AN INVESTIGATION OF MODIFICATIONS TO THE TAC CAMPAIGN MODEL

THESIS

Lieut. [illegible]
[illegible], TUAF

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DEPARTMENT OF THE AIR FORCE
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AIR FORCE INSTITUTE OF TECHNOLOGY

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Efgan Dengür
1LT, TUAF

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Advisor	Dr. James W. Chrissis/ENS	J. W. C
Reader	LtCol James T. Moore/ENS	James T. Moore

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AN INVESTIGATION OF MODIFICATIONS TO THE TAC CAMPAIGN MODEL

THESIS

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Efgan Dengür, B.S.
1LT, TUAF

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Abstract

This research investigates the cost and budget modifications to Mixmaster which is an aggregate version of the Theater Attack Model (TAM). Seven simple cost employment methods, a goal programming approach, a probabilistic approach for determining cost coefficients, and a goal programming version of that probabilistic approach were applied to Mixmaster. Also, Mixmaster was enhanced with an additional leading constraint to incorporate the Air Force tactical considerations in a campaign scenario. The results favor the advantages of the goal programming approaches. In addition, the probabilistic approach introducing the time factor in the computations promises more accurate results, given that the required parameters are estimated accurately. For further enhancements, first, a ranked goal programming application to Mixmaster is recommended upon condition that an efficient software package is used. Second, an investigation of methods to determine better estimates of the probabilistic approach parameters is proposed.

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Preface

The purpose of this research is to modify the Tactical Air Command (TAC) Campaign model that the Joint Study Group (JSG) uses to determine procurement needs. Mixmaster is an aggregate version of the Theater Attack Model (TAM) which is a large scale Linear Program (LP) that the Air Force Center for Studies and Analyses (AFCSA) currently employs for some of their analyses. This research develops four basic cost and budget modifications for the Mixmaster model.

At this opportunity I would like to express my gratitude to my advisor, Dr. James Chrissis, for his efforts. Had he not informed me about this research, I would be a graduate student without a thesis topic. Also, I would like to extend my special thanks to my reader, Lt Col James Moore, for being patient while correcting my spelling and numerical errors. I appreciate the encouragements of my research sponsor, Capt Skip Langbehn; thanks for the hints from an AFIT survivor.

For my best friend Kathy Harrington, it was not that easy to spend two neurotic years with me, but we both survived and my deepest appreciation goes to her. She helped me understand America, what else I needed? I cannot thank enough Debbie Conrad and Buzz Reed for helping me review the grammar and syntax of this paper.

My family, Babacigim, Ruherruh, Abim and Sellum without their support this research could not make the finish line. Thanks also to the generous Turkish Nation for sending me to meet the challenge.

With confidence, I hope this research will be very useful to JSG.

Efgan Dengiz

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Abstract

The Theater Attack Model (TAM) is a large scale linear program (LP) that the Air Force Center for Studies and Analyses (AFCSA) uses to aid the decision makers in making procurement and budget decisions. Through the years, TAM has been modified by the other analysis agencies of the Air Force. Mixmaster is one of the aggregated versions of TAM that the Joint Studies Group (JSG) of the Tactical Air Command (TAC) currently uses. However, Mixmaster does not include cost and budget issues in its current linear configuration.

This research investigates the cost and budget modifications to Mixmaster. Seven simple cost employment methods, a goal programming approach, a probabilistic approach for determining cost coefficients, and a goal programming version of that probabilistic approach were applied to Mixmaster. Also, Mixmaster was enhanced with an additional leading constraint to incorporate the Air Force tactical considerations in a campaign scenario.

The results favor the advantages of the goal programming approaches. In addition, the probabilistic approach introducing the time factor in the computations promises more accurate results, given that the required parameters are estimated accurately.

For further enhancements; first, a ranked goal programming application to Mixmaster is recommended upon condition that an efficient software package is used. Second, an investigation of methods to determine better estimates of the probabilistic approach parameters is proposed.

AN INVESTIGATION OF MODIFICATIONS TO THE TAC CAMPAIGN MODEL

I. INTRODUCTION

1.1 Background

Senior decision makers are concerned about anticipating future needs for procuring aircraft, munitions and spare parts. Accordingly, the United States Air Force (USAF) charges the Air Force Center for Studies and Analyses (AFCSA) to conduct studies associated with various constraints on procurement. These constraints include budget changes; aircraft and munition effectiveness; target values; attrition rates; the cost of current and forecast aircraft; munitions and spares; existing force structure of aircraft and munitions; weather; length of mission and length of conflict. "Currently, AFCSA uses the Theater Attack Model (TAM), a large-scale linear program (LP), in these analyses to evaluate theater level tactical operations in support of procurement decisions" [3:1].

TAM has been exported to other analysis centers and major commands that are seeking a better understanding of the influence of budget and the marginal results of different operational capabilities. There are different versions of TAM in terms of the entity bases (i.e., indices) that are included. The sheer size of TAM makes it one of the largest LPs the Pentagon regularly uses (generally with 3500 constraints and 250000 variables). However, the size of this full version of TAM is not convenient for use by small agencies or sub commands. For their convenience, smaller versions of TAM were developed. They are easier to manipulate; they also provide strong insight despite their small size.

1.2 The TAM Linear Program

To understand TAM as an LP, it is worthwhile to briefly discuss linear programming. For many people, LP is considered among the most important scientific developments of the mid-twentieth century. The foundation of LP goes back to the studies done within the military procurement area during and after World War II with project SCOOP. This project was directed by George Dantzig who later developed the simplex method which facilitates linear programming computations [3:1].

> The impact of LP has been extraordinary. Today, it is a standard tool that has saved many thousands or millions of dollars. What it basically does, is to allocate limited resources among competing alternatives in the best possible way (i.e. optimal). [5:23]

In the mid 1980s, strict weapons effectiveness studies became inadequate due to their lack of real-life conflict parameters. In response, LtCol Robert J. Might developed a model which measures the value of a particular weapon system or munition in the context of an entire conflict in a theater war. The model attempts to provide deeper insight on the following questions:

- How much of a weapon system's effectiveness can be covered by other systems?
- How would attrition affect a current or proposed weapon system?
- How effective is every weapon system and munition combination against every available target type?
- How often will a particular weapon system and munition combination be turned in a war? How many of these sortie types can be realistically supported?
- How long will a weapon system and munition combination remain effective?
- If a new weapon system is not selected for production, will the loss of marginal effectiveness be replaced by another weapon system/munition combination?

- Would the entrance of a new weapon system into the inventory remove all need for an older weapon system? [3:2-3]

Might states that "a strict weapon system analysis cannot answer these questions" [7:55-63] However, TAM, with its inherent parameters such as multi-period conflict, multiple weather bands, multiple sortie-distance, multiple aircraft-munitions and multiple spare resources, computes optimality while accounting for:

- effectiveness of each aircraft and munition combination against each target type,
- expected attrition of each aircraft against each target type,
- daily sortie rates for each weapon system,
- current inventories of aircraft, munitions, and spare parts,
- the numbers and values of enemy targets, day by day, including the effects of replacements,
- procurement costs of new aircraft, munitions and spare parts,
- the value of spare parts to increase, decrease or maintain sortie rates [3:3].

Further, TAM was extended in terms of the entities to include the capability to consider air base operability and the effectiveness of electronic countermeasures to aircraft survivability [3:3].

Unlike the construction of some other sensitive LPs, TAM's construction is very flexible and, as such, may be used for different purposes; ergo, many of the TAC components have modified TAM's construction to meet their study objectives. Among these components, the Joint Studies Group (JSG) has increased its involvement with the USAF Munitions Roadmap and has realized the need for a model which can provide timely, operationally-sound answers while retaining the flexibility to handle a variety of questions. JSG has developed the Mixmaster model, a scaled-down version of TAM, to provide decision makers with an analytical tool for determining munitions requirements. Mixmaster has six parameters representing aircraft type, weapon type, target type, distance band, time, and weather condition. This aggregate model includes fewer parameters than TAM which means less computation time; hence it presents quick solutions even when run on desktop computers. In terms of the weapons in TAC's inventory, with this configuration, Mixmaster is capable of providing adequate insight to the JSG analysts. [4].

1.3 Purpose of the Research

Currently, Mixmaster has constraints associated with aircraft, munitions, and targets. The objective function is to maximize *Target Value Destroyed*. The primary decision variable of the model is *Number of Sorties Flown*. Nevertheless, this construction of the model does not relate the decision variable to any cost figures. Therefore, Mixmaster does not answer questions about cost while optimizing *the Target Value Destroyed*. Moreover, some concerns of decision makers about the model's operational accuracy in a theater conflict questioned the reliability of Mixmaster. Also, the uncertainty of the budget raised another problem for the model. Decision makers were not satisfied by the answers based upon the predetermined budget figure in the model. Regarding these concerns, the purpose of this study is to improve the operational accuracy of Mixmaster and to investigate modifications employing cost and budget figures appropriately.

1.4 Problem Statement

As mentioned, the objective function of Mixmaster maximizes *the Target Value Destroyed*. The LP model solves for *the Number of Sorties* that should be flown by each combination of aircraft and munition for every target, distance band, time, and weather condition. Earlier attempts to modify the objective function with aircraft costs so as to maximize *Target Value Destroyed per Aircraft Dollar* revealed an inconsistency between LP results and air operations expectations. LP optimization techniques treat the cost coefficients as penalties to avoid. Hence, according to the solutions, an aircraft with a cost of $20 million must always be preferred to an aircraft with a cost of $60 million. For instance, the winner of the F-16 versus the F-15 in a scenario where the attrition rates are similar for both will always be the F-16 since its cost is substantially less than the F-15's. This occurs because the difference in yield—target killed per sortie by the aircraft—of these two distinct aircraft remains insignificant when compared to the difference in cost of the two aircraft. The model always produces these kinds of solutions because it does not permit the loss of an aircraft with a substantial cost and it never prefers to use that costly aircraft. Therefore, the model seems to hinder the tactics of decision makers by disregarding the use of more costly, possibly more effective, aircraft.

In addition, Mixmaster does not include a budget constraint in its current configuration. Lacking a budget constraint causes some uncertainties in decision making process since it is probable that the dedicated budget may not suffice to afford procurement needs. If procurement needs are determined accounting for a dedicated budget then the results of Mixmaster will be more sound.

The inconsistency between LP results and air operation requirements reveals another deficiency of Mixmaster. The model disregards the appropriate use of aircraft in a variety of munition target combinations. For instance, results show Mixmaster can allocate the A-10 aircraft deep in enemy territory despite the fact that A-10s are supposed to be allocated for close air support missions [4].

Consequently, this research suggests methods to improve operational accuracy and investigate modifications of the model by using techniques to include costs in the objective function; a goal programming approach ; a probabilistic approach which determines the cost coefficients; and a goal programming version of that probabilistic approach.

1.5 Overview of Subsequent Chapters

The remainder of this paper synthesizes the research and results. The dynamic evolution of TAM currently takes place in various studies and analysis centers of the USAF. Because of classification, documented sources about the applications of TAM are few. Chapter 2 focuses on the developmental phase of TAM and some suggested improvements. Chapter 3 covers the suggested methods for the inclusion of cost in the model. Chapter 4 presents the investigation of the methods applied by analyzing the results. Chapter 5 presents an application of the modifications to two cases. Chapter 6 concludes the research with an examination of the best approaches and presents recommendations for JSG. Appendices contain the inventory levels for aircraft and weapons, the JSG formulation of Mixmaster and the investigated modifications.

II. LITERATURE REVIEW

2.1 Introduction

The literature review focuses on the following topics: a general overview of LP; a developmental and informative review of TAM and information on the methodology. The first topic gives a concise introduction to LP structure and its assumptions. The second topic discusses TAM as an LP application for weapon allocation and procurement. The suggested improvements and the Mixmaster model are reviewed. The third topic reviews the applicability of the suggested methods.

2.2 A General Overview of LP

LP is an optimization technique which involves linear mathematical models. The adjective *linear* means that all the functions in this mathematical model are required to be linear. Hillier and Lieberman further explain *programming* by stating that "the word programming refers to the planning of activities to obtain an optimal result" [5:24].

Although the literature most frequently cites the allocation of resources to activities, LP has a wide range of application to problems whose mathematical model fits the very general format of LP. Simply, resources are usually limited in supply; the objective and limited resources have linear structure. A feasible region is constructed with respect to the "moving" objective function. This general format can be defined as:

$$\begin{aligned} \min(\max)Z &= cx \\ st: Ax &\leq b \\ x &\geq 0 \end{aligned}$$

Hillier and Lieberman mention the implicit feature of assumptions in the model formulation. They describe *proportionality, additivity, divisibilty*, and *certainty* as the basic assumptions of LP. Proportionality considers the activities independently of others. Additivity guarantees that the objective function and the constraints are linear and exclude any interactions between variables; this eliminates cross product terms. Divisibility allows noninteger solutions. Certainty requires all parameters to be known constants [5:31-36].

Bazaraa et al. define several evolving stages in LP. In the first stage, *problem formulation*, they emphasize a detailed study of the system, data collection and the identification of the problem. The second stage, which involves the *construction* and *abstraction* of the problem through a mathematical model, they caution the analysts to make sure the model represents the problem. In the third stage, they suggest using a proper technique to derive a *solution*. In the fourth stage, *model testing*, includes strong insight against what-if questions (i.e., *sensitivity analysis*). In the final stage, they stress that the model should aid the decision making process and not preempt the decision maker's action [1:7-8].

2.3 A Developmental and Informative Review of TAM

The amount of money to be spent on how many items is a paramount question that decision makers always face. From the point of view of an Air Force decision maker, that question becomes "how much of the Air Force procurement budget should be spent on the many different aircraft and how much on the many different munitions" [7:55]?

Might, having reviewed the decisions made at the Department of Defense, implies that whole categories of decisions have been described as unsuitable for the quantitative approach. However, he makes a distinction only in one area where such a quantitative approach has proved useful. That area is the budgeting process of the USAF. The evidence shows that an analytical tool is being used by different components on the Air Force staff to support decisions related to aircraft and conventional munitions. The Air Force staff has been developing munitions procurement options using quantitative analysis—mostly LP—for a number of years. Although the initial process is an important improvement in the Air Force decision making process, Might points out the deficiency that the staff officers do not have the capability to do sensitivity analysis. In addition, the methodology used ignores the existing munitions inventory when maximizing the target value killed per dollar spent. The results always require the procurement of new munitions for every target that is near the top of the target value ranking. Might takes advantage of the existing analytical process which makes the assumptions that are needed to make the objective function linear and the constraints manageable. As the originator of TAM, he approaches the problem in the context of a theater-level conflict to determine the impact of budget, attrition, force structure, targeting decisions and munitions inventories on warfighting capability in a theater scenario. [7:59]. TAM was modified with an additional decision variable and constraint in terms of basic spare parts for aircraft and munitions.

> With the inclusion of spares supportability for sorties flown, the modified model is capable of providing insight to the best allocation of additional budget dollars for procurement of an aircraft, spares and munitions to enhance the capability to destroy targets given appropriate limitations on the resources which are of concern. [3:A-1]

Answering the conceptual questions did not solve the problem of efficiency for TAM. Its enormous size requires substantial CPU-time in the solution phase.

Jackson investigated advanced LP techniques which reduce TAM's CPU-time. He also pointed out that there were redundant constraints and aggregating them based on the requirements of the analysis would save a great deal of CPU-time [3].

In its complete form, TAM has the ability to update the constraints during the conflict. For instance, given that on the first day of a conflict, some of the targets were destroyed, on the second day TAM's construction takes care of restoring the destroyed targets. Similarly, given that there are 90 aircraft available to fly on the first day, it updates the number of aircraft avaiable for the next day by accounting for the aircraft lost on the first day.

Capt Skip Langbehn of JSG approaches TAM by eliminating the implicit updating constraints. Instead, he employs three basic constraints associated with the total available aircraft, the total available munitions, and the total available targets. The construction of his model assumes that the new munitions are already in the inventory. If the solution has positive values for the new munitions, this means there is a need to buy new munitions. Capt Langbehn runs the model for one day, and he begins the next run with the initial solution; hence, he updates the data for each run for each day of conflict. As an example, the scenario starts with 60 targets; at the end of the day the model shows destruction of 30 targets. By accounting for the restored targets—it is assumed that the enemy is rebuilding a percentage of the destroyed targets—he starts with 40 available targets on the next day assuming that the enemy rebuilt 10 of the previousely destroyed targets. This approach does not affect the number of variables, but decreases the number of constraints and thus can reduce CPU-time to solve the model [4].

2.4 Applicability of the Research

The first method presented in Chapter III deals with algebraic forms for individual and combined costs. The second method employed is a Goal Programming approach. Goal programming is an extension of linear or nonlinear programming, whose formulation allows to include multiple goals or objectives. Goal programming enhances the flexibility of linear programming formulation by allowing the inclusion of conflicting goals while still providing the decision maker with an optimal level of achievements for the high priority goals [6:249]. A case study by Schneiderjans and Markland on estimating start-up resource utilization in a newly formed company is worth examining. Sceniderjans and Markland use goal programming combined with input output analysis to solve a multistage, multiproduct production planning problem. The solutions obtained from using the goal programming models provide the finished product production levels for each quarter of the start-up year. Therefore, Sceniderjans and Markland identify excessive inventory levels and future inventory shortages in materials and supplies for planning the start-up year's production operations. Their modeling process is executed on a quarter-by-quarter basis over a one-year time horizon because the production line differs each quarter [9:101-109].

After analyzing this particular case study, a similar multistage, multiproduct approach for a situation where sorties—combinations of aircraft, munition, target, time, distance band and weather— are to be flown to achieve more than one objective suggests modification of Mixmaster's current construction with Goal Programming. In formulating the goal program, an equally weighted linear programming approach is pursued [6:249-282].

Also, work done by Sivazlian has inspired the research in terms of focusing on the sortie modeling concept. Sivazlian developed a methodology for mathematically modeling an aircraft sortie regarding its stochastic features. In this sortie modeling method, Sivazlian starts with two major assumptions:

1. Once an aircraft reaches enemy territory, the time to search, find and acquire a target has a negative exponential distribution with parameter μ.
2. The occurrence of enemy threats against the aircraft is a Poisson process with parameter λ.

Then, Sivazlian develops the Lanchester-type equations to determine various measures of effectiveness [10:127-137]. In the probabilistic approach to determine the required cost coefficients for this study, the same assumptions and Lanchester-type equations are employed.

2.5 Conclusion

LP with its four basic assumptions—divisibility, additivity, proportionality and certainty—has an objective to optimize, subject to a set of constraints. In a mathematical sense, the constraints form a feasible region, and the objective is achieved at one of the corner points of that feasible region. Although LP has a wide variety of applications, only an informative discussion of TAM and some specific suggestions were highlighted as within the scope of this particular research. Therefore, a valuable insight is obtained by presenting the applications implemented by the researchers cited above. Also, the case study of Schneiderjans and Markland employing the Goal Programming approach encourages research in this direction. The sortie effectiveness model that Sivazlian developed gives a promising direc ion for modifications as well. The research and analysis effort is devoted to modifying the aggregated version of TAM discussed by Capt Langbehn. The very same assumptions, structural reasoning—in terms of equations—and the theater level conflict that Might discusses are the foundations of this research.

III. METHODOLOGY

3.1 Introduction

The JSG Mixmaster model makes the following assumptions:

- The war will be fought by "blue" aircraft against "red" ground targets. Air-to-air combat is not modeled
- "Red" does not attack "blue" except to defend their targets. This will cause attrition which the model incorporates
- The basic model is linear

Given these assumptions, the first topic of this chapter discusses the design of the investigation and introduces the software which are used in the research; the current configuration of the model; and the model configuration with the appropriate constraints in terms of air operations. In other words, the baseline model with which the study implements all modifications is introduced. The second topic introduces the mission plan concept that is used. The third topic describes the data collection procedure and data base of the research. The next topic presents the model modifications under the following areas:

1. Simple cost employment in the objective function
    - Including the cost of aircraft
    - Including the cost of munitions
    - Including the cost of sortie generation
    - Including the costs of both aircraft and munitions
    - Including the costs of both aircraft and sortie generation
    - Including the costs of both munitions and sortie generation

- **Including the costs of aircraft, munitions and sortie generation**

2. **Goal programming approach**
3. **Probabilistic approach to determine the objective function cost coefficients**
4. **Goal programming version of the probabilistic approach**

The last topic discusses two case studies implemented using the suggested modifications. The topics and the suggested modifications are presented with details in subsequent sections.

3.2 Design of the Investigation

3.2.1 Time Length of the Campaign. In JSG analyses, the actual time duration issue is handled as the campaign requirements dictate. With the given scenario, if all the targets are destroyed and the objective function level achieved is satisfactory, the campaign is assumed to be over. However, this investigation employs only the first time interval of the given campaign scenario. Whether the campaign is or is not successful, all implementations of the modifications that are suggested in the subsequent sections are tested in only one time interval. The procurement decision issue is discussed based on results of the first time interval. In actuality, the campaign may last longer than expected. The JSG analysts then do successive runs with updated resource values to determine the need for new resources.

3.2.2 Software. This study used the General Algebraic Modeling System (GAMS) to pursue the analyses. GAMS was chosen because the study requires conciseness of expression and generality and portability of the solution methods. Also, GAMS enables the tracking of many of the programming details. In addition, the commercial version of GAMS, consisting of ZOOM and MINOS solvers, was sufficient for the task of investigating modifications within the scope of the study [2] [8].

3.2.3 Current Construction of Mixmaster. Mixmaster is a fairly aggregated version of TAM. The model has three basic constraints. These constraints are the aircraft availability constraint; the weapon availability constraint; and the target availability constraint. The objective function is to maximize *Target Value Destroyed.* The decision variable X_{amkdtw} represents *Number of Sorties Flown* by each combination of aircraft a loaded with weapon m against target k in distance band d at time t and subject to weather condition w:

$$\max TVD = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$Lowerbound_{amkdtw} \leq X_{amkdtw} \leq Upperbound_{amkdtw}$$

where:

- $TGTVAL_{kd}$ represents the target value related to the distance band
- $EXKIL_{amkdtw}$ is the expected number of targets destroyed
- TS_{amkdtw} is defined as the total sorties that can be flown by aircraft a loaded with munition m against target k in distance band d at time t and in weather condition w, calculated as:

$$TS_{amkdtw} = \frac{1 - (1 - ATTRIT_{amkdtw})^{SR_a NDAYS_t}}{ATTRIT_{amkdtw}}$$

- SR_a is the sortie rate for an aircraft

- $DAYS_t$ is the duration of the mission
- $ATTRIT_{amkdtw}$ is the attrition to whichever combination of aircraft a loaded with munition m against target k in distance band d at time t and in weather condition w is subjected
- $TOAC_a$ is the total available aircraft at time t
- $TOWPN_m$ is the total available weapons
- $WPNLD_{am}$ is the number of weapons that an aircraft can carry
- $TOTGT_{kd}$ is defined as the total number of targets in each distance band

Even though Mixmaster does not use another variable identifying which aircraft or weapons should be procured for the success of the mission, it does enable JSG to answer procurement questions. To do that, the JSG analysts relax the aircraft and munitions constraints by assuming that the aircraft and munitions constraints are not binding. After running the model, the JSG analysts compare the levels of use of those relaxed constraints with TAC's inventory level. The difference between TAC's inventory level and the levels of use of the aircraft shows the aircraft or weapons that need to be procured. For instance, if the solution shows that 100 F-16 aircraft are to be used to maximize *Target Value Destroyed* and there are 90 F-16 aircraft in TAC's inventory, then this solution implies that TAC needs to buy 10 additional F-16 aircraft.

3.2.4 Consistency with Air Operations. Decision makers expect Mixmaster to give operationally sound and consistent answers to procurement questions. So far Mixmaster has provided quick answers; nevertheless, questions about its reliability arise because of the inconsistency between the results and operational needs. The model can allocate any type of aircraft munition combination to any target in enemy territory. This behavior of the model may not be consistent with the air operation plans.

The model must include an additional constraint which parallels the planned operation. The operation planners have to describe their mission and help define the additional constraint for Mixmaster. The requirements of a particular mission plan determine the additional constraint. Regarding the characteristics of an aircraft type, the planner determines an upper bound for the number of aircraft to be assigned in a specific part of enemy territory. The aircraft target correspondence is also a major targeting principle to consider. Planners disagree with the results when an A-10 equivalent type of aircraft is allocated to an enemy air base target deep in enemy territory.

3.2.5 Scenario. For the research, the scope is limited to maintain a manageable number of variables. As such, the scenario employs four types of aircraft, five types of weapons, four types of targets, three distance bands, one time period and two weather conditions. The weapon types are assumed to be notional types varying in efficiency of destruction.

A scenario is generated assuming similar aircraft capabilities as follows: aircraft type 1 resembles the F-15; aircraft type 2 matches the F-16; aircraft type 3 matches the F-111; and aircraft type 4 resembles the A-10 aircraft. Target type 1 represents enemy air bases and radar units; target type 2 represents enemy SAM batteries; target type 3 represents supply depots and logistics units and target type 4 represents enemy tank units. The weapons are also classified for both aircraft and target types. Aircraft type 1 can use all weapon types and can be allocated to all target types. Aircraft type 2 can use all weapon types and can be allocated to all target types. Similarly, aircraft type 3 can use all weapon types and can be allocated to all target types. Also aircraft type 4 can use all weapon types and it can be allocated to all target types.

Furthermore, the study assumed that the mission planners determined an allocation plan. This plan is represented by the *leading constraint* of the baseline model and is included in Mixmaster's construction to implement the modifications. The leading constraint co ıcept, as explained in the following section, leads the model's allocation process by limiting the sortie amounts that are to be flown by each aircraft against a particular target type in a particular distance band.

3.2.6 Consistent Configuration. Given the main features of the scenario, the model has to satisfy the following modified constraints:

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \qquad \text{for each } a, k, d$$

where ATD_{akd} is the predetermined sortie percentage of aircraft type a against target type k in distance band d and $TSORTAC_a$ is the total number of sorties that an aircraft type can fly. The total sorties that an aircraft can fly is computed by:

$$TSORTAC_a = \sum_m \sum_k \sum_d \sum_t \sum_w TS_{amkdtw}$$

for each aircraft type a. The desired consistency is achieved by leading the model parallel to a predetermined mission plan. The planners targeting principle limits the number of aircraft allocated to a specific type of target. For example, at most 70% of the total sorties that the A-10 type of aircraft can fly are designated to enemy tanks in the first distance band of the enemy territory. Hence, the upper value of the A-10 mission against tanks in distance band 1 becomes $ATD_{A-10,TANKS,DIST-1}$ $TSORTAC_{A-10} = 0.70 \cdot TSORTAC_{A-10}$. Accordingly, remaining allocations must satisfy the limits on the number of aircraft assigned to a particular target in a particular distance band as well. This percentage should be provided by the mission planner.

3.3 Mission Plan

The mission plan should be determined in terms of the maximum percentage of sorties that may be flown by each aircraft for every target type in a particular distance band. The percentages to be allocated to target types represent the missions such as airbase attack, SAM suppression, logistic suppression—attacking supply depots, railroads, silos—and close air support (CAS). For this study, the mission percentages were determined by the researcher. This plan represented by the percentages is not inviolate. The evaluation of the marginal values obtained from the results may change the percentages as long as their contributions to the objective are significant. This characteristic will help determine whether the mission is planned successfully or not.

3.4 Data Collection

3.4.1 Expected Kill and Attrition Data. Mixmaster's data base is supported by two other models. The first one is the Joint Munitions Effectiveness Model (JMEM). The second one is SABSEL. This model evolved from SABR and SELECTOR, computer models which are no longer supported. The user provides aircraft type, flight profile, target type and weapons load; then JMEM produces expected kills for that particular combination. Similarly, SABSEL produces the attrition rate for each of the combinations given the following inputs: aircraft type, munitions, flight profile, distance, threat in the terminal area, threat on ingress, threat on egress, and delivery profile.

The data base used in the research is independent of the JMEM and SABSEL models. Instead, the research generates notional data for expected kills and attrition by using a flexible random number generator. The random number generator written by Capt Langbehn of JSG is preferred because the program puts the generated data in a format similar to that of a GAMS input file [2].

3.5 Simple Cost Employment in the Objective Function

3.5.1 Including the Cost of Aircraft. This particular approach simply averages the current objective function—*Target Value Destroyed*—in terms of the replacement cost of an aircraft and the attrition rate. Hence, the measure of effectiveness for Mixmaster becomes *Target Value Destroyed per Dollar Risked for an Aircraft*:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{ACCOST_a ATTRIT_{amkdtw}} \right]$$

where $ACCOST_a$ is the cost of aircraft a. In this case the objective function is sensitive only to the aircraft costs and the attrition rates.

3.5.2 Including the Cost of Munitions. Similarly, the costs of munitions are in a product form involving the number of munitions launched during the sortie. The objective function employs that product by modifying *Target Value Destroyed* as *Target Value Destroyed per Dollar Spent for Munitions*:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{COSTM_m WPNLD_{am}} \right]$$

where $COSTM_m$ is the cost of weapon m. In this case, the objective function is sensitive to the munition cost and the amount of munition to be used; the cost of the aircraft flown to launch that munition is disregarded.

3.5.3 Including the Cost of Sortie Generation. The cost of sortie generation is more complicated than the other costs. Actually, the cost of generating one sortie is the sum of the cost of operating an aircraft for the duration of a sortie, the cost associated with the probable loss of that aircraft, and the cost associated with munitions used for that particular sortie. However, JSG uses the constant costs generated for the Munitions Roadmap Working Group. JSG assumes that each sortie, regardless of the duration of the flight, costs nearly the same, and thus can be treated as a constant cost. Then, the cost of sortie generation becomes the divisor of

Target Value Destroyed and the meaning is changed to *Target Value Destroyed per Dollar Spent for a Generated Sortie*:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{SCOST_a} \right]$$

where $SCOST_a$ is the constant sortie cost for aircraft type a.

3.5.4 Including the Costs of both Aircraft and Munitions. This particular approach accounts for both the aircraft cost associated with attrition and the munition cost by combining them in the denominator of the objective function:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{k,d}}{ACCOST_a ATTRIT_{amkdtw} + COSTM_m WPNLD_{am}} \right]$$

and is interpreted as *Target Value Destroyed per Dollar Risked for an Aircraft and Dollar Spent for Munitions.*

3.5.5 Including the Costs of both Aircraft and Sortie Generation. The assumption which JSG makes about the sortie generation cost excludes the cost of attrited aircraft. But, with this particular combination of aircraft and sortie generation costs, the chance that an aircraft can be lost is included. Therefore, accounting for the cost of a sortie not only by flying the aircraft, but also by partially losing it makes more sense in combat circumstances; thus

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{ACCOST_a ATTRIT_{amkdtw} + SCOST_a} \right].$$

The costs are combined in the denominator of the objective function as *Target Value Destroyed per Risked Dollar for an Aircraft and Dollar spent for a Sortie.*

3.5.6 Including the Costs of both Munitions and Sortie Generation. This approach omits the aircraft cost. The imbedded assumption is that the aircraft survives:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{COSTM_m WPNLD_{am} + SCOST_a} \right].$$

The costs of munitions and sortie generation serve as the denominator modifying the objective function as *Target Value Destroyed per Dollar Spent for a Sortie and Munitions.*

3.5.7 Including the Costs of Aircraft, Munitions and Sortie Generation. Intuitively, among the simple cost employment approaches, including all costs is the approach that makes most sense because this method accounts for all expenses associated with one mission. The summed cost is the denominator that translates as the total cost of one mission flown by aircraft type a loaded with weapon m against target k in distance band d at time t and in weather condition w.

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{MISCOST_{amkdtw}} \right]$$

where the mission cost is

$$MISCOST_{amkdtw} = ACCOST_a ATRITT_{amkdtw} + COSTM_m WPNLD_{am} + SCOST_a.$$

Hence, the objective function becomes *Target Value Destroyed per Dollar Spent for Missions.*

3.6 Goal Programming Approach

This particular approach allows incorporation and consideration of multiple objectives or goals within an LP framework. Mixmaster is modified consistently by the inclusion of more than one objective. While the initial decision variables remain the same, this approach requires the definition of additional variables which represent the deviations from the objectives. The decision maker's preferences specify the priorities of the newly defined objectives or *goals*. The research suggests the following goals:

1. Achieve a certain level of target value destroyed per dollar spent for a mission
2. Achieve at least 80% sortie success
3. Avoid overutilization of available aircraft
4. Avoid overutilization of available munitions
5. Kill as many targets as possible

Having specified the goals, the objective function is to minimize the sum of the deviations from the goals. The decision variables that are employed in the objective function are the negative deviate from the constraint of *Target Value Destroyed per Dollar*; the positive deviate from the aircraft constraint; the positive deviate from the sortie success constraint; the positive deviate from the munition constraint and the negative deviate from the target constraint.

In the *Target Value Destroyed per Dollar* goal, only the negative deviate is employed because there cannot be an overachivement for this goal given that there is a limited number of targets. The objective function seeks to minimize the negative deviate. Achieving at least 80% sortie success is equivalent to at most 20% of the sorties failing. So the objective function seeks to minimize the positive deviate from the 20% failure goal. For the aircraft availability goal, the objective seeks to minimize the positive deviate and avoid the overutilization so that the model should use the current inventory. The weapon availability goal is similar to the aircraft availability goal. In the target goal, only the negative deviate is employed because it is not possible to destroy more than the existing number of targets. The objective function seeks to minimize the underachievement of this goal so as to kill as many targets as possible. Consequently, the solver seeks the values of deviates which minimize their sum by allocating the sorties flown by aircraft a loaded with munition m against target k in distance band d at time t in weather condition w—and satisfying the associated constraints simultaneously.

The suggested goal programming construction of Mixmaster is:

$$\min Z = d_1^- + \sum_a dloss_a^+ + \sum_a d_a^+ + \sum_m d_m^+ + \sum_{kd} d_{kd}^-$$

subject to:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_m \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{MISCOST_{amkdtw}}\right] + d_1^- = TVD/\$$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w X_{amkdtw} ATTRIT_{amkdtw} - dloss_a^+ + dloss_a^- = 0.20 \cdot TOAC_a \quad \text{for each } a$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] - d_a^+ + d_a^- = TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} - d_m^+ + d_m^- = TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} + d_{kd}^- = TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$\sum_a d_a^+ ACCOST_a + \sum_m d_m^+ COSTM_m \leq BUDGET$$

$$X_{amkdtw}, d_1^-, dloss_a^+, dloss_a^-, d_a^+, d_a^-, d_m^+, d_m^-, d_{kd}^- \geq 0$$

where d_1^- is the negative deviate from the *Target Value Destroyed Per Dollar* goal; $dloss_a^+$ and $dloss_a^-$ are the positive and negative deviates from the *sortie success* goal; d_a^+ and d_a^- are the positive and negative deviates from the aircraft goal; d_m^+ and d_m^- are the positive and negative deviates from the *waepon* goal; and d_{kd}^- are the negative deviates from the *target* goal. Consequently, the deviates d_a^+ and d_m^+ can also be defined as the procurement variables.

The positive deviates show overachievement and the negative deviates show underachievement of goals. If, for instance, the positive deviation from the munition constraint appears in the solution with a positive value, it means there are insufficient munitions in the inventory given that all the other goals are satisfied. Since the current inventory would not have the required amount of munitions, it would indicate the need to procure new munitions.

As a paramount feature, the goal programming model employs the budget constraint in the configuration. The previously suggested modifications can not employ the budget figure directly since the procurement decision depends on some additional computations. However, the goal programming model answers the procurement needs by employing the procurement variables. As such, the procurement variables and their inherent costs can be employed in a budget constraint. Hence, the procurement cost and the budget restriction can be accomodated in the goal programming approach alleviating the need for further computations.

The priority issue of the goals is considered flexible depending on the decision maker's preference. The research evaluates the goals by giving them the same priority. However, reasonably, different points of views may assess different ranks for the goals. The higher the assessed importance, the higher the priority will be.

3.7 Probabilistic Approach to Determine the Cost Coefficients

3.7.1 Modeling the Sortie. The simplicity of Mixmaster comes from its linear construction. However, the dynamics of combat in real conflicts are mostly nonlinear. The time duration for a sortie, the number of weapons to be launched, and the threat of the enemy are the nonlinear dynamics that Mixmaster treats as a set of linear constraints. Consequently, there exists a trade-off between simplicity and so-called *resolution*, which is the ability to represent real combat situations. In the real-life cost evaluation, time is an important driving factor. The cost of a sortie is directly proportional to the duration of a sortie. The longer the aircraft is in enemy territory, the more it is subjected to attrition. Therefore, a generic approach is developed to capture the time dependent drivers in Mixmaster such as the sortie cost, expected kills, and attrition. The method has the same logical construction as that of Sivazlian [10:127-137]; but, some modifications were necessary in order to apply a stochastic approach to a linear construction.

Sivazlian's method determines various measures of effectiveness for only one aircraft. However, Mixmaster executes the scenario with hundreds of aircraft. In Mixmaster the aircraft may select any one of the targets depending on the contribution of that target to the objective function. In practice, for each aircraft loaded with munitions, there is a possibility of attacking every target. Nonetheless, LP picks the combination of aircraft, munitions, target, distance band, time and weather with the greatest contribution.

3.7.2 Standard Operation Procedures and Assumptions. Standard operation procedures must be explained for the air operation of this study before discussing the assumptions. To acquire and attack a target, a fighter aircraft loaded with either classical or smart weapons flies at 400 nautical miles per hour while in enemy territory. After attacking the target, the aircraft should egress as soon as possible. The aircraft used for close air support missions against enemy tanks flies at 300 nautical miles per hour. The time to acquire a target depends on the speed at which the aircraft flies and on the distance between the target and the point where the aircraft entered enemy territory. Given the standard procedures, the method assumes the following:

1. The time in which an aircraft acquires a target has a negative exponential distribution with parameter μ.
2. The occurrence of the enemy threat is a Poisson process with parameter λ.

The first assumption implies that once a type of aircraft enters enemy territory at time t, there are one or more targets to acquire. Since the targets are located at different distances, the time to acquire one can be different from the time to acquire another. These independent time durations are assumed to be random variables coming from a negative exponential distribution with parameter μ. Thus the average time to acquire a target is $1/\mu$. The attack time is assumed to be included in the target acquisition time. The probability that the target is acquired and hence attacked in the time interval $(t, t + dt)$ is μdt. The probability that a target is killed once attacked is Pk.

The second assumption implies that once a type of aircraft enters enemy territory it is subjected to an enemy threat with a frequency of λ. The enemy threat is assumed to be independent of the targets. Therefore, the probability that an aircraft encounters a threat in time interval $(t, t + dt)$ is λdt. Also, the probability that the aircraft is killed once it encounters an enemy threat is Pa.

The duration of the sortie starts upon entering enemy territory and ends upon leaving enemy territory. Moreover, an aircraft is assumed to be combat ready once it leaves the base.

3.7.3 The Model. Given the assumptions, the sortie can be modeled as a two dimensional Markov chain which has four states in terms of its parameters. $P(i, j, t)$ where $i = 0$ for the target that is killed; $i = 1$ for the target that is not killed and $j = 0$ for the aircraft that is killed; $j = 1$ for the aircraft that is not killed. The initial conditions for this model when $t = 0$, are as follow:

$$P(1,1,0) = \Pi_0; \quad P(1,0,0) = 1 - \Pi_0$$
$$P(0,1,0) = 0; \quad P(0,0,0) = 0$$

where Π_0 is a value of the initial probability; $P(1,1,0)$ is the probability that the aircraft and the target survive; $P(1,0,0)$ is the probability that the target survives, but the aircraft is killed; $P(0,1,0)$ is the probability that the target is killed, but the aircraft survives; and $P(0,0,0)$ is the probability that both the target and aircraft are killed at $t = 0$ and t represents the duration of the sortie

Since the aircraft is assumed to be combat ready once it leaves its base, the initial condition becomes $P(1,1,0) = 1$. To compute these probabilities related to time t, the Lanchester-type equations are set up and solved. For example, $P(1,1,t+dt)$ can be obtained by solving

$$\begin{aligned}
P(1,1,t+dt) &= P(1,1,t)(1-\lambda dt)(1-\mu dt) \\
&+ P(1,1,t)(1-\lambda dt)\mu dt(1-Pk) \\
&+ P(1,1,t)\lambda dt(1-Pa)(1-\mu dt) \\
&+ P(1,1,t)\lambda dt(1-Pa)\mu dt(1-Pk) \\
&+ o(dt)
\end{aligned}$$

where $o(dt)$ represents higher order probabilities that can be omitted.

The results obtained by solving the rest of the equations are:

$$\begin{aligned}
P(1,1,t) &= e^{-(\alpha+\beta)t} \\
P(1,0,t) &= \frac{\alpha}{\alpha+\beta}(1-e^{-(\alpha+\beta)t}) \\
P(0,1,t) &= e^{-\alpha t}(1-e^{-\beta t}) \\
P(0,0,t) &= 1-e^{-\alpha t}-\frac{\alpha}{\alpha+\beta}(1-e^{-(\alpha+\beta)t})
\end{aligned}$$

where $\alpha = \lambda Pa$ and $\beta = \mu Pk$. In the subsequent sections, the probabilities related to a sortie are computed using these results [10:129-131].

