm (i.e. eigenvalue equations).

The FEI discussed above attempts to show in a ratio fashion the damage each side inflicts on the other. This measure is a complex form of loss exchange ratio (LER) and suitable for measuring overall effectiveness of a mixed weapon system (7:Ch 26, 7).

2.5.5 Conclusion. A measure of effectiveness is a quantitative expression of a system's value under its objective (7:Ch 26, 22). Analysts may choose an appropriate way of measuring combat effectiveness depending on the study purpose. A selected measure is appropriate when the quantified value has a meaningful property (2:3). Thus far, measures such as the FEI and LER tell somewhat the story of the battle, but these measures only summarize and do not provide combat perspective or battle field insights. What is need is a more theoretically based measure.

2.6 Lanchester-type Model

2.6.1 General. Combat theory can be investigated by modeling and summarizing the battle in order that "predictions may be made of battle outcomes in terms of the major identifiable factors" (7:Ch 28, 1). Frederick William Lanchester was one of the first to try and describe the dynamics of combat mathematically. He concluded that evaluating a measure of quantitative combat effectiveness could determine the outcome of any battle (20:39-66).

Lanchester-type models are deterministic equations in the sense that a given set of input always arrives at the same output result. Even though combat is a complex random process, such deterministic combat models are commonly used to describe aggregate unit effects within applicable boundaries and have been shown to provide reasonable estimates of combat outcome (25:21).

Many different functional forms for Lanchester-type equations have been developed for different combat situations. In this section two simple differential equation models will be reviewed. These were originally considered by Lanchester in his "Aircraft in Warfare". "These models are fundamental for representing attrition and may still be considered to be a point of departure for modeling combat attrition" (25:21). The following parameters are defined for Lanchester-type models.

B = the size of Blue force

R = the size of Red force

B_0 = initial size of the Blue force

R_0 = initial size of the Red force

β = the constant attrition rate at which Blue forces are attrited by Red

ρ = the constant attrition rate at which Red forces are attrited by Blue

t = battle time

2.6.2 Linear Law. Lanchester initiated his studies by assuming the combatants to be of equal fighting value, and all other combat conditions being equal (20:41). For this circumstance, the attrition rate is assumed to be constant since both sides are using similar weapons and tactics. Lanchester's Linear Law may be defined as (7:Ch 28, 5):

$$\frac{dB}{dt} = -\beta, \qquad \beta, \rho, t > 0 \tag{4}$$

$$\frac{dR}{dt} = -\rho, \qquad B_0 >= B \text{ and } R_0 >= R \tag{5}$$

In these equations, each combatant will always fight against a single opponent, and therefore no useful effect is obtained by concentrating one's forces. Lanchester described this concept as the ancient combat conditions in which combatants have only direct aimed weapon systems.

2.6.3 Square Law. Lanchester's Square Law seemed to fit modern fighting conditions and is known as Lanchester's equations for modern warfare (7:Ch 28, 10; 25:23).

> With modern long-range weapons -- fire-arms, in brief -- the concentration of superior numbers gives an immediate superiority in the active combatant ranks, and the numerically inferior force finds itself under a far heavier fire, man for man, than it is able to return. (20:40-41)

In this model, individual fighting values were assumed to be equal and "the number of men knocked out per unit time will be directly proportional to the numerical strength of the opposing forces" (20:42). This concept is defined by following equations:

$$\frac{dB}{dt} = -\beta R, \qquad \beta, \rho, t > 0 \tag{6}$$

$$\frac{dR}{dt} = -\rho B, \qquad B <= B_0, \; R <= R_0 \tag{7}$$

These equations should meet the following assumptions:

1) Both forces are homogeneous and are continually engaged in combat.
2) Each unit or individual weapon is within the maximum weapon range of all of the opposing units.
3) Collateral damage within the target area is negligible.

4) The attrition coefficients β and ρ also include the probabilities of the target being destroyed when hit.
5) The effective firing rates are independent of the opposing force level.
6) Each unit is aware of the location and condition of all opposing units so that its fire is directed only to live units or functioning weapons. Thus, when a target is destroyed, search begins immediately for a new target.
7) Fire is uniformly distributed over surviving units. (23:73)

Here, the attrition rate is no longer constant, but depends on the opposing force size. Furthermore, These equations are separable (7:Ch 28, 11):

$$\rho B dB = \beta R dR \tag{8}$$

which after integration leads to:

$$\frac{\rho}{2}(B^2 + C_1) = \frac{\beta}{2}(R^2 + C_2) \tag{9}$$

where

C_1 and C_2 = the constants generated by integration

At time t = 0, R = R_o and B = B_o, so that:

$$\frac{\rho}{2}(B_o^2 + C_1) = \frac{\beta}{2}(R_o^2 + C_2) \tag{10}$$

subtraction Eq(9) from Eq(10), produces the following equation called Lanchester's Square Law:

$$\rho(B_o^2 - B^2) = \beta(R_o^2 - R^2) \tag{11}$$

The initial "fighting power" of Blue and Red depend of the squares of the numbers of Blue and Red forces (7:Ch 28, 11). If $\rho B_o^2 > \beta R_o^2$, then Blue has the advantage and will win the battle. Likewise, if $\rho B_o^2 < \beta R_o^2$, then Red has the advantage and when $\rho B_o^2 = \beta R_o^2$, then both Blue and Red are at parity. Figure 3 shows these relationships and a "standoff line" is introduced that indicates that neither side has an advantage (7:Ch 28, 12).

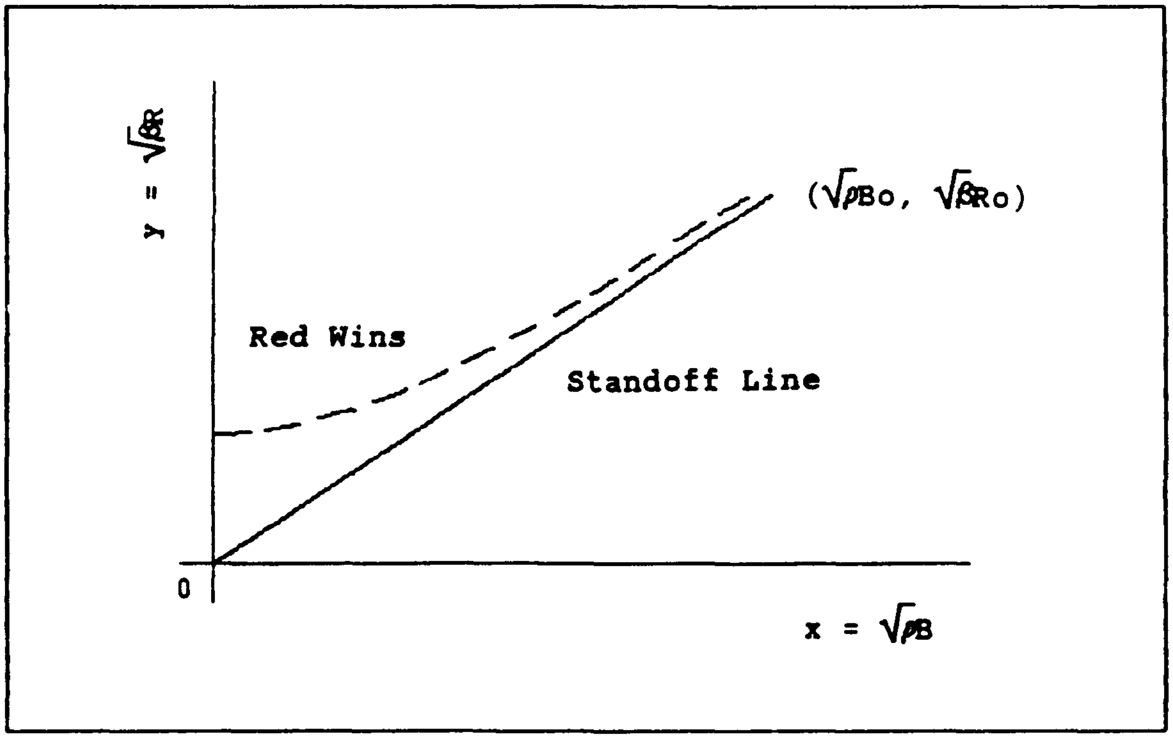


Figure 3. Trajectories of Square Law (7:Ch 28, 12)

2.6.4 Conclusion. Lanchester-type models are easy to compute and interpret, and are widely used in battle analysis. Their objective is to determine the size of force remaining after a set time of conflict and are equations are predictive of the end results of battle. That is, they do not readily address the real time status of the on going conflict.

2.7 Battle Trace

The following discussion introduces the concept of battle trace developed from the above Lanchester's Square Law. This concept is motivated by a need to characterize the battle status throughout time.

2.7.1 Model Development. Bar et al. introduced the term "combat force elasticity" (2:18). This concept is defined as a ratio of two opposing combatants' percentage changes of measured combat effectiveness. Lanchester's Square Law indicates that the initial fighting power, ρB_o^2 or βR_o^2, will decide the battle results. Bar et al. suggested that the ratio ρB^2 / βR^2 to be examined. In any form, the attrition rates, β and ρ, are difficult coefficients to estimate. Barr et al. suggested that the β and ρ values to be substituted by their values from Eqs (6) and (7) (2:15-16). These substitutions can be approximated by delta values, i.e. $dR/dt = \Delta R$ and $dB/dt = \Delta B$. Those substitutions and their approximations are shown on the following equations:

$$
\begin{aligned}
\frac{\rho B^2}{\beta R^2} &= \frac{\rho B}{\beta R}\frac{B}{R} \\
&= \frac{-\frac{dR}{dt}}{-\frac{dB}{dt}}\frac{B}{R} \\
&= \frac{\frac{\Delta R}{R}}{\frac{\Delta B}{B}}
\end{aligned}
\tag{12}
$$

where

ΔB and ΔR = losses of B and R, respectively, in a time interval of duration Δt

The last ratio formed in above equations is similar to the familiar economic concept of elasticity and hence is called the "combat force elasticity (CFE)" (2:18). Now, any battle trace can be represented as the plot of the CFE versus time over the course of a battle. Thus, for the k^{th} CFE, a form of Eq (12) can be rewritten as:

$$
CFE_k = \frac{\Delta R_k}{\Delta B_k}\frac{B_k}{R_k} \tag{13}
$$

where

CFE_k = combat force elasticity at k^{th} battle segment

Relating back to the Lanchester's Square Law, CFE_k in Eq (13) can be viewed as indicating the following: 1) $CFE_k < 1$, the Red force is winning at k^{th} battle segment; 2) if $CFE_k > 1$, the Blue force is winning at k^{th} battle segment; 3) if $CFE_k = 1$, the battle is stalemated at k^{th} battle segment.

In contrast to the Lanchester's Square Law assumptions, Barr et al. suggested that R_k and B_k represent only the active systems in the battle who contribute to combat by identifying targets, the seers so to speak. They advised that only the weapon systems which can see at least one enemy weapon system can contribute the battle results, in modern aimed fire battle. They said, "the intervisibility is a function of the movement, tactics, terrain, and synchronization of battlefield operating systems" (2:20). Therefore, this methodology can contain some real combat phenomena inside.

As a result of above considerations, Barr et al. suggested to allow changes of force size not only due to death by opposing system, but also due to the changes of the number of systems which can "see" opposing systems.

Using delta values, ΔB and ΔR, and counting seers on the battlefield can create condition where the number of systems in these categories can be zero in any time interval. In these cases numerical instabilities can occurred in the battle trace. That is the value of CFE can be infinite or near infinity when any denominator of CFE is zero or near zero. Barr et al. suggested a substitution methodology for this numerical problem. Such substitution methodology includes the following: 1) if ΔB and ΔR are both zeros, set CFE = 1 (the parity value); 2) if $\Delta B = 0$ but ΔR is not zero, set CFE = 2; 3) if either force is completely annihilated (so R = 0 or B = 0), the other side can automatically be scored by qualitative values such as win or lose, and no value of CFE is computed at that time interval; 4) if both R and B are zero, the battle is ended.

2.7.2 Log of CFE Trace. Ratio representations such as Eq (13) have reciprocal symmetry since they are simply a ratio of ratios. The reciprocal symmetry is difficult for decision makers to visualize. In fact, most decision makers are more familiar with additive symmetry than with reciprocal symmetry. Barr et al. suggested a log transformation on the CFE and hence the battle trace of this transformed data is called "log trace" (2:22). The standard battle trace is not symmetric about the standoff line CFE = 1, and plots for a given battle viewed from the B and R point of view are not simply reflections across this line. This means that portions of the battle traces falling between zero and one must be interpreted differently than those portions above the line CFE = 1 (2:22). Plots of the log trace do have symmetric interpretations viewed from B's and R's point of view, and contain an additive sense. The log transformation of the ratio CFE is shown in the following equation:

$$\log(CFE_k) = \log(\Delta R_k) - \log(R_k) - \log(\Delta B_k) + \log(B_k) \tag{14}$$

In this case, a transformed value of CFE greater than zero indicates that the B force is winning, and a negative value indicates that the R force is winning.

Since, the domain of the logarithm function does not include zero, the possible zero values on ΔR_t, R_t, ΔB_t, or B_t are factors of consideration. Barr et al. suggested "adding a small positive quantity to each of these terms before taking their logarithms" (2:22-23). He suggested adding 1.0 to each term, so when a value is zero the log of this value also can be computed as zero. This suggestion is shown in the following equation:

$$\log trace_k = \log(\Delta R_k + 1) - \log(R_k + 1) - \log(\Delta B_k + 1) + \log(B_k + 1) \qquad (15)$$

The value of one that is added to each term can ended up as a series of error terms thereby creating yet another problem. It was suggested by Barr that perhaps the data itself being of a stochastic nature might be made more useful if somehow it could be smoothed out in a time series fashion.

2.7.3 Conclusion. It is difficult to obtain data from actual battles that would allow computation of the battle trace through the course of a real time battle. However, within a simulated battle such as Janus(A) or training exercises such as those at the NTC, such data can be captured (2:20).

The battle trace concept was developed by adopting the Lanchester's Square Law to allowing changes of attrition rates, β and ρ, during a battle process. Such a trace was sufficient to allow Barr to initiate his research. However, numerical instabilities and reciprocal symmetry may be encountered indicating that Eq (13) is not a suitable model for characterizing battle data. The log trace was introduced to transform the reciprocal symmetry of CFE into additive symmetry system, but the approximation by adding one to each component of log of CFE can ended up as a series of error terms. Hence, this research was undertaken to investigate ways to solve these numerical instabilities problems through elimination or data smoothing methods with additive symmetry system of equation.

2.8 Forecasting Methodologies

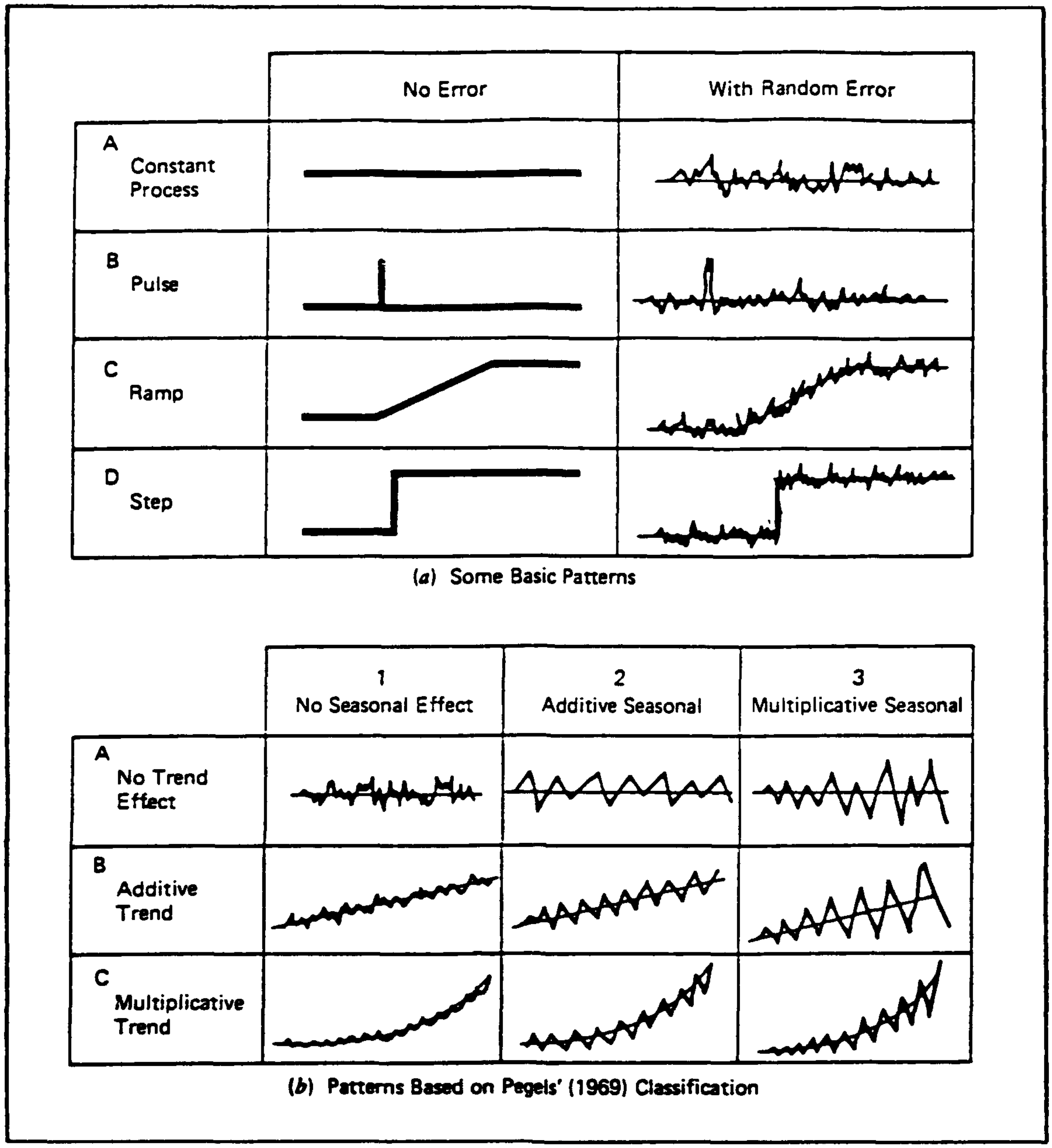
2.8.1 General. Forecasting is a prediction methodology of future values based on known past values (21:889). Two major distinct approaches of forecasting are explanatory forecasting and time series forecasting.

Time series forecasting deals with the system as a "black box" and makes no attempt to discover the factors affecting systems behavior, while explanatory forecasting assumes a "cause and effect relationship" between the inputs and outputs of the system (21:17-18).

Based on the battle trace concept, the time series relationship approach is preferred to the explanatory relationship approach, since the battle trace concept is expected to represent summarized relative attrition as a measure of combat effectiveness throughout the combat stages (2:17-21). Additionally, forecasting methods may support this research based on two reasons. First, the data constituting a battle trace are a time series. Second, the purpose of battle trace is not only for defining the combat status, but also for predicting battle results.

2.8.2 Overall Strategy. Any forecasting data smoothing methodology follows a general strategy: 1) identify the time series of interest; 2) divide the data set into an initialization set and a test set; 3) choose the appropriate forecasting method; and 4) optimize the model structure (21:65-67).

2.8.3 Data Identification. The identification of the appropriate data pattern is important for getting an appropriate data smoothing model. The basic patterns are classified by error terms and the underlying process, while the other patterns are classified based on trend and seasonal effects. Figure 4 shows these two general data classifications.



(a) Some Basic Patterns

(b) Patterns Based on Pegels' (1969) Classification

Figure 4. Data Pattern for Forecasting
(Reprinted from 4:69)

Clearly, an inappropriate forecasting data smoothing model, even when optimized, will not be as effective as choosing an appropriate forecasting model in the first phase (21:68-69).

2.8.4 Averaging Methods. This section will cover several straightforward averaging methods. They are the mean, simple moving averages, double moving averages, and higher order moving averages. The overall purpose of these methods is to make use of past data to develop a forecasting system for future periods.

2.8.4.1 The Mean. This method is called the method of simple averages and is appropriate when an underlying process is constant, has no trend effect, and has no seasonal effect. The total average value of the initial set is used for the forecasting value. As noted below, the average value will include the next data point at period t+1 (21:70-71):

$$F_{t+1} = \frac{1}{t}\sum_{i=1}^{t} X_i \tag{16}$$

$$F_{t+2} = \frac{1}{t+1}\sum_{i=1}^{t+1} X_i \tag{17}$$

where

t = time period

X_t = observation value at period t

F_t = forecasting value

2.8.4.2 Single Moving Average. This method employs the average value of the fixed number of past observations as a moving average. The number of data points in each average remains constant and includes the most recent observations. Depending on the number of periods in the moving average, this method can be used to smooth out seasonal effects. In general, the larger number of periods represent the greater smoothing effect. The forecasting formula of this method can be written as (21:73-74):

$$F_{t+1} = \frac{1}{t}\sum_{i=1}^{t} X_i \tag{18}$$

$$F_{t+2} = \frac{1}{t}\sum_{i=2}^{t+1} X_i \tag{19}$$

where

t = time period

X_t = observation value at period t

F_t = forecasting value

<u>2.8.4.3 Double Moving Average</u>. This method, called the linear moving average, uses the average value of the previous moving average as the moving average. This method is good for modifying a significant trend effect. The general linear moving average procedure is shown in following equations (21:79):

$$S_t^1 = \frac{X_t + X_{t-1} + \ldots + X_{t-N+1}}{N} \tag{20}$$

$$S_t^2 = \frac{S_t^1 + S_{t-1}^1 + \ldots + S_{t-N+1}^1}{N} \tag{21}$$

where

t = time period

S^1_t = single moving average

S^2_t = double moving average

a_t = adjusted smoothing value for period t

b_t = trend component

m = number of periods ahead of t

F_{t+m} = forecasting value obtained m periods ahead of t

2.8.5 Exponential Smoothing Methods. These methods assume that the weights assigned to observations are exponentially decreasing as they are getting older. Many applications of exponential smoothing models have been made to forecast time series (15:1035). In fact, this method is one of the most widely used methods among the explanatory forecasting techniques. Reasons for this are simplicity of the predictor, broad applicability, and computing efficiency (4:445). The general form of the equation used in the method of exponential smoothing substantially reduces any storage problem, because one only needs to store the most recent observation, the most recent forecast, and a value for the smoothing parameter (21:86). One general form is called single exponential smoothing (SES). The basic equation for this method is the following:

$$F_{t+1} = \alpha X_t + (1-\alpha) F_t \qquad (22)$$

where

t = time period

X_t = observation value at period t

F_t = forecast value at period t

α = smoothing parameter

2.8.5.1 Single Exponential Smoothing. An adaptive approach, named "Adaptive-response-rate single exponential smoothing (ARRSES)," allows the value of α shown on Eq (10) to change. This α value will changes when there is a change in the basic data pattern as shown in following equations (21:91):

$$F_{t+1} = \alpha_t X_t + (1-\alpha_t)F_t \tag{23}$$

$$\alpha_{t+1} = \left|\frac{E_t}{M_t}\right| \tag{24}$$

$$E_t = \beta e_t = (1-\beta)M_{t-1} \tag{25}$$

$$M_t = \beta\left|e_t\right| + (1-\beta)M_{t-1} \tag{26}$$

$$e_t = X_t - F_t \tag{27}$$

where

t = time period

X_t = observation value at period t

F_t = forecasting value

α, β = smoothing parameters between 0 and 1

E_t = smoothed error term at time t

M_t = absolute error term at time t

e_t = error term at time t

2.8.5.2 Brown's One Parameter Linear Method. The idea of this method is to use the concept of linear moving averages. This method requires only three data values and one smoothing parameter to be saved. The equations representing in this method can be written as (21:94):

$$S_t^1 = \alpha X_t + (1-\alpha)S_{t-1}^1 \tag{28}$$

$$S_t^2 = \alpha S_t + (1-\alpha)S_{t-1}^2 \tag{29}$$

$$a_t = 2S_t^1 - S_t^2 \tag{30}$$

$$b_t = \frac{\alpha}{1-\alpha}(S_t^1 - S_t^2) \tag{31}$$

$$F_{t+m} = a_t + b_t m \tag{32}$$

where

t = time period

X_t = observation value at period t

F_t = forecasting value

α = smoothing parameter between 0 and 1

$S^1{}_t$ = single moving average

$S^2{}_t$ = double moving average

a_t = adjusted smoothing value for period t

b_t = trend component

m = number of periods ahead of t

F_{t+m} = forecasting value obtained m periods ahead of t

2.8.5.3 Holt's Two Parameter Method. This method uses the double smoothing formula. This provides greater flexibility and allows the trend effect to be smoothed with a different parameter. Therefore, this method is appropriate if there is a trend in the data. The equations for this method can be formulated by using the equations of Brown's one parameter linear method, shown above, and the following three equations (21:97):

$$S_t = \alpha X_t + (1-a)(S_{t-1} + b_{t-1}) \tag{33}$$

$$b_t = \gamma(S_t - S_{t-1}) + (1-\gamma)b_{t-1} \tag{34}$$

$$F_{t+m} = S_t + b_t m \tag{35}$$

where

γ = smoothing parameter for randomness

2.8.5.4 Brown's One Parameter Quadratic Method. When the basic underlying pattern of the data is quadratic or higher order, an additional level of smoothing technique is needed. The equations for quadratic smoothing can be formulated with following equations (9:100):

$$S_t^1 = aX_t + (1-a)S_{t-1}^1 \tag{36}$$

$$S_t^2 = aS_t^1 + (1-a)S_{t-1}^2 \quad (37)$$

$$S_t^3 = aS_t^2 + (1-a)S_{t-1}^3 \quad (38)$$

$$a_t = 3S_t^1 - 3S_t^2 + S_t^3 \quad (39)$$

$$b_t = \frac{\alpha}{2(1-\alpha)^2}[(6-5\alpha)S_t^1 - (10-8\alpha)S_t^2 + (4-3\alpha)S_t^3] \quad (40)$$

$$b_t = \frac{\alpha}{2(1-\alpha)^2}(S_t^1 - 2S_t^2 + S_t^3) \quad (41)$$

$$F_{t+m} = a_t + b_t m + \frac{1}{2}m^2 \quad (42)$$

where

t = time period

X_t = observation value at period t

F_t = forecasting value

α = smoothing parameter between 0 and 1

$S^1{}_t$ = single moving average

$S^2{}_t$ = double moving average

$S^3{}_t$ = triple moving average

a_t = adjusted smoothing value for period t

b_t = trend component

m = number of periods ahead of t

F_{t+m} = forecasting value obtained m periods ahead of t

2.8.6 Comparison of Data Smoothing Methods. A variety of methods have been presented in this chapter. The remaining question is how a forecaster can choose the right model for a data set. This is one of the major questions to be addressed by this study. Identifying an appropriate model is much more important than improving model accuracy (5:559).

One useful suggestion is to try to identify the existence of the nature of trend and seasonality in the data. The overall observations can show the significant pattern by comparing the pattern classified in figure 3. Moreover, the seasonality can be eliminated by plotting the moving averages of the data (21:115).

Another approach is modeling the autocorrelation structure in a stationary time series (12:1244; 16:1237). The correlation of the time series with itself, autocorrelation, can be denoted by correlation coefficient r_k, as follows (21:366):

$$r_k = \frac{\sum_{t=1}^{r-k} (Y_t - \overline{Y})(Y_{t+k} - \overline{Y})}{\sum_{t=1}^{n} (Y_t - \overline{Y})^2} \qquad (43)$$

where

t = time period

Y_t = demand

r_k = correlation coefficient for k time leg

According to a study conducted by Anderson, Bartlett et al. and described by Spyors Markridakis, this autocorrelation coefficients of random data are normally distributed and statistical test can be applied (21:364-365).

