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STABILIZING AND DESTABILIZING CONVENTIONAL WEAPONS

Stephen Biddle
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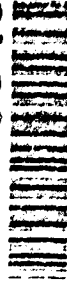
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13. ABSTRACT (Maximum 200 words) The paper is intended to distinguish, if possible, between stabilizing and destabilizing conventional weapons; assess the magnitude of the difference; and provide consequent recommendations with respect to weapon development, force structure, acquisition and arms control policy. Its main conclusion is that force <i>employment</i> , rather than the mix of weapon types, is the most important determinant of conventional stability. While distinctions between stabilizing and destabilizing weapon systems can be drawn, these distinctions are highly sensitive to scenario conditions, and in any case have limited effects on the stability of the theater balance as a whole. That balance, at least for East-West conflict, should be highly stable regardless of plausible additions or subtractions of particular weapon types -- as long as defenders adapt their force employment to suit the changing conditions of the post Cold War European battlefield. Given this, force structure or equipment changes we may wish to make can be based on criteria other than stability.				
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INSTITUTE FOR DEFENSE ANALYSES

Contract MDA 903 89 C 0003
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PREFACE

This brief provides the results of work performed by the Institute for Defense Analyses for the CFE Task Force of the Office of the Under Secretary for Defense for Acquisition (Tactical Warfare Programs) under Contract Number MDA 903 89 C 0003, Task Number T-F6-711, Analytical Support for Conventional Forces in Europe (CFE) Talks. The work described here constitutes one of several sub-elements of the larger task. In particular, this sub-element is intended to distinguish -- if possible -- between stabilizing and destabilizing conventional weapons, and thereby to assist OSD in weapon development, force structure and acquisition decisions, and in identifying weapon types for inclusion in or exclusion from follow-ons to the Conventional Forces in Europe (CFE) talks.

The stability properties of conventional weapons currently are not well understood. In the nuclear arena, distinctions between stabilizing and destabilizing weapons are a matter of broad (if not complete) consensus, and such distinctions have become an important element of U.S. negotiating policy in strategic arms talks with the Soviet Union. In the conventional arena, however, there is much less agreement. Not only do analysts disagree over categorizations of specific weapons, but also there is substantial disagreement whether such distinctions can be drawn at all. In fact, the issue has been debated at least since the 1932 World Disarmament Conference and the dispute between the British military intellectuals Basil Liddell Hart and J. F. C. Fuller over the validity of labelling specific weapons "offensive" or "defensive."¹ In the intervening years, much has been written on the problem of offense and defense (or stability and instability; see below for a more detailed definition). No systematic analysis has been produced, however, which addresses more than a small subset of the relevant issues.² The purpose of this brief is to provide such an analysis, and to contribute thereby to the development of U.S. policy for conventional arms control and force restructuring.

The main conclusions of this analysis are:

- The single most important determinant of conventional stability is *force employment* -- or the doctrinal and operational concepts by which opposing forces are used -- not the mix of weapon types that comprise those forces. While changes in the numbers and types of weapons on the two sides do affect stability, this effect is small relative to the influence of the manner in which those weapons are used.
- Even sizable additions or subtractions of particular weapon types should not materially affect the underlying stability of the East-West military balance, at least for the weapon types considered here -- given that the two sides adapt their force employment to suit the associated changes in weapon mix.
- At least for East-West conflict, the kinds of force structures that are plausible as near- to mid-term policy choices should be highly stable if the two sides maintain sound defensive force employment. This study has not explicitly addressed the possibilities of East-East conflict among the emerging nations of Eastern Europe (where multipolarity may pose very different military problems), or extreme force mix alternatives such as pure infantry or pure Advanced Conventional Munitions (ACM) postures. For East-West conflicts involving some degree of combined arms, however, it should be possible to keep military incentives for attack low, with or without variations in the weapon mix available to the two sides.

The analyses upon which these conclusions are based are described here in annotated briefing format. The main body presents the central arguments of the paper. It is supported by a series of appendices describing in greater detail the results and the methodology upon which those arguments are based. Appendix A presents a series of figures which expand upon themes developed in the main briefing. Appendix B details the theoretical work from which the results presented in the figures were derived, and documents the resulting expansion of the IDA Variable Force eMployment Model (VFM). Appendix C provides the results of a series of combat simulation experiments using the Lawrence Livermore National Laboratory's JANUS model, by which the theoretical construct described in the appendices was tested.

This paper has been reviewed by Dr. Jerome Bracken of Yale University, Professor George Quester of the University of Maryland, and Dr. Frederic Miercort and Mr. John Tillson of the IDA staff. Referencing the format of this document:

When explanatory material exceeds the length of one page of text, the slide to which it applies is repeated so that the reader does not have to flip pages back and forth. Also, end notes are provided for the reader following the main body of the text and Appendices A, B and C.



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CENTRAL ARGUMENTS

The analyses described herein focus on the problems of stability and on the relative contribution of different weapon types to the stability of the military balance in Europe. But why focus on stability, as opposed to cost-effectiveness or some other measure of military capability? Broadly speaking, the answer focuses on the requirements for arms control and mutual security in the post Cold War era; more narrowly, it relates directly to the official mandate for the CFE talks.

For an analysis of arms control and force restructuring, the official mandate under which the CFE negotiations have been conducted should be an appropriate point of departure. The mandate clearly identifies stability and the provision of a more stable balance of military forces as the NATO Alliance's key objective in the talks. Given this, a key criterion for the evaluation of force postures or the inclusion or exclusion of weapon types from the negotiating table should be the degree to which those weapons contribute to military stability in Europe.

WHY STABILITY?

“The objectives of the negotiations shall be to strengthen stability and security in Europe through the establishment of a stable and secure balance of conventional armed forces, which include conventional armaments and equipment, at lower levels; the elimination of disparities prejudicial to stability and security; and the elimination, as a matter of priority, of the capability for launching surprise attack and for initiating large-scale offensive action.”

- Mandate for Negotiation on Conventional Armed Forces in Europe, Palais Liechtenstein, Vienna, January 10, 1989

The CFE mandate was signed in 1989; clearly much has changed since then and much continues to change. Stability remains primary. Indeed, its primacy as a criterion for arms control and force design is growing over time.

The arms control process, for example, is ongoing. While the mandate or even the institutional forum for the talks may change, it is clear that a more stable system of military power on the European continent will remain an important objective for any follow-on talks.

Moreover, both the United States, our allies, and even our potential opponents all are engaged in a major restructuring of conventional forces -- a central aim of which is to reduce military instabilities in the theater.

Finally, the end of the Cold War has forced a re-examination of the institutional framework for collective security in Central Europe. An essential question for any candidate institutional structure, however, is the degree to which the military instruments available for the enforcement of collective security can perform the required tasks. It is essential to know whether particular force postures lend themselves to the larger goals of stability and mutual security -- and if not, how they might be reconfigured to meet those demands more fully in the future.

STABILITY REMAINS IMPORTANT

- Ongoing Conventional Arms Control
- Force Restructuring
- Institutional Design for Post Cold War Europe

Given this, the key questions for this study are threefold: which weapon types are stabilizing; which weapon types are destabilizing; and how large is the difference?

KEY QUESTIONS

- Which Weapon Types are Stabilizing?
- Which Weapon Types are Destabilizing?
- By How Much?

"Stability" is a condition wherein neither of two opposing combatants can obtain a high military payoff by attacking the other, in which neither party has a strong military incentive to strike first in a crisis. Thus we expect that a stable relationship between states will tend to reduce the likelihood of war.

Given this definition of stability, a "stabilizing weapon" is one that, if both sides inventory of that weapon type is increased, the military payoff to attack falls.³ Infantry antitank systems, for example, are often described as "stabilizing" weapons in that, if both sides were more infantry-heavy, many believe the result would be to strengthen the two sides defensive capability, but to add little to either sides offensive potential -- with the result that the potential payoff for either side to attack the other falls.⁴ Conversely, a "destabilizing weapon" is one which *increases* the payoff to attack if given to both sides in equal numbers. Tanks are often cited as an example of a "destabilizing" weapon in that, if both sides were made more tank-heavy, many believe that each sides offense would be strengthened more than their respective defenses -- thus increasing the military payoff for either in an attack on the other.

In using these definitions, three points should be borne in mind. First, stability as defined here is inherently two-sided. This is not necessarily a study of how best to defend NATO; rather, it is a study of how to defend NATO without simultaneously threatening the Soviet Union -- and vice versa. Second, stability is not simply another word for "goodness." Stability is a particular property that may or may not be in the national interest of the United States in any given region of the world at any given time. In Europe, it is a property that is clearly in our national interest (and indeed, it is a major region of the objective of U.S. policy in the region); in the Persian Gulf, on the other hand, it probably is not.⁵ Finally, we will use the term "stability" in a very specific sense -- that is, the relative absence of military incentives to attack a neighboring state. Other commonly used meanings of the term, such as political or arms race stability, while important for other purposes, lie outside our primary focus here and will not be addressed explicitly.

DEFINITIONS

"STABILITY:"

Neither side can obtain high military payoff by attacking the other

"STABILIZING WEAPON:"

Any weapon whose mutual possession decreased both sides' payoff to attack

To determine which weapons are stabilizing and destabilizing in this way, we used the IDA Variable Force employment (VFM) methodology.⁶ The definition of stability given in the previous slide rests on the notion of a "military payoff" to attack. We will define this payoff as the *net territorial gain* a potential attacker could expect were he to initiate a war, where net territorial gain refers to the maximum penetration distance an attacker could take and hold against defensive counterattack.⁷ VFM estimates a combatant's payoff to attack -- or net territorial gain -- as a function of three classes of independent variables: force level, weapon mix, and force employment.

"Force level" refers to the size of the two sides' ground forces, and is measured in units of armored fighting vehicle equivalents (or AFVEs) available in the theater.⁸

"Weapon mix" refers to the composition of those forces (both air and ground for each combatant) in terms of specific weapon types.⁹

"Force employment" refers to the doctrinal and operational concepts by which a given force is actually used in combat. For our purposes, force employment is described abstractly in terms of five key characteristics: the *depth* of the theater defender's deployment (in kilometers), the fraction of the theater defender's total AFVEs that are withheld as operational reserves, the fraction of those reserves that are used for *counterattack* (as opposed to passive reinforcement of the forward committed defenses), the theater attacker's attempted assault *velocity*, and the *frontage* of the theater attacker's advance.¹⁰

For a detailed description of how the VFM methodology interrelates these variables, the reader is referred to IDA P-2380;¹¹ for now, it will suffice to emphasize one characteristic of the way the method treats force employment in particular. That is, VFM considers (at a broad level) the relationship between force employment, force levels, and weapon mixes, and is capable of determining an optimal set of force employment parameters for a given set of conditions.¹²

VARIABLE FORCE EMPLOYMENT MODEL (VFM)

Estimates Attacker's Net Territorial Gain as Function of:

- **Force Level**
- **Weapon Mix**
- **Force Employment**

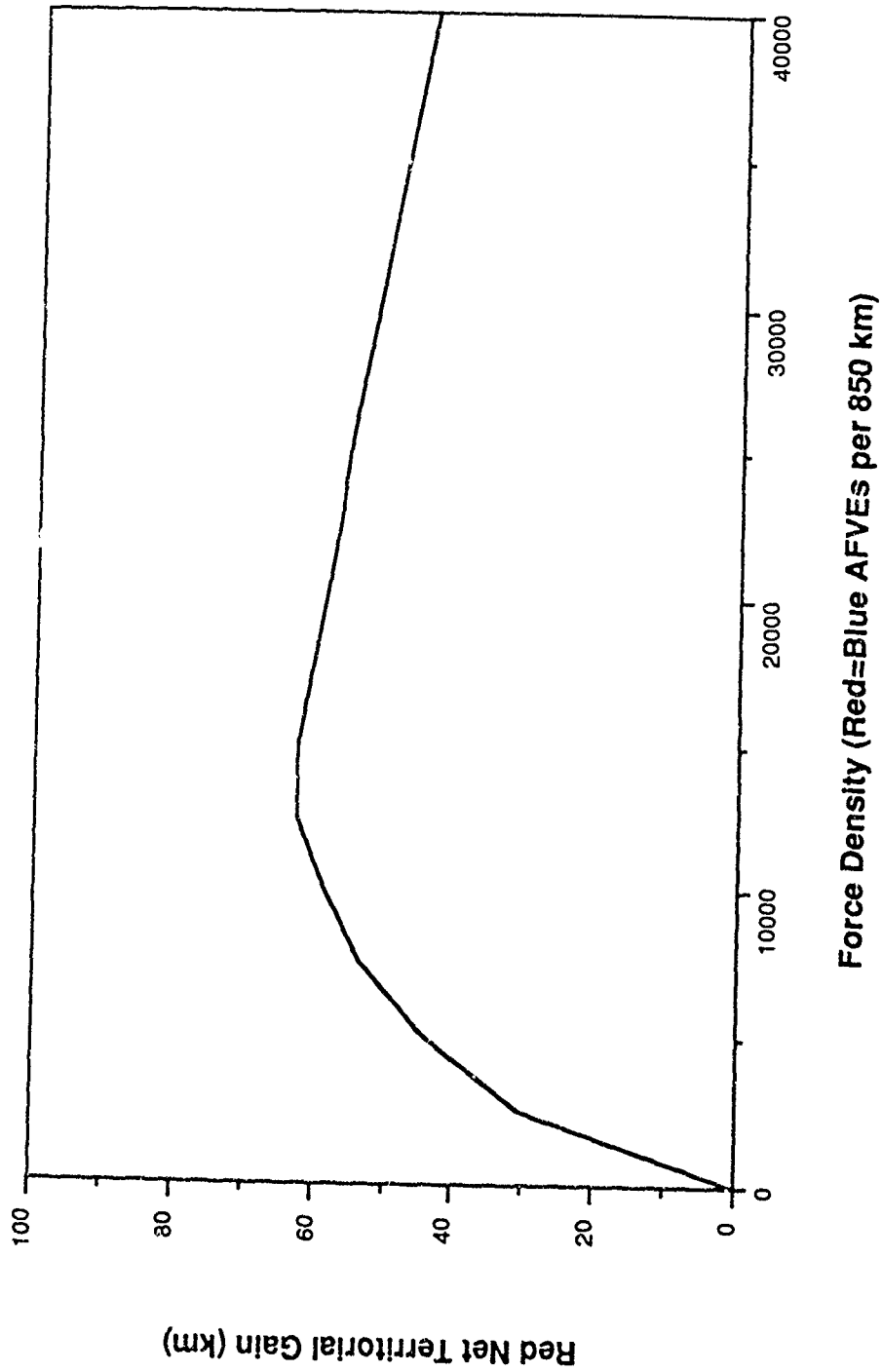
Determines Optimal Employment for Given Force

Slide 6 provides a graphical illustration of typical VFM output. It plots estimated net territorial gain for the putative theater attacker ("red") as a function of theater force level, expressed as ground force density in units of AFVEs per constant 850 kilometers of front. The analysis depicted here assumes theater parity between the putative theater defender ("blue") and theater attacker, optimal force employment by both red and blue, and a weapon mix corresponding roughly to that of the proposed CFE I agreement.¹³

The results suggest that theater level combat outcomes are sensitive to force density. As force density falls below the post-CFE I level (i.e., roughly 35,000 AFVEs), net territorial gain increases, albeit slowly, then crests at a force level somewhat below 50 percent of CFE I and begins to decline. A detailed explanation of this sensitivity of combat results to ground force density is provided in IDA P-2380;¹⁴ for now, it will suffice to note the key role of force employment in this behavior. Under an assumption of theater parity, as force density falls blue has fewer defenders with which to hold a constant front, but red also has fewer forces with which to hold seized territory. Blue thus has an incentive to devote an increasing fraction of his theater forces to counterattack as force density declines, thereby requiring red to divert an increasing fraction of his forces to defensive overhead -- eventually limiting red's overall advance by virtue of insufficient forces for defense of seized ground until, at a force density of zero, no ground can be taken or held since there are no forces with which to do so.¹⁵

In the remainder of the brief, we will use as a benchmark or base case this basic relationship between net territorial gain and force density for a condition of parity, optimal force employment, and a post-CFE I weapon mix. We will then add packages of specific weapon types to both sides' forces. Where the resulting net territorial gain curve is higher than that shown here, the addition of that weapon package increases the payoff to attack, and thus that weapon can be characterized as destabilizing. Where the resulting curve is lower, the addition of that weapon package decreases the payoff to attack, and thus that weapon type is stabilizing.

VFM OUTPUT
THEATER PARITY, POST CFE WEAPON MIX



Slide 6.

Slide 7 lists the specific weapon types we considered. Offensive and defensive counterair are treated explicitly by VFM, but the stability properties of these missions are not considered in detail here. Non-tacair air defense in VFM is disaggregate to the level of HIMADS (HI and Medium altitude Air Defense Systems) and SHORADS (SHOrt Range Air Defense Systems).¹⁶ Advanced conventional munitions are disaggregate to the level of SRACM (Short Range Advanced Conventional Munitions) and LRACM (Long Range Advanced Conventional Munitions).¹⁷

Some of these weapon types were treated explicitly in the initial version of the VFM methodology as developed in IDA P-2380 (specifically, tanks, infantry, artillery, "generic" tacair and barrier defenses). For many, however, it was necessary either to extend the methodology to incorporate new weapon types (e.g., air defense, helicopters, tacair in intratheater transport, and artillery-delivered mines) or to treat previously included weapon types in substantially greater detail (e.g. advanced conventional munitions, combat engineers, or tacair in close air support, interdiction, offensive and defensive counterair missions). These extensions to the VFM methodology are described in detail in Appendices B and C.

WEAPON TYPES CONSIDERED

Tanks

Infantry

Artillery

Tacair

- **Close Air Support (CAS)**
- **Interdiction (INT)**
- **C17 in Intratheater Transport Role**

Air Defense (AD)

Helicopters

- **Attack/Observation Teams (AH)**
- **Utility/Infantry Antitank Teams (UH)**

Barrier Defenses

- **Combat Engineers**
- **Artillery-delivered, Switchable Mines (FASCAM)**

Advanced Conventional Munitions (ACM)

- **Short Range (SRACM)**
- **Long Range (LRACM)**

The list of weapon types given on Slide 7 constitutes a rather sizable menu. A detailed description of the stability properties of individual weapon types is provided in Appendix A; here, however, we will take a somewhat different approach and look at the question of stability in terms of three broad classes of effects that we encounter across many of the individual weapon types listed on the previous slide. In particular, whether a given weapon type is stabilizing or not is often a function of the force density in the theater of war, the size of the available inventory of the weapon in question, and the assumptions made with respect to several key uncertainties in the performance of some weapon types.

STABILITY EFFECTS

- Sensitivity to Force Density
- Sensitivity to Inventory Size
- Sensitivity to Weapon Effectiveness Uncertainties

Slide 8.

17

The first of these effects is sensitivity to ground force density. Slide 9 illustrates this effect by comparing the base case net territorial gain curve (as introduced in Slide 6) with an alternative case representing the effects of increasing each sides' tank fleet by about 8000 tanks.¹⁸

At a post-CFE I force density of about 35,000 AFVEs in the theater, the result of adding this package is to increase net territorial gain from about 48 kilometers in the base case to about 55 kilometers in the alternative, tank-heavier posture. For high force densities, then, larger tank inventories on both sides increase the payoff to attack, and tanks are thus destabilizing.

This is not true for all force densities, however. In particular, for densities below about 15,000 AFVEs in the theater, the result of adding tanks to each side is to *decrease* net territorial gain. At low force densities, larger tank inventories on both sides thus decrease the payoff to attack, making tanks a *stabilizing* weapon for this part of the curve.

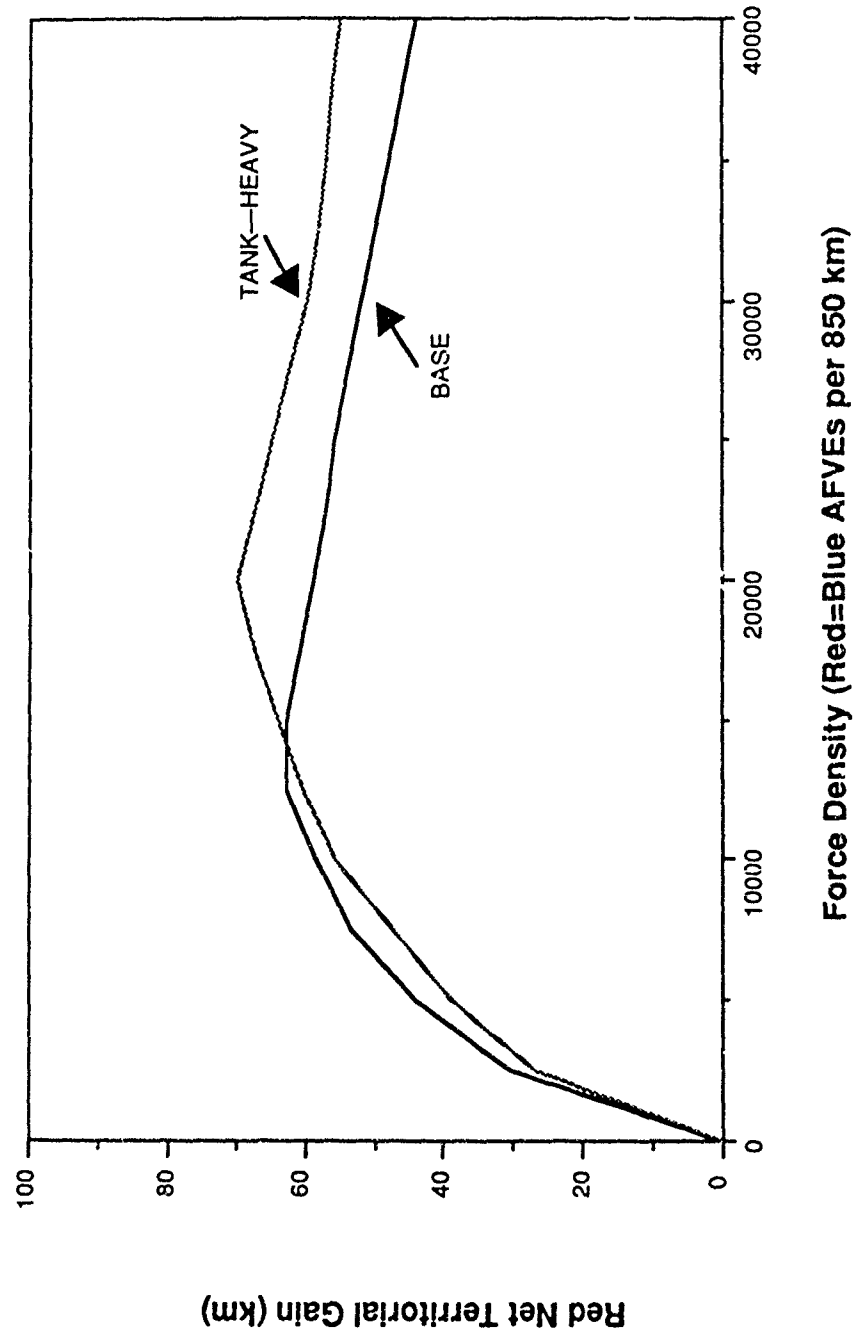
Why do we see this cross-over from destabilizing to stabilizing? The answer has to do with force employment. Tanks are ordinarily best suited for use in the tactical offensive. But as we move from the tactical to the theater level, we see that both sides are engaged in some tactically offensive and some tactically defensive combat actions. Red, for example, uses his forces offensively at a chosen point of attack, but defends elsewhere. Blue typically defends at red's point of attack, but often mounts a counteroffensive against the flanks of the red penetration. It is the particular balance between these two forms of tactical employment in the theater as a whole that determines whether a weapon type best suited for one or the other nets out to the advantage of the invader or the invaded -- and thus whether that weapon is destabilizing or not. If red, for example, emphasizes tactical offense and blue emphasizes tactical defense, then weapons best suited for tactical offense will be relatively more valuable to red than to blue and those weapons will be destabilizing.

The two sides' relative emphasis on tactical offense and defense, however, is not a universal constant; rather, it is a force employment *choice* made by each side to suit the circumstances prevailing at the time of the invasion. When those circumstances change, the force employment incentives of the two sides change, and the relative balance of tactical offense and defense in the theater as a whole changes as well. In particular, as force density falls, blue's incentives are to use a larger and larger fraction of his total AFVEs for counterattack, as opposed to passive defense. This in turn drives red to devote a larger and larger fraction of his total force to flank defense, or else risk the encirclement and possible annihilation of his offensive spearhead in the event that a blue flank attack breaks through.¹⁹

These changing incentives are reflected in the optimal force employment choices computed by VFM. At high force density, blue maintains a relatively modest operational reserve and uses most of this for passive reinforcement of the forward defenses at red's point of attack. Thus the fraction of blue's total AFVEs used for counterattack at a density of 35,000 AFVEs is less than 0.05. Red thus requires only modest off-axis defenses to secure the flanks of his offensive spearhead, which absorbs over 70 percent of red's theaterwide maneuver forces.

SENSITIVITY TO FORCE DENSITY

BASE vs TANK—HEAVY POSTURE



Slide 9.

At low force densities, by contrast, most of blue's force is withheld as a mobile reserve, and most of this reserve is used for counterattack. At a force density of 10,000 AFVEs in the theater, blue devotes almost 80 percent of his total AFVEs to tactically offensive employment; to protect his gains from this threat, red must now divert some 80 percent of his forces to defensive overhead. In effect, taking ground is now relatively easy for red, but holding it, once taken, is not -- indeed, the binding constraint on red's ability to take and hold ground is now the capacity to hold, not the capacity to take.

