AN ANALYSIS OF ESCORT FORMATIONS

THESIS

Mustafa Ilhan
1LT, TUAF

AFIT/GOR/ENS/92M-16

DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY
AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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Mustafa Ilhan, B.S.
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This study used computer simulation to investigate escort fighter formations in an unknown interceptor threat environment. A USAF computer simulation model, TAC BRAWLER, was used to simulate air combat between the escorts and the interceptors in several different combat scenarios. The escort formations were defined by the vertical and horizontal distances of the escort fighters from the main strike body. Interceptors were characterized as one of two types of fighters with dissimilar aircraft frames and maximum missile firing ranges. A central composite design was used to mathematically model the effect of escort formations and interceptor type on the survivability of the bombers and escorts. Three mathematical models were developed to represent the dependent variables of "surviving escorts and bombers", "surviving escort fighters", and "surviving bombers". Within the limits of the experimental design, only the type of the incoming enemy interceptors was found to have a significant effect on the survivability of the friendly fighters.

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Advisor	Major Michael W. Garrambone/ENS	....
Co-Advisor	Major Paul F. Auclair/ENS	
Reader	Dr. Edward F. Mykytka/ENS	....

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Mustafa Ilhan

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Abstract

This study used computer simulation to investigate escort fighter formations in an unknown interceptor threat environment. A USAF computer simulation model, TAC BRAWLER, was used to simulate air combat between the escorts and the interceptors in several different combat scenarios. The escort formations were defined by the vertical and horizontal distances of the escort fighters from the main strike body. Interceptors were characterized as one of two types of fighters with dissimilar aircraft frames and maximum missile firing ranges. A central composite design was used to mathematically model the effect of escort formations and interceptor type on the survivability of the bombers and escorts. Three mathematical models were developed to represent the dependent variables of "surviving escorts and bombers", "surviving escort fighters", and "surviving bombers". Within the limits of the experimental design, only the type of the incoming enemy interceptors was found to have a significant effect on the survivability of the friendly fighters.

AN ANALYSIS OF ESCORT FORMATIONS

I. Introduction

1.1 Overview

One of the major tasks of an air force during combat is the escort mission. This mission involves escorting friendly bombers to their designated target areas or strike zones and protecting them against hostile enemy aircraft called interceptors. This technique of bomber protection was employed extensively during WW II and has been used successfully in conflicts in Korea, Vietnam, and the Middle East.

In order to accomplish this mission, escort fighters are employed in one of several ways. The first and most common method is that of flying the escorts in close proximity to the friendly bombers during their entire mission. A second method involves the use of escorts to precede the strikes in order to sweep the bombers' ingress route. Finally, a third method consists of escort fighters accompanying friendly bombers during specified portions of their flight plan.

An important aspect of the escort mission is the spatial positioning of the escorts with respect to the bombers they are escorting. Interceptor fighters that carry medium to long range guided missiles can fire their weapons at relatively long ranges and seriously threaten the bombers' strike mission. The only way to decrease the effectiveness of such capable interceptors is to attack incoming interceptors while the operating friendly aircraft are still beyond the interceptors' lethal range.

While the proper positioning of the escorts is trivial when the capabilities of the attackers and the direction of the attack are known, planning to defend against unspecified threats is much more complicated. In fact, there are no prescribed plans of defense for the escort mission where the enemy aircraft type, attack direction, and strength are unknown. This type mission, conducted under uncertainty, needs to be examined to discover ways to improve the escort's chance of success.

Two methods which have been used to develop, improve, or evaluate escort tactics are air exercises and computer simulations. Two examples of air combat exercises, are those of the 441st Test and Evaluation Squadron of Nellis AFB, Nevada, which carry out tactical improvement programs for the USAF; and the Red Flag air combat exercises, which annually provide advanced combat training to USAF pilots.

Both the 441st Test and Evaluation Squadron and Red Flag air combat exercises use specialized air combat ranges and air-to-ground weapon delivery ranges. These ranges are equipped with the state-of-art electronic sensors which monitor the aircraft and evaluate their combat performances. Thus, it is easy to see that air exercises require an extensive amount of technical resources.

The second method, computer simulations of combat, have been used since 1960 to analyze military tactics and flight operations. These simulations are performed with computer models containing aircraft engineering data, algorithms of aircraft control and pilot decision logic. These models can portray the dangerous conditions of combat prohibited in actual exercises; can easily capture and replay all aspects of any engagement for later evaluations; and usually cost less to develop, perform and evaluate than air exercises.

1.2 Problem Statement

Although tactics have been defined for situations where the threat interceptors are known, the problem remains that in cases where the threat is not known, prescribed tactics have not been available.

1.3 Purpose of the Research

The purpose of this research has been to investigate the escort mission and to shed light on recommended formations to use when enemy interceptor types are not known. Within this context, the objective of the research is;

1. To provide a framework for investigating the effectiveness of escort tactics and formations.

2. To design an experiment to examine various plausible approaches to escort missions conducted under uncertainty.

3. To, through computer simulation, evaluate alternative approaches, recommending those found most favorable to enhancing the success of the mission.

1.4 Scope of the Research

This research used the USAF's computer model, TAC BRAWLER, to simulate air combat. The study did not intend to justify escort or intercept tactics currently used by any particular nation's air force, although it employed standard escort and interceptor tactics developed from unclassified sources.

Occasionally, judgements for the design of the combat scenario and the employment of combat tactics were made by the author, who is an experienced fighter pilot with over

1000 hours of combat aircraft experience in F-100 and F-4 fighter aircraft.

This research study used unclassified generic data for aircraft, missiles, guns, radars, fire control devices and radar warning receivers.

The data was developed by Aerospace Systems Division at Wright Patterson Air Force Base to be used for in-house testing of the TAC BRAWLER computer model. The aircraft data does not necessarily represent a specific existing air asset. Since this study used only standard tactics and generic data, the specific conclusions of the research may not apply to scenarios other than those used specifically in this research.

1.5 Organization of the Paper

This paper is organized into eight chapters. The first five chapters review the background, techniques, tools and concepts that are used in this study. The last three chapters discuss the experiments, simulation results, analysis, conclusions and recommendations of the research.

1.6 Summary

This chapter defined the concept of the escort mission. It highlighted the significance of the spatial positioning of escorts in encountering attacking interceptors and noted the lack of prescribed tactics to be used when the type of interceptor aircraft are not known. It identified the current methods used to develop, improve, or evaluate escort formations as live exercises and computer simulations. It noted that computer simulations offer the ability of investigating alternative tactics to be used when escorts face unknown interceptors. Finally, this chapter outlined the purpose and the scope of the research and described the remaining organization of the paper.

II. Air Combat Tactics

2.1 Overview

Tactics and maneuvers are two important concepts in air combat. An aircraft maneuver positions an aircraft by actuating its three control vectors of roll, pitch and thrust. An air combat tactic on the other hand is a series of these maneuvers designed to accomplish a specific objective in air combat. The ultimate success of a tactic is dependent upon a number of factors, which include aircraft weapons, avionic systems, threat capability, and combat environment.

In air combat, pilots employ various maneuvers and tactics to achieve their objectives. For example, when the objective is to intercept intruding enemy aircraft and use radar guided missiles to defeat them, pilots will first assess the intercept geometry and select an air tactic to establish contact with the intruders. After acquiring the intruders with their sensors, the interceptors will attempt to place the intruders within the lethal firing envelope of their missiles by using a series of appropriate maneuvers. As the interceptors close in on the intruders, the intruders can be expected to respond with a series of their own tactics. Thus, in this sequence of actions and counteractions, pilots will simply select the appropriate maneuvers, one right after another, to accomplish their mission objectives.

In general, the actual maneuvers and tactics selected in a given air combat scenario depend on the pilot's understanding of his situation. Thus an important concept related to air combat is the pilot decision process. This decision process defines the pilot's perception, his evaluation of the situation, and his choice of actions to support his planned objective.

2.2 Escort Mission

The escort mission protects friendly strikes from enemy interceptors. It can be divided into four general classes: reception escort, remote escort, detached escort and close escort (Shaw, 1985:337).

Reception escort protects withdrawing strike aircraft from enemy interceptors. It is usually done by clearing the egress corridor of the strikes from hostile interceptors. The egress corridor defines the segment of the flight route that strike fighters use to return to a friendly base after attacking their assigned targets.

Remote escort clears the target ingress route of enemy fighters. The ingress corridor is the segment of the flight route that bomber fighters use to fly to their targets.

Remote escorts fly directly ahead of, or on the forward quarters of the main strike body and neutralize hostile interceptors before the strike body passes through the corridor. The escorts usually fly at low altitude to avoid detection by enemy radars. However, their altitude should be consistent with that of the expected threats.

A detached escort is conducted around the main body of the strike fighters. In a detached escort mission, the escort fighters should be positioned on the left and right forward quarters to counter and neutralize a forward-quarter missile attack.

> The optimum positions for detached escorts are dependent on the capabilities of both friendly and hostile weapons and on the nature of the anticipated attacks. Ideally a detached escort is located where it can detect and engage any hostile fighter before it can fire at the aircraft of the strike force. Considerations include the enemy's probable intercept geometry and maximum effective firing range, and escort maneuverability, reaction time, and weapons limitations. (Shaw, 1985: 338)

In detached escort operations, escort fighters should fly in front of the main body at a distance at least as far as the firing range of the enemy's short-range weapons. This distance can be increased to counter the hostile fighter's long range missile threat as well.

Another consideration is the altitude of the escort fighters. The altitude of the detached escort fighters may be lower than that of the strike fighters for better radar detection of the hostile aircraft and improved missile guidance performance. On the other hand, the escort's altitude can also be higher than the strike body to provide quick reaction to short range gun or rocket attacks by enemy aircraft.

To cover the strike body on all sides, the detached escort fighters can also be positioned on the rear quarters and the sides of the main body. The number of escorts covering a side of the main body should be at least two in order to provide mutual support.

Detached escorts should stay with the main body as long as the strike aircraft are not threatened directly. If the main body is threatened, they should engage the hostile fighters offensively and resume their assigned positions after the attack is over.

Close escort operations are conducted over, under, around and among the strike aircraft to provide for the short range defense of the strikes during the terminal phase of an interceptor attack. Close escort fighters, unlike detached escorts, always stay in the immediate vicinity of the main strike body. This tactic allows a response to attacks from any direction and is most effective when combined with a complimentary detached escort operation.

2.3 Summary

This chapter introduced the concepts of air maneuvers and tactics. It highlighted the dynamic nature of air combat, which requires successive quick and correct decisions. Finally this chapter defined the escort mission and briefed four general classes of these missions.

III. Computer Simulation Model

3.1 Overview

In an air combat studies, flight phenomena can be simulated by two means: actual air combat exercises and computer model simulation of air combat. Air combat exercises typically require a large number of costly resources and human effort to simulate the projected scenarios. As a computer model simulates air combat scenarios artificially within a computer, it provides a less expensive, quicker, and less labor intensive means to study the same issues.

Adequately portraying an air combat scenario for a computer simulation study requires many important factors to be simulated in detail. Some of these factors relate to the pilot, aircraft, tactics, weapon systems, onboard equipment, and the combat environment. In a computer simulation, these factors are modeled by mathematical equations or are drawn from tables of operating values stored within the computer.

The computer model should support the purpose of the study effort. For example an aircraft engine model that does not include the effects of turbulent air flow through the air intakes will not simulate engine compressor stall under certain conditions such as high angles of attack. If the purpose of the study is to investigate the high angle-of-attack air combat conditions, this poor engine model would be inadequate and would most likely generate misleading and invalid results.

In the United States Air Force, several computer models such as TAC BRAWLER, PACAM and AASPAM have been used to simulate air combat for various studies. TAC BRAWLER has been a preferred computer simulation model in the USAF and is currently being used by various USAF agencies such as the Air Force Studies and Analysis Agency (AFSAA), the Aeronautical Systems Division (ASD), and the Foreign Aeronautical Science and Technology Center (FAFTC). TAC BRAWLER was selected and used in this study under the sponsorship of ASD at Wright Patterson Air Force Base (WPAFB).

The following sections review the primary modules contained in the TAC BRAWLER air combat simulation model.

3.2 TAC BRAWLER, Air Combat Simulation Model

TAC BRAWLER is a high resolution air combat model. A high resolution combat model is defined as: "...one that includes the detailed interactions of individual combatants or weapon systems..." that "...are resolved at the one-to-one engagement level..." (Hartman, 1985:1-7). TAC BRAWLER simulates air-to-air combat between single aircraft or larger fighter formations. It can model air-to-air missions including intercept, escort, bomb, and fighter sweep.

TAC BRAWLER is a dynamic, time-step, stochastic model. This means that results of certain events are determined by drawing a random number, and calculations are done on a time interval basis. A list of model characteristics are provided in Appendix C.

3.2.1 Physical Sub-Models. TAC BRAWLER uses mathematical equations and table data to model aircraft, missiles, infra-red search and tracking systems, radar warning receivers, radars, fire control systems and on-board electronic counter measure devices.

3.2.2 Human Sub-Models. TAC BRAWLER models a pilot as a decision entity not a physical being. TAC BRAWLER pilots see, understand, and select appropriate actions based on the information they have about their situations. TAC BRAWLER does not model physical pilot factors such as fatigue; "g"[1] induced physical complications; or spatial disorientation[2].

Pilot decision is represented through complex models of

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[1] "g" refers to radial acceleration of aircraft. It occurs when the direction of motion of aircraft is changed. The acceleration acts along the radius of the circle and is directed toward the center of rotation. It shows the ratio of weights: $g=w/w_o$ where w_o is the weight on the surface of the earth and w is the observed weight in the environment. (DeHart, 1987: 204).

[2] Spatial disorientation is the pilots's wrong perception of his orientation with respect to his environment. It is caused by the deceptive effects of relatively sustained linear and angular acceleration over man's sensory system (DeHart, 1985: 330)

Information-Oriented Simulation Architecture and Value-Driven Decision Algorithms.

The Information-Oriented Simulation Architecture models the information flow from the aircraft and other external sources to the pilots. It simulates the situational perception of the pilot and calculates the consequences.

In the Information-Oriented Simulation Architecture, the pilots situational perception is contained in a mental status array. This array differs from the central status arrays where the true physical state of the simulation is kept as state variables. These variables describe aircraft and missile positions, velocities, orientations, fuel and other instrumental status. Pilots are conscious of the events they have in their mental models. The information they have is perfect, but they may not know every possible thing.

The Value-Driven Decision Algorithm models the development of pilot decisions. Pilots reevaluate the state variables in their mental models at specific time steps. These specific time steps are called consciousness events. As a result of this reevaluation they may make a couple of distinct decisions. Their mental models evaluate the possible consequences of each alternative decision, and an evaluation model assigns numerical values for each rated alternative. After all of the possible alternatives have been rated, the one of which has received the highest score is selected and carried out at that time. Pilots can change their decisions only at the next consciousness event. This happens at least once per second for the physical states of the aircraft, and at longer intervals for flight tactics.