3.7.4 Probability of Sortie Success. This study assumes that if the aircraft is not killed and the target is killed, then the sortie is a successful sortie. The probability of sortie succes is

$$P(0,1,t) = e^{-\alpha t}(1 - e^{-\beta t}).$$

3.7.5 Probability of Sortie Failure. This probability can be computed in different ways. The loss of an aircraft can be considered as a failure of the sortie; however, this study considers this failure by the case where the target is not killed. Thus, the probability that the target is not killed with or without an aircraft loss implies the probability of sortie failure and it is computed as

$$P(1,1,t) + P(1,0,t) = \frac{\alpha}{\alpha+\beta} + \frac{\beta}{\alpha+\beta} e^{-(\alpha+\beta)t}.$$

3.7.6 Probability that the aircraft is killed. This can be computed as the sum of the probabilities whose aircraft parameters are zero:

$$PACKIL = P(1,0,t) + P(0,0,t) = 1 - e^{-\alpha t}.$$

3.7.7 Probability That the Target Is Killed. This probability is the sum of the probabilities whose target parameters are zero:

$$PTGTKIL = P(0,1,t) + P(0,0,t) = \frac{\beta}{\alpha+\beta}[1 - e^{-(\alpha+\beta)t}].$$

3.7.8 Expected Number of Targets Killed. Sivazlian defines $N(t)$ as the number of targets killed at time t. Then the expected number of targets killed is:

$$\begin{aligned} ETGTKIL &= \sum_{n=0}^{n=1} nP[N(t) = n] \\ &= P(0,1,t) + P(0,0,t) \\ &= \frac{\beta}{\alpha+\beta}[1 - e^{-(\alpha+\beta)t}]. \end{aligned}$$

3.7.9 Expected Number of Attacks on the Targets. The construction of Mixmaster assumes that once the aircraft attacks a target, it releases all the weapons that it carries in a single pass. For this approach, this study assumes that the target is attacked only once corresponding to one pass over the target. Therefore the probability that a target is attacked in the interval $(0, t)$ is equivalent to the expected number of attacks.

Consequently, the probability that the target is attacked on or before time t is the product of the probability that the aircraft is not killed on or before time x where $x \leq t$, and the probability that the target is acquired in the interval $(x, x + dx)$ [10:132], given by

$$EXATTACK = \int_0^t e^{-\alpha x} e^{-\mu x} \mu dx = \frac{\mu}{\alpha + \mu}[1 - e^{-(\alpha+\mu)t}].$$

3.7.10 Expected Duration of the Sortie. The expected duration of the sortie is limited by the expected time at which the aircraft is killed (i.e., $1/\alpha$). The probability that the aircraft is killed in $(x, x + dx)$ is

$$\lambda \cdot p_a \cdot e^{-\alpha x} dx.$$

Therefore, the duration of the sortie $D(t)$ becomes:

$$D(t) = \begin{cases} x, & 0 \leq x < t \\ t, & t \leq x < \infty. \end{cases}$$

Given $D(t)$, the expected duration of the sortie is

$$\begin{aligned} E[D(t)] &= \int_0^t x\alpha e^{-\alpha x} dx + \int_t^\infty t\alpha e^{-\alpha x} dx \\ &= -te^{-\alpha t} - \frac{1}{\alpha}(e^{-\alpha t} - 1) + te^{-\alpha t} \\ &= \frac{1}{\alpha}(1 - e^{-\alpha t}) \end{aligned}$$

where $(1 - e^{-\alpha t})$ is the probability that the aircraft is killed once it encounters an enemy threat [10:132].

3.7.11 Expected Cost of the Sortie. To compute the expected cost of the sortie, Sivazlian defines four different costs associated with one sortie. They are:

1. The fixed cost realized every time an aircraft gets ready for flight. This cost is similar to the previously defined sortie cost, $SCOST_a$.
2. The expected cost associated with the duration of the sortie. Reasonably, the longer the aircraft is airborne, the more fuel it consumes.
3. The expected cost of an aircraft. This cost is similar to the previously defined aircraft cost, $ACCOST_a ATTRIT_{amkdtw}$; however, in the probabilistic approach, it does not have the time dimension.
4. The expected cost of munitions used. This cost is equal to the cost of munitions used per attack times the expected number of attacks. It is somewhat similar to the previously defined cost of ammunitions, $COSTM_m WPNLD_{am}$.

The sum of these expected costs is the expected cost of the sortie. The fixed cost of the sortie is described with $FIXCOST$. The expected cost associated with the duration of the sortie is $SC_0 \frac{1}{\alpha}(1 - e^{-\alpha t})$ where SC_0 is the cost per unit length of time of the sortie [10:134]. The expected cost of the probable loss of an aircraft is $ACCOST_a(1 - e^{-\alpha t})$ where again $(1 - e^{-\alpha t})$ is the probability that the aircraft is killed once it encounters an enemy threat [10:134]. The expected cost of munitions used is

$$COSTM_m WPNLD_{am} \frac{\mu}{\alpha + \mu}[1 - e^{-(\alpha+\mu)t}]$$

which translates as the expected number of attacks or passes times the weapon load times the weapon cost. This particular cost formula was modified to incorporate the weapon load. Therefore, it differs from Sivazlian's equation. Consequently, the expected cost of the sortie $E[S(t)]$ with the addition of a constant cost K is the sum of all the costs [10:134]:

$$\begin{aligned} ESCOST &= E[S(T)] \\ &= K + SC_0\frac{1}{\alpha}(1 - e^{-\alpha t}) + ACCOST_a(1 - e^{-\alpha t}) \\ &+ COSTM_m WPNLD_{am}\frac{\mu}{\alpha + \mu}[1 - e^{-(\alpha+\mu)t}]. \end{aligned}$$

3.7.12 Parameter Modification. The time spent in enemy territory is easy to incorporate into Mixmaster. The study assumes that the time duration t for an aircraft in enemy territory is related to the distance band value of that territory and the speed of the aircraft. For instance, the average time duration t_{akd} in each distance band is computed by

$$t_{akd} = \frac{DEPTH_{kd}}{ACSPEED_a}$$

where $DEPTH_{kd}$ is the average distance of a target type in enemy territory associated with a distance band and $ACSPEED_a$ is the average speed that the aircraft flies to attack a target in enemy territory. The computed values of time duration t_{akd} for each aircraft target distance-band combination are employed in the formulas to determine the probabilities, the expected number of targets, expected number of passes over targets and the expected costs associated with the sorties.

Furthermore, this method requires reasonable estimates of λ and μ for each type of aircraft. To accommodate Mixmaster, the study evaluates the dimension of the parameters λ and μ as follows: λ_{akd} and μ_{akd}. Hence, the parameters λ and μ are dependent on aircraft type, target type and the distance band. The parameter μ is computed by definition. As previously explained, $1/\mu$ is the average time it takes the aircraft to acquire a target; thus

$$\mu = \frac{1}{averagetime}.$$

The average time is computed as

$$t_{akd} = \frac{DEPTH_{kd}}{ACSPEED_a},$$

so the parameter μ is defined as

$$\mu_{akd} = \frac{1}{t_{akd}}.$$

For computational purposes, the parameter λ is computed by assuming that the existence of the enemy threat when the aircraft is airborne can be expressed in terms of the defense density. This approach computes the frequency of encountering an enemy threat as the percentages of the frequency of acquiring targets. Therefore, λ has the same dimensions as μ. The assumption is that these two processes are independent; however, to ease computations, an aircraft encounters the enemy threat at some percentage of the time that it acquires a target. These percentages are based on the defensive intensity—interdiction—of the enemy. Consequently, λ is computed as

$$\lambda_{akd} = INT_{kd} \cdot \mu_{akd}$$

where INT_{kd} is a percentage value that expresses the density of the enemy threat associated with target k in distance band d.

The parameters α and β are also computed using the new dimensions:

$$\alpha_{amkdw} = \lambda_{akd} Pa_{amkdw}$$

and

$$\beta_{amkdw} = \mu_{akd} Pk_{amkdw}$$

where Pa is the probability that the aircraft is killed once it encounters an enemy threat; and Pk is the probability that the target is killed once acquired.

The expected target kill values, $ETGTKIL_{amkdw}$ are determined from the formula for expected number of targets killed by using

$$ETGTKIL_{amkdw} = \frac{\beta_{amkdw}}{\alpha_{amkdw} + \beta_{amkdw}}[1 - e^{-(\alpha_{amkdw}+\beta_{amkdw})t_{akd}}].$$

The attrition values are determined from the formula for the probability that the aircraft is killed by using

$$ATTRITION_{amkdw} = PACKIL_{amkdw} = 1 - e^{-\alpha_{amkdw}t_{akd}}.$$

The total sorties that can be flown by an aircraft, TS_{amkdtw}, is computed as

$$TS_{amkdtw} = \frac{1 - (1 - PACKIL_{amkdw})^{SR_a NDAYS_t}}{PACKIL_{amkdw}}.$$

The expected number of attacks is computed as:

$$EXATTACK_{amkdw} = \frac{\mu_{akd}}{\alpha_{amkdw} + \mu_{akd}}[1 - e^{-(\alpha_{amkdw}+\mu_{akd})t_{akd}}].$$

Expected duration of the sortie is computed as:

$$EXDUR_{amkdw} = \frac{1}{\alpha_{amkdw}}(1 - e^{-\alpha_{amkdw}t_{akd}}).$$

Given all the parameters, the cost of a mission is defined by:

$$DURATIONCOST_{amkdw} = SCOST_a EXDUR_{amkdw}$$

$$WEAPONCOST_{amkdw} = COSTM_m WPNLD_{am} EXATTACK_{amkdw}$$

$$AIRCRAFTCOST_{amkdw} = ACCOST_a ATTRITION_{amkdw}$$

and with $FIXCOST$ the cost of the mission becomes:

$$\begin{aligned} TSCOST_{amkdw} &= FIXCOST + DURATIONCOST_{amkdw} \\ &+ WEAPONCOST_{amkdw} \\ &+ AIRCRAFTCOST_{amkdw}. \end{aligned}$$

3.7.13 The Modified Model. Using the previous expected costs, expected number of attacks and the inherent probabilities the modified Mixmaster model is:

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} ETGTKIL_{amkdw} TGTVAL_{kd}}{TSCOST_{amkdw}} \right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} EXATTACK_{amkdw} WPNLD_{a,m} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} ETGTKIL_{amkdw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0.$$

3.8 Goal Programming Version of the Probabilistic Approach

The modification of Mixmaster is done on the basis of dimensions. Having different dimensions did not affect the model's construction. Therefore, a similar approach can be implemented to convert the probabilistic modification into another

Goal Program. The basic concept of deviations from the goals applies to this model as well. The previously defined goals are used as is. However, the dimensions of some parameters had to be changed. The goal programming version of the model is:

$$\min Z = d_1^- + \sum_a dloss_a^+ + \sum_a d_a^+ + \sum_m d_m^+ + \sum_{kd} d_{kd}^-$$

subject to:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_m \left[\frac{X_{amkdtw} ETGTKIL_{amkdw} TGTVAL_{kd}}{TSCOST_{amkdw}}\right] + d_1^- = TVD/\$$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w X_{amkdtw} ATTRIT_{amkdw} - dloss_a^+ + dloss_a^- = 0.20 \cdot TOAC_a \quad \text{for each } a$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] - d_a^+ + d_a^- = TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} EXATTACK_{amkdw} WPNLD_{am} - d_m^+ + d_m^- = TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} ETGTKIL_{amkdw} + d_{kd}^- = TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$\sum_a d_a^+ ACCOST_a + \sum_m d_m^+ COSTM_m \leq BUDGET$$

$$X_{amkdtw}, d_1^-, dloss_a^+, dloss_a^-, d_a^+, d_a^-, d_m^+, d_m^-, d_{kd}^- \geq 0.$$

3.9 Case Studies

To gain more insight on the model, the reserach includes two case studies. These cases are run for the last simple cost employment which includes the costs of aircraft, munitions and sortie generation, the goal programming approach, the probabilistic approach and the goal programming version of the probabilistic approach that were explained in the previous sections.

3.9.1 Mission Plan of Case 1. Case 1 is generated by the researcher. It includes different mission percentages than the baseline mission plan. In case 1, aircraft types are allocated missionwise. That is, each aircraft is supposed to fly only a specific type of mission. Aircraft type 1 is dedicated to SAM suppression; aircraft type 2 will fly logistic suppression and CAS. Aircraft type 3 is dedicated to airbase attack. Finally, aircraft type 4 is planned to be used for CAS in the first distance band.

3.9.2 Mission Plan of Case 2. In case 2, the planning is done with respect to the distance bands. The second mission plan employs all types of aircraft for all the missions. However, each aircraft is supposed to fly only in a specific distance band. Aircraft type 1 is planned to fly all the missions in the second distance band. Aircraft type 2 is planned to fly all the missions except CAS in the first distance band. The plan dictates aircraft type 3 fly only in the third distance band. Finally, the plan dedicates aircraft type 4 to fly SAM suppression, logistic suppression and CAS in the first distance band.

3.10 Summary

The research presents four basic modifications: the simple cost employment methods, the goal programming model, the probabilistic method to determine cost coefficients, and the goal programming version of the probabilistic method. Prior to implementing the suggested modifications, the research examines the Mixmaster model and presents a discussion of the marginal values in terms of the procurement decision. Then, the first simple cost employment method is executed with Mixmaster. Following a discussion about the baseline model, all the modifications are implemented with the study's baseline model. In addition, two case studies are run to observe the behavior of the model with modifications. These case studies are used as two different mission plans with which the model will be led to investigate behavioral consistency. The results and all the numerical details are condensed and presented in the fourth and fifth chapters. The summary of the implementations is illustrated in Figure 3.1.

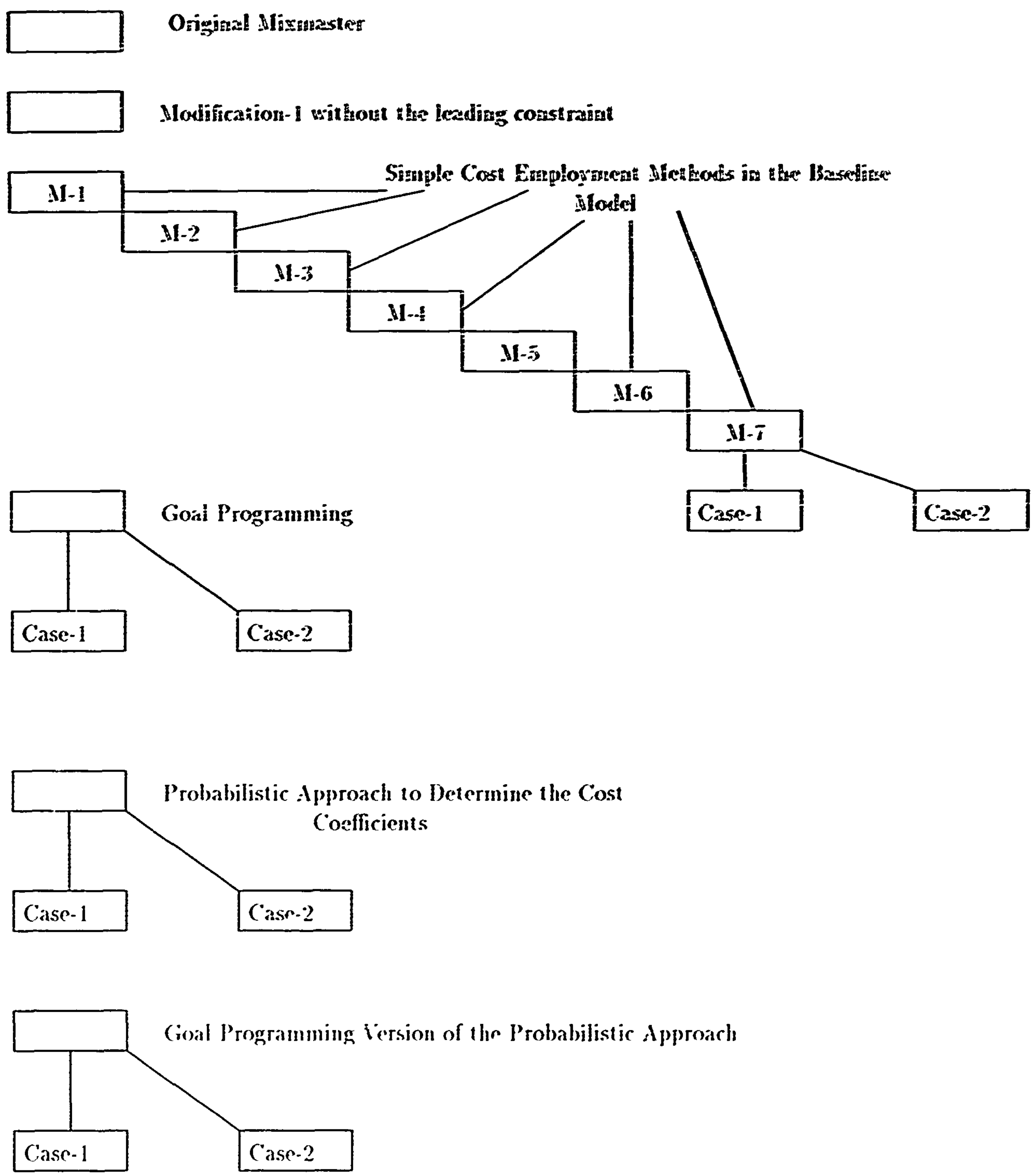

Figure 3.1. Flow Chart of Modifications

IV. SOLUTIONS AND RESULTS

4.1 Introduction

This chapter presents the results obtained from the implementations of the suggested modifications. All modifications were implemented with the same campaign scenario. The simple cost inclusion methods and the goal programming approach employed the very same expected kill and attrition values; whereas, the probabilistic approach to determine cost coefficients in the objective function employed nearly the same expected kill and attrition values.

The procurement decisions had been determined by running the model as if the aircraft and weapons resources were unlimited and then substracting the current aircraft and weapons inventories from the values produced by Mixmaster. If the difference was positive, then it implied that there was a need for new resources. If the difference was negative or zero, then it implied that the current aircraft and weapons inventories were adequate to supply the needs in the hypothetical campaign.

With the exception of the goal programming models, this research uses the same simple method to determine the need for procurement decisions in all the modifications. However, assuming that aircraft and weapons resources are abundant is not adequate to implement the model because Mixmaster produced different sortie allocations when the aircraft and weapons resource levels were large but varied from each other, and when the resource levels were the same large values. That is, Mixmaster's sortie allocations with the right-hand-side values of 1,600,000 and 2,200,000 for aircraft types 1 and 2, respectively; 30,000,000 and 40,000,000 for weapon types 1 and 2, respectively, turned out to be different than the sortie allocations with the right hand-side values of 5,000,000 aircraft for all types and 50,000,000 weapons for all types.

The difference in the allocation process was considered significant because if the right-hand-side values are to be taken as large numbers, they should be all equal to lead the GAMS software unbiasedly; hence, LP selects the variables entering and/or leaving the basis based on their contributions, which are, in fact, expected kill and target value coefficients.

Figure 4.1 presents a flow chart of the models discussed in this chapter. In the

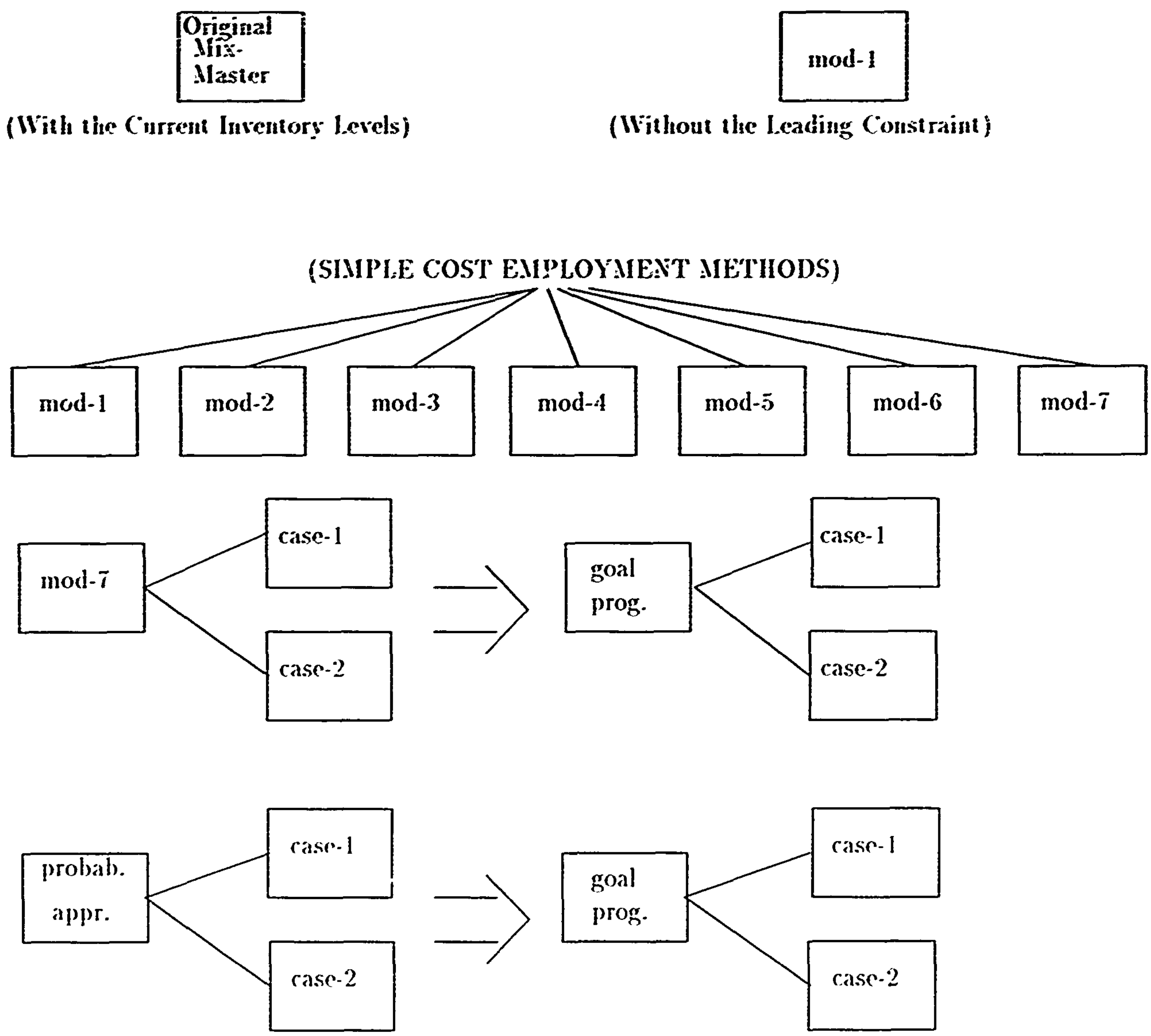

Figure 4.1. Flow Chart of Chapter IV

subsequent sections, the results of Mixmaster without any modification are presented so as to interpret the marginal values. Secondly, the study looks at the first cost modification in the original construction and in the baseline model—with and without the leading constraint—subject to the same very large right-hand-side values. The results were compared to see how the leading constraint in the baseline model leads the solution. Then the simple cost employment in the objective function, which was previously defined as the inclusion of the cost of aircraft, weapons, sorties and their combinations, were implemented in the baseline model.

As far as the research is concerned, the leading constraint, campaign scenario and mission plan are equivalent concepts. To determine a logical and common basis for discussion of the results, the same leading constraint right-hand-side values were used in the succesive modifications. Table 4.1 presents the mission percentages in terms of the maximum sortie number that is allowed for each type of target.

The same campaign scenario was used in the goal programming approach. This method turned out to be very sensitive, since the construction had the same priority for each goal. However, the goal programming results are consistent in aircraft usage with the last simple cost modification.

The probabilistic approach to determine the cost coefficients in the objective function produced very conservative results because the time issue was involved in the computations. The results were somewhat different from those of previous modifications, but the source of this difference was clearly the database. Moreover, this method has incorporated the time dimension as the independent variable of all the probabilities associated with costs. In addititon, to check the consistency a similar goal programming approach was applied to the probabilistic method.

Furthermore, the research presents a case where procurement variables indicate the need to procure. To develop this case, the right-hand-side values of the current aircraft availability constraint were diminished to a considerably lower level for each aircraft. The goal programming approach was run subject to considerably

Table 4.1. Mission Plan In Trems of the Maximum Sortie Percentages

ACFT-1	TGT-1 (Airbase Attack)	TGT-2 (Sam Suppression)	TGT-3 (Logistic Suppression)	TGT-4 (Close Air Support)
DIST-1	NONE	NONE	NONE	NONE
DIST-2	0.20	0.10	0.10	0.10
DIST-3	0.20	0.10	0.10	0.10

ACFT-2	TGT-1	TGT-2	TGT-3	TGT-4
DIST-1	0.15	0.05	0.10	0.10
DIST-2	0.10	0.05	0.05	0.10
DIST-3	0.15	0.05	0.05	0.05

ACFT-3	TGT-1	TGT-2	TGT-3	TGT-4
DIST-1	NONE	NONE	NONE	NONE
DIST-2	0.30	0.10	0.10	NONE
DIST-3	0.30	0.10	0.10	NONE

ACFT-4	TGT-1	TGT-2	TGT-3	TGT-4
DIST-1	NONE	NONE	0.30	0.70
DIST-2	NONE	NONE	NONE	NONE
DIST-3	NONE	NONE	NONE	NONE

low aircraft resources. As such, the goal programming approach generated positive values for the procurement variables indicating the procurement need for aircraft. The results of all these applications are given in the subsequent sections.

4.2 *Results of Original Mixmaster and Interpretation of Marginal Values*

Mixmaster was run in its original construction without the leading constraint. Also, the right-hand-side values were taken from the current aircraft and weapons inventory levels. The model used aircraft types 3 and 4 significantly (Figure 4.2). The summary of the numerical results is presented in Table 4.2.

Table 4.2. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	810.811	0	0	2.027
ACFT-2	678.863	0	0	0	0	6.874
ACFT-3	5321.137	0	0	483.676	0	13.921
ACFT-4	0	0	0	8080.318	0	64.852
WPN. USAGE	6000	0	810.811	8563.99	0	

The model did not allocate any of weapon types 2 and 5 to accomplish the missions. On the other hand the model exhausted weapon type 1 and used weapon type 4 at a significant level (Figure 4.3). The marginal values for the aircraft types turned out to be insignificant. The model loaded weapon type 1 onboard aircraft type 3 and weapon type 4 onboard aircraft type 4 at significant levels (Figure 4.4).

In terms of the procurement decision, the study also interpreted the marginal values. If any of the marginal values were significant, then it would imply that there was a need for procurement. For instance, the objective that was accomplished yielded a target value of 67493. If the the marginal value of the type 1 aircraft constraint had been 10 units, this would translate as the positive contribution of

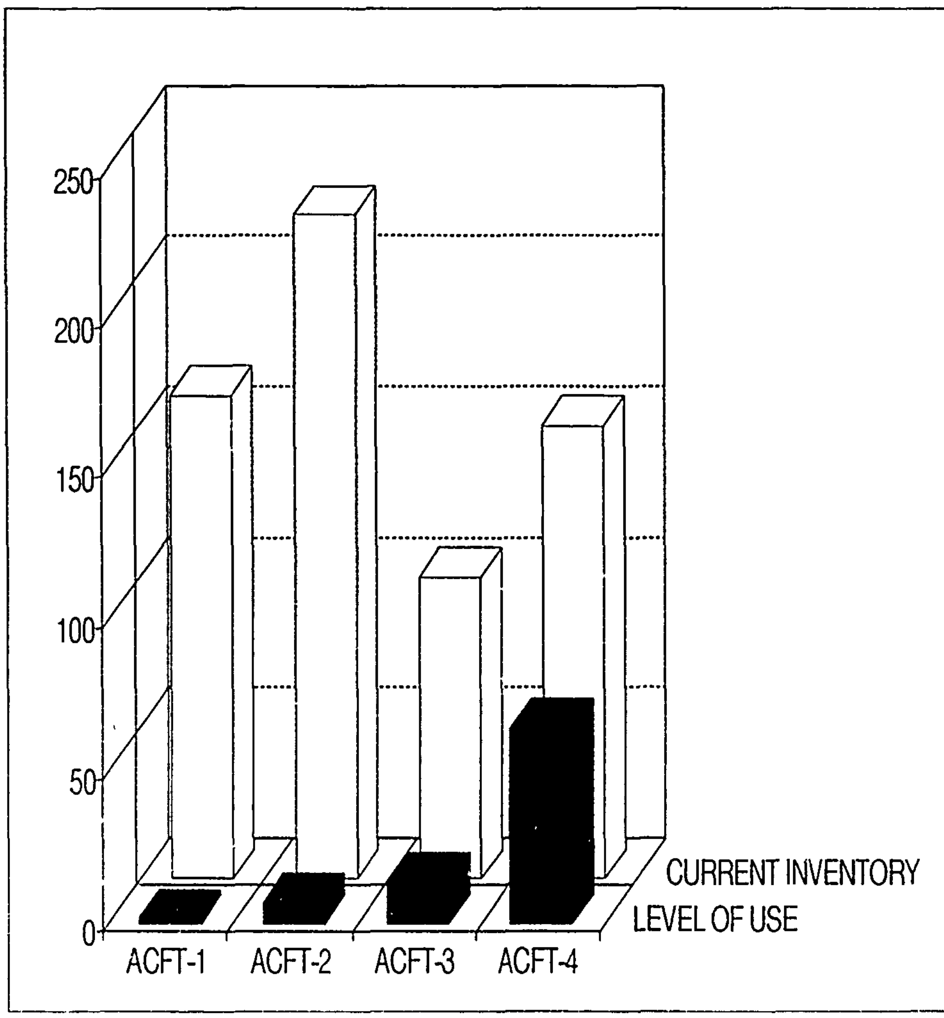

Figure 4.2. Number of Aircraft Per Sortie

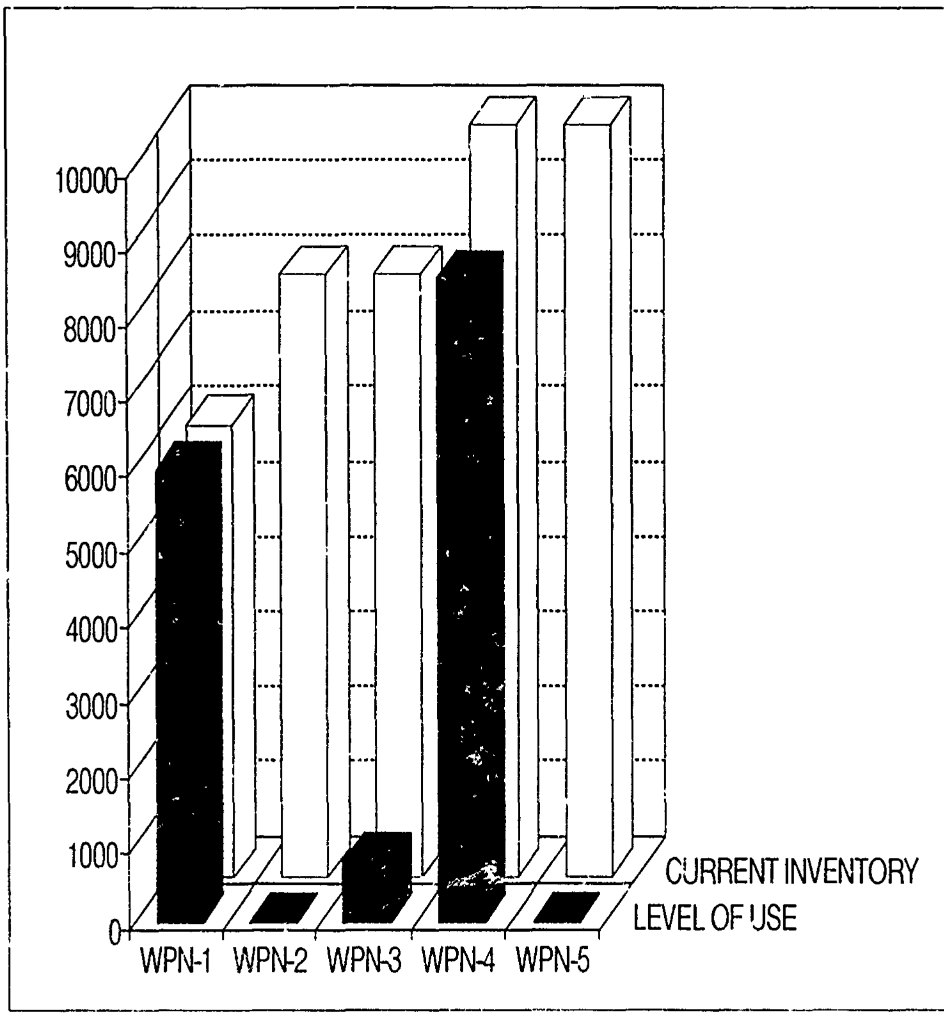

Figure 4.3. Number of Weapons Used

one more aircraft in the inventory. Hence, if the decision maker wanted to achieve a target value level of 68493; this would require 100 more of aircraft type 1.

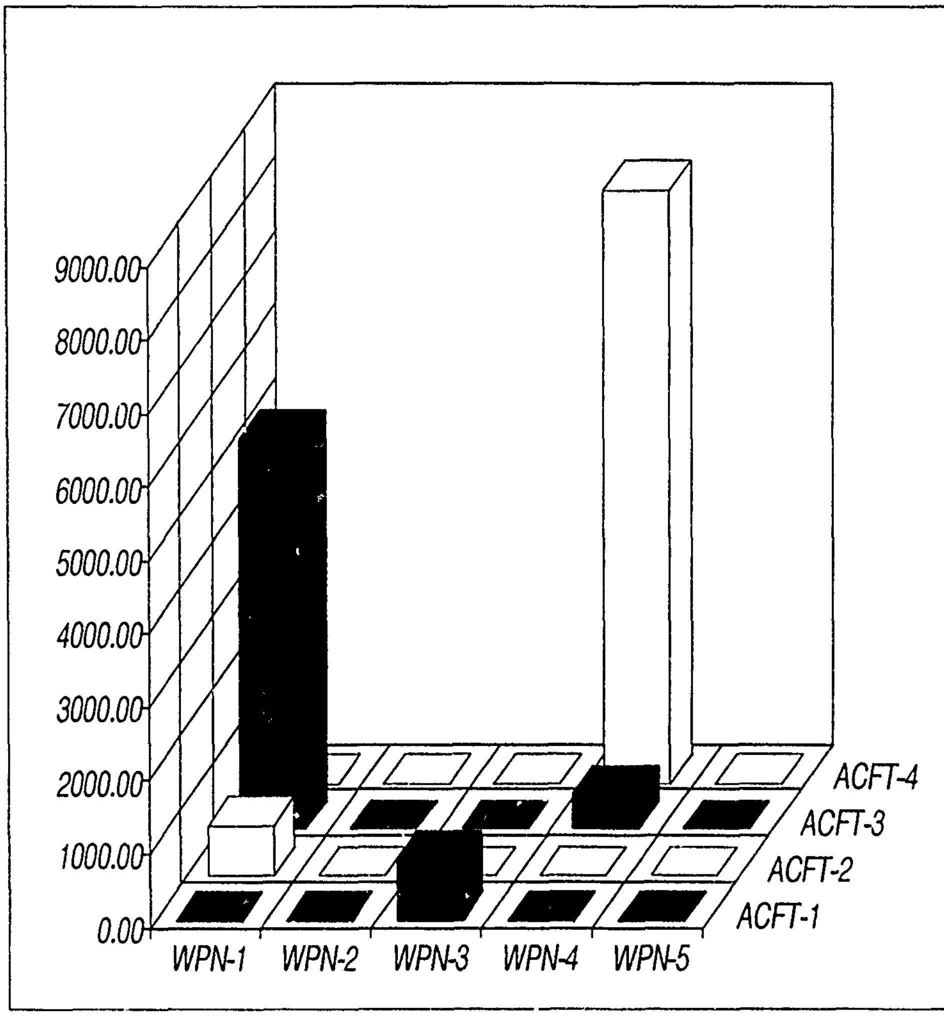

Figure 4.4. Use of Weapons by the Aircraft

4.3 Results of Simple Cost Employment Methods and Comparisons With and Without the Leading Constraint

4.3.1 Aircraft Cost Model Without the Leading Constraint. In this run, the study expected that the model would avoid using the aircraft types associated with substantial costs. Not surprisingly, the solution appeared to be relatively biased against the more costly aircraft types. Aircraft type 1 was not used at all because of its substantial cost (Figure 4.5). On the other hand the model used weapon types 4

Table 4.3. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	0	0	0
ACFT-2	249.221	1793.994	615.385	1004.785	3174.603	19.290
ACFT-3	2732.707	0	0	0	309.119	1.269
ACFT-4	0	0	0	4781.609	669.164	21.949
WPN. USAGE	2981.928	1793.994	615.385	5786.394	4152.886	

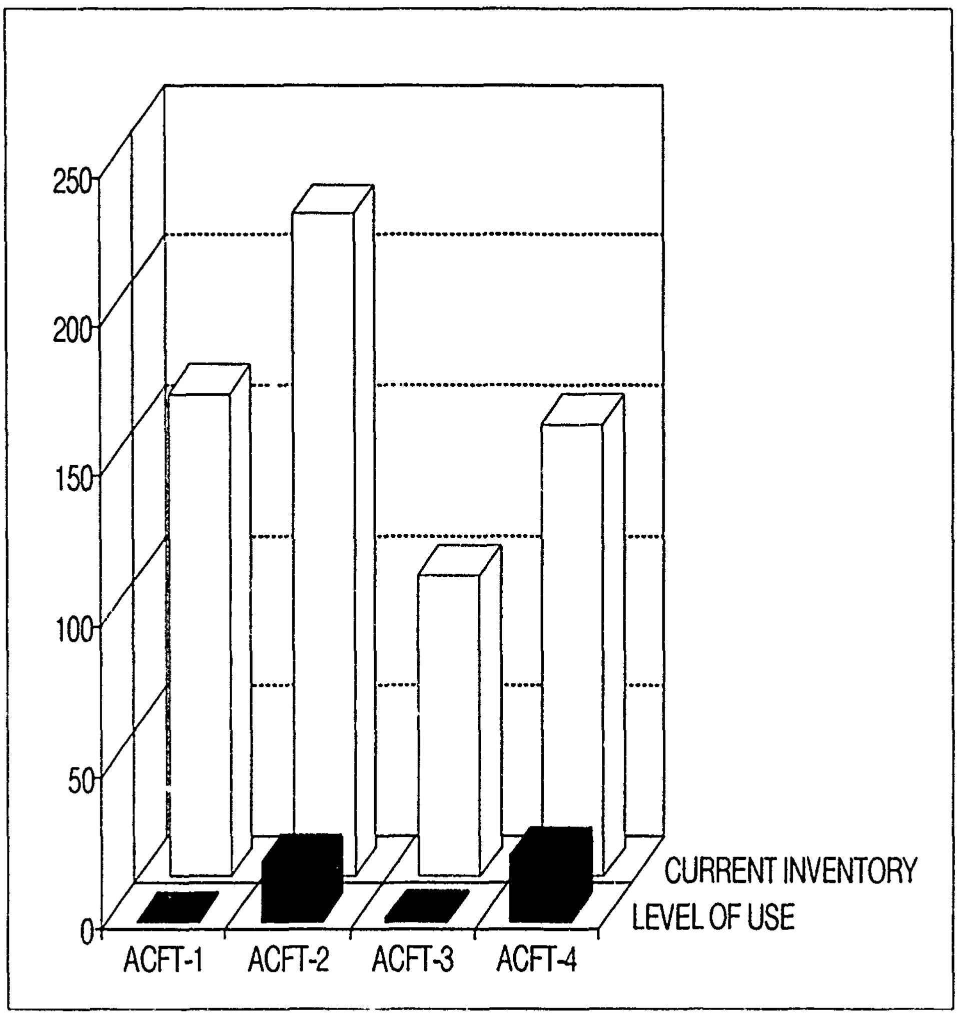

Figure 4.5. Number of Aircraft Per Sortie

and 5 at significant levels to kill all of the targets (Figure 4.6). The summary of the numerical results is presented in Table 4.3.

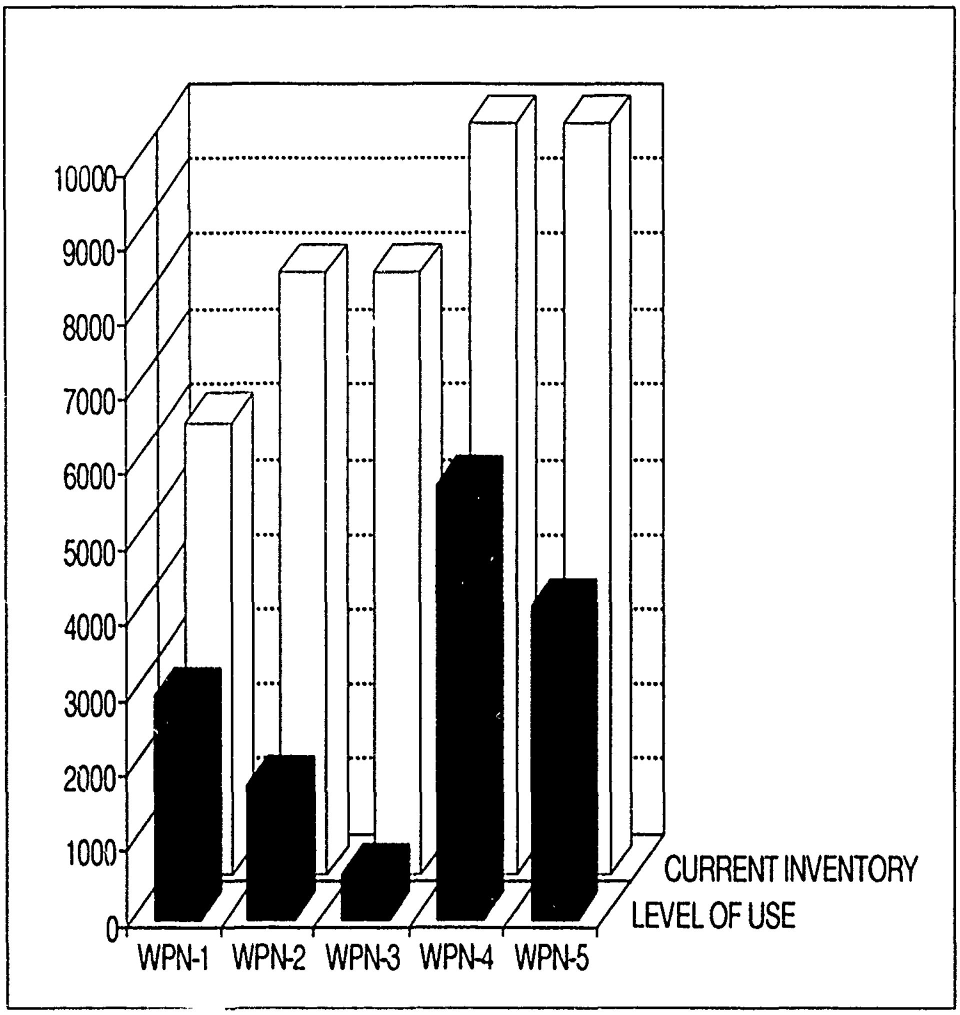

Figure 4.6. The Number of Weapons Per Sortie

In spite of their relatively high cost, the levels of use of weapon types 1 and 2 are remarkable; however, it is not surprising because their costs were not a driving factor in the objective function. Hence the model did not avoid using them (Figure 4.7).