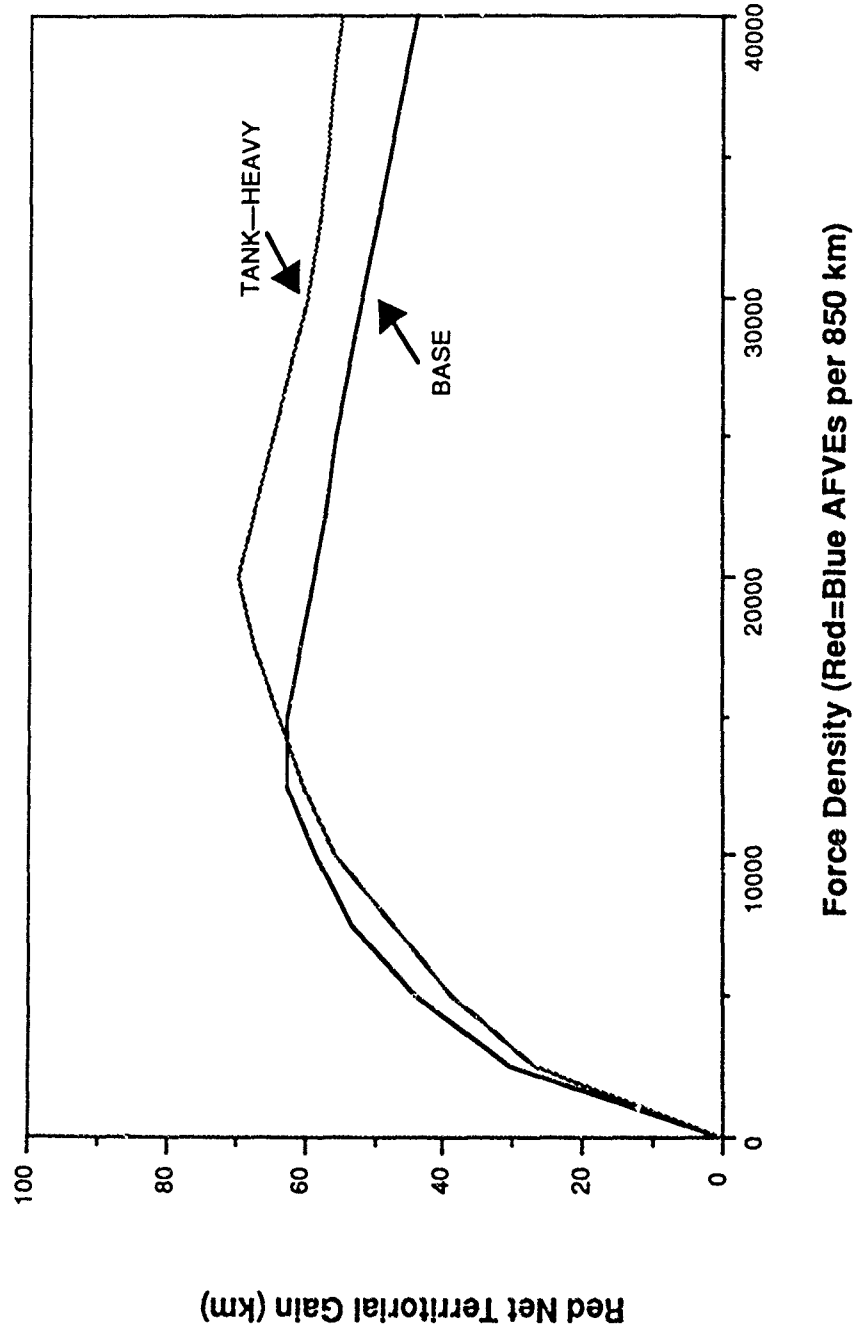
Thus, at high force densities, the lion's share of tactical offense in the theater is conducted by red, and the primary determinant of red's success is the effectiveness of this tactical assault. Increasing the availability of weapons best suited for tactical assault thus benefits red more than blue at high force density, and tanks are therefore destabilizing. At low force densities, however, the primary determinant of red's net territorial gain is the effectiveness of blue's tactical assault against the flanks of the red penetration -- hence increasing the availability of weapons best suited for tactical assault benefits blue more than red at low force density, making tanks stabilizing.

A final point to take away from this slide, however, is that the magnitude of these effects on net territorial gain is quite modest. At high force densities, where the absolute difference between the base case and the tank-heavier alternative is greatest, the actual difference is only about 10 kilometers (or under 20 percent of the base case value). At a density of 10,000 AFVLs in the theater, the absolute difference is less than 5 kilometers.

In short, the stability properties of tanks are a function of force density -- but the difference this makes for the payoff to attack is relatively small.

SENSITIVITY TO FORCE DENSITY

BASE vs TANK—HEAVY POSTURE



Slide 9.

The second class of effects pertaining to stability is sensitivity to inventory size. To illustrate this effect, this slide begins with the two axes from the previous graphs -- red's net territorial gain, and the theater ground force density -- and adds a third axis to represent increasing inventories of long range advanced conventional munitions (LRACM) on both sides.²⁰

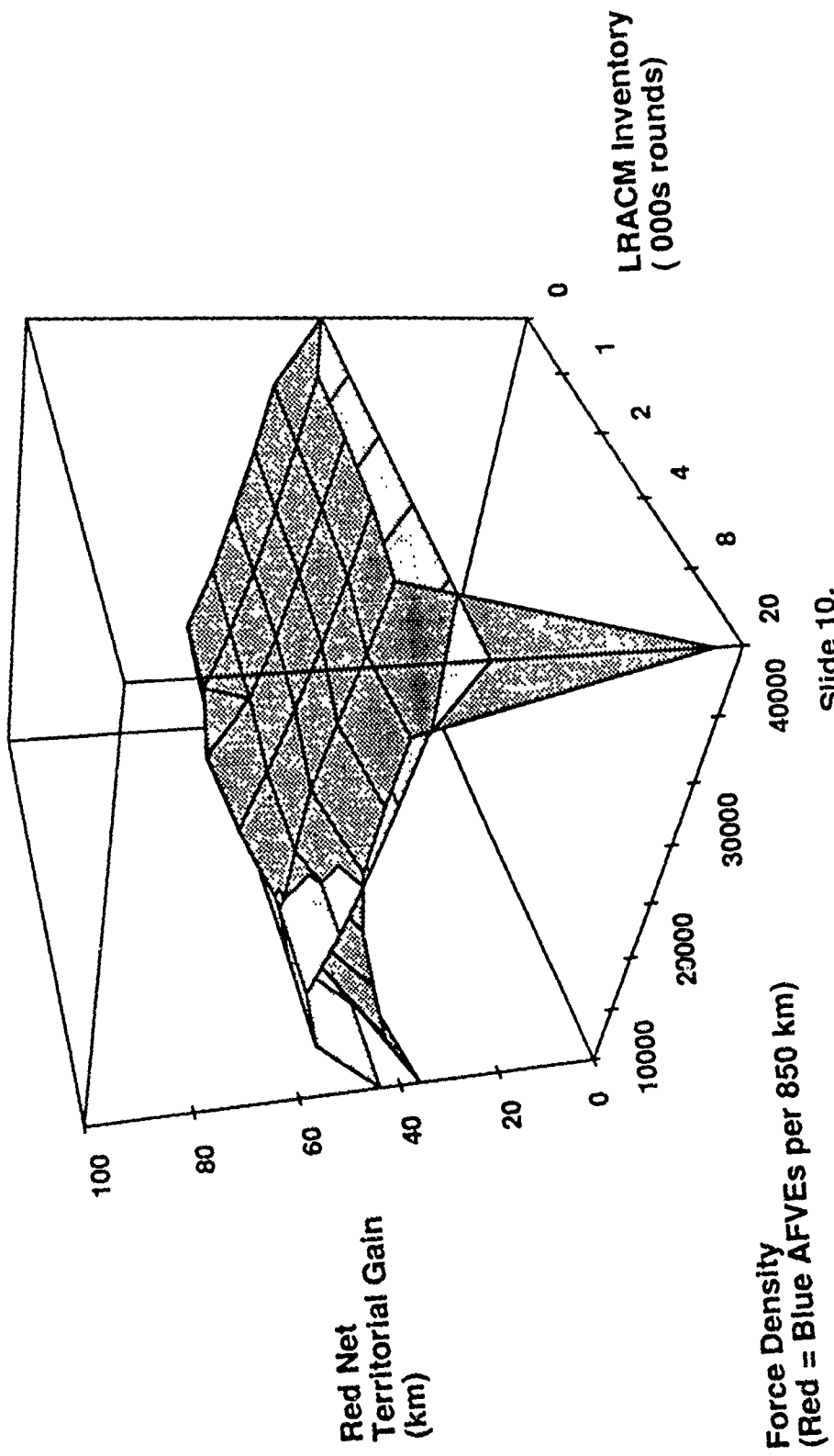
With three dimensions, the base case VFM output from the previous slides is now a surface rather than a line, and is the lighter colored of the two surfaces depicted here. Note that the cross-section of this surface with respect to the net territorial gain vs force density axes is still the same smooth, shallow relationship seen in Slides 6 and 9.²¹ The base case surface is constant with respect to the LRACM inventory axis, providing a zero-LRACM benchmark net territorial gain for the purpose of comparison. This zero-LRACM benchmark surface is compared with an alternative, darker colored surface showing the effects of adding increasing amounts of LRACM to the base case force postures.

At the origin of the LRACM axis, where the LRACM inventories are zero for both the base case and the excursion, the surfaces coincide. But as we add LRACM to the two sides' force postures in the excursion case, the base and excursion surfaces begin to diverge. When red and blue each are given LRACM inventories of 1000 rounds, for example, net territorial gains increase for the excursion case relative to the zero LRACM base (i.e., the darker surface is now above the lighter). Thus, an inventory of 1000 LRACM rounds increases the payoff to attack, and LRACM is destabilizing. As LRACM inventories continue to grow, net territorial gain continues to increase relative to the base case, but if enough rounds are added, net territorial gain eventually turns downward -- and eventually falls below that of the base case. Large inventories of LRACM therefore become stabilizing, especially at very high and very low ground force densities.

Why is the stability of LRACM sensitive to inventory size in this way? The answer has to do with the influence of long range firepower on counterconcentration, and on how this influence differs for theater attackers and defenders. For blue, LRACM provides a source of immediately counterconcentratable combat power that can be brought to bear over theaterwide distances. Successful counterconcentration is essential to the viability of the blue defense; thus the capacity to accomplish this more quickly through very long range fires can be a powerful one for blue. For red, on the other hand, LRACM provides firepower that can be used to reach into the blue rear and interdict the counterconcentration movement of blue ground forces. Where LRACM is available to both sides, but in limited numbers, the resulting aggregate firepower is insufficient for blue to counterconcentrate sufficiently with the use of LRACM fire alone. Yet the ground force movements blue depends upon to make up the difference occur at a modest rate (given the length of the theater front, and thus the distance to be travelled before counterconcentration is complete), and a relatively modest increase in red's long range interdiction capacity (above that already provided by tacair) can be sufficient to reduce substantially blue's reserve arrival rate at the point of attack. For small LRACM inventories, the net effect of these countervailing capabilities is to increase somewhat red's potential territorial gain, and LRACM is destabilizing. If LRACM inventories are large enough, however, then it eventually becomes possible for blue to counterconcentrate sufficiently through LRACM fire alone. At this point, red's ability to interdict the movement of blue ground force reserves is of little value to red, whereas the capacity to counterconcentrate without awaiting the arrival of ground forces is of great value to blue. For large LRACM inventories, the net effect is thus to reduce red's potential territorial gain, and LRACM is stabilizing.

SENSITIVITY TO INVENTORY SIZE

Increasing Quantities of LRACM



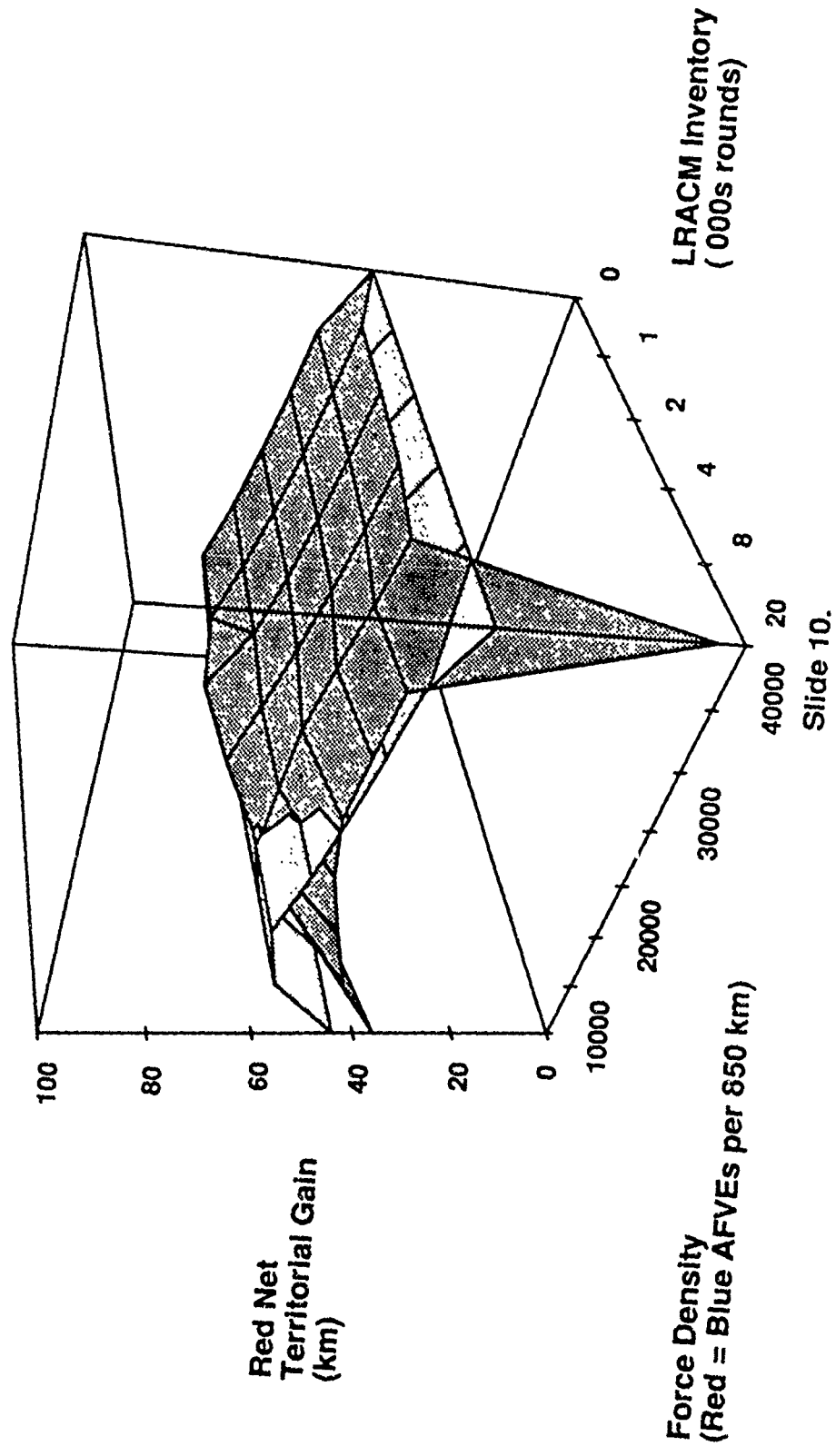
Slide 10.

Finally, note that once again the magnitude of the effect of adding LRACM to both sides' forces is relatively modest for most LRACM inventories. Up to about 10-12,000 rounds, the difference between the base case and excursion net territorial gain is under 15 kilometers.

Thus the stability of LRACM depends on the size of the inventory -- small numbers of rounds tend to be destabilizing, while large numbers can become stabilizing; but for most inventories, the magnitude of the effect is relatively small.

SENSITIVITY TO INVENTORY SIZE

Increasing Quantities of LRACM



The final class of effect pertaining to stability is sensitivity to effectiveness uncertainties. Slide 11 illustrates this effect using the case of short range advanced conventional munitions (SRACM). Again, we begin with the two standard axes from the previous graphs -- net territorial gain and theater force density. Here, however, we add a third axis to represent the relative effectiveness of SRACM against moving and stationary targets. Throughout the graph, we assume a nominal, constant expected kill of 0.4 AFVEs per SRACM rocket against moving targets. At the origin of the third axis, however, we assume that against *stationary* targets, SRACM has virtually no effect, while at the other end of the axis, we assume that SRACM is just as good against stationary targets as it is against movers.

As a rule, SRACM should be at least somewhat better against moving targets. Moving vehicles provide more distinct target signatures in that they typically run hotter and are harder to camouflage with nets or draped foliage. Moving targets also are more difficult to harden against submunition penetration, since it is less practical to add sandbags or other extraneous material to the roofs and decks of maneuvering vehicles.

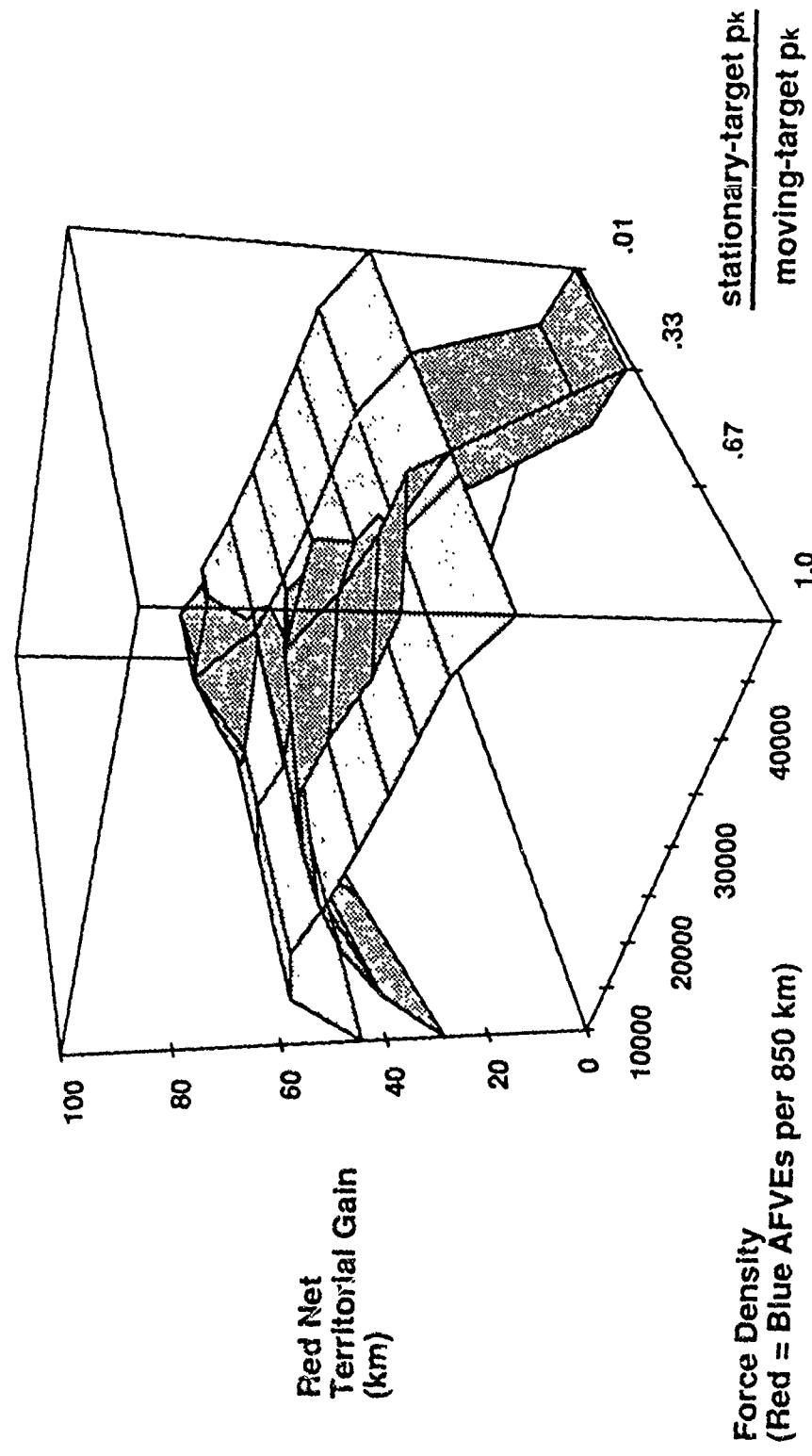
The *degree* of the difference in effectiveness, however, is very difficult to project with confidence. This difference is a function of the particular standing of the ongoing measure-countermeasure race in submunition guidance at the time of the attack, and the outcome of such races can fluctuate radically over time.²² As a result, the precise ratio between SRACM's effectiveness against moving and stationary targets must be regarded as an uncertainty for the purposes of force planning, and this uncertainty matters for the stability of SRACM.

Once again, the lighter colored surface represents the standard base case benchmark, which is constant with respect to relative SRACM effectiveness (there is no SRACM in the base case weapon mix). The excursion case given by the darker colored surface represents the effect of adding to both sides' forces a package of SRACM equivalent to about 30,000 rockets at a force density of 35,000 AFVEs in the theater.²³ This excursion surface is very sensitive to relative effectiveness -- and the direction of the change relative to the benchmark changes at different force densities. When the effectiveness ratio is high, the net effect of adding SRACM to both sides is destabilizing at high force density, but stabilizing when density is low. Conversely, when effectiveness ratio is low, the net effect of adding SRACM to both sides is stabilizing at high force density, but destabilizing when density is low.

Why is this? As with sensitivities to force density, the answer has to do with force employment. As noted above, at high force density, blue employs most of his forces in passive reinforcement or defense at red's point of attack.²⁴ Most of blue's targets are thus stationary at any given time. For SRACM effectiveness ratios near 1.0, SRACM is just as effective against stationary targets as it is against movers. But while both sides' targets are thus potentially vulnerable, red's concentration at the point of attack will provide many more launchers within range than initially will be available to blue (who must counterconcentrate over time). This preponderance can be exploited to destroy a larger fraction of blue's vulnerable targets prior to assault. Red's net territorial gain thus increases and, as a result, SRACM is destabilizing.

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY

SRACM Pk vs. Moving and Stationary Targets



Slide 11.

At low force density, on the other hand, blue's optimal force employment choice emphasizes counterattack, and red's optimal employment emphasizes flank defense to protect gains from that counterattack. As a result, blue now concentrates his own preponderance of SRACM firepower at a chosen point of *counterattack*. Given an SRACM effectiveness ratio near 1.0, red's stationary flank defenders are potentially vulnerable to blue's SRACM, whose large numbers enable blue to destroy a large fraction of red's vulnerable targets prior to assault. As a result, the blue counterattack now constitutes a more powerful threat to red, and thus requires a larger diversion of red combat power into defensive overhead. Red's net territorial gain thus decreases, and SRACM is stabilizing.

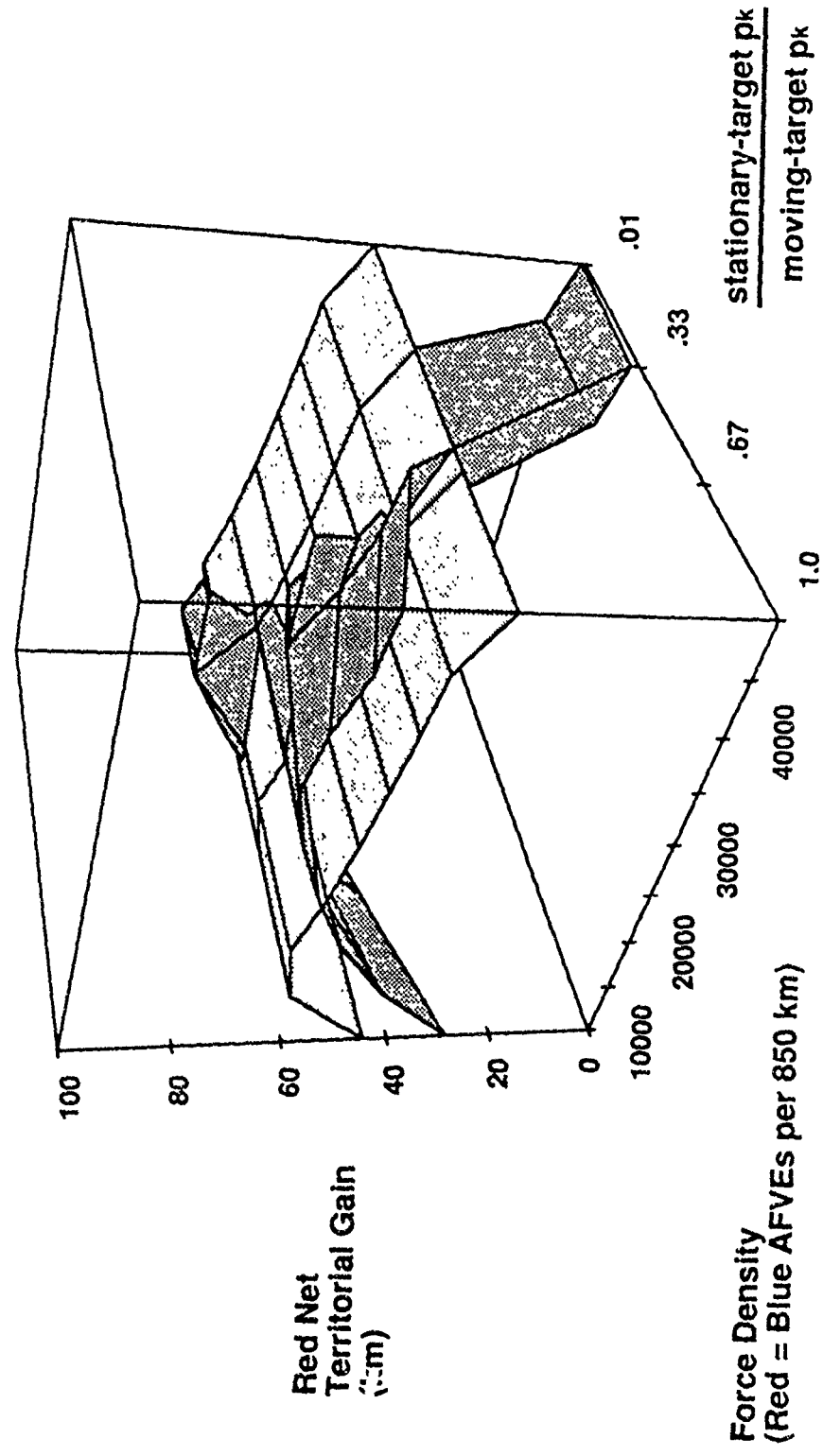
Conversely, where the SRACM effectiveness ratio is near zero, these relationships reverse. Here, SRACM is still highly effective against moving vehicles, but has virtually no effect on stationary targets. Thus, at high force densities -- where blue's targets are mostly stationary and red's are mostly moving -- red's preponderance of SRACM launchers at the point of attack is of little value, yet blue's available SRACM is still effective against red. This increases red's casualties much more than blue's; hence red's net territorial gain falls and SRACM is stabilizing.

At low force densities, where most of blue's forces are moving and more of red's are stationary in flank defense, a SRACM effectiveness ratio near zero now helps red. Blue's SRACM preponderance at the point of counterattack is of little value if the weapons are ineffective against stationary flank defenders, while red's available SRACM is still effective against blue's moving counterattackers. Red now requires fewer defenders to defeat a given counterattack, freeing a larger force for offensive use and thereby increasing red's net territorial gain. Here, SRACM is destabilizing.

In short, then, the stability of SRACM depends on what we assume with respect to the state of an inherently uncertain measure-countermeasure race in ACM submunition guidance (and, thus, the ratio of SRACM effectiveness against moving and stationary targets). SRACM either can be substantially stabilizing or substantially destabilizing, depending on the particular assumptions we make as to the outcome of this race and the force density in the theater of war.

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY

SRACM P_k vs. Moving and Stationary Targets



Slide 11.