2.8.7 Summary. The strategy of estimating data smoothing methods in forecasting follows some general steps such as data identification, smoothing method selection, and optimization. The major advantages of smoothing methods in forecasting are their simplicity and lower cost. In general, the more sophisticated methods are believed to give the higher possibility of achieving good accuracy. However, simpler methods can be found which may give even better accuracy (5:559). The procedure of adaptive or exponential smoothing an easy, quick, and inexpensive (26:198).

2.9 Summary

This chapter discussed the literature relevant to this research. This review revealed that the simulated real world environments of the NTC allows realistic battle data to be generated. Furthermore, it showed that the Janus(A) high resolution combat computer simulation model enables a precise combat simulation to be created. Based on the above, the NTC scenario based Janus(A) combat simulation can provide realistic and accurate combat data output. Finally, the simple Lanchester's combat models and Barr's battle trace concept were reviewed. The next chapter will discuss the database and its conversion to accommodate battle trace usage.

III. Data Base

3.1 Background

3.1.1 Data Source. TRAC-MTRY provided two files of data which contained the trajectories of a single battle for this study. One of the data files, named BTRACE.TST, provides a history of the presents of actively searching weapon systems throughout the battle field. The other data file, named KTRACE.TST, provides the killer-victim information.

The data in these files were extracted from a TRAC-MTRY technical report (TRAC-RDM-191, June 1991), "Comparison of the Janus(A) Combat Model to National Training Center (NTC) Battle Data." This set of battle data was generated by Janus(A), a high resolution combat computer simulation model. The scenario used in Janus(A) was modified from a typical NTC battle scenario described below.

3.1.2 Battle Description. The NTC scenario used in this study was from a simulated battle between five U.S. Army modernized armored task forces and a Soviet motorized rifle regiment that occurred in the Siberia training area of the NTC during 1988 (14:8). The U.S. forces are called "Blue forces" and the Soviet-type forces are called "Red forces" in this study.

3.1.2.1 Missions. The Blue forces' mission was to defend in sector, to destroy enemy forces, and to preclude enemy forces penetration. The commander's intention was to "deceive the enemy as to the location of primary positions and fight an aggressive counter-reconnaissance battle by positioning elements forward" (14:8-10). After the counter-reconnaissance battle had ended, the commander intended to shift these forces into positions in-depth, and to destroy the enemy in selected engagement areas.

The mission of the Soviet motorized rifle regiment, the Red forces, was to conduct a regimental attack. The purpose was to penetrate forward positions, destroy the majority of Blue forces' combat power, and have enough combat power remaining at the objective to conduct a follow-on attack (14:13).

3.1.2.2 Weapon Systems. The BTRACE.TST file contains the types of weapon systems and the corresponding number of systems which "see" at least one enemy system on battlefield. This file contains data on every 20 second interval of the battle. This data file contains 11 different weapon systems for both sides as shown in Table 2.

The Blue weapon systems represent those of a typical U.S mechanized/armored task forces, while the Red equipment is modeled after a Soviet motorized rifle regiment. The Soviet motorized rifle regiment is the basic combined arms

Table 2. System Types

BLUE SYSTEMS (11)	RED SYSTEMS (11)
SP 155mm ARTY	MORTAR 120mm
MORTAR 4.2'	SP 122mm ARTY
TANK	SP 152mm ARTY
TOW	122mm MRL
BRADLEY WITH TOW	BRDM
BRADLEY WITHOUT TOW	T-72 TANK
FASCAM	ZSU
VULCAN	DISMOUNTED SOLDIER
STINGER	BMP
CEV	TRUCK
M113 WITH TOW	SA-7

organization of the Soviet Army (10:Ch 4, 26). Several weapon systems of this Soviet regiment are described below as further reference:

- BRDM, a fully armored, four-wheel drive, amphibious reconnaissance vehicle;
- BMP, a fully armored amphibious infantry combat vehicle;
- T-72 medium tank;
- 152mm self-propelled Howitzer;
- 122mm self-propelled Howitzer;
- 120mm Mortar;
- 23mm ZSU, a fully integrated, self-propelled antiaircraft system;
- 122mm MRL, multiple rocket launcher
- SA-7, a man-portable, shoulder-fired, low-altitude, surface-to-air missile system (10:Ch 5).

The system types of Blue forces and Red forces shown on Table 2 are similar but not exactly the same. That is, this NTC battle is not actually homogeneous, but the nonhomogeneous issue is beyond the scope of this study. For the convenience of this study, their comparative similarity is believed to be enough to satisfy the homogeneity assumption.

3.1.2.3 Combat Strength. The initial combat strength of either force was not clearly provided by the BTRACE.TST data file. Fortunately, the TRAC-MTRY report, "Comparison of the Janus(A) Combat Model to National Training Center Battle Data," provides the initial unit strengths, which are shown in Table 3 below.

Table 3. Key System Starting Strength
(Reprinted from 14:17)

Key BLUEFOR Start Strengths

SYSTEM	DIGITAL TOTAL	ACTUAL TOTAL IN NTC ILLUSTRATIVE MISSION					
		Tm A	Tm B	Tm F	Tm D	TF Ctrl	Total
Tank	40	10	13	4	10	2	39
Bradley(M2/M3)	20	7	0	7	0	4	18
ITV	6	0	0	0	5	0	5
APC	15	3	2	5	3	5	18

Key OPFOR Start Strengths

SYSTEM	DIGITAL TOTAL	ACTUAL TOTAL IN NTC ILLUSTRATIVE MISSION				
		Fwd Det (2d MRB)	1st Ech (3d MRB)	2d Ech (1st MRB)	Regt Ctrl	Total
T-72	52	13	26	16	1	56
BMP	103	21	46	32	9	108
BRDM	4	1	1	0	5	7
MTLB	0	16	0	0	0	16

3.2 Form of Data

As mentioned earlier, the two data files (BTRACE.TST, KTRACE.TST) were collected from the Janus(A) combat simulation output. These two files were to provide the number of systems that have acquired at least one opposing side target during the 20 second interval and a chronological history of battle field deaths referred to as killer-victim data.

3.2.1 Force Size Data. Appendix A shows the form of the given BTRACE.TST data file. This file includes 741 records containing 33 fields of data. The first row of this file provides the weapon system type numbers (eleven system types for both forces) and the task force numbers (five task forces for both sides). The rest of this file is a record of the number of systems of a particular type or in a specific task force that can "see" at least one enemy system at the simulation time specified, every twenty seconds. 741 records of system "seers" were recorded during the total time runs from zero to 247.0005 minute mark.

3.2.2 Killer/Victim Data. Appendix B shows the form of the given KTRACE.TST data file. This file provides the killer/victim data associate with its event time. The total number of record is 203 with the time running from 0.0606 to 150.9589 minutes. This file includes the following data: 1) the simulation time of each kill event; 2) killer type (1 = direct fire, 2 = indirect fire); 3) range of engagement;

4) side of victim (1 = Blue side, 2 = Red side); 5) system type of victim; 6) task force of victim; 7) coordinates of victim when killed; 8) number of systems killed; 9) side of killer (1 = Blue, 2 = Red); 10) system type of killer; 11) task force of killer; 12) coordinates of killer when shot was fired; and 13) type of ammunition fired by killer.

3.3 Initial Data Conversion

3.3.1 Background. The initial data file was converted to fit the data requirements for calculating the battle trace within proper intervals. The data was converted by consolidation, unification of the two data files, and data aggregation.

3.3.2 Data Consolidation. The given two data files were simplified by consolidating only the required data for the battle trace analysis. That is, for both sides one needed only the force strength and the number of death within the battle time segments. In terms of variables, only system deaths and number of seers (ΔB, B, ΔR, and R) are required to apply the battle trace concept. These data requirements are provided by the assumption that the Lanchester attrition rates, β and ρ, can be computed by the historical battle data.

In the BTRACE.TST file, the seers are recorded by system type and task force. Because only the total number of the seers at each battle segment is required to apply the battle trace concept, the last ten columns of BTRACE.TST file, i.e. data by force, can be consolidated. In the KTRACE.TST file, the event time (the first column), the side of victim (the fourth column), the number of systems killed (the eighth column), and the side of killer (the ninth column) needed to be collected.

3.3.3 Data Aggregation and File Unification. Another data conversion was required to aggregate and partition the data recording times of the two data files. The BTRACE.TST data file was posted every twenty seconds, while the KTRACE.TST data file posted kills at each kill event time. One way of matching these two files is to reconstruct the KTRACE.TST file with the same time interval format as the BTRACE.TST data file, that is, sum the number of killed systems in each 20 seconds time interval. This ensures the two files will have the same "time format" according to a chosen interval unifying the two given data files. Now, all file entries correspond to activities captured within a 20 second interval.

A BASIC computer program was developed for the initial data conversion process. The flow chart for this BASIC program is shown on Figure 5 and the BASIC program itself can be found in Appendix C. The converted data is contained in Appendix D.

The following symbols are used in the conversion program:

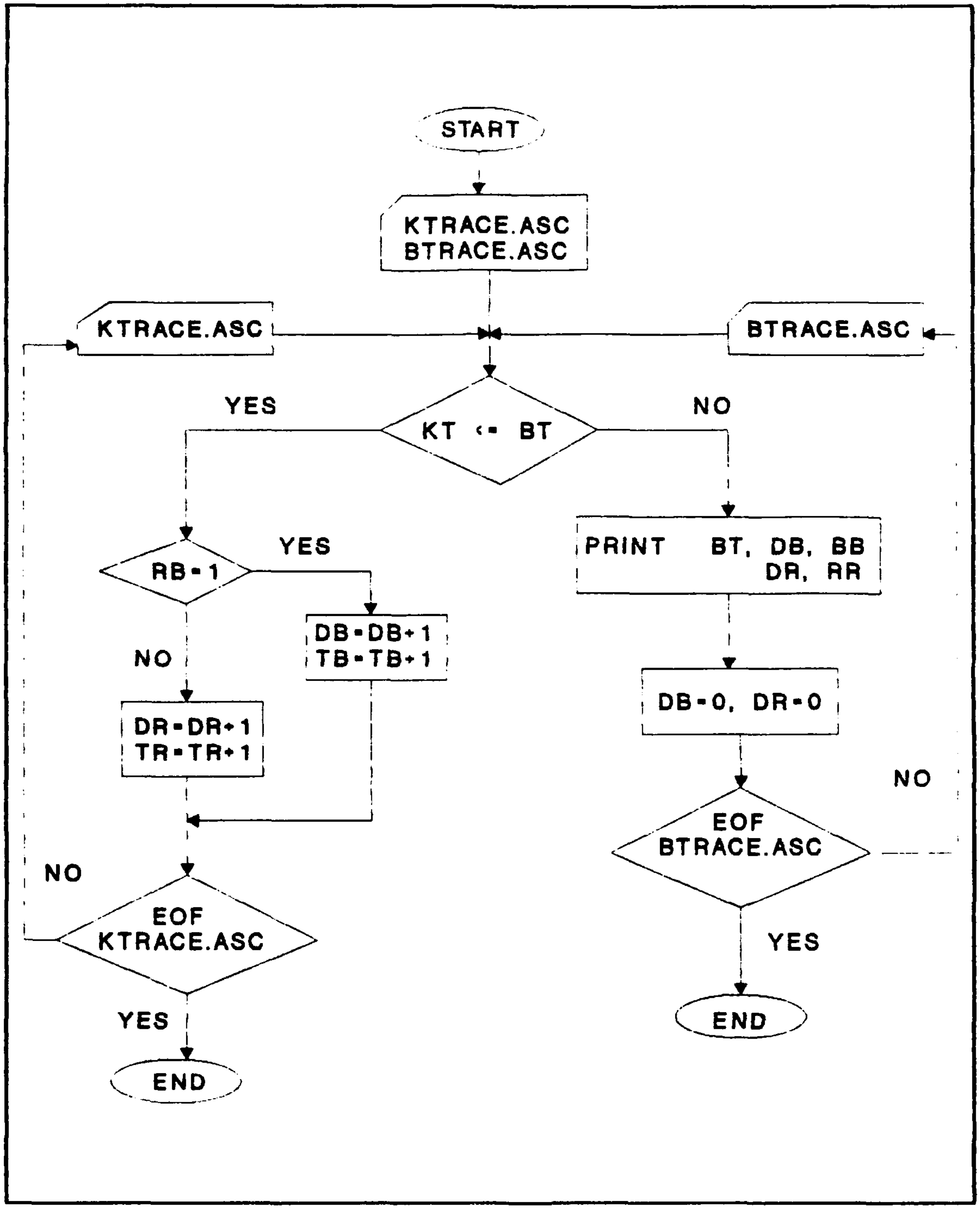


Figure 5. Initial Data Conversion Flow Chart

- START = starting interval;
- KT = kill time;
- CT = battle segment count;

- BT = time that each BTRACE.TST file data was recorded;
- RB = side indicator, 1 is Blue force 2 is Red force;
- ΔB = number of Blue systems killed;
- ΔR = number of Red systems killed;
- TB = cumulated number of Blue systems killed;
- TR = cumulated value of Red system killed;
- ID = end of program identification
- .ASC = American Standard Code for Information Interchange (ASCII) file that was generated from the original .TST file;
- BB, RR = sum of the number of system in each time segment with respect to Blue forces and Red forces.

This program provides both Red and Blue forces' cumulative number of killed and the number of systems that can see at least one opposing system for each battle segment.

3.4 Data Overview

3.4.1 General. The converted data file that was generated from the conversion program enables one to view pieces of the battle over time using some typical extraction methods. In particular, deaths and seers over time, cumulative system deaths, and killer-victim scoreboards are used to see the battle. Data in this form help to more easily track the suggested battle trace concept, and the combat force elasticity. This data can now be applied to review the battle process and examine the efficiency and suitableness of the suggested battle trace methodology.

3.4.2 Cumulative System Deaths. The cumulative number of system deaths through time can give a sense of a units changing battle posture. Figure 6 shows the cumulative number of system deaths of both Blue and Red forces.

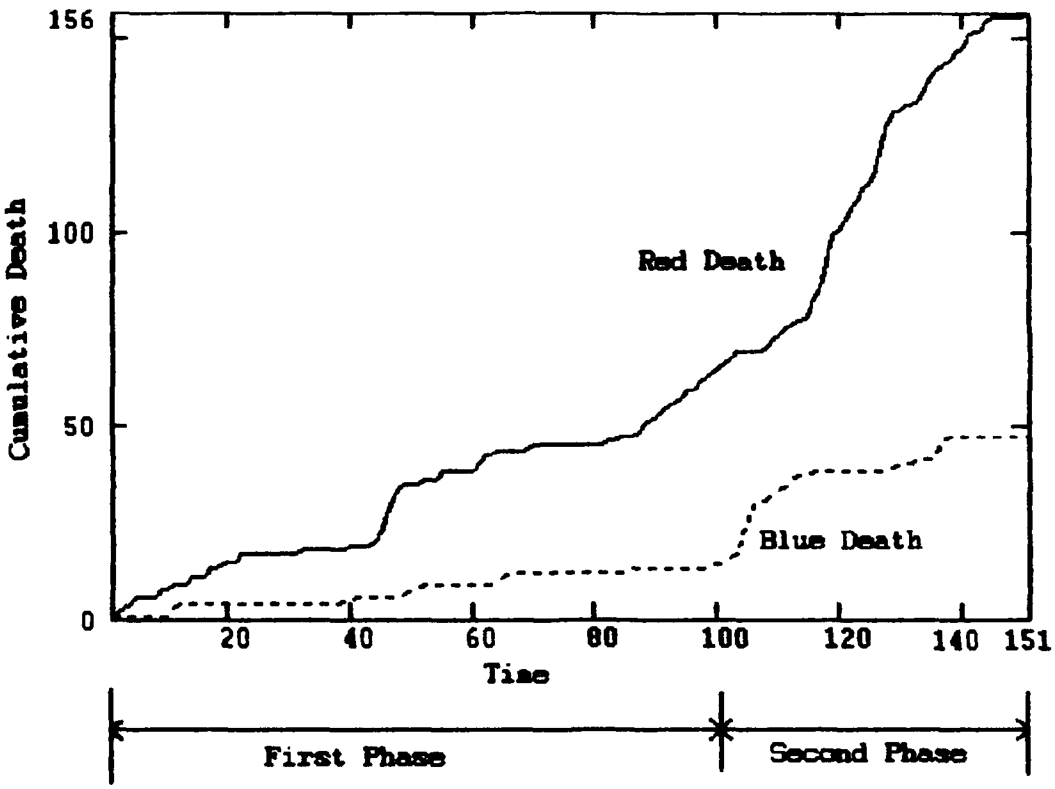


Figure 6. Cumulative System Deaths Over time

As shown on this figure, the system deaths of both Blue and Red forces rapidly increased after 100 minutes and there are nearly zero deaths recorded by either side after

145 minutes (only one death on Red's side at the 151 minute mark). Notice that, the number of Red system deaths is much more significant than the Blue system deaths.

By simple observation, the cumulative system death plots can provide two pieces of information. First, the observations indicate that this battle can be divided into two phases, the first phase is zero to 100 minutes and the second phase is 100 to 145 minutes. And now the two phases can be evaluated separately with other measures of combat effectiveness.

Second, the termination or end point of this battle is at 151 minutes and the analysis will be conducted only from the zero to 151 minute mark.

3.4.3 Deaths and Seers over Time. Based on Barr's interpretation of the Lanchester's Square Law, a battle can be simply represented as a function of both forces' system deaths, force strength (seers), and the attrition rates. By analyzing a battle's history data, that is, plotting the number of system deaths and the number of seers throughout the battle can assist in analyzing the battle from the Lanchester's Square Law perspective. The plots shown in Figure 7 were generated for this purpose.

Obviously, a new plotting method that can aggregate system deaths and seers information throughout the battle process can make for efficient battlefield interpretation and investigation of the battle processes. This requirement

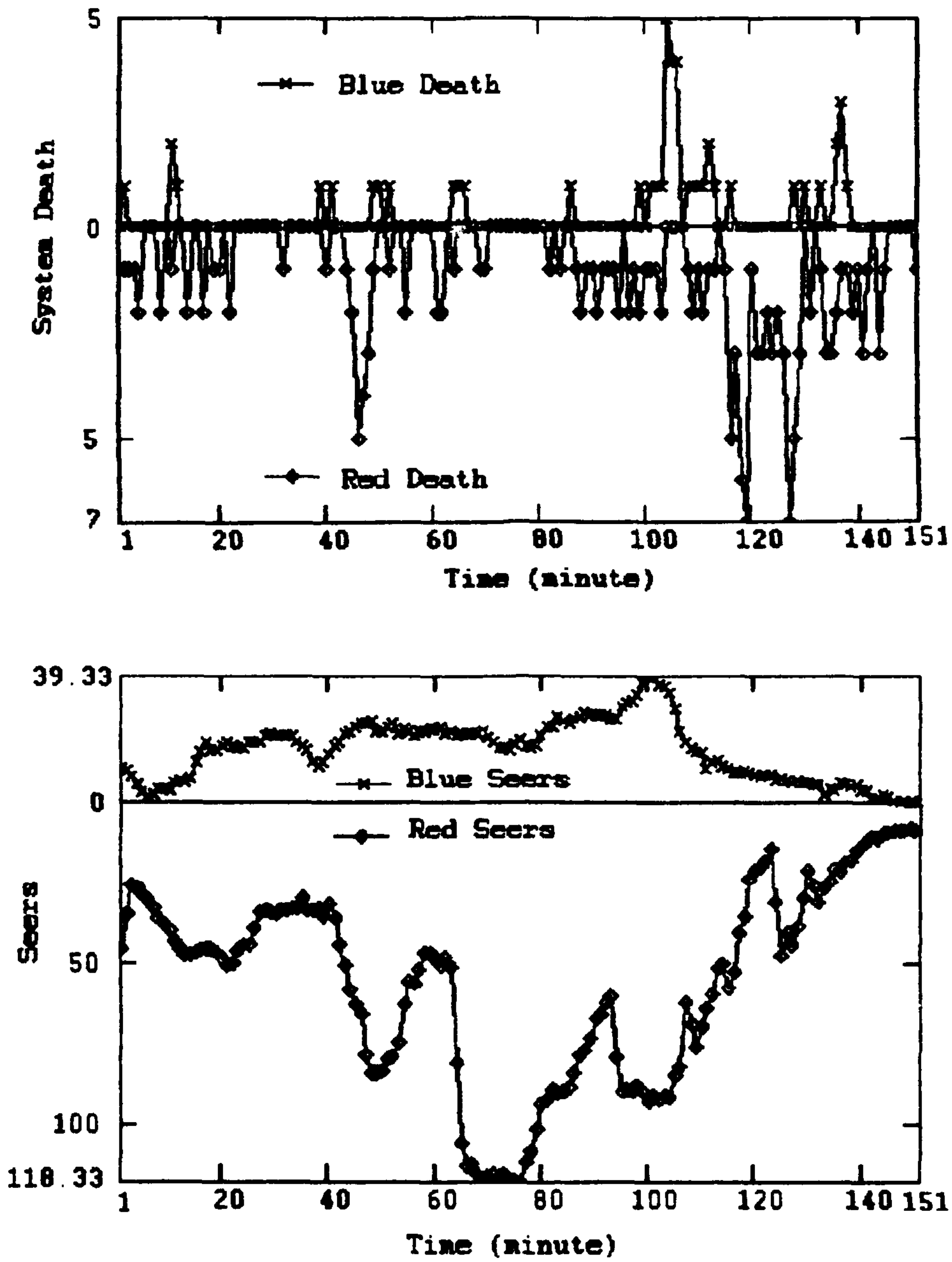


Figure 7. Deaths and Seers Over time

was one of the main forces behind the development of the battle trace concept.

3.4.4 Killer-Victim Scoreboards. The killer-victim scoreboard (K-V scoreboard) is a matrix that shows how many of each weapon system were eliminated by other type weapon systems. The K-V scoreboard can be obtained as a summary chart of an entire engagement or just a portion of the engagement. The total battle was divided into two phases as shown on Cumulative system death plot observations in Chapter 3.4.2. The K-V scoreboard can be made in the form of system-on-system, or task force-on-task force formats. Appendix E include tables for K-V scoreboards. For those scoreboards, killer type weapon systems or task forces are listed on the left side as rows, and killed weapon system types or task forces are listed on each column. These K-V scoreboards do not consider deaths due to friendly forces.

Table 27, in Appendix E, is the K-V scoreboard for the total battle, zero to 151 minutes, in the system-on-system format. This table shows that the M1 tanks and TOW-anti armor weapon are the major killer weapon systems of the Blue forces that contribute to the battle results, while the T-72 tank and BMP infantry combat vehicle are the major killing weapon systems of the Red forces. Blue forces killed 156 total Red force weapon systems, and Blue tanks and TOWs killed 90% of this total. Among the total killed Red systems 92% are T-72 tanks and BMPs. Red forces weapon systems killed 47 Blue systems, and all of the Blue systems were killed by the Red T-72 tanks and BMPs. These observation clearly describe an armor battle.

Table 28, in Appendix E, is the K-V scoreboard for the total battle in a force-on-force format. This table shows that nearly all task forces engage with all other opponent task forces. Major engagements occurred between Blue task force 2 and Red task force 5, Blue task force 3 and Red task force 3, Blue task force 4 and Red task force 2 and 4, and Blue task force 5 and Red task force 4 and 5. Among the Blue task forces, task force 2 and task force 4 are relatively more active than the others. Among the Red task forces, task force 1 is not at all effective, actually this task force killed no Blue weapon system while 17 weapon systems of this task force were killed by Blue.

Table 29 shows the K-V scoreboard in a system-on-system format for phase 1, interval zero to 100 minutes. This table can be viewed with the K-V scoreboard for the phase 2, interval 100 to 151 minutes, shown in Table 30. These two tables show that Blue weapon systems killed 65 Red systems in phase 1 and 91 Red systems in phase 2. Red systems killed fourteen Blue systems in phase 1 and 33 Blue systems in phase 2. These observations indicate that the fiercer engagement was conducted in phase 2.

Table 31 is the K-V scoreboard for phase 1 in force-on-force format. Comparisons of Table 31 with table 32, the K-V scoreboard for phase 2 with force-on-force format are interesting. These two tables show that the Blue task forces 1, 3, and 5 are mostly engaged in phase 1 while the Blue task forces 2 and 4 are mostly engaged in phase 2. It shows that Red task force 3 is mostly engaged in phase 1 while Red task forces 2, 4, and 5 were mostly engaged in phase 2.

3.5 Summary

This chapter discussed the background of the given data, form of data, the data conversion for this study, and provided the data overview using graphical methods and K-V scoreboard. The next chapter will analyze the battle trace methodologies suggested by Barr et al.

IV. An Analysis of the Given Battle Trace Methodologies

4.1 Introduction

Two methodologies of characterizing battle results were introduced by Barr et al. The two methodologies are the combat force elasticity (CFE) and the log of CFE as reviewed in chapter II. The problem with these methodologies is the numerical instability which arises when the force sizes and attrition events are small, sparse, or nonexistent. This problem may be caused by the nature of the battle phenomena or an improper numerical expression of the adopted measure of effectiveness. The purpose of the following discussion is to invoke the instability problem of the suggested methodologies and motivate other methodology developments. The following analysis will use the initially converted battle data which accommodates the battle data for a 20 second time interval.

4.2 Battle Trace of Combat Force Elasticity

4.2.1 Introduction. The initially developed battle trace was a trace of combat force elasticity throughout the battle process. An entire battle was decomposed into a discrete series of smaller time segments and each battle segment was scored by combat force elasticity. As mentioned by TRAC-MTRY, a numerical instability problem was discovered when applying the battle trace methodology.

This numerical instability occurred when any value of the denominator of the CFE is zero or near zero within a battle segment. In such cases, the CFE value turns out to be infinite or near infinity. Numerically, the denominator of Eq. (13), R_k and/or ΔB_k, cannot be zero in computing CFE_k, i.e. the equation generates a singularity point. Here the instability problem is discussed along with some possible ways of handling zero values in the denominators.

4.2.2 Assignment of CFE Value. Barr et al. suggested assigning specific CFE values under certain conditions. Their suggestion included the following assignments: 1) if both ΔR and ΔB values are zero, set CFE = 1; 2) if ΔB is zero but ΔR is not zero, set CFE = 2; 3) If either force is completely annihilated (R = 0 or B = 0), the other side automatically is defined as a winner and no value of CFE is computed for that time interval (2:18). Table 4 shows these suggested assigned values.

Table 4. Assignment of CFE Values

R	B	ΔR	ΔB	CFE(substituted)	Criteria of CFE
> 0	> 0	= 0	= 0	1	<1: Red win
> 0	> 0	> 0	= 0	2	>1: Blue win
> 0	= 0	>= 0	>= 0	Red win	1: Parity
= 0	> 0	>= 0	>= 0	Blue win	

As shown on table 4, qualitative values were assigned for the CFE when one of the force size (seers) was zero.

But, if those qualitative values are assigned to CFE during the battle process, the numerical analysis will be neither efficient nor convenient. Additionally, no value was suggested for the case when both force sizes are zero.

Now, for this assignment methodology to be useful, requires the assumption that R and B are positive during any battle segment at which CFE is evaluated. That is, this methodology cannot allow any force strength value to be zero during the battle process. This assumption is reasonable when the force strength is defined as the total unit strength at evaluation time, since this value will be zero only when the combat forces are annihilated, and the battle automatically ends. However, when the force strength (R or B) is defined as the number of weapon systems that can see at least one enemy system at that time, this force value can be zero anytime during the battle. In fact, within the files provided, only the number of seers is furnished where the seers represent the unit strength.

The initially converted data (Appendix D) shows there exists many battle segments that contain zero value force sizes as the number of seers. The converted data shows that 412 battle segments have zeros on ΔB, 330 battle segments have zeros on ΔR, 17 battle segments have zeros on B, and no zeros are on R out of a total of 452 battle segments. Here alone, 412 battle segments ($\Delta B = 0$) have computational singularity problems.