As previously stated, all of the targets were destroyed. However, the model did not care about the tactical uses of the different aircraft types . For instance, aircraft type 4 was considered to be a Close Air Support (CAS) aircraft; but the model

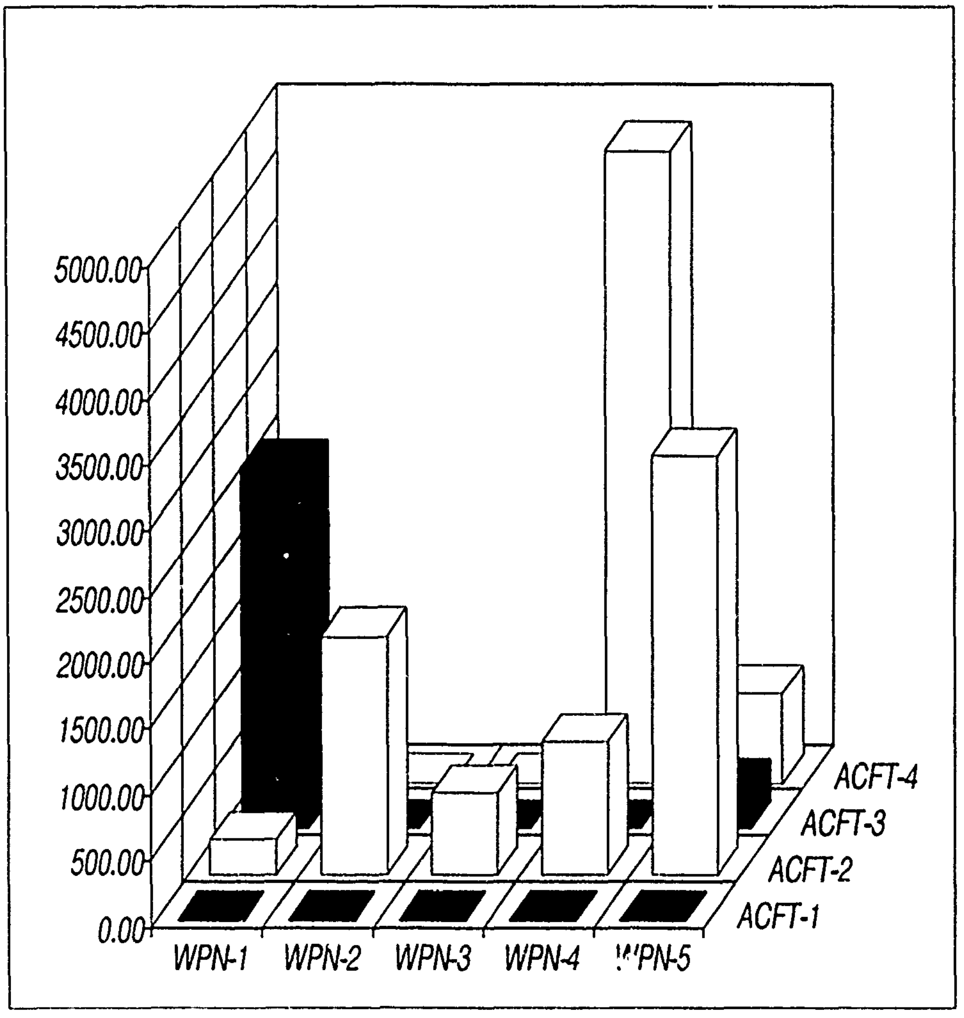

Figure 4.7. Use of Weapons by the Aircraft

allocated it to targets deep in enemy territory which is actually a misallocation of tactical aircraft.

4.3.2 Aircraft Cost Model. The first modification was to include the cost of aircraft in the objective function. This method resulted in somewhat balanced allocations. The model used 21.547 of aircraft type 1, 83.248 of aircraft type 2, 0.927 of aircraft type 3, and 21.331 of aircraft type 4 per sortie (Figure 4.8). The summary of the numerical results is presented in Table 4.4. The study expected the model

Table 4.4. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	4049.311	0	15.496	0	209.424	21.547
ACFT-2	1751.17	4383.169	615.385	4693.61	0	83.248
ACFT-3	0	0	0	0	309.119	0.927
ACFT-4	0	0	0	2245.39	0	21.331
WPN. USAGE	5800.481	4383.169	630.881	6939	518.543	

would avoid using the aircraft types with substantial costs. However, aircraft type 1, despite its relatively high cost, was used as much as the relatively least-cost aircraft, type 1. Because of this behavior of the model, the study perceived that the leading constraint had a remarkable effect over the model on the scenario basis. What the leading constraint did was basically to direct the model parallel to the given scenario that consisted of the predetermined tactical use of aircraft on hand. The percentages of total sorties for each aircraft type were considered as the number and the type of mission which that particular aircraft was supposed to fly. However, the mission-percentage constraint revealed a point of great importance: The evaluation of the marginal values of the mission-percentage constraint is the same as the evaluation of the mission planning in terms of the tactical use of the aircraft. For instance, in this particular run, the model obeyed the tactical use of the aircraft types, but it did

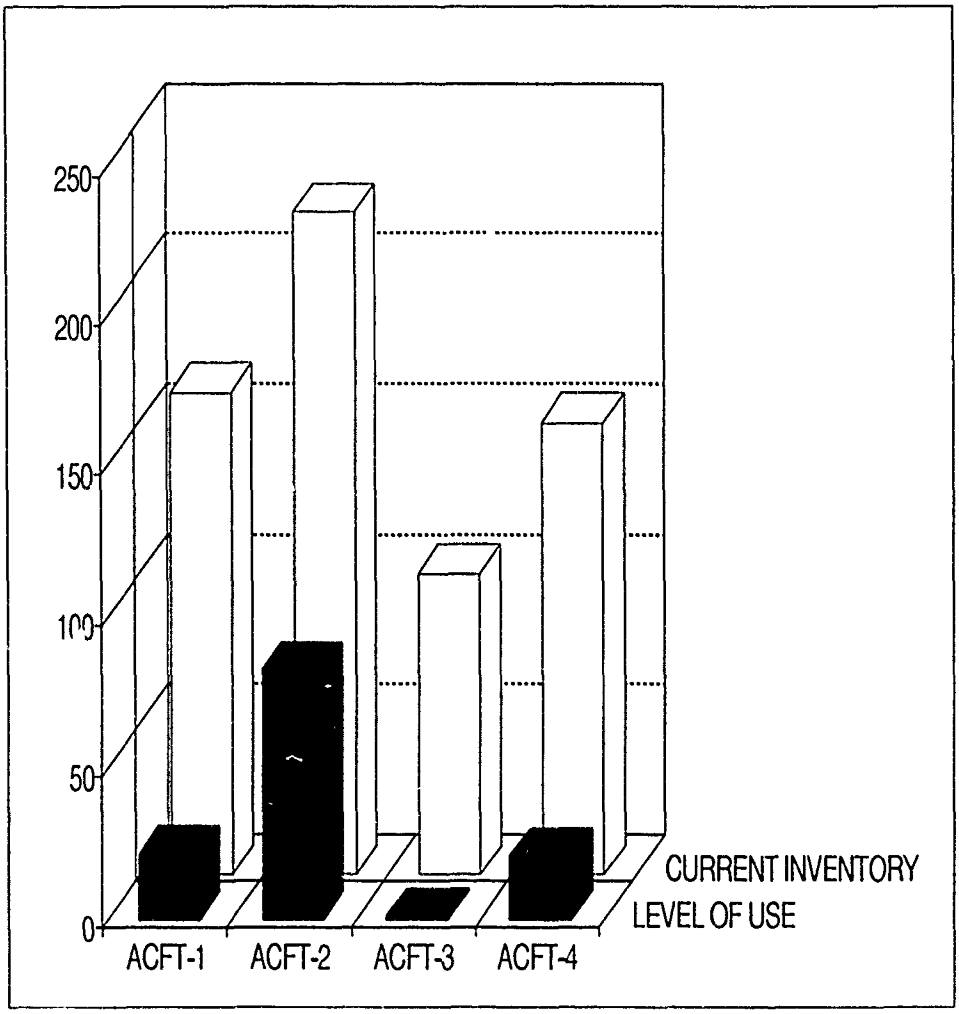

Figure 4.8. Number of Aircraft Per Sortie

not destroy all of the targets. Only 330.153 of 600 type 4 targets in distance band 3 were attacked.

In this modification, the target destruction rate was 85% with 0.34 *Target Value Destroyed per Dollar* as the objective function value. To evaluate the mission planning, the study interpreted the marginal values of the mission-percentage constraint. The most significant marginal value came from the allocation of ACFT-3.TGT-4.DIST-1. If the mission planner had considered increasing the right-hand-side value of this constraint one more unit, then the objective value would have increased by 1.317E–4 unit.

Based on percentages, the results could be interpreted in another way. The total number of sorties that aircraft type 3 could fly was 3605 sorties. 1% of that number is 36 sorties: 10% of that number is 360 sorties. Therefore, if the mission planner considers increasing the sortie limit by 10%, this would contribute 360 times 1.317E–4 [illegible] value to the objective function. However, this decision is not that easy because allocating more sorties to one type of target means allocating fewer to the other types. The decision should account for this kind of trade-off in the best possible way. It is noted that the model did not avoid using the expensive weapons because there was not a weapon cost figure in the objective function (Figure 4.9). The most effective weapons were used regardless of their substantial costs. Weapon types 1 and 2 had the diversity of aircraft weapons allocations (Figure 4.10).

4.3.3 Weapon Cost Model. This modification employed the weapon cost in the objective function. The first expectation was that the model would avoid using weapons with substantial costs and would use any type of aircraft regardless of its cost. Indeed, to achieve 0.7552 *Target Value Destroyed per Dollar*, the model used all aircraft types but aircraft type 3 at significant levels (Figure 4.11). The summary of the numerical results is presented in Table 4.5.

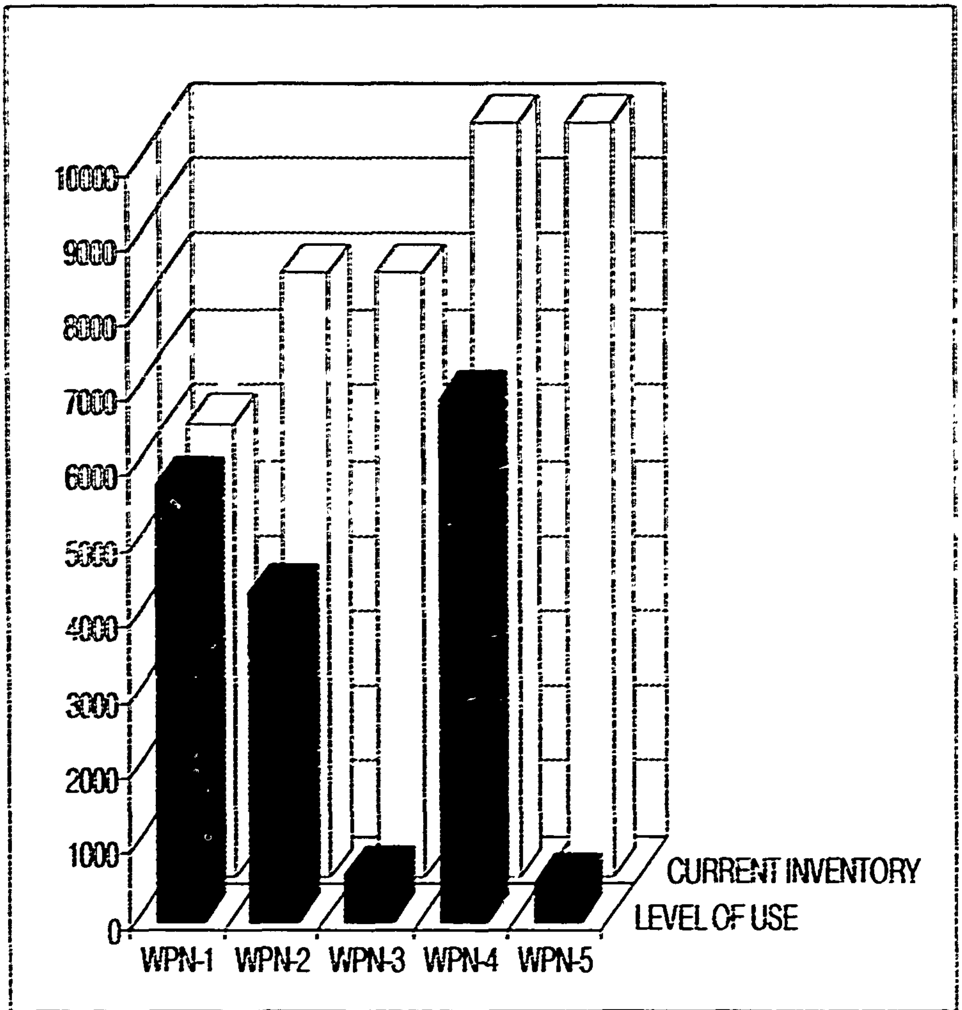

Figure 4.9. Number of Weapons Used

Table 4.5. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	0	1184.335	69.099
ACFT-2	0	0	0	0	2021.307	182.935
ACFT-3	0	0	0	0	309.119	0.927
ACFT-4	0	0	0	0	625.006	37.188
WPN. USAGE	0	0	0	0	4139.767	

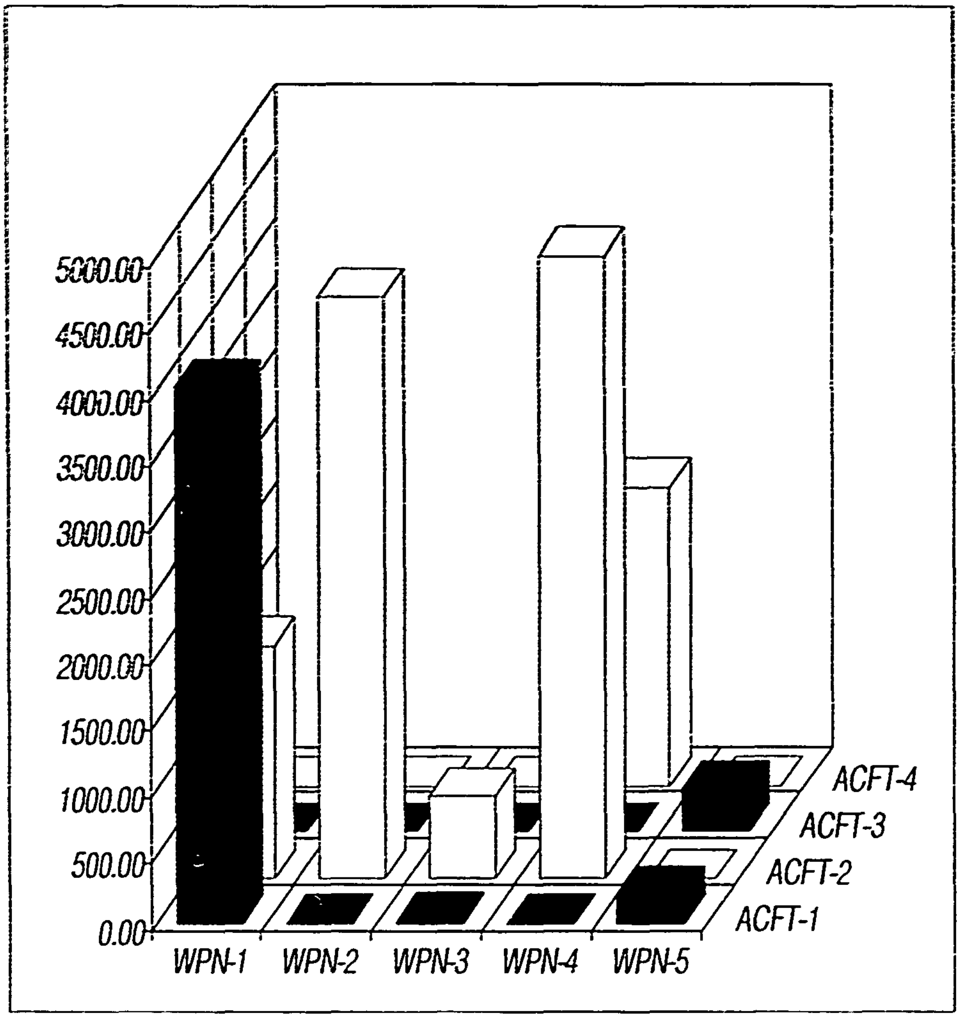

Figure 4.10. Use of Weapons by the Aircraft

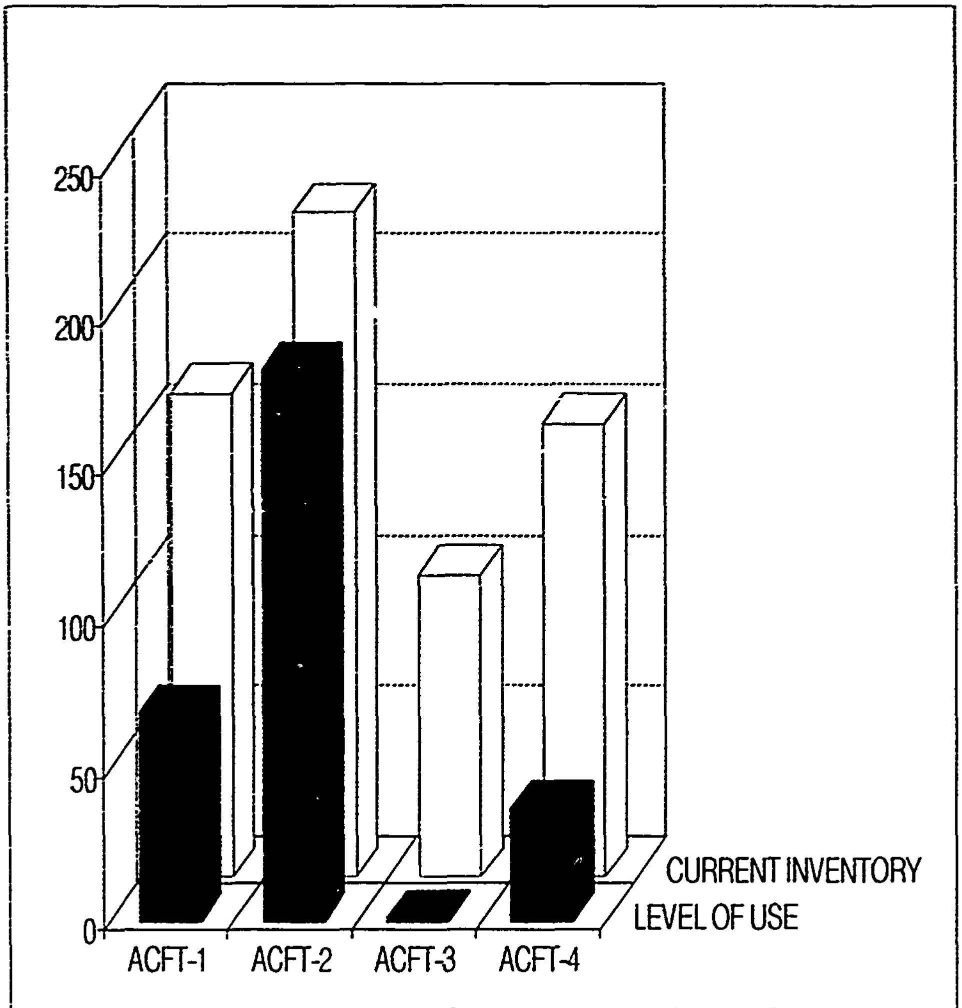

Figure 4.11. Number of Aircraft Per Sortie

It was not surprising that all of these aircraft were loaded with weapon type 5 which was relatively less costly than the first three types of weapons (Figure 4.12). The model might have used the cheapest weapon type; however, the target value per dollar spent for weapon type 4 was not better than that of weapon type 5. With these allocations, all of the first three types of targets were completely destroyed. Only 422.67 of target type 4 in distance band 2, and 512.23 of target type 4 in distance band 3 could not be destroyed.

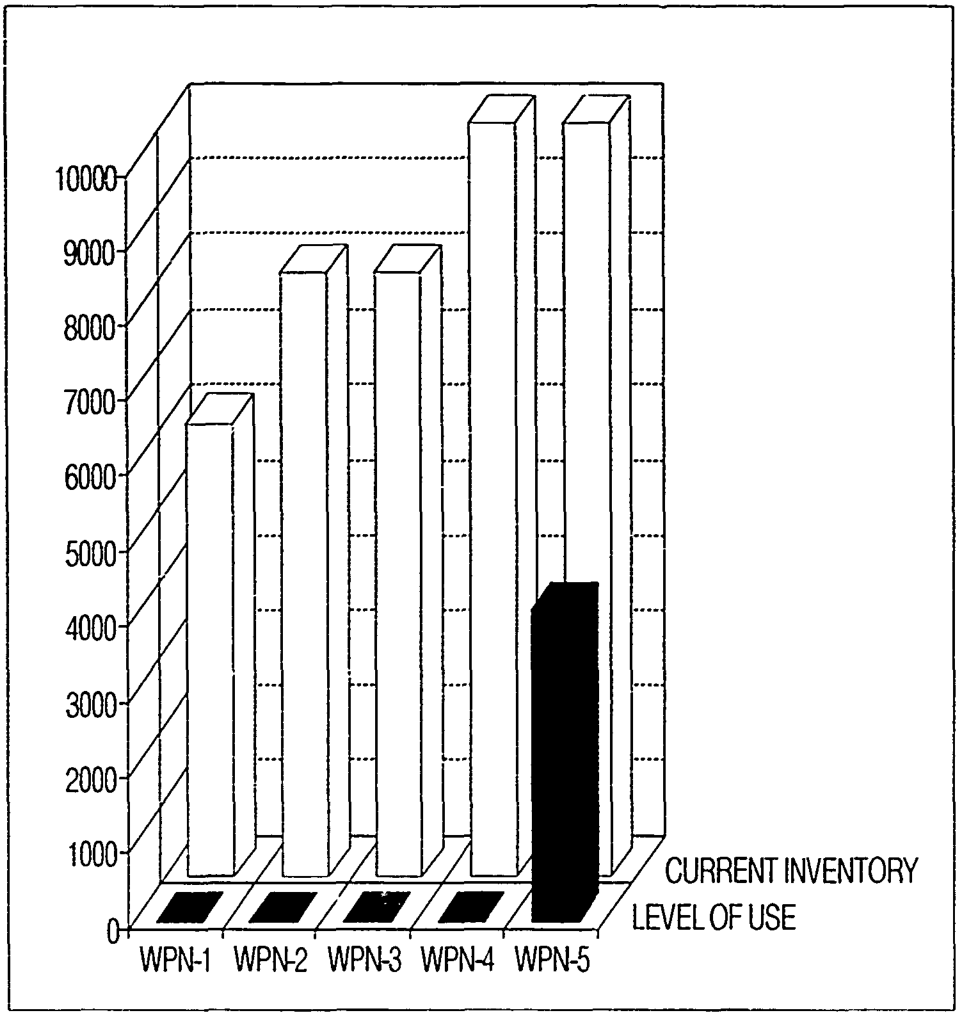

Figure 4.12. Number of Weapons Used

The target destruction rate was 50% and there was not a diversity of aircraft weapons allocations (Figure 4.13). This modification revealed another type of misuse of the tactical aircraft. Since the leading constraint leads the selection of aircraft types, but not weapon types, the leading constraint could not prevent the model from using only one type of weapon.

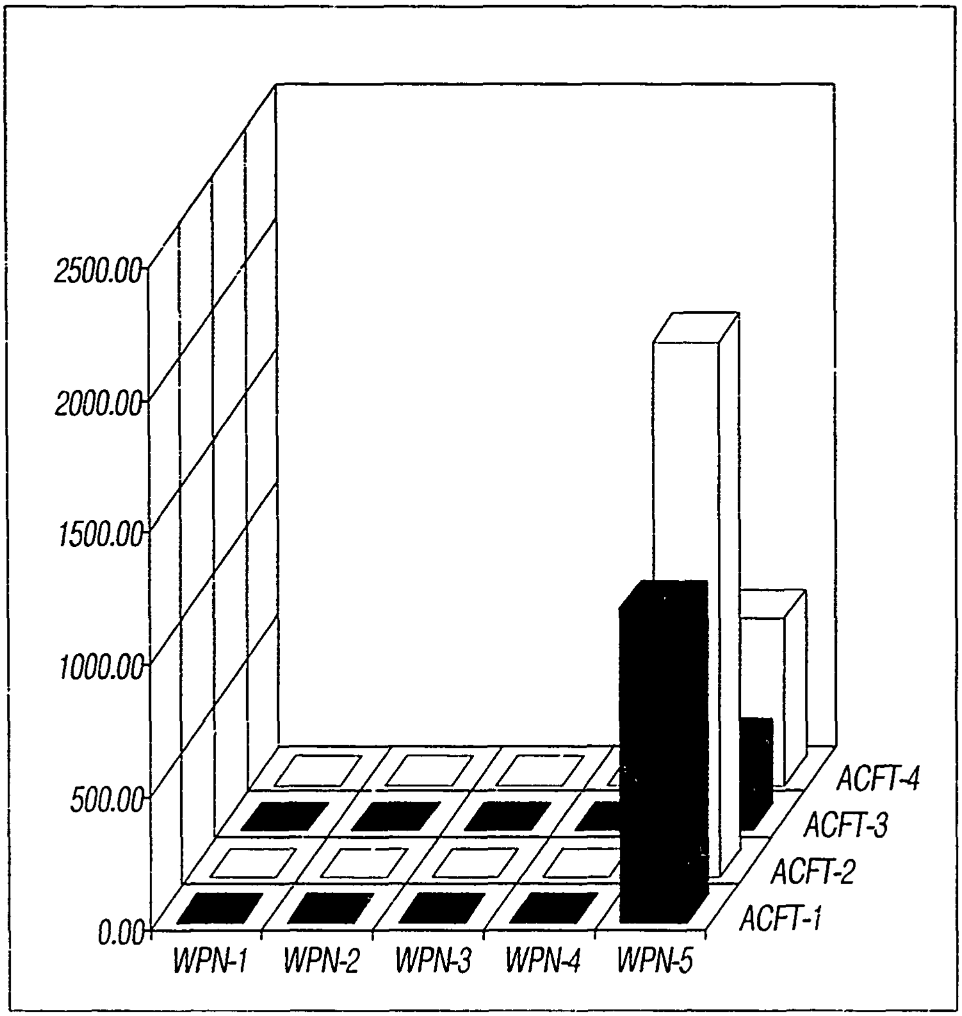

Figure 4.13. Use of Weapons by the Aircraft

The leading constraint did not have a significant impact on the use of weapons. Although the levels of use of the aircraft types per sortie were balanced, from a tactical point of view it was determined that the allocations and the way to accomplish the objective were unsatisfactory. Moreover, the marginal values in the mission-percentage constraint were all insignificant. That would imply that the mission planning was fairly reasonable, but it was not adequate to eliminate the misuse of the tactical aircraft.

4.3.4 Sortie Cost Model. This modification included only the sortie cost realized by each type of aircraft for each mission. It was expected that the aircraft type with the greatest sortie cost would not be selected for the mission. The aircraft type associated with the highest sortie cost was aircraft type 3. The results showed that the model did not employ aircraft type 3 (Figure 4.14). The levels of use for all aircraft types were below the current inventory level so as to imply that there was no need to buy new aircraft. The summary of the numerical results is presented in Table 4.6.

Table 4.6. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	2808.62	0	0	0	155.086	24.21
ACFT-2	4696.676	0	0	1313.386	649.826	164.051
ACFT-3	0	0	0	0	0	0
ACFT-4	2486.619	0	0	2777.184	0	84.021
WPN. USAGE	9991.915	0	0	4090.569	804.912	

Target destruction was 93%; 131 targets of type 4 in distance band 3 were not destroyed. All remaining targets were destroyed. With the allocations presented in Table 4.6, the model achieved 7.8952 *Target Value Destroyed per Dollar*. To achieve this value required a significant use of weapon types 1 and 4. Weapon types 2 and 3 were not loaded (Figure 4.15). The level of use of weapon type 1 was noteworthy. Clearly, aircraft type 1 had a diversity of loads. Also, the high level of use of the most expensive weapon was not unexpected since there was no weapon cost figure which would penalize the objective function (Figure 4.16).

Current inventory of weapon type 1 was 6000. However, the level of use exceeded the current inventory so as to indicate a need to buy more of weapon type 1.

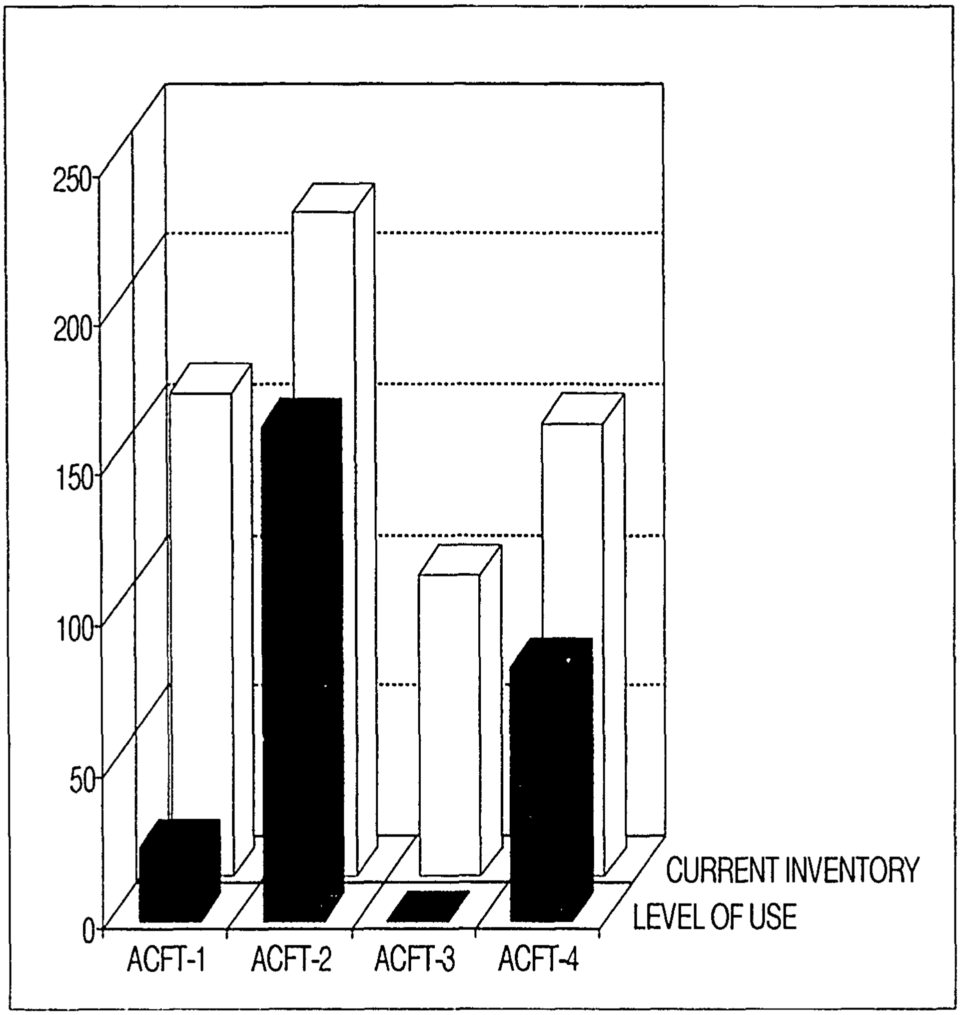

Figure 4.14. Number of Aircraft Per Sortie

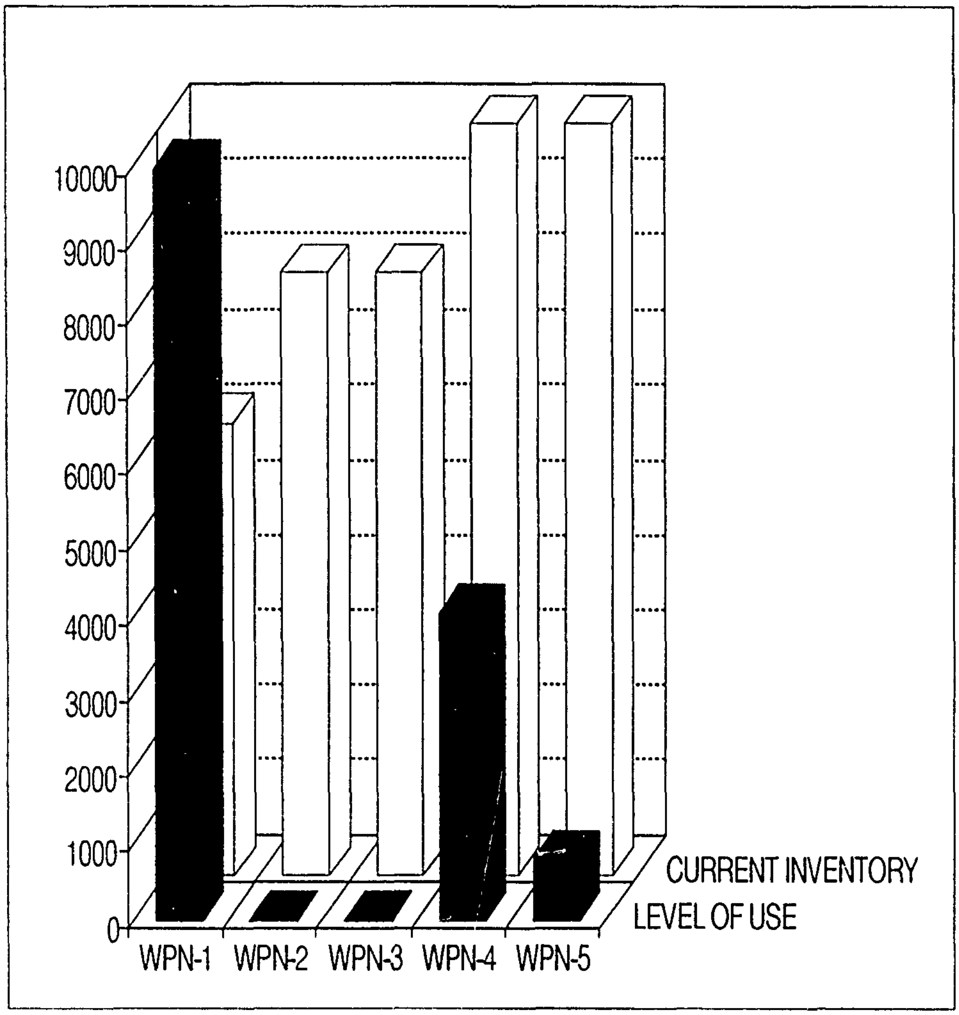

Figure 4.15. Number of Weapons Used

From a tactical point of view, the way that the model avoided using aircraft type 3 was percieved as unnecessary, since the differences in the sortie costs were not substantial. However, the model evaluated the costs based strictly on the differences; it did not care about their magnitude. Moreover, the results showed that the most significant marginal value came out of the ACFT-3.TGT-4.DIST-3 mission-percentage constraint at a level of 9.28E–4 unit. As previously stated, the total number of sorties that aircraft type 3 could fly was 3605. An increase of 30% of the sorties for this mission type would create 9.28E–4 times 1081 which results in a 1.0036 positive contribution to the objective value. This is true if the remaining right-hand-side values were left unchanged. However, this was not the case, because allocating more sorties to a particular mission would require allocating fewer sorties to others. Again, the decision should be made after accounting for the trade-off gains and losses.

Clearly, aircraft type 1 had a diversity of loads. Also, the high level of use of the most expensive weapon was not surprising since there was no weapon cost figure which would penalize the objective function.

4.3.5 Aircraft and Weapon Costs Model. This modification incorporated the costs of aircraft and weapons in the objective function. The model achieved a level of 0.1561 *Target Value Destroyed per Dollar* spent for the aircraft and weapons. 72.23 of target type 4 in distance band 2 and 281 of the same type of targets in distance band 3 could not be destroyed. The target destruction rate was 81%. To achieve this level, the model used all aircraft types (Figure 4.17). The levels of use for all types were below the current inventory level implying that there was no need to buy new aircraft. The summary of the numerical results is presented in Table 4.7.

The results showed that the employments of the aircraft types were tactically reasonable. In this modification the model was forced to avoid using the most expensive aircraft as well as the most expensive weapons. The expectation was that aircraft types 1 and 3 and weapon types 1 and 2 would not be used. However, the

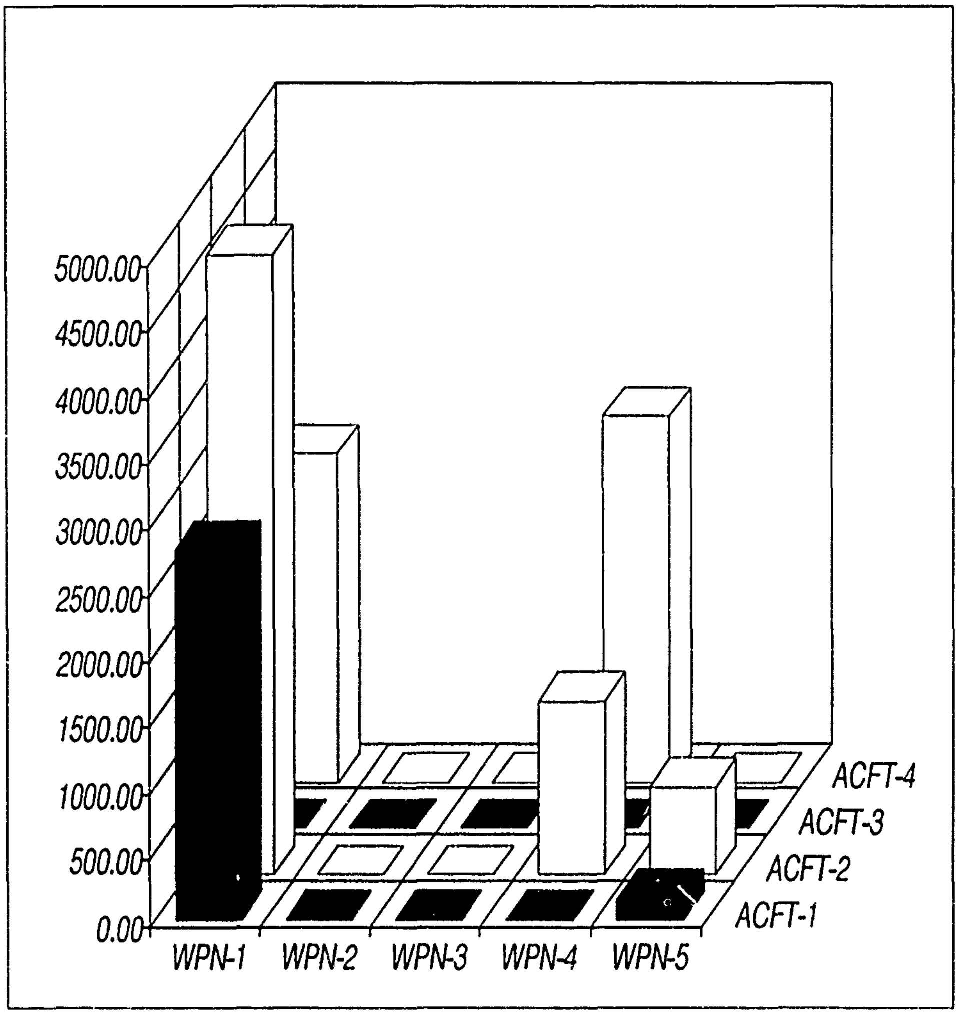

Figure 4.16. Use of Weapons by the Aircraft

Table 4.7. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	2024.655	0	15.496	121.132	462.506	14.186
ACFT-2	2459.217	0	762.712	5430.811	812.246	121.496
ACFT-3	0	0	0	0	309.119	0.927
ACFT-4	0	2721.913	0	2315.559	0	82.946
WPN. USAGE	4483.872	2721.913	778.208	7867.502	1583.871	

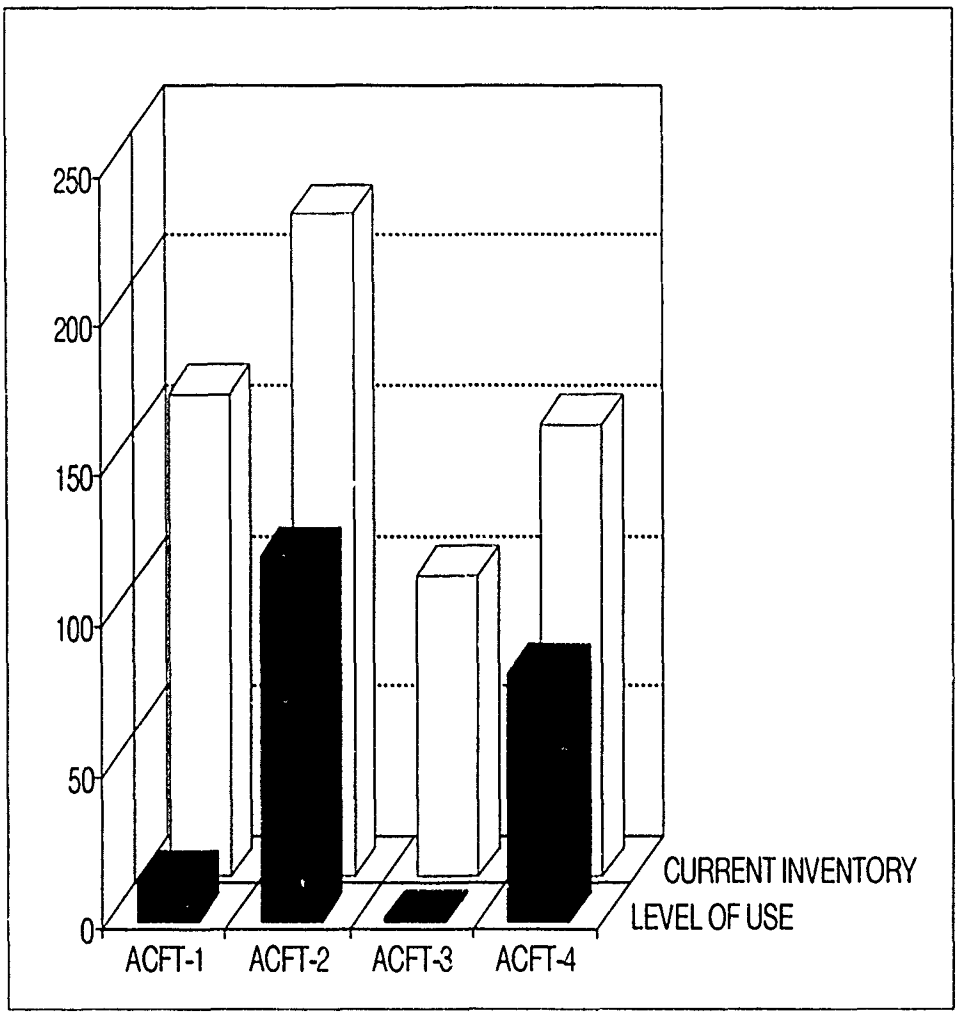

Figure 4.17. Number of Aircraft Per Sortie

leading constraint influenced the selection process of the model on the basis of scenario. The level of use of aircraft type 3 shows that aircraft type 2 was preferred to accomplish most of the missions that aircraft type 3 was supposed to fly. Also, the model used 14.186 of aircraft type 1. In terms of the weapon usage, the expectation was similar. However, weapon type 1 was used at a significant level regardless of its high cost (Figure 4.18). The current weapon inventory level was adequate to supply the requirements, so there was no need to buy new weapons.