These two models provide information flow to the pilots and a means to evaluate alternative actions. Both features provide inputs to the other decision processes.

TAC BRAWLER uses three types of decision procedures: value-driven decisions, traditional decision rules, and production rules. Value-driven decisions have already been discussed. They are used at every level of the decision hierarchy and are especially important for the maneuver decisions, parts of the pilot posture decisions (weapon/target selection and radar mode selection), and the flight posture decisions. Value-driven decisions have limited utility in flight tactic decisions.

Traditional decision rules are composed of software trees that branch according to tests on environmental conditions. Simple decisions such as whether to fire a selected weapon immediately or to wait are made within these decision trees.

Finally, production rules consist of a list of condition-action pairs. These rules are scanned continuously. An action takes place when the condition half of the rule is met. Unlike the other rules, production rules can be designed/written by the user.

TAC BRAWLER uses a value system to evaluate the alternatives. The outcome of the value system can be biased toward the user-written production rules by a weighting factor. The weighting factor may be any real number between zero and one, inclusive, indicating how frequently the production rules will influence a given decision. The net value of an alternative is given by the following equation.

$$V_{NET} = (1-w)\ V_{VALUE} + (w)\ V_{PRODUCTION} \qquad (3.1)$$

where

V_{NET} is the net value of an alternative

V_{VALUE} is the value produced by the value system evaluation of a projected alternative, and

w is the weighting factor

When the weight is zero (w = 0), the production system is not used. When the weight is one (w = 1), the production system is always followed.

A TAC BRAWLER pilot develops his decisions in a hierarchy that has four decision levels. At the first level of the hierarchy, a pilot decides what flight posture is desired. The flight posture decision indicates the general course of action. Currently there are five flight postures in TAC BRAWLER.

1. Mission: This posture has the flight performing routine activities such as flying toward a route point or a series of route points.

2. Attack Immediate: The flight attacks hostiles in the within-visual-range arena.

3. Evade then Reengage: The flight evades the hostiles but does not disengage.

4. Disengage.

5. Close from Long Range: This posture deals with the maneuvers before an attack.

At the second level, a pilot decides the tactic he is going to use for the selected posture. Each flight posture shows a set of appropriate flight tactics. The choice among the tactics is based on user input production rules and default decision rules. A list of flight tactics is given in Appendix A.

At the third level, a pilot terminates his individual posture. Pilot posture refers to simple decisions such as weapon choice, assigning priority to the threats, ground avoidance and selection of the radar.

At the fourth and the lowest level of the decision hierarchy, a pilot decides the maneuvers he is going to perform. Maneuvers are executed over one second decision intervals. A pilot reconsiders his situation at every second. Based on his last evaluation he can cancel a maneuver and start a new one. A list of maneuvers is given in Appendix B. At the fourth level, a pilot also makes decisions for weapon employment, such as which and when to use a specific weapon.

3.2.3 Environment Model. TAC BRAWLER uses a limited environment model. A "billiard ball terrain model" is used to simulate terrain which features only the curvature of the earth. TAC BRAWLER terrain is characterized by its radar reflectivity. Radar reflectivity is defined by a data-input constant that can not be changed during a simulation.

TAC BRAWLER weather model simulates only cloud layers up to 75000 feet for visibility, and different infra-red environments. Weather conditions do not change during a simulation.

3.2.4 Output Options. Outputs that are organized in meaningful formats are the only means to retrieve information from a computer model. TAC BRAWLER provides six different output options.

1. The Run-Time Log is written to the analyst's terminal as the program executes.

2. The Summary Utility produces the time history of a simulation and a statistical summary of the simulation run.

3. The Overview Utility produces a brief summary of the major events.

4. The Statistical Measures of Performance Package gives statistics on selected measures of merit.

5. The Graphics Package produces a graphical reproduction of the simulation on the terminals. TAC BRAWLER is currently installed on a number of machines such as Digital (VAX), IBM, Honeywell (Multics), Gould, Sun (UNIX), CDC and Masscopm (UNIX). Its graphic option can be used on the terminals that has Graphics Compatibility System (GCS) graphics software system and supported by the

mentioned machines.

6. The Histogram Package provides the capability to generate histograms for the engagements.

3.2.5 Model Verification. A true formal code verification of TAC BRAWLER has not been possible due to the magnitude of the computer codes. TAC BRAWLER is composed of approximately one half million lines of computer code written in the FORTRAN programming language. In this study, no effort was spent on model verification.

Model verification of TAC BRAWLER has relied primarily on the utilization of personnel who are familiar with the models or sub models being coded. The personnel who are responsible for the coding of the models are fully knowledgeable about what the model should do. Some of these personnel have been academicians (Ph.D. physicists, and M.S. degreed engineers) and others have been experienced pilots. Therefore modeling of physical processes is believed to correspond with reality. Each system and subsystem of TAC BRAWLER has been verified by running test cases (DSA Analyst Manual, 1988: 4.1-1).

3.2.6 Model Validation. Validation of TAC BRAWLER is based on the observation of the simulation of test scenarios. Simulated pilot behaviors are considered valid when experienced pilots, observing the scenario, consider it reasonable. Graphic capability of the model has been especially helpful for the validation processes.

Most of the data and many of the algorithms used by TAC BRAWLER are provided by various USAF agencies. Some of these agencies are AFSAA, ASD, and FAFTC.

Aircraft and missile performance characteristics modeled in TAC BRAWLER were validated by comparing the results of a test scenario to other simulations conducted by AFSAA.

Radar detection range validation was done by comparing the model results to test data. In a recent study, FAFTC examined and evaluated the radar clutter model of TAC BRAWLER. The conclusion of FAFTC was that the TAC BRAWLER radar mechanization generates acceptably accurate results and is appropriate for simulation of the air combat scenarios.

TAC BRAWLER has been developed by using inputs from many pilots. Therefore TAC BRAWLER decisions are expected to be unbiased. Air combat decisions are considered validated if they are accepted by experienced pilots (ASD Analyst Manual, 1988: 4.2-2).

3.3 Summary

This chapter noted that the selected simulation model should support the study. It reviewed the sub-models of TAC BRAWLER developed for the physical systems, pilot decisions, and environmental factors.

This chapter reviewed the model verification and validation procedures in TAC BRAWLER. It noted that verification relies on the expertise of the coders and the users. It also noted that validation of tactics and pilot behaviors has been evaluated by the expert judgement of experienced pilots. Furthermore, field tests and comparison of the results to other models have also been used for validation.

IV. Simulation Scenario

United States Army Training and Doctrine Command (TRADOC) regulation 71-4 defines a scenario as follows.

> A scenario is a graphical and narrative description of the area, environment, means (political, economic, social, and military), and events of a hypothetical conflict during a future time frame. It reflects currently approved assumptions; the red, blue, and unaligned force structures; terrain; weather; operational art; and tactics...Scenario is a tool that supports the evaluation of a doctrine, training, organization and material.

The scenario plays an important role for the studies done with computer models by affecting the simulation of events. At the initiation of a simulation, the initial status is defined by the contents of the scenario file. Some computer models use these conditions partially or in whole throughout the entire simulation. In these kind of models, the output of a simulation study may rely on the details put into the scenario file.

A well prepared simulation scenario serves the objectives of the study just as a badly prepared scenario may impair its objectives. For example, consider a simulation study that examines the loitering times of tanker aircraft. Suppose that the tankers loiter at a high altitude position over the north Atlantic ocean. To lend credence to the study, the scenario file should include information regarding the jet streams, which are high altitude winds with velocities of about 100 Knots. If the effect of the jet streams is overlooked in the simulation scenario, the study will likely produce false information about the loitering time of the tanker aircraft. In actuality, the jet streams reduce the ground speed of tanker aircraft that fly into it. The reduced ground speed increases flight time and fuel consumption, which leaves less fuel for loitering. Thus, an omission in the scenario file can undermine the intended objectives of a simulation study.

A combat simulation scenario should be produced in three parts; blue operational scenario; red operational scenario; and dynamic scenario (TRADOC,1989:3-4)

4.1 Blue Operational Scenario

The blue operational scenario portrays the blue force's specific and general conditions before the simulation starts. It includes the following details.

1. Force structure of blue forces: The number and type of assets, their weapons, organizational relations, locations, intentions and objectives of the combatants.

2. Environmental settings: Weather conditions, light conditions (day or night), effects of sun, geographical conditions, terrain, and vegetation.

3. Location of the facilities: Location of runways, supply points, communication networks, command posts, and control stations.

4.2 Red Operational Scenario

The red operational scenario portrays the red force's specific and general conditions before the simulation starts. It includes the same components as the blue operational scenario.

4.3 Dynamic Scenario

The dynamic scenario describes the results of the combat between blue and red forces as initiated in the blue and red operational scenarios. It contains the positions of the forces with respect to each other, model constraints, and scenario assumptions.

4.4 Summary

This chapter defined the scenario. It noted the importance of well prepared simulation scenarios when a study uses a computer simulation model. It gave an example of how a scenario should be directed toward the purpose of the study. The example also showed how the misuse of the scenario could lead to false conclusions. Finally, this chapter outlined and reviewed three parts of a combat scenario, red and blue operational scenarios, and dynamic scenario.

V. Statistical Analysis Techniques

5.1 Introduction

The response surface methodology consists of a group of statistical techniques for empirical model building "...It seeks to relate a response or output variable to the levels of a number of predictors, or input variables, that affect it" (Box and Draper, 1987: 1).

The major hypothesis of RSM is that there is a theoretical, or mechanistic, model which defines the actual relationship between the levels of the input variables and the response. The mechanistic model may be represented as

$$E(y) = f(\boldsymbol{\phi}, \boldsymbol{\theta}) \tag{5.1}$$

where

$E(y)$ is a vector of mean responses

$\boldsymbol{\phi}$ is a vector of input variables, and

$\boldsymbol{\theta}$ is a vector of model parameters.

The mechanistic model often is not known due to limited or incomplete knowledge about the studied response. In these cases, it is assumed that the relationship between the response and the input variables would be smooth, at least within a region of the parameter space.

Based on this assumption, the unknown mechanistic model is approximated over a particular portion of the parameter space by an interpolation function of the form

$$y = g(\phi, \beta) \tag{5.2}$$

where

y is a vector of observed responses

ϕ is a vector of input variables, and

β is a vector of estimated parameters.

The interpolation function, or empirical model, can serve a number of purposes.

5.2 Applications

Applications of RSM generally fall into one of the three categories below.

1. Approximate mapping of a surface within a limited region. In this application, the empirical model provides a means to predict the response when the settings of a system are changed within the defined region.

2. Choice of operating conditions to achieve desired specifications. In this application, there are usually multiple requirements that must be satisfied. RSM provides estimated response surfaces for each requirement and defines

the operating conditions for which all the desired specifications are met.

3. Search for optimal conditions. In this application, a local response is approximated by optimizing the empirical model within its defined region.

5.3 Statistical Techniques

The specific techniques encompassed by RSM are experimental designs, the method of least squares estimation, and gradient search procedures. Very simply, an experimental design determines the region of the parameter space where responses will be observed. Properly selected regions will yield the data required to develop an empirical model with the method of least squares. If subsequent empirical models are needed, gradient search techniques aid in determining the location of the associated experimental designs.

The following sections discuss the method of least squares estimation and experimental design at the level of detail required for this research. Since gradient search techniques were not employed in this study, they will not be discussed further. The interested reader is referred to the RSM text by Box and Draper.

5.4 The Method of Least Squares

Given that a set of data is already collected by use of an appropriate experimental design, which will be discussed later in this report, the procedure to fit empirical functions to these data is called the method of least squares.

The method of least squares estimates the parameters of a postulated model so that the sum of the squared differences between the observed and expected responses is a minimum. Consider a set of n responses, $y_1, y_2, \ldots, y_n$, and a postulated model $f(x, \Theta)$ where x is a set of variables and Θ is a vector of parameter values. The sum of squared differences, or errors, may be expressed as

$$S(\theta) = \sum_{i=1}^{n} (y_i - f(x_i, \theta))^2 \qquad (5.3)$$

where "i" is the index of the observations.

A straightforward, but tedious, means of estimating the parameter values is to develop a coarse-grid plot of $S(\Theta)$. The estimate of Θ may be improved to any level of precision by successively refining the plot of $S(\Theta)$ in the vicinity of its minimum value.

A more elegant approach is to determine the β value which minimizes the S(Θ) by use of matrix operations

$$\beta = (X^T X)^{-1} X^T y \tag{5.4}$$

where

β is a px1 vector of the estimated parameters referred to as the regression coefficients

X is the nxp matrix containing the value of the input variables

y is a nx1 vector of the observed responses

n denotes the number of the observations

p denotes the number of the input variables

After the regression coefficients of a model are estimated, a fitted response function can be written as follows.

$$\hat{y} = X\beta \tag{5.5}$$

Equation (5.5) defines the relation between the levels of the input variables and the responses that are modeled mathematically. In actuality there will be differences between the observed and estimated responses for the same levels of the input variables. These deviations are called residuals, and are denoted by

$$e = y - \hat{y} \qquad (5.6)$$

Once again, the sum of the squared residuals is minimized by least squares estimation procedures. The least squares estimates are valid, regardless of the nature of the residuals. However, statistical inferences concerning the adequacy of the model require that the residuals be independent, identically distributed normal random variables.

5.5 Meaning of the Regression Coefficients in First-Order Empirical Models

Regression coefficients of a mathematical model have different meanings for quantitative variables and qualitative variables. In this research one of the variables was a qualitative variable, therefore, models with qualitative and quantitative variables are discussed next.

5.5.1 Quantitative Variables. Given that data obtained from a 2^3 design is to be fit to the mathematical model

$$E(Y) = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \qquad (5.7)$$

by application of the method of least squares, the meaning of the regression coefficients in the model are as follows.

If x_1, x_2 and x_3 are quantitative variables, then the regression coefficients β_1, β_2 and β_3 indicate the rate of change of the response when a factor (x_1, x_2 or x_3) is increased by one unit and β_o represents the intercept on the response. The first order model in Equation (5.7) defines a linear relation between the factors and the responses.

5.5.2 Qualitative Variables. Qualitative variables are categorical in the sense that they represent immeasurable differences in factor levels. For example, the levels might be different colors, types, or configurations. Clearly, qualitative variables can be defined with any two levels such as 0 and 1, or -2 and 5. It follows that regression coefficients of qualitative variables have different meanings from those of quantitative variables as illustrated in the following example.

Suppose that one of the variables in a 2^3 design is a qualitative variable. Let x_3 be the qualitative variable where its levels represent two different immeasurable factors--such as type of aircraft.

When a mathematical model is fit to data, it will look the same as in Equation (5.7). However, when x_3 takes its lower value "-1", the response function becomes

$$E(Y) = (\beta_o - \beta_3) + \beta_1 x_1 + \beta_2 x_2 \qquad (5.8)$$

Equation (5.8) shows that the response function is a plane with slopes β_1 and β_2, and a Y-intercept at $(\beta_o - \beta_3)$.