Another problem of the suggested assignment methodology is that of the assigned CFE value when the ΔB equals zero. Barr suggested assigning the value of two for this case, but this value was intuitively assigned only to give Blue a winning score (CFE > 1). However, there exists some possibility that the value of CFE can be greater than two, even when the Blue winning condition is worse than the situation when ΔB is zero. Also, the CFE value can be smaller than two in cases where the winning condition is better than the situation when ΔB is zero. The following table shows the several examples that the assigned value can be under or overestimated.

Table 5. Example of Under/Overestimation of CFE

Case	ΔR	R	ΔB	B	CFE
1	15	20	0	10	2 (assigned)
2	5	20	1	10	2.5
3	1	100	0	10	2 (assigned)
4	10	100	1	10	1

The table 5 shows the assigned value of case 1 is underestimated compared to case 2. In case 1, Blue forces (with 10 weapon systems) kills 15 Red systems, while Red forces (with 20 weapon systems) kills no Blue systems. The assigned value of CFE for case 1 is 2 as suggested by Barr et al. With the same size of forces, case 2 shows that Blue kills only 5 Red systems, while the Red forces kills one

Blue system. The computed value of CFE in case 2 is 2.5. From the Blue forces point of view, the compared combat effectiveness of case 1 is better than that of case 2. Therefore, the CFE value of case 1 should be somewhat greater than the case 2.

Case 3 shows an example of overestimation compared to case 4. In case 3 Blue forces (with 10 weapon systems) kills one Red system, while the Red forces (with 100 weapon systems) kills no Blue systems. The assigned CFE value for case 3 is two, since the denominator is zero. Case 4 shows, with the same force size as case 3, Blue forces kill 10 Red systems, while the Red forces kill one Blue system. From the Blue forces' point of view, the combat effectiveness of case 3 is worse than that of case 4, even though the estimated value of CFE is greater than case 4. Therefore, the value of case 3 is overestimated compared to the CFE value of case 4.

The possible overestimation or underestimation of CFE values prevents precise numerical analysis of this battle. Obviously, there is no constant value which can be assigned to the undefinable CFE value precisely.

In summary, Barr's suggestion of CFE value assignment is not adequate because of the following three reasons: 1) some qualitative values can be assigned when the battle segment includes zero values in force sizes, R or B; 2) no value was assigned for the case when both force sizes are zero; 3) the suggested value (CFE = 2, when $\Delta B = 0$) may not be proper.

4.2.3 Approximation of CFE. The combat force elasticity equation, Eq (13), cannot be used when any battle segment contains zero values in the denominator. An assignment methodology was suggested by Barr, but this methodology had computational and analytical problems. Another technique suggesting a possible way of tracing the battle with CFE is adding a small value to each absolute value of the numerator and denominator of the equation. The following is the transformed equation of Eq (13).

$$CFE_k = \frac{(|\Delta R_k| + C)(B_k + C)}{(|\Delta B_k| + C)(R_k + C)} \tag{44}$$

where

C = any small constant value

Four constant values (C = 1, 0.1, 0.01, and 0.0001) were examined to see the different effects these had on the CFE values throughout the battle. Table 6 shows the number of battle segments that were classified by battle states of win, lose, or stalemate.

The number of battle segments associated with battle states of winning, losing, or stalemate are listed on first three rows of Table 6. This table shows that the Blue side winning number is decreased while the Blue winning number is increased when the constant C changed from 1 to 0.1, but

Table 6. Number of Battle Segments Classified by the CFE Battle Trace

Battle State	Region of Class	Constant			
		C = 1	C = 0.1	C = 0.01	C = 0.0001
Red win	0 <= CFE < 1	434	346	346	346
Blue win	CFE > 1	17	106	106	106
Stalemated	CFE = 1	1	0	0	0

these numbers were not changed when C = 0.01 and 0.0001.

It can be seen that the battle trace of this approximation methodology shifts depending on the constant C. If the constant C is small the battle segment may be scored differently by this approximated evaluation method. This shift in battle trace due to the constant C may indicate the error of this methodology. Therefore, the analyst should choose a small enough constant C to reduce the expected error. Table 6 indicates values smaller than 0.1 should be selected.

Table 7 shows the maximum and minimum values of CFE in battle trace are dependent on the constant C.

Table 7. Max/Min Value of CFE

Constant	C = 1	C = 0.1	C = 0.01	C = 0.0001
Maximum	1.77	12.00	115.84	11537.92
Minimum	0.05	0.01	0.00	0.00

In this case, there is no doubt that this approximation methodology can manipulate the value of the CFE battle trace when the constant value C is small. Table 7 shows that the maximum value of CFE goes to infinity and the minimum value approaches zero when C value approach zero. That is the CFE values are bouncing between zero and infinity for this small values and may indicate more instability in the battle trace of CFE.

Figures 8,9 and 10 show these approximated battle trace plots with C = 1, 0.1, and 0.01, respectively.

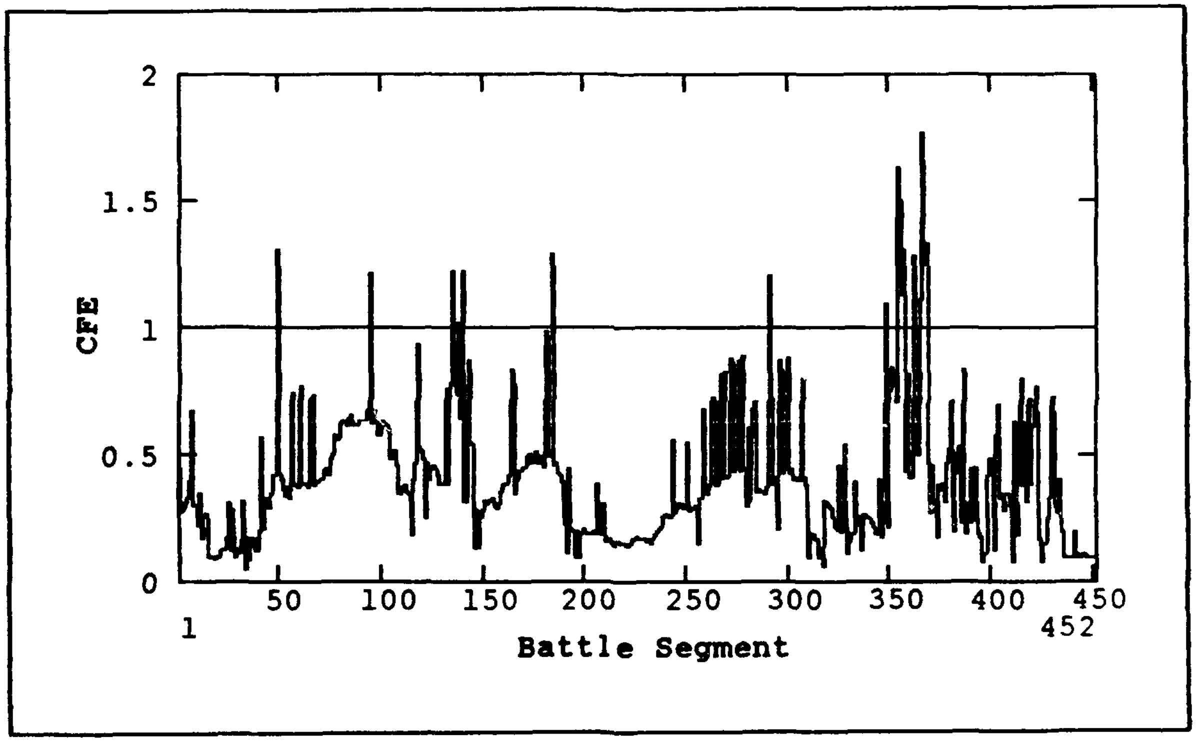


Figure 8. Battle Trace of CFE with C = 1

Here, the battle trace was shifted in the positive direction by the smaller values of C. The region of the CFE spreads out as the constant C becomes small. This unstable behavior of the CFE battle trace becomes obvious when the size of battle segment is small.

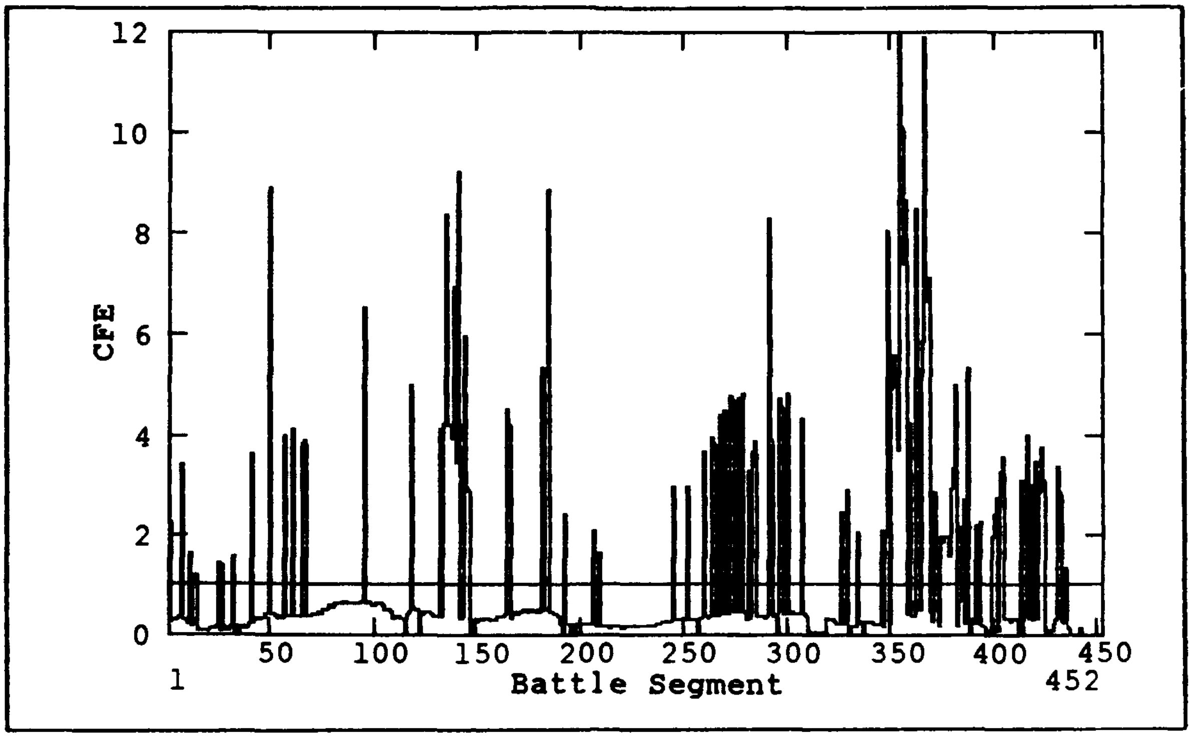


Figure 9. Battle Trace of CFE with C = 0.1

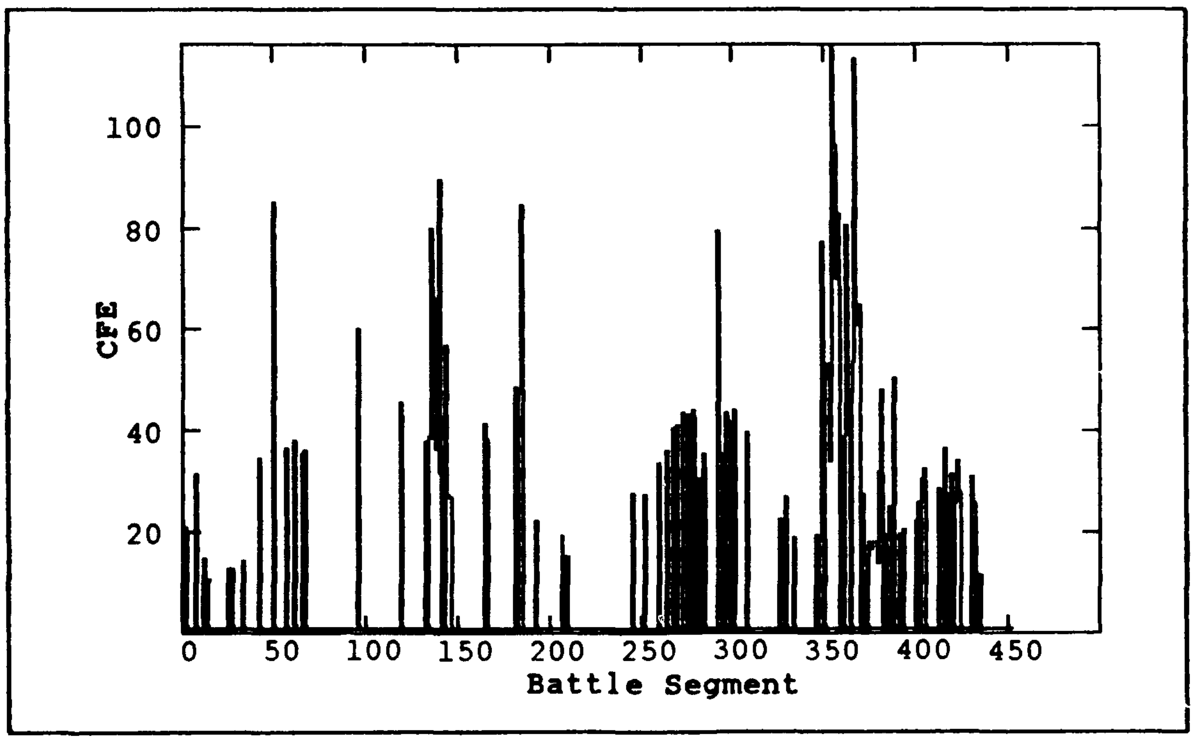


Figure 10. Battle Trace of CFE with C = 0.01

4.2.4 Inverse of the CFE. The previous approximation methodology for the CFE battle trace can be compared to its inverse. An inverse function analysis was conducted to compare the value of the CFE from both Red and Blue points of view. The following is the inverse of Eq (44) with the additive approximation factors:

$$CFE_k^{-1} = \frac{(|\Delta B_k| + C)(R_k + C)}{(|\Delta R_k| + C)(B_k + C)} \quad (45)$$

This inverse function of CFE was investigated with the same constant C (= 1, 0.1, 0.01, and 0.0001) used earlier. Table 8 shows the number of scored battle segments within the battle state classification.

Table 8. Number of Battle Segments Classified by the CFE^{-1} Battle Trace

Battle State	Region of Class	Constant			
		C = 1	C = 0.1	C = 0.01	C = 0.0001
Blue win	0 <= CFE^{-1} <1	17	106	106	106
Red win	CFE^{-1} > 1	434	346	346	346
Stalemated	CFE^{-1} = 1	1	0	0	0

Table 8 shows that the number of "Blue win" battle segments, 0 <= CFE < 1, increases when the constant C becomes smaller, while the number of "Red win" battle segments, CFE > 1, decreases. The behavior of battle trace of CFE^{-1} is nearly the same as that of CFE do to their reciprocal symmetry.

Table 9 shows the maximum values and minimum values of CFE^{-1}.

Table 9. Max/Min Value of CFE^{-1}

Constant	C = 1	C = 0.1	C = 0.01	C = 0.0001
Maximum	19.71	155.26	1505.33	$1.50x10^5$
Minimum	0.57	0.08	0.01	0.00

Table 9 shows the region of CFE^{-1} expands when the constant C is small and indicates the instability of this battle trace. Figures 11, 12, and 13 show the battle trace plots with the constant C = 1, 0.1, and 0.01, respectively.

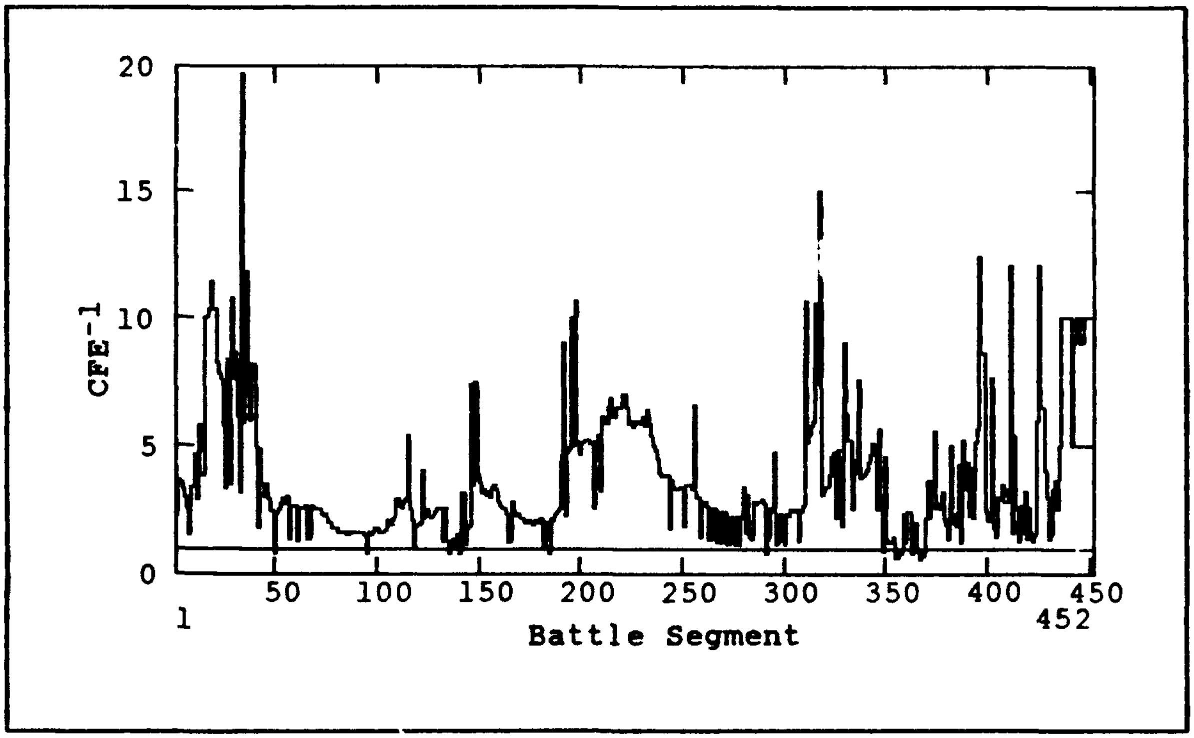


Figure 11. Battle Trace of CFE^{-1} with C = 1

There are considerable differences in the reciprocal changes of regions that decide the battle state. Table 10 shows the

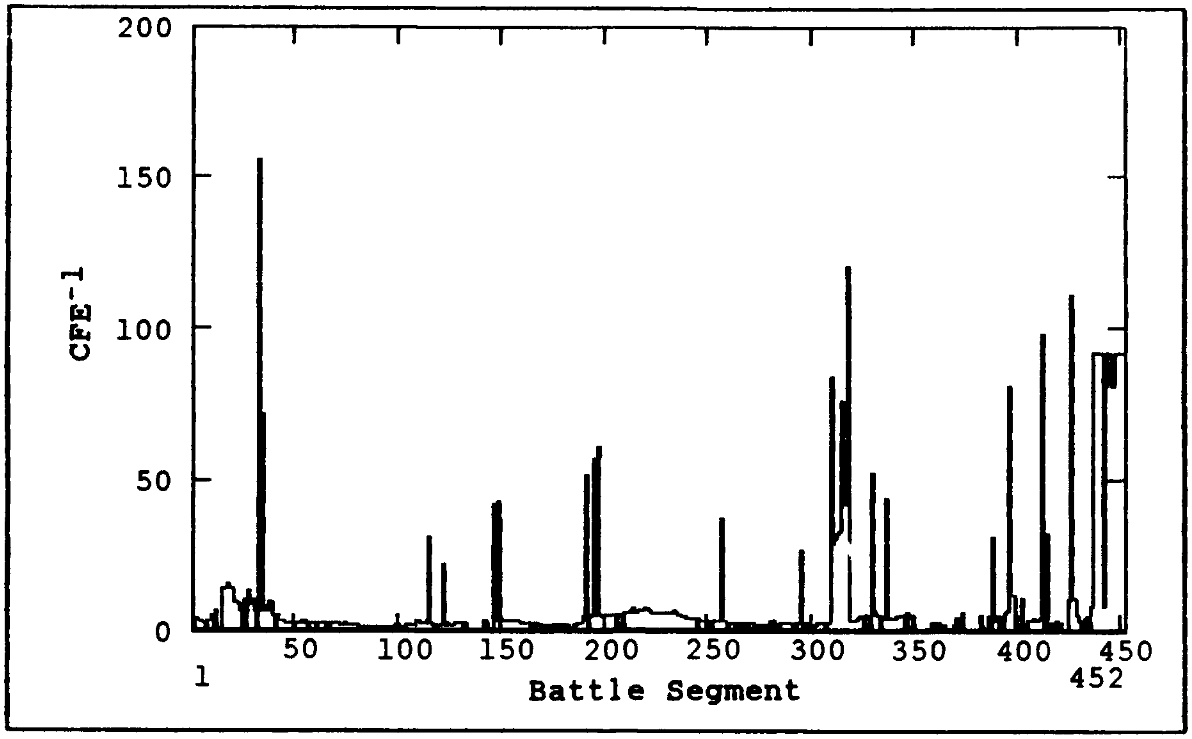


Figure 12. Battle Trace of CFE^{-1} with C = 0.1

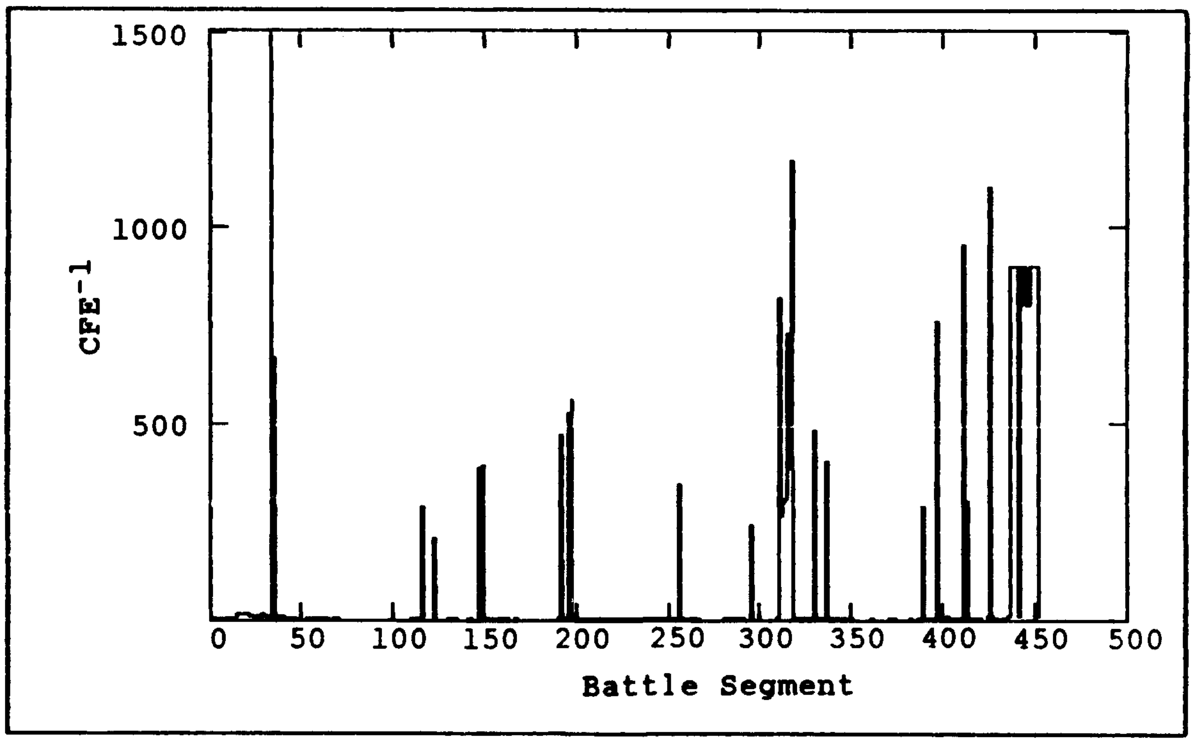


Figure 13. Battle Trace of CFE^{-1} with C = 0.01

Table 10. Decision Region of CFE and CFE^{-1}

Battle state	CFE	CFE^{-1}
Blue win (Red lose)	CFE > 1	0 <= CFE^{-1} < 1
Red win (Blue lose)	0 <= CFE < 1	CFE^{-1} > 1

changes of these decision regions of CFE and CFE^{-1}.

As shown on Table 10, the region in which Blue wins and Red wins is symmetrical with respect to its inverse function. This means the battle trace of CFE has reciprocal symmetry not additive symmetry. Consequently, the geometric mean and the cumulative value of CFE throughout the battle trace may not have meaning. The geometric mean of CFE with a C = 0.01 is 9.253 this mean score of battle indicates the Blue side is winning, while the geometric mean of CFE^{-1} with the same C value is 63.31 indicating Red is winning.

The reciprocal symmetry of this system can be easily observed with the unequally weighted effects of denominators and numerators in the ratio equation. Figure 14 shows the different effects due to changes in denominator and numerator.

In Figure 14, the dependent variable "Y1" shows the effect of the denominator changes, while the dependent variable "Y2" shows the effect of the same amount of numerator changes. This reciprocal symmetry is hard to interpret and analyze within the system.

Effeccts of denominator and numerator

Let, $i := 1..40$

$C := 20$ $\qquad$ $X_i := i$

$Y1_i := \frac{C}{X_i}$ $\qquad$ $Y2_i := \frac{X_i}{C}$

Where,

X = independent variable
Y1, Y2 = dependent variables
C = constant value

Plot to show the effects of denominator changes $\qquad$ Plot to show the effects of numerator changes

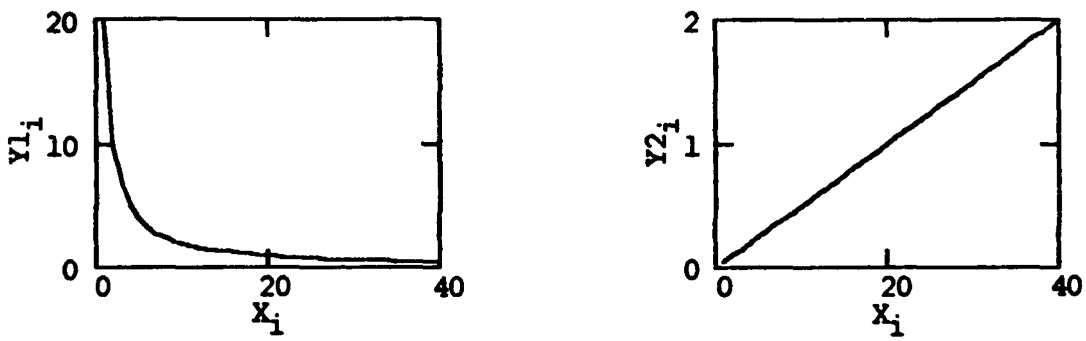


Figure 14. Effects of Denominator and Numerator

The following discussion of log of CFE trace was developed to convert this reciprocal symmetry system to an additive symmetry system.

<u>4.2.5 Log of CFE</u>. The log of CFE (LoCFE) methodology was suggested by Barr et al. The purpose of this methodology was to transfer the reciprocal symmetry of CFE battle trace to an additive symmetry system. This

methodology takes the log of the CFE equation, Eq (14) (2:22). The problem with this methodology is in the domain of the log function. The log function cannot take a zero value which can be provided when the size of battle segment is small.