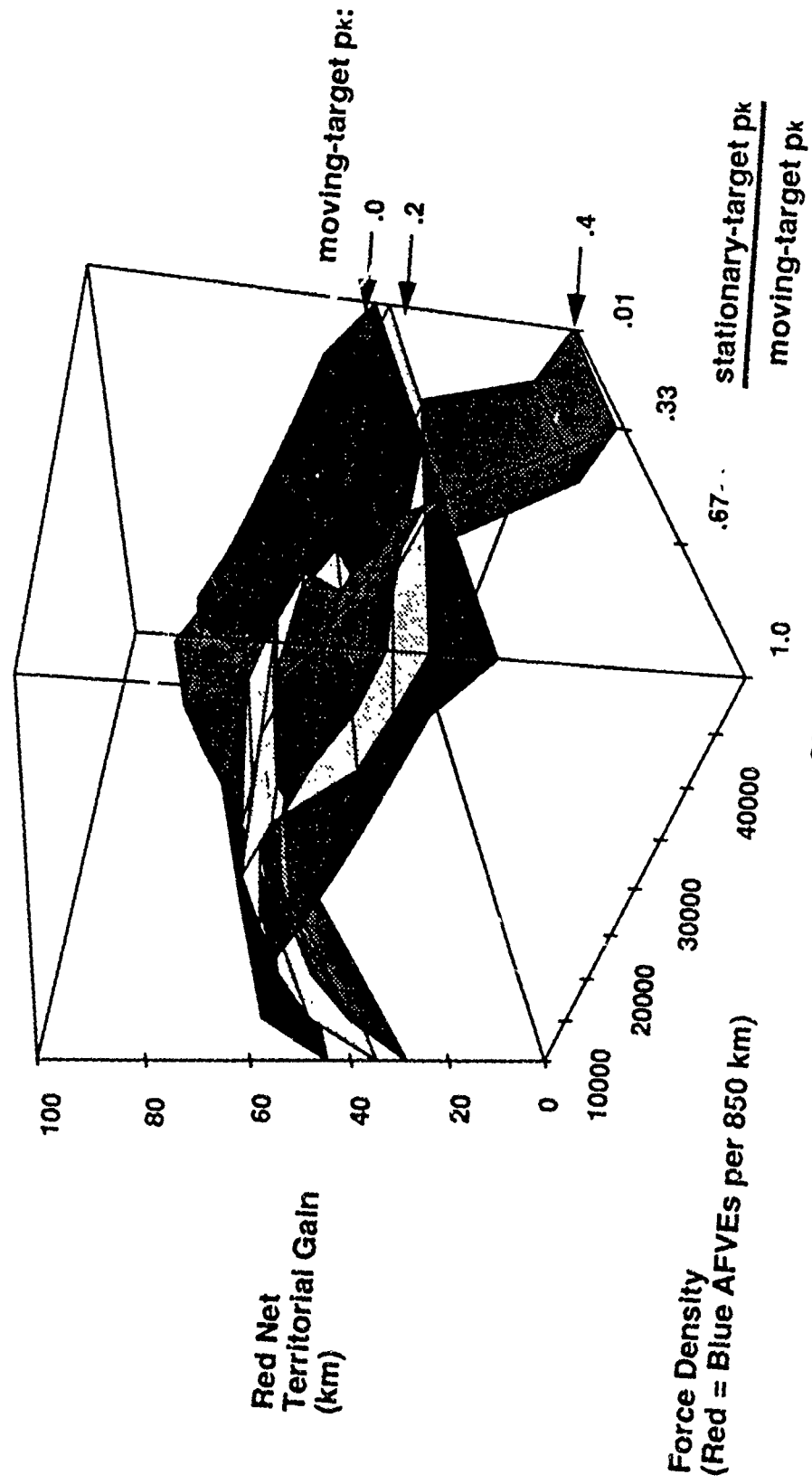
A final point to be made with respect to effectiveness uncertainties concerns the magnitude of the effect. In particular, the degree of difference between the base case payoff to attack and that resulting from the addition of SRACM is itself an uncertainty. The analysis on the previous slide assumed that each SRACM rocket would kill an average of 0.4 moving targets. In fact, however, SRACM's effectiveness against *moving* targets is also subject to variation as the result of the unavoidable measure-countermeasure competition in ACM, with 0.4 AFVE kills per rocket representing an optimistic estimate.²⁵ If we instead consider a more modest estimate of 0.2 moving target kills per rocket, the results are as given by the light surface here (Slide 12). (In effect, the base case, in which no SRACM was present, corresponds to a lower bound assumption of 0.0 AFVE kills per rocket).

Note that the direction of SRACM's effect on net territorial gain is largely the same whether its moving target effectiveness is 0.4 or 0.2 AFVEs per rocket. The magnitude of this effect, however, is generally much smaller, especially at the extremes of force density and relative stationary-moving effectiveness where the magnitude of the effect was greatest in the previous slide. At a force density of 40,000 AFVEs in the theater and a ratio of stationary to moving target effectiveness of 0.01, for example, a moving target Pk of 0.4 causes red net territorial gain to drop by more than 90 percent to a value of less than five kilometers. If SRACM's moving target Pk is only 0.2, however, net territorial gain still falls relative to the no SRACM base case, but the difference is now less than 10 percent of the base case value. Similarly, at a force density of 10,000 AFVEs in the theater and a ratio of stationary to moving target effectiveness of 1.0, net territorial gain also falls for all values of moving target Pk, but the magnitude of the difference is about 40 percent smaller when moving target Pk is 0.2 rather than 0.4. For most points in between, the magnitude of the difference between the base case outcome and the results of adding SRACM to both sides is under 15 percent of the base case net territorial gain.

Overall, then, the magnitude of the effect of SRACM on net territorial gain is modest for most values of force density and SRACM lethality against moving and stationary targets. Under some circumstances (specifically the simultaneous combination of high SRACM effectiveness against moving targets, little or no effectiveness against stationary targets, and high force density in the theater), the result of adding some 30,000 SRACM rockets to both sides can be very pronounced. But for the majority of the range of SRACM effectiveness uncertainties and theater force densities, the effect is much smaller.

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY

SRACM P_k vs. Moving and Stationary Targets



Slide 12.

While we have used specific weapon types to illustrate these sensitivities, in fact these effects characterize many of the weapons addressed in the study. Slide 13 lists the weapons which display each class of effect. Note that there are several weapon types that display all three of these sensitivities (i.e., ACM and CAS), some that display only one or two (e.g., tanks, artillery, or air defense), and a few that are insensitive to any of these effects (i.e., UHs, C17s, interdiction tacair and combat engineers).

The main point of this slide, however, is that for most weapon types, stability is strongly influenced by *context*. Weapons can switch from stabilizing to destabilizing depending on whether the theater ground force density is high or low. Large inventories of weapons can have different stability properties than do small inventories, and it matters whether we assume high or low effectiveness for weapon types whose performance is subject to substantial uncertainty.

TAXONOMY OF STABILITY EFFECTS

Sensitivity to Force Density

Tanks
Infantry
CAS
ACM
AH
Artillery
FASCAM

Sensitivity to Inventory Size

ACM
CAS
FASCAM
AH

Sensitivity to Weapon Effectiveness Uncertainties

ACM
CAS
AD

Relatively Insensitive

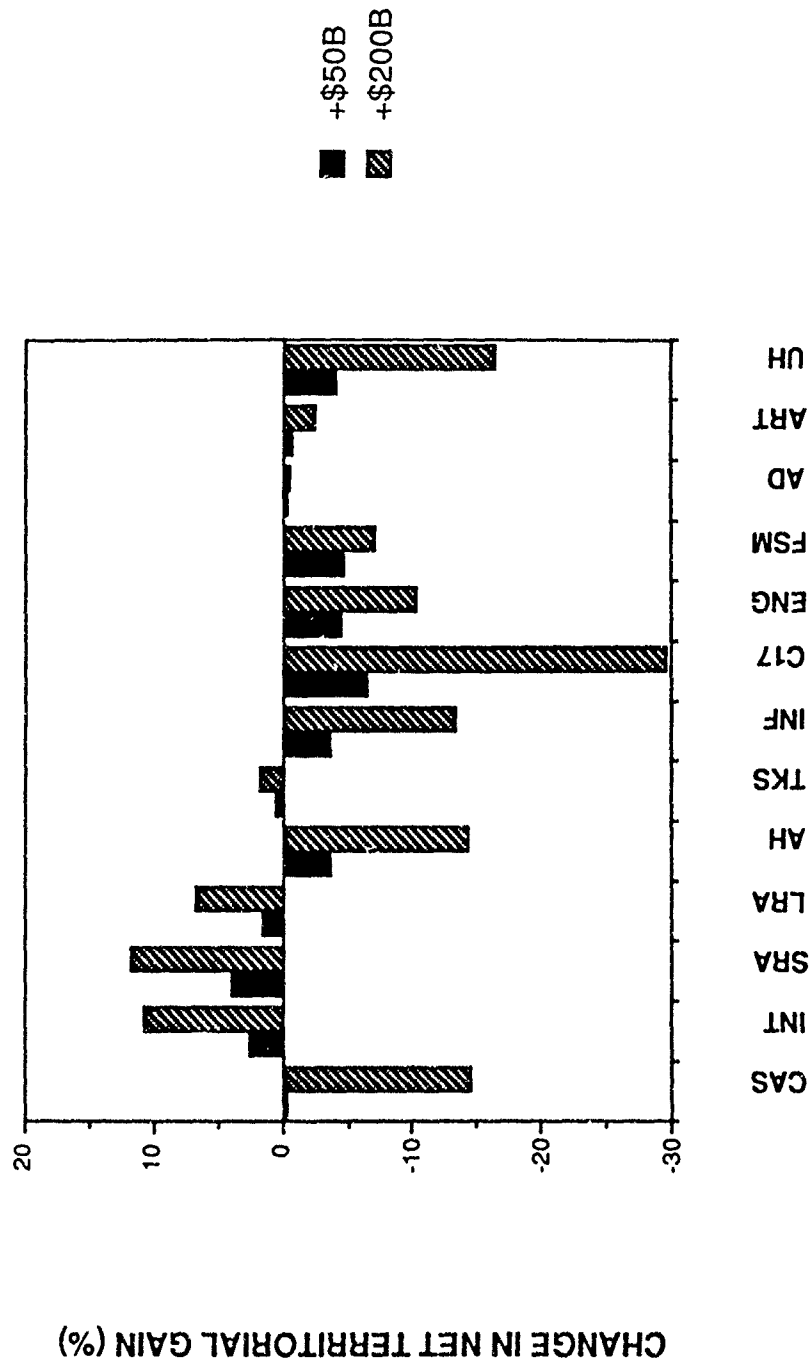
UH/INF
C17
Combat Engineers
Air Interdiction

This does not mean that it is impossible to draw distinctions, however. While it is impossible to label most weapons as stabilizing or destabilizing independent of context, for any *particular* context it is possible to distinguish differences. On Slide 14, for example, ground force density is held constant at a post-CFE I level of 35,000 AFVEs in the theater, effectiveness uncertainties are fixed at roughly mid-range values, and weapon inventories are fixed at two contrasting levels -- the left hand bars show the effects of adding a package equivalent to \$50 billion worth of that weapon type (in 15 year system cost terms), while the right hand bars show the effects of adding a larger, \$200 billion package of that weapon type.²⁶ For each weapon type under study,²⁷ the net territorial gain resulting from adding these packages to each sides' forces was compared to that of the base case. The results plotted here show the resulting percentage change in net territorial gain, and thus the payoff to attack. A positive number means that territorial gain went up when the package was added and thus that weapon type was stabilizing; conversely, a negative number means that territorial gain went down and thus the weapon was stabilizing.

The results suggest that, for these conditions, many weapon types display more or less intuitive stability behavior. Tanks, for example, are stabilizing; infantry is stabilizing.²⁸ Perhaps the most important conclusion to be drawn from this slide, however, is that the magnitude of the illustrated changes in net territorial gain is again quite modest. The most stabilizing weapon type examined under these conditions (the C17 in intratheater transportation) still produced less than a 30 percent difference in net territorial gain for a \$200 billion expenditure by each side. Most produce less than a 20 percent change for the same expenditure, and almost half yield less than a 10 percent difference in territorial gain. When each side adds only \$50 billion, none of the systems examined (for the conditions specified) produced even a 10 percent change in the payoff to attack.

WEAPON SYSTEM STABILITY PROPERTIES

THEATER PARITY, POST CFE FORCE LEVELS



Slide 14.

The results on the previous slide were specific to the context given and, of course, are susceptible to change when circumstances change; as such, they are best regarded as illustrative rather than definitive. But while most weapons' stability properties are sensitive to context, some are not. Moreover, not all contexts are equally relevant for policy purposes. If we step back and take a broader look across the range of conditions likeliest to be of interest for policy planning, some general tendencies emerge. In particular, while stability is at least somewhat sensitive to circumstance or almost all weapon types,²⁹ there are some weapons whose behavior is relatively consistent across the range of conditions that are most likely to be of immediate policy relevance.³⁰ The C17 in intratheater transport, utility helicopters carrying infantry antitank teams, and combat engineers, for example, are stabilizing for most conditions of interest. Tacair in an interdiction role, and long range advanced conventional munitions, by contrast, are generally destabilizing. Given this, how much of a stability improvement might be possible if the two sides' forces were restructured systematically to emphasize those weapon types that are most consistently stabilizing, and to deemphasize those most consistently destabilizing?

SUMMARY OF BEHAVIOR

Consistently Stabilizing

C17
UH
Combat Engineers

Consistently Destabilizing

Air Interdiction
LRACM

Conditional

Tanks
Infantry
Artillery
FASCAM

AD
CAS
SRACM
AH

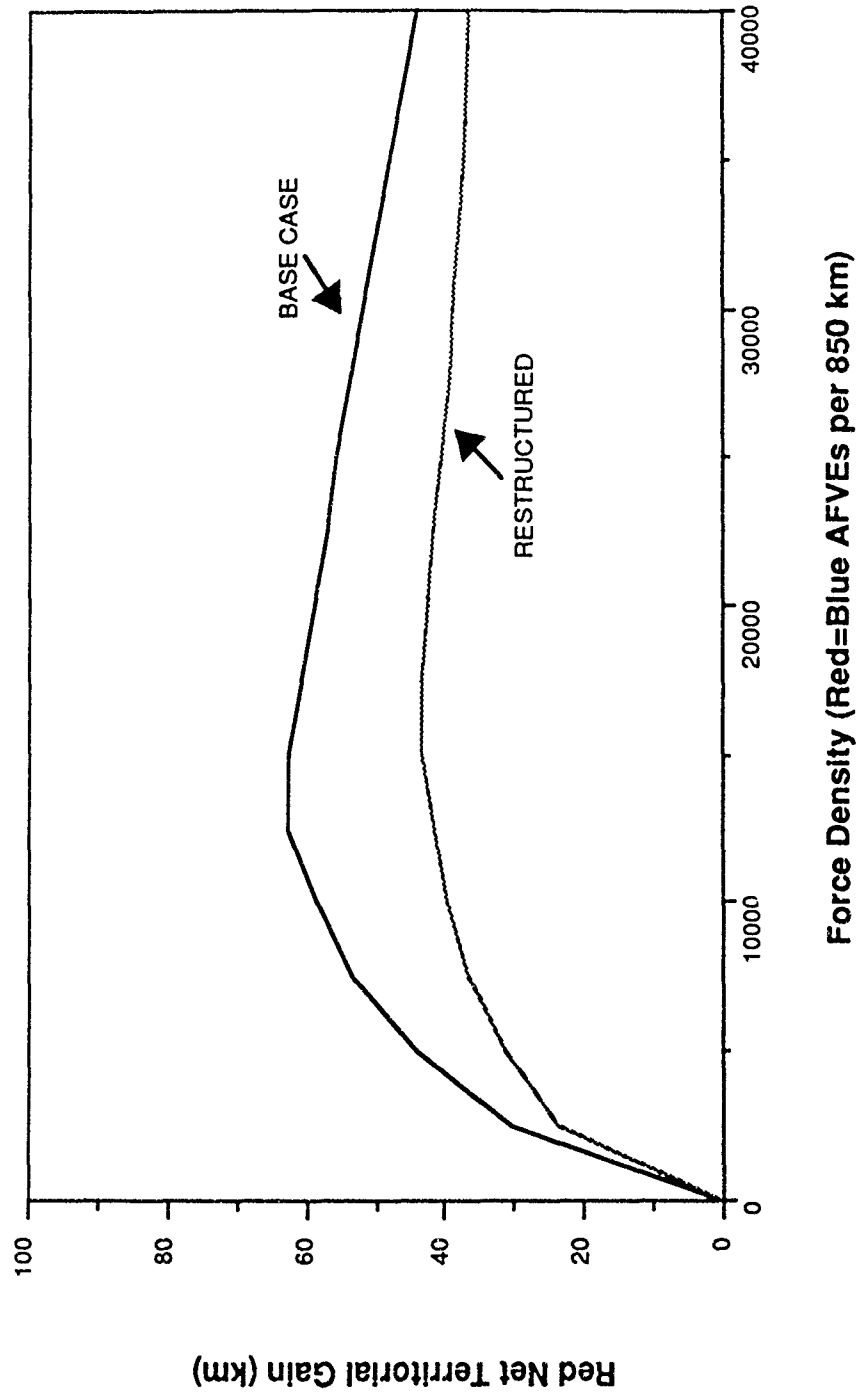
To answer that question, an alternative posture was specified in which neither side deployed any LRACM; both sides eliminated the equivalent of \$100 billion of interdiction aircraft; and both sides invested an additional \$100 billion in intratheater transport aircraft, \$100 billion in utility helicopters with infantry antitank teams, and \$100 billion in combat engine. Thus, the alternative posture represents a net investment of an additional \$200 billion by each side.³¹

The results suggest that the systematically restructured force is indeed stabilizing at all force densities. Again, however, the magnitude of the effect is relatively small. At the post-CFE I force density of 35,000 AFVEs in the theater, the net effect of restructuring is a roughly 20 percent reduction in the payoff to attack; the effect is somewhat greater at lower force densities, but even at 15,000 AFVEs the net change is still only about one-third of the base case net territorial gain (or about 20 kilometers in absolute terms).

It should be emphasized that this is only one possible plan for force restructuring, and many alternatives could certainly be devised. For this particular blueprint, however, the degree of stability improvement obtained from restructuring is relatively modest.

SYSTEMATICALLY RESTRUCTURED FORCES

THEATER PARITY, \$200 B NET INVESTMENT



Slide 16.

The analysis on the previous slide points to a rather consistent property of the results obtained in the study as a whole -- that is, the relative insensitivity of net territorial gain to changes in the weapon mix. Why should this be the case?

There are three reasons (Slide 17). First, at the tactical level, it is a rare weapon that cannot be put to some good use by both attackers and defenders. Tanks, for example, are best suited for the tactical assault, but they also are of substantial value as antitank weapons in the tactical defense. Giving both sides more tanks is thus of some benefit to both. Of course, this benefit is not exactly equal for defenders and attackers -- indeed, few weapons are of identical value in tactical attack and defense. The difference, however, is what matters for stability, and this difference is often not large.³²

Second, at the theater level, both red and blue fight some tactically offensive and some tactically defensive combat actions. Moreover, the relative emphasis between the two roles changes for each side as force density changes. While it will rarely be the case that the two sides' fortunes rest equally on offensive and defensive tactical actions, again it is the difference that matters here, and again the difference is not always large.

Finally, and perhaps most importantly, at both the tactical and the theater level, doctrinal adaptation mitigates the effects of changes in the weapon mix. Each side has the capacity to alter its force employment to adapt to the environmental changes created by changes in weapon availability. If each side is given more tanks, for example, blue can opt to use more of his theater force for counterattack so as to exploit the increased capacity for tactically offensive warfare which this represents. Blue also can increase the depth of his forward deployment so as to forestall red breakthrough while that counterattack takes shape, and withdraw engaged defenses more readily so as to preserve forward forces for a more elastic delaying action in the meantime. Alternatively, if each side is given more barrier defenses, red can opt to slow the pace of his assault to allow more time to clear barriers and suppress overwatching defenders. red also can exploit the capacity of his own barriers away from the point of attack to free a larger force for use in the offensive spearhead. While these doctrinal and operational adaptations rarely compensate completely for the effects of a given change in weapon mix, they can make a major difference in its magnitude.³³ In all, doctrinal adaptation can be an extremely powerful influence on net territorial gain, and thus on stability.

WHY IS STABILITY SO INSENSITIVE TO WEAPON MIX?

At Tactical Level,
Most Weapons of Some Value for Both Offense and Defense

At Theater Level,
Both Invader and Invaded Conduct Both Tactical Offensives and Defenses

Force Employment Adaptation,
At Both Tactical and Theater Level, Mitigates Effects of Changing Weapon Mix

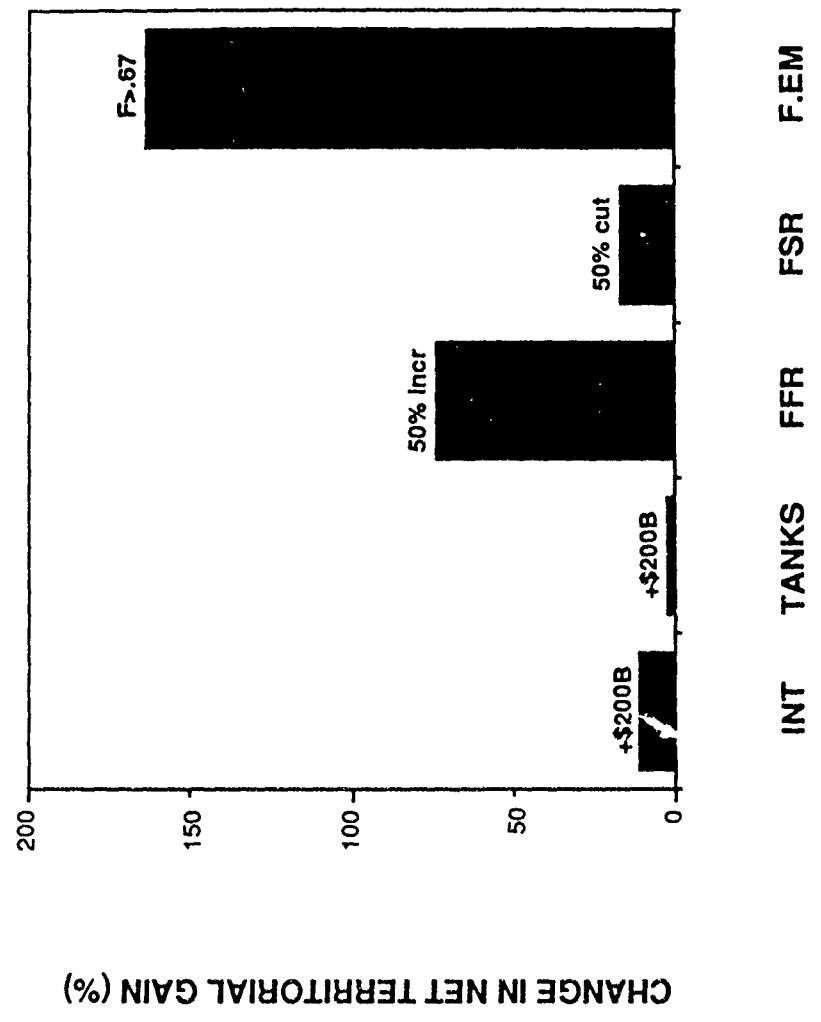
To suggest just how powerful an influence doctrine can be, a series of analyses were conducted to investigate the relative influence on stability of changes in weapon mix, changes in the theater numerical balance, and changes in force employment. Those analyses are summarized on Slide 18.

The slide plots the percentage change in red net territorial gain relative to the base case (Slide 6) resulting from the indicated changes in the two sides' force postures. This base case assumes a post-CFE ground force density of 35,000 AFVEs per 850 kilometers, theater parity between red and blue, and effectiveness uncertainties fixed at mid-range values, yielding a baseline net territorial gain of about 48 kilometers. As with Slide 14, positive values indicate an increase in the payoff to attack and hence a destabilizing effect. Unlike Slide 14, all the changes examined here were destabilizing and thus there are no negative values shown. Five cases were considered. Two involve changes in weapon mix: one (labeled "tanks") in which 50 percent of each sides' infantry are replaced with tanks, and one ("arty") in which each sides' artillery inventory is increased by 50 percent. Two involve changes in the theater numerical balance: one ("fsr") in which the theater ground force density (or "force to space ratio") is reduced by 50 percent, and one ("ffr") in which the force to force ratio (i.e., the ratio of red to blue AFVEs in the theater) is increased 50 percent, from parity to 1.5:1.

Finally, one case (f.em.) involves a change in doctrine, or force employment. Specifically, blue is constrained to make a suboptimal force employment choice. The other cases depicted on this slide -- and indeed, all the other results presented in this briefing -- assume optimal force employment choices by both red and blue. In effect, each side is free to employ its forces in the manner that best serves its military objectives, given the particular circumstances prevailing at the time of putative attack. Here, however, blue's range of potential force employment options has been restricted so as to force a suboptimal choice. In particular, blue is constrained to commit at least two-thirds of his theater AFVEs forward, as opposed to withheld in mobile reserve.

The results show that, as we have seen, variations in weapon mix produce modest changes in net territorial gain. Variations in the theater numerical balance produce somewhat greater changes, but by far the largest effect stems from the constraint on blue's range of force employment choice, which more than doubled red's net territorial gain. In fact, this represents a relatively mild constraint. Many potential restrictions -- especially with respect to the depth of forward defenses -- leave blue unable to prevent red from breaking through, resulting in the catastrophic collapse of the blue defense.³⁴

RELATIVE INFLUENCE ON GROUND GAIN WEAPON MIX, NUMERICAL BALANCE, FORCE EMPLOYMENT



Slide 18.

So far, we have addressed only symmetrical changes in force posture. In all cases, both invader and invaded have been given identical weapon packages, and the base case force postures themselves have been assumed to be largely similar. Symmetric change is a useful conceptual device for understanding the underlying phenomena and fundamental stability properties of different weapon types, especially given the inherently two-sided nature of stability as an arms control criterion. Nevertheless, the real world is not perfectly symmetrical. How would these results differ if, for example, the two sides' weapons are of systematically different quality?

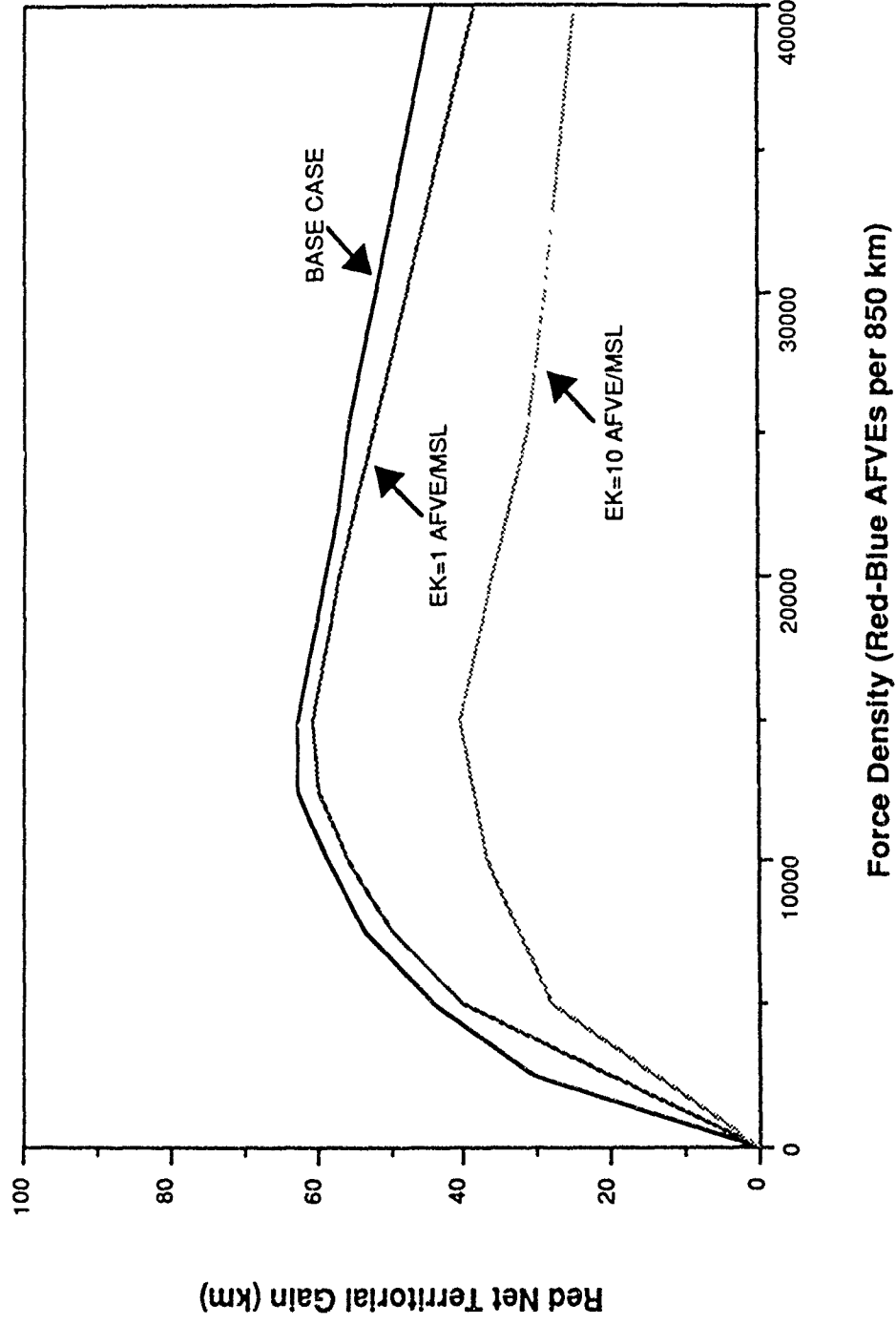
While a systematic examination of effectiveness asymmetries would be beyond the scope of this study, some preliminary analyses have been conducted; the next two slides illustrate the results of one such analysis, pertaining to long range advanced conventional munitions.