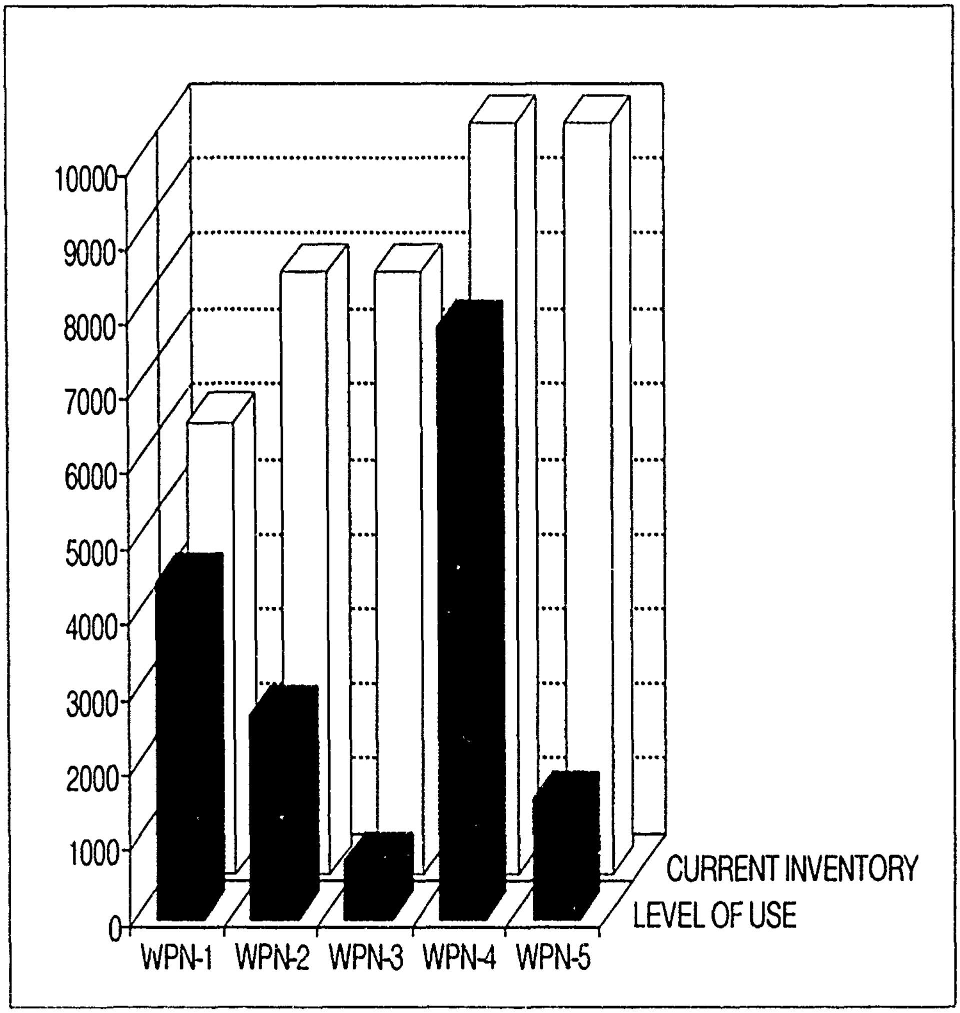

Figure 4.18. Number of Weapons Used

Aircraft types 1 and 2 shared the diversity of loads and hence the diversity of missions (Figure 4.19). The marginal values in the mission-percentage constraint were observed to be insignificant, meaning that the mission plan was well prepared.

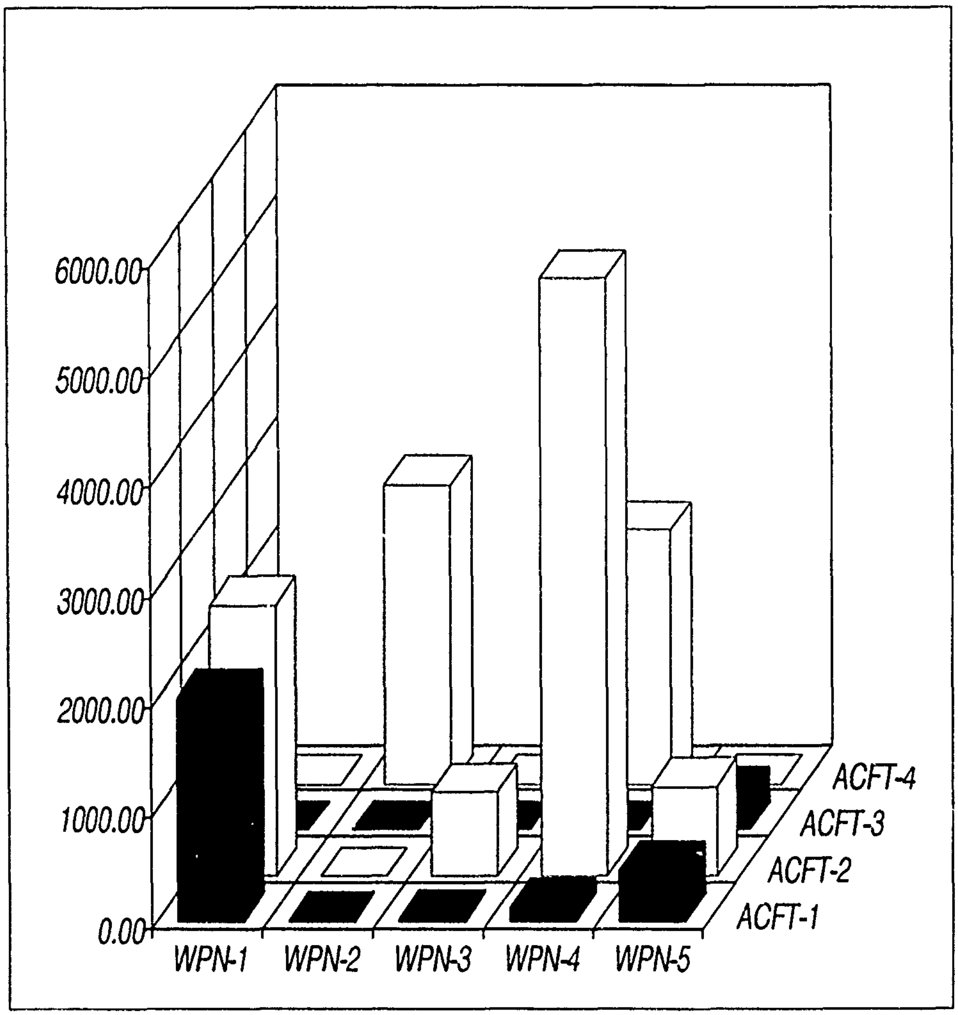

Figure 4.19. Use of Weapons by the Aircraft

4.3.6 Aircraft and Sortie Cost Model. This modification employed both the cost of aircraft and the cost of sortie for each type of aircraft. The model achieved an objective value at a level of 0.3174 *Target Value Destroyed per Dollar* spent for the aircraft and sortie. This configuration could not destroy all the targets. The target destruction rate was 74% with 219 and 269 of target type 4 in distance band 2 and 3, respectively, not attacked.

The model used all aircraft types (Figure 4.20). For this modification, it could be expected that the model would avoid using aircraft types associated with high replacement costs as well as with high sortie costs. However, the model obeyed the tactical use of the aircraft types as the leading constraint required. Despite its substantial replacement cost and high sortie cost, aircraft type 3 was given credit at a level of nearly one (0.927) aircraft per sortie. The levels of use of the aircraft types per sortie were below the current inventory level so that there was no need to buy new aircraft. The summary of the numerical results is presented in Table 4.8.

Table 4.8. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	4049.311	0	15.496	0	209.424	21.547
ACFT-2	249.221	5885.118	615.385	4693.61	0	41.944
ACFT-3	0	0	0	0	309.119	0.927
ACFT-4	0	4854.203	0	2245.39	0	91.717
WPN. USAGE	4298.532	10739.321	630.881	6939	518.543	

The model used the expensive weapons at significant levels because the weapons allocations process was not affected by the aircraft and sortie costs. To destroy 74% of the targets, the model used weapon type 2 extensively (Figure 4.21). The current inventory level of weapon type 2 was 8000. The level of use of weapon type 2 exceeded the current inventory level by 2739 weapons and thus there was a need to buy 2739 additional type 2 weapons. In the weapons allocations process, aircraft type 2 had a diversity of weapons loads and missions (Figure 4.22).

In terms of the marginal values, only the ACFT-3,TGT-4,DIST-1 mission-percentage constraint appeared to be significant, at a level of 1.14E-4. In this particular case, an attempt was made to change the mission percentages as a mission planner might. The model exploited only 30% of the total sorties that aircraft type 3 could have flown. The right-hand-side value of the constraint which gave the most

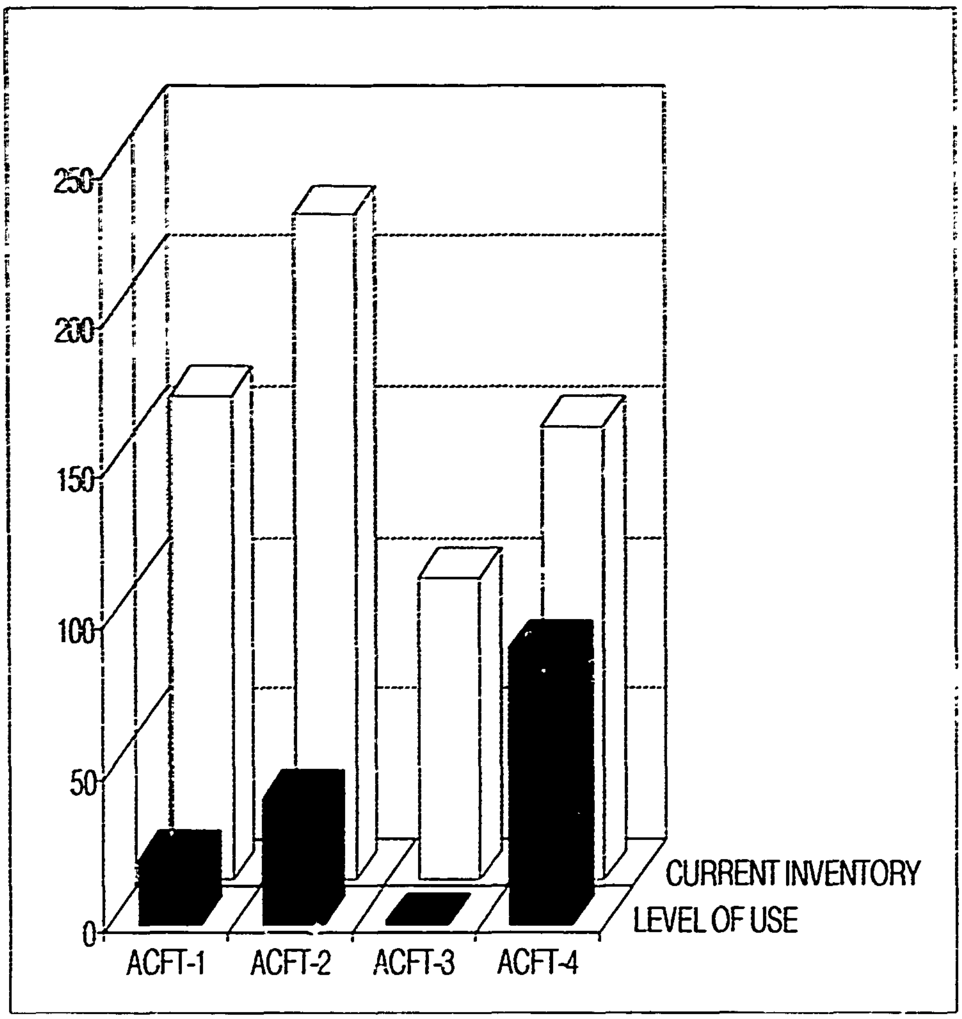

Figure 4.20. Number of Aircraft Per Sortie

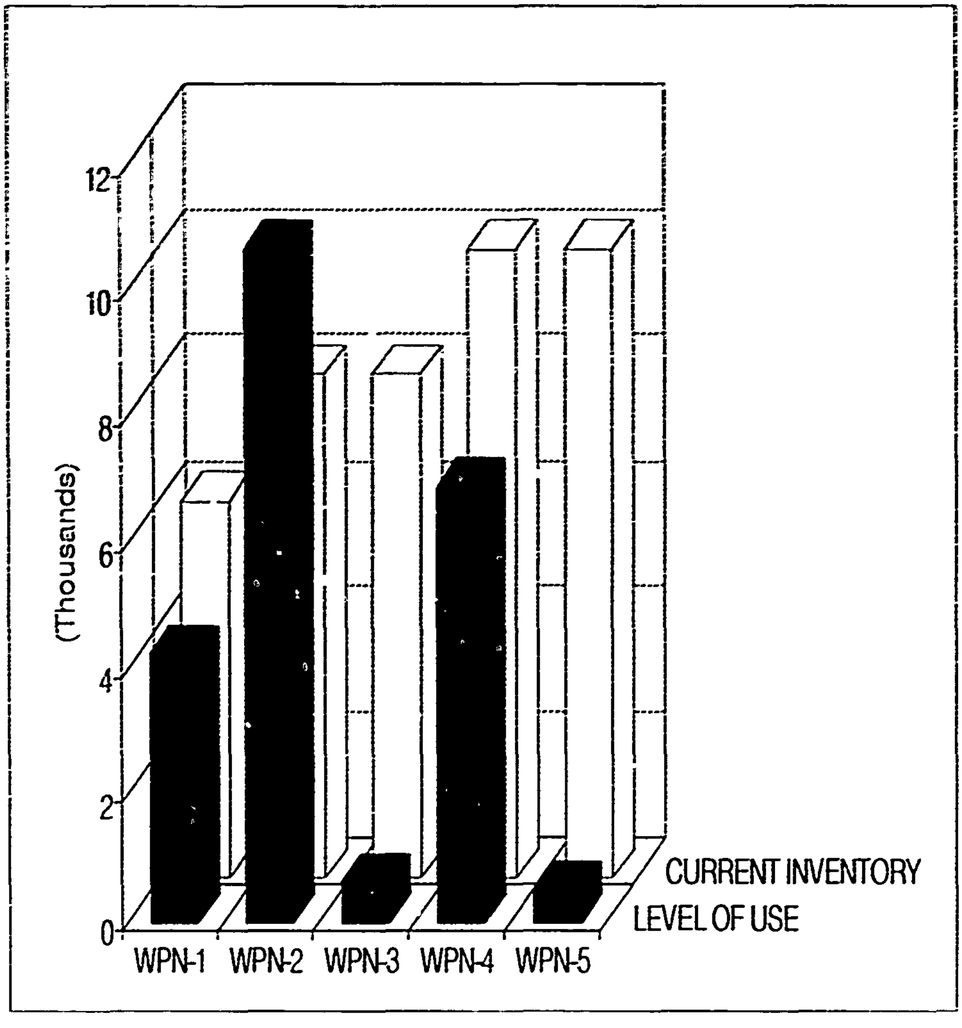

Figure 4.21. Number of Weapons Used

significant marginal value was considered to be suitable for allocating the remaining 70% of the total, computed as 2523.5 sorties. Therefore, the positive contribution turned out to be 2523.5 times 1.14E–4 which is 0.2876. The new *Target Value Destroyed per Dollar* spent for the aircraft and sortie became 0.605. The attempt to force this change was not that difficult because the mission percentages of the other aircraft remained the same; only the sorties not flown were reassigned to generate improvement.

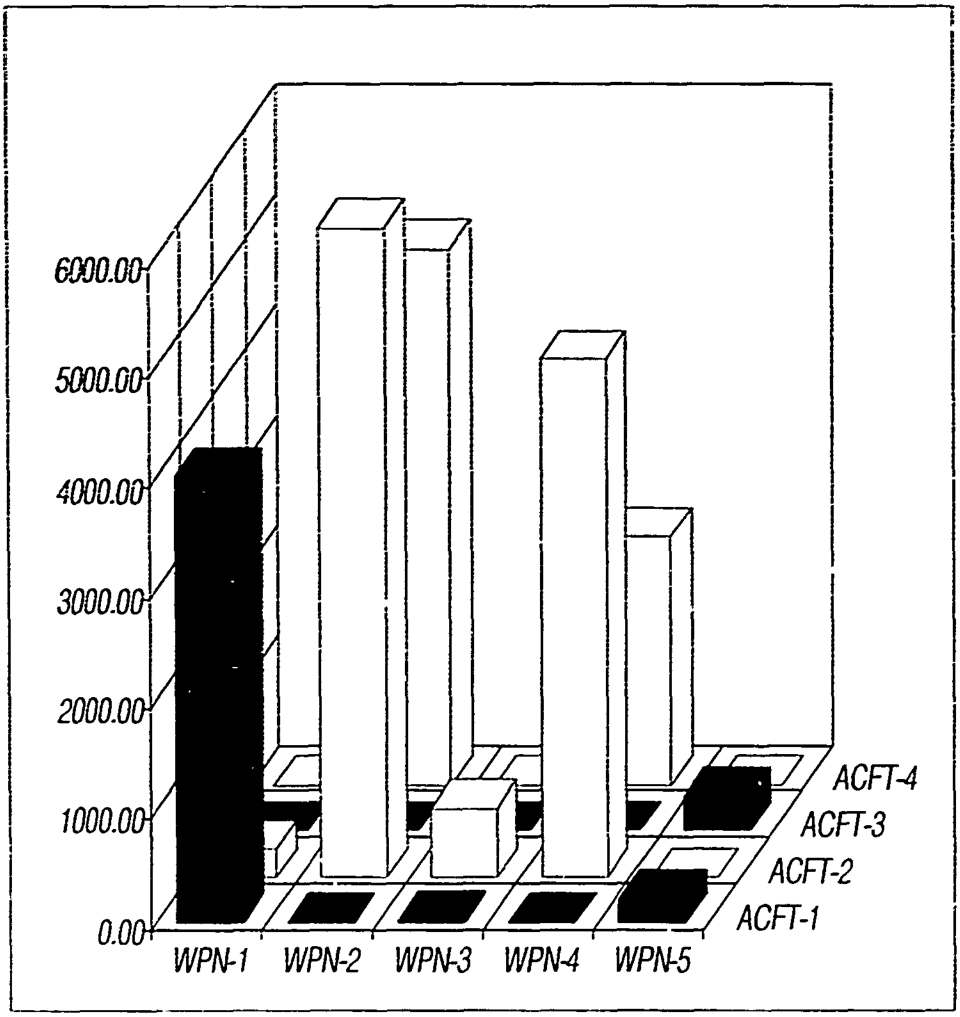

Figure 4.22. Use of Weapons by the Aircraft

4.3.7 Weapon and Sortie Costs Model. In this modification only the weapon and sortie costs were considered as a common denominator. Similarly, the expectation was that the model would avoid using the most expensive weapons and the aircraft types associated with high sortie costs. Indeed, the most expensive type of aircraft, aircraft type 3, was not used at all. Moreover, the model did not allocate any of the first four types of weapons. A level of 0.6835 *Target Value Destroyed per Dollar* spent for weapons and sorties was achieved as an optimal objective function value. The summary of the numerical results is presented in Table 4.9.

Table 4.9. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	0	536.297	17.643
ACFT-2	0	0	0	0	2911.347	267.452
ACFT-3	0	0	0	0	0	0
ACFT-4	0	0	0	0	625.006	37.188
WPN. USAGE	0	0	0	0	4072.65	

A 50% rate of destruction was achieved. The model employed aircraft types 1, 2, and 4, but not aircraft type 3 (Figure 4.23). The level of use of aircraft type 2 was considered to be significant since the current inventory level for aircraft type 2 was 220. Therefore, the results showed that there was a need to buy approximately 47 more of aircraft type 2. In terms of weapons procurement, the levels of use were not high enough to require procurement need (Figure 4.24).

The results did not show any significance in the marginal values of the mission-percentage constraint. To implement the missions, 536.297, 2911.347, 625.006 of weapon type 5 were, respectively, allocated to aircraft types 1, 2 and 4 (Figure 4.25). From a tactical point of view, the results of this modification were interpreted as unsatisfactory because the model employed only one type of weapon to carry out all of the missions. Remaining weapon types were considered to be useless by the model.

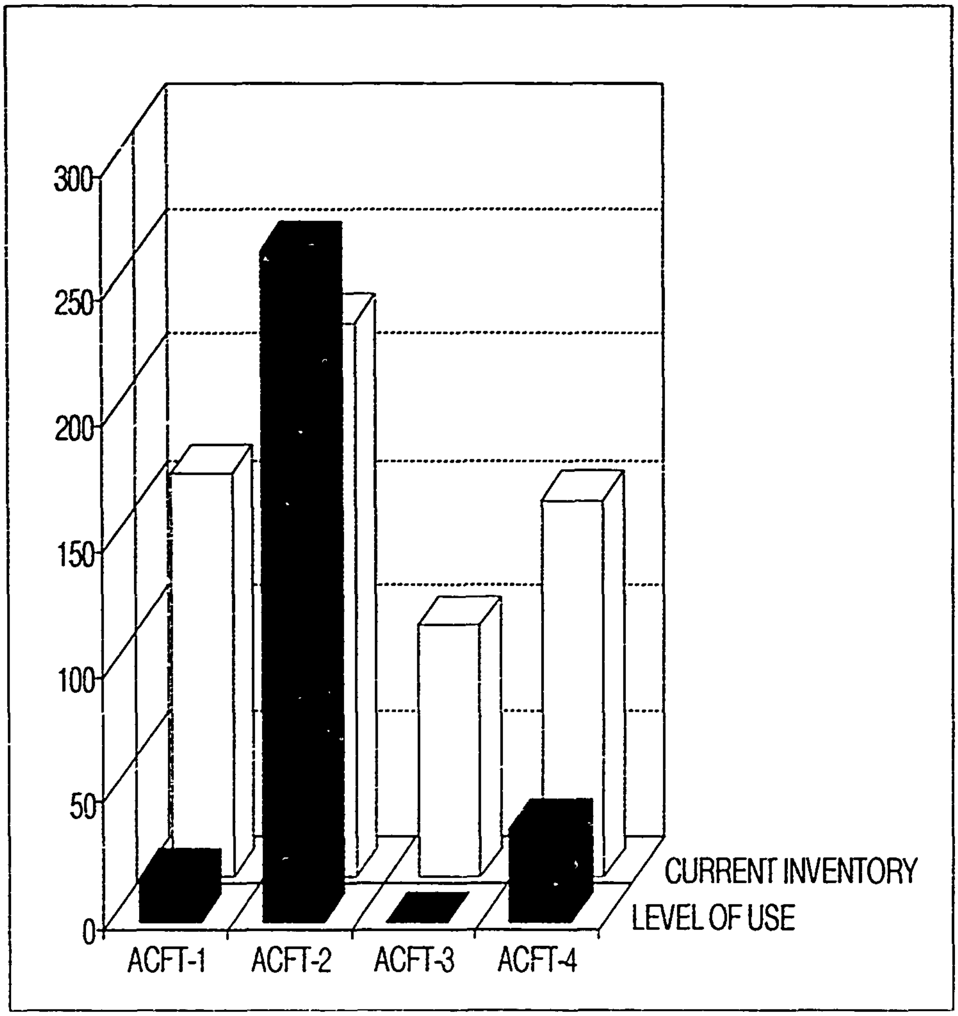

Figure 4.23. Number of Aircraft Per Sortie

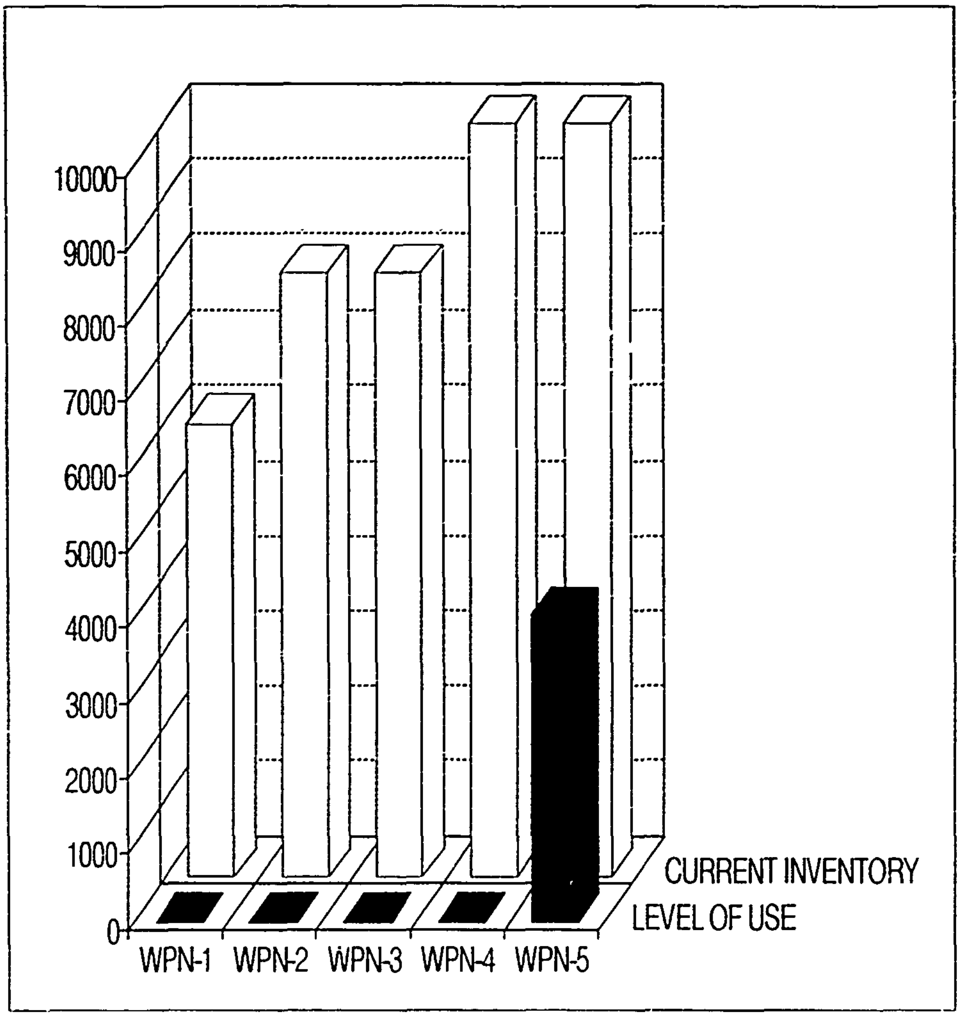

Figure 4.24. The Number of Weapons Per Sortie

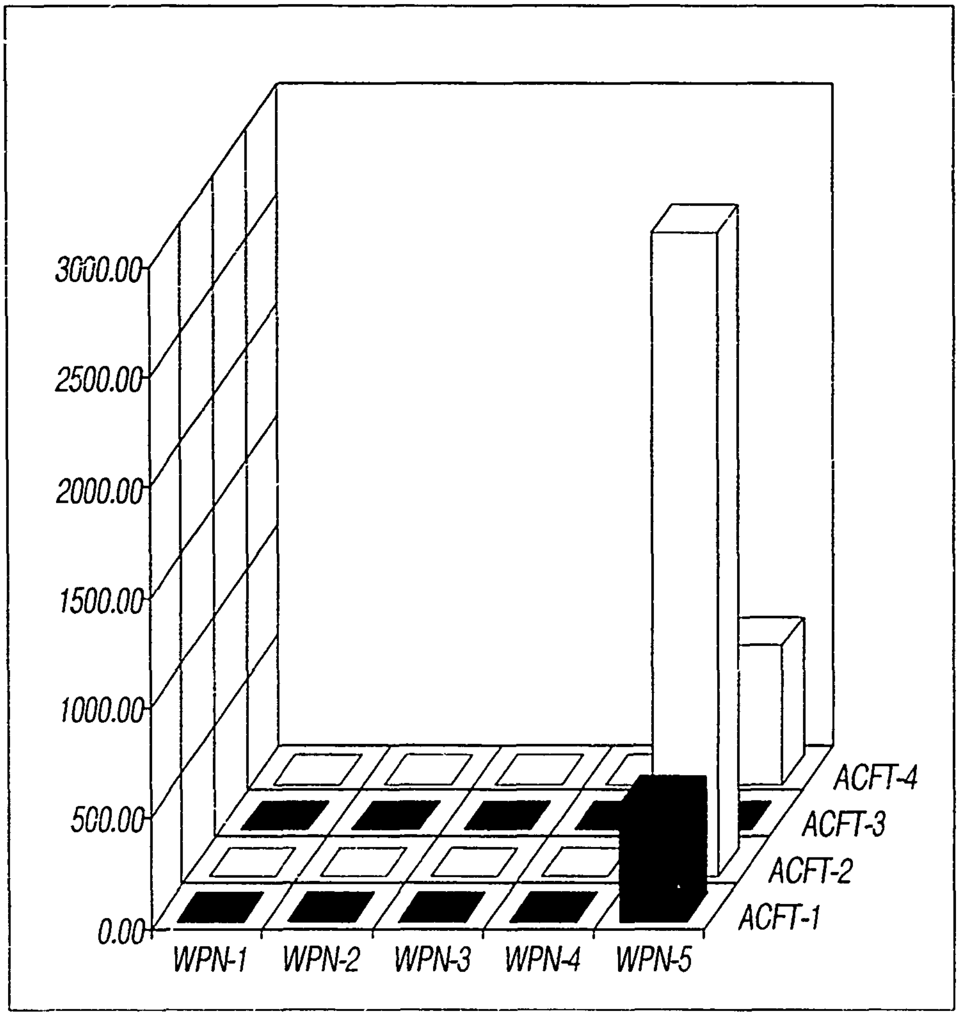

Figure 4.25. Use of Weapons by the Aircraft

4.3.8 Aircraft, Weapon and Sortie Costs Model. The final cost inclusion method incorporated all of the costs and hence the model is referred to as the *complete cost model.* The results were similar to those obtained from the modification which included the costs of aircraft and weapon. The study concluded that the sortie cost was insignificant with respect to the aircraft and weapon costs. Therefore, the model made the same selection as it did when the costs of aircraft and weapons were involved. The model achieved a level of 0.1512 as *Target Value Destroyed per Dollar* spent for the aircraft and weapons. 72.23 of target type 4 in distance band 2 and 281 of the same type of targets in distance band 3 could not be destroyed. The summary of the numerical results is presented in Table 4.10. The destruction

Table 4.10. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	2024.655	0	15.496	121.132	462.506	14.186
ACFT-2	2459.217	0	762.712	5430.811	812.246	121.496
ACFT-3	0	0	0	0	309.119	0.927
ACFT-4	0	2721.913	0	2315.559	0	82.946
WPN. USAGE	4483.872	2721.913	778.208	7867.502	1583.871	

rate of the targets was 81%. To achieve this level, the model used all aircraft types (Figure 4.26). The levels of use for all types were below the current inventory level. Reasonably, this implied that there was no need to buy new aircraft.

The results showed that the employment of aircraft types were tactically reasonable. In this modification, the model was forced to avoid using the most expensive aircraft as well as the most expensive weapons. The expectation was that aircraft types 1 and 3 and weapon types 1 and 2 would not be used. However, the leading constraint influenced the selection process of the model on the basis of scenario. The level of use of aircraft type 3 shows that aircraft type 2 was preferred to accomplish most of the mission that aircraft type 3 was supposed to fly.

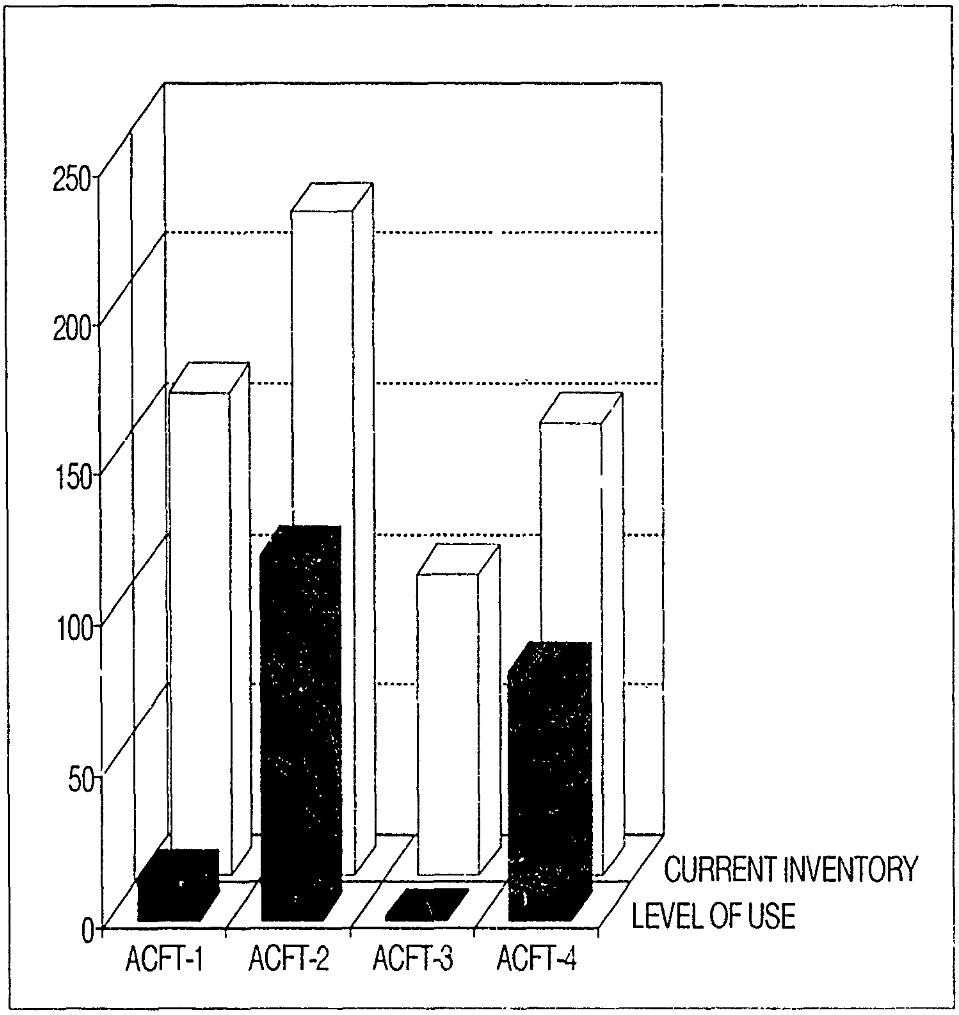

Figure 4.26. Number of Aircraft Per Sortie

In terms of the weapon usage, the expectation was similar. However, weapon types 1 and 4 were used at significant levels (Figure 4.27). In these allocations aircraft types 1 and 2 shared the diversity of loads, hence the diversity of missions (Figure 4.28). The current weapon inventory level was adequate to supply the needs; there was no need to buy new weapons. The marginal values in the mission-percentage constraint were observed to be insignificant. This insignificance implied the mission plan was well prepared. The marginal values in the mission-percentage constraint were observed to be insignificant. This insignificance implied the mission plan was well prepared.

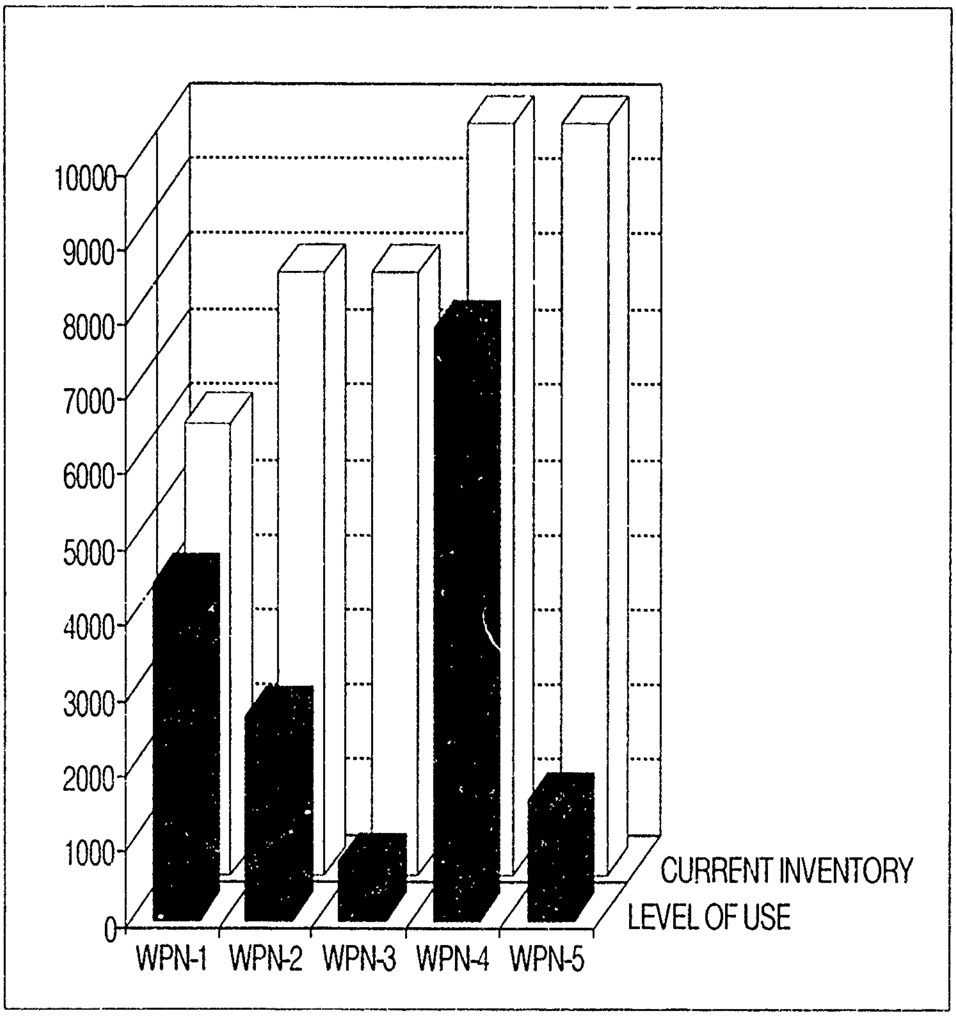

Figure 4.27. Number of Weapons Used

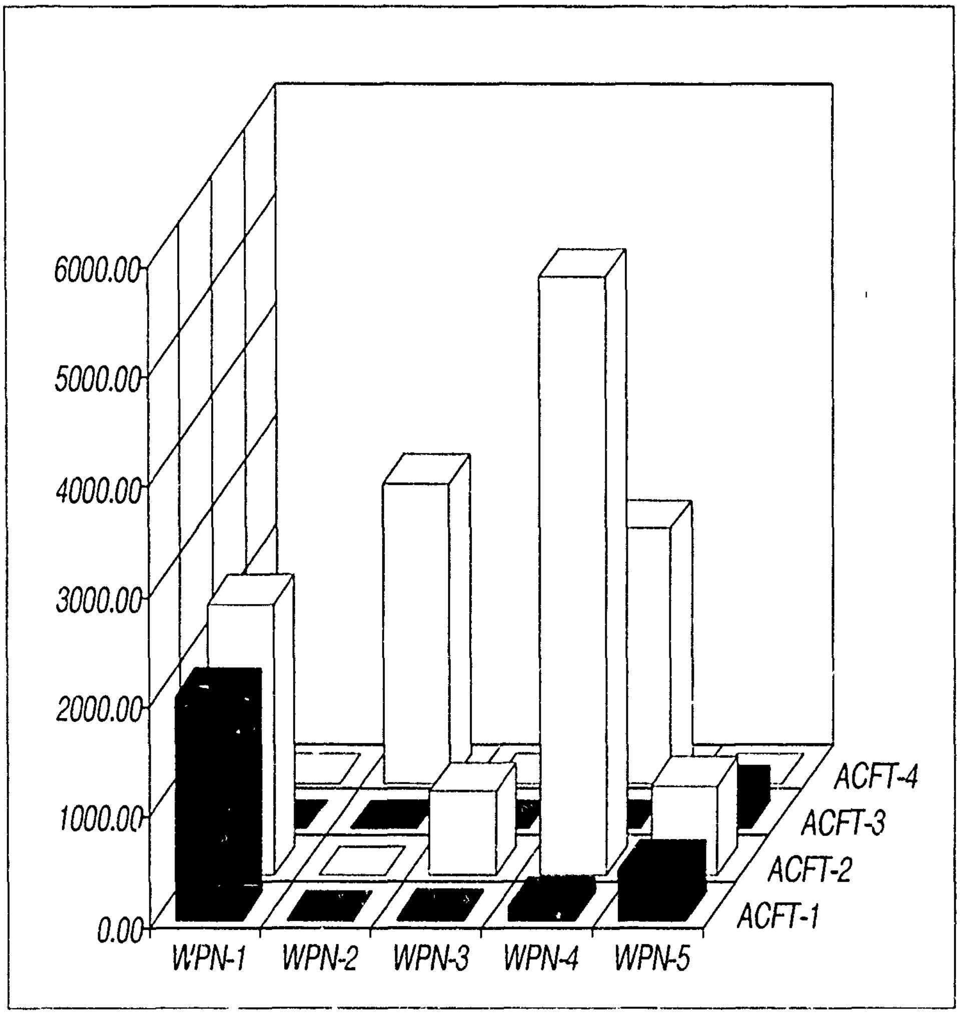

Figure 4.28. Use of Weapons by the Aircraft

4.4 Goal Programming Model

The procurement decision that came from the simple cost employment methods were based on the difference between the level of use of the weapons or aircraft and the inventory level. In the goal programming application, the procurement decisions were represented by two variables; d_a^+, and d_m^+. In addition to these so-defined procurement variables, the set-up of the model also accommodated some other needs. The construction of the model employs a mission success goal and a predetermined level of target value destroyed per dollar spent for a mission. The deviations from these goals also would explain more about the planned mission than the standard model.