Barr et al. suggested adding a small constant value to each element of the equation (2:23). Barr preferred to add 1.0 to each element of the CFE equation before taking the log, so when the value of any element of CFE is zero the log of 1 also is zero, Eq (15). However, there is no specific reason to assign the constant C equal to 1.0. The following is the equation for log of CFE with any arbitrary constant value C:

$$\text{Log of } CFE_k = \log(\Delta R_k + C) - \log(R_k + C) - \log(\Delta B_k + C) + \log(B_k + C) \qquad (46)$$

The scoring of each battle segment with the above equation produces the decision regions in Table 11 (2:22).

Table 11. Battle State Decision Regions of the Log of CFE

Region of log of CFE	Defined battle state
log of CFE > 0	Blue side is winning
log of CFE < 0	Red side is winning
log of CFE = 0	the battle is stalemate

Using C values of 1, 0.1, 0.01, and 0.0001, Table 12 shows the number of battle segments classified by the battle state decision regions shown above.

Table 12. Number of Battle Segments Classified by the Log of CFE

Region of Class	Constant			
	C = 1	C = 0.1	C = 0.01	C = 0.0001
Log of CFE > 0: Blue win	17	106	106	106
Log of CFE < 0: Red win	434	346	346	346
Log of CFE = 0: Parity	1	0	0	0

Table 12 shows that if the constant value C is small enough, the number of battle segments that are classified by the decision region are not changed. However, the number of classified battle segments are different when the battle trace of log of CFE is shifted by the constant C.

Table 13 shows some statistics of this battle trace with different constant C values added.

Table 13. Statistics of Log of CFE

Statistics of Log of CFE	Constant			
	C = 1	C = 0.1	C = 0.01	C = 0.0001
Mean	-0.48	-0.40	-0.27	0.02
Max	0.25	1.08	2.06	4.06
Min	-1.29	-2.19	-3.18	-5.18

The geometric mean value is not an appropriate way of representing the total battle score, but the changes of mean value can indicate that the battle trace is shifted due to C. The min and max values of the log of CFE shows that the battle trace is spread out when the constant value becomes small, but this is not severe compared to the CFE battle trace.

Observations of the above are provided by plots of the battle trace of the log of CFE. Figure 15,16, and 17 show these battle traces with constant C = 1, 0.1, and 0.01, respectively.

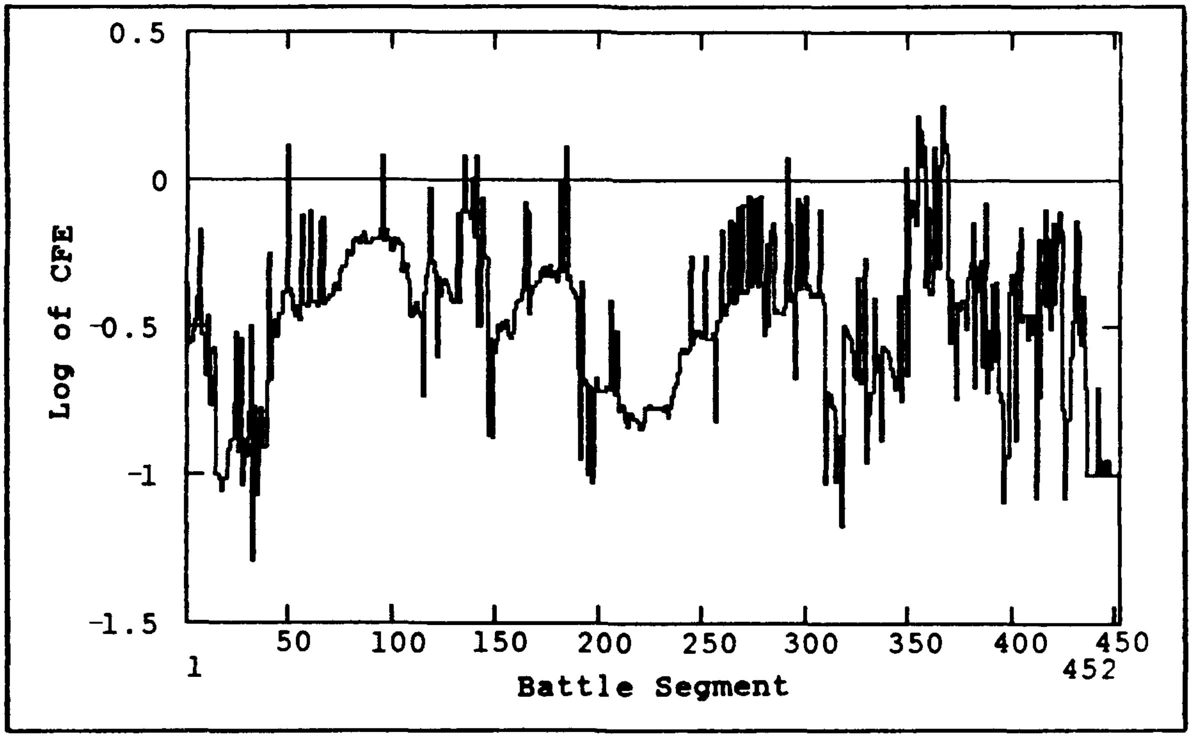


Figure 15. Battle Trace of Log of CFE with C = 1

The function of the log of CFE^{-1} was also examined for system symmetry. The following equation represents the log of CFE^{-1}:

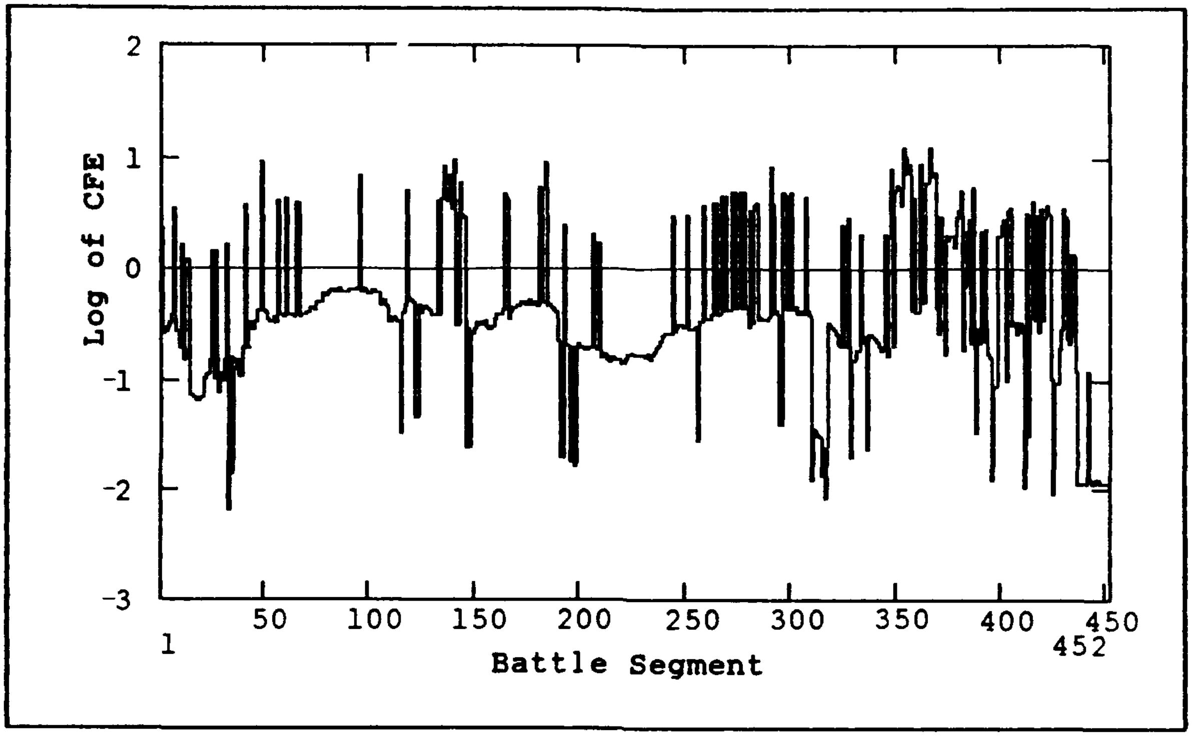


Figure 16. Battle Trace of Log of CFE with C = 0.1

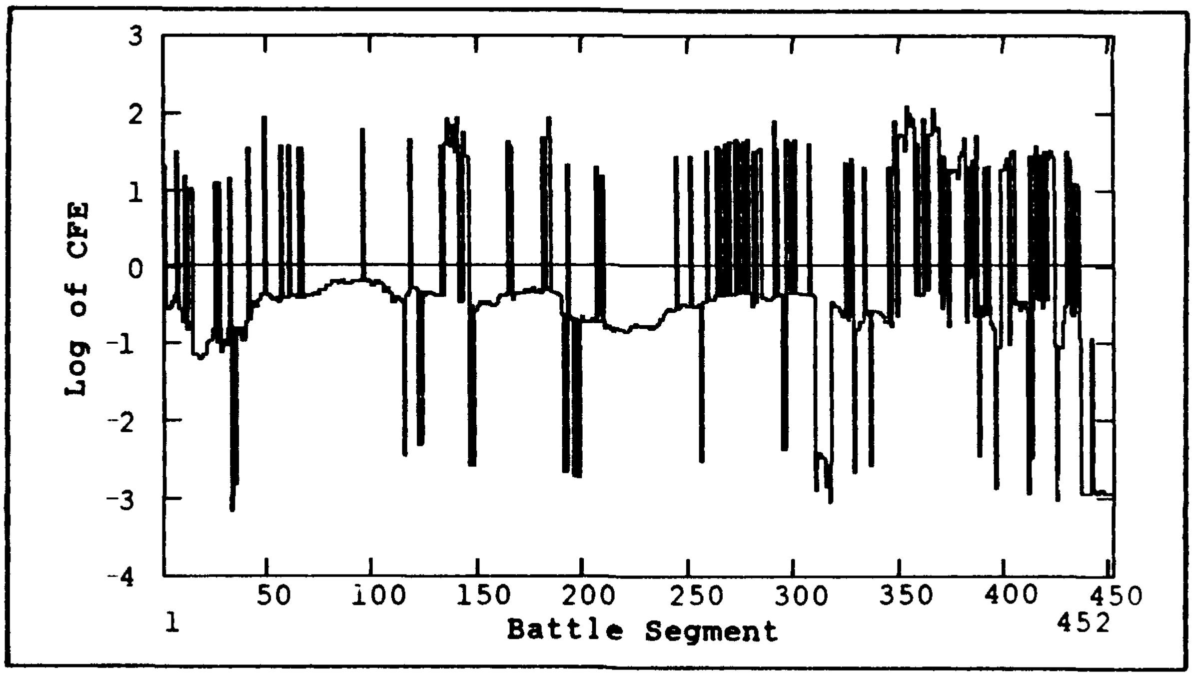


Figure 17. Battle Trace of Log of CFE with C = 0.01

$$\text{Log of } CFE_k^{-1} = \log(\Delta R_k + C)^{-1} - \log(R_k + C)^{-1} - \log(\Delta B_k + C)^{-1} + \log(B_k + C)^{-1}$$
$$= -\log(\Delta R_k + C) + \log(R_k + C) + \log(\Delta B_k + C) - \log(B_k + C) \quad (47)$$

As shown in the above equation, the function of the log of CFE^{-1} is just the negative of the log of CFE and contains additive symmetry.

Table 14 below includes the list of the number of scored battle segments which are classified by the value of the log of CFE^{-1}.

Table 14. Number of Battle Segments Classified by the Log of CFE^{-1}

Battle State	Region of Class	Constant			
		C = 1	C = 0.1	C = 0.01	C = 0.0001
Blue win	$LoCFE^{-1} < 0$	17	106	106	106
Red win	$LoCFE^{-1} > 0$	434	346	346	346
Stalemated	$LoCFE^{-1} = 0$	1	0	0	0

This table shows that the number of wining battle segments is the same as the results shown on Table 12. The only difference is that the Blue forces win when the value of log of CFE^{-1} is less than zero, while the Blue forces win when CFE is greater than zero in Table 12.

Table 15 provides some statistics for comparison with Table 13 and shows that the mean value of the log of CFE^{-1} is the negative of the log of CFE. The max and min of the log of CFE^{-1} is the negative of the min and max of the log of CFE. Confirming that the battle trace of the log of CFE has additive symmetry.

Table 15. Statistics of Log of CFE^{-1}

Statistics of Log of CFE^{-1}	Constant			
	C = 1	C = 0.1	C = 0.01	C = 0.0001
Mean	0.48	0.40	0.27	-0.02
Max	1.29	2.19	3.18	5.18
Min	-0.25	-1.08	-2.06	-4.06

Figures 18, 19 and 20 show the plots of log of CFE^{-1} battle traces with C = 1. 0.1, and 0.01, respectively.

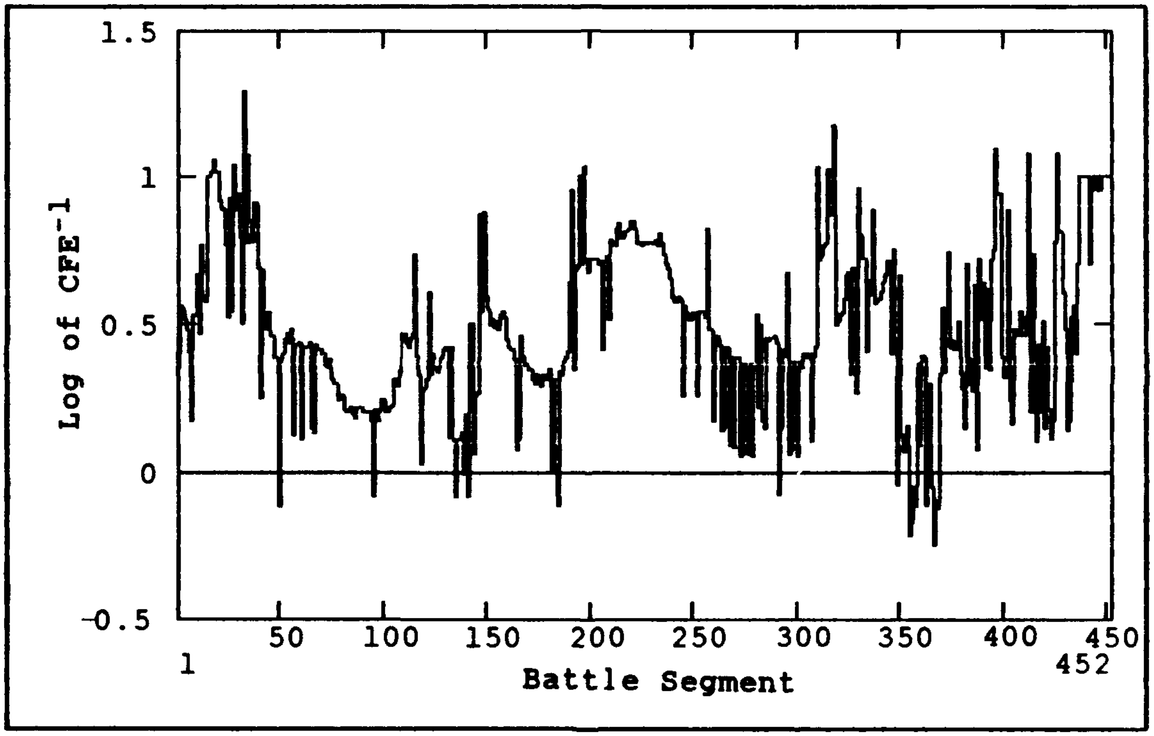


Figure 18. Battle Trace of Log of CFE^{-1} with C = 1

The three battle traces show the different effects due to various selection of the constant C. Those plots provide he following observations. The battle trace of the log of CFE has additive symmetry as expected and the values of the log of CFE are bounded within a certain range. This bounded

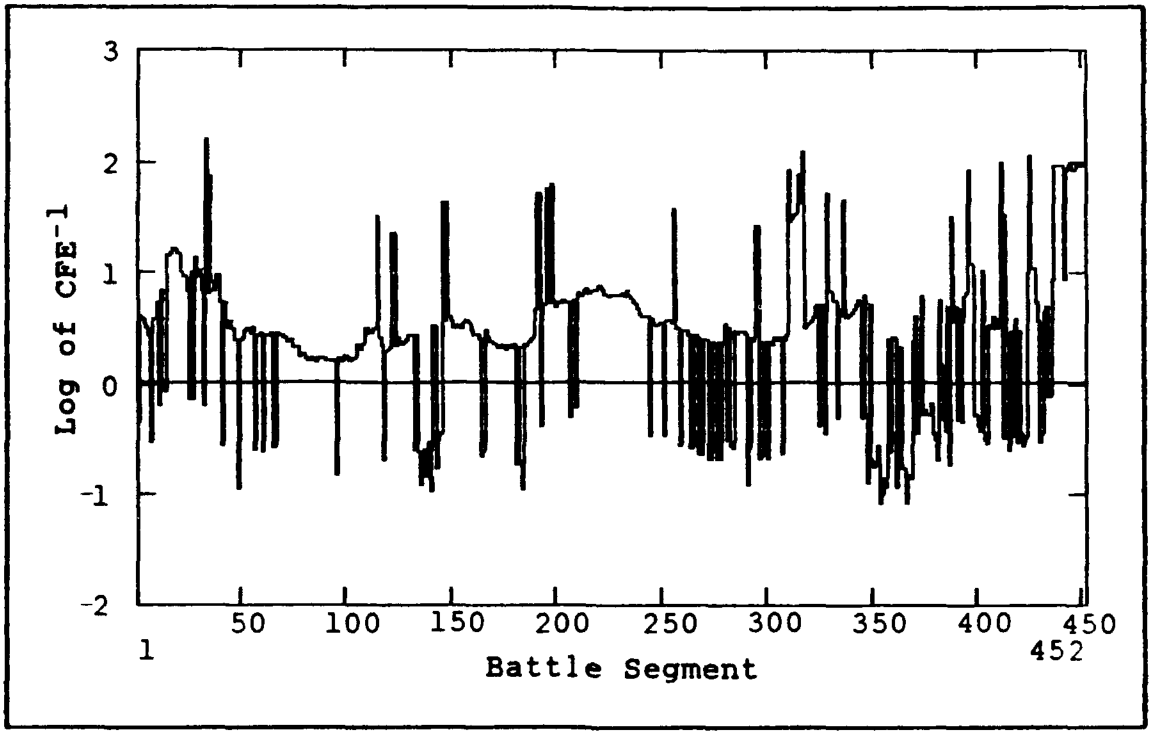


Figure 19. Battle Trace of Log of CFE^{-1} with C = 0.1

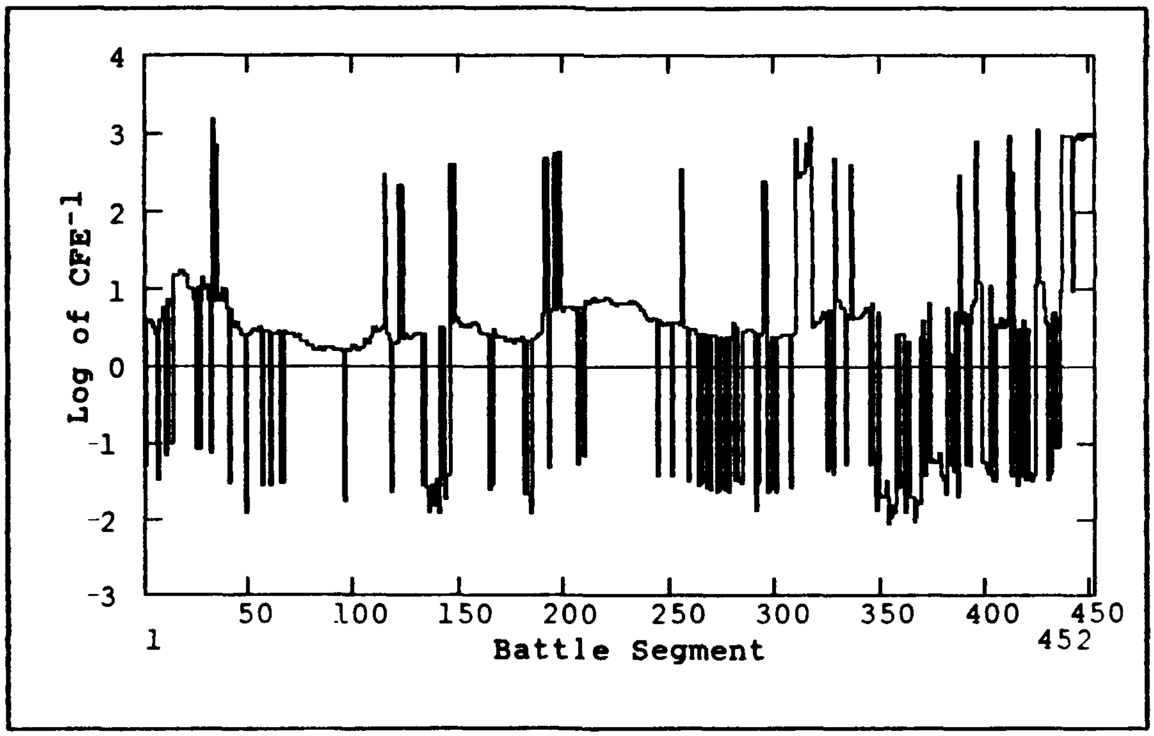


Figure 20. Battle Trace of Log of CFE^{-1} with C = 0.01

range is much smaller than that of simple CFE, however, the behavior of that the battle trace shifts depending upon the constant C. The methodology of adding a constant C to each element of the log function adds to the problem the necessity of deciding what constant C value to use in this process.

4.2.6 Conclusion. The battle trace with CFE was discussed along with the problem of numerical instability when the force size in any battle segment are too small or zero. Barr et al. suggested assigning some "reasonable" values to circumvent computational problems when the CFE values are undefinable. The problems with this methodology are that some quantitative values (win or lose) can be inappropriately assigned during the battle, and the suggested value (CFE = 2 when $\Delta B = 0$) can under or overestimate when compared to nearby circumstances of other computed CFE values.

In the approximation methodology, a constant value C was added to each element of the CFE. In the analysis of this methodology, the battle trace of CFE was found to display reciprocal symmetry which has interpretational and analytical difficulties and may introduce error due to the selection of the constant C.

The battle trace of log of CFE was suggested to convert the reciprocal symmetry in CFE to an additive symmetry. The analysis of log of CFE battle trace demonstrated the additive symmetry of this system, but the battle trace criteria shifts due to the value of the constant C which was introduced to eliminate the original domain problem of the log function.

The above analysis of battle trace used the initially converted battle data which was generated from the 20 seconds time interval raw database. Now, if the interval of the battle segment was made larger than the original 20 seconds interval data, then the number of battle segments that included numerical instability was expected to be reduced. This methodology of choosing an appropriate interval size became an important issue in this study and is the subject of the following section.

4.3 Time Interval analysis

4.3.1 Introduction. The problem as discussed in above analysis of CFE battle trace is with zero or near zero values in any element of CFE (ΔB, B, ΔR, and R). This problem can be solved by increasing the size of battle segments. That is, by increasing the time interval, each battle segment can now include more battle activities causing many CFE components to assume nonzero values. Moreover, using larger time intervals can aggregate more battle data, such that these battle trace graphs can be "smoothed out" in comparison to the graphs of the smaller time intervals. Another reason to increase the length of the time interval is that the original time interval, 20 seconds, is far too quick to see and record the battle in any real sense and has no actual combat analysis appeal.

However, the purpose of battle trace was to provide a battle characterizing methodology throughout the battle process rather than the typical end-of-combat analysis result. This suggests that the interval size selected should capture a sufficient number of intervals to make up a meaningful time series battle trace.

The following section will cover the time interval selection process that provided better views of the battle trace.

4.3.2 Data Conversion for Interval Analysis. The Data conversion process to any specific time interval required that a data conversion program be developed. The first requirement was to develop an algorithm that aggregated the force size data (B and R) and the number of killed data (ΔB and ΔR).

While deaths, ΔB and ΔR, can accumulate over any interval of time similar to the technique used in the initial program. Several methods to represent the force size in that interval are possible. Two ways to collect data are the maximum value recorded in the interval, and the mean value of force size within the interval. Barr et al. arbitrarily used the maximum value of the force size in each time interval. The data conversion BASIC program used for this interval analysis employed the mean value of the aggregated "seer" forces within the interval specified. Figure 21 represents the flow chart of the original BASIC data conversion process for the interval analysis.

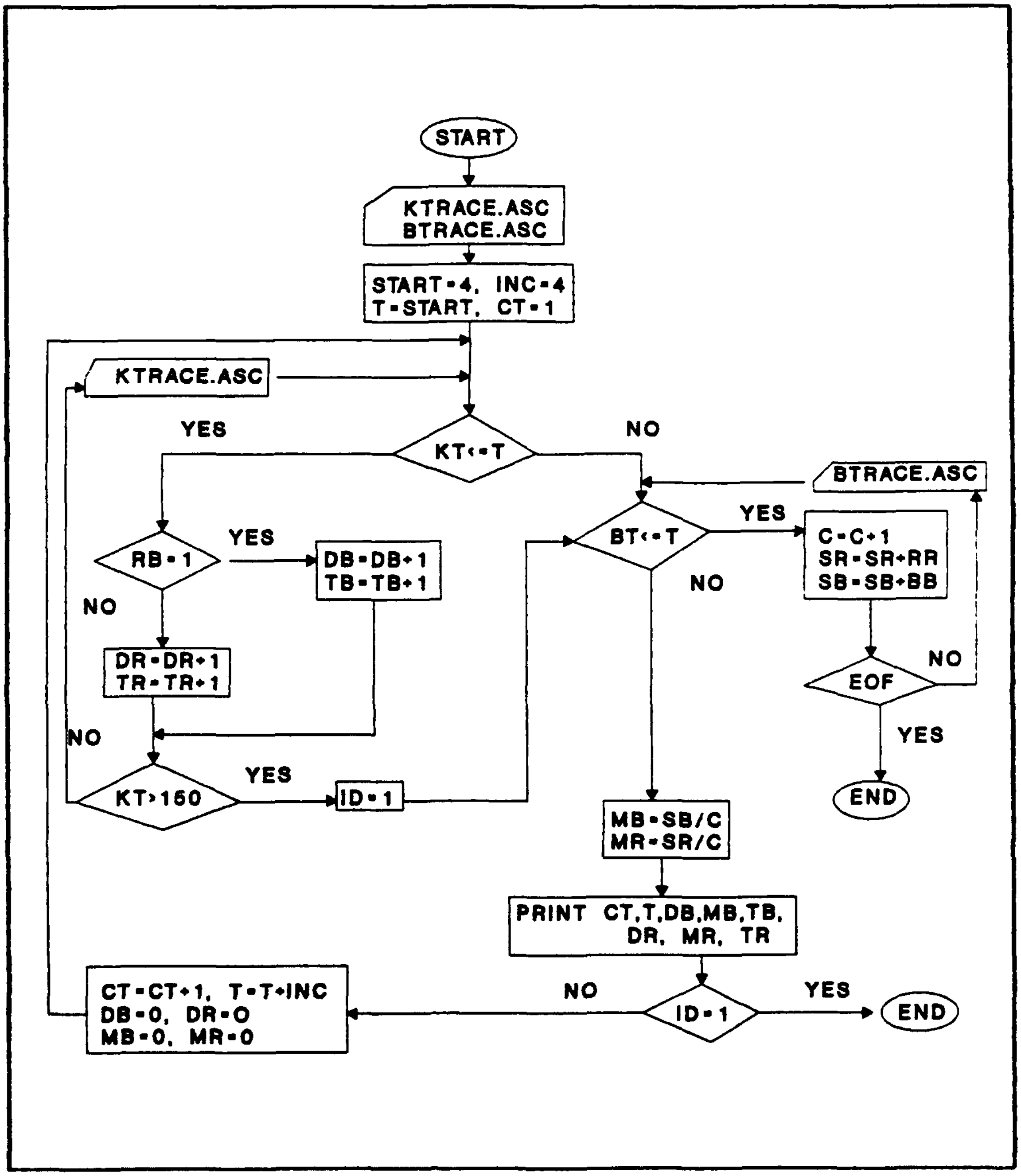


Figure 21. Flow Chart for Interval Analysis

The BASIC program for this and the data generated by this program appear in Appendix F and G, respectively.