In particular, what if only NATO were to deploy LRACM? In this slide(Slide 19), we assume that NATO is the theater defender, and we estimate the results of a potential attack by a Soviet force (at theater parity) which has no LRACM against a NATO defense buttressed by a package of LRACM equivalent to about \$50 billion of ATACMS missiles at a force density of 35,000 AFVEs in the theater.³⁵ Since the effectiveness of all ACM is subject to the outcome of an inherently uncertain measure-countermeasure race, the results have been expressed as a range of outcomes for LRACM effectiveness values varying between a minimum of one AFVE kill per missile and a maximum of 10 kills per missile.

The results suggest that if we assume high LRACM effectiveness, then unilateral possession of LRACM by NATO can indeed reduce the Soviets' ability to take and hold NATO territory. For an expected kill of 10 AFVEs per missile, for example, net territorial gain falls by almost 50 percent relative to the no-LRACM base case at all force densities. Unilateral possession of effective LRACM is thus clearly advantageous to NATO. But recall that for the purposes of *stability*, it is necessary that *neither* side be offered a high potential payoff to attacking the other. How much does this same technology increase NATO's potential capacity to take and hold *Soviet* territory?

SYSTEMATIC FORCE ASYMMETRIES

UNILATERAL BLUE LRACM



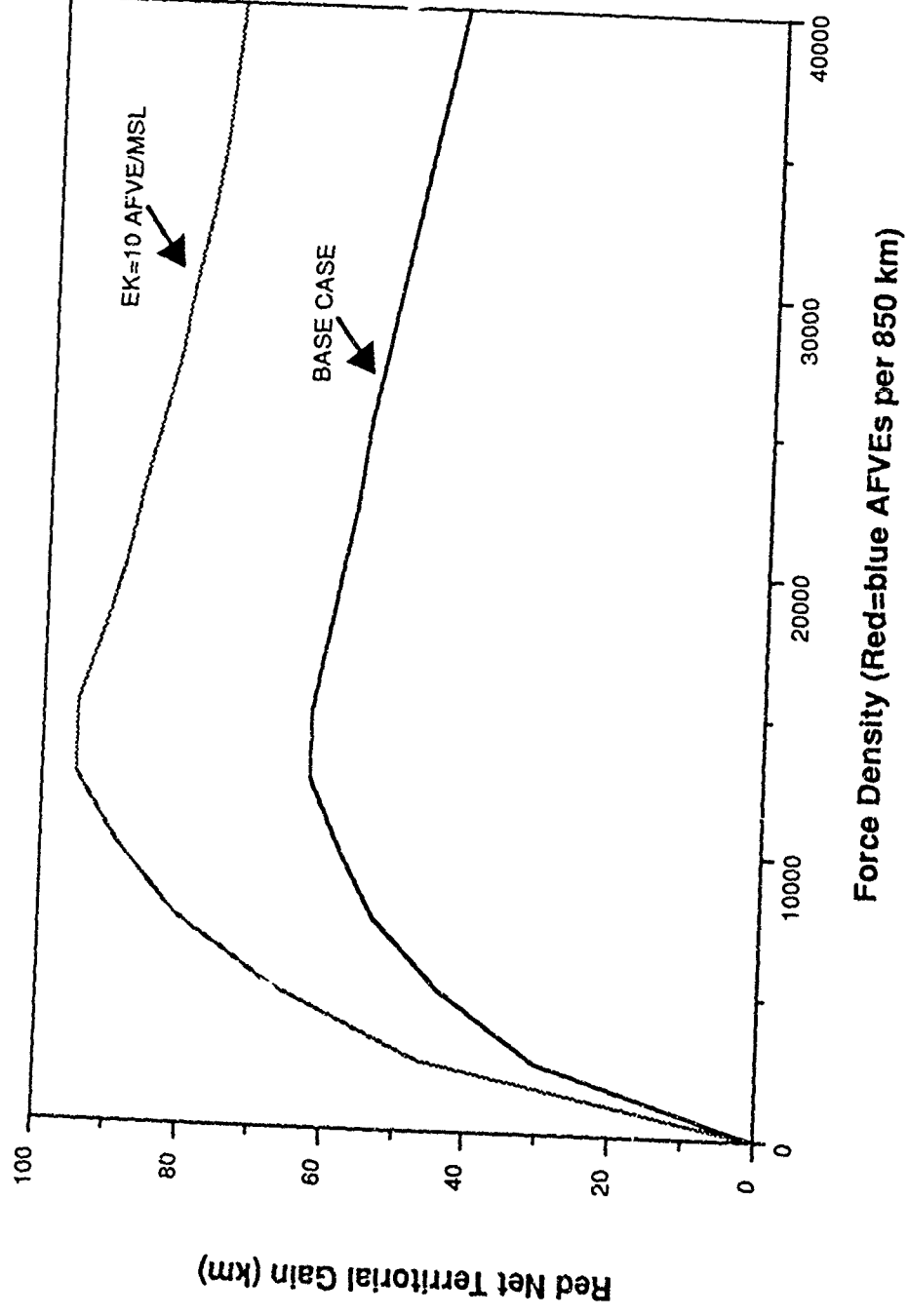
Slide 19.

Slide 20 addresses this other side of the stability problem by assessing the degree to which a unilateral NATO LRACM capability increases the potential payoff to a hypothetical NATO attack on the Soviet Union. Here "red" -- the theater invader -- is assumed to be NATO, which alone deploys an LRACM force equivalent to \$50 billion of ATACMS at a force density of 35,000 AFVEs in the theater.³⁶ Of course, the effectiveness of this LRACM is again subject to uncertainty; only the assumed maximum value of 10 AFVE kills per missile is given here (the others would lie proportionally between this maximum and the no-LRACM base case as per the previous slide).

The results suggest that unilateral deployment of LRACM could substantially increase NATO's potential payoff to an attack on the Soviet Union if we assume high effectiveness for the deployed missiles. Given theater parity in non-LRACM forces, adding LRACM with an effectiveness of 10 AFVE kills per missile to the theater invader's inventory could increase potential net territorial gain by about 40 percent relative to a no-LRACM base case. While it is difficult to say whether this increase would ever be sufficient to undermine deterrence in a crisis, this is nevertheless the largest single increase in the potential payoff to attack of any weapon package considered in the course of this study. Thus, weapons that can be highly effective in increasing NATO's ability to defend against Soviet attack are not necessarily *stabilizing* per se -- stability and the unilateral pursuit of maximum security are not necessarily compatible aims.³⁷

SYSTEMATIC FORCE ASYMMETRIES

UNILATERAL RED LRACM



Slide 20.

What conclusions, then, can be drawn from these analyses? To begin with, the single most important conclusion of this study is that *force employment* -- not the weapon mix -- is the most important single determinant of stability. While changes in the numbers and types of weapons on the two sides do affect stability, this effect is small relative to the influence of the manner in which those weapons are used.

A second conclusion is that for East-West conflict, the kinds of force structures that are plausible as near to mid term policy choices should be highly stable if the two sides maintain sound defensive force employment.-- and in particular, if armies deepen their prepared defenses, withhold larger reserves as fractions of their total available forces, and use an increasing fraction of those reserves for counterattack as force density falls.³⁸ This study has not explicitly addressed the possibilities of East-East conflict among the emerging nations of Eastern Europe (where multipolarity may pose very different military problems), or extreme force mix alternatives such as pure infantry or pure ACM postures. For East-West conflicts involving some degree of combined arms, however, it should be possible to keep military incentives for attack low even if force densities fall substantially.

A related conclusion is that even sizable additions or subtractions of particular weapon types should not materially affect the underlying stability of the East-West military balance -- at least for the weapon types considered here -- given that the two sides adapt their force employment to suit the associated changes in weapon mix.

Finally, this suggests that changes we may wish to make as a result of arms control or unilateral modernization can be based on criteria other than stability. Such criteria might include differences between weapons' relative values as insurance against greater or different than expected threats, differences between weapons' relative utility in non-European contingencies, or the relative ability of different weapons to replace manpower with materiel, or to facilitate inter- or intra-theater transportation.

MAIN MESSAGES

Force employment—not weapon mix—is most important single determinant of stability

Practical future force structures in Europe will be highly stable if both sides maintain sound defensive force employment

This stability will not be significantly affected by plausible additions or subtractions of any of the types of weapons available in the near-term

Changes in weapon mix via arms control or modernization can be based on factors other than stability:

- **Value of weapons as insurance**
- **Utility in non-European contingencies**
- **Traditional U.S. interest in substituting materiel for manpower; ease of transportation; etc.**

There are some possible exceptions to these general conclusions, however. It is always possible, for example, that the emergence of a new technology or radical new weapons design could lead to different results than those described here; the conclusions on the previous slide necessarily pertain only to the weapon types considered in the study. It is also possible, however, that the weapon effectiveness uncertainties discussed above could have significant ramifications for stability.

In particular, if we assume upper bound effectiveness values for tacair and advanced conventional munitions, then it is possible that either weapon type could have a significant effect on stability. Tacair in the interdiction role, or long range ACM, for example, could be significantly destabilizing if effectiveness values are very high. Tacair in close air support, on the other hand, could become either significantly stabilizing or significantly destabilizing, depending on the theater force density at the time of attack. Short range ACM could likewise be either one, depending on ground force density, and on the relative effectiveness of its submunitions against moving and stationary targets at the time of the attack.

POSSIBLE EXCEPTIONS

Tacair, ACM Effectiveness Uncertainties Could Be Significant for Stability

Given Upper Bound Effectiveness:

- Air Interdiction, Long Range ACM Could be Significantly Destabilizing
- CAS Could be Either, Depending on Force Density
- Short Range ACM Could be Either, Depending on Force Density; Relative Effectiveness vs Stationary Targets

Barring these exceptions, however, the results here suggest a number of other implications for U.S. policy. First, with respect to conventional arms control generally, the reduction in numerical imbalances as a result of CFE is clearly beneficial. Nevertheless, continued arms control per se is unlikely to be decisive for stability. This is because the single most important determinant of stability is force employment: it is extremely difficult to mandate a particular operational concept or military doctrine by negotiated agreement.

Second, the process of doctrinal adaptation to changes in weapon technology and theater numerical balances is crucial. If doctrine fails to keep pace with changes in force density or weapon design, then the consequences of such changes for stability could be much more severe than those described above. More generally, if the key to stability is how the forces are used in battle, then it is clear that providing the best possible doctrine to guide that use is of the greatest importance.

Finally, this analysis suggests some implications for the ratification of CFE. First, the net territorial gain estimates for the post-CFE scenarios we have been considering throughout this study have been quite low in absolute terms. This suggests that the potential military payoff to a Soviet attack on NATO (or a NATO attack on the Soviet Union) should be small in a post-CFE environment. This does not mean that there can be no military instabilities in a post-CFE Europe -- multipolarity and political turmoil in Eastern Europe and the Soviet Union (among other possibilities) could give rise to regional coalitions that could provide dangerous military incentives to potential attackers, especially with respect to East-East conflict. The East-West balance, on the other hand, should be highly stable (as long as the force to force ratio between the Soviet Union and NATO does not radically increase as a result of non-negotiated asymmetric reductions in forces), and this suggests that the draft CFE treaty is, at the very least, consistent with fundamental U.S. objectives for the region.³⁹

The specific cuts in treaty limited items mandated by the treaty are a mixture of slightly stabilizing (e.g., tank limits) and slightly destabilizing (e.g., armored troop carrier limits). The benefits resulting from theater numerical parity, however, are quite substantial, and outweigh any moderately stabilizing or destabilizing tendencies in the specific weapon ceilings.

With respect to verification, an implication of the relative insensitivity of combat outcomes to small changes in weapon inventories is that low levels of cheating ought not to pose an enormous military problem. Modest increases in theater weapon inventories or force levels can increase an invader's potential net territorial gain, but it would require large changes in forces for that increase to be militarily or geopolitically significant, or to constitute a serious threat of breakout from the treaty limits.⁴⁰

BOTTOM LINES

**Arms Control Can Have Beneficial Effects, But Unlikely to be Decisive
for Stability**

**Force Employment Adaptation to Changing Weapon Technology,
Numerical Balance is Crucial**

Ratification of CFE:

- **Post CFE Force Posture Should Be Highly Stable**
- **CFE I TLI Cuts a Mixture of Slightly Stabilizing and Slightly
Destabilizing; Parity a Substantial Benefit**
- **Verification Uncertainties Can Be Lived With**

END NOTES

- 1 On the World Disarmament Conference, see Marion William Boggs, Attempts to Define and Limit "Aggressive" Armament in Diplomacy and Strategy (Columbia, MS: University of Missouri, 1941), University of Missouri Studies, Vol. 16, No. 1. For the Liddell Hart/Fuller debate, see Basil H. Liddell Hart, "Aggression and the Problem of Weapons," English Review, July 1932, pp. 71-78; and J.F.C. Fuller, "What is an Aggressive Weapon?" English Review, June 1932, pp. 601-5.
- 2 For a review and critique of the relevant literature, see Stephen D. Biddle, The State of Knowledge on the Determinants of Offensiveness and Defensiveness in Conventional Ground Forces (Alexandria, VA: Institute for Defense Analyses, September 1989), IDA P-2295.
- 3 Given an equal increase on each side. For the purposes of this briefing, we focus on the special case of theater level parity between potential combatants, symmetrical base case weapon mixes, and a geographically undifferentiated theater of war. Given these simplifying assumptions, we can therefore exclude from consideration the classification of weapons which, if given to both sides in equal numbers, increases the payoff to combatant A in an attack on B, but does not increase the payoff to B in an attack on A (or increase that payoff to a lesser degree). Under these assumptions, any weapon that increases the payoff to an attack for one side would likewise increase the payoff were the attack to be made by the other side instead.
- 4 For a survey of published views on the stability properties (or offensiveness and defensiveness) of particular conventional weapon types, see IDA P-2295, op. cit.
- 5 While it could be argued that a state of extreme military stability -- in which Saddam Hussein's original invasion of Kuwait would itself have been militarily impossible -- would serve U.S. interests in the Persian Gulf, it is far from clear that it is realistic to expect a degree of stability sufficient to enable the tiny Kuwaiti army to repel an Iraqi invasion. In the absence of such an extreme, counter-invasion will remain the only feasible option for response to such aggression on the part of remotely located security guarantors, and successful counter-invasion is made more difficult in conditions of relatively high military stability.
- 6 The VFM methodology was introduced in S.D. Biddle, et al., Defense at Low Force Levels (Alexandria, VA: Institute for Defense Analyses, December 1990), IDA P-2380, esp. Appendices C, D, and E. This methodology was substantially extended in the course of the analyses described below; for a detailed description of this extension, see Appendices B and C.
- 7 Note that this definition of the payoff to attack does not take explicit account of possible critical thresholds for ground gain (e.g., the depth of the theater as a whole, or the distance between a national capital and the international border). The territorial gains reported below, however, are typically small relative to either potential threshold values.

- 8 An AFVE is simply a convenient, "generic" measure of force size selected to facilitate testing of the theory underlying the VFM methodology (see S.D. Biddle, et al., IDA P-2380, op. cit., esp. Appendix D). A single main battle tank represents one AFVE (regardless of nationality, make or model). A single armored troop carrier with its infantry complement is also scored as one AFVE. A carrier without its infantry is one-half an AFVE; the infantry without the carrier is half an AFVE. Armored antitank, command, or reconnaissance vehicles also are one-half an AFVE. Non-maneuver forces are accounted for separately (see Appendix B), and thus are excluded from the AFVE totals per se.
- 9 Where "weapon types" include, for example, tanks, infantry, artillery, air defense, attack helicopters, utility helicopters, etc. (for a complete list, see Slide 7). The VFM methodology currently does not distinguish between different models within a weapon type (e.g., M1 vice M60 vice T72).
- 10 Of these variables, force employment is perhaps the most challenging to describe analytically. Any operational doctrine represents a broad and often subtle collection of guidelines for employing a force -- and even if the totality of these official guidelines could be pinned down, individual commanders in the field ultimately determine how those official guidelines become actual practice. (By "operational doctrine" we follow the definition given by the U.S. Military Academy at West Point: "The [officially accepted] body of ideas . . . concerning the use of available military resources to attain strategic ends in a theater of war. As the link between tactics and strategy, it governs the manner in which operations are designed to meet strategic ends and the way in which campaigns are conducted." John I. Alger, *Definition and Doctrine of the Military Art* (Wayne, NJ: Avery Publishing, for the Department of History, United States Military Academy, West Point, New York, 1985, pp. 7, 5). Operational doctrine is thus neither Alliance strategy, e.g., Forward Defense and Flexible Response in NATO, nor small unit tactics, e.g., assault formation or the siting of weapons for maximum engagement range. Where possible, however, we will use the more specific term "force employment" as is defined in greater detail below.)
- Yet force employment is clearly a central issue for conventional combat dynamics. To capture the effects of changing weapon inventories on an attacker's potential net payoff to attack, it is thus important that the interactions between force employment and changing weapon holdings be represented explicitly.
- One way to do this would be to simulate (by direct imitation) the detailed movements of the two sides' forces over time as an operation unfolds. Sand-table games, field maneuvers, and map exercises, for example, all trace the stops, starts, turns and dispositions of individual units over three-dimensional terrain in ways that enable the form of any given maneuver to be recognized directly. Such techniques enable analysts to trace out the consequences of any particular sequence of movements and countermovements, and offer the flexibility to represent a wide range of different sequences. But while such techniques can show how any given sequence might play out, it is effectively impossible to examine all potential combinations of all potential turns, starts, and stops; alternatively, attempts to develop "rules" by which to identify the one right or best sequence of individual movements have been unsuccessful to date. As a result, although there are many uses for such techniques, they are ill-suited for making general observations about the effects of weapon mix on combat outcomes.
- An alternative approach would be to step back from the detailed movements of individual units and develop instead a more abstract description of the relationship between broader classes of alternative operational concepts and their effects on combat outcomes. Rather than literally walking individual units around a hypothetical battlefield, we would instead isolate a discrete set of key dimensions along which to distinguish the practical force employment alternatives and then describe how differences with respect to those key dimensions of force employment affect combat outcomes -- and how their effects change as weapon holdings change.
- But how are we to identify such a set of key dimensions, or key aspects, of a force employment concept? Fortunately, the military theoretical literature eases our task somewhat by emphasizing the importance of a small number of essential operational issues for the effectiveness of defenses at the theater level. (For surveys of relevant literature, see IDA P-2380, op. cit., Appendices A and B; also IDA P-2295, op. cit.) While this is no guarantee that these aspects of doctrine are sufficient for our purposes, the experience embodied in the literature at least provides us with a sound point of departure.

These key aspects of force employment are five: depth, reserves, counterattack, "tempo," and concentration. Respectively, these are defined for our purposes as the distance from the initial line of contact to the defender's rear defense line (in kilometers); the fraction of the defender's total forces withheld from contact for use as mobile reserves; the fraction of those reserves used for counterattack (as opposed to passive reinforcement); the attacker's assault velocity (in kilometers per hour); and the frontage of attack (in kilometers). (Defensive "depth" is further separated into *predeployed* depth -- the number of prepared defensive positions initially manned by forward forces -- and *rolling* depth -- a function of the fraction of the defender's forces in any given forward position that are withdrawn for use in secondary positions behind the predeployed lines -- as is discussed in greater detail below. These combine according to a functional relationship described in Appendix C to determine the ultimate depth of the defense as a whole. For a more rigorous definition of these variables, see Appendix C.)

In these terms, the U.S. Army's 1970s doctrine of "Active Defense" can be characterized as one with a small fraction of total forces held in reserve, a small fraction of reserves used for counterattack, and a limited deployment depth for committed forces. The Army's current AirLand Battle doctrine, by contrast, is one with a higher fraction of total force in reserve, a higher fraction of those reserves used for counterattack, and a deeper forward deployment. A "blitzkrieg" offensive doctrine is distinguished by high velocity in a narrow front; a more cautious offensive would be conducted at lower attempted velocity on a broader front. On the distinction between Active Defense and AirLand Battle, see for example John L. Romjue, From Active Defense to AirLand Battle: The Development of Army Doctrine, 1973-1982 (Ft. Monroe, VA: Historical Office, U.S. Army Training and Doctrine Command, 1984), esp. pp. 3-22, 51-74; and Huba Wass de Czege and L.D. Holder, "The New FM 100-5," Military Review, July 1982, pp. 53-70. the term "blitzkrieg," while commonly used, is rarely defined in such a manner as to make possible systematic comparisons with plausible alternatives. For a definition that goes beyond "sinning quickly," see Mearsheimer, Conventional Deterrence, op cit., pp. 35-43; for historical examples, see e.g., Charles Messenger, The Art of Blitzkrieg (London: Ian Allen, Ltd., 1976).

11 op. cit. in note 4.

12 That is, VFM identifies a set of force employment parameters that solves a two-person, zero sum game where the two players are the putative theater attacker ("red") and theater defender ("blue"), the player strategies are unique sets of force employment choices, and the payoffs are net territorial gains for red in the event of attack, given specific combinations of force employment choices by the two sides. By constraining the range of potential strategies for one or both sides, VFM also can explore the consequences of suboptimal force employment choices for net territorial gain, but an important characteristic of the methodology is its ability to treat force employment as an endogenous independent variable.

13 For more specific values, see David G. Gray, IDA Unclassified Conventional Forces Database (Alexandria, VA: Institute for Defense Analyses, 1989), IDA D-708.

14 op. cit. in note 4.

15 It should be emphasized that these incentives, and the resulting optimal force employment choices, are computed automatically by the VFM model as force density changes. For a more detailed discussion of the relationship between force employment optima and theater force density, see IDA P-2380, op. cit., esp. figs. I-3 to I-5.

16 HIMADS representing systems such as Patriot; SHORADS representing systems such as Chaparral, Vulcan, or Singer.

17 Where LRACM represents systems such as the ATACMS/TGSM (Army Tactical Missile System with Terminally Guided Submunition warheads), and SRACM represents systems such as the MLRS/TGW (Multiple Launch Rocket System with Terminally Guided Warheads).

18 At the post-CFE I force level of 35,000 AFVEs in the theater. Note that this increase is scaled to the size of the theater ground forces as a whole so as to hold constant the relative balance of armor and other weapon types across force densities. Thus, for a force density of 5,000 AFVEs in the

theater, the net increase in each sides' tank fleet corresponds to about 1,100 vehicles on each side. Without such scaling, the weapon mix would become almost tank pure at low force densities as a result of adding a constant number of tanks to a smaller and smaller base of other weapon types.

19

For more detailed argument, see IDA P-2380, op. cit. in note 4, esp. Appendix C.

20

We assume here for purposes of illustration that each LRACM round is capable of destroying one opposing AFVE, although this quantity is subject to substantial uncertainty: see Slide 12.

21

Note that for purposes of graphical clarity, the force density axis is truncated here, with its origin at a density of 5,000 AFVEs in the theater (rather than zero as in Slides 6 and 9); thus the minimum net territorial gain depicted in this slide corresponds to a force density of 5,000 -- rather than zero -- AFVEs, and is consequently greater than the minimum value (zero) in Slides 6 and 9.

22

For more detailed discussions of the problem of uncertainty in measure-countermeasure races, see Stephen Biddle and Victor A. Utgoff, The Battlefield of the Future (Alexandria, VA: Institute for Defense Analyses, January 1989), IDA P-2194; also Stephen Biddle, "Can Conventional Forces Substitute for Battlefield Nuclear Weapons?" in Stephen Biddle and Peter D. Feaver, eds., Battlefield Nuclear Weapons: Issues and Options (Lanham, MD: University Press of America, 1989), pp.67-96.

23

The size of this package is scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, about 17,000 rockets are added to each side.

24

Of course, since optimal force employment changes when circumstances change, the addition of SRACM to either sides' force posture will change both sides' force employment incentives, and thus the optimal choices will differ from those of the base case. In general, however (and specifically with respect to SRACM), blue's optimal allocation of force to counterattack will still increase significantly as force density falls (and thus red's optimal allocation to flank defense will rise). For any given force density, blue will allocate less force to counterattack if counterattack is made less attractive by, for example, SRACM, but counterattack allocations will still generally be higher at low density than at high. Note, however, that blue's ability to alter his force employment will tend to reduce the ultimate impact of SRACM on net territorial gain relative to an alternative condition in which blue's force employment is held constant. The results we see in the slide are thus the net results given both sides' adaptation to the new weapon package.

25

This value was chosen to provide as clear as possible an exposition of the effects of variation in ACM's relative effectiveness against moving and stationary targets -- an uncertainty capable of reversing the stability properties of SRACM (whereas uncertainty in moving target effectiveness alone primarily determines the magnitude, but not the direction, of SRACM's contribution to net territorial gain). As an optimistic estimate, however, it does not necessarily represent the most *likely* value for SRACM's moving target effectiveness. On the general question of effectiveness uncertainty in advanced conventional munitions, see IDA P-2194, op. cit.; also Biddle, "Can Conventional Forces Substitute for Battlefield Nuclear Weapons?" op cit., pp.67-96.

26

For a detailed survey of relevant cost data, see David G. Gray, Selected Official Cost Estimates for Near to Mid Term Weapon Systems (Alexandria, VA: Institute for Defense Analyses, 1990), unpublished manuscript.

27

Weapon types are abbreviated as follows: CAS, tactical air support; INT, tactical air interdiction; SRA, short range advanced conventional munitions; LRA, long range advanced conventional munitions; AH, attack helicopters; TKS, tanks; INF, infantry; ART, artillery; FSM, artillery-delivered switchable mines; AD, air defense; UH, utility helicopters carrying infantry anti-tank teams; and ENG, combat engineers.