The goal programming approach offers two more advantages. First, the mission planner would know how many aircraft were lost due to attrition by looking at the value of the deviate $dloss_a^+$ or if the deviate $dloss_a^+ = 0$, by simply subtracting the negative deviate $dloss_a^-$ from the level of use for the mission success. Secondly and more importantly, the goal programming formulation includes a budget constraint which accounts for the possibility of procurement need. This feature could not be employed directly in any other modifications, but the goal programming formulation allows inclusion of the budget figure since it employs the procurement variables directly.

However, given all these advantages, there does exist a shortcoming. The goal programming approach applied in this study is a linear program with some additional variables. There is no prioritization of the goals. Therefore, the model does not distinguish the goals in terms of their priority. As a quick reaction, prioritizing the goals by including a scalar coefficient for each of them might seem to be reasonable. Unfortunately, this way of ranking the goals might not assure the desired results at optimality with available solvers. If the goals were ranked, the optimal solution could achieve the higher priority goals which might have been satisfied at the expense of the lower priority goals. However, with equal ranking, it would not be possible to have that kind of optimal solution. From the point of view of the researcher, ranking the goals does not cause any problem as long as there is a solver capable of solving ranked goal programs. Considering all these circumstances, the study maintained the equally-ranked goal programming approach.

For this particular application, the study input 0.16 as the desired level of *Target Value Destroyed per Dollar* spent for the mission. This value was drawn from the complete cost model that included the cost of aircraft, weapons, and sorties. The results were very similar to those obtained from the complete cost model. The sum of the deviations was 160.9284, and the desired level of target value destroyed was underachieved at a level of 1%. But even at that level, it was equal to the value that was achieved in the complete cost model. Furthermore, the mission success goal appeared to be nonbinding and was satisfied by the solution. The model used all aircraft types (Figure 4.29). The summary of the numerical results is presented in Table 4.11.

Table 4.11. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	2311.175	1738.136	15.496	121.132	209.424	19.276
ACFT-2	3688.825	0	3075.109	3118.414	607.311	177.587
ACFT-3	0	0	0	0	309.119	0.927
ACFT-4	0	2721.913	4909.394	2162.603	0	93.741
WPN. USAGE	6000	4460.048	8000	5402.149	1125.855	

The levels of use of aircraft types were similar to those obtained from the complete cost model. Only the level of use of aircraft type 2 was significantly higher than that of the last modification, but the difference was compensated for by the use of different weapons. Weapon types 1 and 3 were exhausted in the goal programming results, whereas only 75% of weapon type 1 and 54% of weapon type 3 were used in the last simple cost employement (Figure 4.30). The target destruction rate was 91%

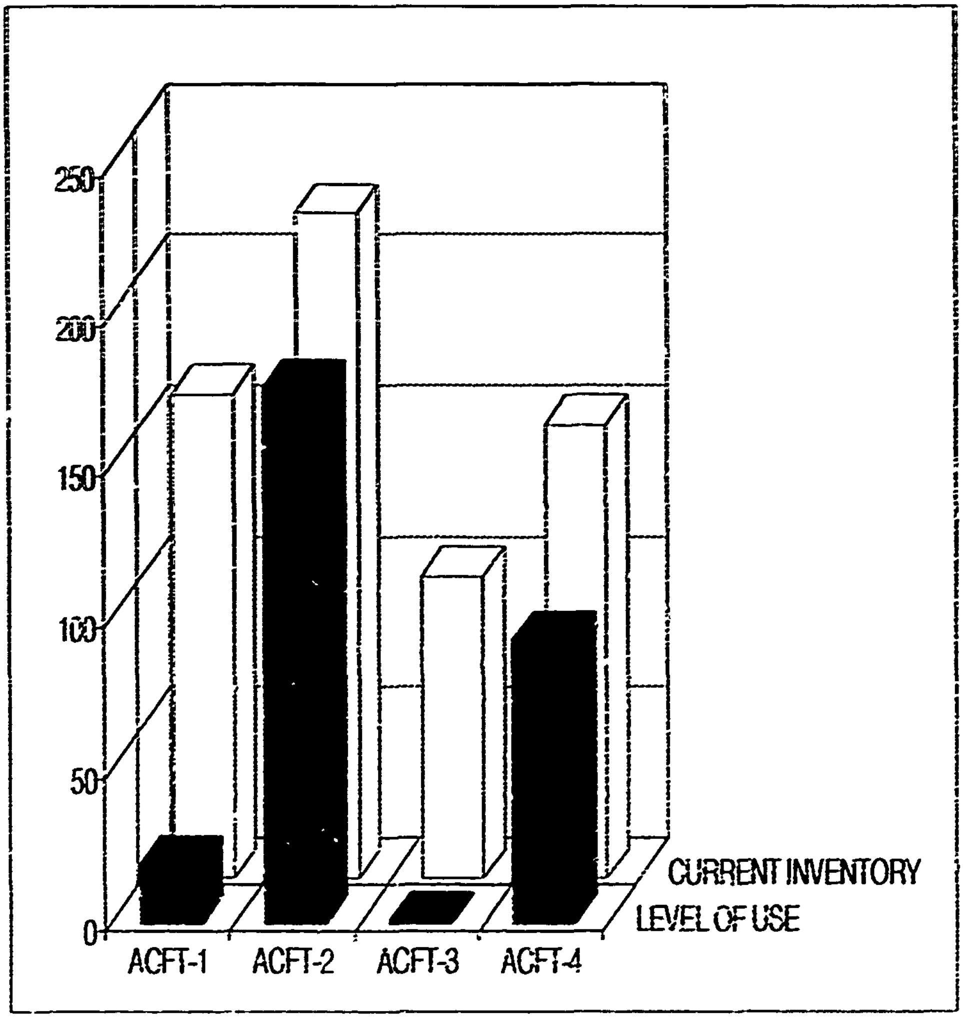

Figure 4.29. Number of Aircraft Per Sortie

which is higher than that of the complete cost model. Only 160.918 of type 4 targets could not be destroyed. The weapons allocation was observed to be balanced; here was a diversity of loads for aircraft types 1, 2, and 4 (Figure 4.31).

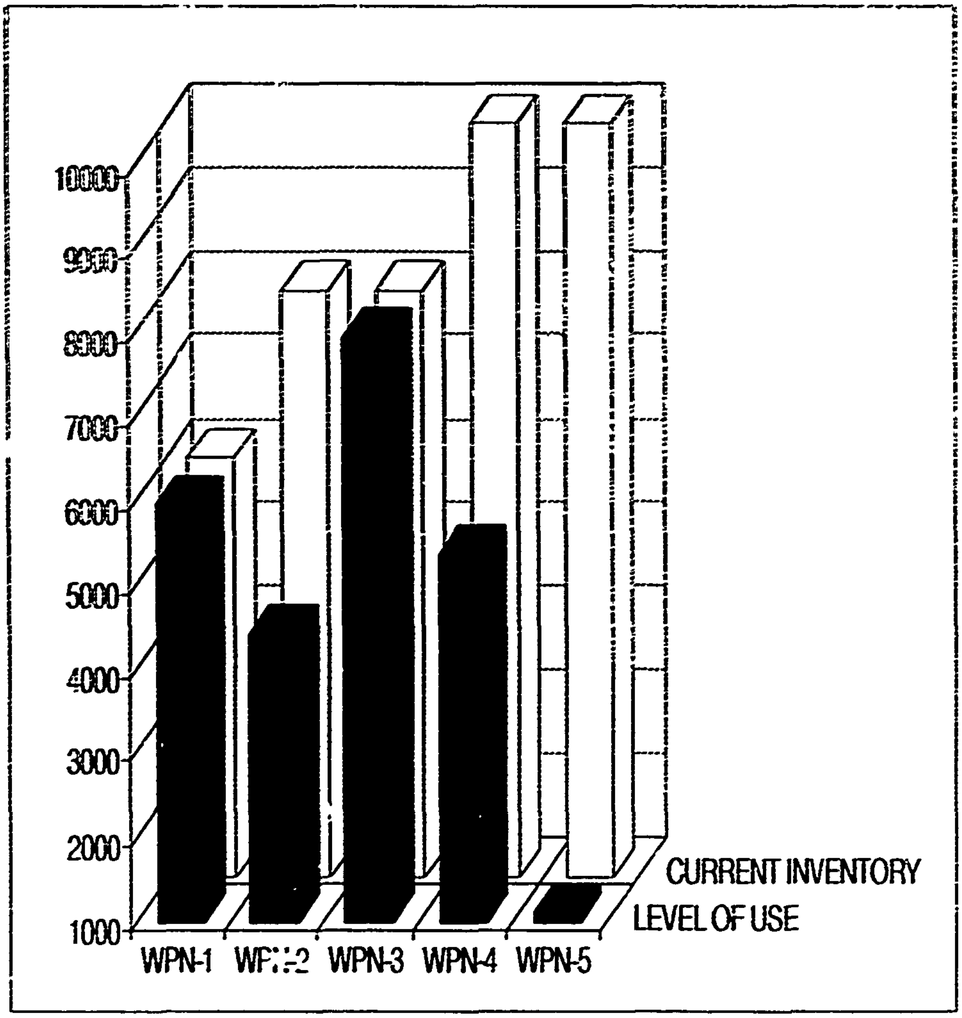

Figure 4.30. Number of Weapons Used

In terms of the mission-percentage constraint, the marginal values of the ACFT-1.TGT-1.DIST-2 and 3 constraints were more significant than the other marginal values; however, the tactical use of aircraft type 4 did not permit use of the aircraft deep in enemy territory. Therefore, the lower significance levels were taken into account such as -0.264 and -0.314 from the ACFT-1.TGT-1.DIST-2 and 3 mission-percentage constraints. The study attempted to change the mission percentages because 10% of

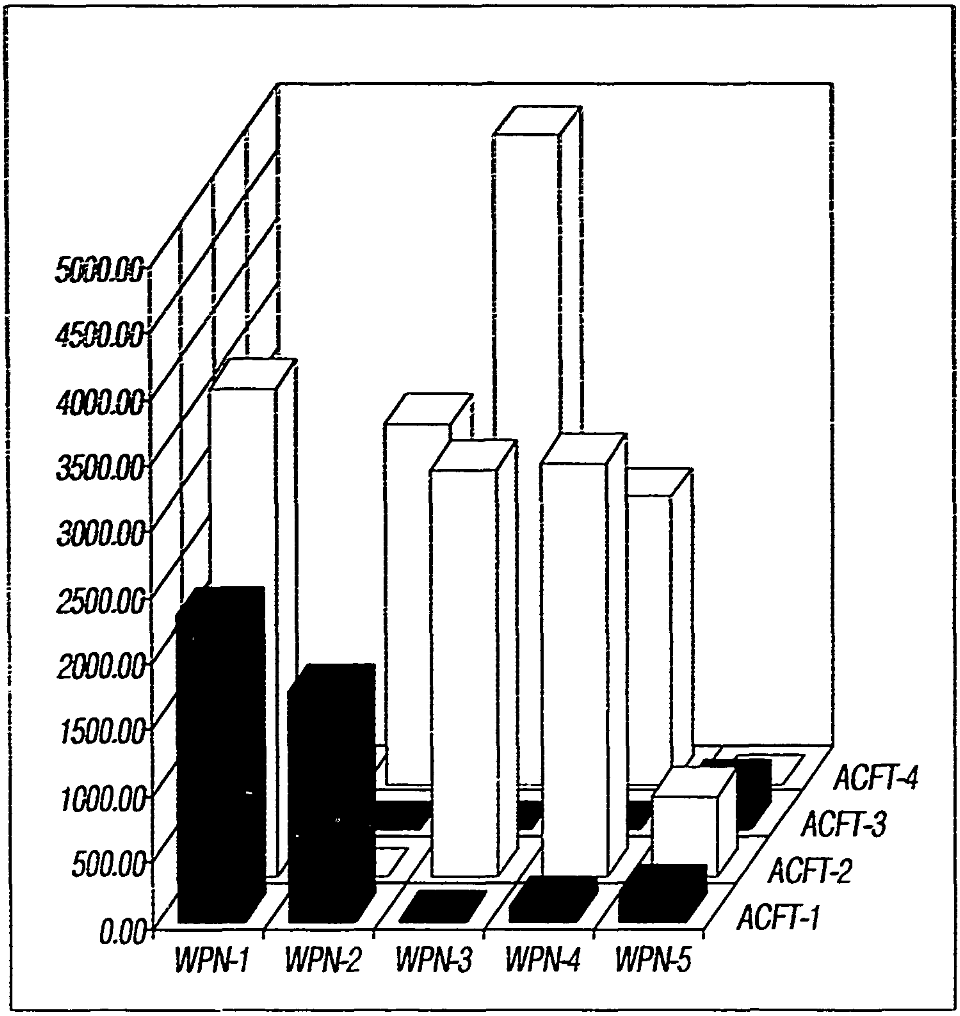

Figure 4.31. Use of Weapons by the Aircraft

the total sorties that aircraft type 1 was supposed to fly were not used. Therefore, the initial 40% allocation of sorties was diverted to the marginally most significant mission. The objective value was expected to be very small. Indeed, it turned out to be 0.0084. In terms of the deviations, that was considered to be very insignificant.

The goal programming method assures that the model will make the best allocations with what is in the inventory, and if the current inventory level is not adequate to support the missions, the procurement variables d_c^+ and d_m^+ have positive values in the solution implying a need to buy more aircraft or weapons. Another remarkable feature of this method is that the model does not show any significant bias in aircraft/weapon allocation. The study shows that the goal programming method exhausted weapon type 1 in spite of its substantial cost.

4.5 Probabilistic Parameter Model

This approach first determined the cost of the mission employing the stochastic concept of the sortie and then modified the dimensions of the parameters to make them consistent with Mixmaster. As previously stated, this particular method used a similar data base with differences in parameter dimensions. In addition, there is a time issue involved in this method. In cost computations, time is the independent variable. The study used predetermined time values for each type of aircraft, against a target, in a distance band. The employment of the time variable generated a great deal of conservative behavior in terms of the allocation process. The results showed that the number of targets destroyed were nearly the same as in the other models, however the model achieved that level with fewer aircraft and weapons. The time variable affects the probability that the aircraft is killed, the probability that the target is destroyed, the expected number of passes, and the expected sortie durations. Hence, within the limitations of the assumptions, the time-dependent computations were considered to be accurate and realistic.

The model achieved the objective at a level of 0.1653 *Target Value Destroyed per Dollar*. This level is very close to those obtained in the goal programming model and the last simple cost employment models. The conservative behavior of the model appeared first in the aircraft allocation process where the model tried to avoid using the expensive aircraft. Consequently, aircraft types 1 and 3 were used at levels of 0.795, and 1.174 per sortie, respectively. Most of the missions were executed by aircraft types 2 and 4 (Figure 4.32). The summary of the numerical results is presented in Table 4.12.

Table 4.12. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	0	1059.535	0.795
ACFT-2	0	0	273.595	301.525	357.332	20.705
ACFT-3	0	0	0	0	79.61	1.174
ACFT-4	0	0	0	0	574.04	25.005
WPN. USAGE	0	0	273.595	301.525	2070.517	

The model continued the same conservative behavior in the weapons allocations process. The weapons with subtantial costs were not used at all. Only weapon types 3, 4, and 5 were employed to accomplish the missions (Figure 4.33). The levels of use for the resources were considered to be very low with respect to those of the former models.

To decide whether there was a need to buy more aircraft or weapons the levels of use for the aircraft and weapons constraint were compared with the current inventory levels. The levels of use were well below current inventory levels implying that the current inventory levels were adequate to accomplish the missions.

In the probabilistic method, the study observed that the leading constraint was not as effective as it was in the former models. For example, in the complete cost model the objective function included all the costs, but the model did not avoid

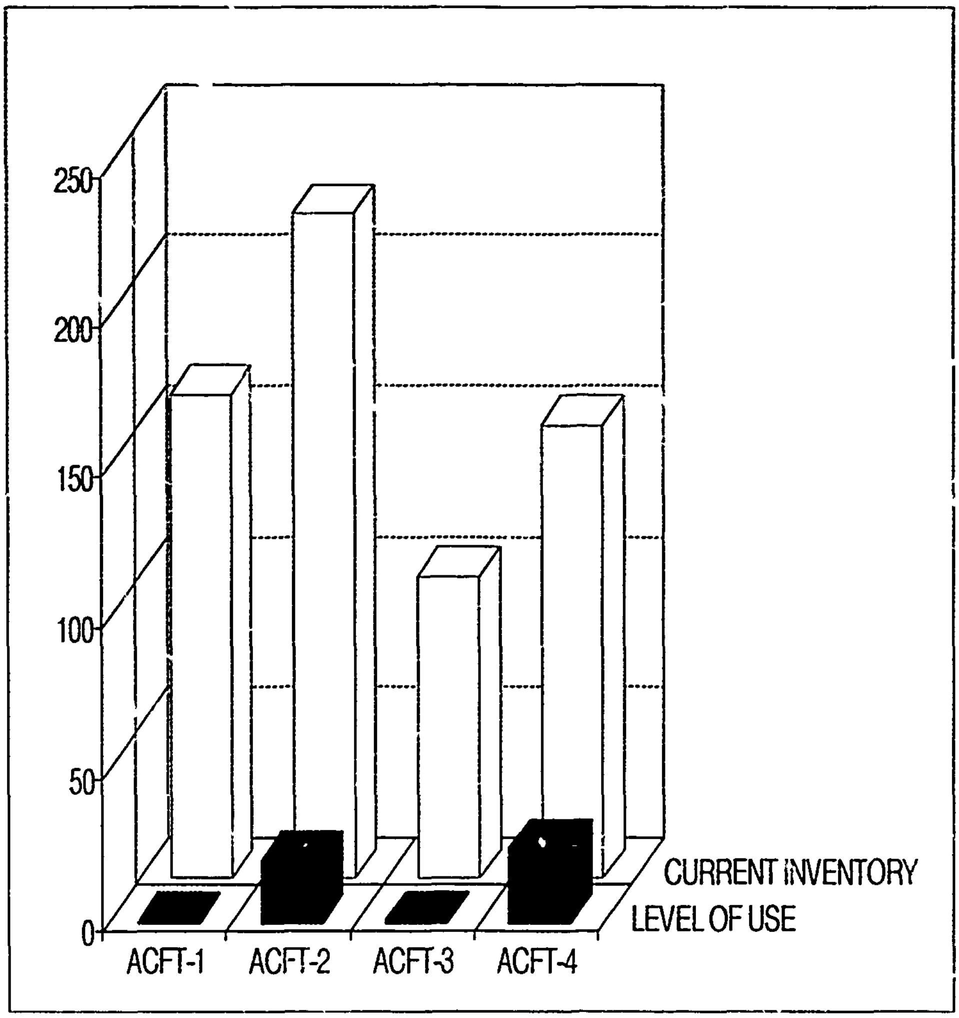

Figure 4.32. Number of Aircraft Per Sortie

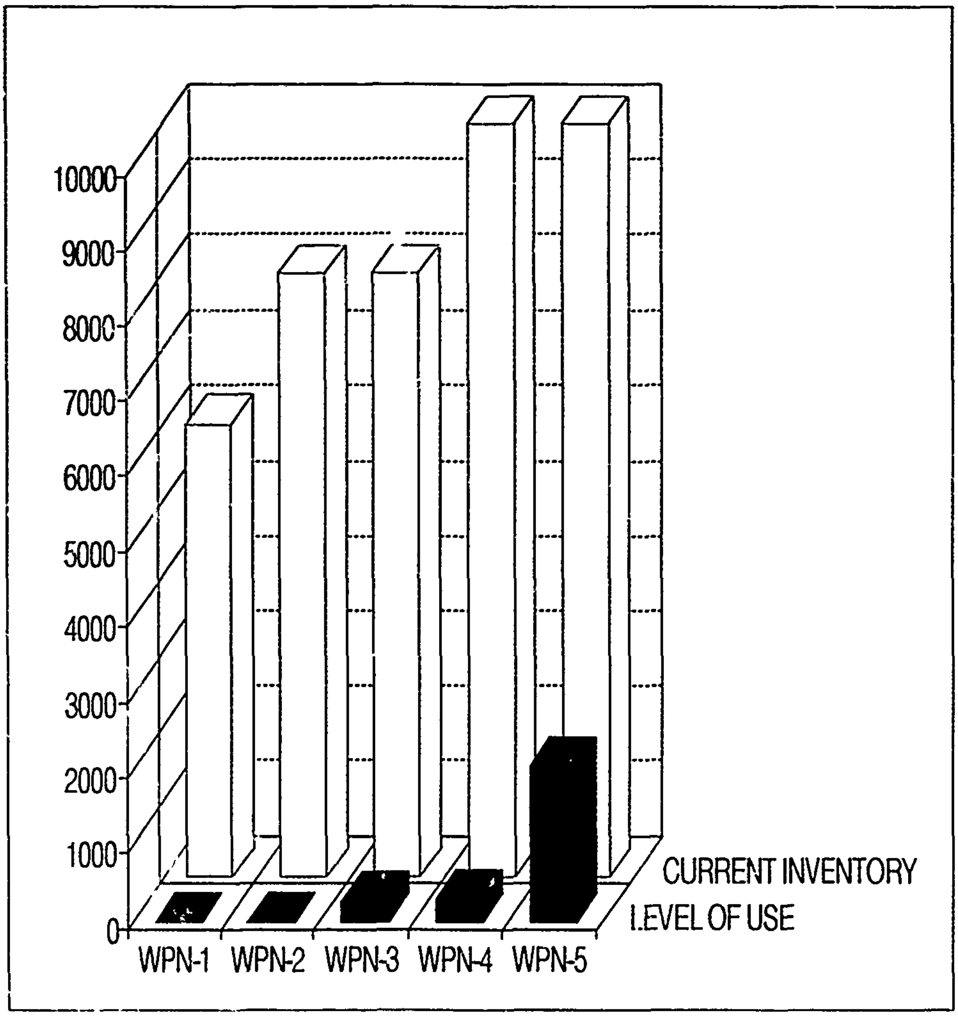

Figure 4.33. Number of Weapons Used

using one of the most expensive aircraft types at a level of 14.186 per sortie. Also, two of the most expensive weapons, weapon types 1 and 2, were used at significant levels. The probabilistic method used the same aircraft at an insignificant level and did not use either of the same expensive weapons. Only 15.24 of target type 4 in distance band 3 remained unattacked. The model loaded a diversity of weapon types to aircraft type 2; the remaining aircraft types were loaded with weapon type 5 (Figure 4.34).

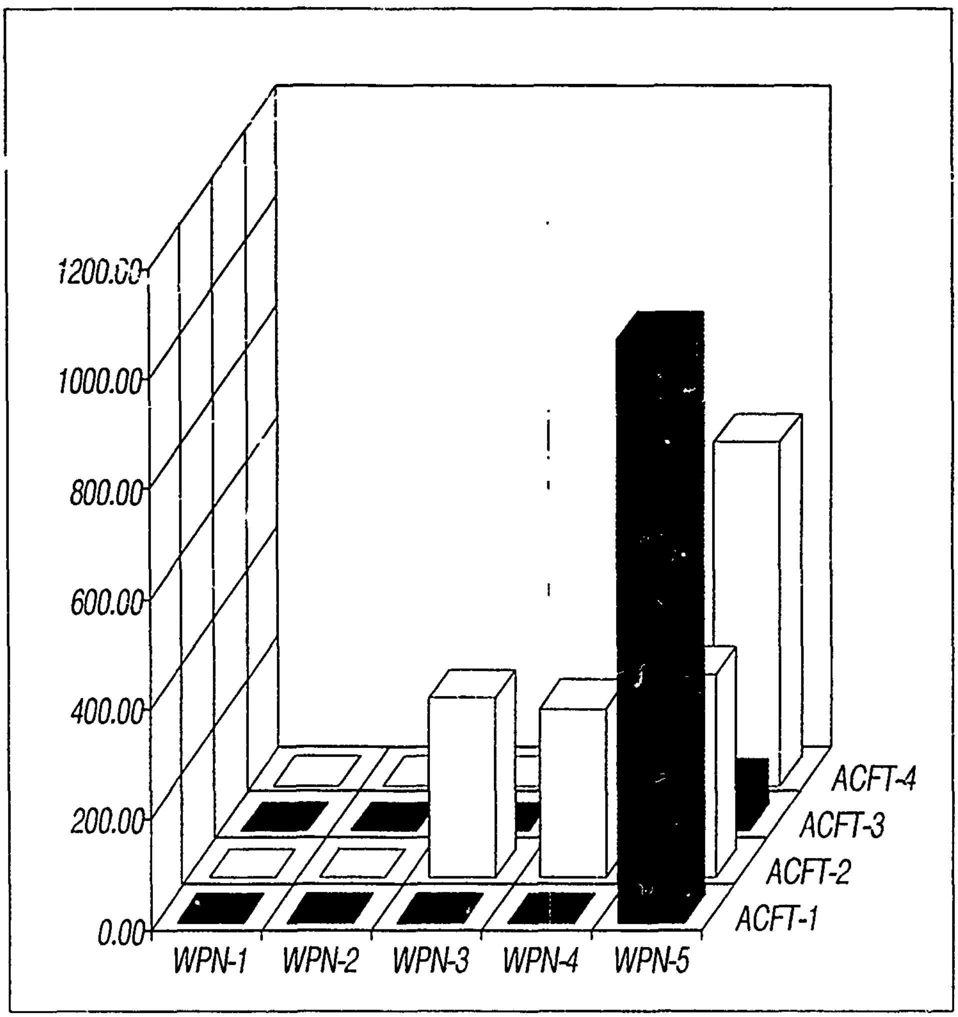

Figure 4.34. Use of Weapons by the Aircraft

In terms of the mission percentages, none of the marginal values turned out to be significant. However, after analyzing the marginal values of the mission-percentage constraint, it could be concluded that in the first distance band, the use of aircraft type 1 against target types 2, 3 and 4 would contribute more than the present allocation did.

4.6 Goal Programming Form of the Probabilistic Model

None of the modifications that the study implemented changed the linear construction of Mixmaster. The idea was to investigate the possible ways to incorporate cost and budget issues in Mixmaster. Given that the probabilistic approach to determine the cost coefficients did not upset the linear construction of Mixmaster, the study followed the same logic used in the goal programming method. The probabilistic method showed a biased and conservative behavior in the aircraft/weapon selection process. It was expected that the goal programming construction would execute the selection process without bias, because the goals are equally weighted. Therefore, the selection process in the goal programming would not preempt the use of expensive aircraft and weapons. However, this behavior does not mean that the selection process is done without regard to the cost of the aircraft and weapons. On the contrary, the selection process accounts for the achievement of desired levels for the goals associated with costs, and to achieve the levels specified by the goals, the model must implement the best allocations. For example, to achieve a level of 0.16 for *Target Value Destroyed per Dollar*, the model must find the best combinations that would sum to 0.16 for *Target Value Destroyed per Dollar.* Selecting only the most expensive aircraft and weapons would underachieve this goal unless the model destroys the desired number of targets with fewer aircraft weapon combinations. Indeed, the results show that, unlike the probabilistic method, the goal programming version allocated the most expensive weapons in doing the missions.

The goal programming version achieved a level of 0.14 for the objective function value and used all aircraft types (Figure 4.35). The level of use for aircraft type 3 was not significantly different than that of the probabilistic model, but aircraft type 1 was used three times more in the goal programming form. In addition, the level of use for aircraft type 4 was observed to be more significant than that of the probabilistic model—almost four times more. The summary of the numerical results is presented in Table 4.13.

Table 4.13. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	4218.436	0	0	0	529.741	3.292
ACFT-2	367.303	0	273.595	301.525	245.201	21.45
ACFT-3	0	0	0	0	79.61	1.174
ACFT-4	0	1932.452	0	1994.064	0	89.183
WPN. USAGE	4586.739	1932.452	273.595	2295.589	854.552	

The substantial difference came from the weapons allocation. The goal programming model selected mostly the weapons with substantial costs. The model executed the selection process without bias. Indeed, weapon type 1, and weapon type 2 were used at significant levels compared to the probabilistic model (Figure 4.36). Similar to the probabilistic model, a 99% target destruction rate was achieved. Only 11.513 of target type 4 in the third distance band remained unattacked. The model presented a diversity in weapons allocations process for aircraft type 2 (Figure 4.37).

The mission-percentage constraint produced some significant marginal values. For example, the marginal value from the ACFT-1.TGT-4.DIST-3 mission-percentage constraint was -0.995; the one from the ACFT-3.TGT-4.DIST-3 mission-percentage constraint was -0.978, the one from the ACFT-4.TGT-4.DIST-2 was -0.948. This means that to allocate one more sortie to one of these missions would decrease the deviations in the objective function by aproximately one unit. For the goal programming model, the marginal values imply a unit decrease in the objective function if the related resource is increased by one unit. So the marginal evaluation affects the deviations from the goals, and hence, their levels of achievement

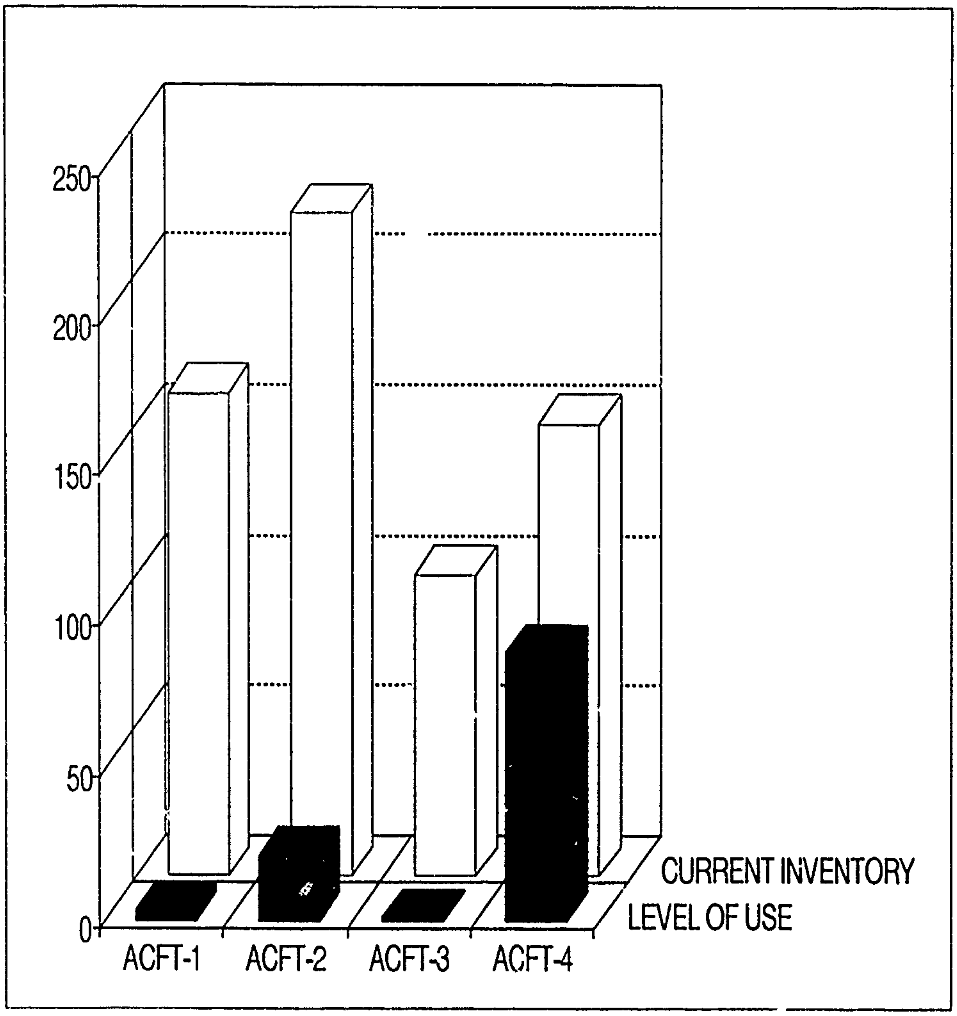

Figure 4.35. Number of Aircraft Per Sortie

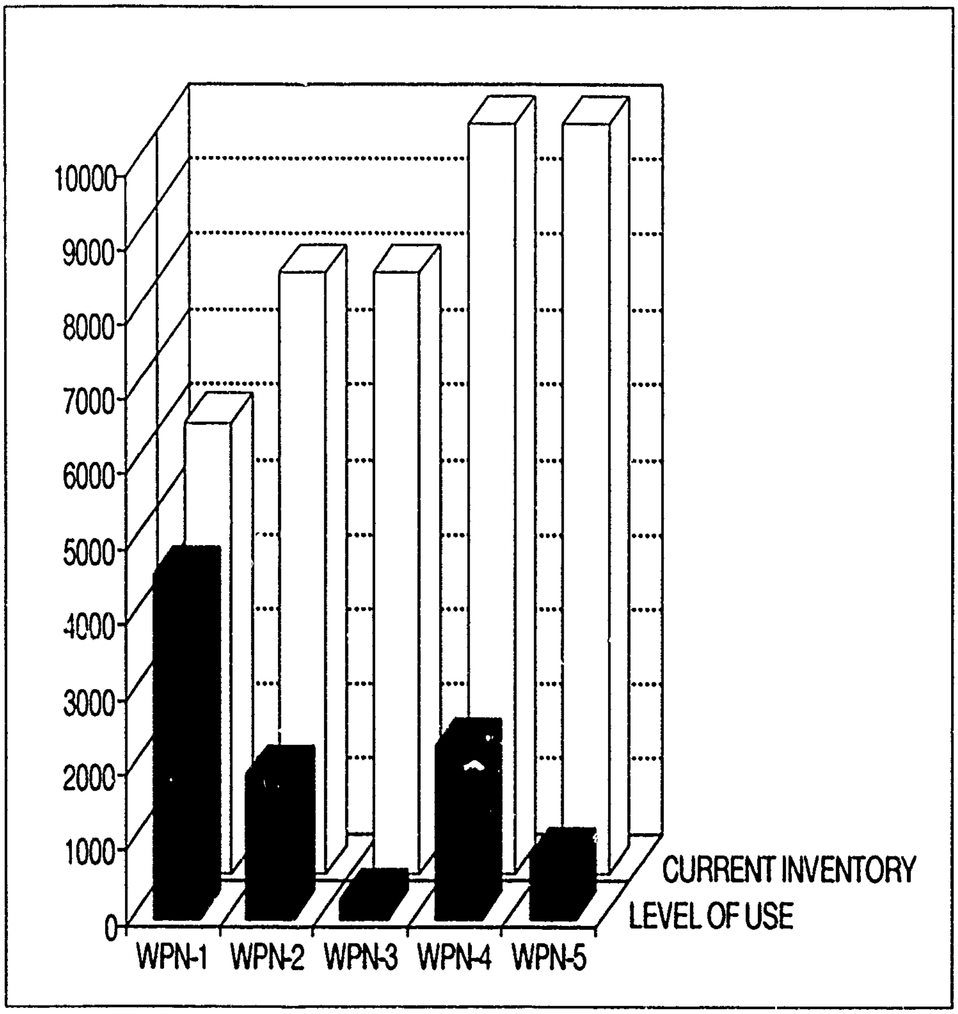

Figure 4.36. Number of Weapons Used

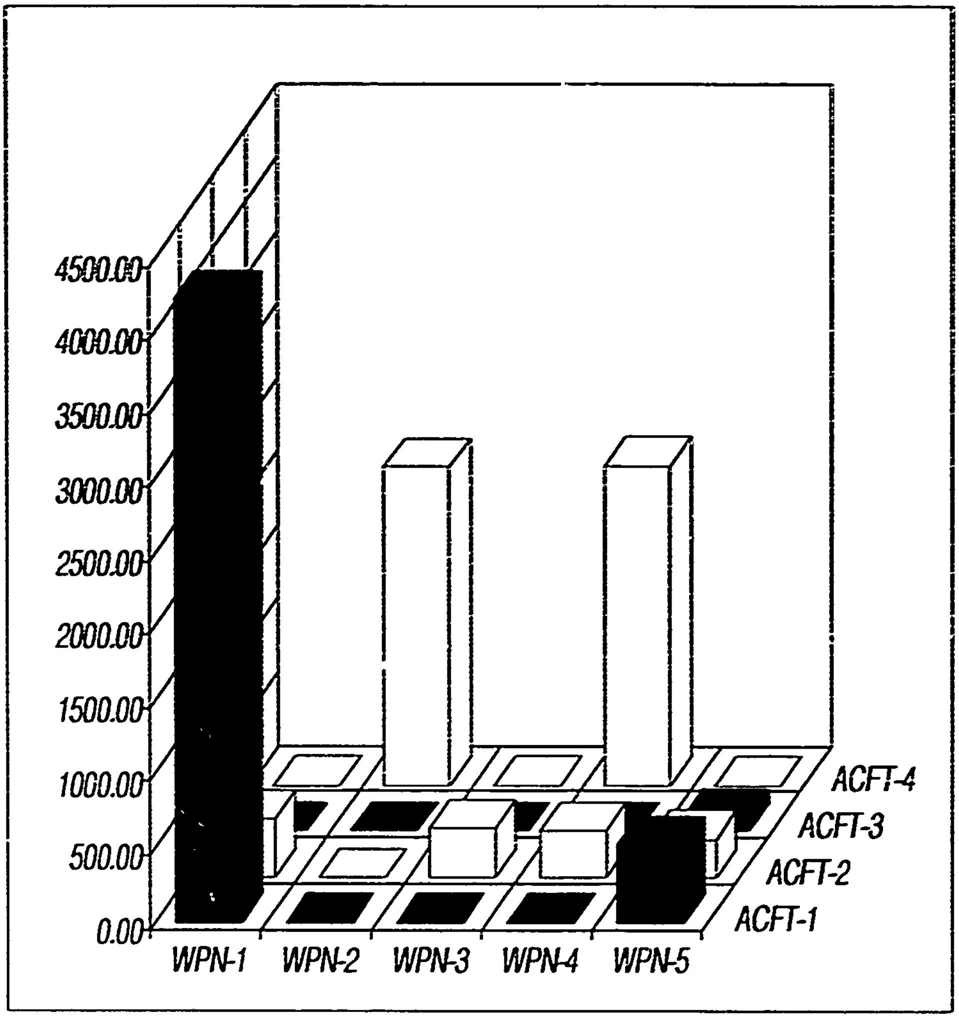

Figure 4.37. Use of Weapons by the Aircraft

As in the first goal programming model, to decide whether there was a need to buy more aircraft or weapons the values of the procurement variables were checked. All of the procurement variables turned out to be zero, implying that there was no need to buy more aircraft or weapons. The current inventory levels were adequate to accomplish the missions.

4.7 *Procurement Variables*

A case was generated to determine whether the goal programming method would, in fact, allow the procurement variables to be in the solution. To generate this case, the right-hand-side values of the aircraft availability constraint were diminished to a considerably low level for each type of aircraft. The goal programming application of Mixmaster was run subject to low aircraft resources. The levels of the weapon resources were kept unchanged. The assumed inventory for the aircraft is presented in Table 4.14.

Table 4.14. Assumed Inventory Levels of Aircraft

ACFT-1	10
ACFT-2	20
ACFT-3	10
ACFT-4	15

The model underachieved the desired level of *Target Value Destroyed per Dollar* by approximately 50%. The level of achievement was 0.088 and the desired level was 0.16. But most importantly the model used the aircraft resources completely; in addition, it employed the aircraft procurement variables to accomplish the missions. The current weapon inventory was adequate for the requirements of the campaign. The solutions showed that the model employed 16.451 of aircraft type 1, 20.030 of aircraft type 2, 10 of aircraft type 3, and 23.030 of aircraft type 4 per sortie (Figure 4.38).