<u>4.3.3 Number of Zeros</u>. The number of zeros in the converted data set with the original interval of 20 seconds and 1 to 21 minute time interval lengths are shown on Table 16.

Table 16. Number of Zeros Depending on the Interval Size

Interval	Number of Interval Reports	Number of Zeros Components in			
		ΔB	ΔR	B	R
20 sec	452	412	330	17	0
1 min	151	119	71	3	0
2 min	76	53	24	1	0
3 min	51	32	9	0	0
4 min	[illegible]	21	6	0	0
5 min	31	14	5	0	0
6 min	26	14	2	0	0
7 min	22	10	1	0	0
8 min	19	5	1	0	0
9 min	17	6	1	0	0
10 min	16	4	1	0	0
11 min	14	3	0	0	0
16 min	10	2	0	0	0
21 min	8	0	0	0	0

Clearly, the larger interval sizes contain the smaller number of zeros. An observation of Table 16 shows the following: 1) there are no zero values on the number of seers of Red forces for all examined intervals; 2) if the interval is greater than or equal to 3 minutes, no zero values were found for Blue seers; 3) if the interval is greater than or equal to 11 minutes, no zero values were found on Red system deaths; 4) if the interval is greater than or equal to 21 minutes, no zero values were recorded.

To eliminate all possible zeros in the given battle data, the interval should be greater than or equal to 21 minutes. Unfortunately with an interval of 21 minutes the system contains only 8 records of data. This low number of battle segments may not be sufficient to even trace the battle.

The number of data time segments for the battle trace is determined by the size of the battle segments. An appropriate size of battle segments is dependent upon the battle activity. If the battle has greater combat interactions, the time interval for the battle segment can be made smaller than when the battle contains sparse activity. Thus far, however, no clear decision rules have been developed to select the appropriate interval size.

In the 3 minute time interval battle, no zero values were found for R and B. Assuming that the time intervals of 4 and 5 minutes have a sufficient number of data points for battle trace, 3 to 5 minutes time intervals were selected for the following discussion.

4.3.4 Plots of Deaths and Strengths. Since the force size and the number of deaths are the components of the battle trace, plots of those components will be used in examining the battle trace. Appendix H includes these plots when the interval sizes are 20 seconds, 3 minutes, and 5 minutes. These plots show that there are no significant shape changes to the diagrams within these different time intervals indicating that 3 to 5 minute intervals are acceptable.

4.3.5 Conclusion. The interval size should be examined before the battle trace is analyzed. The interval size must be selected in a way to satisfy the following two requirements: 1) the size of interval should be large enough to include sufficient battle activities in each battle segment; 2) the total number of data points should be big enough to trace the battle flow. However, no clear decision rule has thus far been provided and analyst can choose the interval size intuitively within the above considerations.

4.4 Battle Trace Smoothing with Forecasting Methodology

Forecasting is a prediction technique based on past known values. The forecasting methodology was expected to provide a way to smooth the battle trace. However, the predicted values can be no better than the true history values. Even though, the battle trace with predicted values can be smoother than the battle trace with simply historical data, this methodology may cause the loss of some insight to battle behavior and, therefore, the forecasting methodology for smoothing was tried but not found analytically satisfying to track the stochastic battle trace.

4.5 Conclusion

Several methodologies of battle trace were discussed in this chapter. Those methodologies were based on the concept of combat force elasticity. The problem of numerical instability was identified, where force sizes and attrition events are small, sparse, or nonexistent. The assignment methodology suggested by Barr et al. was introduced, when the equation for the CFE can not take any zero values in the numerator. This methodology had inappropriately assigned values in several cases. A technique which added a small constant C value to each element of CFE shows the battle trace of CFE may still have numerical instability and possible error, and may still not circumvent issues of reciprocal symmetry.

Time interval analysis was discussed to reduce numerical instability when an appropriate selection of the interval size can minimizing this problem. However, too much aggregation caused by increasing the interval size may cause the battle trace to lose the power of representing battle dynamics.

The next chapter discusses alternate transform methodologies also based upon Lanchester's Square Law.

V. Constructing Battle Traces Employing the Lanchester Transformation

5.1 Introduction

Thus far, the suggested battle trace methodologies have had both computational and interpretational problems. The instabilities when applying these methodologies may be due to the improper expression of the numerical models or the instabilities which arise due to the nature of the common stochastic battle field phenomena. It is the problem with the numerical instabilities of the model which should be solved before analyzing the battle.

This chapter represents an attempt to discover a reasonable measure for comparing two combat forces combat effectiveness. Other possible transformation methodologies of the Lanchester's Square Law will be presented in this chapter. These methodologies are expected to allow zero values in the components, be computationally efficient, have meaningful interpretations with additive symmetry, and be statistically sound.

5.2 Constant in the Lanchester's Square Law

5.2.1 Introduction. The Lanchester's Square Law is the basis of the battle trace measurements where the constant C was created in the Lanchester's Square Law integration process. The value of the constant C computed by the Lanchester's Square Law alone can decide the battle state (2:10-12). The following discussion contains the model development of the battle trace with this constant.

5.2.2 Model Development. The following equation, Eq(48), is ano er form of the Lanchester's Square Law.

$$\beta R^2 - \rho B^2 = C \tag{48}$$

Since, the attrition rates, ρ and β, cannot be estimated exactly, the attrition rates are substituted by the attrition rates in Eq (6) and (7). The following equation results from this substitution.

$$\beta R^2 - \rho B^2 = \frac{-\frac{dB}{dt}}{R} R^2 - \frac{-\frac{dR}{dt}}{B} B^2 = C \tag{49}$$

The derivatives of B and R are associated with time (dB/dt and dR/dt) and are always zero or negative. For the discrete series of battle segments, these derivatives can be approximated by the difference of the force size when the interval size is small. Therefore, this value was assigned as the number of system deaths in that time interval. The number of system deaths is always zero or positive, that is the -dB/dt and -dR/dt can be substituted by ΔB and ΔR, respectively. So, Eq (49) can be approximated by the following equation:

$$\begin{aligned} \beta R^2 - \rho B^2 &= \frac{\Delta B}{R} R^2 - \frac{\Delta R}{B} B^2 \\ &= \Delta B R - \Delta R B \end{aligned} \tag{50}$$

This substitution is useful when the components, ΔB, ΔR, B, and R are known values. The battle can now be decomposed into a discrete series of battle segments. Each battle segment can be considered as an individual brief engagement. Therefore, Eq (50) for the K^{th} battle segment would be defined in the following equation:

$$\Delta B_k R_k - \Delta R_k B_k = C_k \tag{51}$$

Each battle segment can be scored by the constant C_k and the value of ΔB, ΔR, B, and R can be zero when employing this equation, so no computational problem will arise.

The battle state decision regions are listed below.

Table 17. Decision Region of Lanchester Constant C

Battle state	C_k
Blue win (Red lose)	$C_k < 0$
Red win (Blue lose)	$C_k > 0$
Parity	$C_k = 0$

The decision regions for both side's battle states are the same and indicate that the equation, Eq (51), has additive symmetry and that the contributions for ΔB and ΔR or B and R are the same to the battle score, i.e. the constant C.

Therefore, the following analysis concentrates on investigating the model sensitivity associated with the changes in the battle interval size.

5.2.3 Analysis. Performing time interval analysis is essential to see how the model sensitivity changes with changes of the battle time interval size. The initial 20 seconds, 3 minute, and 5 minute interval battle data were used for this analysis. Figures 22, 23, and 24 shows the battle traces with the different interval sizes.

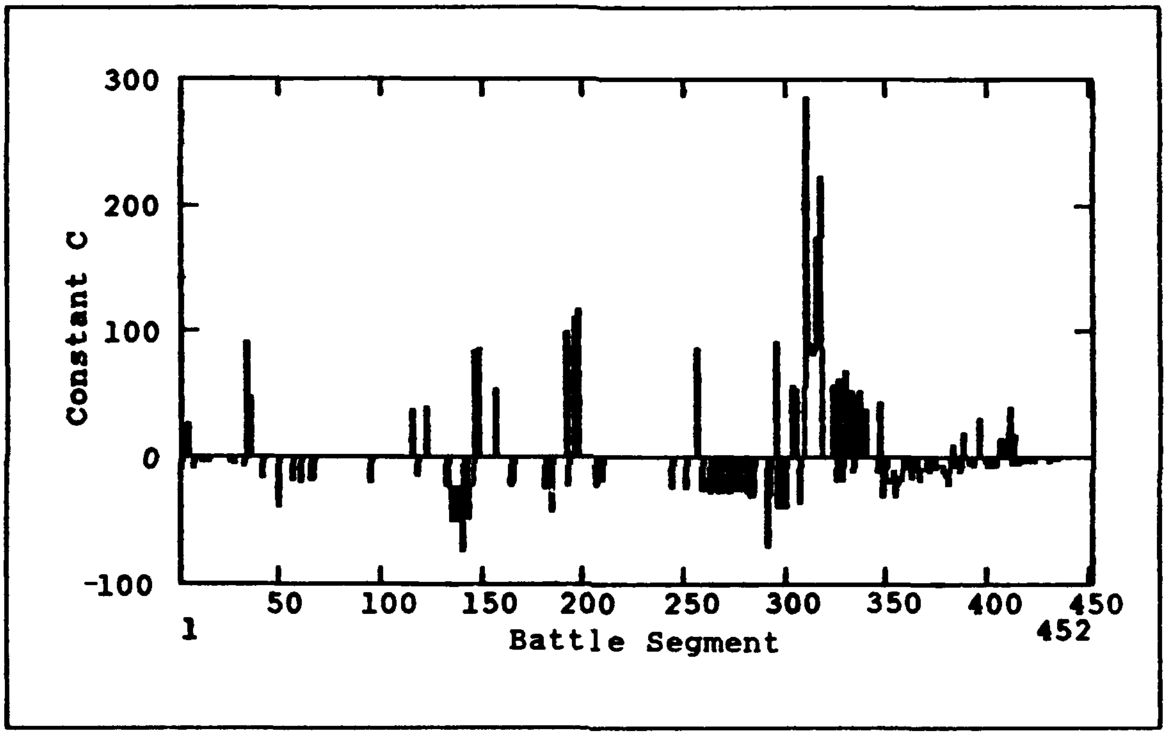


Figure 22. Battle Trace of Lanchester Constant C with Interval Size 20 Seconds

The first observation is in the proportional changes of the number of battle segments (classified by the battle status, win, lose, or parity) associated with different interval sizes. The battle segments were classified by the computed value of the constant C as shown on Table 17 and the number of battle segments categorized by these constant

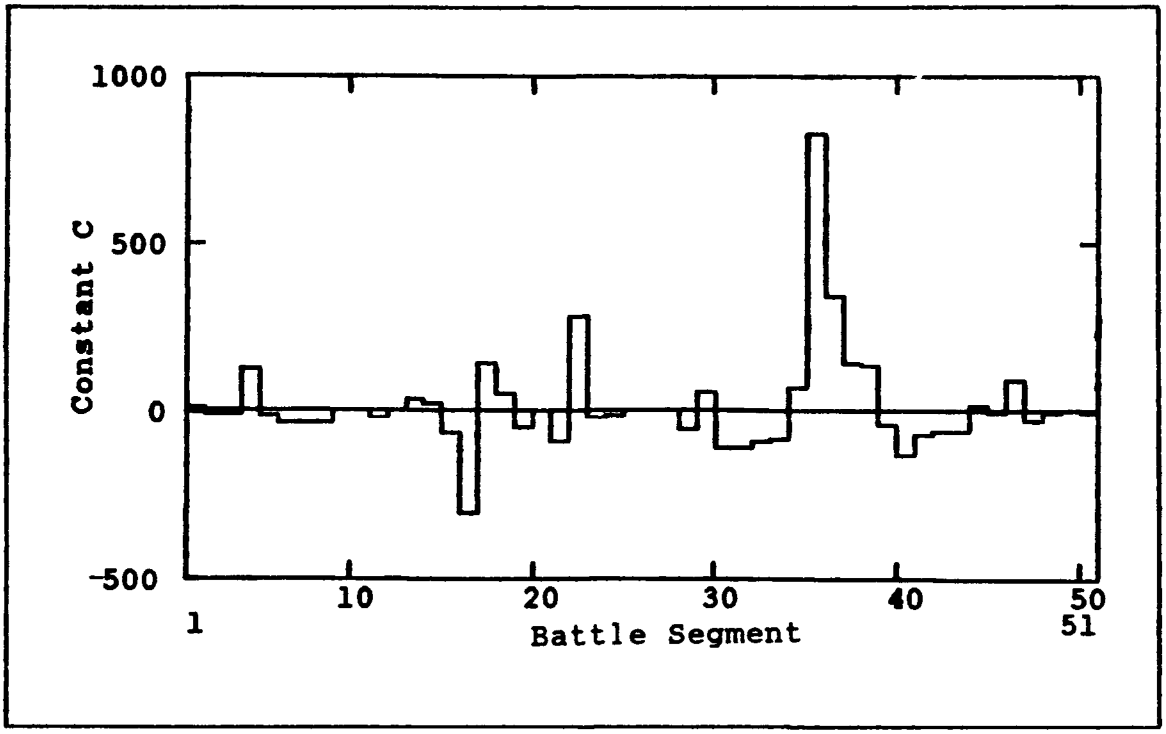


Figure 23. Battle Trace of Lanchester Constant C with Interval Size 3 Minutes

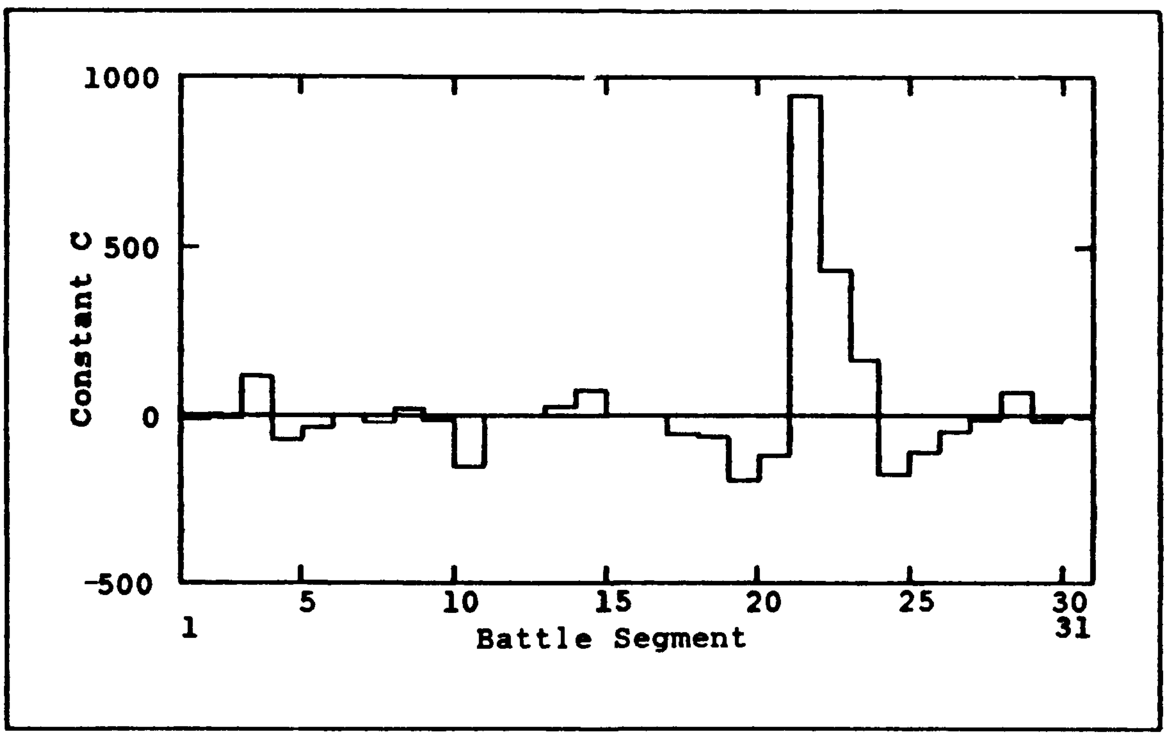


Figure 24. Battle Trace of Lanchester Constant C with Interval Size 5 Minutes

C values are listed in Table 18.

Table 18. Number of Battle Segments Classified by the Lanchester Constant Trace

Region	Interval		
	20 second	3 minute	5 minute
C > 0: Red win	40 (8.85%)	15 (29.41%)	9 (29.03%)
C < 0: Blue win	106 (23.45%)	28 (54.90%)	17 (54.84%)
C = 0: Parity	306 (67.70%)	8 (15.69%)	5 (16.13%)
Total	452 (100%)	51 (100%)	31 (100%)

This table shows the percent of the number of battle segments that were classified as parity during a 20 second interval battle (67.70%) is much higher than the 3 minute and 5 minute interval battles (15.96% and 16.13%, respectively). Negligible percentage changes were observed between the 3 minute and the 5 minute interval battle. Therefore, the aggregation of battle data by increasing interval size reduces the number of battle segments that are scored as parity conditions and makes the battle trace less erratic in terms of battle scoring.

Some other statistics of the changes of Lanchester constant C during the battle process with interval size 20 seconds, 3 minutes, and 5 minutes are listed in Table 19. The variance was computed based upon the same assumption that appealed with overall battle trace such that each battle segment is independently associated with any other battle's time and place. This table shows the magnitudes

Table 19. Statistics of Lanchester Constant C

Statistics of Constant C	Interval		
	20 seconds	3 minute	5 minute
Mean	1.94	15.95	25.23
Max	284	830.04	946.96
Min	-72	-300	-190.89
Variance	817.27	22914.30	41572.91

of mean, max, min, and variance increased when the interval size is increased. Obviously, the integration of the Lanchester's Square Law for larger interval sizes may capture more information. This may cause the variance of the value C to be increased. The trend in the mean value shows that the battle score can grow larger as the interval size becomes larger.

Another consideration is the cumulative value of the constant C. From the Eq (51), the integration for all of the discrete series of battle segments is shown as:

$$\sum_{1}^{N} \Delta B_k R_k - \sum_{1}^{N} \Delta R_k B_k = (\Delta B_1 R_1 - \Delta R_1 B_1) + \dots + (\Delta B_k R_k - \Delta R_k B_k) + \dots$$
$$+ (\Delta B_{N-1} R_{N-1} - \Delta R_{N-1} B_{N-1}) + (\Delta B_N R_N - \Delta R_N B_N) \qquad (52)$$
$$= C_1 + C_2 + \dots + C_k + \dots + C_{N-1} + C_N$$

where

N = the total number of battle segments

k = the k^{th} battle segment

C = the constant generated during the Lanchester's Square Law integration

Let, the cumulative integration of C (= CC) at the k^{th} battle segment be defined as:

$$CC_k = C_1 + C_2 + \ldots + C_{k-1} + C_k \tag{53}$$

where

CC_k = the cumulative integration of C at k^{th} battle segment

The cumulative C is derived from the Lanchester's Square Law notation, and the difference between the original Lanchester's notation and Eq (53) is only the substitution of the attrition rate coefficients, β and ρ. These attrition rate coefficients were substituted from Eqs (6) and (7).

The K^{th} cumulative constant C (CC) is depicted in Figures 25, 26, and 27 with interval sizes 20 seconds, 3 minutes, and 5 minutes, respectively. These battle traces show the cumulative values of Lanchester constant C is not significantly different across different interval sizes. This may indicate that the changes of the mean, max, and variance associated with the different interval sizes do not significantly affect the results of the battle trace

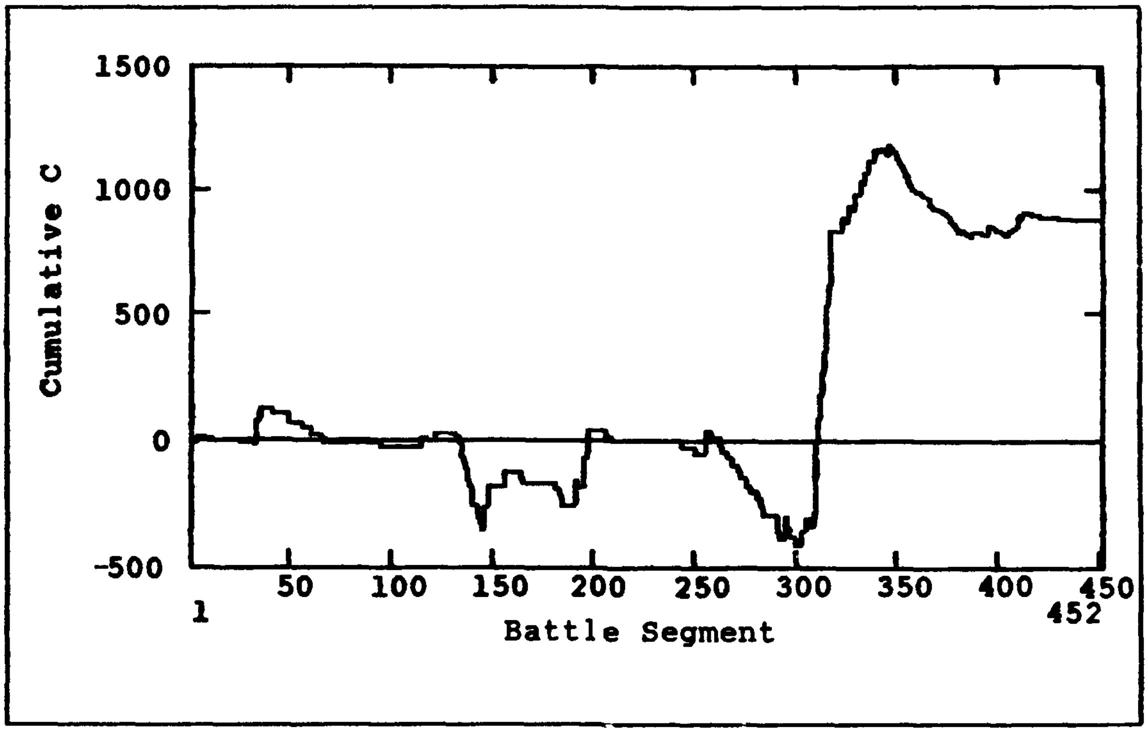


Figure 25. Plot of Cumulative Lanchester Constant C with Interval Size 20 Seconds

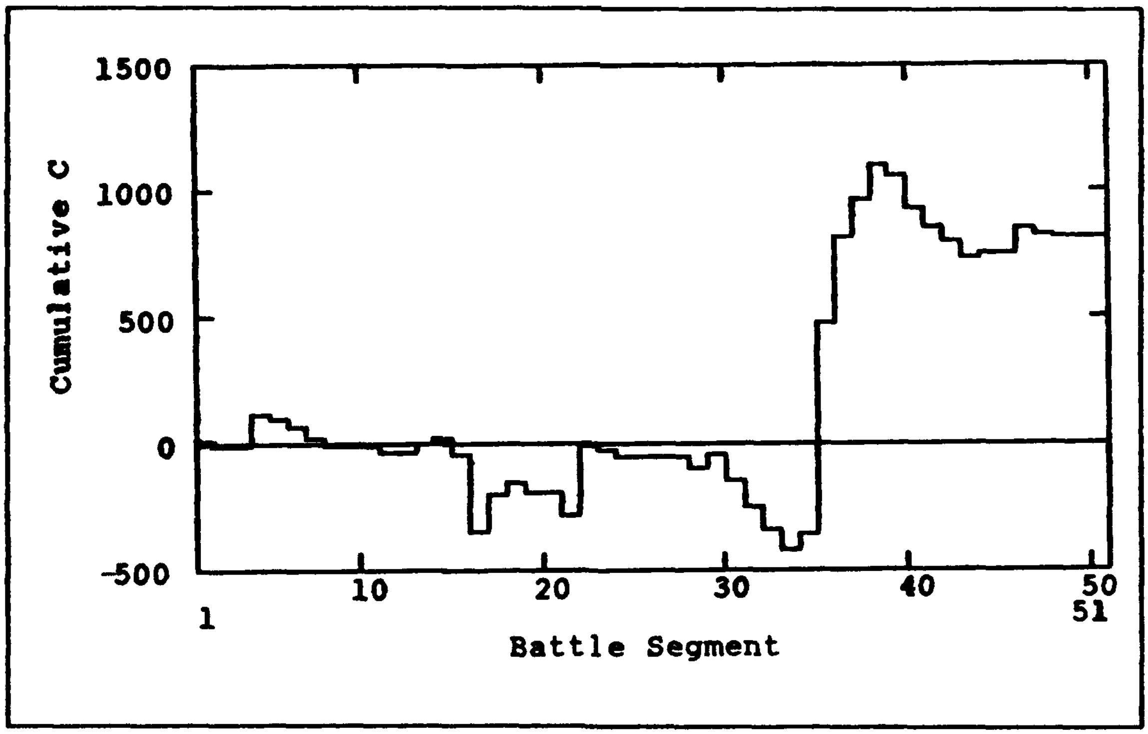


Figure 26. Plot of Cumulative Lanchester Constant C with Interval Size 3 Minutes

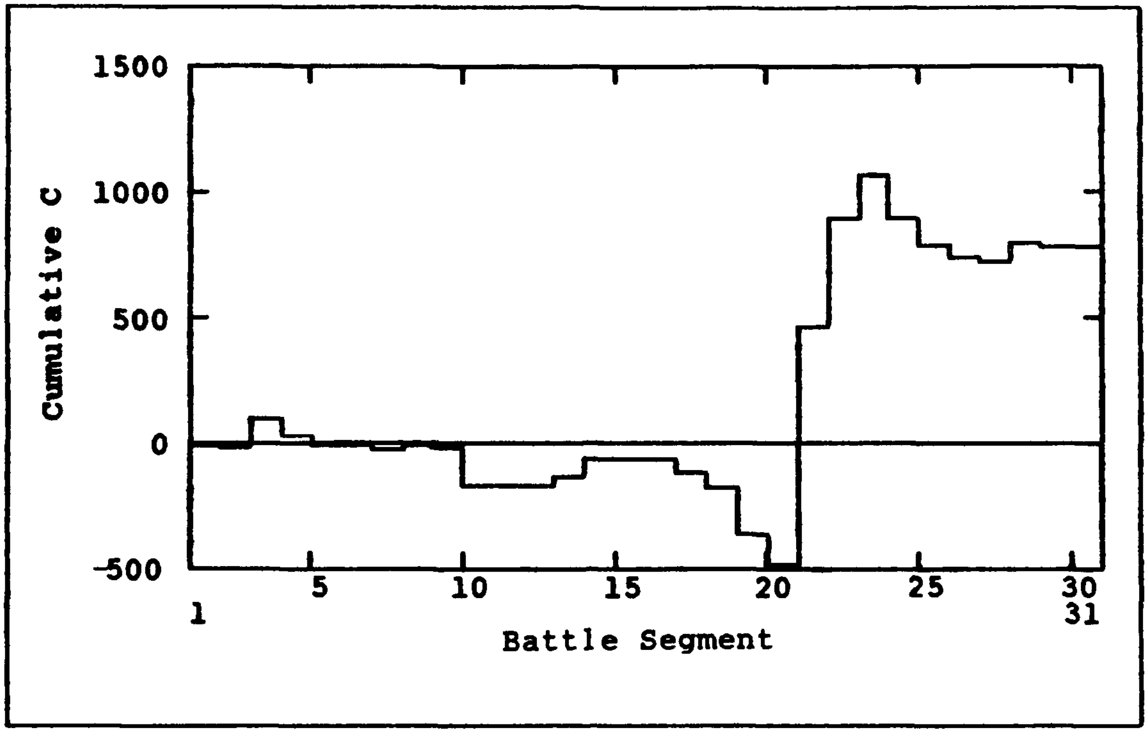


Figure 27. Plot of Cumulative Lanchester Constant C with Interval Size 5 Minutes

analysis. the values of the different increments can be explained in a sense that the overall cumulative constant C values are almost the same regardless of the sizes of interval. For example, the sum of the K-1th and Kth battle segments' constant C in interval size 2 minutes can be the same as the K/2th battle segment's constant C in interval size 4 minutes. Therefore, every 2 battle segments' constant C in the interval size 2 minutes battle will be summed up in the interval size 4 minutes battle.