When x_3 takes its upper value "1", the response function becomes

$$E(Y) = (\beta_o + \beta_3) + \beta_1 x_1 + \beta_2 x_2 \qquad (5.9)$$

Equation (5.9) shows that new response function is also a plane, parallel to the (5.8) with the same slope β_1 and β_2, but with a different intercept $(\beta_o + \beta_3)$.

5.6 Designs

An experimental design specifies the levels of the input variables for every observed response. A class of experimental designs called factorial designs is especially useful in RSM experiments.

A complete factorial design in "k" factors, or input variables, is built by choosing n_i levels for each factor i and selecting all possible combinations of the factor levels. For example, a design with three factors having four, two and five levels, respectively, would be described as a 4x2x5 factorial experiment. This design has 40 combinations of the levels of the three factors, where each combination of factor levels is referred to as a design point.

In the analysis of computer generated simulation data, two-level factorial designs have been found most useful (Law and Kelton, 1991: 656). In a two level factorial design, each variable occurs at only two levels. A two-level factorial design in k factors is called a 2^k design.

5.6.1 First Order 2^k Designs. First-order, two-level full factorial designs with k factors have 2^k design points. An example of a first order full factorial design is shown in Figure 5-1.

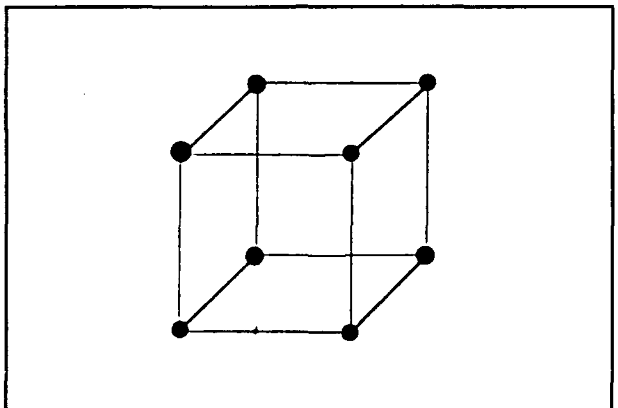

Figure 5-1. First Order Design in Three Variables

In the example, the design has three factors with two levels each, $(x_1, x_2, x_3) = (\pm1, \pm1, \pm1)$. Consequently, there are eight design points ($2^3 = 8$) that appear as the vertices of a cube. These design points are called "corner points" or "cube points." A matrix for the factor values of the example design is given in Table 5-1. In the matrix, Y values indicate the responses to the experiments done at the corresponding design points. As these designs sample the response at two levels of each factor, they are particularly well-suited for systems with linear relationships between the factors and the response.

Table 5-1

Estimation Columns of 2^3 Design

Y	I	x_1	x_2	x_3
.	1	-1	-1	-1
.	1	1	-1	-1
.	1	-1	1	-1
.	1	1	1	-1
.	1	-1	-1	1
.	1	1	-1	1
.	1	-1	1	1
.	1	1	-1	1

<u>5.6.2 Second Order Designs</u>. When the factors have nonlinear effects on the response, linear models are generally inappropriate. To account for the nonlinearities in the response, higher order models such as a second-order polynomial are required. Developing such higher order models relies on the corresponding higher order designs to generate the appropriate data. The higher order design used in this research is a second order design.

A second-order design must include at least three levels of each factor to allow for the possibility of quadratic terms in the empirical model. Although, there are a number of three-level designs available, central composite designs (CCD) offer a more efficient alternative (Box and Draper, 1987: 304-309). A diagram of the levels of one of the factors in three factor CCD is given in Figure 5-2. The design point in the center is called the "center point" and designated by "o". The points on the axes emanating from the center are called "axial points" or "star points" and are designated by "α". Finally, design points on the vertices of the cube are referred to as "corner points" or "cube points" (-1,1) as in the two-level designs.

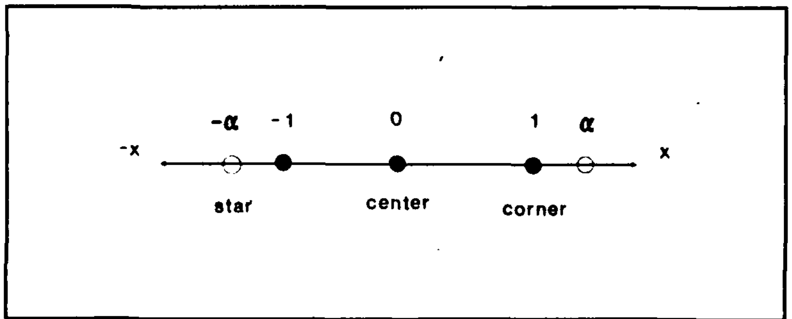

Figure 5-2. Augmented Two Level Variable Vector

Thus, the CCD is actually a two-level factorial design that is augmented with center and axial points. This composition makes the CCD especially useful in sequential experimentation that begins with a hypothesized linear model. In cases where the linear model is inadequate, it may be augmented to provide a second-order CCD.

Central composite designs are clearly appropriate for models that include only quantitative variables. In such designs, the axial and center points can easily be related to the levels of the variables. For example, suppose that temperature is a quantitative variable having cube points of positive 100° F and negative 10° F.

When the vector temperature factor is augmented with the axial and center points of a central composite design, the five temperature values will be $-\alpha$, -10, 45, 100, and α as shown in Figure 5-3.

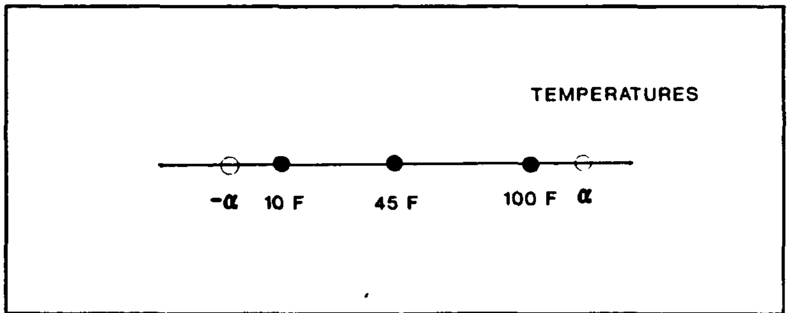

Figure 5-3. Augmented Quantitative Variable Vector, Daily temperatures

Clearly, each level of the factor represents a different daily temperature.

Unlike quantitative variables, qualitative variables may not have meaning beyond the original two levels. For example, if the factor is the type of fighter aircraft employed, the two levels may be an F-15 for one and an F-16 for the other. In this case, the axial points and center point cannot be associated with aircraft types. They simply do not have any meaning in this context.

A practical solution to handle this inconvenience has been proposed by Draper and John. In their study of the subject matter, Draper and John suggested fitting the same model at every level of the qualitative variables with a minimum number of experiments. They assumed that "...the same basic first or second order response surface will fit at any choice of (qualitative factor level) except for a change in response level" (Draper and John, 1988: 425).

To illustrate Draper and John's approach, suppose that a two level factorial design in three variables is to be augmented with axial and center points of the three variables, let x_1 and x_2 be quantitative variables and x_3 a qualitative variable.

A design that fits the same model at every level of x_3 is presented in Figure 5-4.

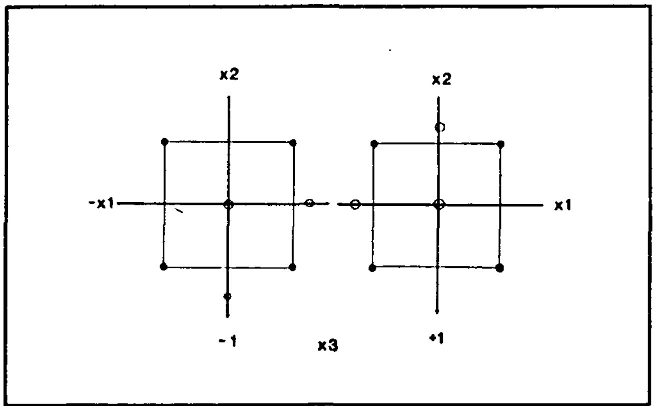

Figure 5-4. First Order Two Level Design for Qualitative Variable

When the designs at each level of the qualitative variable are superimposed. They take on the form of a central composite design as demonstrated in Figure 5-5

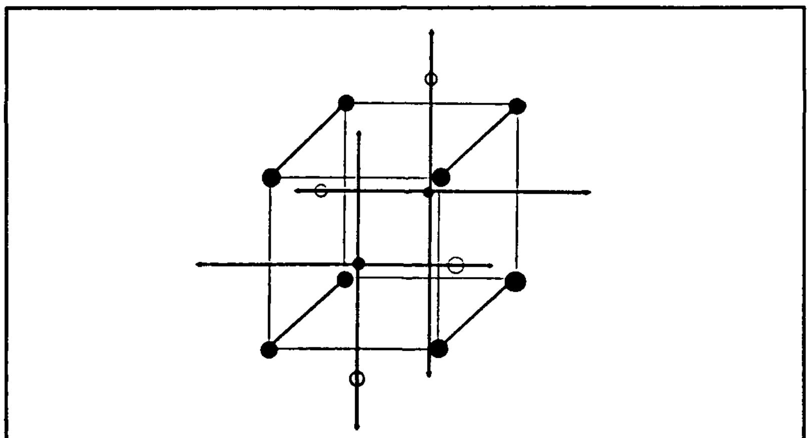

Figure 5-5. Central Composite Design With a Qualitative Variable

5.7 Orthogonal and Rotatable Experimental Designs

There are a number of design properties that should be considered in a given experiment. Those properties relating to the variance of the model's estimated parameters or responses are particularly important. Two classes of designs that yield desirable variance properties in the experimental region are orthogonal and rotatable designs. Orthogonality became an important design consideration due to the work of R.A. Fisher and F Yates, while G.E.P Box and J.S. Hunter introduced the concept of rotatability.

5.7.1 Orthogonal Designs. First order designs are orthogonal if all pairs of coefficients are uncorrelated, which occurs when the variance-covariance matrix is diagonal. In first-order orthogonal designs, the coefficient of a particular variable in the model remains the same, regardless of which other variables are included in it. Additionally, the joint confidence interval of the parameters is generally much smaller than that of a nonorthogonal design (Box and Draper, 1987: 79).

For second-order designs, it is not possible to obtain a diagonal variance-covariance matrix. Thus, second-order designs are considered orthogonal if the covariance between the quadratic terms is zero.

Central composite designs can be made orthogonal by replicating experiments at the center of the design. The number of the central experiments required to satisfy the orthogonality is given by the following equation.

$$N_o = \frac{4\alpha^2 \ (N_c + \alpha^2)}{N_c} - N_A \tag{5.10}$$

where

N_o defines the number of the experiments in the center,

N_A defines the number of the experiments on the axis,

N_c defines the number of the experiments in the corners of the design and,

α = the distance from the center of the design of the axial point.

If Equation (5.10) produces a non-integer solution, that is the number of required center points is close to N_o the design is said to be nearly orthogonal.

5.7.2 Rotatable Designs. A design is rotatable if the variance of the estimated response function at any point depends on the distance, but not the direction, from the design center. Having equally precise estimates in every direction is especially useful in sequential experiments, where the direction of advance to the following design center is usually not known before experimentation.

Orthogonality is a necessary and sufficient condition for rotatabity in first order designs. However, a central composite design can be made rotatable only through properly selecting the coordinates of the axial point from the design center, which is computed by

$$\alpha^2 = N_c^{\frac{1}{2}} \tag{5.11}$$

Central composite designs with integer "α^2" values can be both orthogonal and rotatable. When α^2 is not an integer, the design cannot be both orthogonal and rotatable; both properties can be approximated or one of the two can be attained at the expense of the other.

5.8 Model Selection Criteria

In an experiment there can be many candidate models to represent the "best" empirical model. As the number of the factors in the experiment increases, the number of the candidate models will also increase drastically. However, not all of these models will have the same power to explain the variation in the response. In practice, among those possible mathematical models, the most parsimonious is selected as the best empirical model.

A parsimonious model is considered to be one that adequately describes the variation in the response with as few variables as possible. Consequently, some evaluation criteria are needed to designate one of the candidate models as best.

The adequacy of a model is first determined with a "lack-of-fit test". Three other commonly used criteria for model evaluation are the coefficient of multiple determination, variance inflation factor and C_p criteria.

5.8.1 Lack-of-fit Test. A lack-of-fit test determines whether or not a specified regression function adequately fits the data. It requires repeated observations at one or more design points. At such points, the sum of squared error partitions into the sum of squares due to pure error and the sum of squares due to lack of fit. The sum of squared error terms is denoted by SSE and is calculated by

$$SSE = \sum_{i} (Y_i - \hat{Y}_i)^2 \tag{5.12}$$

The "pure error sum of squares" is denoted by SSPE and calculated by

$$SSPE = \sum_{j} \sum_{i} (Y_{ij} - \overline{Y}_j)^2 \tag{5.13}$$

where

j = denotes unique design points, and

i = denotes the observations

The difference between SSE and SSPE represents the deviation between the observations and the model due to inadequacies in the model. This difference is called sum of squares due to lack of fit and denoted by SSLF.

The ratio of SSLF to SSPE, each divided by its respective degrees of freedom yields a test statistic denoted by "$\dot{F}$".

$$\dot{F} = \frac{SSLF \div (m - p)}{SSPE \div (n - m)} \tag{5.14}$$

where

n = denotes the total number of the observations

p = denotes the number of the parameters included in the model, and

m = denotes the number of unique design points

$\dot{F}$ follows an F distribution under the null hypothesis of no lack of fit. A large test statistic relative to a tabulated F value implies that the model does not adequately fit the data. If the model does not exhibit a significant lack of fit, then additional testing to develop a parsimonious model is appropriate.

5.8.2 R^2 Criterion. The Coefficient of Multiple Determination (R^2) is computed by taking the ratio of the variation about the mean explained by the model (SSR) to the total variation about the mean in the observed responses (SSTO). "R^2 is a measure of the usefulness of the terms, other than β_o, in the model" (Draper and Smith, 1981: 91).

$$R_p^2 = \frac{SSR_p - n\overline{Y}^2}{SSTO - n\overline{Y}^2} \qquad 0 \le R_p^2 \le 1 \qquad (5.15)$$

where

SSE_p is error sum of squares

SSR_p is regression sum of squares

SSTO is total sum of squares.

Equation (5.15) indicates that if the model explains most of the variation in the responses, the ratio of the SSR to SSTO will approach one. A model that does not account for much of the variation in the responses will have a lower value of R^2.

Adding more terms into the model will increase the value of the coefficient of multiple determination. However, this approach might include more terms in the model than is necessary. Therefore, rather than maximizing R^2, it is preferred to find the point where adding more terms to the model only marginally improves its explanatory ability.

The goal is to find the number of factors after which the increase in the coefficient of multiple determination is not worthwhile.

5.8.3 VIF Criterion. The variance inflation factor, designated by VIF is used to check for multicolinearity. The variance inflation factor for variable X_i, VIF_i, is produced by

$$VIF_i = (1 - R_i^2)^{-1} \qquad i = 1,2,\ldots,p-1 \qquad (5.16)$$

where R_i^2 is the coefficient of multiple determination when x_i is regressed on the remaining (p-2) other x variables in the model.