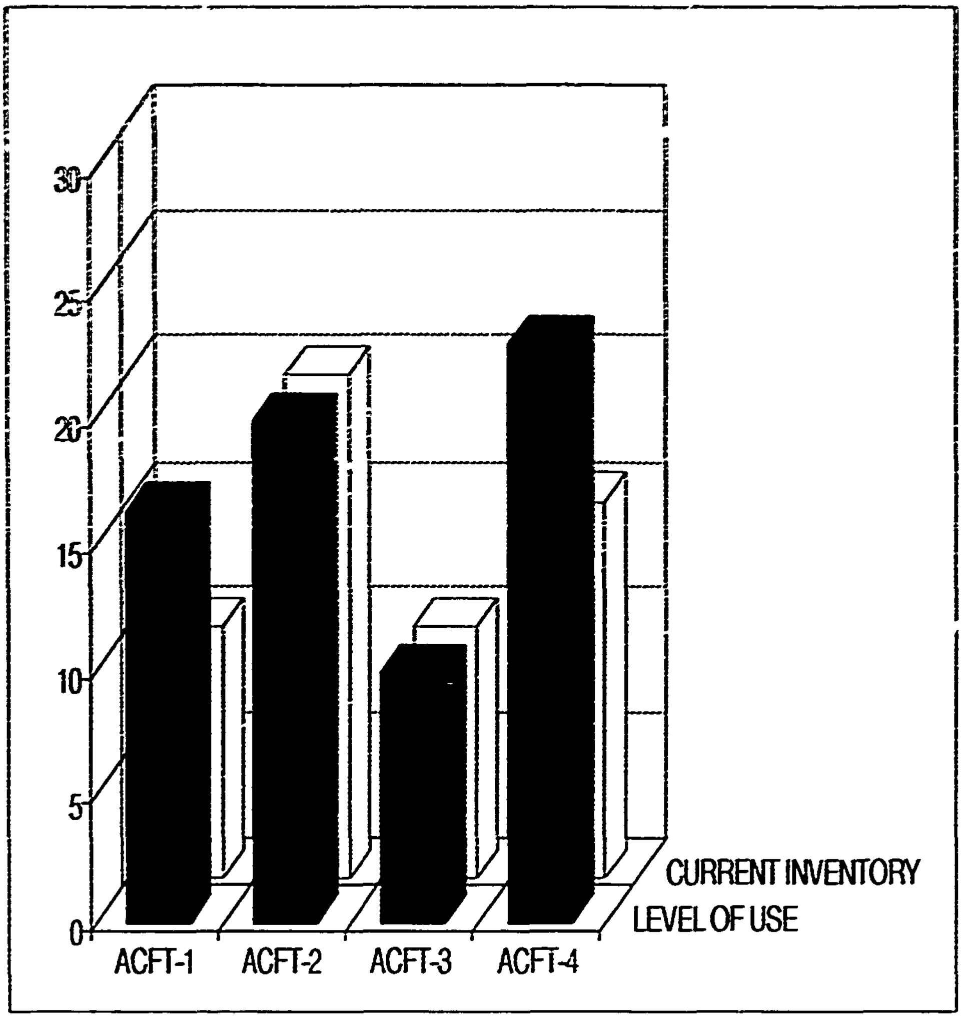

Figure 4.38. Number of Aircraft Per Sortie

If the current method of JSG had been implemented to determine the procurement needs, the levels of use for aircraft would have been obtained by computing the difference from the current aircraft inventory. However, in the goal programming approach, the procurement variables would present the difference that would have been computed under the current approach. Indeed, in this case, the procurement variables are:

$$d^+_{ACFT-1} = 6.451$$

$$
\begin{aligned}
d^{+}_{ACFT-2} &= 0.030 \\
d^{+}_{ACFT-3} &= 0 \\
d^{+}_{ACFT-4} &= 8.030
\end{aligned}
$$

indicating that there is a need to buy 6.451 more aircraft of type 1, 0.030 of type 2, and 8.030 of type 4. Consequently, a decision should be made taking these procurement variables into account.

4.8 Summary

This chapter presented the results obtained from the implementations of modifications. Since the simple cost employment methods did not include the cost and budget figures in the configuration of the model simultaneously, they proved to be unsatisfactory except for one feature that the last simple cost employment method provided. This feature is that the last simple cost employment method introduces the maximum *Target Value Destroyed per Dollar* that can be achieved within the given data base and the mission plan. The research adopted that value to use in the goal programming approach as the desired level of the *Target Value Destroyed per Dollar*.

The goal programming application generated important features. First, it allowed the inclusion of both cost and budget figures in the configuration simultaneously. Secondly, since it employs the procurement variables directly in the configuration, there were no requirements for further calculations to determine the procurement needs. Finally, the aircraft and weapons allocations process was not biased by the substantial cost differences.

The probabilistic approach introduced the time issue. The probabilities, the expected kill values and the costs were all time-dependent variables. Consequently, the probabilistic method proved to be a conservative method because the model achieved almost the same target destruction rate by using considerably fewer aircraft and weapons than the previous methods. The reason was interpreted to be the time factor. For example, in the previous method, an aircraft was launching all the weapons that were loaded. But in the probabilistic method, the expected number of attacks, and hence, the number of weapons launched are time-dependent. Therefore, the research also noted that the probabilistic method would produce more accurate numbers given that the parameters λ and μ are accurate. However, the probabilistic method did not include both the cost and budget figures simultaneously. Follow-on calculations were required to determine the procurement needs. Due to these disadvantages, the research developed a goal programming version of the probabilistic methods.

The goal programming version of the probabilistic approach also employed the procurement variables directly in the configuration. The cost and budget figures were involved simultaneously. No further calculations were required to determine procurement needs. The model was not influenced by the substantial cost differences which could cause a bias. Furthermore, introducing the time variable in the calculations promised more accurate results given that the parameters λ and μ are estimated accurately.

The following chapter discusses the case studies. Two different mission plans are employed in the complete cost model, the goal programming model, the probabilistic model and the goal programming version of the probabilistic model. The effects of the leading constraint and the consistency of the modifications are investigated.

V. CASE STUDIES

5.1 Introduction

This chapter reports the behavior of the model subject to various mission plans. For investigating the overall consistency of the modifications and the suggested models, the study executed two case studies. These case studies employed different mission percentages for each type of aircraft in the scenario. Changing the mission percentages—they are actually the percentages of the total sorties that should not be exceeded—meant changing the leading constraint. This constraint, as explained previously, has already been leading the model as the scenario requires. The last simple cost modification, the goal programming approach, the probabilistic approach to determine the cost coefficients in the objective function and its goal programming version were run for these two cases. The first case is the mission plan that defines the upper bounds of sortie allocation to targets for each aircraft by mission. The second mission plan defines the same upper bounds distance-wise. The aircraft fly against all targets but in only one distance band.

In case 1, aircraft type 1 is dedicated to SAM suppression. 50% of the total sorties that aircraft type 2 fly are planned for logistic suppression; the remaining 50% are planned for close air support (CAS). Aircraft type 3 is dedicated to airbase attack. Finally, aircraft type 4 is planned to be used for CAS in the first distance band. These allocations are presented in Table 5.1.

In case 2, aircraft type 1 is planned to fly airbase attack, SAM suppression, logistic suppression and CAS only in the second distance band with equal numbers of sorties. Aircraft type 2 is planned differently. Aircraft type 2 flies 35% of its total sorties for airbase attack, 35% for SAM suppression and 30% for logistic suppression only in the first distance band. Aircraft type 2 does not fly CAS missions. The plan dictates aircraft type 3 to fly equal numbers of sorties for airbase attack, SAM suppression, logistic suppression, and CAS in the third distance band. Finally, the plan dedicates 20% of the total sorties that aircraft type 4 flies to SAM suppression, 20% to logistic suppression and 60% to CAS, in the first distance band. The allocations of case 2 are described in Table 5.2.

These cases were modeled using the complete cost model (i.e., including the aircraft, weapon and sortie costs), for the goal programming method, for the probabilistic approach and for the goal programming version of the probabilistic approach.

5.2 Case 1

5.2.1 Simple Cost Employment Method. This model is the complete cost model whose objective function includes the aircraft, weapon and sortie costs. The model achieved a level of 0.0889 *Target Value Destroyed per Dollar.* The level of use for aircraft type 3 was considered to be remarkable since this aircraft type was the most expensive type (Figure 5.1). The summary of the numerical results is presented in Table 5.3.

The target destruction rate was 78% with 488 targets of type 4 in the third distance band unattacked. As a tactical suggestion, the plan confined aircraft type 4 in the first distance band against target type 4. The number of sorties assigned to this aircraft type was more than what is required to destroy target type 4. The solution showed that aircraft type 4 employed 19% of its total effort. This percentage was sufficient to perform the mission. However, most of the same type targets in the third distance band remained unattacked. The upper bound on sorties that aircraft type 2 was supposed to fly against target type 4 in the third distance band was not adequate to permit destruction of all type 4 targets. Furthermore, employing another aircraft type was implausible in the mission plan. The model acted with bias in weapon usage. Although it allocated the most expensive weapon, which was weapon type 1, at a level of 2652.175, it did not use weapon type 2 (Figure 5.2). The weapon that the model used most was weapon type 4. Figure 5.3 shows that aircraft type 2 had the diversity of weapon allocations.

Table 5.1. Mission Plan of Case 1

		Airbase Attack TGT-1	Sam Suppression TGT-2	Logistic Suppression TGT-3	Close Air Support TGT-4
ACFT-1	DIST-1	NONE	0.30	NONE	NONE
	DIST-2	NONE	0.30	NONE	NONE
	DIST-3	NONE	0.40	NONE	NONE
ACFT-2	DIST-1	NONE	NONE	0.20	NONE
	DIST-2	NONE	NONE	0.10	0.25
	DIST-3	NONE	NONE	0.20	0.25
ACFT-3	DIST-1	0.25	NONE	NONE	NONE
	DIST-2	0.35	NONE	NONE	NONE
	DIST-3	0.40	NONE	NONE	NONE
ACFT-4	DIST-1	NONE	NONE	NONE	1.00
	DIST-2	NONE	NONE	NONE	NONE
	DIST-3	NONE	NONE	NONE	NONE

Table 5.2. Mission Plan of Case 2

		Airbase Attack TGT-1	Sam Suppression TGT-2	Logistic Suppression TGT-3	Close Air Support TGT-4
ACFT-1	DIST-1	NONE	NONE	NONE	NONE
	DIST-2	0.25	0.25	0.25	0.25
	DIST-3	NONE	NONE	NONE	NONE

		TGT-1	TGT-2	TGT-3	TGT-4
ACFT-2	DIST-1	0.35	0.35	0.30	NONE
	DIST-2	NONE	NONE	NONE	NONE
	DIST-3	NONE	NONE	NONE	NONE

		TGT-1	TGT-2	TGT-3	TGT-4
ACFT-3	DIST-1	NONE	NONE	NONE	NONE
	DIST-2	NONE	NONE	NONE	NONE
	DIST-3	0.25	0.25	0.25	0.25

		TGT-1	TGT-2	TGT-3	TGT-4
ACFT-4	DIST-1	NONE	0.20	0.20	0.60
	DIST-2	NONE	NONE	NONE	NONE
	DIST-3	NONE	NONE	NONE	NONE

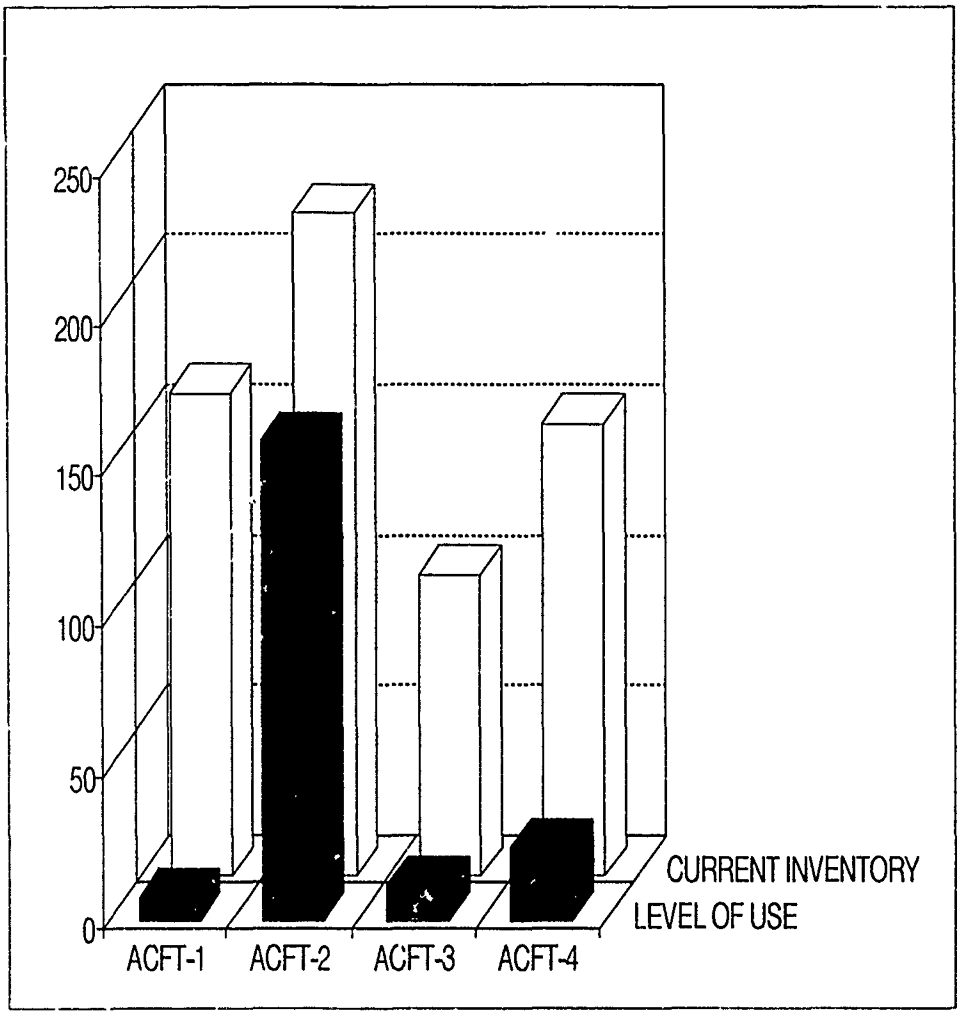

Figure 5.1. Number of Aircraft Per Sortie

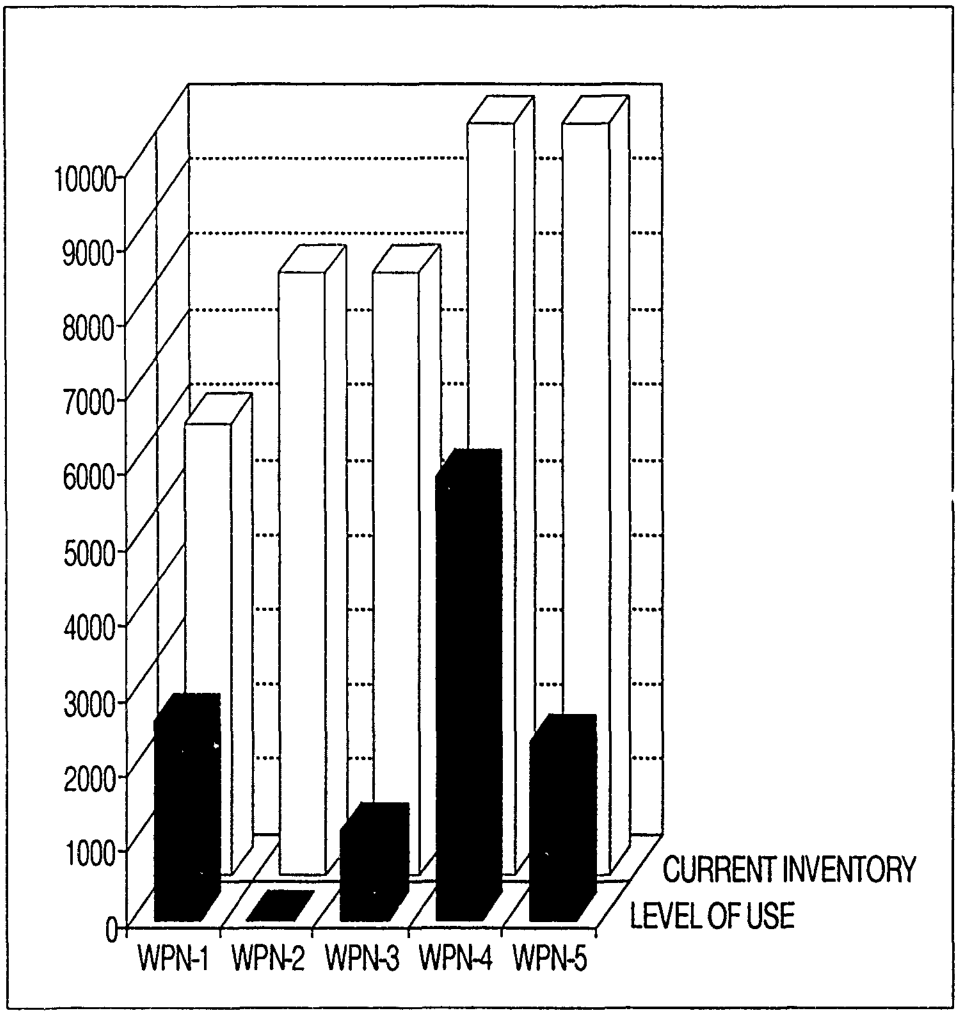

Figure 5.2. Number of Weapons Used

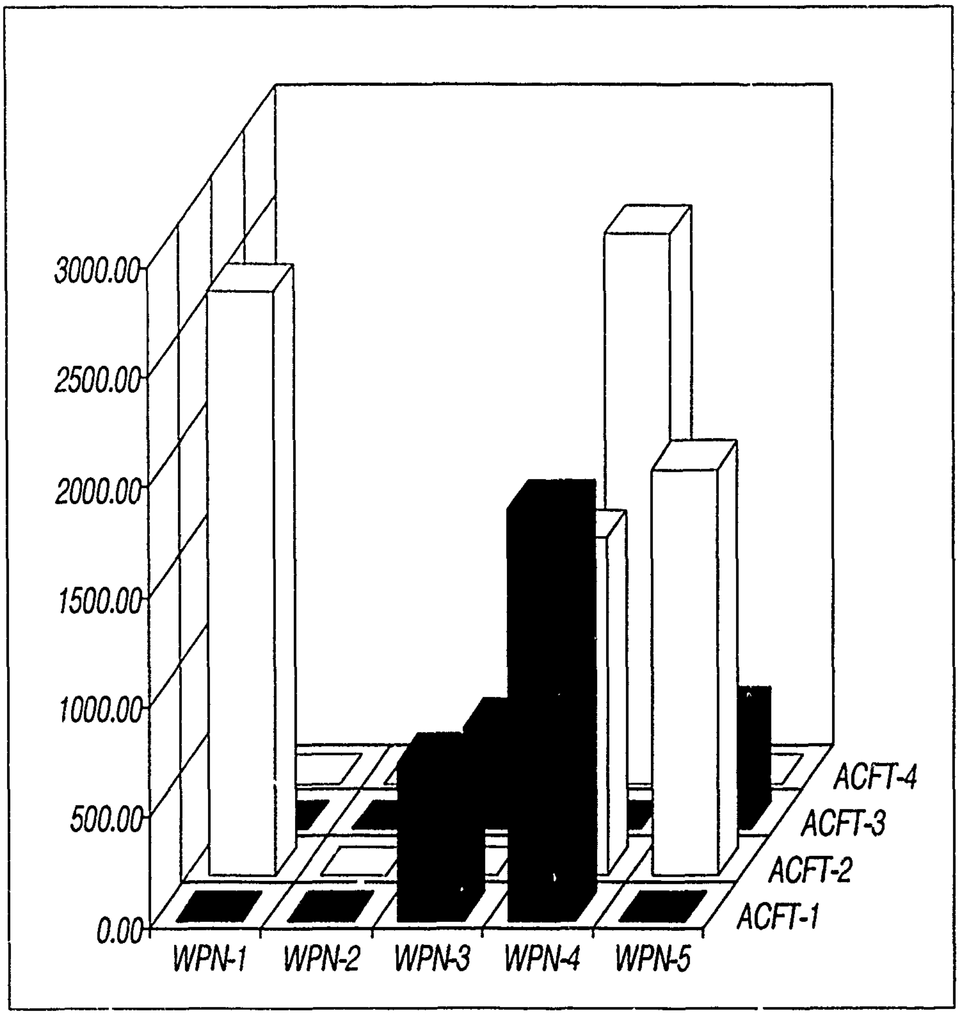

Figure 5.3. Use of Weapons by the Aircraft

Table 5.3. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	722.892	1873.922	0	7.494
ACFT-2	2652.175	0	0	1532.167	1843.028	159.631
ACFT-3	0	0	454.545	0	518.909	11.915
ACFT-4	0	0	0	2500	0	24.167
WPN. USAGE	2652.175	0	1177.437	5906.088	2361.937	

The marginal values obtained from the mission-percentage constraint were insignificant, but some of the sortie allocations were not used at all. With a close examination of the upper bounds for sortie allocations, the mission planner can easily assign the sorties which were not flown to missions that would contribute more to the objective. For instance, in the mission-percentage constraint the sortie allocations of aircraft types 1 and 2 that remained idle could be used for airbase attack, SAM suppression, and CAS missions in each distance band.

In terms of the procurement decision, the levels of use of aircraft and weapon types implied that there was no need to buy more. The current inventory levels were adequate to supply the mission requirements during the first time interval of the campaign.

5.2.2 Goal Programming. The goal programming results were similar to those obtained in the complete cost model. Indeed, even with 50% underachivement, the value of *Target Value Destroyed per Dollar* appeared to be nearly the same. The negative deviate d_1^- from the first goal was 0.074, and the desired level of achievement was 0.16; therefore, the first goal was underachieved. The actual value of *Target Value Destroyed per Dollar* was 0.086. However, the same level of achievement of the goals did not mean the same allocations of aircraft and weapons. Aircraft types 2 and 4 were used more significantly than the other types (Figure 5.4). The summary of the numerical re^ults is presented in Table 5.4

Table 5.4. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	722.892	1873.922	0	7.494
ACFT-2	6000	12.748	0	1532.167	1282.933	204.301
ACFT-3	0	0	454.545	0	518.909	11.92
ACFT-4	0	634.318	6822.563	2424.242	0	49.28
WPN. USAGE	6000	647.07	8000	5830.33	1801.84	

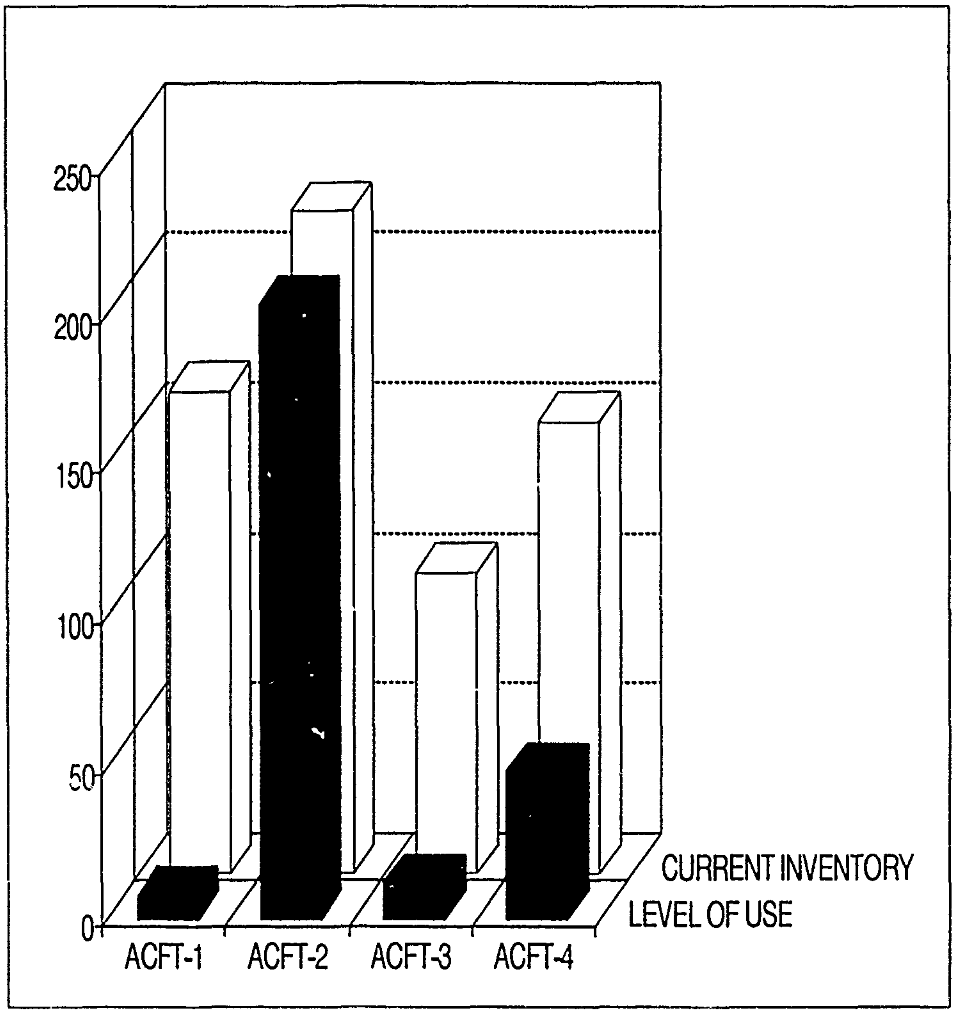

Figure 5.4. Number of Aircraft Per Sortie

The target destruction rate was 100%, whereas 40% of target type 4 remained unattacked in the complete cost model. But to destroy all the targets, the model used more weapons including the most expensive ones. Weapon types 1 and 3 were exhausted by the model (Figure 5.5). It is noteworthy that the model used the most expensive weapon in its allocation process. This behavior indicated that the goal programming structure worked better with the leading constraint.

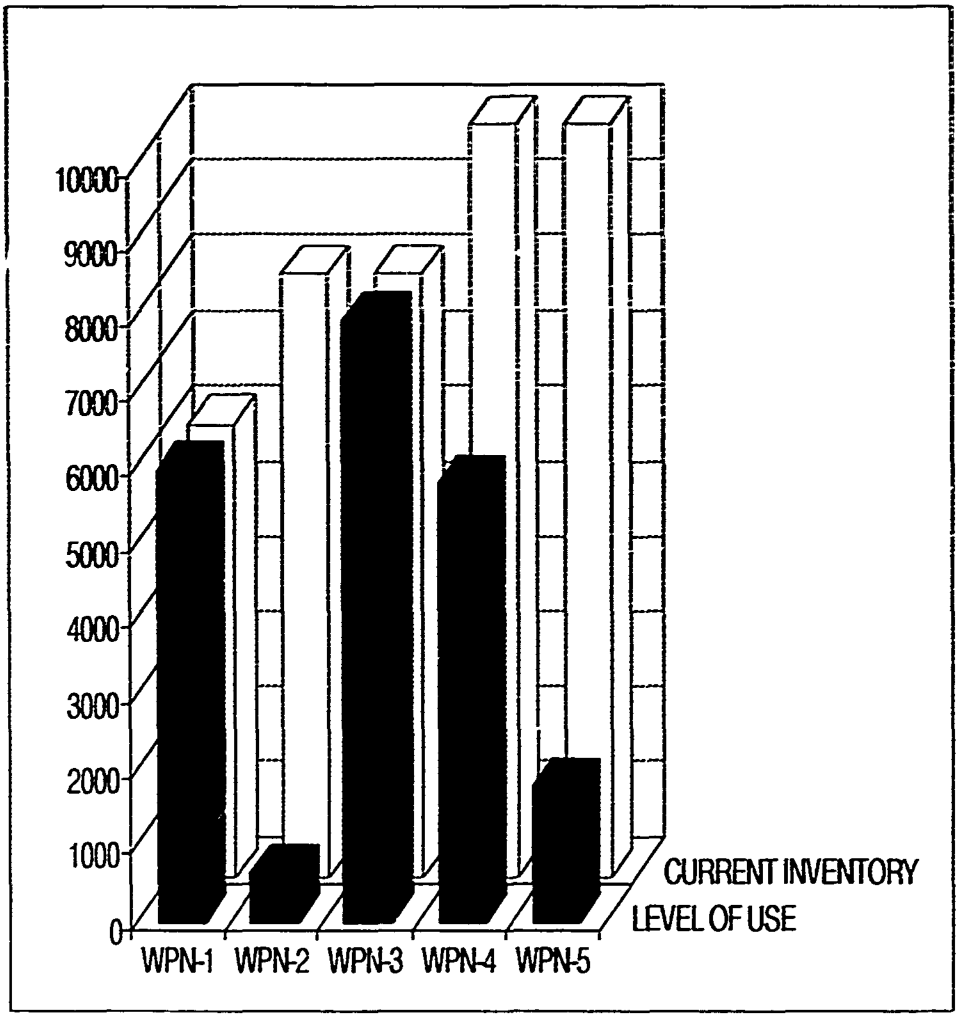

Figure 5.5. Number of Weapons Used

The deviations from the goals were not present in the solution except for the negative deviate from the first goal d_1^-. This implied the satisfaction of all the goals except the first one. But even this level of achievement for the target value destroyed was considered to be satisfactory since all the targets were destroyed.

In the evaluation of the marginal values from the mission-percentage constraint, no significant marginal values were observed. However, as far as the study was concerned, the number of sorties remaining idle could be allocated to aircraft type 3 against target type 4. There were other sorties to allocate to aircraft type 4 against target types 1 and 2, but the assignment of aircraft type 4 to target types 1 and 2 would be a tactical misuse of the aircraft. Aircraft types 2 and 4 had the diversity of weapon allocations. All aircraft and weapon types were employed by the model (Figure 5.6).

In terms of the procurement decision, the study concluded that there was no need to buy more aircraft or weapons because the procurement variables d_a^+ and d_m^+ did not appear in the solution. Furthermore, within the given time scenario, the model produced a solution that destroyed all the targets using the resources in the current inventory.

5.2.3 Probabilistic Method. The probabilistic method results of the case 1 plan were conservative as they were with the original plan. However, the levels of use of aircraft types were significantly different than those obtained with the baseline plan. A level of 0.0356 *Target Value Destroyed per Dollar* was achieved as the objective function value. The missionwise allocation of the aircraft made the model assign even the most expensive aircraft, which is aircraft type 1 (Figure 5.7). The summary of the numerical results is presented in Table 5.5.

It was expected that the model would not use expensive weapons. Indeed, weapon types 1, 2, and 3 were not employed at all. Overall evaluation of the weapon use was that the model's weapon selection was not sound from the tactical point of view (Figure 5.8). There was no diversity of weapon usage. All of the missions were accomplished by using only two weapon types (Figure 5.9).

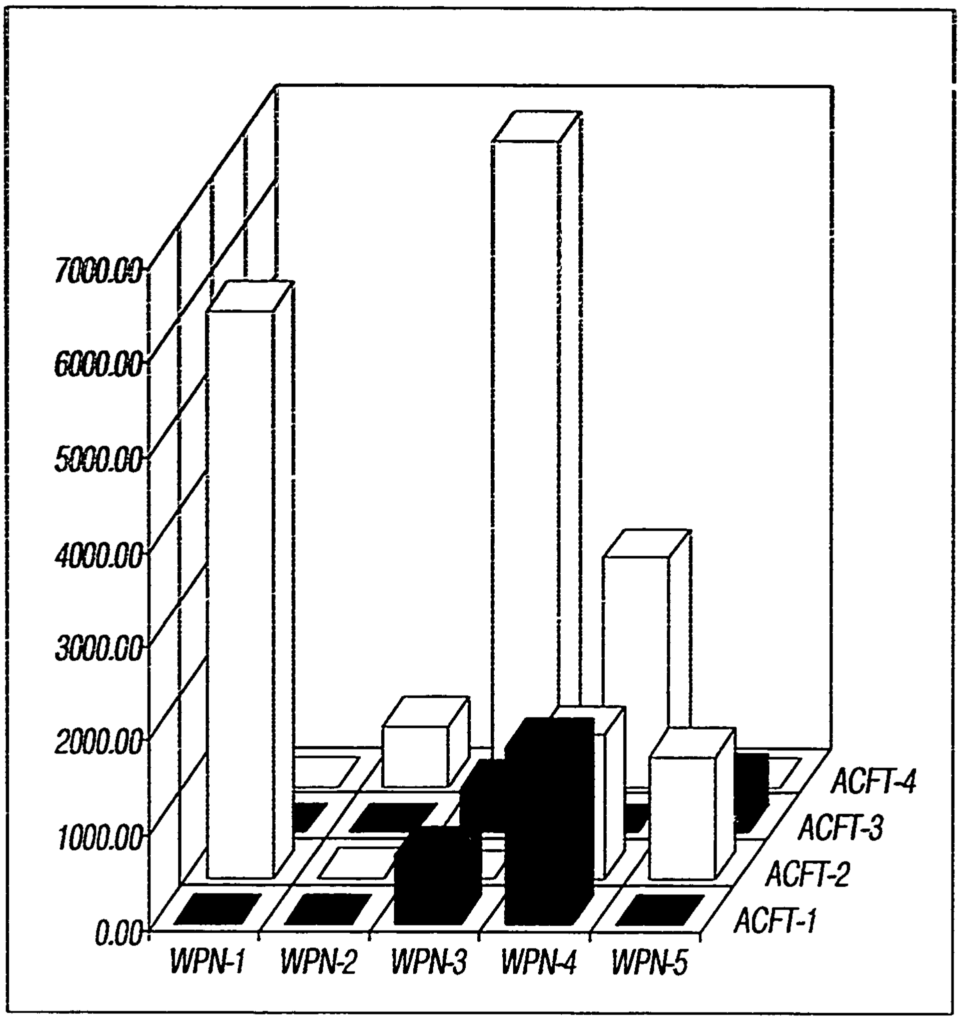

Figure 5.6. Use of Weapons by the Aircraft

Table 5.5. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	852.351	0	11.338
ACFT-2	0	0	0	0	778.311	18.1
ACFT-3	0	0	0	0	66.208	1.913
ACFT-4	0	0	0	0	796.083	34.678
WPN. USAGE	0	0	0	852.351	1640.602	

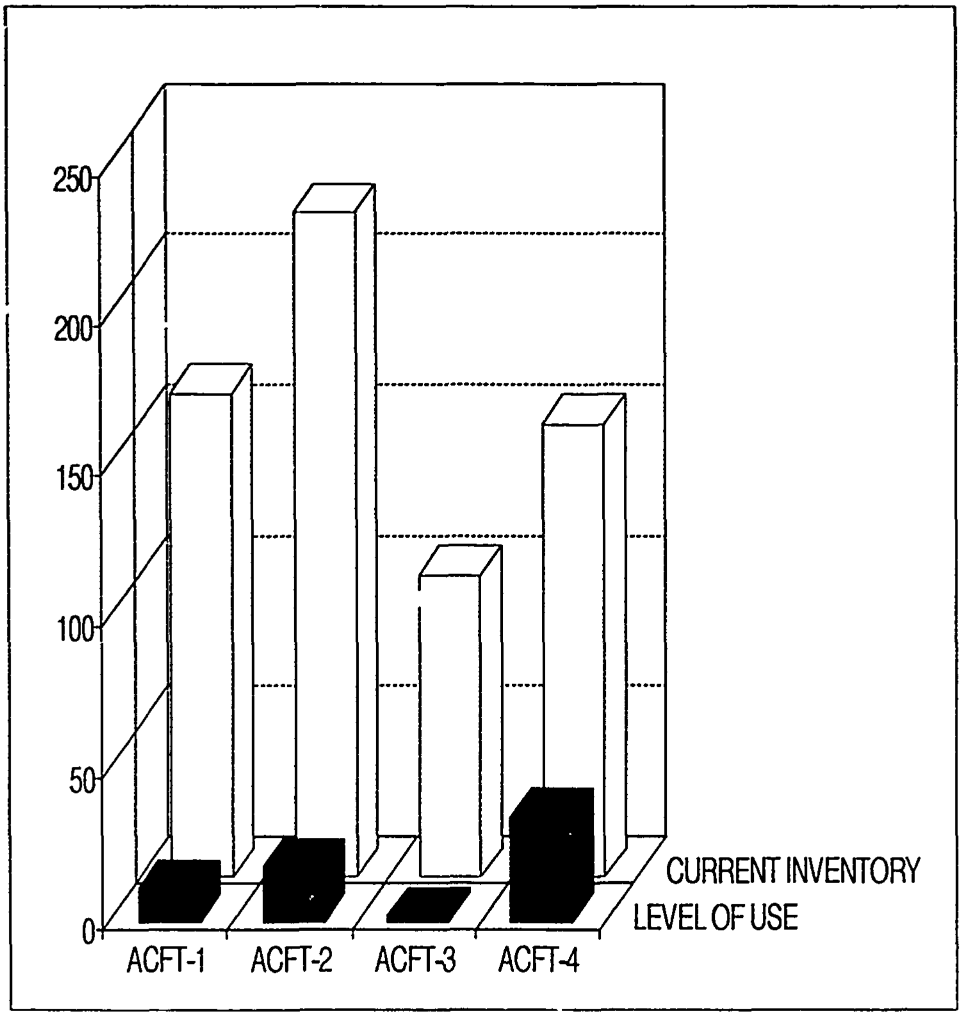

Figure 5.7. Number of Aircraft Per Sortie

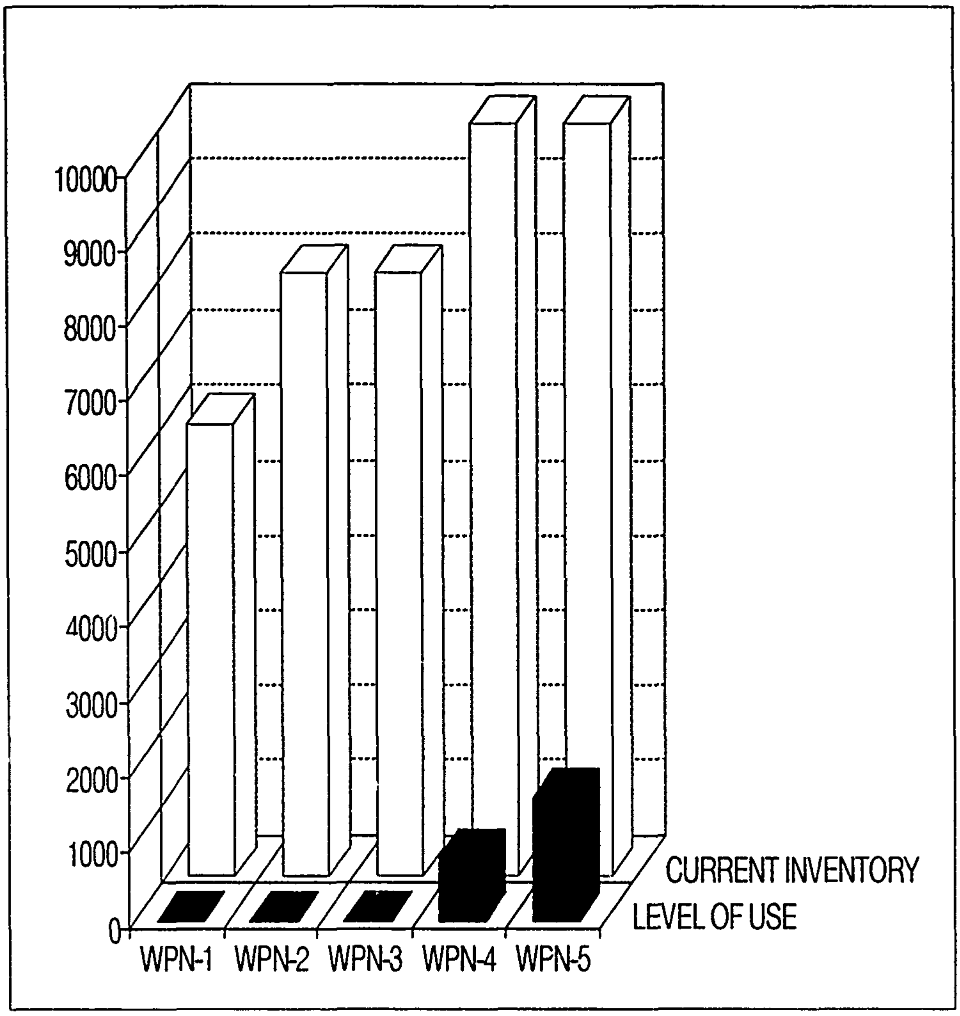

Figure 5.8. Number of Weapons Used

The target destruction rate was 69%, and only 51% of target type 4 could be destroyed. The number of sorties assigned to aircraft type 2 against target type 4 in the second and third distance bands was not sufficient. The marginal values from the mission-percentage constraint implied that the case 1 mission plan was not well planned. There were very significant marginal values. The values from the ACFT-2.TGT-1.DIST-1, 2, and 3 mission-percentage constraints were the most significant marginal values at a level of 0.002. These significant marginal values implied that the use of aircraft type 1 against target type 1 would contribute more than the current allocation did. In the missionwise planning, the numbers of sorties that aircraft types were supposed to fly were not equally distributed. Consequently, there were idle sorties that were not flown at all. By observing the marginal values, the results suggested the assignment of the remaining—not flown—sorties to the missions with high marginal values.

In terms of the procurement needs, the levels of use for aircraft and weapon types were not more than the current inventory levels. There was no need to buy new aircraft or weapons. The current inventory was sufficient to supply the requirements of the campaign. Furthermore, it should be noted that the evaluation of the marginal values should be done before the successive runs of the models to determine the actual procurement needs.