- 28 See IDA P-2295, op. cit., note 2, esp. pp.37-8 for a summary of existing literature on the "offensiveness" or "defensiveness" of particular weapon types.
- 29 One hypothesis under study, but not yet proven, holds that stability is conditional for all weapon types if the range of circumstances is not somehow constrained.
- 30 For example, force densities between CFE I levels and perhaps 50 percent of CFE I; weapon investments of \$200 billion or less; and generally mid-range effectiveness assumptions for tacair and ACM.
- 31 For the post-CFE I force level of 35,000 AFVEs in the theater. Note that these investments were scaled to the size of the theater force as a whole. Thus, for a force density of 5,000 AFVEs in the theater, the assumed net additional investment was only \$28.6 billion (without such scaling, the two force structures would have been nothing but C17s, UHs and combat engineers for the lower force densities depicted on the slide).
- 32 Of course, for those weapon types which are *purely* offensive or *purely* defensive at the tactical level, this difference will be much larger. Buried mines, for example, are purely defensive at the tactical level (although not at the theater level, where minefields away from the point of attack enable invaders to divert more forces to offensive use). Few such "pure" weapon types exist, however. Infantry, for example, can be used in the tactical assault (and indeed, is ordinarily a major component of any modern combined arms offensive. Even used alone, infantry can succeed in the assault under the right circumstances, as the Germans demonstrated in March and April of 1918, and as the British demonstrated in the Falklands). It is thus unavoidable that the great bulk of any army's forces will be at least somewhat capable of both offensive and defensive tactical employment.
- 33 "Structural incapacity to attack" as has been advocated by proponents of Nonoffensive Defense is thus unfeasible in the literal sense; "relative unsuitability for attack" is the most that can be secured for any achievable force posture. More generally, while it is possible to improve stability on the margin by restructuring conventional forces, the analyses conducted here suggest that the degree of change one can expect from such restructuring will be relatively modest. By the same token, however, the analyses conducted here also suggest that any negative military consequences of restructuring forces along such lines also are unlikely to be large, given the relative insensitivity of combat outcomes to variations in weapon mix. It has been argued by some that such a restructuring could serve useful non-military ends independent of its effect on defensive capability; to the extent that this were so, the analyses conducted here suggest that the military cost of such a program need not necessarily be prohibitive. For critical reviews of the Nonoffensive Defense debate, see for example Jonathan Dean, "Alternative Defense: Answer to NATO's Central Front Problems?" *International Affairs*, Winter 1987/88 (Vol.64, No.1), pp.61-82; David Gates, "Area Defense Concepts: The West German Debate" *Survival*, July/August 1987 (Vol.29, No.2), pp.301-17; Stephen J. Flanagan, "Nonprovocative and Civilian-Based Defenses" in Joseph S. Nye, Graham T. Allison, and Albert Carnesale, eds., *Fateful Visions: Avoiding Nuclear Catastrophe* (Cambridge, MA: Ballinger, 1988), pp.93-109; Hans W. Hoffmann, Reiner K. Huber, and Karl Steiger, "On Reactive Defense Options" in Reiner K. Huber, ed., *Modelling and Analysis of Conventional Defense in Europe* (New York and London: Plenum, 1986), pp.97-140; and IDA P-2295, op. cit. in note 2, pp.27-29.
- 34 For a more detailed discussion of the effects of doctrinal adaptation on net territorial gain, see IDA P-2380, op. cit. in note 4, esp. Appendix C.
- 35 For examples, see IDA P-2380, op. cit. in note 4, esp. Table 1.
- 36 The size of this package is scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, about \$30 billion of LRACM missiles were added to the theater defender's forces.

- 36 Again, the size of this package is scaled down proportionally to the size of the supported ground forces as density falls. The forces present on both sides and all weapon performance values are assumed to be the same as those in the base case although NATO is attacking and the Warsaw Pact is defending. All other factors such as preparation times and the terrain in which the offensives take place are assumed to be the same for both attacks
- 37 "Stability" is a particular property that may or may not be in any given state's national interest at any given time -- it is not simply another word for "goodness" with respect to arms control or force planning. As a consequence, it will not necessarily be adopted as a strategic objective by any given state. Where opposing states (or alliances) each opt to maximize their own security unilaterally, however, the result can be what Robert Jervis has termed the *Security Dilemma*, wherein each side's efforts to increase its own security reduces that of its neighbors. For a more detailed discussion of this issue, and its consequences for the arms race and crisis stability, see Robert Jervis, "Cooperation Under the Security Dilemma," *World Politics*, January 1978 (Vol. 30, No. 2), pp. 167-214.
- 38 For a more detailed treatment of the problem of force density, see IDA P-2380, op. cit. in note 4.
- 39 Assuming, of course, the successful resolution of outstanding differences between the Soviet Union and N," with respect to treaty implementation.
- 40 For a detailed treatment of the sensitivity of net territorial gain to variations in force to force ratios (and thus, to one-sided increases -- or decreases - in force levels), see IDA P-2380, op. cit., esp. Figure I-6 and associated text.

Appendix A

WEAPON SYSTEM STABILITY SENSITIVITIES

Mr. Stephen D. Biddle

This Appendix provides a more detailed treatment of the stability properties described in the main briefing. In particular, it illustrates each of the sensitivities displayed by each of the weapon types identified in the taxonomy given here (and in the main briefing above). Where weapon types are subject to more than one of these sensitivities, they will be addressed here more than once; where particular weapon types and stability effects have been treated in the main briefing, they will not be addressed here.

TAXONOMY OF STABILITY EFFECTS

Sensitivity to Force Density

Tanks
Infantry
CAS
ACM
AH
Artillery
FASCAM

Sensitivity to Inventory Size

ACM
CAS
FASCAM
AH

Sensitivity to Weapon Effectiveness Uncertainties

ACM
CAS
AD

Relatively Insensitive

UH/INF
C17
Combat Engineers
Air Interdiction

As for the first of these: infantry, like armor (as treated in the main briefing, see Slide 9), is a weapon type whose stability is sensitive to the density of forces in the theater of war. Slide A-2 illustrates this property by contrasting the base case with an excursion in which each side is given a package of additional (mechanized) infantry equivalent to about 10,000 armored troop carriers with their associated rifle squads at a force density of 35,000 AFVEs in the theater.¹

At force densities above about 11,000 AFVEs in the theater, the addition of some 10,000 AFVEs of infantry to both sides' forces decreases the potential payoff for either side to attack the other, and infantry is thus stabilizing. At force densities below 11,000 AFVEs in the theater, however, increasing both sides' infantry strength increases the potential payoff to attack relative to the less infantry-heavy base case, and infantry is destabilizing. Thus, the stability of infantry is sensitive to force density.

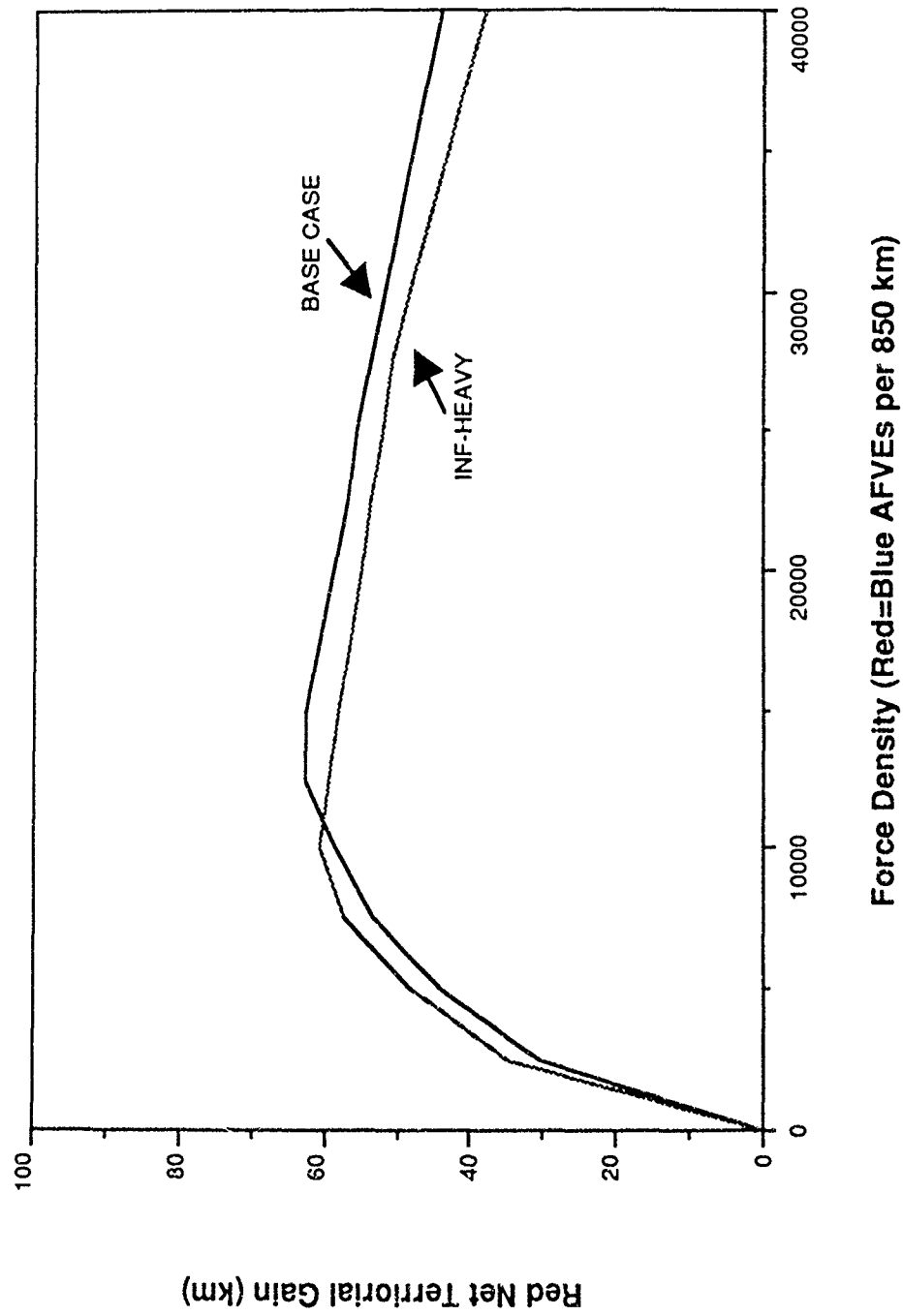
The reason for this outcome is similar to that of the armor case discussed in the main briefing and concerns changes in the two sides' optimal force employment. Infantry, unlike armor, is best suited to the tactical defensive. At high force density, blue's optimal choice is to commit a large fraction of his total forces to tactical defense in predeployed forward positions. Red thus faces little threat of counterattack, and can devote a large fraction of his total forces to tactical assault at the point of main effort. At high force density, giving both sides more forces specialized for tactical defense helps blue more than red, and net territorial gain falls.

At low force density, however, blue's optimal choice is to hold a larger reserve, and to use a large fraction of that reserve for counterattack. Red's optimal choice in response to this is to devote a larger fraction of his total forces to tactically defensive use on the flanks of his penetration. Under these circumstances, the binding constraint on red's ability to take and hold blue territory becomes the "overhead cost" of defending a lengthening flank against this blue counterattack threat. Here, giving both sides more forces specialized for tactical defense thus helps red more than blue, and net territorial gain rises.

Again, however, the magnitude of the effect is quite small at all force densities. At a density of 30,000 AFVEs in the theater, the infantry-heavy excursion decreases net territorial gain by only about 6 kilometers, or roughly 10 percent of the base case value; at a density of 5,000 AFVEs in the theater, the excursion increases net territorial gain by under 10 kilometers, or about 20 percent of the base case value.

SENSITIVITY TO FORCE DENSITY

BASE vs INF—HEAVY POSTURE



Slide A-2.

A-5

Attack helicopters (AHs) are also a weapon type whose stability properties are sensitive to the density of forces in the theater of war. Slide A-3 illustrates this effect by contrasting the base case with an excursion in which each side is given a package of additional AHs equivalent to about 2,000 attack and 1,500 associated observation helicopters at a force density of 35,000 AFVEs in the theater.²

At force densities above about 10,000 AFVEs in the theater, the AH-heavy case decreases the potential payoff for either side to attack the other, relative to the base case outcome, and AHs are thus stabilizing. At force densities below 10,000 AFVEs in the theater, however, increasing both sides' AH inventory increases the potential payoff to attack relative to the less AH-heavy base case, and AHs are destabilizing. Thus, the stability of attack helicopters is sensitive to force density.

Again, the reason for this sensitivity to force density has to do with force employment and the changing relative incentives for tactical assault and defense on the two sides as force density changes. AHs, like most weapon types, have both tactically offensive and tactically defensive missions. Of the primary offensive roles for AHs, however, one, close support, is both extremely costly and only marginally effective.³ The other, pursuit, can be highly effective, but does not necessarily require AHs for its execution.⁴ AHs, while useful, are thus not ideally suited for tactically offensive employment.

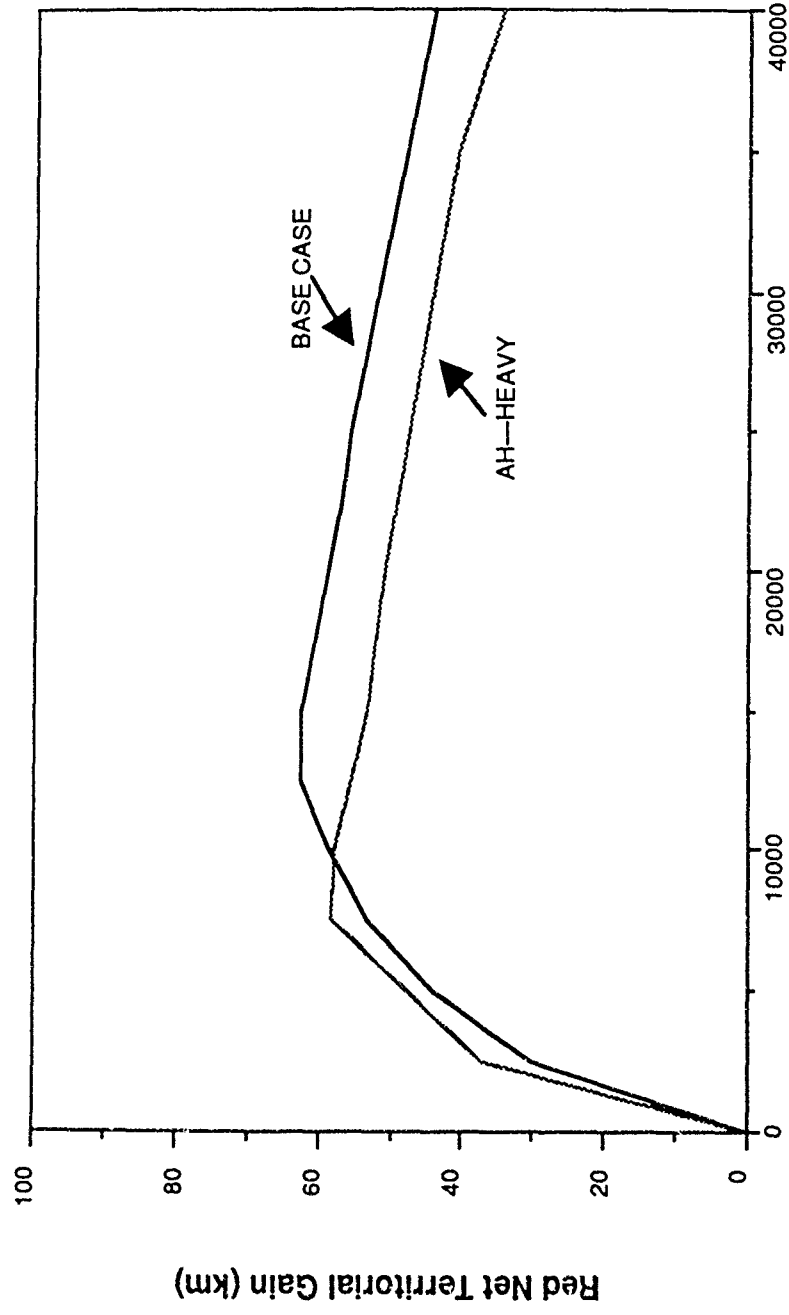
On the tactical defensive, on the other hand, AHs can use a combination of long range ATGMs and terrain masking to provide a major increase in the antiarmor firepower of a defended position, given appropriate conditions.⁵ Moreover, the high speed of rotary wing aircraft gives AHs the ability to counterconcentrate very rapidly over potentially theaterwide distances. These two capabilities -- high antiarmor firepower in support of defended positions and rapid counterconcentration -- make AHs potentially very effective weapons in the tactical defense. Combined with their limited effectiveness in tactically offensive roles, the AH is, thus, on balance, more useful on the tactical defense than the tactical offense.

Consequently, at high force density, where blue's optimal force employment tends to emphasize the tactical defense and red's tends to emphasize the tactical offense, giving both sides more of a weapon best suited to the tactical defense aids blue more than red, and net territorial gain falls. At low force density, where blue's optimal force employment tends to emphasize counterattack and where red is heavily dependent on the success of its flank defense against that counterattack, greater availability of weapons specialized for tactical defense aids red more than blue, and net territorial gain rises.

Again, however, the magnitude of the effect is modest at all force densities. At a density of 30,000 AFVEs in the theater, for example, net territorial gain falls by less than 10 kilometers, or about 15 percent of the base case value; while at a density of 5,000 AFVEs in the theater, the difference is under 5 kilometers, or about 8 percent of the base case.

SENSITIVITY TO FORCE DENSITY

BASE vs AH—HEAVY POSTURE



Force Density (Red=Blue AFVEs per 850 km)

Slide A-3.

A-7

Close air support (CAS) aircraft are another weapon type whose stability is sensitive to the density of forces in the theater of war. Slide A-4 illustrates this property by contrasting the base case with an excursion in which each side is given a package of additional CAS equivalent to about 2,700 aircraft at a force density of 35,000 AFVEs in the theater.⁶

At force densities above about 25,000 AFVEs in the theater, the CAS-heavy excursion decreases the potential payoff for either side to attack the other, and CAS is thus stabilizing. At force densities below 25,000 AFVEs in the theater, however, increasing both sides' CAS inventory increases the potential payoff to attack relative to the less CAS-heavy base case, and CAS is destabilizing. Thus, the stability of CAS aircraft is sensitive to force density.

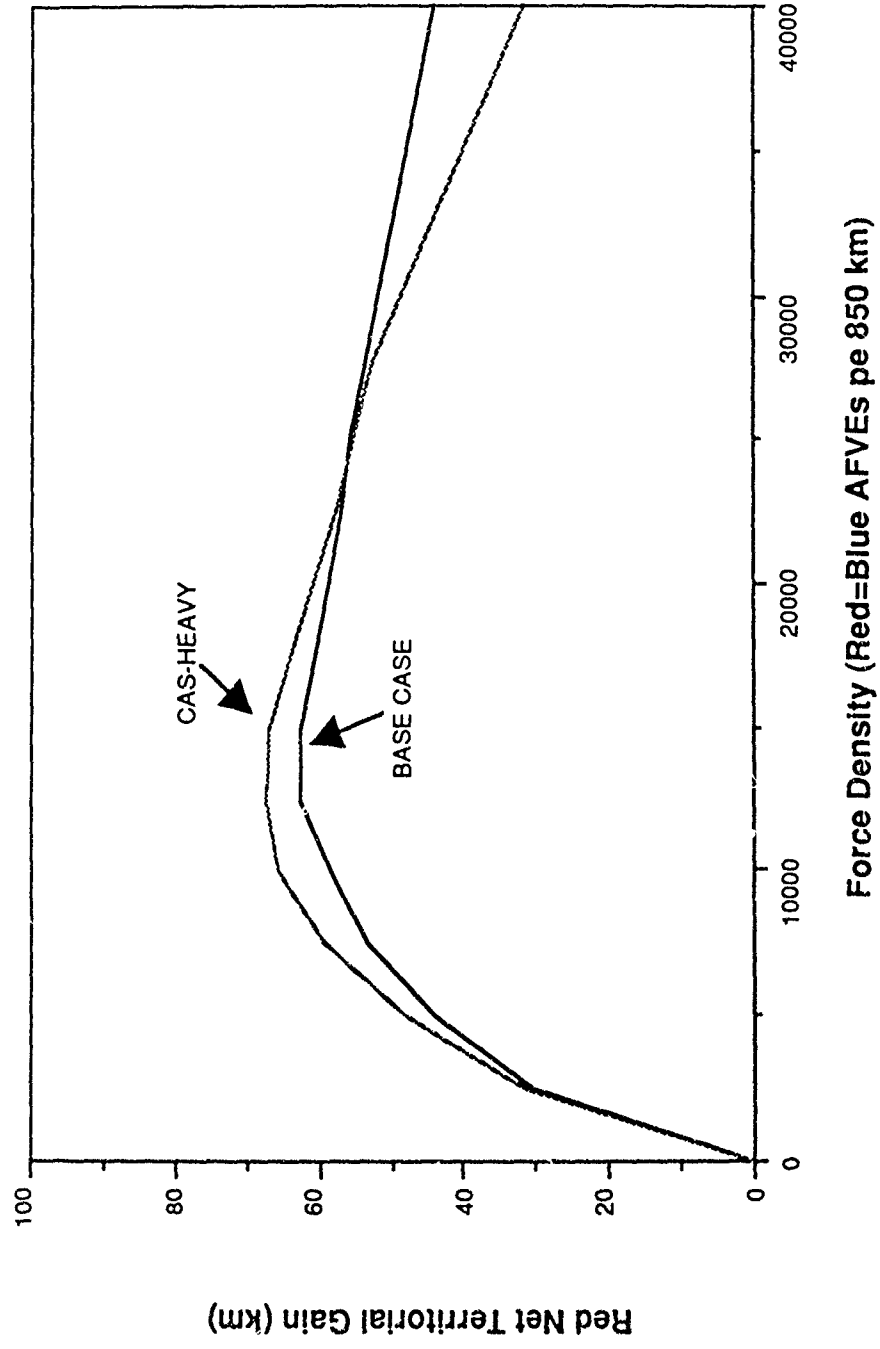
As with tanks, infantry, and AHs, the reason for this sensitivity to force density again has to do with force employment and the changing relative incentives for tactical assault and defense on the two sides as force density changes. On a sortie-for-sortie basis, CAS is typically less effective in support of tactically offensive ground forces. This is because CAS is most lethal when it can deliver precision ordnance (e.g., PGMs such as Maverick, or gunfire such as the GAU-8 30mm cannon) against specific, acquired targets. At high speed and low altitude, however, it can be difficult for pilots to acquire concealed, stationary ground forces for engagement by such weapons -- and it is exactly this sort of target that confronts CAS sorties flown in support of an assault against a prepared defense-in-place. CAS sorties flown in support of the tactical defense, on the other hand, have a much higher likelihood of finding their targets moving and exposed in the open -- the conditions that most favor the employment of precision ordnance by high performance aircraft.

Because CAS is most effective when supporting the tactical defense, it tends to contribute relatively more to blue than red at high force densities, when it is the success of blue's tactical defense at the point of attack that most determines red's net territorial gain. At high force density, CAS is thus stabilizing. At low force densities, where the principle driver of red's territorial gain is the success of blue's tactical offensives against red's flank defenses, CAS thus tends to contribute relatively more to red than blue, and CAS is destabilizing.

Again, however, the magnitude of the effect is relatively modest. Even at a high force density of 35,000 AFVEs, where its effect is relatively large in percentage terms, the net effect of adding 2,700 aircraft to both sides is still only about a 25 percent reduction in the base case territorial gain. At its most destabilizing point, the indicated CAS package adds no more than about 10 percent to the base case territorial gain.

SENSITIVITY TO FORCE DENSITY

BASE vs CAS—HEAVY POSTURE



Slide A-4.

A-9

Artillery delivered mines (or FASCAM, for the Army's Family of Scatterable Mines program)⁷ are also weapons whose stability is sensitive to the density of forces in the theater of war. Slide A-5 illustrates this property by contrasting the base case with an excursion in which each side is given a package of additional artillery delivered mines equivalent to about 1,200 additional FASCAM rounds per artillery tube.

At force densities above about 18,000 AFVEs in the theater, increasing each side's inventory of scatterable mines decreases the potential payoff for either side to attack the other, and FASCAM is thus stabilizing. At force densities below 18,000 AFVEs in the theater, however, increasing both sides' scatterable mine inventory increases the potential payoff to attack relative to the base case, and FASCAM is destabilizing. Thus, the stability of scatterable mines is sensitive to force density.

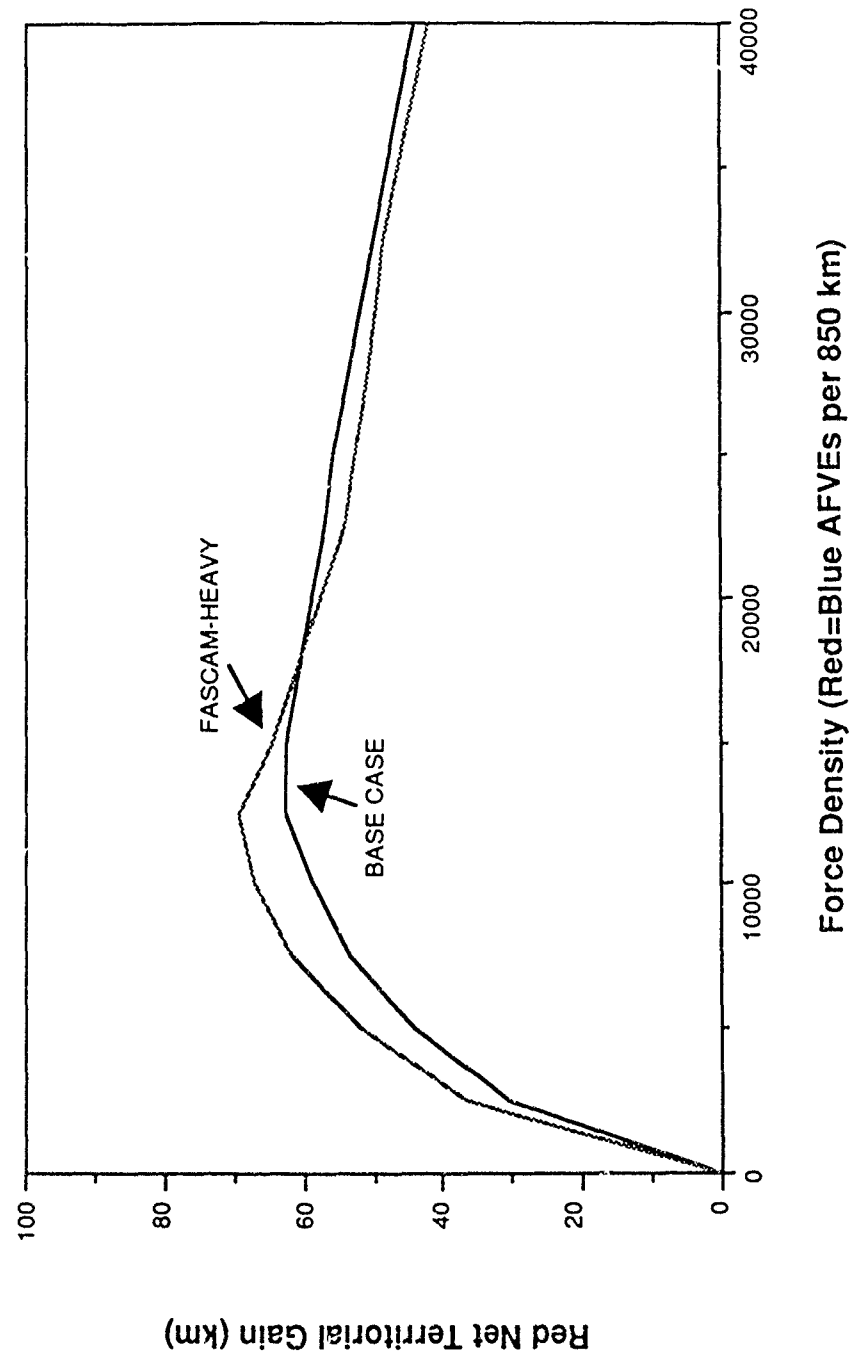
This sensitivity is another consequence of the changing incentives for force employment determined by the model as force density changes -- and thus the changing effects of each sides' use of the mines available to it. Scatterable mines can be used in several ways. Tactical defenders, for example, can use them as rapidly deployable barriers to strengthen whatever mine and obstacle system they may have deployed around their positions. These barriers, however, can be used both by blue's tactical defenders at the international border, and by red's tactical defenders on the flanks of red's offensive spearhead. The two uses are not necessarily of equal value, however. Blue, whose defenders own the ground on which they fight, has ample time and resources for barrier construction in advance of the red offensive. While helpful, FASCAM is thus only one of many available means for the construction of blue barriers at the international border. Red must also defend its side of the international border in sectors away from his chosen attack frontage, and here FASCAM, while helpful, is likewise only of incremental value. On the flanks of the red penetration, however, red must defend ground only recently obtained as a result of the advance of the red spearhead. Here, preparation time and labor availability are much less than for either red or blue's defenders at the international border. FASCAM thus provides red's flank defenders a capability that would be difficult to produce by other means, and is therefore of special value.