VIF_i is one when $R_i^2 = 0$, indicating that x_i is not linearly related to the predictor variables. When there is a perfect linear relation between x_i and other variables in the model, VIF_i is unbounded.

VIF values greater than ten are considered an indication of severe multicolinearity in the model. They show that the estimates of the regression coefficients are influenced by the multicolinearity and may change depending on the predictors added to the model (Neter, 1990: 408-410).

5.8.4 C_p Criterion. The C_p criterion focuses on "the total mean squared error" of the n fitted values for each subset regression model (Neter, 1990: 448). It is calculated by

$$C_p = \frac{SSE_p}{s^2} - (n - 2p) \qquad (5.17)$$

where

SSE_p is the error sum of squares for the fitted model with p parameters, (p-1) predictors

s^2 is the mean square error of the model that includes all possible predictors (s^2 = MSE)

n is total number of the observations

p is the number of the parameters including the intercept value.

The C_p criterion exploits the fact that the MSE is the sum of a model's variance and the square of its bias. The development of the C_p expression assumes that s^2 is an unbiased estimator of σ^2. Thus, if the equation with p parameters is adequate, $E(SSE_p) = (n-p)\sigma^2$ and the expected value of C_p may be approximated by

$$\frac{(n-p)\ (\sigma^2)}{\sigma^2} - (n-2p) = p \qquad (5.18)$$

(Draper and Smith, 1981: 299)

Since SSE_p may be expressed as $(MSE_p)(n-p)$, the C_p criterion evaluates the degree of bias in the mean square error of the model with p parameters. For models with little or no bias in MSE_p, the value of C_p tends to be not much greater than p. Larger values of C_p indicate a greater bias component in the MSE.

5.9 Summary

This chapter reviewed response surface methodology techniques. It discussed how to fit a model to a set of data by the method of least squares. It introduced two level factorial designs and central composite designs. It noted how to utilize central composite designs when some of the input variables are immeasurable. It also discussed two variance properties of a central composite design. Finally, selection criteria for the most parsimonious model and a formal test for model adequacy were introduced.

VI. A Description of the Experiment

6.1 Introduction

The purpose of this study was to investigate the escort mission and to shed light on which escort formations to use when enemy interceptor types are not known. To do this, survivability of the friendly fighters was observed as the output in the computer simulations of the air combats. Survivability of the friendly fighters in such missions may depend on many factors such as the number of the incoming interceptors, aircraft performances, avionics, weapons, and command and control of the missions. Only three variables were investigated in this research, range of the escorts to the front edge of the strike body, altitude of the escorts from the ground, and the type of the interceptor fighters.

Within the given variables of the study it is hypothesized that in an armed confrontation the surviving number of the friendly escort and bombers would be dependent on the flight formation of the escorts and the capabilities of the incoming enemy interceptor aircraft. Formation of the escort fighters was defined simply by the range of the escorts to the strike body, and escorts' flight altitude. Specific reasons for the selection of the range and the altitude of the escorts in this study are as follows.

Rationale For Range. It is reasonable to believe that the survivability of the bombers will increase if the escort fighters can neutralize the interceptors before the bombers are within the interceptors' lethal weapon range. On the other hand, once initially engaged, the survivability of the escorts should depend on the capability of the incoming interceptors rather than the escorts' range to the interceptors. To what degree range forward of the strike package is a factor was to be determined.

Rationale For Altitude. When the escorts fly close to the surface of the earth, reflected radar waves from the ground can create acquisition and guidance problems (clutter) on some interceptors' radar systems. This might cause late acquisition of the escorts by the interceptors giving a tactical advantage to the escorts. Tactical advantage refers to having a radar or visual contact with the adversaries before they have it. It is generally assumed that the side who has the tactical advantage will capture the initiative in combat and use it for its own benefit. To what degree altitude plays in this survivability issue is also of concern.

6.2 Measure of Effectiveness

To investigate the relationship between range, altitude and the interceptor type, this research observed three measures for each combat simulation the number of surviving escorts and bombers, the number of surviving escorts only, and the number of surviving bombers only.

6.3 Organization of the Research

The research was organized into a set of sequential stages. The conceptual stage of the study established its purpose, objectives, hypothesis and measures of effectiveness. The scenario stage defined the context for the combat simulations, while the data generation stage yielded the results of simulating the research scenario. Finally, the analysis stage examined the simulation data through graphical and RSM techniques. The following sections discuss the phases of the experiment in detail.

6.4 Research Scenario

A USAF computer model, TAC BRAWLER, was used to simulate the escort, strike, and intercept missions. The scenario of the study was developed with the following conditions.

6.4.1 Scenario Components.

1. Combat Environment:

Air combat took place over a flat terrain where the constant radar reflectivity of the terrain was defined as 0.25 (25% of the incident energy is reflected back). Missions were executed in daylight conditions with clear weather and visibility of 23 Km. Sun effects were not modeled.

2. Combatants and Their Configurations:

Each battle consisted of two red interceptors against a blue air strike package containing two escort fighters configured around two bombers.

Each combatant carried a machine gun, a radar warning receiver, an air interceptor radar, and one fire control system. In addition, blue escorts and red interceptors carried four infrared (IR) and four semi-active radar guided missiles.

3. Missions of Combatant:

The red interceptor's mission was to intercept and destroy blue strikes and escorts, with priority against the strike bombers. When one of the interceptors was killed, the remaining interceptor would continue to fight until further threatened. When clearly outnumbered, the interceptor would disengage and withdraw to the north.

Escorts defended strikes by engaging the interceptors that attack the strike body. After deterring or destroying the interceptors escorts resumed their assigned positions around the strike body.

4. Command and Control:

The TAC BRAWLER model allows pilots to pass information, such as threat warnings and specific tactical instructions. Neither electronic warfare nor command control stations (air or ground) were modeled.

5. End Game Conditions:

The simulation was terminated when:

- All of the aircraft on either side were killed
- All of the combatants expended their weapons
- Simulation time reached six minutes of combat time (approximately thirty minutes of clock time).

6.4.2 Scenario Assumptions.

1. All players had enough fuel to complete their missions.

2. Escorts and interceptors used the same shot strategy to increase the probability of kill on a single target. Each would commit a second missile before determining the results of the first missile.

3. All players used their sensors continuously. Aircraft were initially within one another's radar perception ranges. Radars had a fifty percent probability of acquiring a target with a radar-cross section of two square meters at fifty seven nautical miles. The distance between the strikes and the interceptors was initially fifty nautical miles and closing.

5. There was no anti-aircraft artillery or surface-to-air missile threat to blue fighters over enemy air space.

6. Fratricide was possible due to faulty missile acquisition or the misperception of hostile action.

7. Damaged aircraft and mid air collisions were not possible.

8. The interceptor attacks were expected only from the front quarter of the strike body.

6.4.3 Base Scenario.

Blue operational scenario. The blue bombers fly Mach 0.85 in level flight at 1000 feet at a heading of 090°. The lateral separation of the aircraft is 1.5 nautical miles (NM). The escorts are in level flight 10 NM in front of the fighter bombers at the 2000 feet level, mach 0.85 and on the same heading. The lateral separation of the escorts is 3 NM.

Escorts are to provide air defense protection to the bombers whose plan is to penetrate into the enemy air defense zone and fly at a low altitude to their target.

Red operational scenario. Two Interceptors are positioned 50 NM east of the blue strikes. They fly at 5000 feet on a heading of 270° at Mach 0.85. Their lateral separation distance is 1.5 NM abreast.

There are two different types of interceptor aircraft, but each simulation run contains two aircraft of the same type. Interceptors are tasked to neutralize the incoming bombers and escorts.

Dynamic scenario. The simulation begins at time zero with the escorts and bombers flying the defined formation within the enemy air defense zone with their radar and radar warning receivers on.

At the start of the simulation, the escort and bomber flight has been detected by the enemy air defense radar, and the interceptors have been directed to the blue intruders. The interceptors also have their radars and radar warning receivers on. Neither side, however, had necessarily made radar contact with one another at this time.

The blue and the red fighters are approaching each other at a rate of approximately 1700 feet per second. When blue and red detect and acquire one another on their sensors, each attempts to establish the required parameters to fire their radar-guided missiles. If this initial attack fails, they will attempt a close range infra-red guided missile attack. If the latter fails, they will employ their machine guns to attack the opposing aircraft.

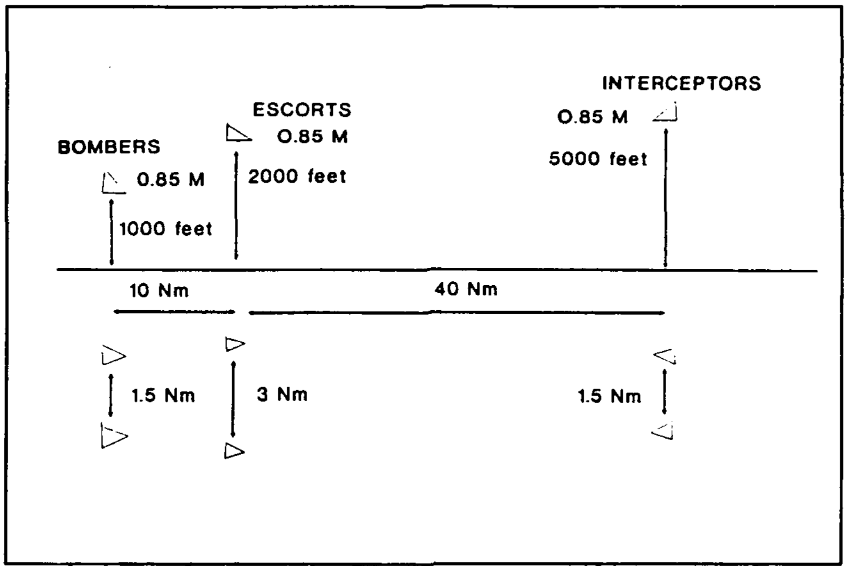

Figure 6-1. The Base Research Scenario

6.4.4 Tactics of the Combatants. The specific flight tactics and maneuvers selected by the combatants were determined by the decision modeling algorithms in TAC BRAWLER. The alternative air maneuvers and flight tactics, given by default, are listed in Appendix A and Appendix B.

The value model of the decision modeling algorithm was biased by user written production rules that were coded for each strike, escort, and interception mission. The weight of the bias was defined as fifty percent. That is, the net decision was biased fifty percent of the time toward the user defined production rules:

$$V_{NET} = (0.5)\ V_{VALUE} + (1-0.5)\ V_{PRODUCTION\ RULES} \qquad (6.1)$$

Production Rules. User written production rules define the basic ideas of the three missions in the scenario.

1. Escort production rules dictate that the escorts will maintain their assigned positions as long as interceptors do not engage the strike body. Escorts engage the interceptors only if the interceptors attack the bombers. After the attack, escorts will resume their assigned positions and will not pursue withdrawing interceptors.

2. Bomber production rules dictate that bombers fly at their initial altitudes of 1000 feet to their target if they are not attacked directly by the interceptors. However, when they detect the interceptors, they will descend to a altitude of 200 feet to avoid premature radar acquisition and frontal attacks by the interceptors.

If the interceptors directly engage the bombers, the bombers will descend to 100 feet and increase their speed to Mach 0.89 (approximately 890 feet per second). At this time, they will remain flying on their straight and level initial course. Should the interceptors converge onto bombers' rear quarters, the bombers will start making level weaves (they change their heading as much as thirty degrees off their original flight path) at 100 feet, Mach 0.89. After an attack is neutralized, the bombers will resume a straight heading and maintain their latest altitudes.

3. Interceptor production rules dictate that the interceptors should attack the bombers first. This priority was defined by a data-input coefficient in the scenario file (bombers were given the higher priority coefficient).

Interceptors carry out their mission as a coordinated flight. Whenever they lose mutual support, the remaining interceptor considers disengaging. Mutual support is considered to be lost when one element of the flight is killed. However, since it would be unrealistic to disengage the fight when there is no direct threat to the remaining interceptor, a single interceptor may continue to fight until he is threatened directly by the escort fighters.

Combat conditions for interceptors to continue to fight are defined in the interceptor production rule file. A single interceptor will fight when:

3.1 It has a positive radar lock on one of the blue fighters and is within the firing envelope of his weapon, and

3.2 Escorts do not have radar lock on the interceptor or the interceptor is outside of the firing envelopes of the escorts.

<u>6.4.5 Data and Effectiveness of Combatants</u>. The escorts and interceptors were assigned different levels of combat effectiveness by using two different generic aircraft characteristic files and assigning different maximum missile

firing ranges (rmax) to these combatants. The value of rmax is a constant set to 0.7 for the escorts, and 0.9 and 0.5 for the types of the interceptors in TAC BRAWLER. The maximum firing ranges of different missiles are defined in terms of rmax.

The assignment of the generic aircraft, avionic and weapon data, and "rmax" coefficients are presented in Table 6-1.

Table 6-1

Generic Data Sets Used by the Players

Generic Data Sets	Participating Aircraft			
	Blue Escort	Blue Strike	Int.Type 1	Int.Type 2
AC 1 Data	✓	✓	✓	---
AC 2 Data	---	---	---	✓
RWR Data	✓	✓	✓	✓
Radar Data	✓	✓	✓	✓
FCS Data	✓	✓	✓	✓
MSLR Data	70 % rmax	---	90 % rmax	50 % rmax
MSLI Data	70 % rmax	---	90 % rmax	50 % rmax
Gun Data	✓	✓	✓	✓

✓ indicates the data is used by the corresponding aircraft.

Table 6-1 shows that the escorts and the first flight of the interceptors (Type-1) used the same aircraft 1 data, and the second flight of the interceptors (Type-2) used aircraft 2 data. Furthermore, the escorts could commit their missiles when they were at 70% rmax where as the first interceptor and aircraft of the second interceptor had rmax values of 90% and 50% respectively.

In conclusion, the first type interceptor aircraft was identical to the escort aircraft except for having a superior "rmax" value. The second interceptor aircraft was different from the escort aircraft, had identical avionics, and inferior "rmax" values.

6.4.6 Pretest Inspection of the Production Rules and Base Scenario. The scenario and the production rules were pretested for their performances in the simulation. The objective of the inspection was to ensure that the scenario and the rules were acceptable for the study. The inspection followed the simulation of the combat through the data found in Run-Time Log files, where events were recorded at every five seconds of combat time. In addition, scenarios were visually verified through the use of a three dimensional graphical output option of the model.

6.5 Design of the Research Experiment

The experimental design of the study formed a structure to carry out the experiments and enhanced the following analysis of the simulation responses. The components of the experimental design are discussed below.

Input Variables. Three input variables were selected for investigation in the research scenarios.

1. X_1, the range of the escorts to the strike body, a quantitative variable, represented the offset distance of the escorts to the strike body.

2. X_2, the formation altitude of the escorts, a quantitative variable, represented the altitude of the escorts above ground level.