5.2.4 Goal Programming Version of the Probabilistic Method. The goal programming version of the probabilistic method produced very similar results to those obtained in the preceding section. The same value of 0.03 was achieved as the *Target Value Destroyed per Dollar*. Actually, the target value goal was underachieved. The desired level was 0.25 and the results showed that the negative deviate from the target value goal, d_1^-, was 0.22; hence, the difference was the level of achievement for this goal as 0.03 *Target Value Destroyed per Dollar*. The behavior of the goal programming model was not as conservative as it was in the preceding section. The goal programming construction eliminated the bias that was occurring in the

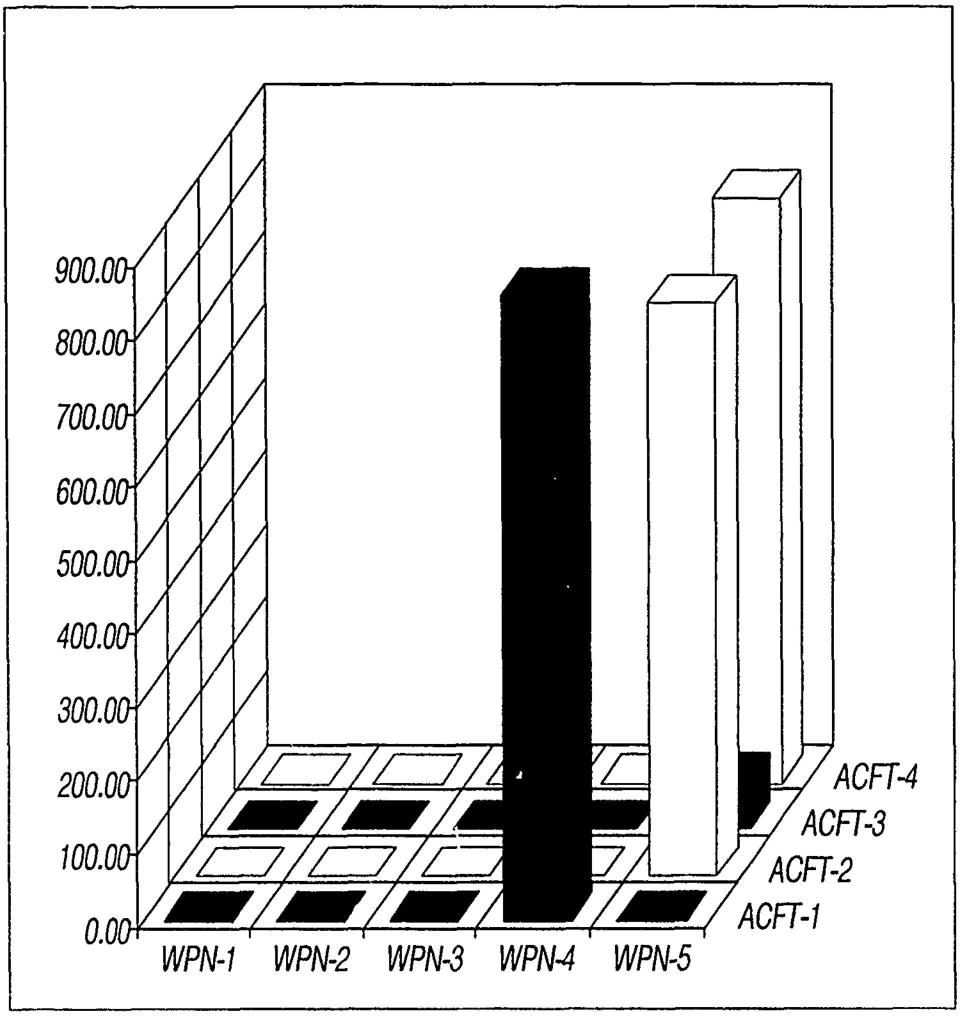

Figure 5.9. Use of Weapons by the Aircraft

Table 5.6. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	852.351	0	11.34
ACFT-2	3657.704	0	741.765	0	0	44.72
ACFT-3	0	0	0	0	66.208	1.91
ACFT-4	0	1685.915	0	2411.008	0	80.37
WPN. USAGE	3657.704	1685.915	741.765	3263.359	66.208	

aircraft/weapon selection process. In this construction, the model used all types of aircraft and weapons regardless of their substantial costs.

The model employed all aircraft types (Figure 5.10). The levels of use for aircraft types 2 and 4 were significant. However, the mission plan constrained the use of aircraft type 2 against target type 4 in the second and third distance bands. In addition there were substantial numbers of sorties that remained idle. The summary of the numerical results is presented in Table 5.6.

With this mission plan a level of 70% was achieved as the target destruction rate. Similarly, only 52% of target type 4 could be destroyed. There was a 1% increase compared to the results of the preceding model. To accomplish the missions with 70% target destruction rate, the model employed all weapon types (Figure 5.11). Clearly, the weapon that the model used most was weapon type 1 which, at the same time, was the most expensive weapon. The study interpreted that behavior as strong evidence for an unbiased selection process.

The marginal values from the mission-percentage constraint seemed to suggest the same thing. The marginal values from the ACFT-2.TGT-4.DIST-2 and 3 mission-percentage constraint were the most significant at levels of -0.932 and -0.940 respectively. The study concluded that if the number of preassigned sorties to be flown by aircraft type 2 against target type 4 in the second and third distance bands had been increased, the deviations from the goals would have been decreased. Also,

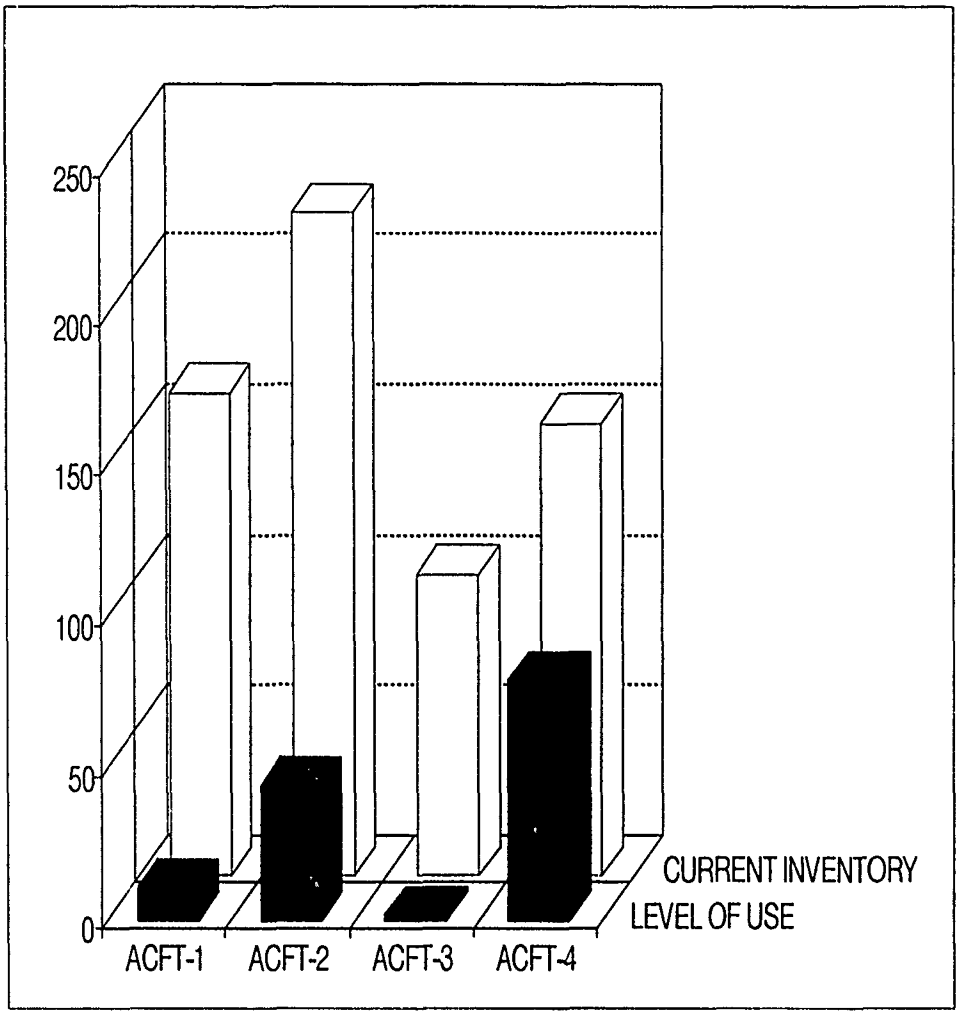

Figure 5.10. Number of Aircraft Per Sortie

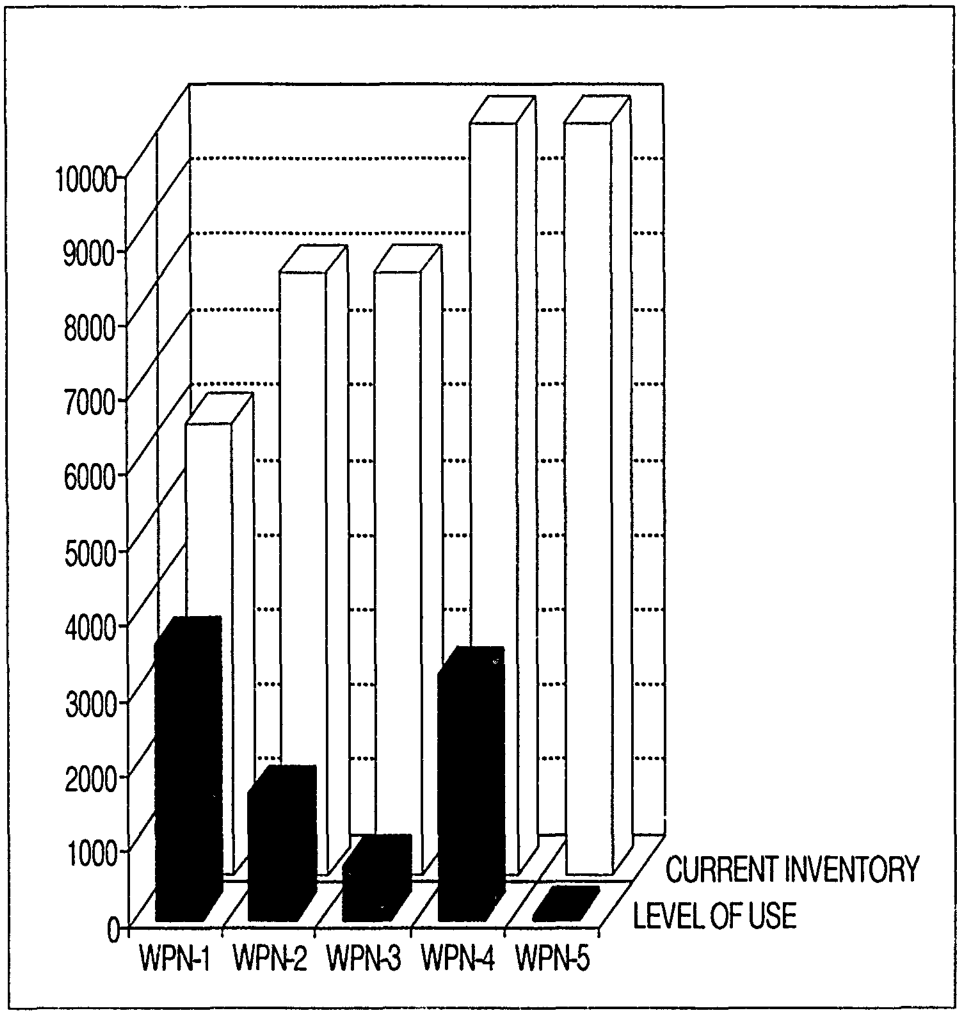

Figure 5.11. Number of Weapons Used

the study observed that a substantial number of sorties for aircraft type 1 were assigned unnecessarily. It was suggested that those idle sorties should have been allocated to missions with high marginal values. Therefore, the levels of achievement for the goals would have increased.

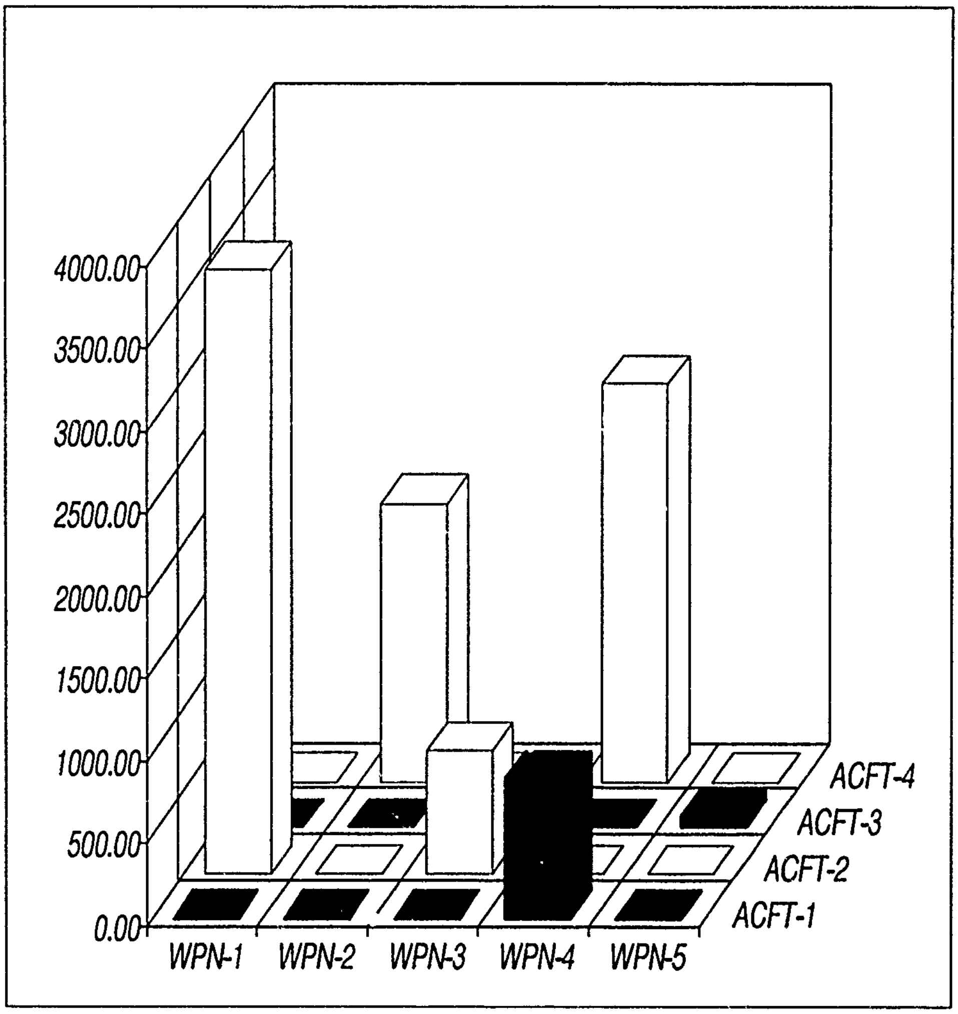

Figure 5.12. Use of Weapons by the Aircraft

The aircraft/weapon allocations presented more diversity than the probabilistic model did. In Figure 5.12, the allocations showed that there was a diversity in weapon usage. This diversity can be interpreted as the significant difference from the preceding model.

Although the goal programming model used more weapons, it did not incur any shortage in the current aircraft and weapon inventories. The goal programming model used more weapons than the probabilistic model in order to achieve the goals. This meant that a significant number of weapons were allocated although they had small contributions to the achievement of the goals. There was no need to buy new aircraft or weapons. The current inventory levels were adequate to supply the requirements for the campaign. In addititon, the evaluation of the marginal values should preempt the decision for further runs by updating the constraint with the initial results because the marginal values may suggest better missions.

5.3 Case 2

5.3.1 Simple Cost Employment Method. The mission plan of case 2 was made distancewise. Each aircraft type flew all the missions but only in one particular distance band. The complete cost model produced reasonable results that helped highlight how the leading constraint affected the model. The objective value of this method with the case 2 mission plan was 0.1382 *Target Value Destroyed per Dollar.* It was considerably higher than the value obtained with the case 1 mission plan. However, the case 2 mission plan was not as successful as the case 1 mission plan in terms of the number of targets destroyed. The target destruction rate was 54% which was significantly lower than the rate achieved with the case 1 mission plan. As another result of the case 2 mission plan, 73% of target type 4 could not be destroyed. To destroy 54% of the targets, the model employed all weapon types (Figure 5.13). It was remarkable that the leading constraint affected the selection process of the model because the level of use for aircraft type 3 was quite different from those of the modifications previously presented. The case 2 mission plan forced the model to use the aircraft for all the missions and preassigned them distancewise. The summary of the numerical results is presented in Table 5.7.

Table 5.7. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	1603.369	817.32	27.926
ACFT-2	0	0	0	1004.785	402.377	5.366
ACFT-3	0	0	935.673	0	1415.505	16.73
ACFT-4	0	0	0	2500	0	24.167
WPN. USAGE	0	0	935.673	5108.153	2635.202	

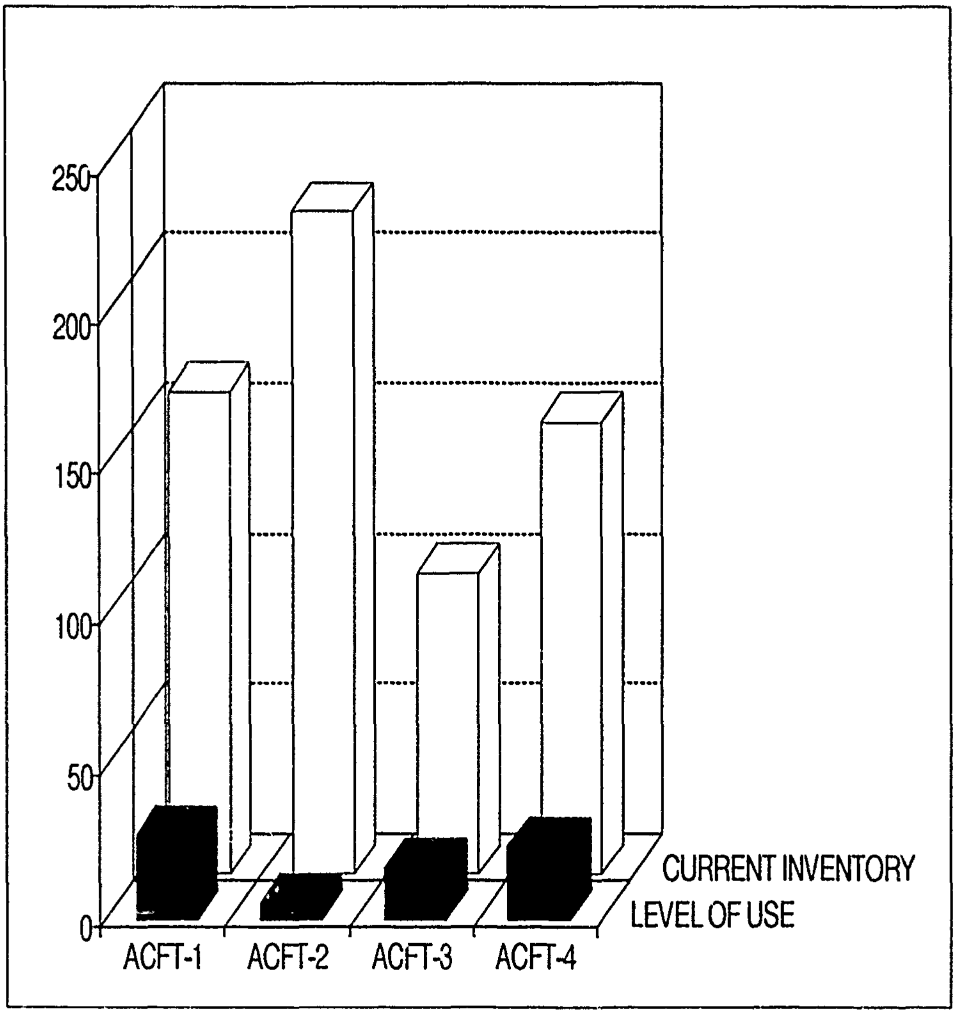

Figure 5.13. Number of Aircraft Per Sortie

The model did not use any of the relatively expensive weapons. The behavior that the model displayed in the weapon selection process was expected since the leading constraint did not have any effect on the weapon selection process (Figure 5.14).

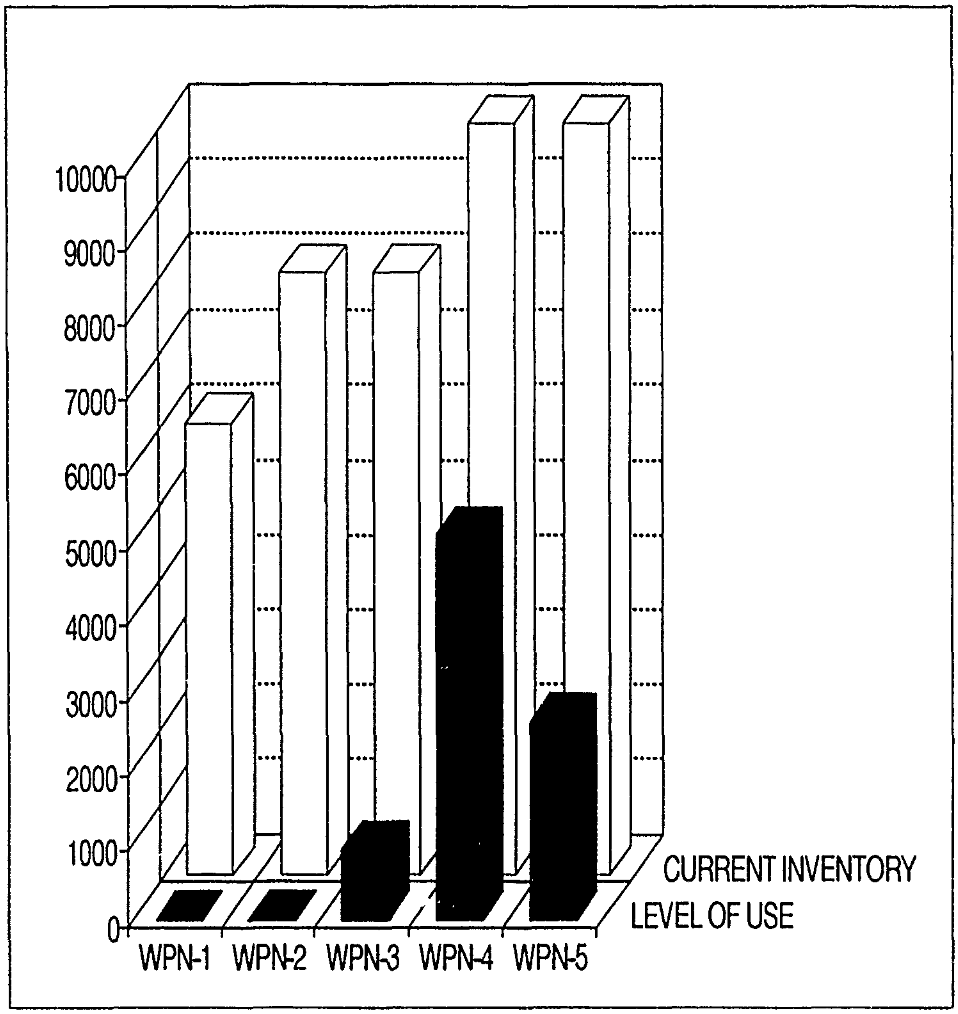

Figure 5.14. Number of Weapons Used

The marginal values from the mission-percentage constraint were insignificant. However, the model could not destroy 73% of target type 4 in the second and third distance bands. The case 2 mission plan led the model to use aircraft type 1 in the second distance band and aircraft type 3 in the third distance band. The number of sorties preassigned to aircraft types 1 and 3 were not adequate to accomplish the destruction of the remaining type 4 targets. Furthermore, there were other missions where the very same aircraft flew a fewer number of sorties than preassigned. Therefore, a considerable amount of sorties remained idle. The study suggested changing the upper bound of sorties that aircraft types could fly by taking the marginal values into consideration. It should be noted that each run may produce more profitable suggestions for the mission plan.

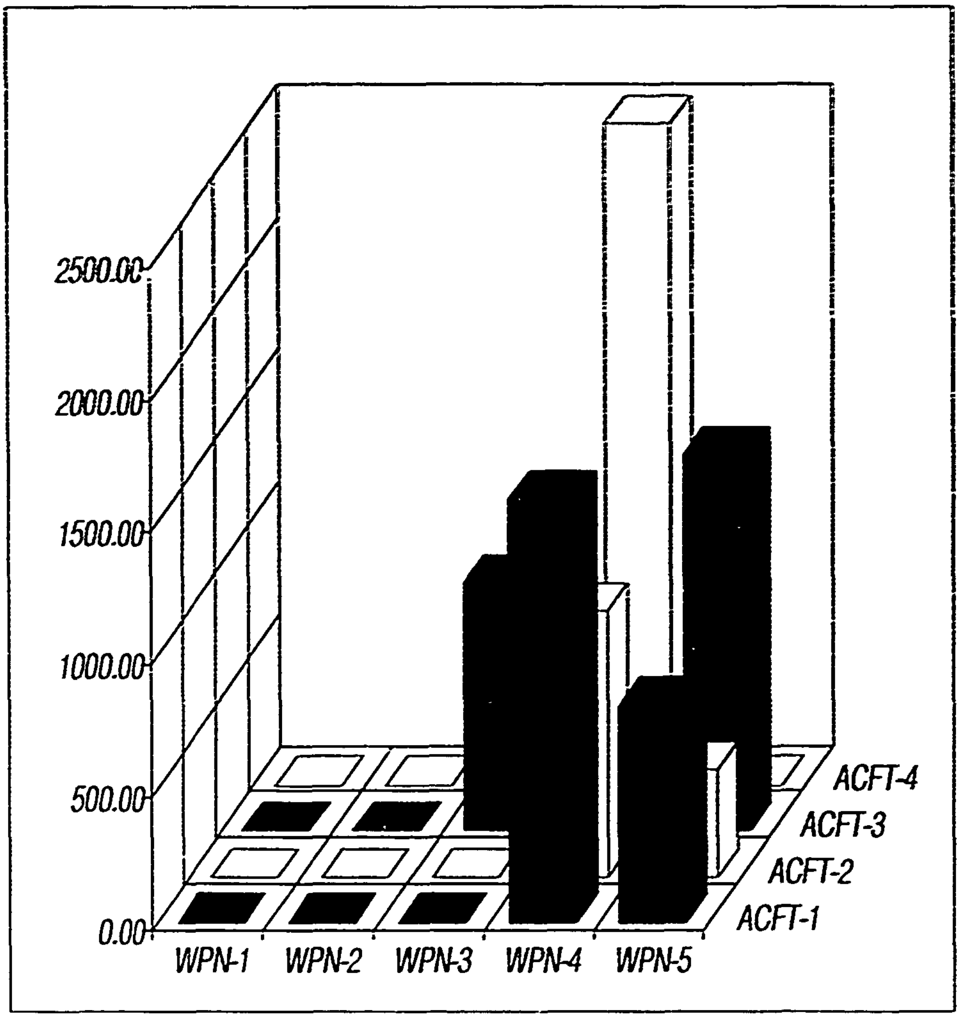

Figure 5.15. Use of Weapons by the Aircraft

In terms of aircraft/weapon allocation, it should be noted that the model's aircraft allocation process had a tendency toward costly, yet effective, aircraft types wheras the weapons allocations process did not favor the costly, yet effective weapons (Figure 5.15).

To decide whether there was a need to buy new aircraft and weapons, the levels of use for aircraft and weapons were compared with the current inventory levels. The study concluded that there was no new procurement needs. The current inventory levels were adequate to supply the requirements of the campaign. However, if there were still some undestroyed targets, even after accounting for the changes caused by the interpretations of the marginal values, then additional runs would be necessary to implement all the missions (i.e., the way JSG approaches the problem). Consequently, the levels of use shown in the additional runs would dictate whether there would be a need to buy new resources or not.

5.3.2 Goal Programming. The goal programming approach was expected to eliminate bias from the model. Although the results revealed that the model ignored the substantial cost differences in the aircraft and weapons allocation process, the model did not ignore the costs of aircraft and weapons. The desired level of achievement for the *Target Value Destroyed per Dollar* goal was 0.16. However, the negative deviate, d_1^-, appeared in the solution as 0.025 implying the underachievement of that goal. Then, the level of achievement for the goal was determined by the difference which was almost the same as the value achieved in the complete cost model. This difference was 0.135 target value detsroyed per dollar.

Target destruction rate was a 92% and only 136.822 of target type 4 in the second distance band remained unattacked. The reason for this appears to be the limitations caused by the case 2 mission plan. Aircraft type 1 was preassigned to the second distance band to fly all the missions. But the upper bound on the sorties that this aircraft was supposed to fly was not adequate to permit destruction of all target type 4 in this particular distance band.

With the case 2 mission plan, the model employed all aircraft types (Figure 5.16). It should be noted that the levels of use were very significant. Aircraft types 1 and 3 were employed despite their substantial costs. The summary of the numerical results is presented in Table 5.8.

Table 5.8. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	2470.727	2590.911	0	1603.369	184.615	68.723
ACFT-2	0	0	0	1004.785	402.377	5.336
ACFT-3	3529.273	4139.703	907.029	0	309.119	100
ACFT-4	0	313.406	7092.971	2500	0	110.862
WPN. USAGE	6000	7044.019	8000	5108.153	896.111	

The model demonstrated an expected behavior in the weapons allocations process. The results showed that weapon type 1 and 3 were exhausted by the model. Obviously, the substantial cost differences did not affect the allocation process since weapon types 1, 2 and 3 were used at significant levels. This behavior indicated the unbiased behavior of the goal programming model (Figure 5.17). There was a diversity of weapon usage (Figure 5.18).

The evaluation of the marginal values revealed the need for more sorties against target type 4 in the second distance band. The marginal value from the ACFT-1.TGT-4.DIST-2 mission-percentage constraint was -0.264. This value was considered to be very significant. Also, similarly, there were some idle sorties left over. The study suggested raising the upper bounds of mission-percetages by taking the marginal values into account.

The procurement variables d_a^+ and d_m^+ did not appear in the solution. Furthermore, the interpretation of the marginal values should preempt any premature procurement decision. The initial assignment process for the upper bounds on sorties—mission plan- would likely generate a better solution, the new results then could be considered to determine whether there would be a need to buy new aircraft and weapons.

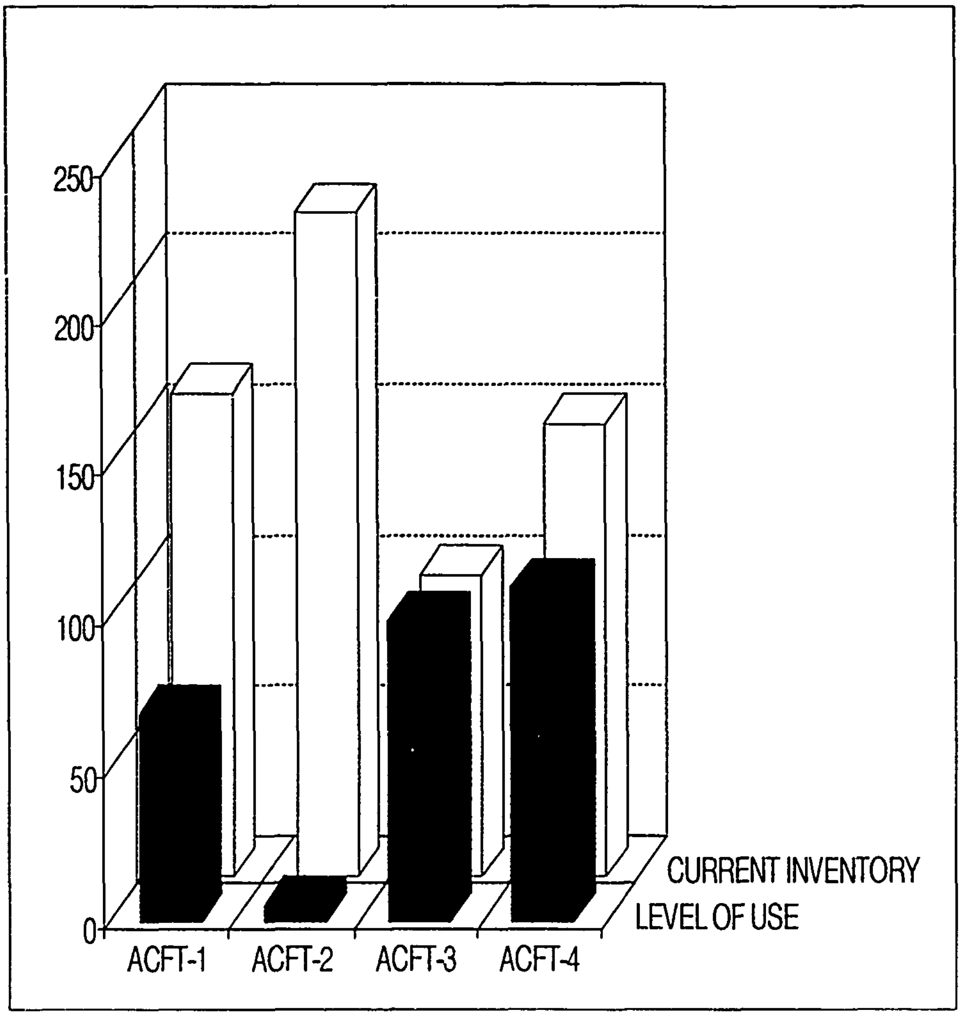

Figure 5.16. Number of Aircraft Per Sortie

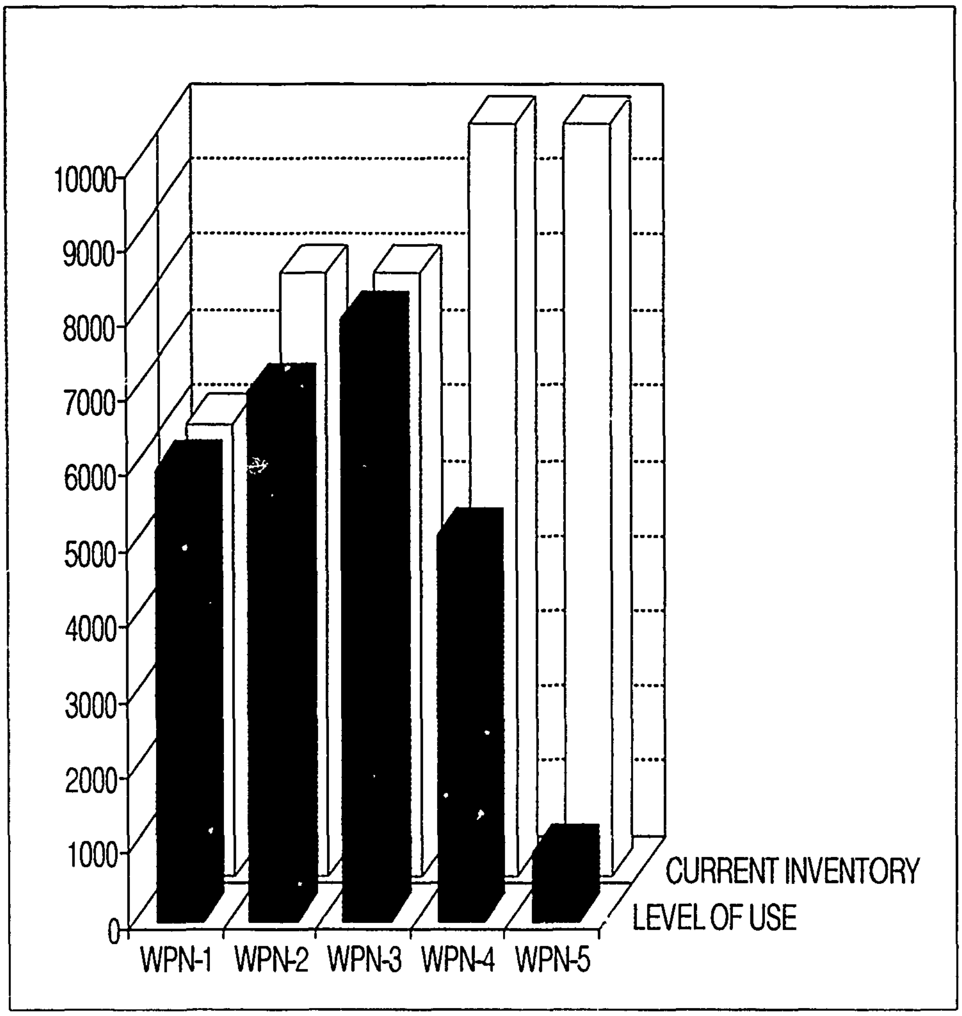

Figure 5.17. Number of Weapons Used

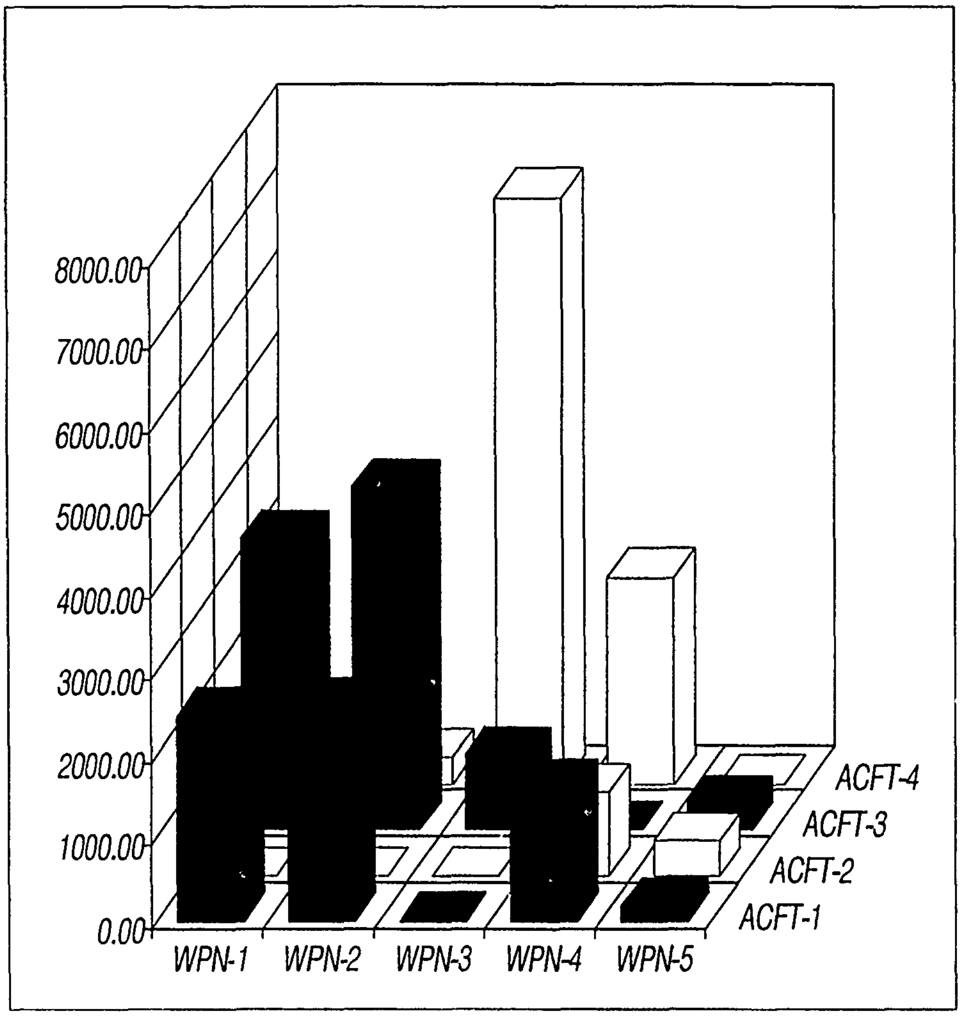

Figure 5.18. Use of Weapons by the Aircraft

5.3.3 Probabilistic Method. In the probabilistic method the case 2 mission plan achieved an objective of 0.078 *Target Value Destroyed per Dollar.* The anticipation was that the model would again act conservatively. Indeed, the results were conservative implying that fewer aircraft and weapons were used. However, the effect of the case 2 mission plan was remarkable. The leading constraint strongly led the model.

The target destruction rate was 88% and only 212.088 of target type 4 in the third distance band remained unattacked because, again, the mission plan constrained aircraft type 3 against target type 4 in that particular distance band. On the other hand, there were some idle sorties that aircraft type 3 did not need to fly against some other targets since those targets were already destroyed by fewer sorties.

The model employed all aircraft types (Figure 5.19). The effect of the case 2 mission plan could be easily observed from the level of use for aircraft type 3. This level of use was considerably different from that presented for the case 1 mission plan. The summary of the numerical results is presented in Table 5.9.

Table 5.9. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	275.117	637.477	9.114
ACFT-2	0	0	0	76.985	48.2	2.222
ACFT-3	0	0	3910.227	0	70.412	29.69
ACFT-4	1002.273	0	0	245.452	796.083	110.855
WPN. USAGE	1002.273	0	3910.227	597.554	1552.172	

The model's behavior in the weapon allocation was biased because the level of use for weapon type 1 was not considered as significant. Furthermore, weapon type 2 was not used at all. To achieve an 88% target destruction rate, the model used 1002.273 of weapon type 1, 3910.227 of weapon type 3, 597.554 of weapon type 4

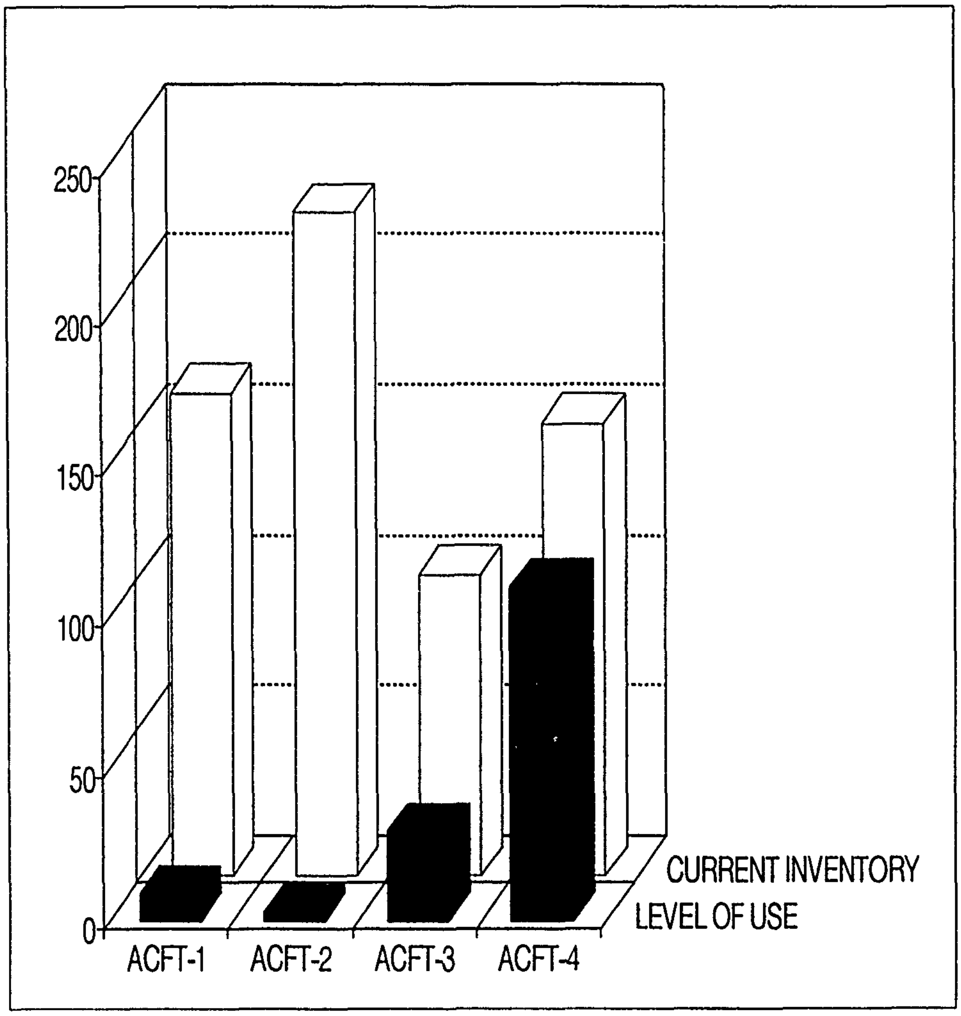

Figure 5.19. Number of Aircraft Per Sortie

and 1552.172 of weapon type 5 (Figure 5.20). In aircraft/weapon allocations, aircraft type 4 had more diversity than the other aircraft types (Figure 5.21).