The analyst must be careful to use the same interval size when comparing several battle traces.

5.2.4 Conclusion. The battle trace of the Lanchester constant C does not have computational problems or numerical instabilities. The instabilities found in the battle trace can be strictly related to the instabilities associated with the rapidly changing battle dynamics. The interpretation of this battle trace methodology is the same as the concept of the Lanchester's Square Law. The sensitivity to the different interval sizes can be accommodated by the cumulative C value, but interval sizes should be kept constant when several battles are to be compared.

The cumulative value of constant C represents the cumulative scores of the combat effectiveness up to the time specified. The cumulative constant C (CC) trace throughout the battle process is a useful battle trace. This battle trace is not sensitive to the interval size and has no computational problems. This methodology only requires a sufficient number of battle segments to trace the battle.

5.3 Battle Trace of Lanchester Arc-tangent

The Lanchester's Square Law concept implies that the attrition rate β and ρ are constant throughout the battle. Although the attrition rates may change during a battle in a complicated modern warfare environment, this idea may be reasonable when using a fairly small time intervals. Therefore, it can be supposed a smaller time interval will allow a more efficient application of the Lanchester's Square Law.

The trajectories of the Lanchester's Square Law can be depicted in a force sizes (B, R) plane as hyperbolas when the constant C is not zero (2:11, 7:Ch 28,12). When C equals zero the battle is at the parity condition and the trajectory is on the standoff line. The trajectories of the Lanchester's Square Law are shown in the Figure 28.

The angle of the standoff line (parity) of k^{th} battle segment is dependent on ρ_k and β_k. The initial battle position of each battle segment is defined by the value of R and B at that time. If the k^{th} battle position is above the k^{th} standoff line, the value of C_k ends up positive and the Red forces will tend to win within the time interval as depicted in Figure 28.

One way to measure the compared combat effectiveness between Blue forces and Red forces is to compare the radian difference between the k^{th} standoff line and the k^{th} battle position line, i.e. the line between the battle position (B_k, R_k) and the origin (1). This radian is denoted as Q_k in Figure 28. The value of Q_k can be computed by the following equations:

$$Q_k = S_k - P_k \tag{54}$$

where

Q_k = radian measure between the k^{th} standoff line and the k^{th} battle position line

S_k = radian measure between the k^{th} battle

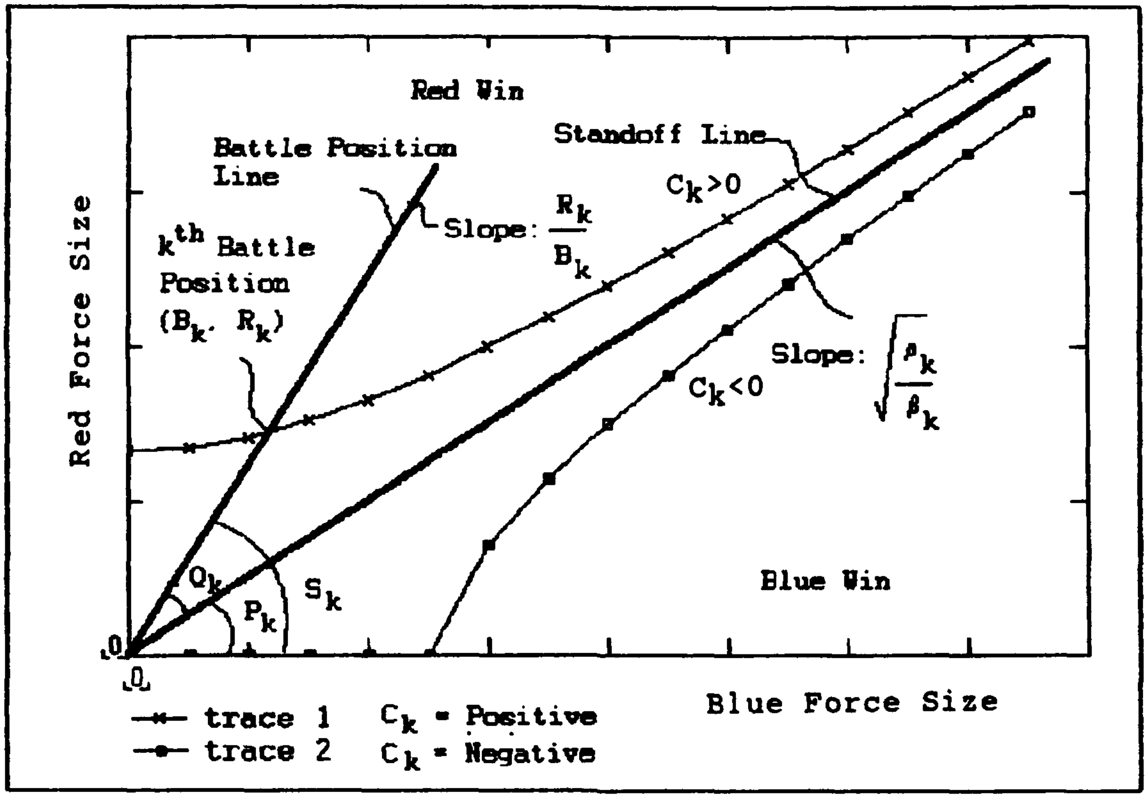


Figure 28. Trajectories of the Lanchester's Square Law Arc-tangent

position line and the B-axis

P_k = radian measure between the k^{th} standoff line and the B-axis

This radian measure can be computed by utilizing the arc-tangent function as follows:

$$Q_k = \tan^{-1}\left(\frac{R_k}{B_k}\right) - \tan^{-1}\sqrt{\left(\frac{\rho_k}{\beta_k}\right)} \quad (55)$$

and substituting the attrition rates ρ_k and β_k into $\Delta R_k/B_k$ and $\Delta B_k/R_k$, respectively.

$$Q_k = \tan^{-1}\left(\frac{R_k}{B_k}\right) - \tan^{-1}\sqrt{\left(\frac{\frac{\Delta R_k}{B_k}}{\frac{\Delta B_k}{R_k}}\right)} \quad (56)$$

The arc-tangent(x,y) is the angle (in radians) from positive x-axis to the point (x,y) in x-y plane. Therefore, Eq (56) can be rewritten as:

$$Q_k = \text{angle}(B_k, R_k) - \text{angle}\left(\sqrt{\frac{\Delta B_k}{R_k}} \cdot \sqrt{\frac{\Delta R_k}{B_k}}\right) \quad (57)$$

where

angle(x,y) = the angle (in radians) from positive x-axis to the point (x,y) in the x-y plane.

This equation illustrates that the force size cannot be zero, because the ratio cannot have zero in the denominator and the angle cannot be defined when both components are zero. The battle intervals greater than or equal to 3 minutes battle do not include any zero values in the force size data for B and R. However, these battles include zero values in ΔB and ΔR. Therefore, if the interval sizes 3 and 5 minutes are taken for this study only one problem will arise when ΔB and ΔR both are zero.

One possible way to solve this computational problem is to assign the same constant value to both ΔB and ΔR when they are both zero. If the same constant value is assigned to both ΔB and ΔR the slope of the standoff line would remain the same. That is, angle[a(x,y)] = angle (x,y), where a is any arbitrarily chosen constant. Hence the equation for Q_k when ΔB and ΔR both are zero can be described as:

$$Q_k = \text{angle}(B_k, R_k) - \text{angle}\left[\frac{a}{\sqrt{R_k}}, \frac{a}{\sqrt{B_k}}\right] \tag{58}$$

where

a = any arbitrarily chosen nonzero constant

Moreover, if the values of ΔB and ΔR are the same, including (0, 0), the slope of the standoff line only depends on the values of R and B. Therefore, any same constant values can be assigned to ΔB and ΔR when they are equal *including* the cases when they are both zero. In this study, the values one and one were chosen for the constant value "a" when both ΔB and ΔR are zero.

Each battle segment can be scored by the values of Q in Eq (55). The regions of battle status classification are listed in Table 20.

Table 20. Decision Region of Lanchester Arc-tangent

Arc-tangent (Q)	Battle state
$Q > 0$	Red win
$Q < 0$	Blue win
$Q = 0$	parity

This table indicates this equation has symmetry. That is, both sides' combat effectiveness are equally weighted in the computation of the arc-tangent. The Lanchester arc-tangent with interval sizes 3 minutes and 5 minutes are shown in Figures 29 and 30, respectively.

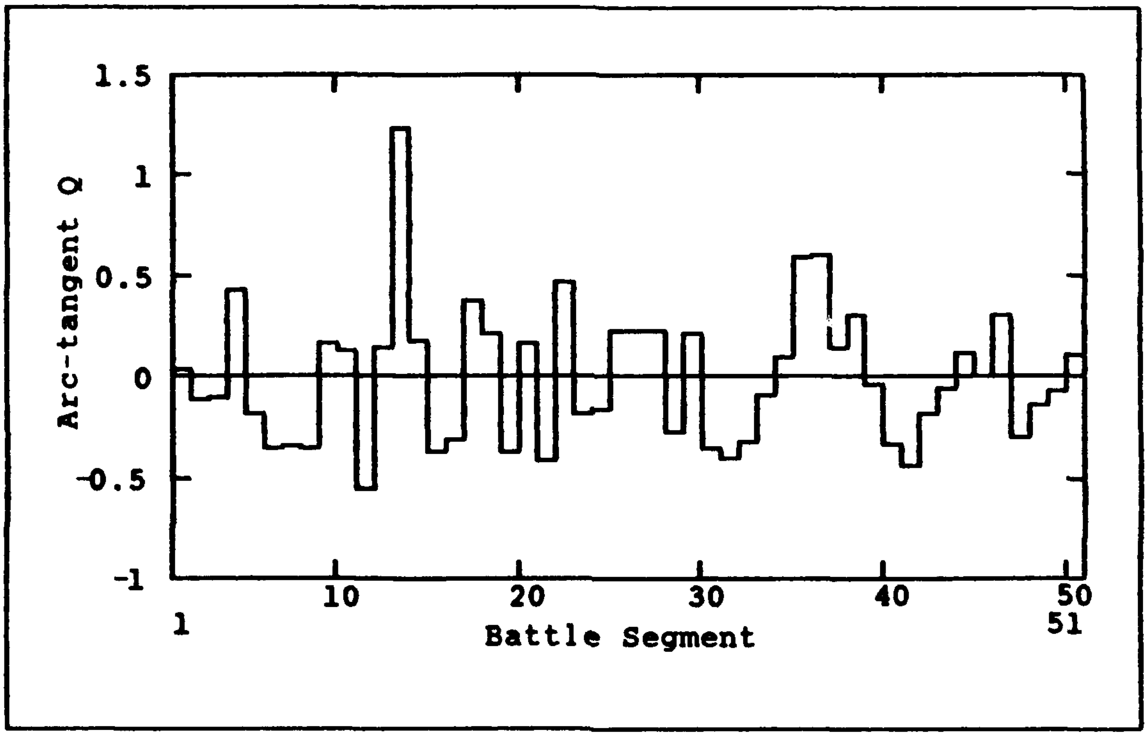


Figure 29. Battle Trace of Lanchester Arc-tangent with Interval Size 3 Minutes

The number of battle segments classified by battle state are shown in Table 21. The percentages of the classified battles vs. total number of battle segments is almost unchanged with respect to the interval size. Therefore, the battle trace of the arc-tangent of the Lanchester's Square Law is steady in terms of this battle state classification. Some statistics on Q for this data

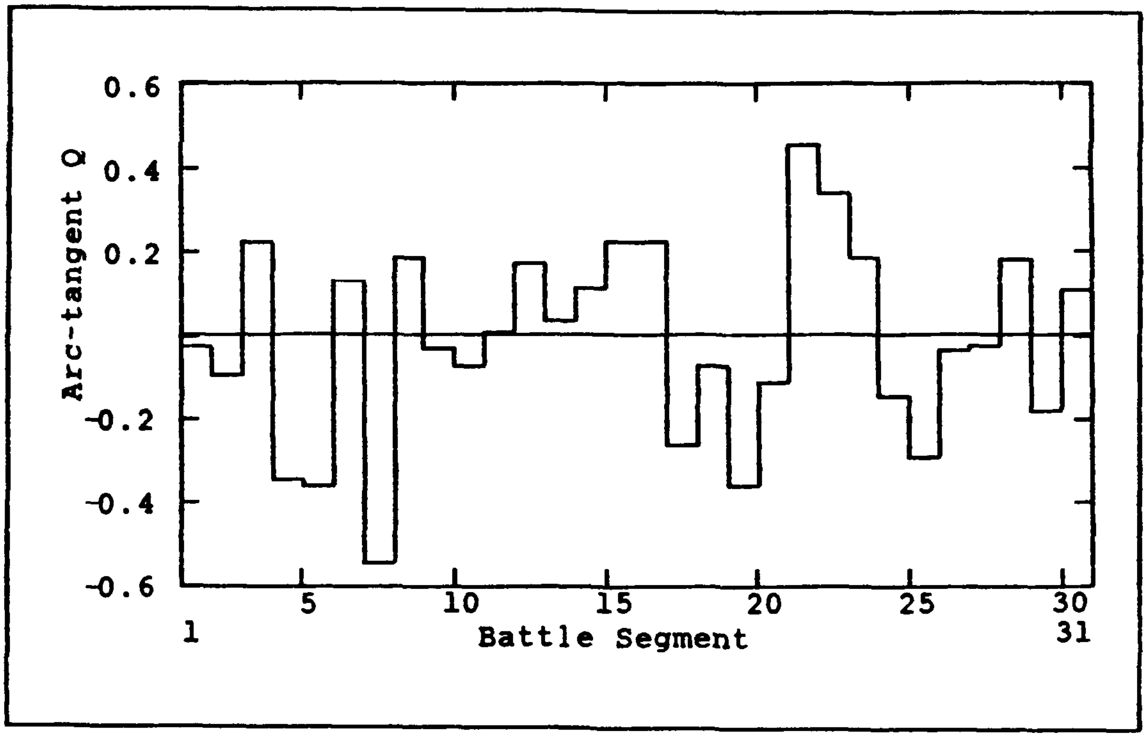


Figure 30. Battle Trace of Lanchester Arc-tangent with Interval Size 5 Minutes

Table 21. Number of Battle Segments Classified by the Lanchester Arc-tangent

Arc-tangent(Q)	Interval size	
	3 minute	5 minute
Q > 0 : Red win	23 (45.1 %)	14 (45.2 %)
Q < 0 : Blue win	28 (54.9 %)	17 (54.8 %)
Q = 0 : Parity	0	0
Total	51 (100 %)	31 (100 %)

are shown in Table 22. The mean in the table is the geometric mean of the arc-tangent of the battle starting off point and the standoff line. The variance was computed with an assumption that the arc-tangent is independently associated with battle time and place. Table 22 shows that

Table 22. Statistics of Arc-tangent of the Lanchester's Square Law

Statistics of Q	Interval size	
	3 minute	5 minute
Mean	-0.01	-0.02
Max	1.23	0.45
min	-0.56	-0.55
Variance	0.11	0.05

the statistics changed very little with changes from the 3 minutes to 5 minutes interval.

In conclusion, the battle trace of arc-tangent of the Lanchester's Square Law is not very sensitive to the related time interval when the interval size is large. When the size of battle segment is large and does not include zero values in R and B, this methodology can be applied on the battle trace. However, application of this method should be preceded by time interval analysis to reduce the likelihood of a singularity problem in attrition rate computation and a reduction in the number of points where the undefinable values of the arc-tangent exists. Since the value of arc-tangent is the magnified interpretation of the Lanchester's Square Law, this methodology can provide a reasonable combat interpretation.

5.4 Conclusion

In this chapter two battle trace methodologies were discussed. The battle trace of the Lanchester constant employed the constant C in the Lanchester's Square Law as the measure of comparing two forces' combat effectiveness. The battle trace of arc-tangent of the Lanchester's Square Law employed the angle between initial battle point in the B-R plane and the standoff line.

The battle trace of the Lanchester's Square Law constant has no computational problems or instabilities cause by the battle dynamics. This methodology is directly related to the Lanchester's Square Law interpretation itself.

The battle trace of arc-tangent of the Lanchester's Square Law has a computational problem when any value of force size is zero or both side's number of system deaths are zero within a time interval. These problems can be solved by taking appropriate interval sizes and assigning the same constant values to ΔB and ΔR when both are zero.

Intuitively, the battle trace of Lanchester constant C is a somewhat better method to employ than that of arc-tangent trace, since it is computationally more efficient. However, no clear decision rule was provided to decide which was the better methodology. Reasonable decision making requires enough experimental analysis with the accessible output processes. It is the case that both methodologies reduced the numerical instabilities problem that were significant in the originally suggested battle trace methodologies.

VI. Results, Conclusions, and Recommendations

6.1 Results

The approach of measuring the combat effectiveness of a series of small battle segments and plotting these measured values of combat scores is one reasonable way to characterize the battle process. Several battle trace methodologies were examined in this study to provide ways to compute relative combat effectiveness for a series of battle segments. The analysis of these methodologies focused on the following factors: 1) efficiency of computation; 2) numerical stabilities; 3) combat interpretation.

Barr's battle trace of combat force elasticity provided the impetus for this research. The interpretational difficulty due to the reciprocal symmetry suggested the log of CFE trace be employed to transform the reciprocal symmetry to additive symmetry. However, each methodology tried, developed a computational problem, when any zero value occurred in the denominator of CFE or as any argument of the log of CFE. To solve this computational problem, Barr suggested assigning judgmental battle status values to the undefinable values. However, these assigned values are qualitative and can be unsuitable compared with other nearby computed quantitative values. These computational problems indicate the numerical instability of both methodologies. Another method of handling this computational problem was adding small values to each component of CFE. However, this methodology also contains error due in fact to the added values. This computational problem indicates that these combat evaluation methodologies also contain numerical instabilities.

As a conclusion to this analysis, it is believed that these methodologies contain numerical instabilities not only because of the battle instabilities but also because of their mathematical properties. Therefore, other Lanchester's Square Law transformations were developed to eliminate the numerical computational problem and serve as alternatives to the data smoothing approaches.

Two Lanchester's Square Law based relative combat evaluation methodologies were developed. These trials were, specifically, the battle trace of the Lanchester's Square Law constant and the battle trace of the Lanchester's Square Law arc-tangent.

The battle trace of the Lanchester's Square Law constant is the trace of the constant C computed during the integration of the Lanchester's Square Law. This methodology does not have any computational problems. The instability in the battle trace plot of using this measure probably represents the true nature of the battle phenomena when the size of battle interval is properly set. The measured relative combat effectiveness can misrepresent the battle status when the interval size is too small.

Another advantage of this battle trace methodology was the ability to meaningfully interpret cumulative values of the Lanchester constant. These cumulative constant values can be interpreted as the total score of the relative battle effectiveness until that computed time. The trace of cumulative constant C (CC) can be a useful battle trace without any computational problem and sensitivity to various interval sizes.

The battle trace of the Lanchester's Square Law arc-tangent is the plot of the arc-tangent values that represent the radians between the standoff line and force size values marked in the Blue force size and Red force size plane. This methodology has a computational problem when either side's force size is zero or both of the number of deaths are zero in that time interval. Battle segment interval sizes greater than or equal to 3 minutes were analyzed since no zero value was found in those interval battle segments. The problem when the delta values, ΔB and ΔR, were both zero, was solved by assigning the same constant value for ΔB and ΔR. The assignment was deemed reasonable because the slope of standoff line depended only on the values of B and R when ΔB and ΔR are the same value. The analysis demonstrates this battle trace methodology is not very sensitive to the interval size when that size is sufficiently large. Since this methodology uses the

Lanchester's Square Law interpretation, the interpretation of this battle trace is similarly reasonable.

6.2 Conclusions

Combat is a stochastic process where many unexpected combat factors can effect combat results. These stochastic and unexpected combat factors can be reflected in the battle trace plot, but may show up as instabilities within the battle trace. If however, the numerical instability problem can be eliminated, the remaining instabilities can be construed as the true combat phenomena itself. Hence, an appropriate way to measure the relative combat effectiveness that is computationally sound and expected to contain no numerical instability due to computational problems can provide the measure that can represent the unstable stochastic nature of battle.

Two measures of relative combat effectiveness were developed by adopting the Lanchester's Square Law. The battle trace of the Lanchester's Square Law constant was computationally efficient and can provide a proper interpretational sense based on the Lanchester's Square Law since the constant is generated by the Lanchester's Square Law integration itself. However, the interval analysis indicated that if the size of interval is too small, the battle status can be miss scored. It is important that a proper time interval size should be selected.

The battle trace of the Lanchester's Square Law arc-tangent has a computational problem when either R or B is zero. But again, a proper selection of interval size seemed to solve this problem. This arc-tangent methodology also provides a reasonable interpretation based on the Lanchester's Square Law.

After comparing these two battle trace methodologies, it can be concluded that the Lanchester constant trace is a better methodology than the arc-tangent trace because the battle trace of Lanchester constant C does not have any computational problems and has meaning when used as the cumulative value of constant C. However, this conclusion is based only on common sense observations since no objective criterion was developed for methodology selection. Clearly, the efficiency of any battle trace methodology depends on the methodology of measuring the relative combat effectiveness and the frequency of repeating battle activities facilitated by proper time interval size selection.

6.3 Recommendations

There are several recommendations that can be made for future enhancements to battle trace methodology.

First, an enhanced methodology needs to be developed to consider additional combat factors in the measurement of combat effectiveness. The battle trace in this research concerned itself with only the Lanchester's Square Law based measurement where the attrition rates were computed based upon battle results. That is, the battle trace depended only on simple combat factors (the system deaths and force sizes). However, the results of battle are not only dependent on the force size, but are also depended on the level of training, troop motivation, leadership, determination, audacity, etc. It is recommended that a similar study be performed utilizing these factors.

Second, model verification requires analysis of a sufficient number of duplicated experimental data sets. The statistical analysis on a number of data sets will enable the process of establishing that the developed model executes as intended. Unfortunately, only one set of data was provided for this study. Therefore, it is recommended that several sets of data be evaluated in future studies.

Third, the model requires further analysis to see if a desired accuracy or correspondence exists between the characterized (scored) battle status and the real battle status. This analysis can be accomplished by having a precise time description associated with the unfolding of the battle scenario which can enable users to identify the development of the controlling combat factors. Analyzing changes in these combat factors and the corresponding changes in the battle trace will enable the analyst to recognize the sensitivity of the battle trace to these combat factors. It is recommended that a dynamic scenario be provided for this analysis.

Next, it is recommended that a forecasting method be developed in terms of battle results prediction. This research did not focus on predicting battle results or examine a forecasting method. However, for battle prediction "during battle", this methodology may be worthwhile.

Finally, further battle trace analysis is recommended to include additional battle trace comparison methodologies. Comparison of two distinct battles can contribute to understanding the nature of the differences in the battles and can provide a way to identify significant combat factors that effect combat results (2:24). Although no single measure of combat effectiveness can capture all explanatory combat phenomena, this possible application of battle trace can assist the analyst in seeing the results of the battle comparison.

Appendix A: Format of the BTRACE.TST File

Table 24 shows the format of the given BTRACE.TST data file. This file includes 741 records that represent the number of systems of a particular type or in a specific task force that can "see" at least one enemy system within a 20 second battle interval time. The first row of the original file identifies the weapon-system-type-numbers and the task force numbers. The first column marks the simulation time and the remaining columns list the number of seers from the weapons corresponding to the codes found in Table 23 below.

Table 23. Contents of the BTRACE.TST File

Blue side		Red side	
Heading (Column Number)	Description	Heading (Column Number)	Description
B1 (2)	SP 155mm ARTY	R1 (13)	MORTAR 120 mm
B2 (3)	MORTAR 4.2"	R2 (14)	SP 122mm ARTY
B3 (4)	Tank	R3 (15)	SP 152mm ARTY
B4 (5)	TOW	R4 (16)	122mm MRL
B5 (6)	BRADLEY with TOW	R5 (17)	BRDM
B6 (7)	BRADLEY without TOW	R6 (18)	T-72 Tank
B7 (8)	FASCAM	R7 (19)	ZSU
B8 (9)	VULCAN	R8 (20)	Dismounted soldier
B9 (10)	Stinger	R9 (21)	BMP
B10 (11)	CEV	R10 (22)	Truck
B11 (12)	M113 with TOW	R11 (23)	SA-7
BF1 (24)	Task Force 1	RF1 (29)	Task Force 1
BF2 (25)	Task Force 2	RF2 (30)	Task Force 2
BF3 (26)	Task Force 3	RF3 (31)	Task Force 3
BF4 (27)	Task Force 4	RF4 (32)	Task Force 4
BF5 (28)	Task Force 5	RF5 (33)	Task Force 5

Table 24. BTRACE.TST Data Form

T	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	B11	R1	R2	R3	R4	R5	R6	R7	R8	R9	R10	R11	BF1	BF2	BF3	BF4	BF5	RF1	RF2	RF3	RF4	RF5
0.0000	3	5	51	53	58	59	60	61	63	86	133	2	3	4	6	56	58	59	65	73	8 1	19	1	2	3	4	5	1	2	3	4	5
0.3333	0	0	4	0	1	1	1	0	0	0	3	0	0	0	0	3	10	0	7	29	0	0	2	1	1	5	1	11	0	38	0	0
0.6667	0	0	4	1	1	1	1	0	0	0	3	0	0	0	0	3	9	0	7	24	0	0	2	1	1	5	2	11	0	32	0	0
1.0167	0	0	4	1	1	1	1	0	0	0	3	0	0	0	0	3	7	0	7	25	0	0	2	1	1	5	2	11	0	31	0	0
1.3500	0	0	4	1	0	1	1	0	1	0	2	0	0	0	0	3	5	0	7	20	0	0	2	1	1	4	2	11	0	24	0	0
1.6833	0	0	4	1	0	1	0	0	1	0	2	0	0	0	0	3	5	0	7	14	0	0	1	1	1	4	2	11	0	18	0	0
2.0167	0	0	4	1	0	1	1	0	1	0	2	0	0	0	0	3	4	0	7	13	0	0	2	1	1	4	2	11	0	16	0	0
2.3500	0	0	4	0	0	1	1	0	0	0	2	0	0	0	0	2	4	0	7	13	0	0	1	1	1	4	1	10	0	16	0	0
2.6833	0	0	3	0	0	1	1	0	0	0	2	0	0	0	0	2	4	0	7	13	0	0	1	1	1	3	1	10	0	16	0	0
3.0167	0	0	3	0	0	1	1	0	0	0	2	0	0	0	0	2	5	0	7	12	0	0	1	1	1	3	1	10	0	16	0	0
3.3500	0	0	2	0	0	1	0	0	0	0	2	0	0	0	0	2	5	0	7	13	0	0	0	1	1	3	0	10	0	17	0	0
3.6833	0	0	2	0	0	1	0	0	0	0	1	0	0	0	0	2	4	0	7	15	0	0	0	1	1	2	0	9	0	19	0	0
4.0167	0	0	2	0	0	1	0	0	0	0	1	0	0	0	0	2	5	0	7	14	0	0	0	1	1	2	0	9	0	19	0	0
4.3333	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0	1	5	0	7	17	0	0	0	1	0	2	0	8	0	22	0	0
4.6667	0	0	2	0	0	1	0	0	0	0	0	0	0	0	0	0	5	0	7	17	0	0	0	1	0	2	0	7	0	22	0	0
5.0000	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	7	17	0	0	0	0	0	2	0	7	0	22	0	0
5.3333	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	7	16	0	0	0	0	0	2	0	7	0	22	0	0
5.6667	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	6	0	7	17	0	0	0	0	0	2	0	7	0	23	0	0
6.0000	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	7	0	7	19	0	0	0	0	0	2	0	7	0	26	0	0
6.3334	0	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	5	0	7	18	0	0	0	0	0	2	0	7	0	23	0	0

Appendix B: Format of the KTRACE.TST File

Table 26 shows the format of the given KTRACE.TST file. This file provides the killer/victim data associate with its event time. The total number of records is 203 with the time running from 0.0606 to 150.9589 minutes. The contents of this file are listed in Table 25.