Tactical attackers, on the other hand, can use FASCAM to pin defenders in place and prevent their withdrawal. By firing FASCAM into and immediately behind defended positions just as the assault nears completion, attackers can make it very costly for defenders to evacuate positions rapidly, yet the mines can be switched off as the attackers themselves sweep over the position. In this way, the attacker can limit the effective depth of the blue defense by reducing the defender's ability to add "rolling depth" to the defense through withdrawal under pressure.⁸

FASCAM is thus of some value to both tactical attackers and tactical defenders, but it is not equally valuable under all circumstances. In particular, its value to defenders is greatest in flank defense, and its value to attackers is greatest where the defender is most dependent on withdrawal. The importance of flank defense and withdrawal to theater outcomes, however,

SENSITIVITY TO FORCE DENSITY

BASE vs FASCAM—HEAVY POSTURE



Slide A-5.

A-11

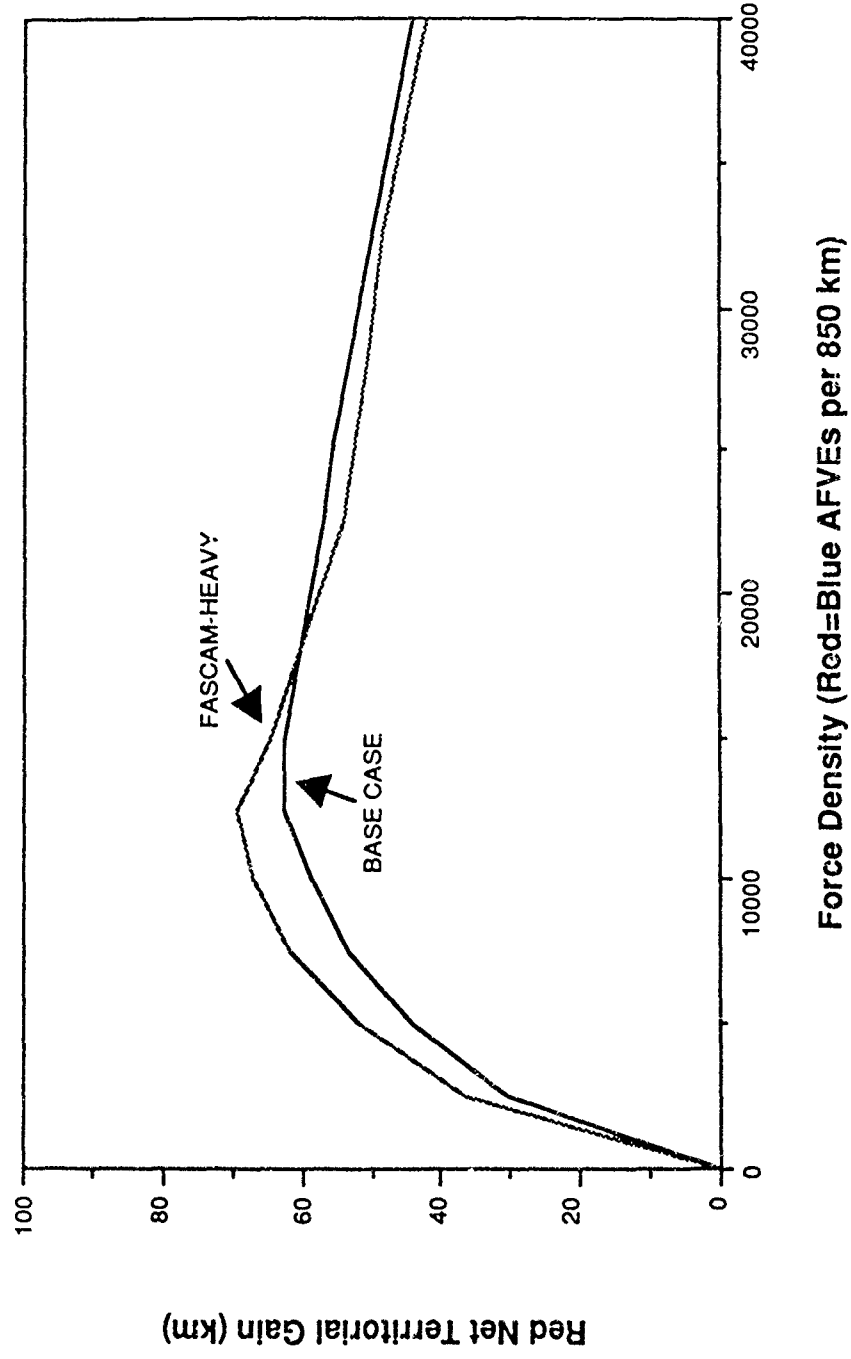
varies with force density. In particular, at low force densities blue depends heavily on counterattack and a withdrawal-oriented elastic delay-in-depth at red's point of attack. Under these circumstances, giving both sides more FASCAM provides red with an ideal means of improving flank defense and denying blue the ability to add depth by withdrawal, while providing blue only the ability to augment the performance of existing barrier creation assets.⁹ The net result helps red more than blue, and FASCAM is destabilizing.

At high force densities, on the other hand, blue's optimal force employment choices emphasize defense in place at the red point of attack, with less emphasis on counterattack, and less reliance on withdrawal. Here, giving both sides more FASCAM again stiffens somewhat the blue defensive line at the international border, but now offers no special advantage to red beyond the benefits of a somewhat more effective defense of its own side of the border away from the point of attack. While not without value to red, this contribution is outweighed by the increase in the strength of blue's barrier system. The net result thus helps blue more than red, and FASCAM becomes stabilizing.

Again, however, the magnitude of the effect is modest. At a relatively low force density of about 10,000 AFVEs, where the effect is greatest in absolute terms, red's net territorial gain is increased by less than 20 percent relative to the base case value. At a higher force density of 40,000 AFVEs, the difference in territorial gain is only about five percent of the base case value.

SENSITIVITY TO FORCE DENSITY

BASE vs FASCAM—HEAVY POSTURE



Slide A-5.

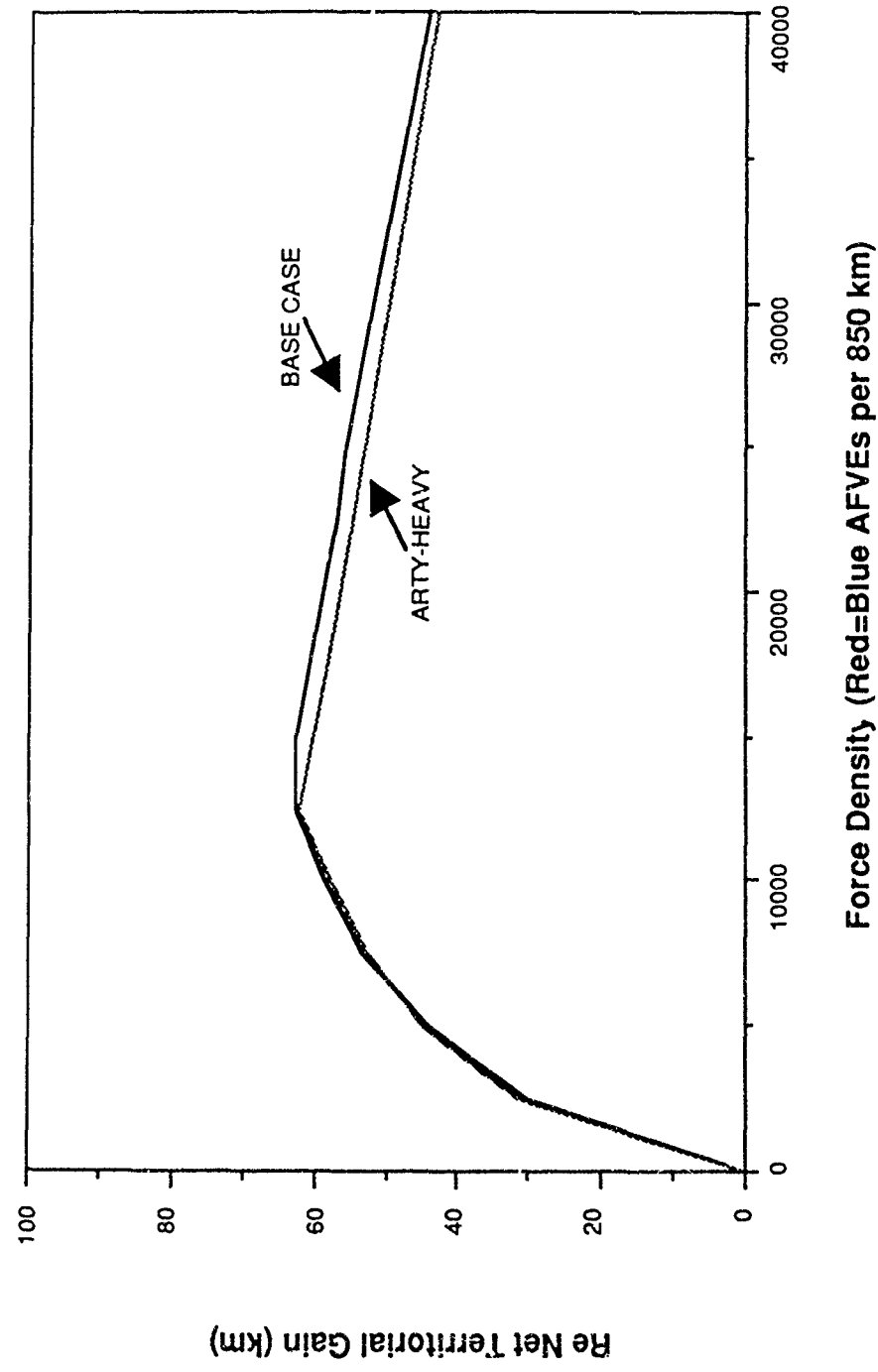
A-13

The stability properties of artillery demonstrate a very modest sensitivity to force density in the analyses conducted for this study. Inasmuch as artillery effectiveness differs systematically in the tactical offense and the tactical defense, this result will tend to obtain (as a result of the changing incentives for tactically offensive and defensive employment of available forces as force density changes, as discussed above). In particular, in the analyses summarized here, a package of additional artillery equivalent to about 4,200 tubes on each side at a force density of 35,000 AFVEs in the theater tended to favor the tactical defense, with the result that for force densities above about 10,000 AFVEs in the theater, artillery proved stabilizing, with the opposite outcome for lower densities.

These results, however, should be interpreted with some caution. The statistical fits obtained for the artillery parameters in VFM were weaker than those for most other weapon types and, in particular, resulted in the use of a functional form for the representation of casualties that demonstrates diminishing marginal returns to increased artillery inventories for red, but constant marginal returns for blue.¹⁰ While this form is the best representation of the data derived from the JANUS runs conducted, and while this outcome is not strongly at odds with available theory on artillery effectiveness, neither is it supported by any persuasive *a priori* theoretical argumentation. As such, there is some risk that this form is a statistical artifact rather than a true representation of the underlying properties of artillery as a weapon system. Pending further JANUS experimentation, this form is better than available alternatives,¹¹ but caution should be exercised in interpreting results dependent on the particular nature of this functional form. In particular, the results described in Slide A-6 are sensitive to this choice of functional form, since it will tend to push loss exchange ratios in tactical engagements in favor of defenders for increases in artillery inventories above those of the base case. While these results are thus more reliable than alternative conclusions given the current state of experimental work, they must be regarded as tentative in nature for the time being.

STABILITY PROPERTIES OF ARTILLERY

BASE vs ARTY—HEAVY POSTURE



Slide A-6.

A-15

The second contextual variable, or class of sensitivity, that must be considered in assessing the stability properties of individual weapon types is the size of the two sides' inventories of that weapon. Short range advanced conventional munitions (SRACM), for example, like the LRAM treated in the main briefing, can switch from stabilizing to destabilizing as a function of the size of the inventories considered. To illustrate this effect, Slide A-7 contrasts the base case with two excursions representing increasing inventories of SRACM: one in which each side is given a package corresponding to about 70,000 SRACM rounds at a force density of 35,000 AFVEs, and one in which each side is given a package of about 300,000 rounds at 35,000 AFVEs.¹² A constant per-round effectiveness of 0.4 AFVEs per rocket fired is assumed throughout; the ratio of stationary to moving target effectiveness is held constant at 0.5.

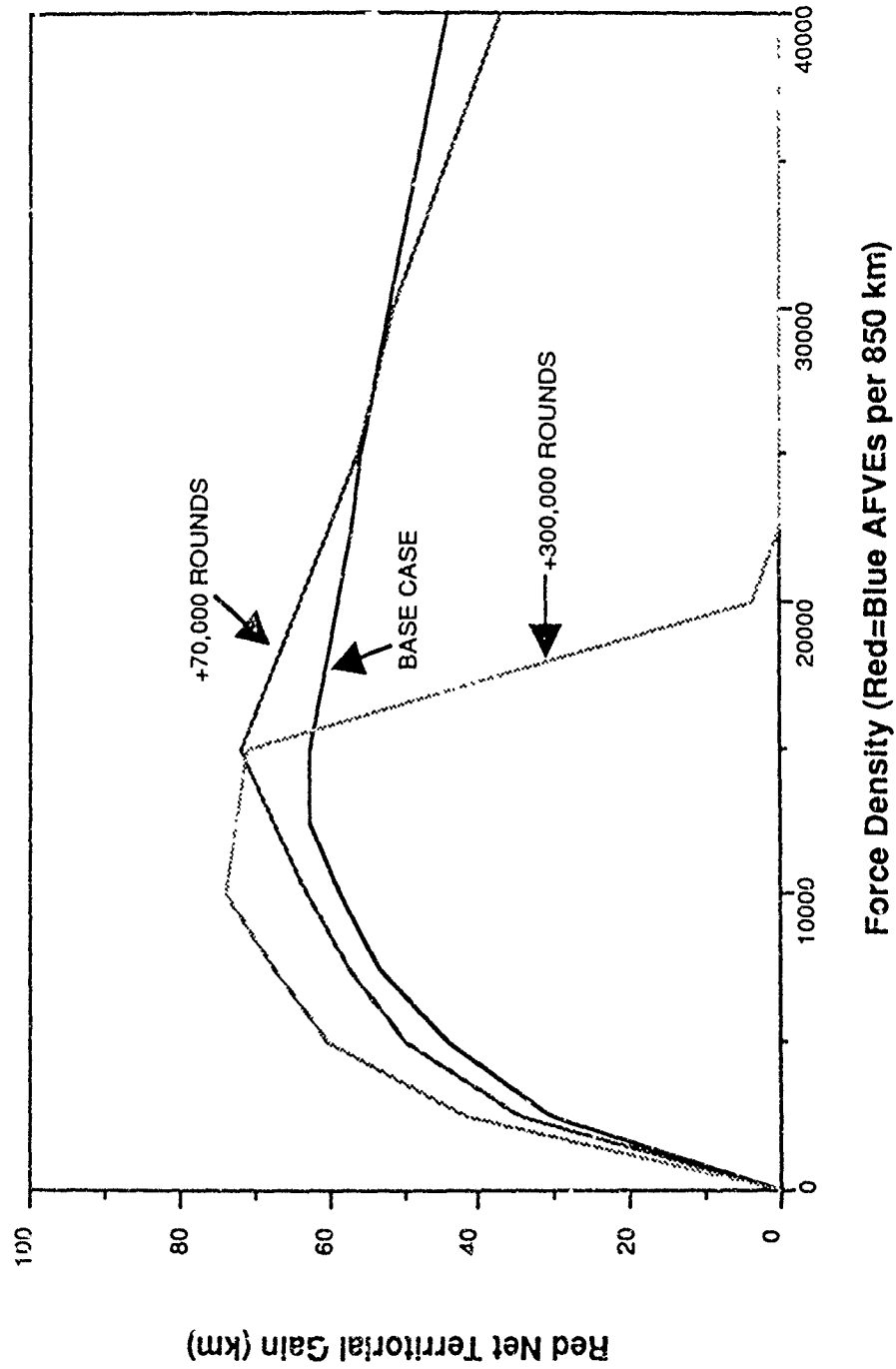
The results suggest that for force densities between about 15,000 and 28,000 AFVEs in the theater, adding the smaller of the two packages is modestly destabilizing. Adding a larger inventory of SRACM, however, eventually becomes stabilizing, as shown by the larger of the two packages illustrated here.¹³

In effect, if enough (effective) SRACM firepower is available to the defender, a tactical assault eventually can be halted as a result of SRACM fire alone. If this is so, then the attacker's ability to destroy defending AFVEs with SRACM of his own is of diminishing consequence -- in the limit, an outcome in which both sides are annihilated by SRACM fire leaves red without forces to take and hold territory which otherwise belongs to blue, and thus net territorial gain as defined here disappears. The particular rate at which actual outcomes approach this theoretical limiting condition varies as a function of other circumstances (e.g., force density as depicted here), but if inventories are high enough, the ability of such firepower to deny the seizure of ground eventually outweighs other considerations and the net effect of adding such firepower becomes stabilizing regardless of other circumstances.

Note, however, that the inventories required for such effects may be quite large (depending, of course, on assumptions with respect to per-round effectiveness). Moreover, it should also be noted that for most inventories and most force densities depicted here, the magnitude of SRACMs net effect on the payoff to attack is modest. For the smaller of the packages considered here, for example, changes in net territorial gain amount to less than 20 percent of the base case value for all force densities.

SENSITIVITY TO INVENTORY SIZE

INCREASING QUANTITIES OF SRACM



Slide A-7.

A-17

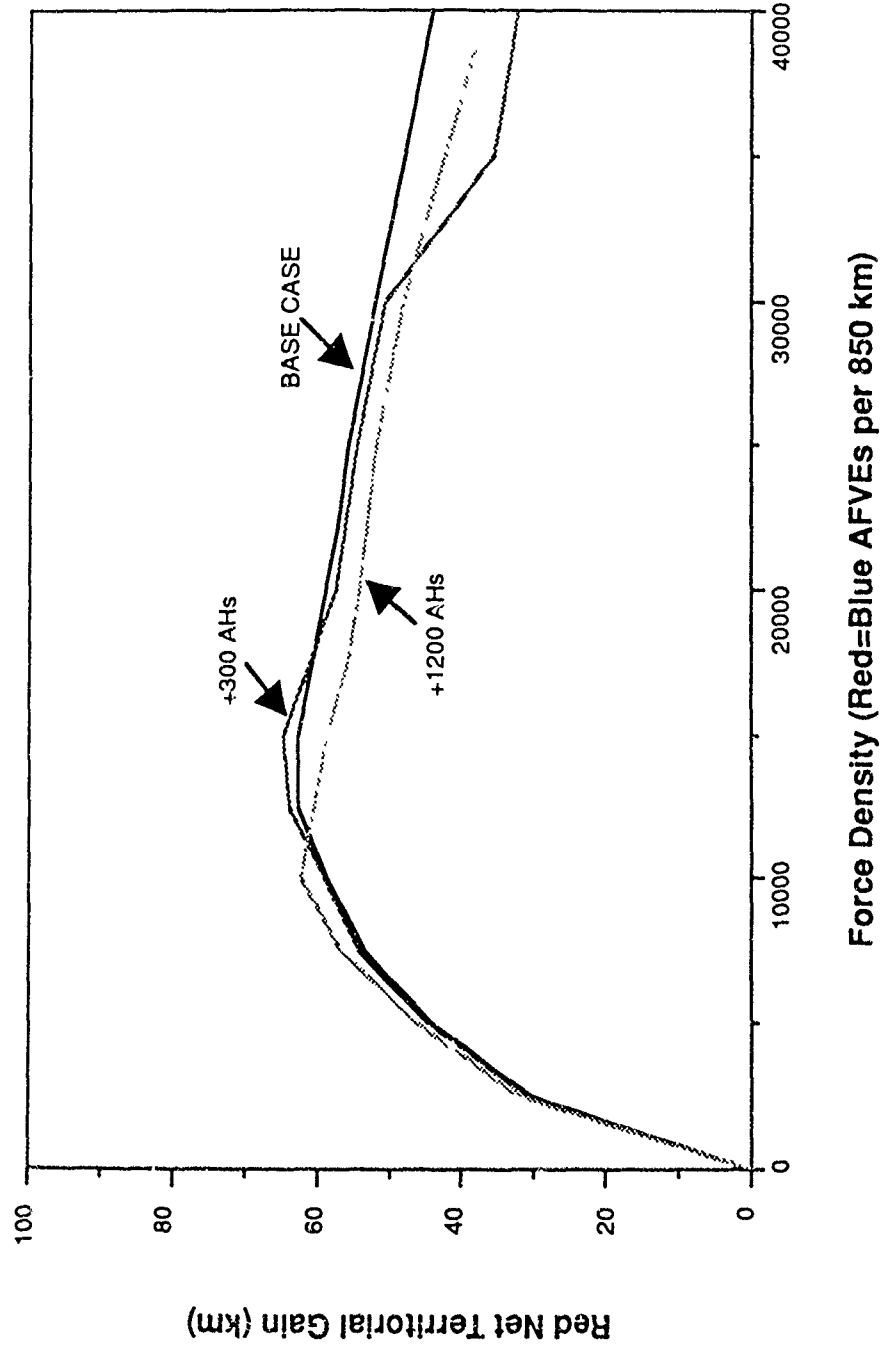
Attack helicopters (AHs) are another weapon type whose stability properties are sensitive to inventory size. To illustrate this effect, Slide A-8 compares the base case with a pair of excursions in which increasing inventories of AHs are added to each sides' base case attack helicopter force.¹⁴ The results suggest that for some force densities, adding small packages of AHs can be stabilizing, while adding larger packages can be destabilizing. At a density of 15,000 AFVEs in the theater, for example, adding the smaller of the two packages considered here to each side increases net territorial gain relative to the base case; thus the package is stabilizing at this force density. For the same theater force level, however, the larger package causes territorial gain to fall relative to the base; hence the larger package is stabilizing.

Increased availability of helicopters has two principle effects at low force densities. As we have seen, additional AHs can be used by blue to strengthen tactical defenses at red's point of attack; they can also be used by red to strengthen flank defenses against blue counterattack. If enough helicopters are added to each side, blue can eventually mass a sufficient force of AHs at the red point of attack to limit red's potential territorial gain without being forced to rely heavily on counterattack. Where inventories are large enough for this effect to obtain, the ability of red's additional AHs to thwart blue counterattack is of little use to red, while the ability of blue's additional AHs to stiffen a thinly manned forward defense can be a significant advantage to blue. Where the AH increase is too small to decrease significantly blue's reliance on counterattack -- as is the case for the smaller inventory illustrated here -- the net result is to reduce the effectiveness of the counterattack which constitutes the primary constraint on red's ability to take and hold ground, and thus net territorial gain rises.

Again, however, the magnitude of these effects is quite modest. At the 15,000 AFVE force level, even the larger of the two packages -- which effectively doubles the base case inventories on each side -- still produces only about a ten percent reduction in red's potential net territorial gain. The smaller package produces changes of less than five percent.

SENSITIVITY TO INVENTORY SIZE

INCREASING QUANTITIES OF AHs



Slide A-8.