3. X_3, the type of interceptor flight was the qualitative variable (either Type 1 or Type 2) that represented the two different types of the interceptors.

Levels of The Variables. Two levels were selected for each variable in the design. These levels, lower and upper, were selected by the author, Table 6-2.

Table 6-2

Predictors and Predictor Levels

Predictors		Range	Altitude	Intercept.
Predictor Levels	Upper	10 NM	2000 feet	Type 1
	Lower	5 NM	500 feet	Type 2

Transformation of The Predictors. The quantitative variables were transformed into binary variables (1,-1) by use of transformation equations,

$$x_1 = (X_1 - 6) \div 4 \qquad x_1 = \pm 1 \qquad (6.2)$$

$$x_2 = (X_2 - 1250) \div 750 \qquad x_2 = \pm 1 \qquad (6.3)$$

where

x_1 is the transformed range variable and

x_2 is the transformed altitude variable

Selection of The Experimental Design. The central composite design discussed in section 5.6.2 was used in this experiment since nonlinear relationships were possible.

An ordinary two level full factorial design (having eight corner points) for the two variables, Range and Altitude, was expanded by four star points and two central points at each level of the third variable, Interceptor Type, Figure 6-2.

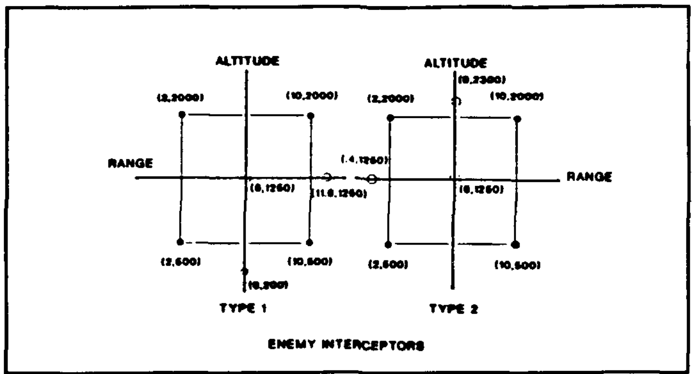

Figure 6-2. Research Design

When the two designs for the levels of the interceptors "x_3" are superimposed, the central composite design of the research is apparent. This design allowed the fitting a second order model to the simulation data at each interceptor level. The resulting design matrix is shown in Table 6-3.

Table 6-3

Central Composite Design Matrix

Run	x_1	x_2	x_3	x_1^2	x_2^2	x_1x_2
1	-1	-1	-1	1	1	1
2	1	-1	-1	1	1	-1
3	-1	1	-1	1	1	-1
4	1	1	-1	1	1	1
5	0	0	-1	0	0	0
6	α	0	-1	α^2	0	0
7	0	$-\alpha$	-1	0	α^2	0
8	$-\alpha$	0	1	α^2	0	0
9	0	α	1	0	α^2	0
10	0	0	1	0	0	0
11	-1	-1	1	1	1	1
12	1	-1	1	1	1	-1
13	-1	1	1	1	1	-1
14	1	1	1	1	1	1

Using equations 6.2 and 6.3, the coded values correspond to the variable settings shown below.

Range(x_1)	$-\alpha$ 0.4 NM	-1 2 NM	0 6 NM	1 10 NM	α 11.6 NM
Altitude(x_2)	200 ft	500 ft	1250 ft	2000 ft	2300 ft

Two of the variance properties of the research design, rotatability and orthogonality, were analyzed. It was found that the design in Table 6-3 could not be rotatable, but could be made orthogonal by having extra experiments at the two center points.

<u>Rotatability of the Research Design</u>. Rotatability would ensure that the variance of predicted responses at equal distance from the center of the design (6 Nm, 1250 feet) would be equal. The necessary condition for rotatability stipulated that the distance of the axial experiments should be approximately 1.68 units away from the center of the design.

$$\alpha = (N_c)^{\frac{1}{4}} = 8^{\frac{1}{4}} = 1.6817928 \approx 1.68 \qquad (6.4)$$

The values of the uncoded variables corresponding to $\alpha = 1.68$ is given in Table 6-4.

Table 6-4

Predictor Values at Star Points For $\alpha = 1.68$

	$\alpha = 1.68$	$-\alpha = -1.68$
Range	12.72 Nm	-0.72 Nm
Altitude	2510.00 feet	-10.00 feet

As is seen in Table 6-4, the lower values of the star points for both predictors assume negative values. These negative values indicate that the escorts should fly 0.72 Nm behind the strike body and 10 feet below the ground level. The altitude value is physically impossible, and the range value falls outside of the experiment region. Therefore the coded distance of the axial points was set at $\alpha = 1.4$ to maintain realistic variable settings and as high a degree of rotatability as possible.

Table 6-5

Predictor Values at Star Points For $\alpha = 1.4$

	$\alpha = 1.4$	$-\alpha = -1.4$
Range	11.6 Nm	0.4 Nm
Altitude	2300.0 feet	200.0 feet

2. Orthogonality of the Research Design. An orthogonal design would ensure that the regression coefficients of the range and altitude factors in the mathematical model are not influenced by the existence or absence of the other.

The necessary condition for the orthogonality required number of the runs in the center of the design to be approximately eleven, Equation (6.5). Therefore, the central experiment was repeated eleven times.

$$N_o = \frac{4 \cdot N_C^{\frac{1}{2}} (N_C + N_C^{\frac{1}{2}})}{N_C} - N_A = \frac{4 \cdot 8^{\frac{1}{2}} \cdot (8 + 8^{\frac{1}{2}})}{8} - 4 \approx 11.31 \tag{6.5}$$

In conclusion the experimental design of this research study was not rotatable and could not be made so within the experiment region. However, it was made approximately orthogonal by conducting a total of eleven experiments at the center of the design. The eleven central runs were shared arbitrarily between the levels of the interceptor type as five runs at Type-1 and six runs at Type-2.

6.6 Simulation and Experimentation

Since TAC BRAWLER is a stochastic model, it can yield different responses to the identical scenarios. Therefore, instead of relying on the response of a single simulation run, the arithmetic average of a number of independent trials or repetitions was used for analysis in this research study.

Computation of the Replication Size. The minimum number of repetitions required to calculate the average response was calculated empirically as an analytical approach focusing on reducing the variance about the mean response required more replications than could be accomplished.

The base scenario was run one hundred times using a different random number seed for each run to ensure the independence between the runs. The resulting one hundred independent observations were summed cumulatively in sets of one to one hundred output values. Each sample was then divided by the total number of observations in that group. The resulting means were calculated as shown below and plotted in Figure 6-3.

$$m_1 = \frac{z_1}{1}, m_2 = \frac{z_1 + z_2}{2}, \ldots, m_{100} = \frac{(z_1 + z_2 + \ldots + z_{100})}{100} \qquad (6.6)$$

where

z_i is the output of the "ith" run, and

m_i is the mean of the first i observations

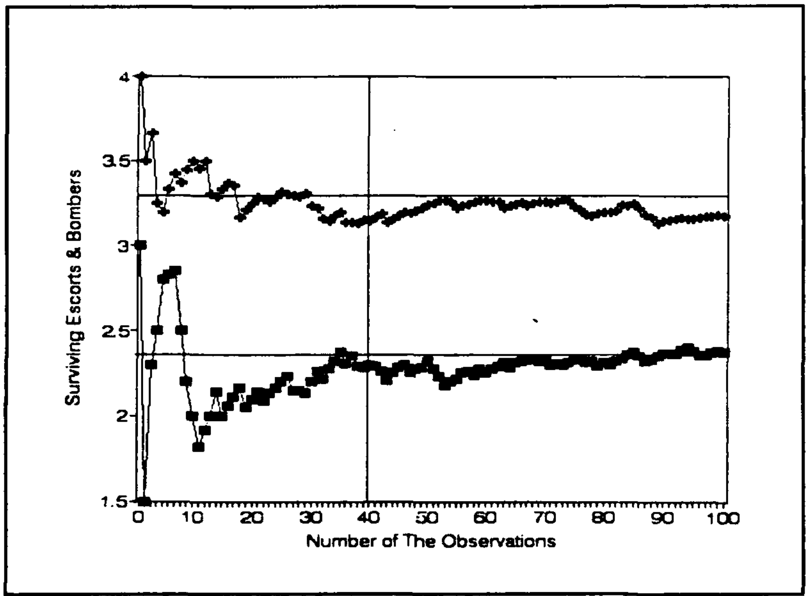

Figure 6-3. Sample Means

The same examination was repeated at another design point to confirm this process where the range, altitude, and the interceptor type were different. In a graphical sense, the point after which more repetitions had little marginal contributions to the accuracy of the mean response determined the number of the replications used per design point. Each experiment point was simulated 40 times.

<u>The Experiments</u>. Twenty three experiments were conducted in all eight corner, four axial, and eleven center points. For each simulation point three measures were collected: "average number of the surviving escorts and bombers," "average number of surviving escorts," and "average number of surviving bombers."

With three output measures on twenty three observation points, the nine distinct cases below were considered in the investigation of the variable effects.

1. Escort and Bombers versus Type 1 and Type 2 interceptors.
2. Escorts versus Type 1 and Type 2 interceptors.
3. Bombers versus Type 1 and Type 2 interceptors.

4. Escort and Bombers versus Type 1 interceptors.
5. Escorts versus Type 1 interceptors.
6. Bombers versus Type 1 interceptors.

7. Escorts and Bombers versus Type 2 interceptors.
8. Escorts versus Type 2 interceptors.
9. Bombers versus Type 2 interceptors.

The first three of these cases were used to evaluate the research hypothesis. The remaining cases were used to inspect the individual effects of the interceptors.

Inspection of the Simulation Output Data. Cases 4 through 9 were able to be inspected graphically for they had two dependent and one independent variable. These six cases were plotted in three dimensions where the horizontal axis of the plots represented the range and altitude variables, and the vertical axis represented the observations. These bar charts provided useful insights into the possible effects of the variables on the response for each of the cases considered.

Statistical Analysis. The final investigation was carried out through statistical analysis of the results obtained for the nine cases. An empirical mathematical model was developed that related the response to the levels of the input variables.

For each model, an iterative model evaluation method was employed. This method tested all possible combinations of the variables, their quadratic terms, and their two-factor interaction. Based on the evaluation, only statistically significant terms were included into the model.

The statistical significance for the inclusion of a term into the model was defined as fifteen percent or less of making a "type I error."

The resulting candidate models were first tested for lack of fit. Models with significant lack-of-fit were eliminated from further evaluations. The remaining "significant" mathematical models were compared with each other based on their "coefficients of multiple determination", "variance inflation factors", and "Mallow's Cp statistics." In addition, regression coefficients of the models were tested to see if they were statistically different from zero.

6.7 Summary

This chapter identified the hypothesis of the study and outlined the measures to be observed in the experiments. It also discussed the assumptions, the scenario, and the tactics of the combatants.

The latter part of this chapter discussed the experiments. It defined the experimental design, selection of variables, and the rationale for selecting the central composite design. The chapter closed by describing the cases considered and the various analysis methods employed. The next chapter will discuss the results of the experiment.

VII. Experimental Findings and Analysis

7.1 Experimental Results

Every experiment of the research was repeated forty times as determined before. Each simulation run was given a different random number seed to ensure the independence between these replications. At every run, three measures were collected: "total number of the surviving escorts and the bombers", "total number of the surviving escort fighters", and "total number of the surviving bombers." Average responses of the twenty three experiments (see Section 6.6) are shown in Table 7-1.

Table 7-1.
Results of The Experiments

CODED VARIABLES				AVERAGE NUMBER SURVIVING		
Run No	x_1	x_2	x_3	Totals	Escorts	Bombers
1	-1.0	-1.0	-1	2.47917	1.04160	1.43750
2	1.0	-1.0	-1	2.50000	1.15618	1.34375
3	-1.0	1.0	-1	2.06250	0.91667	1.14583
4	1.0	1.0	-1	2.37000	1.03000	1.34000
5	0.0	0.0	-1	2.91489	1.27659	1.63830
6	0.0	0.0	-1	2.40816	1.10204	1.30612
7	0.0	0.0	-1	2.20000	1.02000	1.18000
8	0.0	0.0	-1	2.64000	1.24000	1.40000
9	0.0	0.0	-1	2.33000	1.12000	1.21000
10	1.4	0.0	-1	2.58300	1.04167	1.54167
11	0.0	-1.4	-1	2.50000	1.16667	1.33333
12	-1.0	-1.0	1	3.14583	1.50000	1.64600
13	1.0	-1.0	1	3.20830	1.52100	1.68800
14	-1.0	1.0	1	3.08300	1.37500	1.70800
15	1.0	1.0	1	3.25000	1.43800	1.81300
16	0.0	0.0	1	3.13000	1.44445	1.68888
17	0.0	0.0	1	3.12000	1.58000	1.54000
18	0.0	0.0	1	3.24000	1.50000	1.74000
19	0.0	0.0	1	3.30000	1.65000	1.65000
20	0.0	0.0	1	3.24000	1.58000	1.66000
21	0.0	0.0	1	3.32000	1.62000	1.70000
22	-1.4	0.0	1	3.47458	1.72881	1.74576
23	0.0	1.4	1	3.15000	1.50000	1.65000

7.2 Graphical Inspection of the Simulation Responses

This inspection was applied only to the cases four, five, six, seven, eight and nine since it was possible to plot them on three dimensional plots, where axes of the plots represent range, altitude and response. First three cases had four variables, three independent and one dependent, therefore they could not be plotted in three dimensions. The purpose of this inspection was to examine the behavior of the three performance measures for each type of interceptor and to gain introductory information for the succeeding statistical analysis.

Responses of the simulation runs for each case are shown in Table 7-2 through Table 7-7. Figures from 7-1 to 7-12 shows the graphical relation between each combination of the variables, and their variations about the mean values.

In bar charts vertical axes represent the number of the surviving blue fighter aircraft (escorts, bombers or escorts and bombers), and two horizontal axes represent the distance between the bombers and the escorts (NM), and the altitude of the escorts above ground (feet) respectively.

In high-low graphs edges of the solid lines represent two standard deviations and the marks represent one standard deviation from the mean of the corresponding combination of variables.

Case 4. Surviving total number of escorts and bombers when the incoming fighter was a Type 1 interceptor.