Table 25. Contents of KTRACE.TST File

Heading	Description
T	Simulation time kill
TK	Type of kill: 1=Blue, 2=Red
RE	Range of engagement
SV	Side of victim: 1=Blue, 2=Red
STV	System type of victim
TFV	Task force of killer
CV	Coordinate of victim when killed
NK	Number of system killed
SK	Side of killer: 1=Blue, 2=Red
STK	System type of killer
TFK	Task force of killer
CK	Coordinates of killer when shot was fired
TA	Type of ammunition fired by killer

Table 26. KTRACE.TST Data Form

T	TK	RE	SV	STV	TFV	CV	NK	SK	STK	TFK	CK	TA
0.0606	1	1.033	2	73	1	552.814 910.724	1	1	51	4	552.026 910.056	17
1.1403	1	1.744	2	56	1	555.419 903.612	1	1	51	5	553.712 903.966	18
1.2714	1	1.145	1	58	1	548.430 897.909	1	2	58	3	547.488 898.561	30
2.3382	1	0.606	2	56	1	549.828 909.047	1	1	59	2	549.284 908.781	6
3.4362	1	1.898	2	73	1	550.454 909.282	1	1	51	4	552.200 910.024	17
4.1220	1	0.397	2	56	1	552.461 910.324	1	1	51	4	552.200 910.024	18
4.5812	1	0.333	2	56	1	548.929 909.034	1	1	59	2	549.215 908.864	6
8.2542	1	0.236	2	58	3	548.680 898.104	1	1	60	1	548.473 898.216	12
8.9527	1	0.260	2	73	3	547.249 897.999	1	1	60	1	547.208 898.255	12
10.4854	1	0.197	2	73	3	547.222 898.059	1	1	60	1	547.208 898.255	12
10.8399	1	0.668	1	51	4	550.188 899.250	1	2	58	3	550.140 898.583	30
10.9217	1	0.861	1	51	4	550.225 899.375	1	2	58	3	550.853 898.786	30
11.3812	1	0.227	1	63	1	550.516 898.706	1	2	58	3	550.332 898.573	8
13.3568	1	0.504	2	56	1	555.328 903.729	1	1	58	3	554.917 903.439	6
13.4529	1	0.239	2	58	3	548.417 897.984	1	1	60	1	548.473 898.216	12
16.1046	2	9.668	2	81	4	547.985 897.396	1	1	3	1	556.475 902.022	4
16.1388	2	10.246	2	73	3	548.343 897.783	1	1	3	1	557.361 902.647	4
18.5213	1	0.295	2	58	3	548.732 898.076	1	1	60	1	548.473 898.216	12
19.8433	1	1.960	2	56	1	554.807 905.280	1	1	51	5	552.895 904.847	18
21.5546	1	0.296	2	81	4	548.760 898.142	1	1	60	1	548.473 898.216	12
21.8447	1	2.075	2	56	1	554.611 905.645	1	1	51	5	552.748 904.733	18
31.3753	1	2.488	2	58	3	553.860 900.676	1	1	51	3	555.175 902.788	17
38.0253	1	1.240	1	58	3	554.625 901.850	1	2	58	3	554.729 900.614	30
39.1043	1	2.079	2	73	3	554.507 900.823	1	1	51	3	555.400 902.700	17
40.4477	1	1.518	1	58	1	541.163 900.888	1	2	58	5	540.120 899.785	30
43.9700	1	1.988	2	73	3	554.426 900.967	1	1	51	3	555.400 902.700	17
44.5055	1	2.199	2	73	3	554.310 900.790	1	1	51	3	555.400 902.700	17
44.7380	1	2.120	2	73	3	554.212 901.043	1	1	51	3	555.188 902.925	17
45.0021	1	1.873	2	73	3	554.325 901.167	1	1	51	3	555.400 902.700	17
45.2697	1	1.964	2	58	3	554.156 901.110	1	1	51	3	555.175 902.788	17

Appendix C: Initial Data Conversion Basic Program

The below BASIC program was used to convert (consolidate, aggregate, and unify) the BTRACE.TST and KTRACE.TST files.

```
100 ' ***********************************************
101 ' ********** DATA CONVERSION PROGRAM 1*************
102 ' THIS PROGRAM IS FIRST THE BASIC DATA CONVERSION
      PROGRAM FOR 20 SECONDS TIME INTERVALS' SYSTEM
      DEATH AND SEERS
103 ' THE INPUT ASCII FILE, BTRACE.ASC CONTAINS THE
      BATTLE SEERS DATA
105 ' THE INPUT ASCII FILE, KTRACE.ASC CONTAINS THE
      KILLER-VICTIM DATA
106 ' ***********************************************
107 '
120 OPEN "F:\BASIC\INDATA\KTRACE.ASC" FOR INPUT AS #1
130 OPEN "F:\BASIC\INDATA\BTRACE.ASC" FOR INPUT AS #2
140 OPEN "F:\BASIC\OUTDATA\1STCON.ASC" FOR OUTPUT AS #3
150 '
155 ' **  READ KTRACE.ASC AND BTRACE.ASC DATA  **
160 GOSUB 700
165 GOSUB 800
167 '
168 ' **  COMPARE THE BTRACE.ASC INTERVAL AND
          KTRACE.ASC EVENT TIME  **
170 IF KT <= BT THEN GOTO 200
171 GOTO 400
190 GOTO 500
193 '
195 ' **  SUM THE NUMBER OF SYSTEM DEATH IN EACH BATTLE
          SEGMENT  **
200 IF RB = 1 THEN DB = DB + 1
210 IF RB = 2 THEN DR = DR + 1
220 IF EOF1 THEN 500
230 GOSUB 700
240 GOTO 170
300 '
310 ' **  CREATE OUTPUT FILE **
400 PRINT#3, BT, DB, BB, DR, RR
410 DB = 0: DR = 0
420 IF EOF2 THEN 500
425 GOSUB 800
430 GOTO 170
440 '
```

```
500 CLOSE 1
510 CLOSE 2
520 CLOSE 3
600 '
610 ' **  READ KILL EVENT TIME AND SIDE IDENTIFICATION
          IN KTRACE.ASC FILE  **
700 LINE INPUT#1, K$
710 KT = VAL(MID$(K$,1,8))
720 RB = VAL(MID$(K$,26,1))
730 RETURN
740 '
750 ' ** READ SYSTEM SEERS IN BTRACE.ASC FILE AND SUM
          IT **
800 LINE INPUT#2, B$
810 BT = VAL(MID$(B$,1,8))
820 B1 = VAL(MID$(B$, 9,4)): R1 = VAL(MID$(B$,29,4))
830 B2 = VAL(MID$(B$,13,4)): R2 = VAL(MID$(B$,33,4))
840 B3 = VAL(MID$(B$,17,4)): R3 = VAL(MID$(B$,37,4))
850 B4 = VAL(MID$(B$,21,4)): R4 = VAL(MID$(B$,41,4))
860 B5 = VAL(MID$(B$,25,4)): R5 = VAL(MID$(B$,45,4))
870 BB = B1+B2+B3+B4+B5:  RR = R1+R2+R3+R4+R5
880 RETURN
890 ' ************************************************
900 ' **** THE OUTPUT INCLUDE THE FOLLOWING DATA ******
910 ' ** 1ST COLUMN (BT): TIME (BATTLE SEGMENT)
920 ' ** 2ND COLUMN (DB): NUMBER OF BLUE SYSTEM DEATH
930 ' ** 3RD COLUMN (BB): SUM OF BLUE SEERS IN EACH
          BATTLE SEGMENT (20 SECONDS INTERVAL)
940 ' ** 4TH COLUMN (DR): NUMBER OF RED SYSTEM DEATH
950 ' ** 5TH COLUMN (RR): SUM OF RED SEERS IN EACH
          BATTLE SEGMENT (20 SECONDS INTERVAL)
960 ' ************************************************
```

Appendix D: Data Generated by the Initial Data Conversion Program

The following data file was generated by the initial data conversion BASIC program. This converted data file identifies the system deaths and seers recorded for Red and Blue at each 20 second interval of time. The time column is in units of 1 minute.

TIME	BLUE DEATH	BLUE SEERS	RED DEATH	RED SEERS
.333	0	10	1	49
.6667	0	11	0	43
1.0167	0	11	0	42
1.35	1	10	1	35
1.6833	0	9	0	29
2.0167	0	10	0	27
2.35	0	8	1	26
2.6833	0	7	0	26
3.0167	0	7	0	26
3.35	0	5	0	27
3.6833	0	4	1	28
4.0167	0	4	0	28
4.3333	0	3	1	30
4.6667	0	3	1	29
5	0	2	0	29
5.3333	0	2	0	29
5.6667	0	2	0	30
6	0	2	0	33
6.3334	0	2	0	30
6.6667	0	2	0	30
7	0	4	0	40
7.3334	0	4	0	38
7.6667	0	4	0	37
8	0	5	0	34
8.3334	0	4	1	32
8.666701	0	4	0	41
9	0	5	1	41
9.3334	0	3	0	42
9.666701	0	4	0	37
10	0	4	0	42
10.3333	0	6	0	42
10.6667	0	6	1	43
11	2	6	0	45

11.35	0	7	0	46
11.6833	1	7	0	46
12.0167	0	6	0	48
12.35	0	7	0	47
12.6833	0	5	0	48
13.0166	0	5	0	48
13.35	0	9	0	48
13.6833	0	8	2	47
14.0166	0	9	0	47
14.35	0	13	0	47
14.6833	0	15	0	47
15.0166	0	13	0	47
15.3499	0	15	0	45
15.6833	0	19	0	47
16.0166	0	19	0	47
16.3499	0	19	2	45
16.6833	0	19	0	46
17.0166	0	18	0	46
17.3499	0	16	0	46
17.6832	0	15	0	46
18.0166	0	16	0	45
18.3499	0	15	0	47
18.6832	0	18	1	50
19.0165	0	18	0	49
19.3499	0	18	0	49
19.6832	0	18	0	50
20.0165	0	19	1	51
20.3499	0	19	0	51
20.6832	0	19	0	51
21.0165	0	19	0	51
21.3498	0	18	0	50
21.6832	0	17	1	49
22.0165	0	17	1	48
22.3498	0	17	0	47
22.6832	0	17	0	45
23.0165	0	17	0	45
23.3498	0	17	0	44
23.6831	0	17	0	44
24.0165	0	19	0	45
24.3498	0	19	0	44
24.6831	0	18	0	44
25.0165	0	19	0	40
25.3498	0	19	0	39
25.6831	0	19	0	39
26.0164	0	19	0	34
26.3498	0	19	0	35
26.6831	0	19	0	35
27.0164	0	20	0	33
27.3498	0	21	0	34
27.6831	0	21	0	34
28.0164	0	21	0	35
28.3497	0	22	0	35

28.6831	0	22	0	34
29.0164	0	21	0	35
29.3497	0	21	0	35
29.6831	0	21	0	35
30.0164	0	21	0	34
30.3497	0	21	0	34
30.683	0	21	0	34
31.0164	0	21	0	34
31.3497	0	23	0	35
31.683	0	19	1	32
32.0163	0	20	0	33
32.3497	0	21	0	32
32.683	0	21	0	34
33.0163	0	19	0	34
33.3497	0	19	0	34
33.683	0	19	0	31
34.0163	0	19	0	31
34.3496	0	18	0	30
34.683	0	17	0	29
35.0163	0	15	0	32
35.3496	0	17	0	36
35.683	0	17	0	34
36.0163	0	15	0	34
36.3496	0	11	0	34
36.6829	0	11	0	34
37.0163	0	12	0	34
37.3496	0	12	0	33
37.6829	0	11	0	33
38.0163	0	11	0	34
38.3496	1	13	0	37
38.6829	0	14	0	36
39.0162	0	15	0	33
39.3496	0	14	1	31
39.6829	0	16	0	31
40.0162	0	16	0	32
40.3496	0	18	0	38
40.6829	1	19	0	39
41.0162	0	20	0	44
41.3495	0	18	0	46
41.6829	0	19	0	44
42.0162	0	21	0	47
42.3495	0	22	0	51
42.6829	0	22	0	54
43.0162	0	22	0	58
43.3495	0	22	0	59
43.6828	0	22	0	58
44.0162	0	23	1	62
44.3495	0	24	0	64
44.6828	0	24	1	63
45.0162	0	25	2	63
45.3495	0	5	1	66
45.6828	0	25	1	70

46.0161	0	25	2	76
46.3495	0	25	1	80
46.6828	0	24	3	81
47.0161	0	25	0	81
47.3494	0	27	0	88
47.6828	0	24	2	85
48.0161	0	23	1	86
48.3494	0	22	1	84
48.6828	1	22	0	84
49.0161	0	23	0	83
49.3494	1	22	0	85
49.6827	0	21	0	84
50.0161	0	22	0	80
50.3494	0	25	0	80
50.6827	0	24	0	81
51.0161	0	25	0	79
51.3494	0	24	0	79
51.6827	0	26	0	80
52.016	1	22	1	76
52.3494	0	21	0	75
52.6827	0	22	0	75
53.016	0	23	0	66
53.3494	0	23	0	62
53.6827	0	23	0	61
54.016	0	22	0	58
54.3493	0	23	0	56
54.6827	0	22	1	54
55.016	0	20	1	53
55.3493	0	20	0	59
55.6827	0	23	0	58
56.016	0	23	0	54
56.3493	0	22	0	52
56.6826	0	22	0	50
57.016	0	22	0	50
57.3493	0	22	0	46
57.6826	0	22	0	45
58.016	0	22	0	48
58.3493	0	23	0	48
58.6826	0	23	0	46
59.0159	0	23	0	50
59.3493	0	23	0	50
59.6826	0	23	0	48
60.0159	0	23	0	52
60.3493	0	25	1	52
60.6826	0	23	0	48
61.0159	0	22	1	47
61.3492	0	21	2	50
61.6826	0	22	0	49
62.0159	0	23	0	50
62.3492	0	21	0	50
62.6825	0	22	0	54
63.0159	0	21	0	55

63.3492	0	20	0	91
63.6825	1	21	0	98
64.0159	0	22	1	102
64.3493	0	22	0	109
64.6827	0	21	0	108
65.0161	1	21	0	109
65.3495	0	22	0	114
65.6829	1	21	0	116
66.0163	0	21	0	113
66.3497	0	23	0	112
66.6831	0	21	0	113
67.0165	0	22	0	118
67.3499	0	21	0	115
67.6833	0	21	0	115
68	0	22	0	115
68.33351	0	22	0	117
68.6669	0	22	1	117
69.0003	0	22	0	117
69.3337	0	21	0	117
69.6671	0	17	1	116
70.00051	0	18	0	115
70.3339	0	19	0	116
70.6673	0	19	0	116
71.0007	0	17	0	117
71.3341	0	16	0	116
71.66751	0	18	0	116
72.0009	0	18	0	116
72.3343	0	17	0	116
72.6677	0	17	0	116
73.0011	0	17	0	117
73.33451	0	16	0	118
73.6679	0	16	0	118
74.0013	0	17	0	117
74.3347	0	19	0	117
74.6681	0	19	0	119
75.0015	0	20	0	120
75.3349	0	19	0	118
75.66841	0	19	0	117
76.0018	0	18	0	113
76.3352	0	18	0	112
76.6686	0	18	0	111
77.002	0	17	0	109
77.33541	0	18	0	110
77.6688	0	16	0	108
78.0022	0	17	0	105
78.3356	0	18	0	102
78.669	0	19	0	99
79.00241	0	19	0	96
79.3358	0	20	0	94
79.6692	0	23	0	93
80.0026	0	24	0	93
80.336	0	24	0	93

80.6694	0	23	0	92
81.0028	0	23	0	91
81.3362	0	24	1	89
81.6696	0	24	0	90
82.003	0	26	0	91
82.3364	0	27	0	91
82.6698	0	27	0	91
83.00331	0	26	0	91
83.3367	0	26	0	91
83.6701	0	24	1	90
84.0035	0	25	0	90
84.3369	0	25	0	89
84.67031	0	25	0	89
85.0037	0	26	0	89
85.3371	1	25	0	85
85.6705	0	26	0	80
86.0039	0	26	0	80
86.33731	0	26	1	79
86.6707	0	27	0	78
87.0041	0	27	0	79
87.3375	0	29	0	78
87.6709	0	27	1	76
88.0043	0	26	1	75
88.3377	0	28	0	75
88.67109	0	27	0	72
89.0045	0	27	1	68
89.3379	0	27	0	67
89.6713	0	27	1	67
90.0047	0	27	0	67
90.33821	0	27	0	66
90.6716	0	28	1	65
91.005	0	27	1	64
91.3384	0	27	0	63
91.6718	0	27	0	61
92.00521	0	26	1	61
92.3386	0	26	0	61
92.672	0	26	1	60
93.0054	0	26	0	60
93.3388	0	25	0	87
93.6722	0	27	1	91
94.0056	0	29	0	92
94.339	0	30	1	91
94.6724	0	31	1	89
95.0058	0	31	0	89
95.3392	0	32	0	90
95.6726	0	31	0	90
96.00599	0	31	0	90
96.3394	0	31	0	91
96.6728	0	33	0	91
97.0062	0	35	2	89
97.3396	0	31	1	88
97.67299	0	34	0	89

98.0065	0	36	0	89
98.3399	1	38	0	91
98.6733	0	39	1	91
99.0067	0	39	1	95
99.34011	0	40	0	95
99.6735	0	39	0	90
100.0069	0	39	1	90
100.3403	0	39	0	92
100.6737	0	40	0	92
101.0071	1	37	1	94
101.3405	0	37	0	94
101.6739	1	37	1	91
102.0073	0	36	0	91
102.3407	0	36	1	92
102.6741	0	37	0	92
103.0075	1	34	1	91
103.3409	3	35	0	95
103.6743	1	34	0	90
104.0077	1	32	0	88
104.3411	1	28	0	83
104.6745	1	28	0	85
105.0079	2	24	0	87
105.3414	1	23	0	88
105.6748	3	19	0	74
106.0082	0	19	0	61
106.3416	0	19	0	63
106.675	0	18	0	63
107.0084	0	18	0	64
107.3418	0	18	0	72
107.6752	1	15	1	72
108.0086	0	15	0	75
108.342	0	17	1	77
108.6754	1	15	1	77
109.0088	0	16	0	75
109.3422	0	18	1	69
109.6756	1	14	0	67
110.009	0	10	0	68
110.3424	0	11	0	62
110.6758	1	11	1	62
111.0092	0	12	1	65
111.3426	1	13	1	61
111.676	0	13	0	54
112.0094	1	13	0	52
112.3428	0	13	0	51
112.6763	0	13	0	52
113.0097	1	12	1	49
113.3431	0	12	0	51
113.6765	0	11	0	51
114.0099	0	11	0	55
114.3433	0	11	0	60
114.6767	0	11	0	59
115.0101	0	12	1	64

115.3435	1	8	1	50
115.6769	0	8	1	45
116.0103	0	10	3	39
116.3437	0	9	0	45
116.6771	0	9	2	39
117.0105	0	10	2	38
117.3439	0	9	2	36
117.6773	0	11	1	33
118.0107	0	10	3	26
118.3441	0	11	2	23
118.6775	0	8	2	23
119.0109	0	9	2	22
119.3443	0	9	0	22
119.6777	0	8	1	21
120.0112	0	8	0	21
120.3446	0	8	0	21
120.678	0	8	2	20
121.0114	0	9	1	19
121.3448	0	9	0	19
121.6782	0	9	1	17
122.0116	0	9	2	16
122.345	0	9	1	15
122.6784	0	9	1	14
123.0118	0	6	0	14
123.3452	0	9	0	35
123.6786	0	6	2	45
124.012	0	7	1	48
124.3454	0	8	0	49
124.6788	0	8	1	46
125.0122	0	7	1	43
125.3456	0	7	1	40
125.679	0	7	1	40
126.0124	0	6	1	44
126.3458	0	6	2	44
126.6792	0	7	2	45
127.0126	0	7	3	44
127.3461	0	7	0	39
127.6795	1	6	4	34
128.0129	0	6	1	32
128.3461	0	7	1	29
128.6794	0	6	0	29
129.0126	0	6	2	24
129.3459	1	7	0	20
129.6791	0	6	0	22
130.0124	0	5	0	25
130.3456	0	5	1	26
130.6789	0	6	0	28
131.0121	0	6	1	30
131.3454	0	5	0	30
131.6786	0	5	0	33
132.0119	1	4	0	30
132.3451	0	2	0	25

132.6784	0	2	0	25
133.0116	0	4	1	23
133.3449	0	6	1	28
133.6781	0	3	2	24
134.0114	0	2	0	22
134.3447	0	3	2	20
134.6779	0	7	1	22
135.0112	0	7	0	23
135.3444	1	7	1	22
135.6777	0	5	0	20
136.0109	1	6	1	19
136.3442	0	6	0	19
136.6774	1	5	1	19
137.0107	2	4	0	19
137.3439	0	5	1	18
137.6772	1	6	0	18
138.0104	0	6	0	17
138.3437	0	5	1	14
138.6769	0	5	0	15
139.0102	0	4	1	15
139.3434	0	4	0	15
139.6767	0	4	1	13
140.0099	0	4	0	12
140.3432	0	3	1	12
140.6764	0	4	1	12
141.0097	0	3	1	11
141.3429	0	0	1	11
141.6762	0	0	0	11
142.0094	0	1	0	12
142.3427	0	1	0	12
142.6759	0	2	0	11
143.0092	0	3	0	11
143.3425	0	3	1	10
143.6757	0	1	2	8
144.009	0	2	0	9
144.3422	0	2	0	10
144.6755	0	1	1	9
145.0087	0	1	0	9
145.342	0	0	0	9
145.6752	0	0	0	9
146.0085	0	0	0	9
146.3417	0	0	0	9
146.675	0	0	0	9
147.0082	0	1	0	9
147.3415	0	0	0	8
147.6747	0	0	0	9
148.008	0	0	0	9
148.3412	0	0	0	8
148.6745	0	0	0	8
149.0077	0	0	0	9
149.341	0	0	0	9
149.6742	0	0	0	9

150.0075	0	0	0	9
150.3407	0	0	0	9
150.674	0	1	0	9

Appendix E: Killer-Victim Scoreboards

The following tables are the killer-victim scoreboards (K-V scoreboards) used for the given data overview.

Table 27. K-V Scoreboard for Total Phase
(System on System)

1. Blue killed Red

	MT120	SP122	SP152	MRL122	BRDM	T72	ZSU	DMS	BMP	TRU	SA-7	*SUM*
SP155	0	0	0	0	0	0	0	0	1	1	0	*2*
MORTAR4.2"	0	0	0	0	0	0	0	0	0	0	0	*0*
TANK	0	0	0	0	4	43	4	0	70	0	0	*121*
TOW	0	0	0	0	0	7	0	0	13	0	0	*20*
BRAD/W TOW	0	0	0	0	1	1	0	0	0	0	0	*2*
BRAD/WO TOW	0	0	0	0	2	0	0	0	0	0	0	*2*
FASCAM	0	0	0	0	0	3	0	0	5	1	0	*9*
VULCAN	0	0	0	0	0	0	0	0	0	0	0	*0*
STINGER	0	0	0	0	0	0	0	0	0	0	0	*0*
CEV	0	0	0	0	0	0	0	0	0	0	0	*0*
M113	0	0	0	0	0	0	0	0	0	0	0	*0*
SUM	*0*	*0*	*0*	*0*	*7*	*54*	*4*	*0*	*89*	*2*	*0*	*156*

2. Red killed Blue

	SP155	MT4.2	TANK	TOW	B/W/T	B/WO/T	FASC	VULC	STIN	CEV	M133	*SUM*
MT120	0	0	0	0	0	0	0	0	0	0	0	*0*
SP122	0	0	0	0	0	0	0	0	0	0	0	*0*
SP152	0	0	0	0	0	0	0	0	0	0	0	*0*
MRL122	0	0	0	0	0	0	0	0	0	0	0	*0*
BRDM	0	0	0	0	0	0	0	0	0	0	0	*0*
T72	0	0	23	5	5	0	0	3	1	0	1	*38*
ZSU	0	0	0	0	0	0	0	0	0	0	0	*0*
DISM SOLDIER	0	0	0	0	0	0	0	0	0	0	0	*0*
BMP	0	0	0	0	2	1	0	0	2	0	4	*9*
TRUCK	0	0	0	0	0	0	0	0	0	0	0	*0*
SA7	0	0	0	0	0	0	0	0	0	0	0	*0*
SUM	*0*	*0*	*23*	*5*	*7*	*1*	*0*	*3*	*3*	*0*	*5*	*47*

Table 28. K-V Scoreboard for Total Phase (Force on Force)

1. Blue killed Red

	R/TF1	R/TF2	R/TF3	R/TF4	R/TF5	*SUM*
B/TF1	2	1	6	2	1	*12*
B/TF2	6	7	1	5	24	*43*
B/TF3	3	4	22	0	0	*29*
B/TF4	3	19	1	13	9	*45*
B/TF5	3	3	0	14	7	*27*
SUM	*17*	*34*	*30*	*34*	*41*	*156*

2. Red killed Blue

	B/TF1	B/TF:	B/TF3	B/TF4	B/TF5	*SUM*
R/TF1	0	0	0	0	0	*0*
R/TF2	2	2	1	4	0	*9*
R/TF3	2	0	9	2	0	*13*
R/TF4	1	0	0	1	8	*10*
R/TF5	2	0	0	2	11	*15*
SUM	*7*	*2*	*10*	*9*	*19*	*47*

Table 29. K-V Scoreboard for Phase 1
(System on System)

1. Blue killed Red

	MT120	SP122	SP152	MRL122	BRDM	T72	ZSU	DMS	BMP	TRU	SA-7	*SUM*
SP155	0	0	0	0	0	0	0	0	1	1	0	*2*
MORTAR4.2'	0	0	0	0	0	0	0	0	0	0	0	*0*
TANK	0	0	0	0	4	8	0	0	18	0	0	*30*
TOW	0	0	0	0	0	7	0	0	13	0	0	*20*
BRAD/W TOW	0	0	0	0	1	1	0	0	0	0	0	*2*
BRAD/WO TOW	0	0	0	0	2	0	0	0	0	0	0	*2*
FASCAM	0	0	0	0	0	3	0	0	5	1	0	*9*
VULCAN	0	0	0	0	0	0	0	0	0	0	0	*0*
STINGER	0	0	0	0	0	0	0	0	0	0	0	*0*
CEV	0	0	0	0	0	0	0	0	0	0	0	*0*
M113	0	0	0	0	0	0	0	0	0	0	0	*0*
SUM	*0*	*0*	*0*	*0*	*7*	*19*	*0*	*0*	*37*	*2*	*0*	*65*