A-19

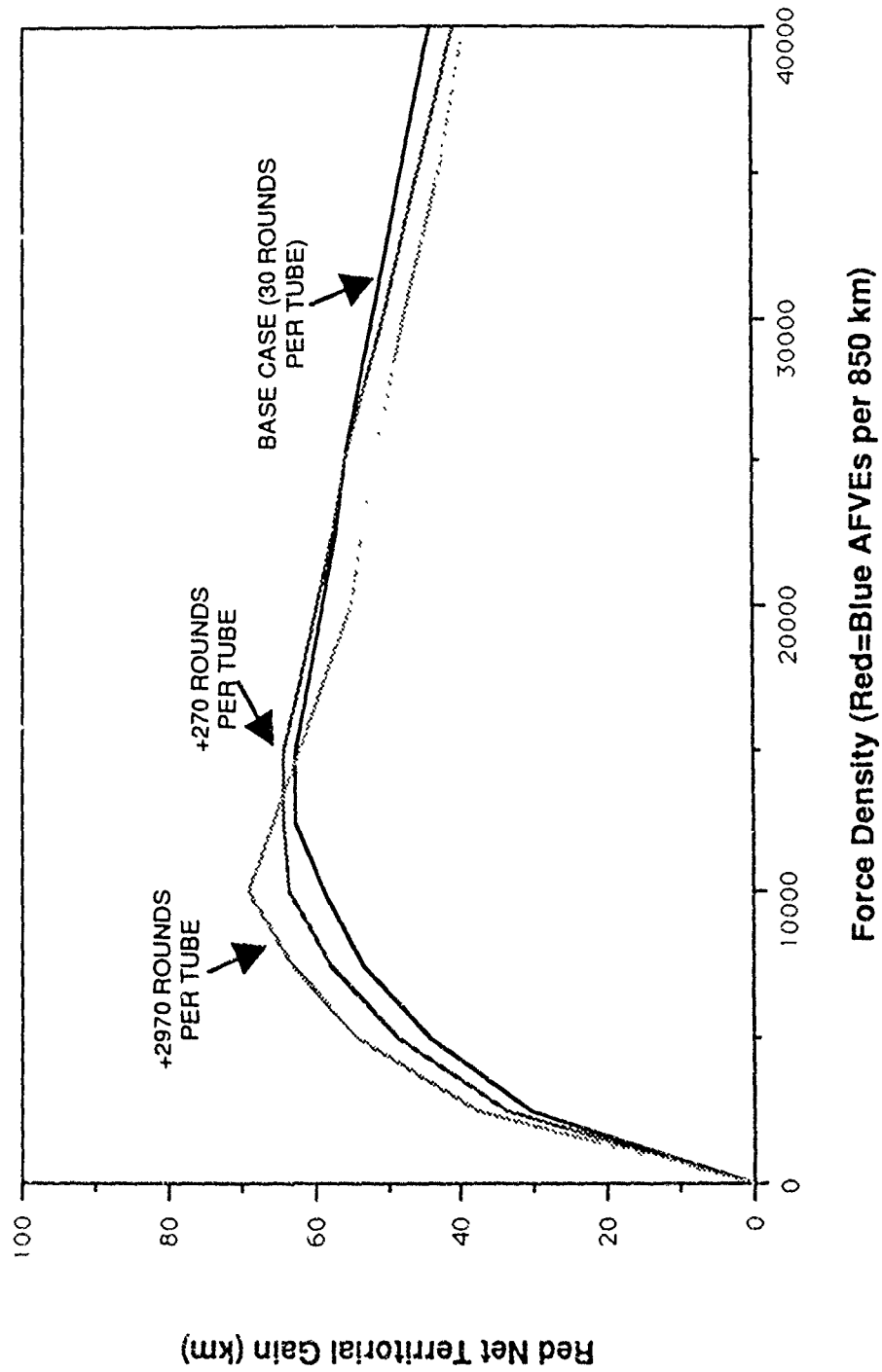
FASCAM is also a weapon type whose stability properties are sensitive to inventory size. To illustrate this effect, Slide A-9 compares the base case with a pair of excursions in which increasing inventories of FASCAM are added to each side's base case stockpiles. The results suggest that for some force densities, adding small packages of FASCAM can be destabilizing, while adding larger packages can be stabilizing. At a density of 15,000 AFVEs in the theater, for example, adding to each side about 300 additional FASCAM rounds per tube of artillery increases net territorial gain relative to the base case; thus the package is destabilizing at this force density. For the same theater force level, however, a larger package of about 3,000 additional FASCAM rounds per tube of artillery causes territorial gain to fall relative to the base; hence the larger package is stabilizing.

This effect is a consequence of FASCAM's ability to change blue's optimal force employment choices if present in sufficient quantity. In modest amounts, FASCAM does not increase the potential effectiveness of blue's forward defenses by a sufficient margin to enable blue to lessen significantly his reliance on counterattack or withdrawal -- yet the ability of red's FASCAM to reduce the effectiveness of blue's counterattack and withdrawal degrades the overall performance of the blue defense and enables red to take and hold somewhat more ground. Given enough FASCAM, however, blue can erect a barrier of sufficient strength to enable blue to succeed with a defense based on a heavy forward allocation of forces with little withdrawal or counterattack. Such a force employment profile denies red the most profitable uses of red's own FASCAM while making best use of blue's, and in the process reduces red's potential net territorial gain to less than that attainable in the base case at this force density. Thus, the relative contribution of FASCAM to red and to blue -- and thus the stability of FASCAM as a weapon type -- depends on the quantity present in the theater at the time of attack.

Once again, however, the magnitude of the effect is relatively modest. The difference between the base case and the excursion is greatest at a force density of 10,000 AFVEs in the theater and a massive FASCAM inventory increase of about 3,000 rounds per artillery tube, but even here the difference is still only about 30 percent of the base case value -- and this is for a FASCAM inventory almost 100 times larger than that of the base case (which assumes 30 FASCAM rounds per artillery tube on each side). Elsewhere, the effect is much smaller -- with the difference falling to as little as five percent of the base case value at a force density of 40,000 AFVEs in the theater and an inventory increase of about 300 FASCAM rounds per artillery tube.

SENSITIVITY TO INVENTORY SIZE

INCREASING QUANTITIES OF FASCAM



Slide A-9.

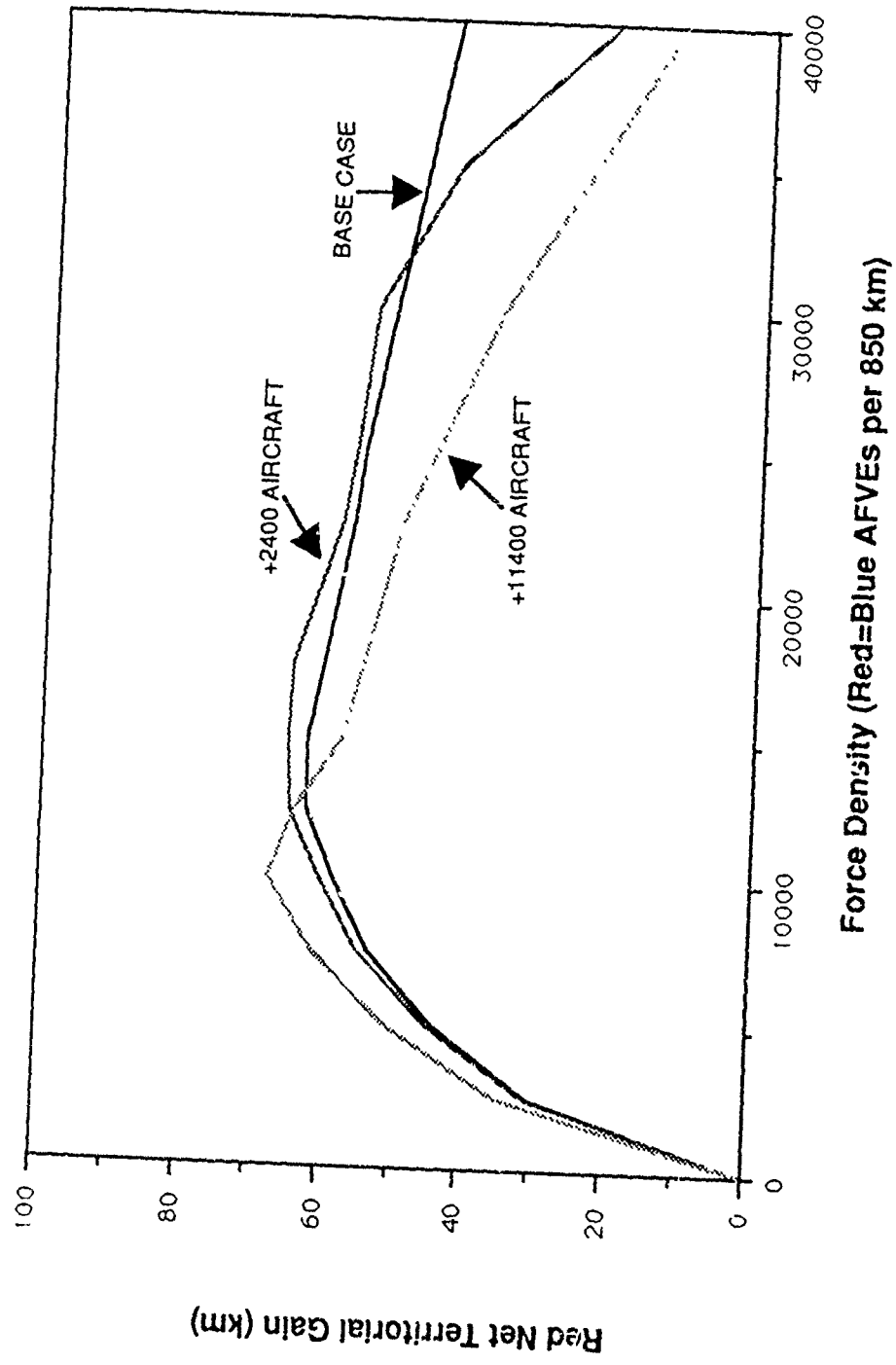
A-21

CAS is another weapon type whose stability properties are sensitive to inventory size. To illustrate this effect, this slide compares the base case with a pair of excursions in which increasing inventories of CAS are added to each sides' base case air forces.¹⁵ The results suggest that for some force densities, adding small packages of CAS can be destabilizing, while adding larger packages can be stabilizing. At a density of 20,000 AFVEs in the theater, for example, adding to each sides' forces the smaller of the two packages considered here increases net territorial gain relative to the base case; thus the package is destabilizing at this force density. For the same theater force level, however, the larger package causes territorial gain to fall relative to the base; hence the larger package is stabilizing.

Like attack helicopters, increased availability of CAS has two principle effects at low force densities. As with AHs, additional CAS aircraft can be used by blue to strengthen tactical defenses at red's point of attack; they can also be used by red to strengthen flank defenses against blue counterattack. If enough CAS is added to each side, blue can eventually mass sufficient airpower at the red point of attack to limit red's potential territorial gain without being forced to rely heavily on ground counterattack. Where inventories are large enough for this effect to obtain, the ability of red's additional CAS to thwart blue counterattack is of little use to red, while the ability of blue's additional CAS to stiffen a thinly manned forward defense can be a significant advantage to blue. Where the CAS increase is too small to decrease significantly blue's reliance on counterattack -- as is the case for the smaller package illustrated here -- the net result is to reduce the effectiveness of the counterattack, which constitutes the primary constraint on red's ability to take and hold ground, and thus net territorial gain rises somewhat.

Once again, for most force densities the magnitude of the effect is not great. At 20,000 AFVEs in the theater, even the larger package produces only about a ten percent change in the base case net territorial gain. At 40,000 AFVEs in the theater, the magnitude of the effect is greater as a percentage of the base (reducing territorial gains to less than 30 percent of the base case value), but is still relatively modest in absolute terms (amounting to a difference of about 32 kilometers).

SENSITIVITY TO INVENTORY SIZE INCREASING QUANTITIES OF CAS



Slide A-10.

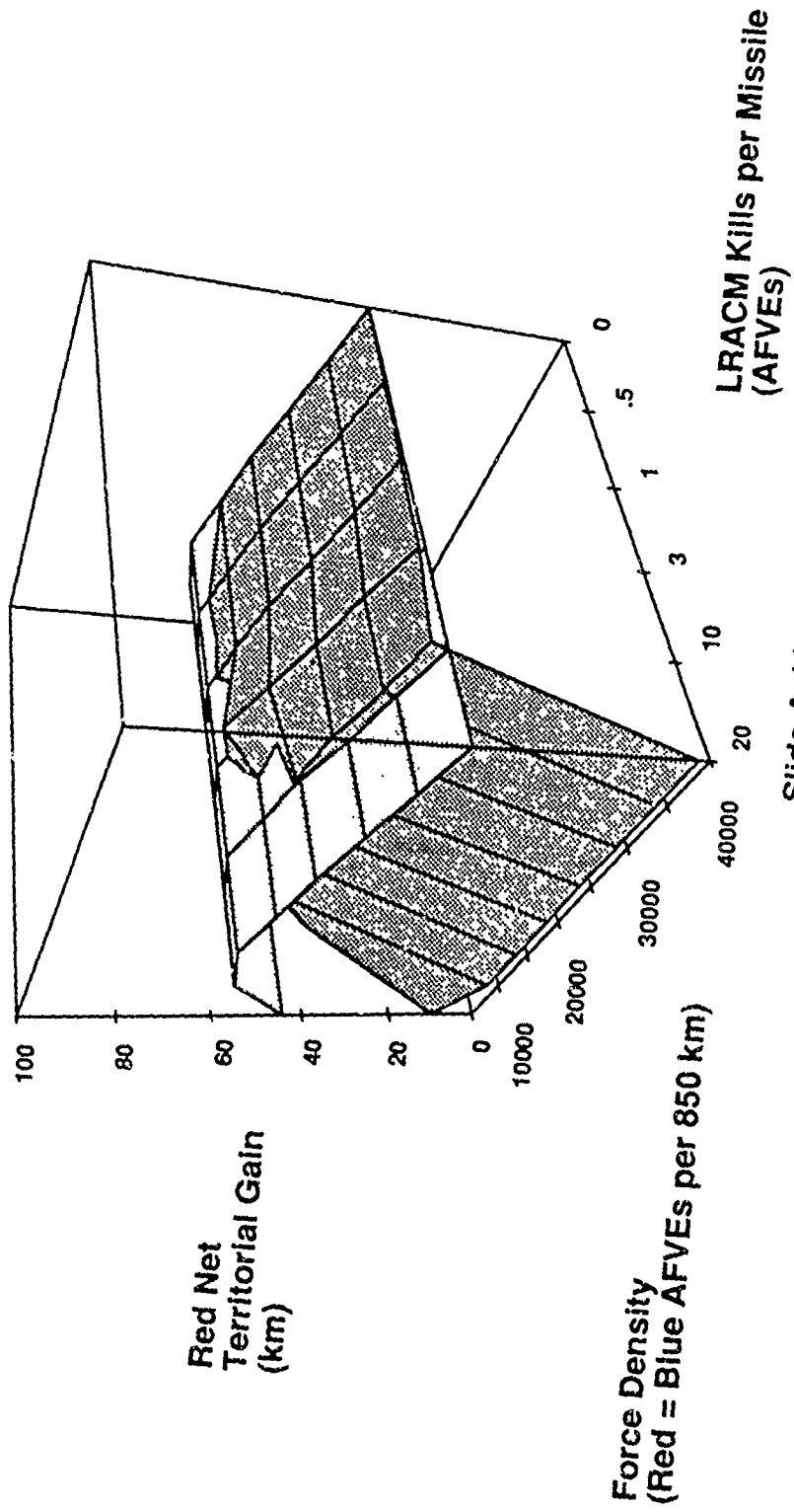
A-23

The final contextual variable, or class of sensitivity, to be considered in assessing the stability properties of individual weapon types is the effectiveness assumptions to be made with respect to certain weapon types whose performance is subject to substantial uncertainty. Advanced conventional munitions systems, as a class, tend to be subject to such uncertainties.¹⁶ Sensitivity to effectiveness uncertainties for SRACM is discussed in the main briefing; here we extend that analysis to long range ACM.

Slide A-11 compares the standard base case with an excursion surface in which a package of about 1,600 rounds of LRACM with varying per-round effectiveness is added to the two sides at a force density of 35,000 AFVEs.¹⁷ The lighter, base case surface assumes no LRACM on either side; this surface is thus constant with respect to the LRACM effectiveness axis. The darker, excursion surface, however, is sensitive to LRACM effectiveness. In particular, for low per-round effectiveness assumptions, the excursion surface lies above the base case, suggesting that the addition of LRACM with assumed effectiveness of less than about 10 AFVE kills per missile fired increases potential net territorial gain and, thus, that LRACM is destabilizing for the inventory considered here. If we assume higher per-round effectiveness, however, net territorial gains crest and eventually decrease -- falling below those of the base case for per-round effectiveness above about 10 AFVEs per missile fired. The given inventory of LRACM is thus stabilizing for high per-round effectiveness assumptions.

LRACM has several effects. For blue, it provides a source of immediately counterconcentratable combat power that can be brought to bear over theaterwide distances. Successful counterconcentration is essential to the viability of the blue defense; thus the capacity to accomplish this more quickly through very long range fires can be a powerful one for blue. For red, on the other hand, LRACM provides firepower that can be used to reach into the blue rear and interdict the counterconcentration movement of blue ground forces. Where LRACM is available to both sides, but with limited effectiveness, the resulting aggregate firepower is insufficient for blue to counterconcentrate sufficiently with the use of LRACM fire alone. Yet the ground force movements blue depends upon to make up the difference occur at a modest rate (given the length of the theater front, and thus the distance to be travelled before counterconcentration is complete). A relatively modest increase in red's long range interdiction capacity (above that already provided by tacair) can be sufficient to reduce substantially blue's reserve arrival rate at the point of attack. For low LRACM effectiveness, the net effect of these countervailing capabilities is to increase somewhat red's potential territorial gain, and LRACM is destabilizing. If LRACM is effective enough, however, then it eventually becomes possible for blue to counterconcentrate sufficiently through LRACM fire alone. At this point, red's ability to interdict the movement of blue ground force reserves is of little value to red, whereas the capacity to counterconcentrate without awaiting the arrival of ground forces is of great value to blue. For high LRACM effectiveness, the net effect is thus to reduce red's potential territorial gain, and LRACM is stabilizing.

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY LRACM Kills per Missile

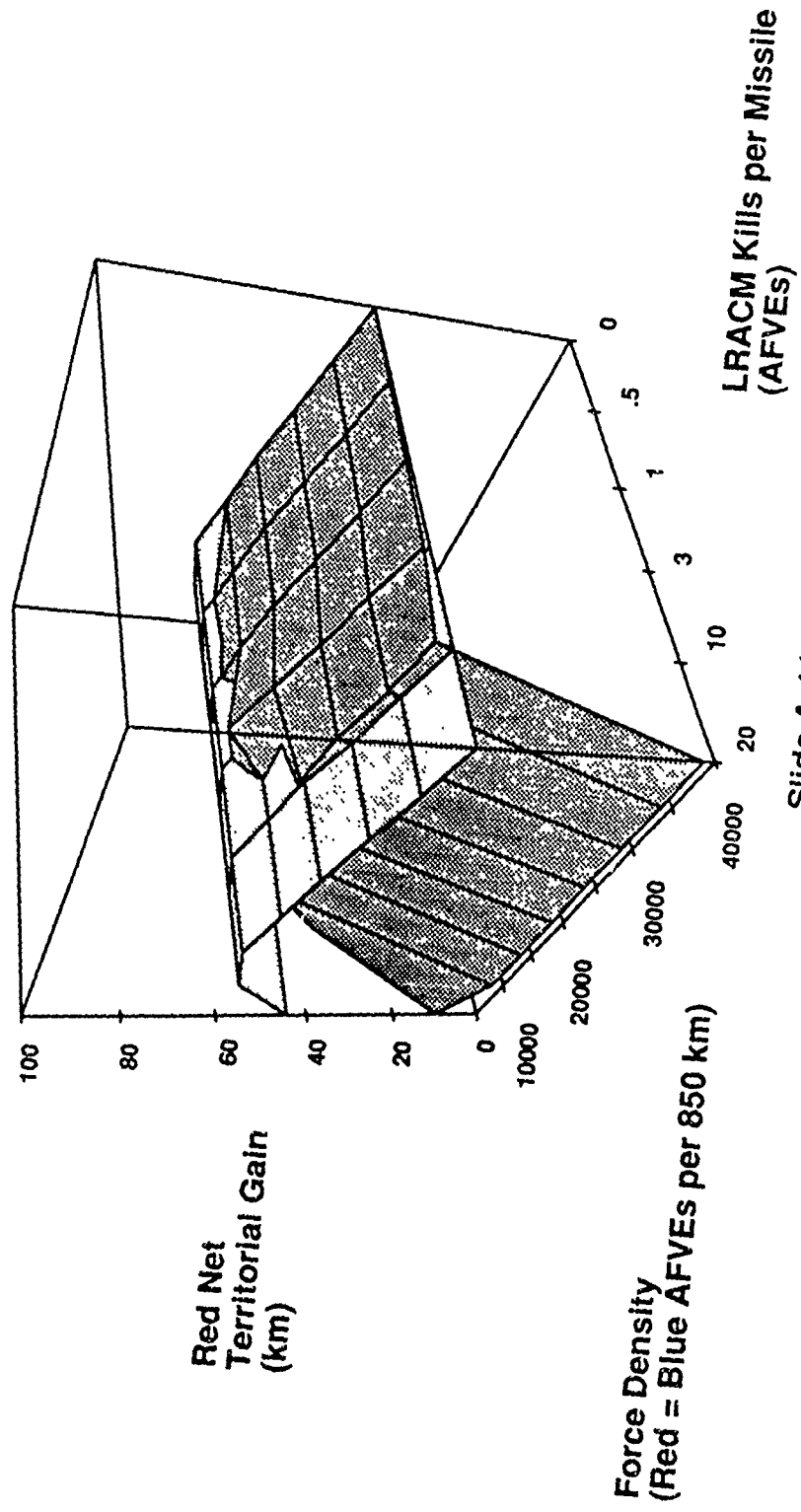


Slide A-11.

A-25

Note, however, that for much of the potential range of LRACM effectiveness, the magnitude of these effects is quite modest. To illustrate the nature of the effect, we have shown results for per-round effectiveness up to 20 AFVE kills per missile fired; a more plausible upper bound, however, might be closer to 10.¹⁸ Yet for effectiveness estimates of around 12 kills or less, the resulting changes in net territorial gain amount to less than 10 percent of the base case value for all force densities. While the magnitude of change can be substantially greater if we assume very high effectiveness levels, the net effects are quite modest for a substantial fraction of the range of uncertainty at issue here.

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY LRACM Kills per Missile



Slide A-11.

A-27

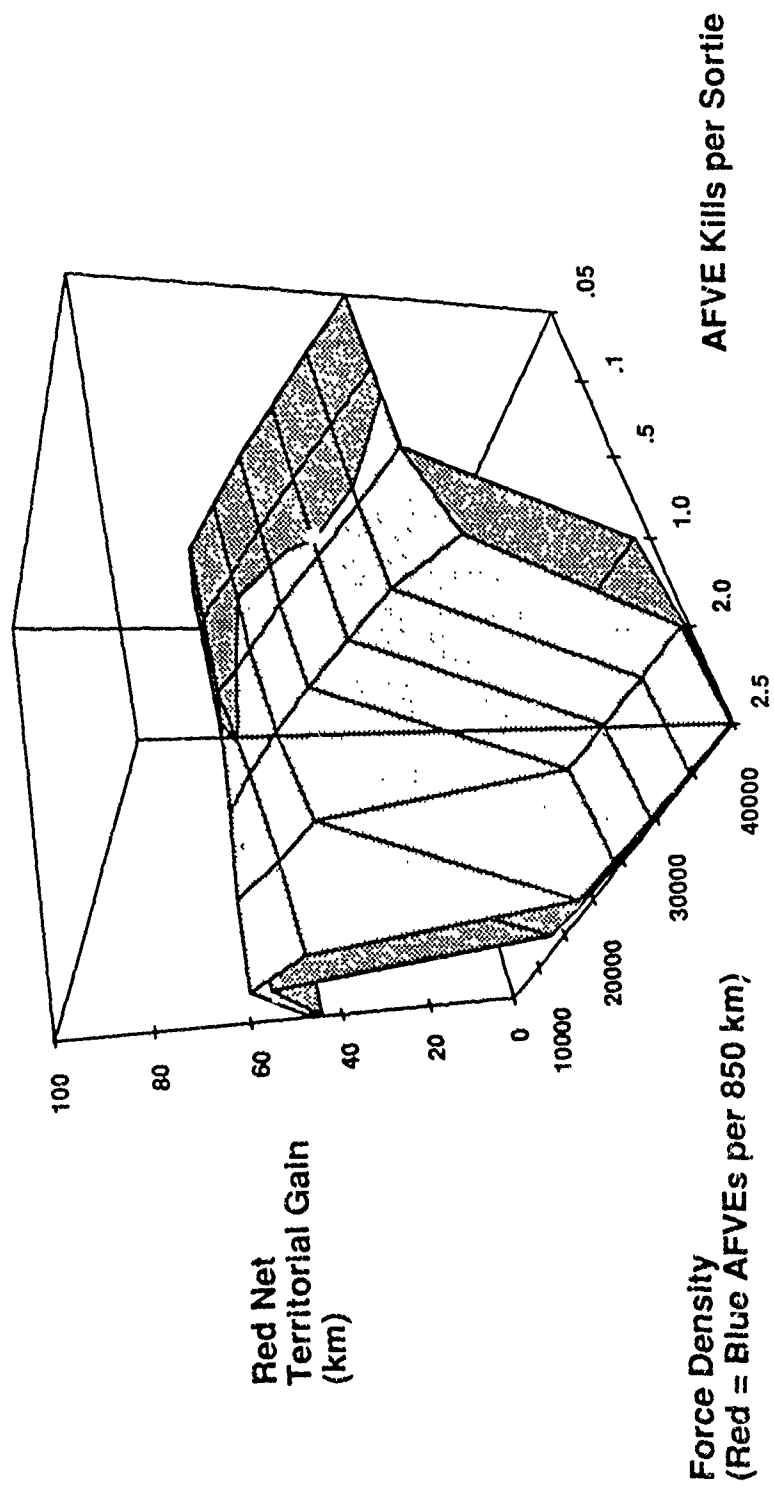
Another weapon type whose stability properties are sensitive to effectiveness uncertainties is close air support aircraft (CAS). Published estimates of the combat performance of CAS vary widely. To illustrate the effect of such variation on the stability properties of CAS, Slide A-12 compares the net territorial gain corresponding to the base case's CAS inventories (i.e., the lighter surface) with the net territorial gain resulting from the provision of an identical package of about 700 additional CAS aircraft to both sides (i.e., the darker surface),¹⁹ as a function of the assumed potential AFVE kills per sortie for CAS aircraft.²⁰ Note that since the base case force structures contain significant CAS inventories, variations in CAS effectiveness assumptions affect both the excursion surface and the base case results.

The results suggest that low effectiveness assumptions produce increases in net territorial gain as a consequence of increasing each sides' CAS inventory relative to the inventories present in the base case, and thus CAS is destabilizing. For high effectiveness assumptions, on the other hand, the same package of additional CAS reduces the payoff to attack relative to the base, and CAS becomes stabilizing.

This sensitivity is due in large part to differences in the way tactical attackers and defenders use CAS. For attackers, potential targets are typically dug in and concealed; often only general locations are known in advance of the request for support, and specific targets may not be visible to pilots upon their arrival over the target area. CAS in support of tactical attacks is thus often unable to exploit fully the capacity of anti-armor PGMs, and must resort to area munitions such as traditional high explosives. For defenders, on the other hand, targets are often exposed and moving, and directly visible for precision engagement by CAS pilots. Such circumstances maximize the effectiveness of modern PGMs. For a tactical attacker, an increase in the potential kills per sortie afforded by more effective munitions is thus not necessarily realizable in the field, given the difficulty of acquiring suitable targets; for tactical defenders, target acquisition is less of a constraint, and increases in effectiveness are exploited more completely.

Again, however, it should be noted that the magnitude of these effects is relatively small. For all but a rather narrow range of effectiveness estimates and force densities, the magnitude of change associated with the provision of a package equivalent to 700 additional aircraft at a force density of 35,000 AFVEs is less than five percent of the base case net territorial gain. It should be pointed out, however, that base case outcomes themselves are potentially quite sensitive to variations in assumed CAS effectiveness; for a combination of kill per sortie estimates of greater than about 1.5 and force densities in excess of about 25,000 AFVEs in the theater, net territorial gain can fall substantially, relative to the results corresponding to the 0.5 AFVE kills per sortie assumed in the main briefing. Near upper bound effectiveness estimates for CAS can thus have an important effect on the projected potential payoff to attack. Given the uncertainty attending these estimates today, this result suggests the importance of reducing these uncertainties through further study.

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY CAS Kills per Sortie



Slide A-12.