Table 7-2.
Design Points and the Average Responses For Case 4

RUN	RANGE	ALTITUDE	TYPE	RESPONSE
1	2	500	1	2.47917
2	10	500	1	2.50000
3	2	2000	1	2.06250
4	10	2000	1	2.37000
5	6	1250	1	2.91489
6	6	1250	1	2.40816
7	6	1250	1	2.20000
8	6	1250	1	2.64000
9	6	1250	1	2.33000
10	11.6	1250	1	2.58300
11	6	200	1	2.50000

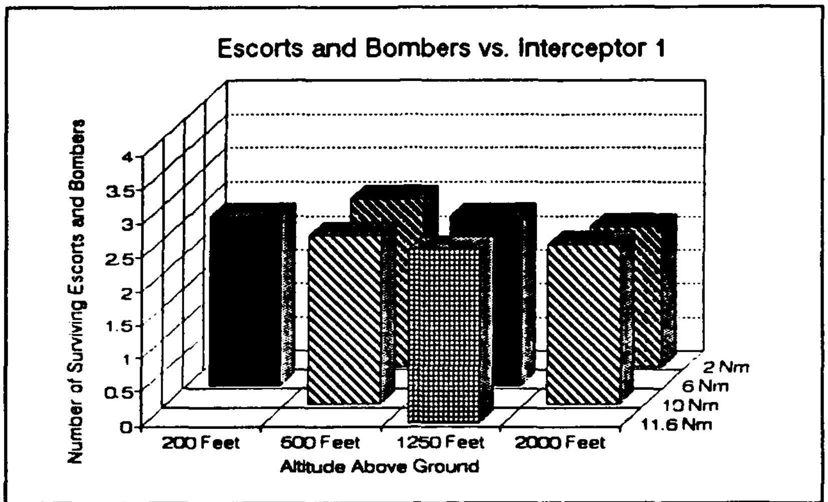

Figure 7-1. Bar Chart For Case 4

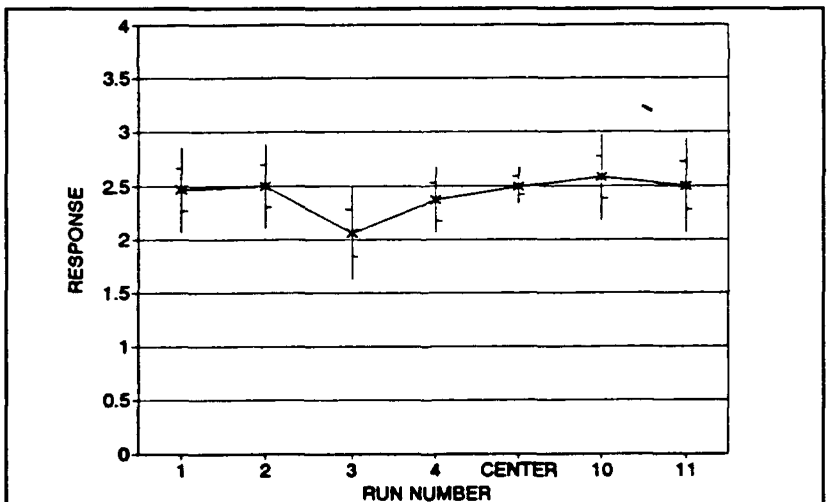

Figure 7-2. Variation About the Means

Case 5. Surviving number of the escort fighters when the incoming fighter was a Type 1 interceptor.

Table 7-3.
Design Points and the Average Responses For Case 5

RUN	RANGE	ALTITUDE	TYPE	RESPONSE
1	2	500	1	1.04167
2	10	500	1	1.15618
3	2	2000	1	0.91667
4	10	2000	1	1.03000
5	6	1250	1	1.27659
6	6	1250	1	1.10204
7	6	1250	1	1.02000
8	6	1250	1	1.24000
9	6	1250	1	1.12000
10	11.6	1250	1	1.04167
11	6	200	1	1.16667

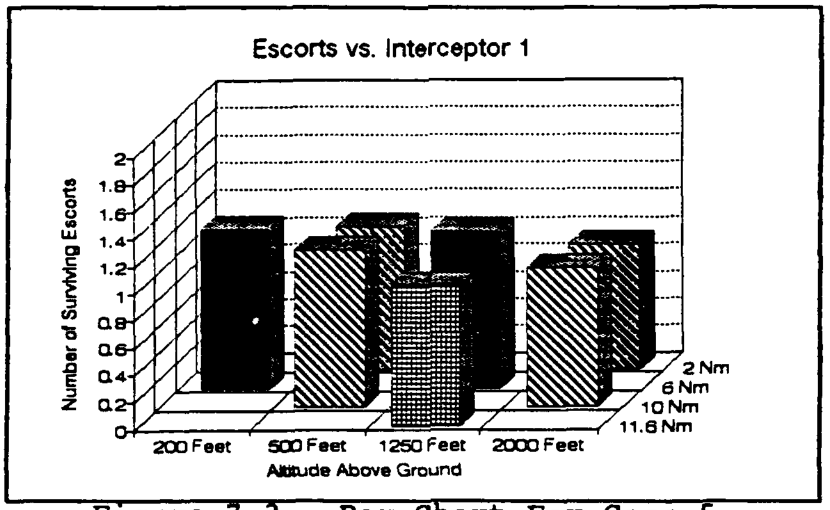

Figure 7-3. Bar Chart For Case 5

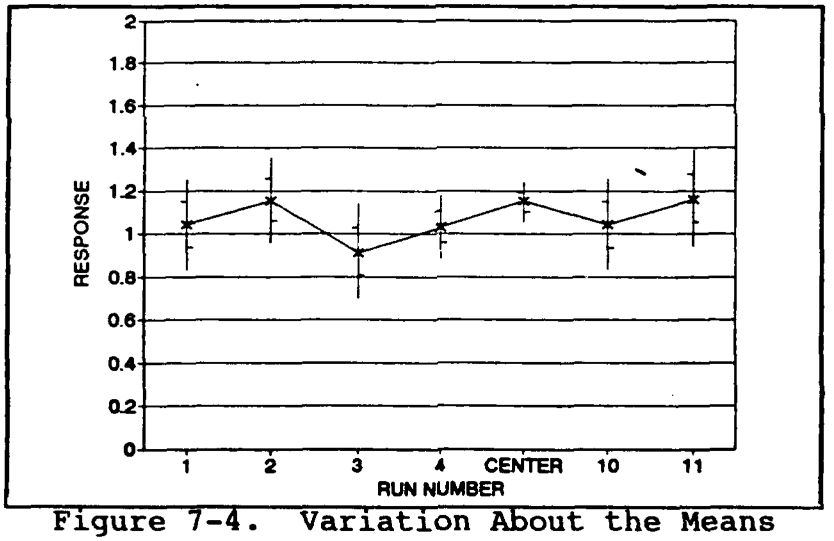

Figure 7-4. Variation About the Means

Case 6. Surviving number of the bombers when the incoming fighter was a Type 1 interceptor.

Table 7-4.
Design Points and the Average Responses For Case 6

RUN	RANGE	ALTITUDE	TYPE	RESPONSE
1	2	500	1	1.43750
2	10	500	1	1.34375
3	2	2000	1	1.14583
4	10	2000	1	1.34000
5	6	1250	1	1.63830
6	6	1250	1	1.30612
7	6	1250	1	1.18000
8	6	1250	1	1.40000
9	6	1250	1	1.21000
10	11.6	1250	1	1.54167
11	6	200	1	1.33333

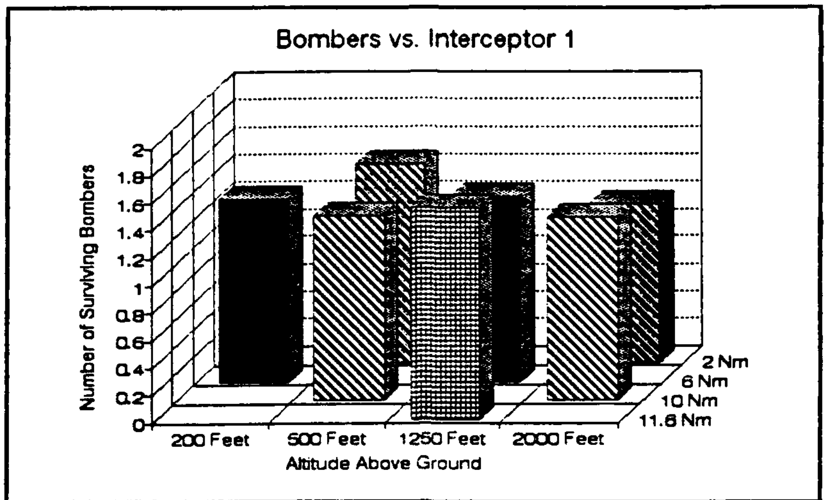

Figure 7-5. Bar Chart For Case 6

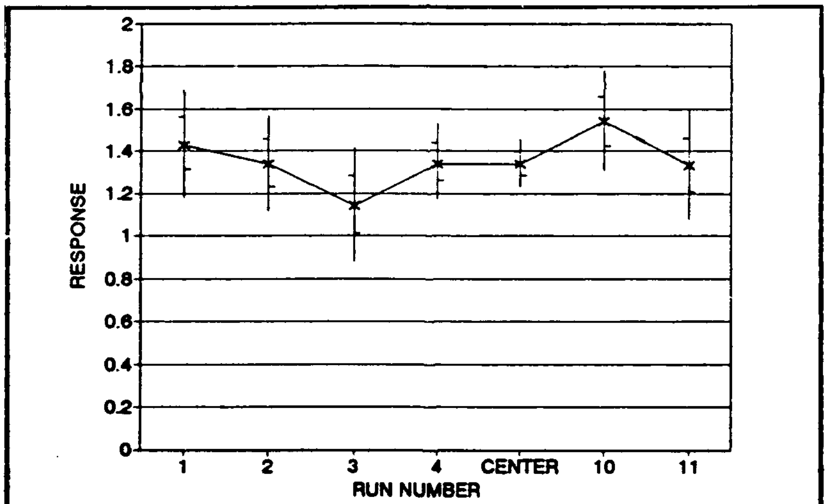

Figure 7-6. Variation About the Means

Case 7. Surviving total number of escort and bombers when the incoming fighter was a Type 2 interceptor.

Table 7-5.
Design Points and the Average Responses For Case 7

RUN	RANGE	ALTITUDE	TYPE	RESPONSE
12	2	500	2	3.14583
13	1.0	500	2	3.20830
14	2	2000	2	3.08300
15	1.0	2000	2	3.25000
16	6	1250	2	3.13000
17	6	1250	2	3.12000
18	6	1250	2	3.24000
19	6	1250	2	3.30000
20	6	1250	2	3.24000
21	6	1250	2	3.32000
22	0.4	1250	2	3.47458
23	6	2300	2	3.15000

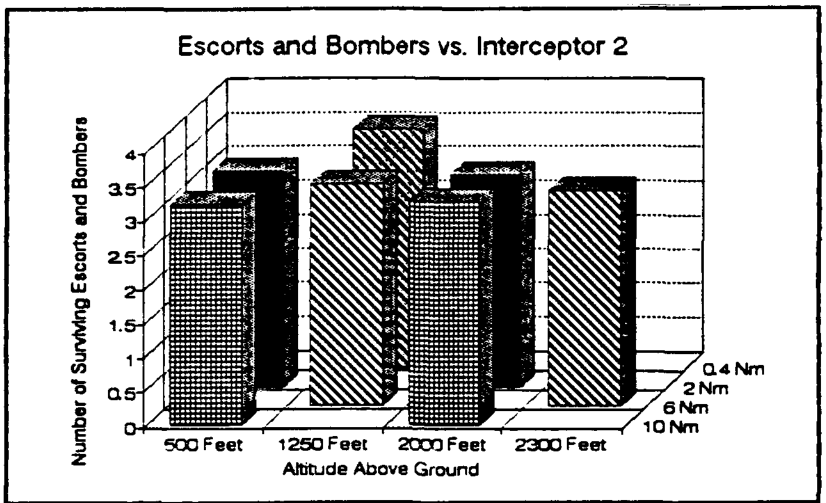

Figure 7-7. Bar Chart For Case 7

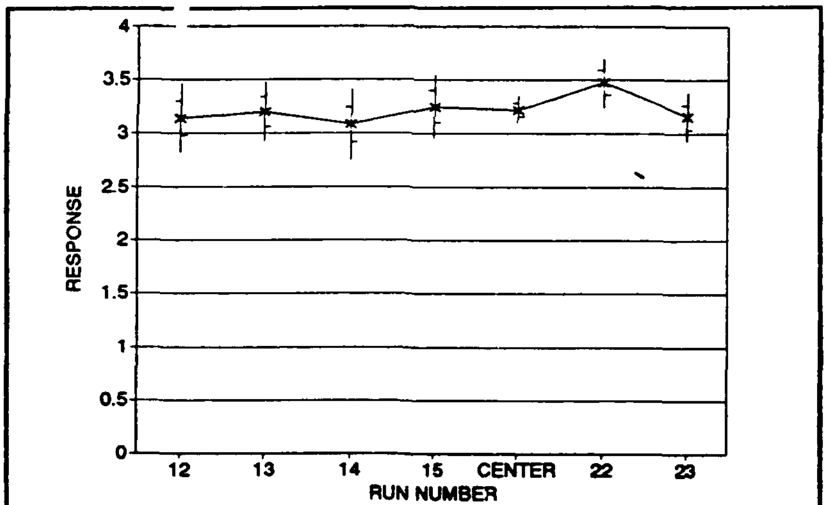

Figure 7-8. Variation About the Means

Case 8. Surviving number of escort fighters when the incoming fighter was a Type 2 interceptor.

Table 7-6
Design Points and the Average Responses For Case 8

RUN	RANGE	ALTITUDE	TYPE	RESPONSE
12	2	500	2	1.50000
13	1.0	500	2	1.52100
14	2	2000	2	1.37500
15	1.0	2000	2	1.43800
16	6	1250	2	1.44445
17	6	1250	2	1.58000
18	6	1250	2	1.50000
19	6	1250	2	1.65000
20	6	1250	2	1.58000
21	6	1250	2	1.62000
22	0.4	1250	2	1.72881
23	6	2300	2	1.50000

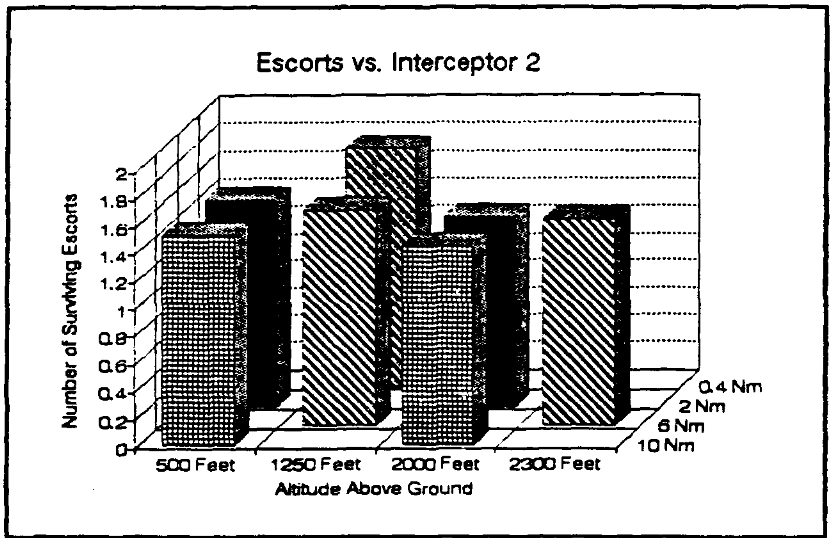

Figure 7-9. Bar Chart For Case 8

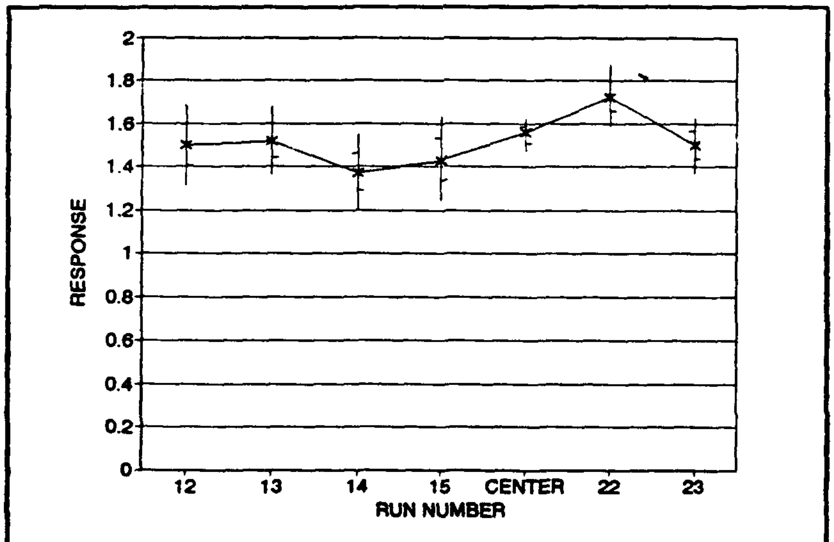

Figure 7-10. Variation About the Means

Case 9. Surviving number of bombers when the incoming fighter was a Type 2 interceptor.