2. Red killed Blue

	SP155	MT4.2	TANK	TOW	B/W/T	B/WO/T	FASC	VULC	STIN	CEV	M133	*SUM*
MT120	0	0	0	0	0	0	0	0	0	0	0	*0*
SP122	0	0	0	0	0	0	0	0	0	0	0	*0*
SP152	0	0	0	0	0	0	0	0	0	0	0	*0*
MRL122	0	0	0	0	0	0	0	0	0	0	0	*0*
BRDM	0	0	0	0	0	0	0	0	0	0	0	*0*
T72	0	0	4	0	5	0	0	0	1	0	0	*10*
ZSU	0	0	0	0	0	0	0	0	0	0	0	*0*
DISM SOLIDER	0	0	0	0	0	0	0	0	0	0	0	*0*
BMP	0	0	0	0	2	1	0	0	0	0	1	*4*
TRUCK	0	0	0	0	0	0	0	0	0	0	0	*0*
SA7	0	0	0	0	0	0	0	0	0	0	0	*0*
SUM	*0*	*0*	*4*	*0*	*7*	*1*	*0*	*0*	*1*	*0*	*1*	*14*

Table 30. K-V Scoreboard for Phase 2
(System on System)

1. Blue killed Red

	MT120	SP122	SP152	MRL122	BRDM	T72	ZSU	DMS	BMP	TRU	SA-7	*SUM*
SP155	0	0	0	0	0	0	0	0	0	0	0	*0*
MORTAR4.2"	0	0	0	0	0	0	0	0	0	0	0	*0*
TANK	0	0	0	0	0	35	4	0	52	0	0	*91*
TOW	0	0	0	0	0	0	0	0	0	0	0	*0*
BRAD/W TOW	0	0	0	0	0	0	0	0	0	0	0	*0*
BRAD/WO TOW	0	0	0	0	0	0	0	0	0	0	0	*0*
FASCAM	0	0	0	0	0	0	0	0	0	0	0	*0*
VULCAN	0	0	0	0	0	0	0	0	0	0	0	*0*
STINGER	0	0	0	0	0	0	0	0	0	0	0	*0*
CEV	0	0	0	0	0	0	0	0	0	0	0	*0*
M113	0	0	0	0	0	0	0	0	0	0	0	*0*
SUM	*0*	*0*	*0*	*0*	*0*	*35*	*4*	*0*	*52*	*0*	*0*	*91*

2. Red killed Blue

	SP155	MT4.2	TANK	TOW	B/W/T	B/WO/T	FASC	VULC	STIN	CEV	M133	*SUM*
MT120	0	0	0	0	0	0	0	0	0	0	0	*0*
SP122	0	0	0	0	0	0	0	0	0	0	0	*0*
SP152	0	0	0	0	0	0	0	0	0	0	0	*0*
MRL122	0	0	0	0	0	0	0	0	0	0	0	*0*
BRDM	0	0	0	0	0	0	0	0	0	0	0	*0*
T72	0	0	19	5	0	0	0	3	0	0	1	*28*
ZSU	0	0	0	0	0	0	0	0	0	0	0	*0*
DISM SOLIDER	0	0	0	0	0	0	0	0	0	0	0	*0*
BMP	0	0	0	0	0	0	0	0	2	0	3	*5*
TRUCK	0	0	0	0	0	0	0	0	0	0	0	*0*
SA7	0	0	0	0	0	0	0	0	0	0	0	*0*
SUM	*0*	*0*	*19*	*5*	*0*	*0*	*0*	*3*	*2*	*0*	*4*	*33*

Table 31. K-V Scoreboard for Phase 1 (Force on Force)

1. Blue killed Red

	R/TF1	R/TF2	R/TF3	R/TF4	R/TF5	*SUM*
B/TF1	1	1	6	2	1	*11*
B/TF2	2	1	0	0	0	*3*
B/TF3	2	0	22	0	0	*24*
B/TF4	3	0	0	0	0	*3*
B/TF5	3	3	0	11	7	*24*
SUM	*11*	*5*	*28*	*13*	*8*	*65*

2. Red killed Blue

	B/TF1	B/TF2	B/TF3	B/TF4	B/TF5	*SUM*
R/TF1	0	0	0	0	0	*0*
R/TF2	0	0	0	0	0	*0*
R/TF3	2	0	9	2	0	*13*
R/TF4	0	0	0	0	0	*0*
R/TF5	1	0	0	0	0	*1*
SUM	*3*	*0*	*9*	*2*	*0*	*14*

Table 32. K-V Scoreboard for Phase 2
(Force on Force)

1. Blue killed Red

	R/TF1	R/TF2	R/TF3	R/TF4	R/TF5	*SUM*
B/TF1	1	0	0	0	0	*1*
B/TF2	4	6	1	5	24	*40*
B/TF3	1	4	0	0	0	*5*
B/TF4	0	19	1	13	9	*42*
B/TF5	0	0	0	3	0	*3*
SUM	*6*	*29*	*2*	*21*	*33*	*91*

2. Red killed Blue

	B/TF1	B/TF2	B/TF3	B/TF4	B/TF5	*SUM*
R/TF1	0	0	0	0	0	*0*
R/TF2	2	2	1	4	0	*9*
R/TF3	0	0	0	0	0	*0*
R/TF4	1	0	0	1	8	*10*
R/TF5	1	0	0	2	11	*14*
SUM	*4*	*2*	*1*	*7*	*19*	*33*

Appendix F: Data Conversion Basic Program for Interval Analysis

The following data conversion BASIC program provides data for various interval sizes. This program was used for the time interval analysis portion of the study.

```
100 ' *************  DATA CONVERSION PROGRAM 2 *************
110 ' THIS IS THE SECOND BASIC DATA CONVERSION PROGRAM FOR
     THE DATA CONVERSION FOR INTERVAL ANALYSIS
112 ' THE INPUT ASCII FILE, BTRACE.ASC CONTAINS THE BATTLE
     SEERS DATA
113 ' THE INPUT ASCII FILE, KTRACE.ASC CONTAINS THE
     KILLER-VICTIM DATA
114 ' *****************************************************
115 '
125 OPEN "B:\INDATA\KTRACE.ASC" FOR INPUT AS #1
135 OPEN "B:\INDATA\BTRACE.ASC" FOR INPUT AS #2
145 OPEN "B:\OUTDATA\THREMIN.ASC" FOR OUTPUT AS #3
146 '
150 ' **INITIATE THE SIZE OF BATTLE SEGMENTS; DECIDE THE
TIME      INTERVAL **
151 START=3: INC=3: T=START : CT = 1
156 '
157 ' ** READ KTRACE.ASC AND BTRACE.ASC FILES **
160 GOSUB 700
165 GOSUB 800
167 '
168 ' ** MATCH THE KTRACE INTERVAL WITH THE INITIATED
     INTERVAL **
170 IF KT <= T THEN GOTO 200
180 GOTO 260
190 '
191 ' ** SUM THE NUMBER OF DEATH IN GIVEN TIME INTERVAL **
200 IF RB = 1 THEN DB = DB + 1
210 IF RB = 2 THEN DR = DR + 1
211 ' ** GIVE THE BATTLE DURATION **
220 IF KT > 150 THEN 255
230 GOSUB 700
240 GOTO 170
250 '
251 ' ** RECOGNIZE THE BATTLE END POINT **
255 ID=1
256 '
257 ' ** MATCH THE BTRACE INTERVAL WITH THE INITIATED
     INTERVAL **
```

```
260 IF BT <= T THEN GOTO 300
265 GOTO 346
270 '
271 ' ** COUNT THE DATA RECORD NUMBER OF BTRACE.ASC  IN THE
     INTERVAL **
300 C = C + 1
301 '
302 ' ** SUM THE SEERS IN EACH TIME INTERVAL **
310 SR=SR + RR   : SB=SB + BB
320 IF EOF(2) THEN 500
330 GOSUB 800
340 GOTO 260
344 '
345 ' ** TAKE THE MEAN VALUE OF SEERS **
346 MR=SR/C: MB=SB/C
390 ' ** CREATE OUTPUT FILE **
400 PRINT#3,USING "###  ###.###   ###  ####.##    ###
####.##";CT;T;DB;MB;DR;MR
402 IF ID=1 THEN  500
408 CT = CT + 1
410 DB = 0: DR = 0:C = 0: SR = 0: SB = 0 :MR = 0:MB = 0
415 ' ** NEXT INTERVAL TIME **
420 T = T + INC
430 GOTO 170
440 '
500 CLOSE #1, 2, 3
510 END
520 '
600 ' ** READ KTRACE.ASC, TIME AND SIDE IDENTIFICATION **
700 LINE INPUT#1, K$
710 KT = VAL(MID$(K$,1,8))
720 RB = VAL(MID$(K$,26,1))
725 '    PRINT KT, RB
730 RETURN
740 '
750 ' ** READ BTRACE.ASC AND SUM ALL SEERS **
800 LINE INPUT#2, B$
810 BT = VAL(MID$(B$,1,8))
820 B1 = VAL(MID$(B$, 9,4)): R1 = VAL(MID$(B$,29,4))
830 B2 = VAL(MID$(B$,13,4)): R2 = VAL(MID$(B$,33,4))
840 B3 = VAL(MID$(B$,17,4)): R3 = VAL(MID$(B$,37,4))
850 B4 = VAL(MID$(B$,21,4)): R4 = VAL(MID$(B$,41,4))
860 B5 = VAL(MID$(B$,25,4)): R5 = VAL(MID$(B$,45,4))
870 BB = B1+B2+B3+B4+B5:  RR = R1+R2+R3+R4+R5
880 RETURN
900 '
910 ' *******************************************************
920 ' ****** THE OUTPUT INCLUDE THE FOLLOWING DATA ********
930 ' ** 1ST COLUMN (CT): RECORD COUNT
940 ' ** 2ND COLUMN (T): EVENT TIME (MINUTES)
950 ' ** 3RD COLUMN (DB): NUMBER OF BLUE SYSTEM KILLED
960 ' ** 4TH COLUMN (MB): MEAN VALUE OF THE BLUE SEERS
```

```
970 ' ** 5TH COLUMN (DR): NUMBER OF RED SYSTEM KILLED
980 ' **6TH COLUMN (MR): MEAN VALUE OF THE RED SEERS
990 ' ***************************************************
```

Appendix G: Data Generated by the Interval Data Conversion Program

This appendix contains the data extracted for time intervals of 3 and 5 minutes. The first column (CT) provides the battle segment number, the second column (T) represent the simulation time. The remaining columns are denoted by DB, MB, DR, and MR. These columns contain the Blue deaths, the mean value of the Blue force seers, Red deaths, and the mean value of the Red force seers, respectively.

G.1 Interval Size 3 Minutes Data

CT	T	DB	MB	DR	MR
1	3.000	1	9.50	3	34.63
2	6.000	0	3.40	3	28.90
3	9.000	0	3.78	2	35.89
4	12.000	3	5.38	1	42.88
5	15.000	0	8.56	2	47.44
6	18.000	0	17.00	2	46.11
7	21.000	0	17.78	2	49.22
8	24.000	0	17.33	2	47.00
9	27.000	0	18.89	0	39.44
10	30.000	0	21.11	0	34.44
11	33.000	0	20.89	1	33.56
12	36.000	0	17.78	0	32.33
13	39.000	1	12.22	0	34.33
14	42.000	1	17.22	1	37.56
15	45.000	0	22.44	3	57.33
16	48.000	0	25.00	12	76.67
17	51.000	2	22.67	1	83.00
18	54.000	1	23.22	1	72.56
19	57.000	0	21.89	2	54.89
20	60.000	0	22.56	0	47.89
21	63.000	0	22.44	4	50.22
22	66.000	3	21.22	1	100.22
23	69.000	0	21.67	1	115.00
24	72.000	0	18.56	1	116.22
25	75.000	0	17.33	0	117.11
26	78.000	0	18.11	0	113.11

27	81.000	0	20.78	0	96.33
28	84.000	0	25.22	2	90.56
29	87.000	1	25.67	1	84.33
30	90.000	0	27.22	4	73.00
31	93.000	0	26.78	4	63.11
32	96.000	0	29.11	3	86.56
33	99.000	1	34.22	5	89.89
34	102.000	2	38.56	3	92.56
35	105.000	10	33.33	2	89.67
36	108.000	5	19.22	1	71.56
37	111.000	3	14.11	5	70.22
38	114.000	3	12.44	2	54.00
39	117.000	1	9.89	9	50.67
40	120.000	0	9.44	14	27.11
41	123.000	0	8.67	8	18.00
42	126.000	0	7.22	8	40.00
43	129.000	1	6.44	15	37.78
44	132.000	1	5.67	2	26.44
45	135.000	1	3.67	7	24.33
46	138.000	6	5.67	4	19.67
47	141.000	0	4.33	6	13.89
48	144.000	0	1.56	4	10.78
49	147.000	0	0.67	1	9.11
50	150.000	0	0.11	0	8.67
51	153.000	0	0.33	1	8.33

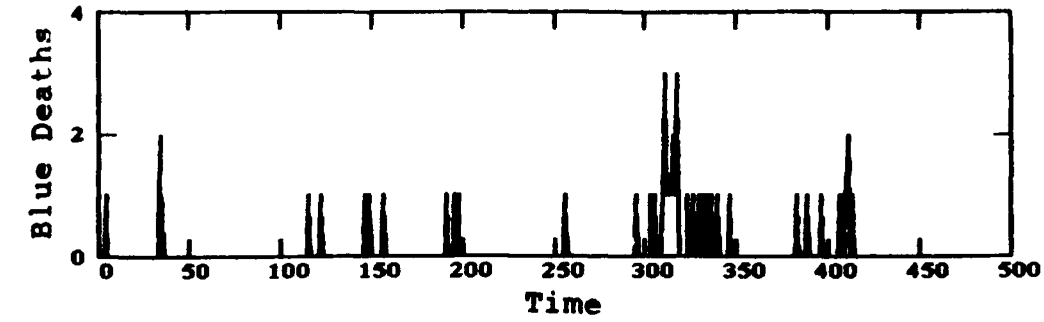
G.2 Interval Size 5 Minutes Data

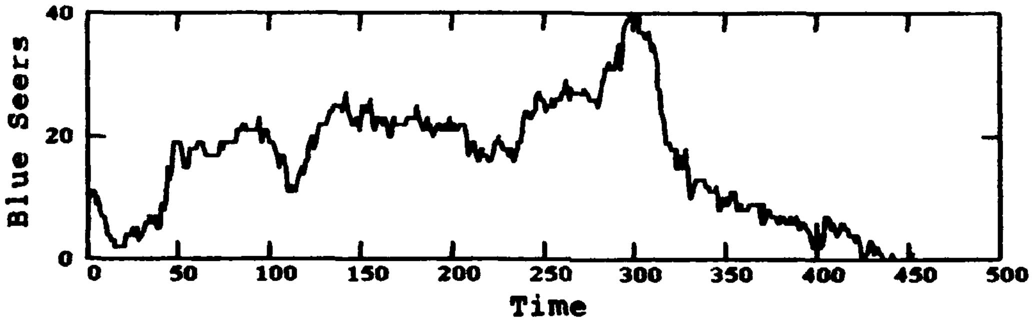
CT	T	DB	MB	DR	MR
1	5.000	1	6.93	6	31.60
2	10.000	0	3.40	2	35.73
3	15.000	3	7.79	3	46.36
4	20.000	0	17.07	4	47.00
5	25.000	0	17.93	2	47.27
6	30.000	0	20.27	0	35.47
7	35.000	0	19.93	1	32.73
8	40.000	1	13.60	1	33.73
9	45.000	1	20.80	3	50.60
10	50.000	2	23.87	13	79.73
11	55.000	1	23.13	3	70.80
12	60.000	0	22.20	0	50.47
13	65.000	2	21.93	5	67.67
14	70.000	1	21.27	2	114.93
15	75.000	0	17.53	0	116.67
16	80.000	0	18.60	0	107.13
17	85.000	0	24.87	2	90.73
18	90.000	1	26.73	5	76.53
19	95.000	0	27.27	7	71.87
20	100.000	1	34.67	6	90.53
21	105.000	12	35.27	4	90.67
22	110.000	7	17.87	4	72.27

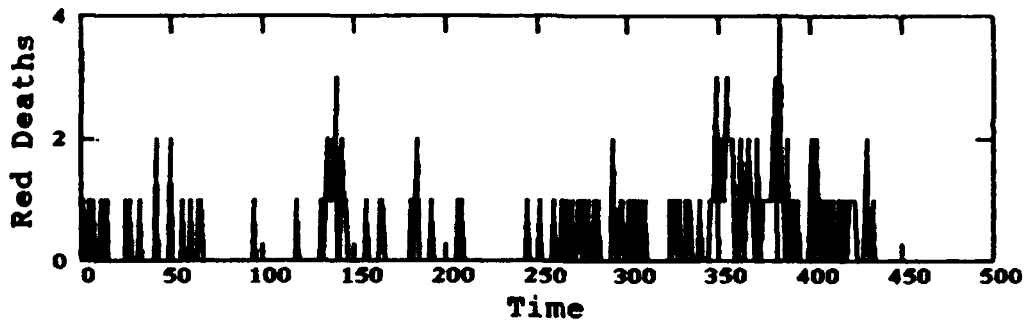
23	115.000	4	11.80	5	56.80
24	120.000	1	9.40	22	35.07
25	125.000	0	8.13	13	26.60
26	130.000	2	6.53	18	35.27
27	135.000	1	4.33	9	26.07
28	140.000	6	5.27	7	17.73
29	145.000	0	2.00	8	10.73
30	150.000	0	0.13	0	8.80
31	155.000	0	0.20	1	8.20

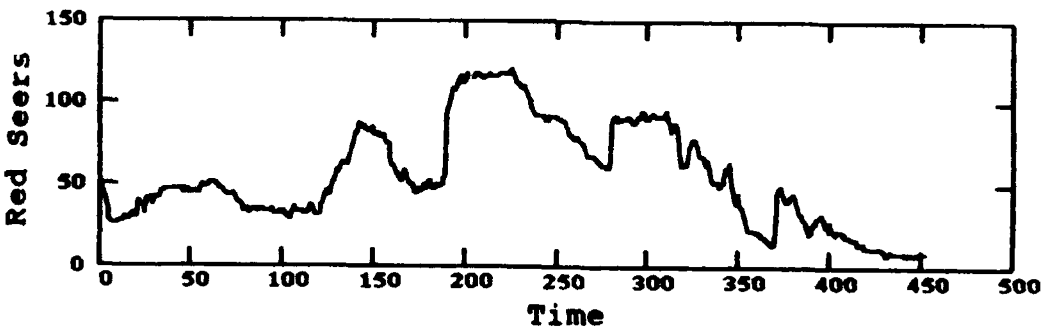
Appendix H: Plots of Deaths and Force Sizes throughout Battle Process for Interval Analysis

H.1 Plots of Interval Size 20 Seconds

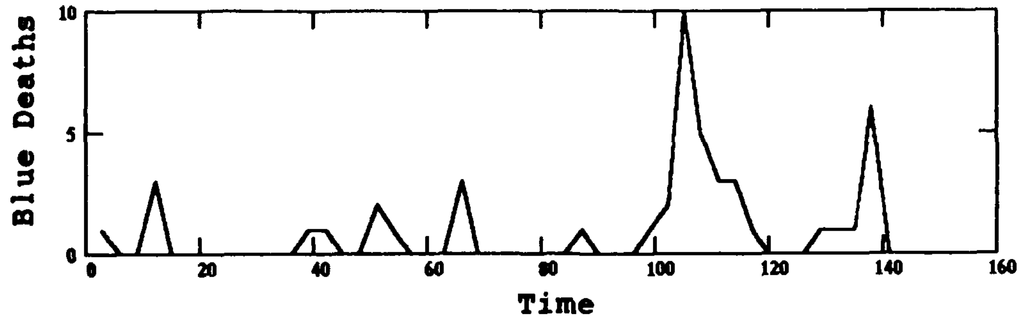


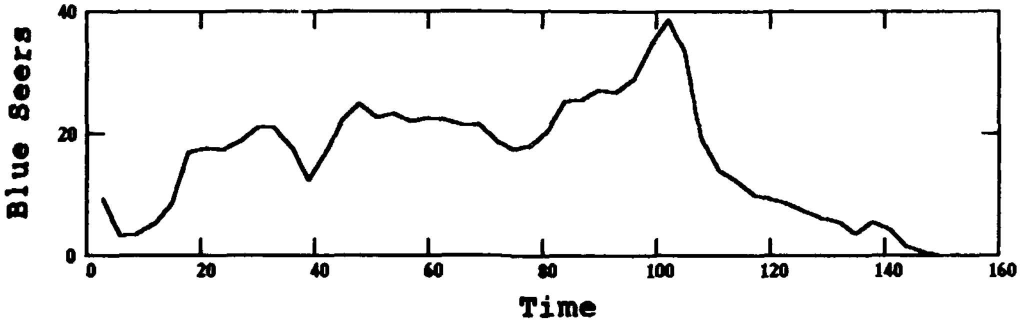


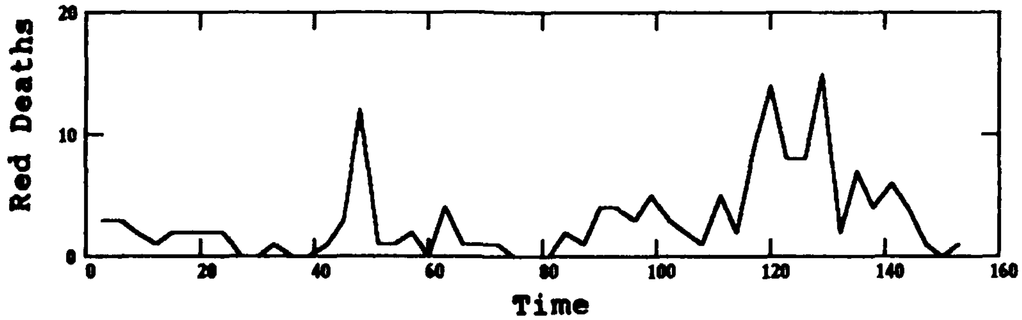


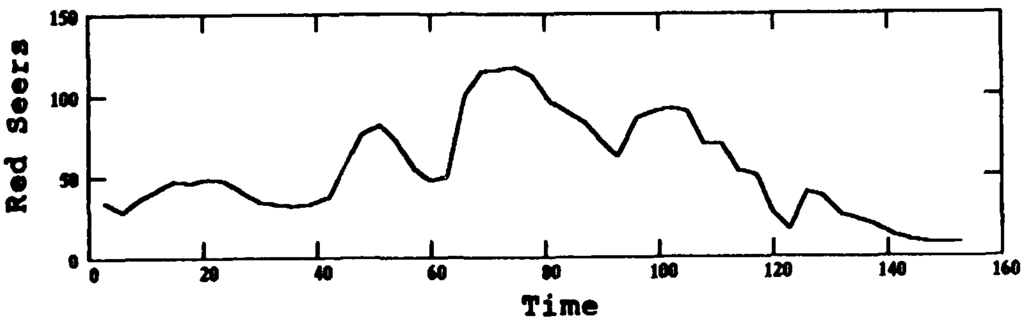


H.2 Plots of Interval Size 3 Minutes

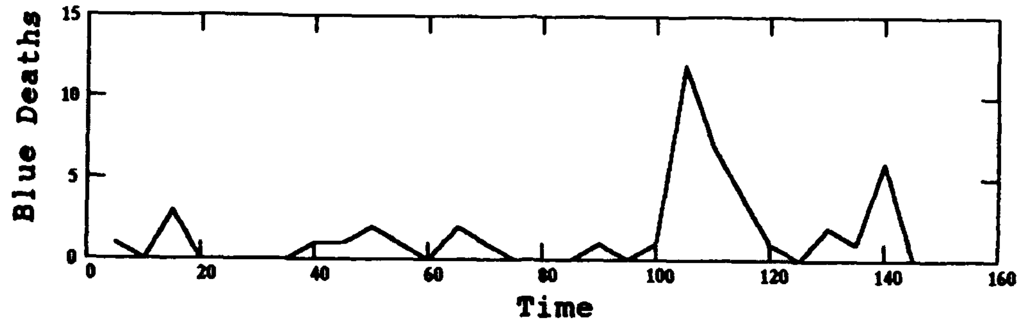


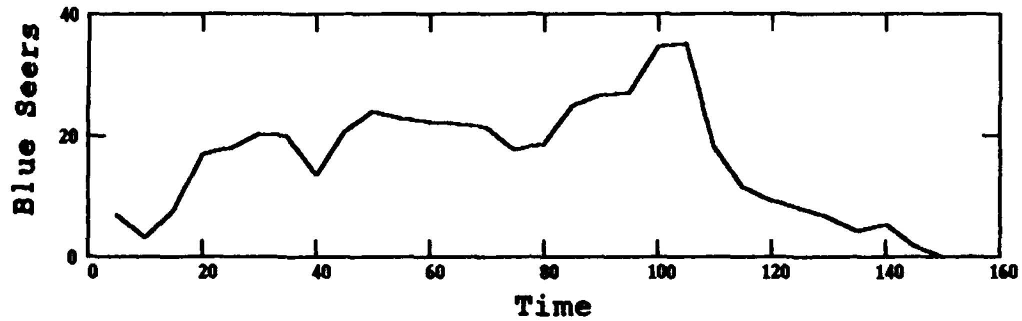


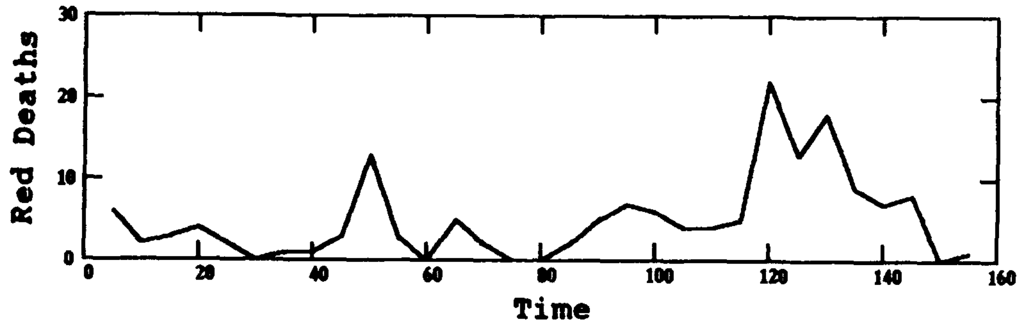


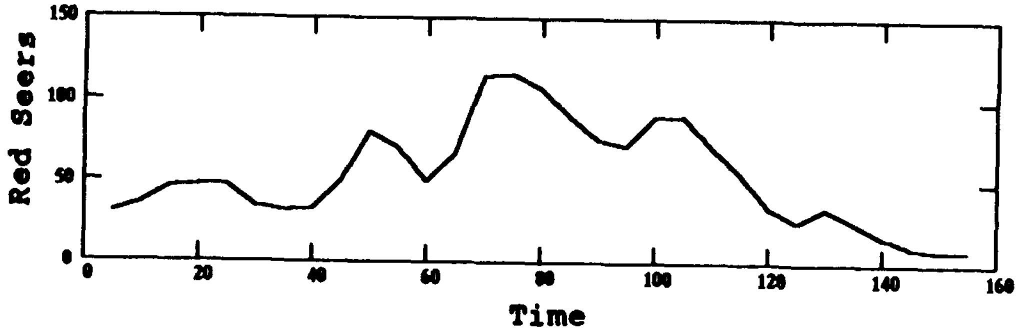


H.3 Plots of Interval Size 5 Minutes









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13. ABSTRACT (*Maximum 200 words*)

The purpose of this study was to find an appropriate way of characterizing battle data as a function of time. This study was initially designed to remove the numerical instability problem cited in Barr's "battle trace" methodology as suggested by TRAC-MTRY. The research focused on the instability problem and identifying/recommending a technique that improved the efficiency of computation, and enhanced an analyst's ability to meaningfully interpret the battle trace. Two Lanchester's Square Law based methodologies are introduced and analyzed. The results of this analysis indicate that the battle trace of a constant value generated by Lanchester's Square Law integration seemed to be the best measure to characterize battle data as a function of time. This methodology has no computational problems, is numerically stable and efficient, and has a battlefield interpretation with the proper interval size selection.

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