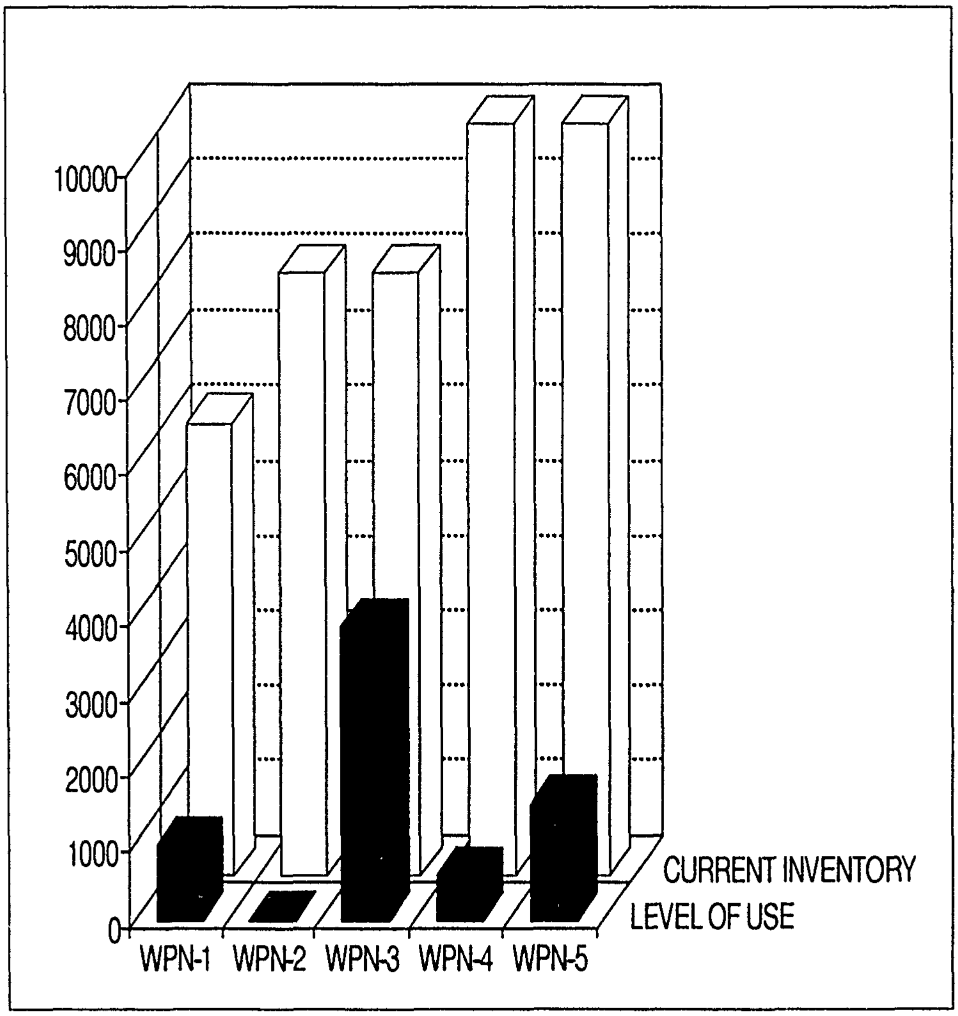

Figure 5.20. Number of Weapons Used

The marginal values from the mission-percentage constraint were all insignificant. However, the study analyzed the levels of use for the mission-percentage constraint. As previously stated, the case 2 mission plan constrained aircraft type 3 against target type 4 in the third distance band. The study suggested that the idle sorties should have been reassigned by increasing the upper bound for the ACFT-3.TGT-4.DIST-3 mission-percentage constraint if a higher objective function value was sought.

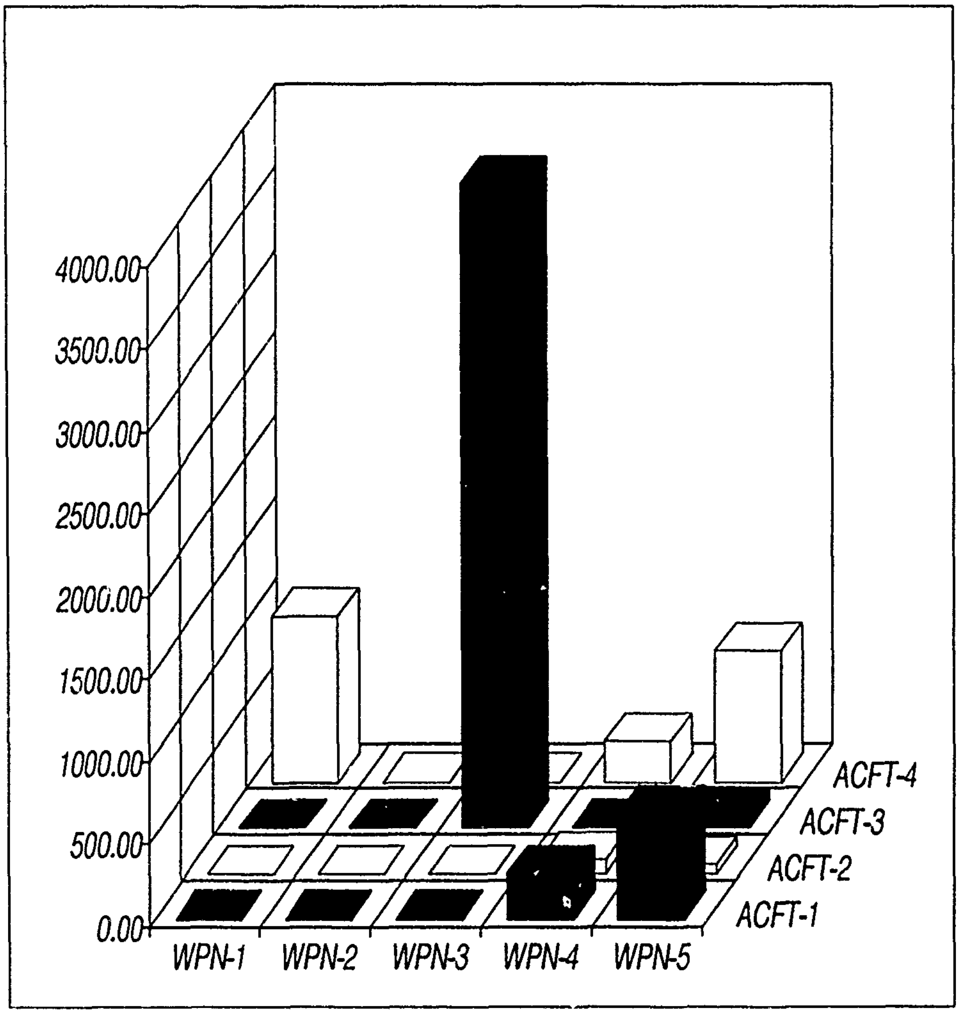

Figure 5.21. Use of Weapons by the Aircraft

To determine the need for procurement, the levels of use were compared with the current inventory levels. The results indicate that with this conservative behavior, the probabilistic model would hardly generate any need to buy more of the resources within the scenario of this study. Indeed, there was no need to buy new aircraft or weapons. The current inventory levels were adequate to supply the requirements of the campaign. Prior to making procurement decisions, one should evaluate the marginal values of the mission-percentage constraint since they may suggest changes in the mission plan.

5.3.4 Goal Programming Version of the Probabilistic Method. The goal programming version of the probabilistic method achieved more success than the original probabilistic model did with the case 2 mission plan. The target destruction rate was 92% and the model kept the same value of the objective at 0.078 *Target Value Destroyed per Dollar*. In this configuration, the model could destroy only 88% of target type 4. Even this level of destruction against target type 4 was better than that of the conventional linear model.

The model was expected to act unbiasedly against substantial cost differences. Indeed, the levels of use of the expensive aircraft and weapons presented the proof that the goal programming version of the model facilitated the guidance of the leading constraint. Moreover, the resources were employed as they were required, regardless of their substantial cost differences. The model used all aircraft types (Figure 5.22). The summary of the numerical results is presented in Table 5.10.

Another superior behavior appeared in the weapons allocations process. The model employed the first two types of weapons despite their high costs. The model achieved the same level of *Target Value Destroyed per Dollar* as the probabilistic model did; however, the goal programming version used the weapon resources in a more balanced fashion (Figure 5.23). It noteworthy that the probabilistic model had not used weapon type 2 whereas the goal programming version used it at a significant

Table 5.10. Summary of Aircraft-Weapon Allocations

	WPN-1	WPN-2	WPN-3	WPN-4	WPN-5	ACFT. USAGE
ACFT-1	0	0	0	533.508	588.073	9.114
ACFT-2	0	0	0	76.985	0	0.147
ACFT-3	3704.775	0	336.695	0	70.412	12.656
ACFT-4	0	1320.45	0	500.986	796.083	104.098
WPN. USAGE	3704.775	1320.45	336.695	1111.479	1454.568	

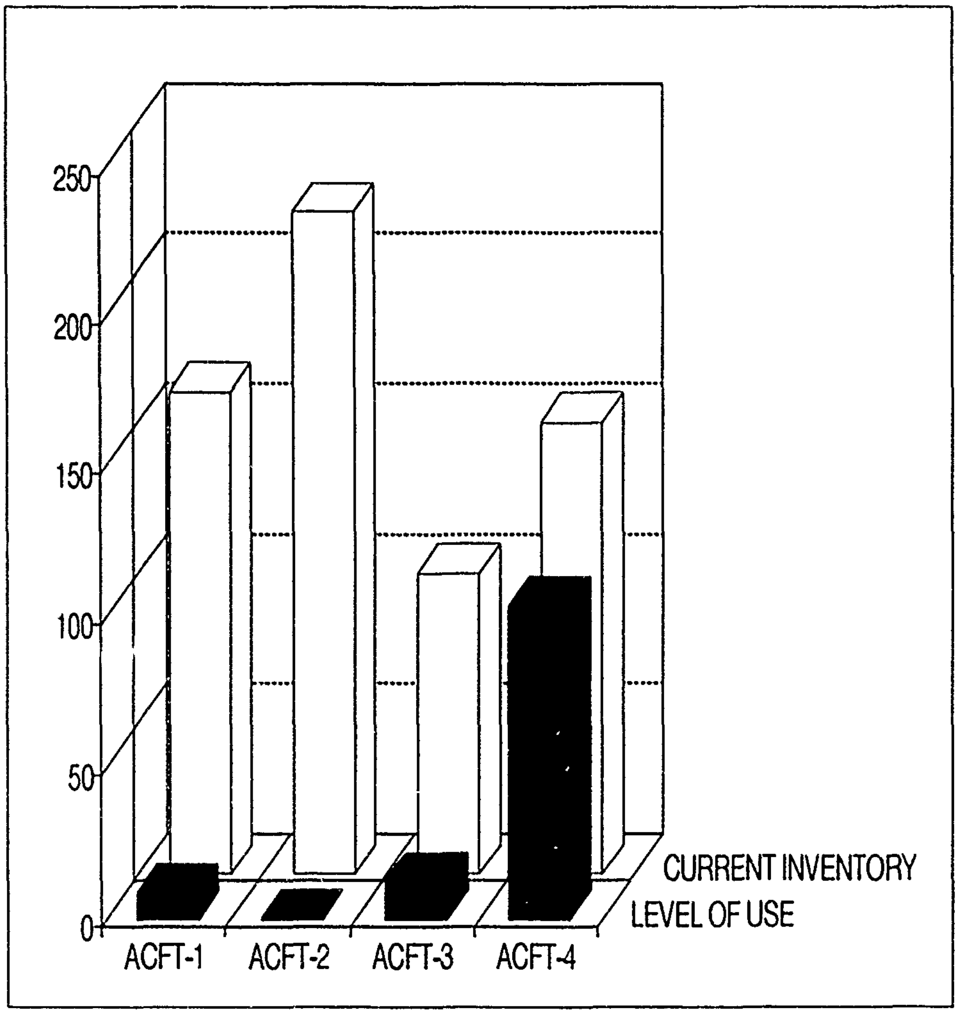

Figure 5.22. Number of Aircraft Per Sortie

level. With the case 2 mission plan, the allocation process of the goal programming approach presented more diversity than the probabilistic model did (Figure 5.24).

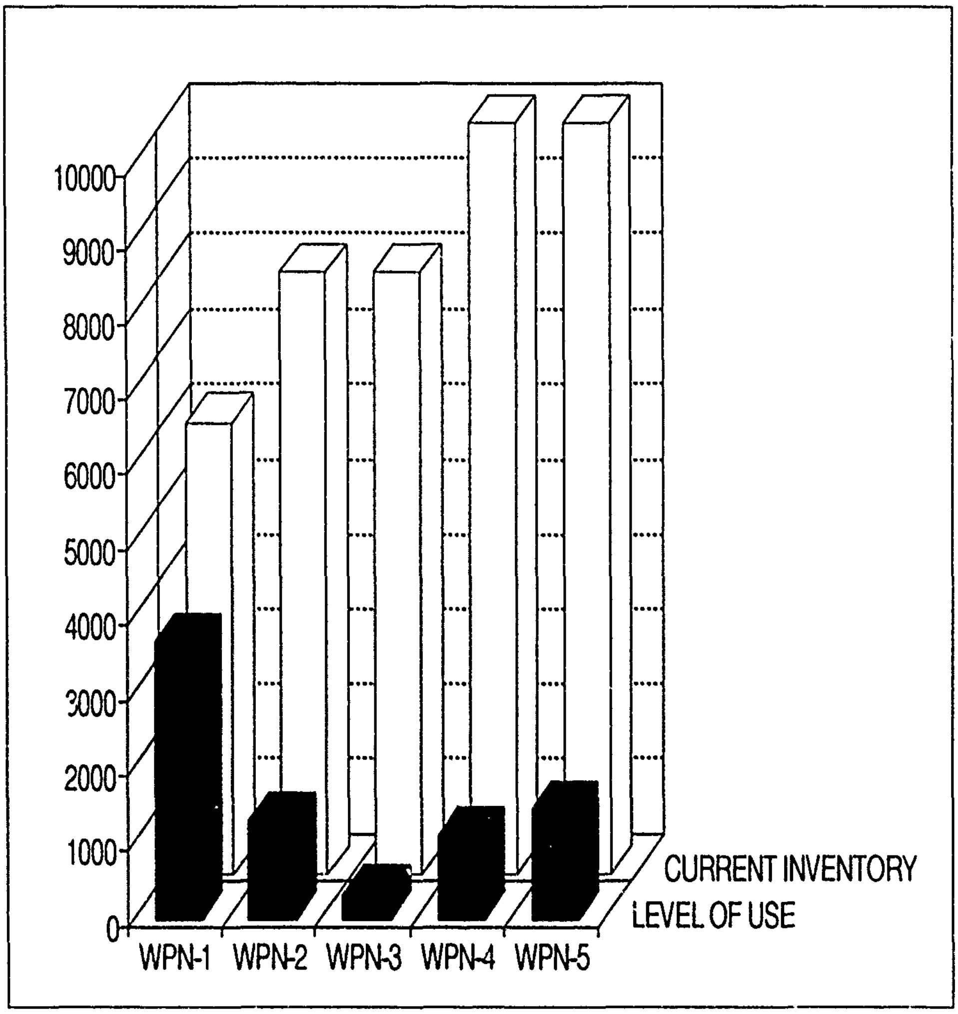

Figure 5.23. Number of Weapons Used

Analyzing the marginal values from the mission-percentage constraint revealed that the case 2 mission plan constrained aircraft type 3 against target type 4 in the third distance band. There were some idle sorties that aircraft type 3 did not fly because the preassigned missions were already accomplished with fewer sorties than initially allocated. The study observed that the marginal value from the ACFT-3.TGT-4.DIST-3 mission-percentage constraint was very significant at a level of -0.978. This significant value implied that one additional sortie preassigned to aircraft type 3 against target type 4 in the third distance band would decrease the sum of the deviations from the goals by −0.978 unit. The results suggested that the idle sorties should be reallocated to aircraft type whose marginal value is significant.

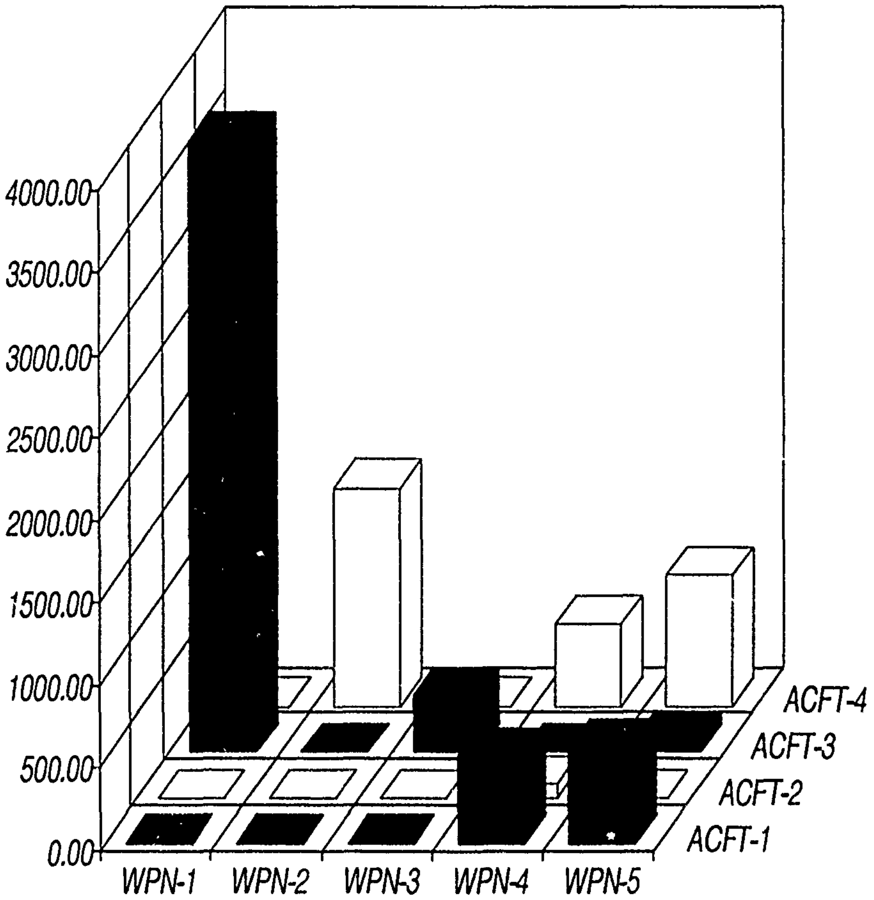

Figure 5.24. Number of Weapons Used

Consequently, to decide whether there was a need to buy new aircraft and weapons, the study checked the values of the procurement variables d_a^+ and d_m^+. They did not appear in the solution, implying that there was no need for new aircraft and weapons. The results support the argument stated in the preceding section that the conservative behavior of the probabilistic method would rarely produce a solution where the procurement variables have significant positive values. The current inventory levels were adequate to supply the requirements of the campaign. Again, it should be noted that revising the mission plan after considering the marginal values should precede successive runs to determine the procurement needs.

5.4 Summary

The case studies were implemented with the last simple cost employment, the goal programming approach, the probabilistic method, and the goal programming version of the probabilistic method. The case 1 mission plan consists of missionwise initial allocations, whereas the case 2 mission plan initially allocates aircraft distancewise. The results obtained from the implementations showed that the mission plan—leading constraint—has a considerable effect on the model.

As observed in Chapter 4, the superior features of the goal programming construction such as unbiased allocations, providing direct answers for procurement, and including the cost and budget figures simultaneously was remarkable. Furthermore, the probabilistic method kept its conservative behavior in aircraft/weapon allocations for both mission plans.

VI. CONCLUSIONS AND RECOMMENDATIONS

6.1 Review of the Research

Given a campaign scenario, the JSG analysis efforts in support of procurement decisions do not include the costs of aircraft, weapons or sorties. Moreover, when a decision is made to procure aircraft and weapons, the analyses do not directly address the question of whether the designated budget would be adequate to allow the procurement needs. Therefore, JSG needs a modified Mixmaster model: one which incorporates the cost and budget issues within its construction. In this way the analyses are more sound and timely since the results are evaluated on the basis of cost.

Initial attempts at modifying the Mixmaster model for direct inclusion of cost in the objective function caused the Mixmaster model to execute the campaign scenario inconsistently with operational and tactical needs due to its aggregation level and linear construction. This inconsistency is manifest as allocating an aircraft beyond its operating range or allocating an aircraft with improper tactical characteristics against a target. Such problems were encountered when the cost of aircraft was included in the objective function. The model considers substantial cost differences between two aircraft types to be significant. Consequently, the results obtained from the model showed a remarkable bias in favor of the least costly aircraft. However, in the inventory there are already different types of aircraft and weapons, and prior to procurement of more of the least costly aircraft, TAC should use what it has in the inventory. Allocating the least costly aircraft makes sense, but not if the allocated aircraft does not meet operational and tactical needs.

Having stated the problem, the research objective was first to suggest alternative modifications to the model and then to investigate them. A corollary objective was to assure the consistency of the model with the operational and tactical needs.

Therefore, the research generated the concept of a *leading constraint*. This constraint allocates the number of sorties that an aircraft should not exceed when flying against a target type determined by the mission planner. The maximum numbers of sorties are determined as the percentages of the total sorties that an aircraft can fly given the attrition rates, sortie rates, and duration of the conflict. With this constraint, the model is led parallel to the given operational and tactical requirements. Consequently, a baseline model was developed with the addition of the leading constraint to the the Mixmaster model. Indeed, the application of the first modification with and without the leading constraint demonstrated that the leading constraint had a considerable effect on the model's results.

The research suggested four main alternative modifications where the first alternative has seven variations They are:

1. Simple cost employment in the objective function
   - Including the cost of aircraft
   - Including the cost of munitions
   - Including the cost of sortie generation
   - Including the costs of both aircraft and munitions
   - Including the costs of both aircraft and sortie generation
   - Including the costs of both munitions and sortie generation
   - Including the costs of aircraft, munitions and sortie generation
2. A goal programming approach
3. A probabilistic approach to determine the cost coefficients in the objective function
4. A goal programming version of the probabilistic approach

All modifications were applied to the baseline model. The data base and the mission plan—right-hand-side values of the leading constraint—were generated by the researcher, and two case studies were performed to investigate the operational consistency with two different mission plans. Given that JSG addresses "real-life" Air Force procurement problems, and the number of constraints is fewer than the number of variables in the linear configuration of the Mixmaster model, it was expected that the results of the Mixmaster model will give more than one optimal solution. Indeed, the results obtained from the runs presented alternate optimal solutions. Unfortunately, it is not yet possible to predetermine the number of alternate solutions when the problem size is quite large (i.e., problem size that requires a computer program). Thus, the research interest focused on the solutions obtained with the first runs. The research conclusions are presented in the subsequent sections.

6.2 Simple Cost Employment Method

The simple cost employment methods provided the researcher with an understanding of the behavior of the model when including different costs. The bias generated by the evaluation of substantial cost differences was eliminated to a remarkable degree by means of the leading constraint. But, to some extent, due to the linear programming construction of the problem, the effect of including different costs generated expected behavior from the model. For example, including the cost of aircraft generated an allocation pattern from less costly to more costly aircraft while still obeying the leading constraint. Also, the research did not generate a constraint that would lead the allocation of weapons. The reason for this is the weapon allocation should be performed on an expected kill basis - the weapon's ability to destroy the target. For the model, the differences in weapon costs were much more significant than the differences in expected kills. Including only the aircraft cost ignores the weapon and sortie costs; whereas, including only the weapon cost ignores the aircraft and sortie costs. When the costs were combined, the effect of sortie cost on the solution was insignificant. In particular, when the aircraft, weapon and sortie costs were combined into one cost, the results were exactly the same as the ones produced by only including the aircraft and weapon costs. The objective function value was somewhat different since there was an additional cost.

The overall evaluation of the simple cost employment methods proved unsatisfactory for various reasons. First, the model did not answer the procurement question directly; to determine the need for procurement, further computations were required. Second, when procurement was required, the model did not permit inclusion of the budget figure designated for procurement. As previously explained, the right-hand-side values for the aircraft and weapon availability constraints were assumed to be extremely large. Therefore, the model was not constrained by the current inventory, rather by the number of targets and the sortie rates. After running the model, the levels of use were compared with the current inventory levels. If the current inventory levels were less than the levels of use, the difference would indicate the need for procurement. If not, then the current inventory would be adequate. Moreover, even if the difference indicated a need for procurement, the procurement issue was not subjected to a budget constraint.

However, the last simple cost employment method, complete cost model, provided an important feature. Since the model included all costs and was run by relaxing the aircraft and weapon availability constraints, the level of achievement for the objective resulted in the maximum value that the model could produce. On the other hand, producing the maximum value did not assure that the allocations would be sound because in the original linear construction of the Mixmaster model, there was no constraint which forced the model to use primarily the resources in the inventory. But, it was possible to exploit the simple cost employment method, especially the complete cost model.

6.3 Goal Programming

The goal programming formulation of the complete cost model produced the most satisfactory results. The effect of the leading constraint was very significant in forcing the model to make allocations parallel to the mission plan. The allocations were operationally and tactically sound. Not using the cost figures in the objective function eliminated the bias that the linear programming inevitably had in its solution procedure. Five goals were defined:

1. Achieve a Level of Target Value Destroyed per Dollar Spent for a Mission
2. Achieve at least 80% Sortie Success
3. Avoid Overutilization of Available Aircraft
4. Avoid Overutilization of Available Munitions
5. Kill as Many Targets as Possible

The objective function employed positive or negative deviates in terms of the desired underachievement or overachievement of the goals. The desired level of target value destroyed was found by means of the last simple cost employment because this value was the maximum achievable *Target Value Destroyed per Dollar* for the given scenario. Reasonably, the thought for pushing the maximum yield above the achieved level might come to mind. However, with the equally-ranked goal programming model, increasing the desired level for target value destroyed per dollar did not change the level of achievement since it was already at the maximum.

Goal programming provided a solution for total sortie success that the decision maker seeks to achieve. According to the results, the level of sortie success was not binding for the given scenario. However, there may be some cases where the desired level of success is binding. Hence, the decision maker would readjust the sortie success level. By means of the underachivement variable which is the negative deviate from the sortie success goal, $dloss_a^-$, it is possible to determine the number of aircraft expected to be lost during the campaign.

As its most attractive feature, the goal programming model directly provides the procurement variables. The procurement variables, the positive deviates from the aircraft and weapon availability goals, d_a^+ and d_m^+, are directly shown in the solution. Consequently, providing this information eliminated the need for the computations that JSG performs to determine procurement needs. In addition, the goal programming model uses current inventory levels for right-hand-side values; there was no need to relax the aircraft and weapon availability constraints and to make the right-hand-side values as large as JSG assumed for procurement computations.

Also, the goal programming model allows the analyst to include the budget figure in the model and make the budget constraint affect the procurement needs. Hence, the analyst was aware of whether or not the budget designated to procurement needs was adequate. Changing the budget figures and then running the model also provided further insight in terms of simulating budget uncertainties.

6.4 Probabilistic Method

The probabilistic method presented a critical concept: the concept of time. In the the Mixmaster model, the time concept was treated as the duration of the campaign specified by the analyst for computing the total sorties that an aircraft flies as related to the attrition and sortie rates. The costs, therefore, were fixed costs and did not include the time element. However, the main driver of real-life conflict is time. Most of the variables related to conflicts depend on the time element. Moreover, there is an uncertainty issue associated with time. Under these circumstances, the research applied Sivazlian's stochastic approach, with necessary modifications, to the Mixmaster model. The formulas that included time as a main driver were based on two primary assumptions:

1. The time in which an aircraft acquires a target has a negative exponential distribution with parameter μ.

2. The occurrence of the enemy threat is a Poisson process with parameter λ.

In the probabilistic method, the Mixmaster model keeps its linear construction; however, the probabilities, the expected kills and the costs are computed with a dependence on time.

The probabilistic approach allowed the analyst to play the scenario with different aircraft speeds. The speed factor was directly related to the time in which the aircraft was subjected to a threat. The time issue affects the attrition rates, the expected kills, and hence, the level of achievement for the objective function. However, it was noted that the absolute requirement to produce sound and accurate results was to have good estimates of the parameters λ and μ.

The time dependency increased the accuracy and credibility of the numbers that were used in the Mixmaster model. Furthermore, the time dependency caused the probabilistic method to produce considerably conservative results. When the results were compared, the probabilistic approach destroyed more targets than the simple cost employment did with significantly fewer aircraft and weapons. The research attributed this behavior to the conservative behavior of the probabilistic model. Although the probabilistic model introduced the time factor in the computations of the probabilities, expected kills and costs, the procedure to determine procurement needs was the same as in the simple cost employment method. Therefore, the probabilistic approach did not improve the model in terms of determining procurement needs; however, the cost figures were more reliable because of the time dependency being reflected.

6.5 Goal Programming Version of the Probabilistic Model

The research applied the goal programming structure to the probabilistic model. The goals were the same as previously defined. The desired level of achievement for target value destroyed per dollar was borrowed from the probabilistic method. Similarly, the model kept its conservative behavior in terms of the numbers of aircraft and weapons used to destroy the targets. That behavior is considered as an advantage which avoids wasting resources. However, it was noted that this advantage was valid only as long as the parameters λ and μ were accurate.

The goal programming version of the probabilistic model also provided the procurement variables. The positive deviates from the aircraft and weapon availability constraints, d_a^+ and d_m^+, defined the procurement requirements. The right-hand-side values of the aircraft and weapon availability constraints were the current inventory levels. Therefore, the analyst obtained direct insight on the procurement needs.

The budget figure was included in the model directly so that the procurement variables associated with their inherent costs could be subjected to a budget constraint. In this manner, the analyst could evaluate whether or not the designated budget was adequate to meet procurement requirements.

6.6 Conclusions

The research concludes that the leading constraint proved to be useful. The leading constraint helps the analyst assess the mission. Significant marginal values from the leading constraint provide insight for improving the mission plan. The simple cost employment modifications supported the utility of the leading constraint. The complete cost model makes more sense than the other simple cost employment models since it includes all the costs. However, to determine the need for procurement, the simple cost employment model requires further computations, whereas the goal programming model can provide the analyst with the procurement needs directly. Also the goal programming model employs the budget figure in its construction. In addition, the goal programming construction eliminated the bias that the conventional linear programming approach generates when subjected to substantial cost differences. The goal programming model has added flexibility by allowing the analyst to define desired levels of achievement.

The probabilistic model introduces the critical time issue. As long as the parameters λ and μ are estimated accurately, the results obtained from the probabilistic model will make more sense since the time issue is involved. Furthermore, the goal programming construction of the probabilistic method offers more advantages by incorporating the time issue, the procurement variables and the budget figure. Also, the goal programming approach allows the analyst to define additional goals and the ability to strive for overachievement or underachievement of the goals.

Finally, given that JSG uses the Mixmaster model, modifying Mixmaster as a goal programming model is the most appropriate way to incorporate cost and budget. The objectives of this research are met by employing *a leading constraint* for tactical and operational accuracy and by applying a goal programming approach which includes the procurement variables subject to cost and budget limitations.

6.7 Further Recommendations.

The research applied the modifications successfully. The results showed that there are various ways to solve the problems that JSG faces. However, the research indicates that two further enhancements can assist the model in generating more certain and more precise results. These two enhancements are:

1. Ranked goal programming
2. Better estimates for the parameters λ and μ

6.7.1 Ranked Goal Programming. The goal programming construction that the research developed herein has equally-ranked goals. Therefore, one of the goals cannot be preferred to another; all five goals are evaluated equally. But in some circumstances, some goals can have higher priorities than others, and there can be a case where the decision maker wants to achieve a particular goal even at the expense of the other goals. In that case, the goal programming model can be modified by ranking and weighting the goals according to the desires of the decision maker. However, there is a key requirement in order to address this issue—software which can be used to solve ranked goal programming problems. The research recommends that the goal programming model be raked and run using a ranked goal programming solver.

6.7.2 Better Estimates for λ and μ The probabilistic method introduced the time concept on which most of the computations depend. The research claims that the computations depending on time are more accurate and realistic. However, the formulas that include time as a main driver are based on two main assumptions:

1. The time in which an aircraft acquires a target has a negative exponential distribution with parameter μ.
2. The occurrence of the enemy threat is a Poisson process with parameter λ.

No doubt, the probabilistic model works as long as the parameters justify the assumptions. Therefore, the research also recommends further investigations to improve the estimates for the parameters λ and μ by taking the "real life" combat dynamics into account.

Appendix A. *Inventory Levels For Aircraft and Weapons*

CURRENT INVENTORY LEVELS

AIRCRAFT TYPE 1	160
AIRCRAFT TYPE 2	220
AIRCRAFT TYPE 3	100
AIRCRAFT TYPE 4	150
WEAPON TYPE 1	6000
WEAPON TYPE 2	8000
WEAPON TYPE 3	8000
WEAPON TYPE 4	10000
WEAPON TYPE 5	10000

Appendix B. *Current Configuration of Mixmaster*

$$\max TVD = \sum_{a}\sum_{m}\sum_{k}\sum_{d}\sum_{t}\sum_{w} X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}$$

subject to:

$$\sum_{m}\sum_{k}\sum_{d}\sum_{t}\sum_{w}\left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] \leq TOAC_{a} \quad \text{for each } a$$

$$\sum_{a}\sum_{k}\sum_{d}\sum_{t}\sum_{w} X_{amkdtw} WPNLD_{am} \leq TOWPN_{m} \quad \text{for each } m$$

$$\sum_{a}\sum_{m}\sum_{t}\sum_{w} X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$Lowerbound_{amkdtw} \leq X_{amkdtw} \leq Upperbound_{amkdtw}$$

Appendix C. *Baseline Model of the Research*

$$\max TVD = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

where

$$ATD_{akd} TSORTAC_a$$

is the upper bound of sorties that an aircraft can fly (determined by the mission planner).

Appendix D. *Simple Cost Employment Methods*

D.1 Inclusion of the Aircraft Cost

$$\max TVD/\$ = \sum_{a}\sum_{m}\sum_{k}\sum_{d}\sum_{t}\sum_{w}\left[\frac{X_{amkdtw}EXKIL_{amkdtw}TGTVAL_{kd}}{ACCOST_{a}ATTRIT_{amkdtw}}\right]$$

subject to:

$$\sum_{m}\sum_{k}\sum_{d}\sum_{t}\sum_{w}\left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] \leq TOAC_{a} \quad \text{for each } a$$

$$\sum_{a}\sum_{k}\sum_{d}\sum_{t}\sum_{w} X_{amkdtw}WPNLD_{am} \leq TOWPN_{m} \quad \text{for each } m$$

$$\sum_{a}\sum_{m}\sum_{t}\sum_{w} X_{amkdtw}EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_{m}\sum_{t}\sum_{w} X_{amkdtw} \leq ATD_{akd}TSORTAC_{a} \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

D.2 Inclusion of the Weapon Cost

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{COSTM_m WPNLD_{am}} \right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPX_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

D.3 Inclusion of the Sortie Cost

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{SCOST_a} \right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

D.4 Inclusion of the Aircraft and Weapon Costs

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{k,d}}{ACCOST_a ATTRIT_{amkdtw} + COSTM_m WPNLD_{am}} \right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

D.5 *Inclusion of the Aircraft and Sortie Costs*

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{ACCOST_a ATTRIT_{amkdtw} + SCOST_a}\right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

D.6 *Inclusion of the Weapon and Sortie Costs*

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{COSTM_m WPNLD_{am} + SCOST_a} \right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{MISCOST_{amkdtw}} \right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] \leq TOAC_a \qquad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} \leq TOWPN_m \qquad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} \leq TOTGT_{kd} \qquad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \qquad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

where the mission cost is

$$MISCOST_{amkdtw} = ACCOST_a ATRITT_{amkdtw} + COSTM_m WPNLD_{am} + SCOST$$

Appendix E. *The Goal Programming Model*

$$\min Z = d_1^- + \sum_a dloss_a^+ + \sum_a d_a^+ + \sum_m d_m^+ + \sum_{kd} d_{kd}^-$$

subject to:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_m \left[\frac{X_{amkdtw} EXKIL_{amkdtw} TGTVAL_{kd}}{MISCOST_{amkdtw}} \right] + d_1^- = TGTVALDES$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w X_{amkdtw} ATTRIT_{amkdtw} - dloss_a^+ + dloss_a^- = 0.20 \cdot TOAC_a \quad \text{for each } a$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}} \right] - d_a^+ + d_a^- = TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} WPNLD_{am} - d_m^+ + d_m^- = TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} EXKIL_{amkdtw} + d_{kd}^- = TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$\sum_a d_a^+ ACCOST_a + \sum_m d_m^+ COSTM_m \leq BUDGET$$

$$X_{amkdtw} \geq 0$$

$$d_1^-, d_a^+, d_a^-, dloss_a^+, dloss_a^-, d_m^+, d_m^-, d_{kd}^- \geq 0$$

where the mission cost is

$$MISCOST_{amkdtw} = ACCOST_a ATRITT_{amkdtw} + COSTM_m WPNLD_{am} + SCOST$$

Appendix F. *The Probabilistic Model*

$$\max TVD/\$ = \sum_a \sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw} ETGTKIL_{amkdw} TGTVAL_{kd}}{TSCOST_{amkdw}}\right]$$

subject to:

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] \leq TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} EXATTACK_{amkdw} WPNLD_{a,m} \leq TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} ETGTKIL_{amkdw} \leq TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$X_{amkdtw} \geq 0$$

where the mission cost is

$$\begin{aligned} TSCOST_{a,m,k,d,w} &= FIXCOST + DURATIONCOST_{amkdw} \\ &+ WEAPONCOST_{amkdw} \\ &+ AIRCRAFTCOST_{amkdw} \end{aligned}$$

Appendix G. *The Goal Programming Version of the Probabilistic Model*

$$\min Z = d_1^- + \sum_a dloss_a^+ + \sum_a d_a^+ + \sum_m d_m^+ + \sum_{kd} d_{kd}^-$$

subject to:

$$\sum_a \sum_m \sum_k \sum_d \sum_t \sum_m \left[\frac{X_{amkdtw} ETGTKIL_{amkdw} TGTVAL_{kd}}{TSCOST_{amkdw}}\right] + d_1^- = TGTVALDES$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w X_{amkdtw} ATTRIT_{amkdw} - dloss_a^+ + dloss_a^- = 0.20 \cdot TOAC_a \quad \text{for each } a$$

$$\sum_m \sum_k \sum_d \sum_t \sum_w \left[\frac{X_{amkdtw}}{TS_{amkdtw}}\right] - d_a^+ + d_a^- = TOAC_a \quad \text{for each } a$$

$$\sum_a \sum_k \sum_d \sum_t \sum_w X_{amkdtw} EXATTACK_{amkdw} WPNLD_{am} - d_m^+ + d_m^- = TOWPN_m \quad \text{for each } m$$

$$\sum_a \sum_m \sum_t \sum_w X_{amkdtw} ETGTKIL_{amkdw} + d_{kd}^- = TOTGT_{kd} \quad \text{for each } k, d$$

$$\sum_m \sum_t \sum_w X_{amkdtw} \leq ATD_{akd} TSORTAC_a \quad \text{for each } a, k, d$$

$$\sum_a d_a^+ ACCOST_a + \sum_m d_m^+ COSTM_m \leq BUDGET$$

$$X_{amkdtw} \geq 0$$

$$d_1^-, d_a^+, d_a^-, dloss_a^+, dloss_a^-, d_m^+, d_m^-, d_{kd}^- \geq 0$$

where the mission cost is

$$\begin{aligned} TSCOST_{amkdw} &= FIXCOST + DURATIONCOST_{amkdw} \\ &+ WEAPONCOST_{amkdw} \\ &+ AIRCRAFTCOST_{amkdw} \end{aligned}$$

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Vita

1LT Efgan Dengür was born on 5 March 1965 in Adana TURKEY. He graduated from the ISIKLAR Military High School in 1983 and entered the Turkish Air Force Academy. 1LT Dengür graduated from the Academy as a Second Lieutenant on 30 August 1987.

After graduating from the Turkish Flight School in 1989, he attended the War Readiness Training School at 3.AFB Konya. 1LT Dengür was assigned to 9.AFB Balikesir for his follow-on training in the F-104 as a wingman in 193.Fighter Squadron.

Before completing his training in the F-104, 1LT Dengür was selected for the Postgraduate Education Program. He entered the School of Engineering, Air Force Institute of Technology, WPAFB, OH in 1990.

Permanent address: 96 Sok. No:40/1 Teksin
Apt. Göztepe IZMIR
TURKEY