Table 7-7.
Design Points and the Average Responses For Case 9

RUN	RANGE	ALTITUDE	TYPE	RESPONSE
12	2	500	2	1.64600
13	10	500	2	1.68800
14	2	2000	2	1.70800
15	10	2000	2	1.81300
16	6	1250	2	1.68888
17	6	1250	2	1.54000
18	6	1250	2	1.74000
19	6	1250	2	1.65000
20	6	1250	2	1.66000
21	6	1250	2	1.70000
22	0.4	1250	2	1.74576
23	6	2300	2	1.65000

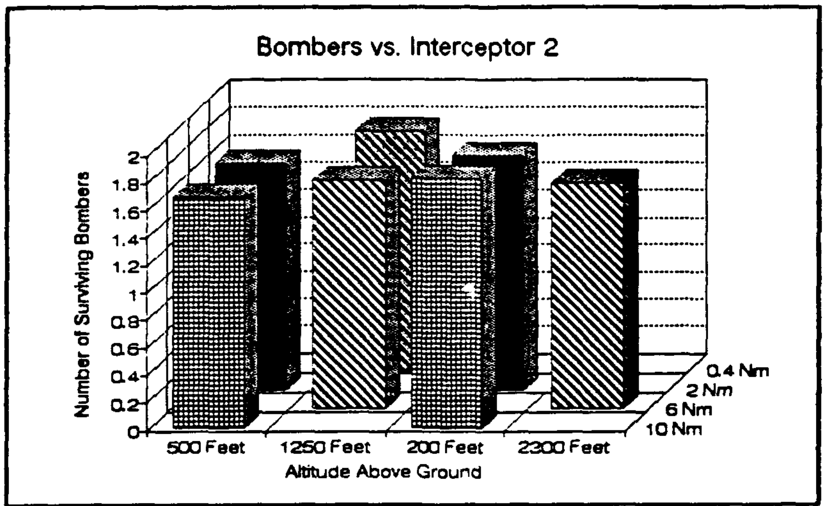

Figure 7-11. Bar Chart For Case 9

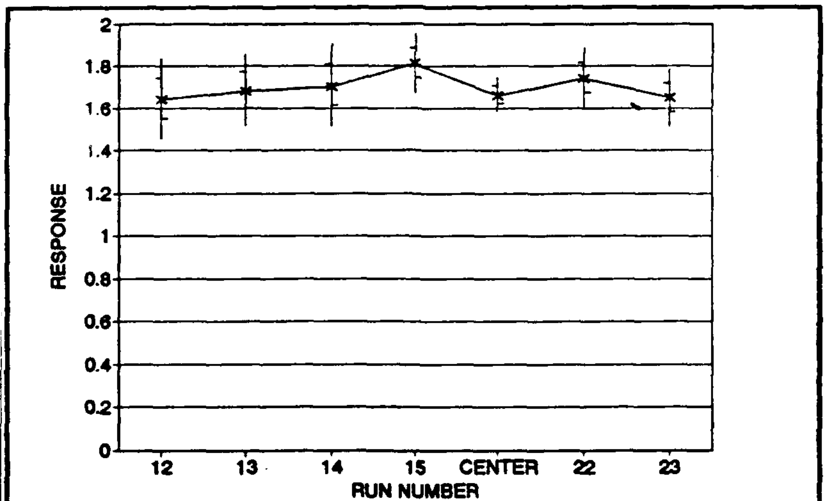

Figure 7-12. Variation About the Means

Conclusion of the Inspections. Graphical inspection showed that the variation in the simulation responses is quite large. Although there seem some differences between the responses of different combinations, it is not clear whether these differences were caused by the noise in the model or by the true effects of the variables on the response. Therefore, it was not possible to make sound inferences about the behavior of the variables in different combinations of range, altitude and interceptor types.

7.3 Statistical Analysis

Final analysis of the simulated combat data was carried out using statistical techniques. An iterative model evaluation procedure was used to select the most parsimonious mathematical models for the nine cases (see Section 6.6).

The first three cases were used to assess the combined effects of the interceptor flights over the blue fighters, which was one of the objectives of this study. The remaining six cases were used to support the first three by exploring the relation of range and altitude to the responses at each level of the interceptors.

Results of the evaluations presented statistically significant mathematical models. These mathematical models are shown in Table 7-8. In the following table, Table 7-9, values of the three evaluation criteria, test statistics of lack-of-fit tests, and P-values of t-tests for the significance of the model terms are presented for the corresponding response functions.

Table 7-8

Response Functions for Nine Research Models

Model 1	$y = 2.83 + 0.38\ x_3$
Model 2	$y = 1.31 + 0.21\ x_3$
Model 2*	$y = 1.34 - 0.05\ x_2 - 0.05\ x_2^2 + 0.22\ x_3$
Model 3	$y = 1.51 + 0.16\ x_3$
Model 4	$y = 2.45$
Model 5	$y = 1.14 - 0.08\ x_1^2$
Model 6	$y = 1.35$
Model 7	$y = 3.22$
Model 8	$y = 1.57 - 0.07\ x_2^2$
Model 9	$y = 1.66 + 0.04\ x_1^2$

x_1 denotes Range of the escorts to the bombers

x_2 denotes Altitude of the escorts above ground

x_3 denotes the type of the incoming enemy interceptors

Table 7-9
Properties of the Selected Models

Response Functions	R^2	VIF	C_p	$\dot{F}$	95 % F-val	P-val. $x_i=0$
Model 1	0.84	1.00	1.15	0.63	3.07	Int .0001 x_3 .0001
Model 2	0.83	1.00	3.16	1.28	3.07	Int .0001 x_3 .0001
Model 2*	0.87	0.97	4.00	1.28	3.14	Int .0001 x_2 .0710 x_2^2 .0874 x_3 .0001
Model 3	0.70	1.00	1.60	0.62	3.07	Int .0001 x_3 .0001
Model 4	0.00	0.00	***	0.47	6.16	***
Model 5	0.27	1.00	1.96	0.78	6.26	Int .0001 x_1^2 .0960
Model 6	0.00	0.00	***	0.46	6.16	***
Model 7	0.00	0.00	***	2.35	4.95	***
Model 8	0.27	1.00	-1.4	1.69	5.05	Int .0001 x_2^2 .0797
Model 9	0.21	1.00	1.77	0.80	5.05	Int .0001 x_1^2 .1263

R^2 is the Coefficient of Multiple Determination

VIF is the Variance Inflation Factor

C_p is the Mallow's Cp Statistic

P-val. Ho is the results of the F tests, showing the probability of making Type I error in testing where the fitted mathematical models are significant.

$\dot{F}$ is the test statistic for lack-of-fit test

95% F-val. shows value of F at the 95th percentile

Interpretation of the Mathematical Models. The mathematical models show that range and altitude did not have significant effects over the survivability of the escorts and the bombers in general. On the other hand, the type of the interceptor fighters determined the outcome of a confrontation in the experiments.

The mathematical models for cases one, two, and three show this strong relation between the interceptor types and the responses by having only the interceptor variable in the models. Although, the second Model 2^* could be defined by a quadratic model as shown in Table 7-9, the quadratic model has high P-values as compared to 95% confidence level (probability of a type-1 error was defined as $\alpha = 0.05$ in this study) for the quadratic term and the altitude variable. Accordingly, the linear model, Model 2, was preferred to represent the second case.

When the effects of altitude and range were analyzed for each interceptor type, in Model 4 to Model 9, it was seen that they did not have any impact on the results of the air combat. That is, when blue fighters encountered a specific type of interceptor, their chance of surviving the battle depended on the missile capability of the incoming interceptor, not the spatial positions of the escorts.

As in model two, the mathematical models for cases five, eight and nine have high P-values for their quadratic terms and low explanatory power. Therefore, these research models (Model 5, Model 8 and Model 9) were represented by the mean responses of the simulations conducted at respective design points. Analysis of variance tables, parameter estimates, covariance of estimates and t-tests for the model variables are shown in Appendix D through Appendix F for cases 1, 2 and 3.

7.4 Summary

This chapter presented the experimental findings and the corresponding analysis.

Twenty three experiments were conducted over fourteen points of the experimental design. The resulting simulation responses were analyzed graphically and statistically.

Graphical inspection of the simulation responses showed that response variation was large and mean responses, calculated at each design point, were close to one another. Therefore graphical analysis did not provide startling information about the relationships of the variables.

Statistical analysis, on the other hand, showed that there were not significant relation between the variables, except between type of the interceptors, and the output survivability response.

VIII. Conclusion and Recommendation

8.1 Summary of Experiments

The purpose of this study has been to investigate the escort mission and to shed light on recommended formations to use when enemy interceptor types are not known. The investigation was conducted using the TAC BRAWLER air combat simulation model which simulated air-to-air combat between enemy interceptors and friendly escort fighters in various combat scenarios. Based on the initial data gathered from these combat scenarios a series of experiments were conducted using a composite design. For each design point, forty simulation runs were conducted from identical initial starting conditions. To ensure independence between the replications, different random number seeds were employed. The mathematical average of the responses corresponding to the simulation runs at each design point provided the basis for the analysis.

To gain preliminary information, the mean simulation responses of the twenty three design points were analyzed by graphical analysis. Then, a parsimonious mathematical model for each measures of effectiveness that had been initially selected was developed through the use of regression analysis.

The variables remaining in these models were identified as the statistically significant factor in the research scenarios that effected the outcome of the air combat between the escorts, bombers and the enemy interceptors.

8.2 Analysis

Graphical Analysis. Graphical analysis showed that the variation of the responses at a specific combination of range and altitude was large and the values of the means were close to each other.

When the variation in the responses for one and two standard deviations from their mean were inspected, it was seen that the simulation responses had remarkable variation within the given number of repetitions (40 repeats). Therefore it was not possible to make an inference about the effects of the range and the altitude based on the graphical inspection.

Statistical Analysis. The experimental data was analyzed using statistical techniques to develop nine mathematical models for explaining the effects of the research variables on the response variables. At this stage the nine different cases were evaluated, and a mathematical model developed for each case.

Six of these nine models represented the effects of the range and the altitude of the escorts on the survivability of the friendly fighters, survivability of the escorts, and survivability of the strikes for each enemy interceptor.

The statistical analysis of the six models showed that the number of the surviving friendly fighters was not dependent on the range and the altitude of the escorts in cases where the incoming interceptors had the same capabilities as suggested by the graphical inspection too.

The findings in these six mathematical models (for cases 4 through 9) supported the mathematical models of the first three cases (models 1 through 3 which included the effects of different interceptor capabilities). The three mathematical models had only one statistically significant input variable, type of the enemy interceptor and is shown below with the equation intercept values.

Model 1: (Number of The Surviving Escorts and Strikes) = 2.83 + 0.38 (Type of Incoming Interceptors)

Model 2: (Number of The Surviving Escorts) = 1.31 + 0.21 (Type of Incoming Interceptors)

Model 3: (Number of The Surviving Strikes) = 1.51 + 0.16 (Type of Incoming Interceptors)

Interpretation of the Results. It is important to acknowledge that this study used a simplified research scenario, computer model default flight tactics, and an unclassified generic model data. The results of the statistical analysis were unable to show that when the friendly fighters are escorting a strike body, their ranges and altitudes with respect to the strike body can change the outcome of an armed confrontation with enemy interceptors. The study did indicate that the type of incoming interceptor will determine the chances of survival for the friendly fighters.

8.3 Conclusion of the Research

This research study showed that the investigated escort formation did not have statistically significant effects on the survivability of the escort fighters and bombers within the bounds of the experiment.

More important than the particular conclusion reached in the studied research scenario, is that the research proved that the TAC BRAWLER computer simulation along with RSM techniques could be used in the investigation of similar combat issues.

8.4 Recommendations for Further Research

This study used generic data, default air tactics, and simplified combat scenarios. Consequently, possible effects of the remaining many other factors were not investigated. The below factors are recommended for further investigation.

1. This research made a couple of important assumptions that meant other important aspects of an escort mission were not considered at all. Some of these aspects are the number of the incoming interceptors, number of the escort fighters, specialized air combat tactics, missiles, avionic systems, etc.. A statistical screening of these factors would uncover the significant factors and prepare the ground work for thorough analysis of the escort mission.

2. This research used generic data that was a serious handicap. Using actual systems' data would bring added reality into the simulation. Thus, the performances of the real world systems could be evaluated in light of the defined combat scenarios.

3. One additional area recommended for further research is in the TAC BRAWLER model itself. TAC BRAWLER is a very powerful model in which to simulate air combats between flights of aircraft. It provides a valuable option to the user to incorporate his/her own rules into the model's decision processes. During this research however, it was only possible to use a small portion of this option. It is recommended that this option be used to code formally approved air tactics into the model, which would enhance the investigation of their performance.

Appendix A-1: TAC BRAWLER Flight Tactics Alternatives

Taken from TAC BRAWLER Air Combat Simulation Analyst Manual.

Tactic Name	Comments
BRAWL_ATTACK	This is a within-visual-range tactic designed to let the aircraft "run free", except that hostile values are adjusted by orders to encourage proper sorting. No special pilot posture interpretation is required.
CROSS	This tactic is similar to SPLIT_MUSUP, except that the attackers are sent to the "wrong" side, effecting a cross. Pilot posture interpretation is performed by the subroutine spbvi.
CROSS_LOW	Similar to cross, except that, in case, the attackers "go down to the deck", while in the non-LOW case they use the smaller of their current altitude and the target altitude. Pilot posture is interpreted by subroutine spbvl.
DEFAULT_MISSION	This tactic is appropriate for flights not interacting with hostiles; but it is also used by bomber mission when the interaction with hostiles is to be left to the fighter escorts. It pays attention primarily to route point values (for flight leaders) and to formation flying (for wingmen). No special pilot posture is required.
DISENGAGE_FLIGHT	This tactic is intended to get everyone to escape. Pilot posture interpretation is performed by subroutine spdis, although the disengagement maneuver generator actually does the work of setting up the pilot maneuver value function.
END_RUN	In this tactic, all attackers attempt to engage from the same flank of the hostile formation. The "easiest flank to reach" is chosen. Pilot posture interpretation is performed by subroutine spbvri.

Appendix A-2: TAC BRAWLER Flight Tactics Alternatives

Tactic Name	Comment
END_RUN_LEFT	In this tactic all attackers attempt to engage from the right flank of the hostile formation. Pilot posture interpretation is performed by subroutine spbvri.
END_RUN_LOW	This tactic is similar to END_RUN except that, in this case, the attackers "go down to the deck", while in the non-LOW case they use the smaller of their current altitude and the target altitude. Pilot posture is interpreted by subroutine spbvrl.
END_RUN_RIGHT	In this tactic, all attackers attempt to engage from the left flank of the hostile formation. Pilot posture interpretation is performed by subroutine spbvri.
ESCORT_TACTICS	This tactic is intended for use by bomber escorts. The basic idea is to keep the escorts close to the bombers, ignoring hostiles, until the pose a significant threat to the bomber group. Even then the efforts is made to commit as few escorts as possible, holding back the others as a reserve. Pilot posture interpretation is performed by subroutine spesc.
EVADE_REENGAGE-FLT	This is a "slashing attack" in which a flights acts much as in HIT_AND_RUN, but will reengage after 5 nmi. separation has been achieved. Pilot posture interpretation is performed by subroutine evdrng.

Appendix A-3: TAC BRAWLER Flight Tactics Alternatives

Tactic Name	Comments
HIT_AND_RUN	This within-visual-range tactic is designed to get in and out quickly it is appropriate for situations where the fight is out numbered but wants to engage anyway, perhaps because it has positional advantage. Pilot posture interpretation is performed by subroutine hitrun which acts by assessing whether the current phase is hit or run and setting offensive and defensive values appropriately.
HOOK_DRAG	This tactic is used when there is a significant threat to some member of the flight but there exist significant discrepancies in the degree to which various members are jeopardized. This tactic has the more threatened members of the flight run, while the others attack. The "runners" are made to run from the hostiles at a 135 degree relative heading. When the attacker portion is further away from the hostiles than the "runners" the heading is modified so as to turn towards the attacker portion of the flight (so that they don't get taken out of the action); in the opposite situation the heading takes the "runners" away from the attacker portion, to prevent the hostiles from being able to point at both sets of friendlies simultaneously. Pilot posture interpretation takes places in subroutine spbvrh.
LAUNCH_AND_LEAVE	The attacker fires one or more active missiles at a single target and disengages without getting into the hostile weapon envelopes. Pilot posture interpretation is performed by subroutine launlv which emphasizes the defensive element in having a long range missile.

Appendix A-4: TAC BRAWLER Flight Tactics Alternatives

Tactic Name	Comments
LOOP	In this tactic, the forwardmost element does an ENDRUN tactic while the others do a 360 degree turn. The idea is to obtain more separation and set up a later HOOK_DRAG. Pilot posture interpretation is performed by subroutine spbvri.
LOOP_LOW	This tactic is similar to LOOP, except that, in this case, the attackers "go down to the deck", while in the non-LOW case they use the smaller of their current altitude and the target altitude. Pilot posture is interpreted by subroutine spbvrl.
MULTI_LAUN_LV	This is repetitive variation on the LAUNCH_AND_LEAVE tactic, in which the disengage is more to the beam than directly away from the hostiles, and the pilot pitches back in to reattack whenever he judges that he can do so without getting into the hostile weapon range.
MULTI_TARGET	This tactic represents the opposite side of the active radar missile coin. The idea here is to take advantage of the fact that the lock does not need to be maintained on a single target with this missile, allowing several targets to be attacked simultaneously. Pilot posture interpretation is performed by mlttgt.
POINT	In this tactic all aircraft go straight at the targets. Pilot posture is interpreted by subroutine spbvrl.
POINT_LOW	This tactic is similar to POINT, except that, in this case, the attackers "gp down to the deck", while in the non-LOW case they use the smaller of their current altitude and the target altitude. Pilot posture is interpreted by subroutine spbvrl.

Appendix A-5: TAC BRAWLER Flight Tactics Alternatives

Tactic Name	Comments
SPLIT_MUSUP	In this case, all the attackers of the same target attack from the same (most convenient) side, but the attackers of different targets may choose different flanks. Pilot posture interpretation is performed by subroutine spbvri.
SPLIT_MUSUP_LOW	This tactic is similar to SPLIT-MUSUP, except that, in this case, the attackers "go down to the deck", while in the non-LOW case they use the smaller of their current altitude and the target altitude. Pilot posture is interpreted by subroutine spbvrl.
SPLIT_SPLIT	This tactic forces attackers which are assigned to the same target to attack it from opposite sides. Each group of attackers is treated independently of those assigned to other targets within a group attack, direction depends upon which side of a line the attacker lies on; this line joins the target and the group center of mass. Pilot posture interpretation is performed by the subroutine spbvri.
SPLIT_SPLIT_LOW	This tactic is similar to SPLIT_SPLIT, except that, in this case, the attackers "go down to the deck", while in the non-LOW case they use the smaller of their current altitude and the target altitude. Pilot posture is interpreted by subroutine spbvrl.

Appendix B-1: TAC BRAWLER Maneuver Alternatives

Taken from TAC BRAWLER Air Combat Simulation Analyst Manual.

Maneuver Name	Comments
AIM_MISSILE	Point at a computed intercept point, using maximum instantaneous g capability.
DIRECT MANEUVER	Special maneuver for production rules and interactive pilot for generating constant g turns.
DISENGAGE	Attempt to fly a vector velocity computed to achieve disengagement from a hostile.
EVADE_MISSILE	One of two maneuvers. At long times-to-impact, this maneuver runs away from the missile. At short times-to-impact, the maneuver generated attempts to brake out of the plane defined by "my" velocity and the line-of-sight to the missile.
FORCE_OVERSHOOT	Intended to force a hostile to overshoot.
FORMATION_FLY	Attempts to achieve a configuration co-velocity with a leader at a specified offset from him.
GRAND_AVD_UP/LEFT/RIGHT	These three attempt maximum instantaneous g pullups; one directly up, and the others rolled slightly to the left or right. The last two may do better for other value components than the straight-up pull.
ILLUMINATE	Turn to 30 degree from the line-of-sight to the target being illuminated for semi-active missile or an active radar missile that has not acquired.
LEFT_BREAK_EVADE	Break left to evade a hostile.

Appendix B-2: TAC BRAWLER Maneuver Alternatives

Maneuver Name	Comments
LOW_SPD_RECOVER	If node is very high, this maneuver pulls down at moderate g. If the climb angle is no more than 45 degree, this maneuver unloads and maintains present heading. Both versions use maximum afterburner.
LOW_SPDR-LEFT45	Turn to the left 45 degree to recover speed.
LOW-SPDR_RIGHT45	Turn to the right 45 degree to recover speed.
NEGATIVE_VELOCITY	Turn on negative velocity vector of hostile.
PULL UP AT GMAX_SUST	Pull up at maximum sustainable g's. Intended to achieve a roll-over-the-top maneuver, initial phase.
PURE 1V1 MANEUVER	Special one-versus-one maneuver subject to being in special one-versus-one mode.
RIGHT_BRAKE_EVADE	Break right to evade a hostile.
ROLL-15&PULL	Roll 15 degree to the left of the current bank angle and pull maximum sustained g's. Intended as a perturbation on the current turn.
ROLL+15&PULL	Roll 15 degree to the right of the current bank angle and pull maximum sustained g's. Intended as a perturbation on the current turn.
ROUTE_MANEUVER	Flies toward a routepoint at a specified speed and altitude.
RUN_AWAY	Get on negative line-of-sight vector to hostile.
SLIGHT_LEFT_TURN	Perform a slight left turn. For use with BVR maneuvers to achieve small course corrections.

Appendix B-3: TAC BRAWLER Maneuver Alternatives

Maneuver Name	Comments
SLIGHT_RIGHT_TURN	Perform a slight right turn. For use with BVR maneuvers to achieve small course corrections.
SLOW_&_PULL_CUR_PLN	Slows and pulls up while remaining in the current maneuver plane.
SLOW_&_PULL_LEFT	Slows and pulls up and left at maximum sustained g's; 15 degree change in maneuver plane.
SLOW_&_PULL_RIGT	Slows and pulls up and right at maximum sustained g's; 15 degree change in maneuver plane.
SLOW_IN_CURRENT_DIR	One of several maneuvers designed to prevent overshoot; this one reduces speed while maintaining current direction.
STATIONARY	Remain stationary, used in the implementation of ground-based players.
STRAIGHT_AND_LEVEL	Performed at current speed. a default maneuver which is always considered.
STRAIGHT_MAX_AB	Maintain current direction, including climb/dive angle, with maximum afterburner.
STRT&LEV,MX SPD	Fly straight and level on the current heading at full afterburner.
SUPPORT_BOMBERS	A formation-fly maneuver for escorts relative to bomber formation.
TAIL_ATTACK	Attempt to get on the tail of a specified hostile, co-velocity. The distance behind the target depends upon the weapon type selected.
VECTORED_FLIGHT	Attempt to fly a specified vector velocity.

Appendix C-1: Characteristics of The TAC BRAWLER Air Combat Simulation Model

PURPOSE: TAC BRAWLER is both a research and evaluation tool and an operation support tool (decision aid). The model represents the effects of hardware and tactics on air-to-air combat at the flight-versus-flight level. Each aircraft, avionic system, and missile is explicitly represented in the simulation.

DOMAIN: Air.

SPAN: Local.

FORCE COMPOSITION: Component

SCOPE OF CONFLICT: Conventional air-to-air combat.

MISSION ARENA: Virtually any combination of current or proposed air-to-air weapon systems to include airframes, engines, missiles, guns, and avionics.

LEVEL OF DETAIL OF PROCESS AND ENTITIES: Individual aircraft and weapon systems.

HUMAN PARTICIPATION: Not required.

TIME PROCESSING: Dynamic (treat time dependent process), time-step (passing of time calculated on time basis).

Appendix C-2

TREATMENT OF RANDOMNESS: Stochastic, Monte Carlo (outcomes of the same event probabilistic, that is, they are determined by drawing a random number from a distribution function).

SIDEDNESS: Two sided, asymmetric (sides may have different assets), both sides reactive.

INPUT: Airframe aerodynamics data, avionics data, RSC data, engine data, scenario files, and rule files.

SECURITY CLASSIFICATION: Secret

Preceding information was taken from the J 8 Catalog of Wargaming and Military Simulation Models, 11 Edition DTIC AD-A213 970, page T-3. A complete list of the characteristics of the model can be found in the above referenced document.

Appendix D-1: Statistical Findings of Model 1

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	3.38842	3.38842	110.988	0.0001
Error	21	0.64112	0.03053		
C Total	22	4.02954			

Root MSE	0.17473	R-square	0.8409
Dep Mean	2.85432	Adj R-sq	0.8333
C.V.	6.12149		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEPT	1	2.837619	0.03646759	77.812	0.0001
X3	1	0.384190	0.03646759	10.535	0.0001

Covariance of Estimates

COVB	INTERCEPT	X3
INTERCEPT	0.0013298848	-0.000057821
X3	-0.000057821	0.0013298848

Appendix D-2

T-tests for The Model Variables

Intercept value

Numerator:	184.8478	DF:	1	F value:	6054.7218
Denominator:	0.03053	DF:	21	Prob>F:	0.0001

Variable "Interceptor type".

Numerator:	3.3884	DF:	1	F value:	110.9883
Denominator:	0.03053	DF:	21	Prob>F:	0.0001

Appendix E-1: Statistical Findings of Model 2

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	1.08796	1.08796	104.034	0.000
Error	21	0.21961	0.01046		
C Total	22	1.30757			

Root MSE	0.10226	R-square	0.8320
Dep Mean	1.32821	Adj R-sq	0.8240
C.V.	7.69934		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	T for H0: Parameter=0	Prob > |T|
INTERCEPT	1	1.318741	0.02134352	61.786	0.0001
X3	1	0.217698	0.02134352	10.200	0.0001

Covariance of Estimates

COVB	INTERCEPT	X3
INTERCEPT	0.0004555458	-0.000019806
X3	-0.000019806	0.0004555458

Appendix E-2

T-tests for The Model Variables

Intercept Value

Numerator:	39.9232	DF:	1	F value:	3817.5701
Denominator:	0.010458	DF:	21	Prob>F:	0.0001

Variable "Interceptor Type"

Numerator:	1.0880	DF:	1	F value:	104.0340
Denominator:	0.010458	DF:	21	Prob>F:	0.0001

Appendix F-1: Statistical Findings of Model 3

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Prob>F
Model	1	0.63792	0.63792	49.258	0.0001
Error	21	0.27196	0.01295		
C Total	22	0.90988			

Root MSE	0.11380	R-square	0.7011
Dep Mean	1.52635	Adj R-sq	0.6869
C.V.	7.45573		

Parameter Estimates

Parameter Variable	DF	Standard Estimate	Error	T for H0: Parameter=0	Prob > |T|
INTERCEPT	1	1.519106	0.02375158	63.958	0.0001
X3	1	0.166698	0.02375158	7.018	0.0001

Covariance of Estimates

COVB	INTERCEP	X3
INTERCEPT	0.0005641377	-0.000024528
X3	-0.000024528	0.0005641377

Appendix F-2

T-test for The Model Variables

Intercept Value

Numerator:	52.9764	DF:	1	F value:	4090.6401
Denominator:	0.012951	DF:	21	Prob>F:	0.0001

Variable "Interceptor Type"

Numerator:	0.6379	DF:	1	F value:	49.2576
Denominator:	0.012951	DF:	21	Prob>F:	0.0001

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Vita

First Lieutenant Mustafa Ilhan was born in Zonguldak, Turkey on 1 May 1961. He completed his elementary and middle school education in his hometown Zonguldak. He obtained his high school education at Kuleli Military High School, in Istanbul. In 1979 he entered the Turkish Air Force Academy in Istanbul where he majored in Management Science. Upon his graduation in 1983, he started pilot training in Izmir. He was awarded his wings in 1984. Between 1984 and 1990 he served in various fighter squadrons in the Turkish Air Force.

In May 1990 he was selected to attend the Graduate Operations Research course, in the School of Engineering, at the U.S. Air Force Institute of Technology.

First Lieutenant Mustafa Ilhan is married and has one daughter born in 1991, while he was a student at the Air Force Institute of Techology.

Permanent Address: Terakki Mahallesi
Dumlupinar Sokak No:15/B
Zonguldak/Turkey