A-29

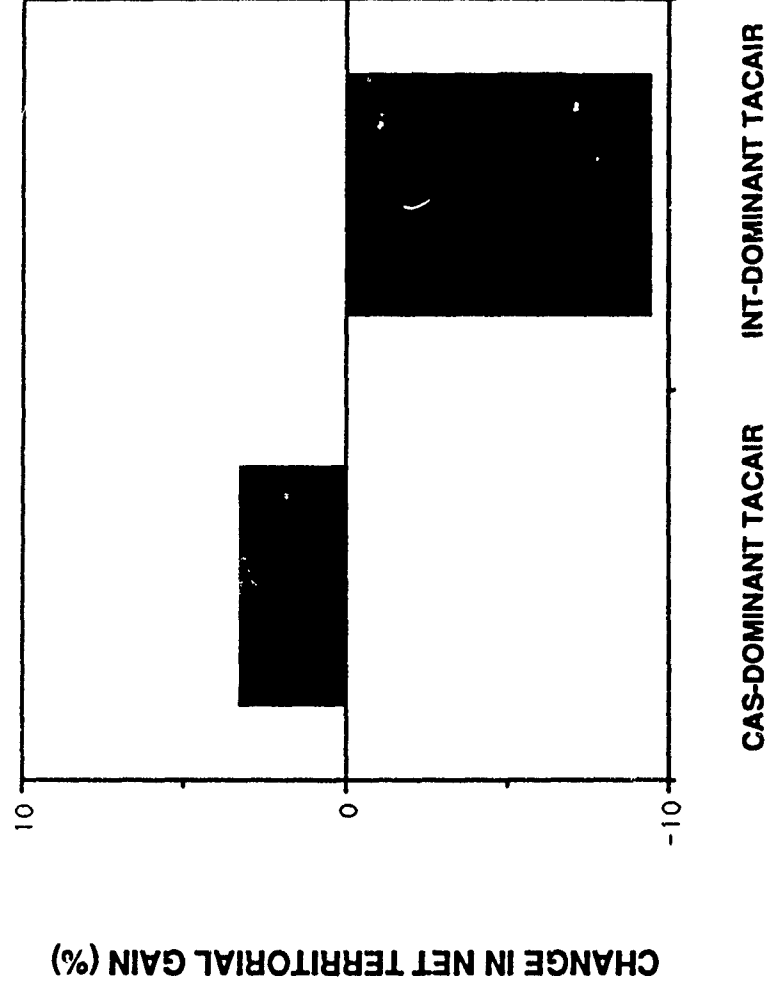
Air defense (AD), like CAS, is a weapon type whose stability properties are sensitive to effectiveness uncertainties. Published estimates of the combat performance of air defenses vary widely.²¹ Moreover, for the purpose of stability, the properties of AD are directly related to those of the aircraft against which those air defenses are used. If, for example, the bulk of the tacair contribution to a given confrontation is in the form of interdiction -- which is uniformly destabilizing -- then, *ceteris paribus*, an increase in AD inventories on both sides will tend to be stabilizing. If the bulk of the tacair contribution is in the form of CAS -- and if CAS effectiveness, inventories, and the force density in the theater are such that CAS is stabilizing -- then, *ceteris paribus*, an increase in AD inventories on both sides will tend to be destabilizing (and vice versa if circumstances are such that CAS is destabilizing). The relative contribution of CAS and interdiction tacair, however, is itself uncertain, depending as it does on the uncertain effectiveness of each type of tacair. As a consequence, the stability properties of air defense are correspondingly uncertain.

To illustrate this interrelated sensitivity, Slide A-13 compares two excursions in which a package of 10,000 additional SHORADS (Short Range Air Defense Systems) and 500 additional HIMADS (HI and Medium altitude Air Defense Systems) was added to each side's base case forces. In one excursion, (represented by the left hand bar) CAS effectiveness is assumed to be substantially higher than that of interdiction, with the result that the bulk of the tacair AFVE kills are contributed by CAS.²² In the other excursion, this relationship is reversed: interdiction effectiveness is substantially higher than that of CAS, with the result that the bulk of the tacair AFVE kills are contributed by interdiction.²³ In both excursions, a force density of 25,000 AFVEs in the theater was assumed.²⁴ In each case, the net territorial gain resulting from the excursion was compared to that of the base case; the values given represent the percentage change in net territorial gain resulting from the addition of some 10,500 AD systems under the indicated assumptions as to weapon effectiveness uncertainties.²⁵ A positive value indicates an increase in net territorial gain as a result of the additional AD deployments; a negative value indicates that net territorial gain fell when the AD package was added.

The result suggests that the stability of AD systems is indeed sensitive to the effectiveness assumptions made with respect to tacair performance. Where we assume high CAS performance and largely ineffective interdiction, as in the left hand bar, increasing the number of AD systems increases red's potential net territorial gain relative to a base case in which CAS performance is also high, but air defenses are less numerous on both sides. Given these effectiveness assumptions, AD is therefore destabilizing. Conversely, where we assume low CAS performance but highly effective interdiction, as in the right hand bar, adding AD systems decreases red's potential net territorial gain, and AD is stabilizing.²⁶

SENSITIVITY TO EFFECTIVENESS UNCERTAINTY

AIR DEFENSE



Slide A-13.

A-31

END NOTES

- 1 The size of this package is scaled down proportionally as density falls; at a density of 20,000 AFVEs in the theater, for example, about 5,700 ATCs and associated infantry squads were added to each side's forces. Note also that adding infantry AFVEs to each side's ground forces increases the force density in the theater (as scored by the AFVEs per 850 km index measure used here) as well as changing the composition of the forces. A direct "before and after" comparison of the base case and the result of adding a package of infantry is thus between the 20,000 AFVE point on the base case and the 25,700 AFVE point on the excursion curve (i.e., 20,000 + 5,700 AFVEs of infantry). Comparison of the 20,000 AFVE point on the base case curve and the 20,000 AFVE point on the excursion curve thus does not directly relate the effects of adding the indicated package; instead it relates the effects of changing the weapon mix while controlling for the effect of force density. In controlling for the effects of density per se, this "constant density" comparison is arguably more instructive, and thus has been used in the text above.
- 2 The size of this package is scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, about 1,200 AHs were added to each side's forces. Attack helicopters ordinarily operate as part of an Attack helicopter-Observation helicopter team, in which observation helicopters act as scouts and target acquisition elements for the associated AHs. For a more detailed discussion, see Appendix B.
- 3 See Appendix B for a more detailed discussion.
- 4 The purpose of tactical pursuit -- to cut off the withdrawal of the defenders occupying the position under assault -- can likewise be accomplished by, e.g., artillery-delivered, scatterable mines. The presence or absence of AHs is thus not necessarily decisive for the attacker to deny withdrawal to the defense, depending on the availability of alternative means. See Appendix B for a more detailed discussion.
- 5 Such as a mounted, exposed attacker. Use of obscurants, covered approach routes, dismounted infantry, anti-helicopter overwatch tactics or forward massing of anti-aircraft systems all can reduce the effectiveness of defending AHs, albeit at the price of reduced rate of closure with the objective (i.e., a reduced assault velocity). See Appendix B for a more detailed discussion.
- 6 The size of this package is scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, about 1,500 were added to each side's forces.
- 7 While the acronym "FASCAM" technically pertains only to a specific set of U.S. Army munitions, we will use the term here as a general description of any artillery deliverable, remotely switchable mine system.
- 8 For a more detailed discussion of rolling depth, predeployed depth, and their interactions, see IDA P-2380, Appendix C.
- 9 Blue's tactical offensive on the flanks of the red spearhead faces a defense with an inherently limited capacity to fight an extended delay-in-depth by virtue of the limited width of the penetration corridor; flank engagements thus tend to be fought in place with modest reliance on defensive

withdrawal. Thus, the capacity of FASCAM to deny withdrawal is of limited value to blue flank attackers. For a more detailed discussion, see IDA P-2380, Appendix C.

- 10 For a more detailed discussion, see IDA P-2380, Appendices C and D.
- 11 For a discussion of the alternatives examined, see IDA P-2380, Appendix D.
- 12 The sizes of these packages are scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, the excursions represent the addition to each sides' forces of 40,000 and 171,000 SRACM rounds, respectively.
- 13 Note that the particular intersection points depicted here are sensitive to the specific assumptions made with respect to per-round effectiveness against moving targets, and the ratio of moving target effectiveness to stationary target effectiveness (see the discussion accompanying Slide 11 in the main briefing).
- 14 The sizes of these packages are scaled down proportionally to the size of the supported ground forces as force density falls; the inventory numbers given here refer to AH inventories at a force density of 35,000 AFVEs in the theater.
- 15 The sizes of these packages are scaled down proportionally to the size of the supported ground forces as force density falls; the inventory numbers given here refer to CAS inventories at a force density of 35,000 AFVEs in the theater.
- 16 See, for example, S.D. Biddle and V.A. Utgoff, *The Battlefield of the Future* (Alexandria, VA: Institute for Defense Analyses, 1989), IDA P-2194; also S.D. Biddle, *How to Think About Conventional-Nuclear Substitution* (Alexandria, VA: Institute for Defense Analyses, 1986), IDA P-1884.
- 17 The sizes of these packages are scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, the excursions represent the addition to each sides' forces of 912 LRACM rounds.
- 18 A single ATACMS, for example, is currently planned to carry 30 terminally guided submunitions. Even if each ATACMS were delivered perfectly to the intended aimpoint, and even if each submunition were 70 percent likely to acquire, and 70 percent likely to kill its target, and each target is unique (i.e., perfect distribution of submunitions to targets, and high target density), the resulting AFVE kill per ATACMS fired would still be under 15. Of course, future designs may differ, but for the near to mid term, it would seem that an estimate of 10 AFVE kills per LRACM round fired would constitute a reasonable upper bound.
- 19 The size of this package is scaled down proportionally to the size of the supported ground forces as density falls; at a density of 20,000 AFVEs in the theater, for example, the excursions represent the addition to each sides' forces of 400 CAS aircraft.
- 20 Where "potential" means kills given acquisition of suitable targets by the pilot. In effect, this axis reflects the performance of the munitions themselves, and not the aggregate performance of the complete intelligence-target-acquisition-delivery-vehicle-munition system as a whole.
- 21 For a survey, see Deitchman, op. cit.
- 22 Specifically, the left hand bar assumes 1.0 AFVE kills per CAS sortie and 0.1 AFVE kills per interdiction sortie.
- 23 Specifically, the right hand bar assumes 0.1 AFVE kills per CAS sortie and 1.0 AFVE kills per interdiction sortie.
- 24 For illustrative purposes, expected kills per encounter for air defense systems was reduced from 0.35 to 0.1; interceptor aircraft kills per encounter was reduced from 0.2 to 0.1; the assumed cross-country speed of target vehicles in the open was reduced from 20 kilometers per hour to about 10; and the maximum aircraft density over a potential target area was increased from .001 aircraft per minute per square kilometer to .002. The result of these changes is to reduce the incidence of degenerate results (e.g., where all aircraft are killed at either the base case or the excursion case number of ADUs, and thus the effect of uncertainty on stability is masked or damped), and thereby to increase somewhat the magnitude of the

reported effects -- although the direction of the effects remains the same. The reported magnitudes, however, are best regarded as upper bounds for the given force density and inventory levels.

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In all cases, air defense effectiveness was assumed sufficient to provide an expected kill of 0.1 aircraft per AD encounter; variations in air defense effectiveness per se (as opposed to variations in *tacair* effectiveness -- and the influence of such variations on the stability of AD -- as illustrated here) merely increase or decrease the magnitude of the effects illustrated here. While such variations can produce meaningful differences in magnitude, they do not affect the *direction* of the change in net territorial gain. Whether AD systems are stabilizing or destabilizing is thus insensitive to variations in AD system effectiveness per se, although the stability of AD as a weapon type is sensitive to uncertainties in the performance of other weapons -- i.e., *tacair*, as discussed above.

Note also that not all changes in context that affect *tacair* contributions produce corresponding changes in the stability of air defense systems. Force density, for example, has very different effects on *tacair* (which can counterconcentrate almost immediately at even theaterwide distances, but is potentially subject to a variety of constraints on air traffic density and target availability) and air defense (which is tied to the dispositions and tactical densities of far less mobile, but typically more densely massable ground forces). Thus, while AD exhibits a relatively pronounced sensitivity to variations in *tacair* effectiveness, its stability properties are substantially less sensitive to variations in force density.

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Some analysts also have suggested that the mobility characteristics of air defenses could materially affect their stability properties. In particular, it is sometimes argued that mobile air defenses, which could accompany offensive forces as they advance into the defender's territory, are destabilizing, whereas immobile air defenses which could defend only stationary forces, are stabilizing. As we have argued above, however, it is not necessarily the case that tactically offensive systems are destabilizing; at low force densities precisely the opposite is often the case. Moreover, it is not clear that any militarily practical air defense system could really be called "immobile" in the sense that it could not be displaced forward to support advancing ground forces. Truly fixed site air defense systems are extremely vulnerable to modern long range weapons, whether in the form of *tacair*, surface to surface missiles, or even land-based special forces teams. It is thus difficult to see how such systems could contribute materially to the success of either a defender or an attacker. Even relatively slow-moving non-fixed site systems such as the U.S. Patriot are nevertheless mobile enough to keep up with advance rates of the sort that either VFM or historical combat results suggest are likely in modern mechanized warfare (typically a few miles per day to a few tens of miles per day). We have therefore focused here on *tacair* effectiveness uncertainties which do appear to affect significantly the stability properties of modern air defenses, rather than on mobility properties, which do not.

Appendix B

THEORY AND MODEL DOCUMENTATION

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THEORY AND MODEL DOCUMENTATION

I. INTRODUCTION

This appendix is intended to explain the modeling of new weapon types and the modifications made to the modeling of existing weapon types in the Variable Force eMployment (VFM) theater combat model. Specifically, the new equations used to represent the interactions of the forces, along with brief explanations of their derivation, are contained herein. As this effort was an augmentation of the capabilities of the old VFM code, frequent reference will be made to it. The reader is directed to "Force to Space Ratios and Defense Effectiveness: A Theoretical Approach to the Analysis of Deep Cuts in Conventional Forces," Stephen D. Biddle, et al.,¹ for an in-depth discussion of the development of the VFM code.

The additions and changes to the code have been grouped by weapon type and are presented in the following sections: II. Helicopters, III. Tactical Aircraft and Air Transports, IV. Advanced Conventional Munitions, and V. Barriers and Mines. Limitations to this approach are presented in Section VI. Helicopters and air transports were the only truly new systems added to the code; the others existed in the code previously but are now modeled in a more concrete and detailed manner.

II. HELICOPTERS

Rotary-wing aircraft have unique qualities that give them the potential to have a significant impact on modern combat operations. Attack helicopters combine high speed, mobility and firepower, making them potent anti-armor weapons, but they also are fragile, being vulnerable to nearly every weapon system deployed on the modern battlefield.² Utility or transport helicopters have the capability to move troops around the battlefield rapidly, allowing a commander to quickly deploy forces in

response to enemy action, but they, too, are fragile and their employment must consider this fragility.³ This addition to the VFM code represents the effects of the use of attack and utility helicopters upon modern combat operations.

Our approach to modeling the use of helicopters consisted of four steps, resulting in the equations given in Sections II.A and II.B. The first step was to conduct a review of literature on the helicopter doctrine of the armies of the world, with emphasis on U.S. and Soviet doctrines. The next step was to identify, from the literature, the roles and missions that are planned for helicopters on the modern battlefield. Having identified those missions, the third step was to form hypotheses regarding the effects of helicopters upon modern combat and their interactions with other forces. Specifically, we formulated equations representing the behavior of helicopters to be used in VFM. The last step was to test and confirm or reject some or all of the hypotheses. The confirmation or rejection was carried out using the JANUS combat simulation of Lawrence Livermore National Laboratory. The end result was the equations presented in the following sections.

A. Attack Helicopters

Attack helicopters may be employed generally in two ways on the modern battlefield: to counter an enemy thrust by flying to the point of attack and destroying attacking enemy armored vehicles, primarily through the use of long-range anti-tank guided missiles (ATGMs);⁴ or to exploit a breakthrough by pursuing and/or enveloping the defeated enemy force.⁵ Helicopters are much less effective in the deliberate attack because of their vulnerability to enemy ground fire. On the defensive, attack helicopters can employ pop-up tactics so that they are exposed to the enemy only during that short time in which they acquire and fire on advancing enemy vehicles.⁶ On the attack, stationary enemy vehicles are difficult to acquire and therefore the helicopters must expose themselves to enemy fire for an extended period of time while they acquire targets. Nearly every weapon on the battlefield has the potential to engage and destroy a hovering helicopter if it is in range. Even at the typical engagement ranges of ATGMs, hovering helicopters are vulnerable to surface-to-air missiles (SAMs), tank guns and enemy ATGMs themselves.⁷ Similarly, helicopters do not perform as well in the face of a slow, deliberate enemy attack, as the enemy has time to take measures to reduce their effectiveness. Artillery may be used to lay down smoke screens to cut off the lines of sight to the attacking vehicles, or to bombard positions from which the helicopters could fire. Air defense units may be deployed specifically

to suppress helicopters, or some portion of the attacking vehicles may be held in overwatch to engage the helicopters as they pop up.⁸

1. Defensive Employment

The first mission of attack helicopters to consider is their use as anti-armor weapons at the point of attack, either by blue forces defending against the red attack or by red forces on red's flank defending against a blue counterattack. In the VFM code, the red advance into blue territory is modeled as a series of discrete engagements in which the red force assaults and takes successive identical blue lines until the red force is reduced to the point that it can be stopped by the blue reserves that have accumulated by that time in the operation. The effectiveness of defending weapons is expressed in terms of the casualties C incurred by an attacker in the course of taking a single defender's line (either red taking a blue line or a blue counterattack taking a red flank line):

$$C = \frac{\lambda_{\text{ATK}}}{2} \left[\alpha \gamma \left(\frac{\hat{B}_{\text{LI}}}{\hat{R}_{\text{ECH}}} \right) \left\{ 14.32\beta V \phi_i + 11.80\phi_T + 1.448\hat{H}_B V + 0.22 \frac{\hat{B}_{\text{ARTLI}}}{(0.01 + V)} + \frac{693.6}{\hat{R}_{\text{ARTECH}}(1 + V)} \right\} + \mu_B \right] \quad (1)$$

where:

- λ_{ATK} = the total frontage of the attack (km)
- α = a factor accounting for the reduction of the attacker's casualties caused by early defender withdrawal
- γ = a factor accounting for the disruption of an attacking echelon as it takes successive lines
- $\hat{B}_{\text{LI}}$ = the number of defending AFVEs per 2 km of line
- $\hat{R}_{\text{ECH}}$ = the initial strength, in AFVEs, of an attacking echelon per 2 km of line
- μ_B = casualties caused by ACM and tacair (see Sections III and IV)

β = factor accounting for the effects of defensive barriers (see Section V)
 ϕ_i = the sum of the fractions of the attacking and defending forces that are infantry
 V = the average velocity of the attack (km/hr)
 $\hat{H}_B$ = the number of defending helicopters per 2 km of line
 ϕ_T = the sum of the fractions of the attacking and defending forces that are tanks
 $\hat{B}_{ARTLI}$ = the number of defending artillery tubes supporting each 2 km of line
 $\hat{R}_{ARTECH}$ = the number of attacking artillery tubes supporting each 2 km of attack frontage.

Equation 1 was derived from the results of experiments performed using the JANUS combat simulation of Lawrence Livermore National Laboratory. The experiments consisted of simulated red assaults upon blue lines under varying conditions of force composition, force to force ratio and tactics (red attack velocity and blue withdrawal). The reader is directed to Biddle, et al.,⁹ for further reference.

The effectiveness of attack helicopters on the defensive is given by the term $1.448 \hat{H}_B V$ where the numerical coefficient was calculated from the data taken from the JANUS simulations. The results of the simulations confirmed the hypothesis that the effectiveness of attack helicopters would decrease as the attacker slowed down and executed a more deliberate assault (employing dismounted infantry and more artillery preparation); thus the proportionality of casualties caused by helicopters to attack velocity. The trade-off between the attacker's velocity and the casualties he takes may be shown by plotting the casualties as a function of the velocity. The component of the casualty-velocity curve due to helicopters is illustrated in Figure A-1.

During the JANUS runs an attempt was made to simulate the use of attack helicopters by the attacker in support of his assault upon the defending line. It has been written that Soviet doctrine is to use helicopters, rather than fixed-wing aircraft, in the close air support (CAS) role to attack defending infantry with area fire weapons such as rockets, and to attack defending AFVs with ATGMs.¹⁰ The results of the simulations showed that on a modern, high-density battlefield, this tactic would be

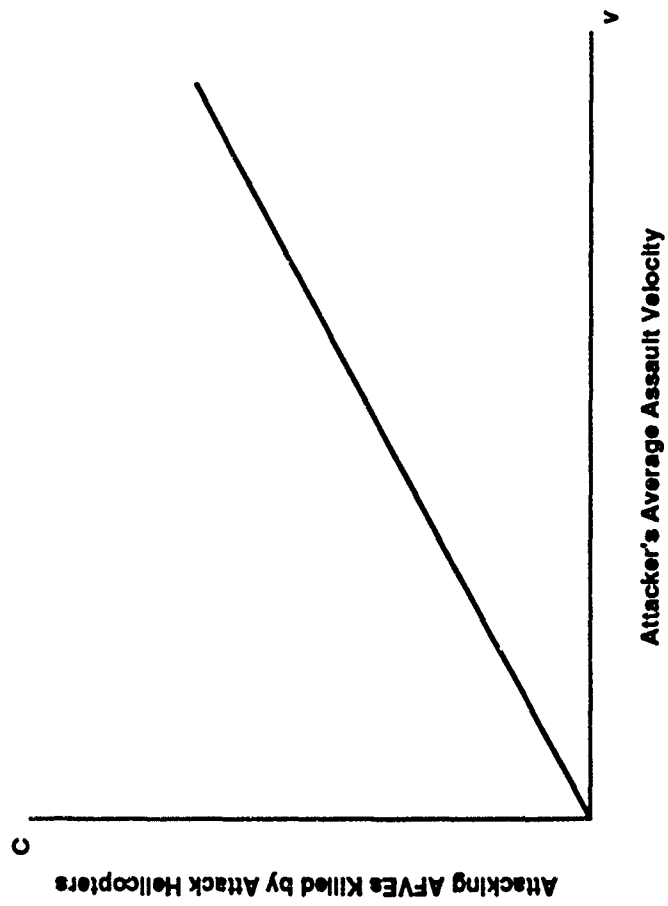


Figure A-1. Relationship between the Attacker's Average Assault Velocity and his Casualties due to Attack Helicopters¹¹

highly ineffective, resulting in heavy helicopter losses and few casualties inflicted upon the defenders. Close observation of the simulations indicated that the ineffectiveness was due primarily to the relative inability of the attacking helicopters to acquire stationary defending targets, especially dug-in infantry, and the helicopters' extended exposure to defending fire while flying near the defender's positions.

In order to complete the treatment of attack helicopters on the defensive, we must address their distribution in the theater and their availability for combat. We assume that red allocates attack helicopters either to the defense of his flanks against blue counterattacks or to the pursuit of withdrawing blue forces in the attack sector. We further assume that red allocates his helicopters before the operation, and that the allocation does not change during the operation. Over the course of this work it has been found that red's pursuit of withdrawing defenders is generally more effective than red's flank defense in that, while blue may or may not launch a large counterattack, he almost always depends upon withdrawal to provide some rolling depth to his theater defense. Therefore red assigns most of his helicopters to the offensive pursuit mission, which will be discussed in the next section.

Red's flank defense is modeled in the same manner as blue's theater defense in that the red defenders are deployed in lines and the blue counterattackers fight through each line in succession until, as with the red attackers driving through blue theater defensive lines, they are stopped at the last red line of defense by the reserves that red has deployed in reaction to the blue counterattack.¹² Red reserves arrive over the elapsed time of blue's counterattack minus some reaction time in which red recognizes the location of the blue counterattack and decides to deploy his reserves. red helicopters used for flank defense are assumed to fly to the last line of defense and deploy with the accumulated reserves. All of the red helicopters dedicated to flank defense are assumed to arrive immediately following the red reaction time.

Blue's deployment of his helicopters is related to his deployment of ground forces in the theater. It is assumed that blue deploys some fraction of his forces ϕ_{FWD} forward, in a number n_L of prepared lines, and the remainder (a fraction $1-\phi_{FWD}$) in reserve, either to be used in counterattack or in defense against the red thrust.¹³ Blue is assumed to deploy an equal fraction ϕ_{FWD} of his attack helicopters forward to support the defending troops, and a fraction $1-\phi_{FWD}$ in reserve. Once the war begins, it is assumed that blue immediately deploys his reserve helicopters to support those defending ground forces in the attack sector

over a number of lines n_{LI} . Thus the number of attack helicopters per two kilometers of blue line facing the red attack $\hat{H}_B$ in Equation 1 above, is given by:

$$\hat{H}_B = \frac{2H_B}{\lambda_{THR}^{n_{LI}}} + \frac{2H_B(1-\phi_{FWD})}{\lambda_{ATK}^{n_{LI}}} \quad (2)$$

where H_B is the total number of blue attack helicopters in the theater. We assume here that, because the flight speed of an attack helicopter is much greater than the ground speed of the blue reserves, blue can immediately deploy his helicopters across the red attack sector. (The ground speed of reserve units would be approximately 10 km/hr,¹⁴ while the cruising speed of a CH-47, for instance, is approximately 245 km/hr.¹⁵) In reality there would be some delay between the start of the attack and blue's recognition of its location and his decision to deploy reserves, but it would be short compared to the total length of the campaign. We also assume that the number of blue helicopters per two kilometers of defended line remains constant over the length of the campaign, as the helicopters are assumed to withdraw with the ground forces they are supporting or to be destroyed as they are destroyed. The VFM code assumes that all blue lines before the last one, on which all accumulated blue reserves are deployed, are identical, and that the same fraction of the initial force withdraws from each line before being destroyed in combat.¹⁶

The last issue to be addressed regarding the defensive employment of attack helicopters is their availability over the course of the campaign. Attack helicopters consume fuel at a high rate and do not carry a large amount of ammunition. Therefore they must spend a considerable amount of time moving from the battlefield to (in the case of the U.S. Army) a Forward Area Rearming and Refueling Point (FARRP), rearming and refueling and moving back to the battlefield. The U.S. Army estimates that helicopters will spend approximately two-thirds of their time at the front out of direct combat, moving to and from the FARRP.¹⁷ We assume that, in the case of red's defending against a blue counterattack, and owing to the relatively high velocity of the counterattacking forces, combat occurs nearly continuously, all day for a short time. Therefore the availability of red helicopters on the defensive is taken to be one-third, as there are no lulls in the fighting in which the helicopters can rearm and refuel "for free." In the case of blue defending against the red attack, availability is not so simply determined. The red rate of advance along the ground is described by the following equation:¹⁸

$$\psi_{ROA} = \frac{D_L}{\frac{D_L}{V} + n_{ECH} \left(t_{OPREP} + \frac{n_{ECH} D_{ASY}}{2V_{RSV}} \right)} \quad (3)$$

where:

- D_L = the depth of a blue line (km)
- V = the average attack velocity (km/hr)
- n_{ECH} = the number of red echelons required to take a blue line
- t_{OPREP} = the preparation time before each red echelon is committed to action
- D_{ASY} = the separation distance between red echelons
- V_{RSV} = the velocity of red echelons moving to contact.

It can be seen from Equation 3 that the combat time (D_L/V) is only part of the total time required for red to take a blue line. The rest of the time is taken up by red preparation for the attack (t_{OPREP}) and by the movement of echelons to contact with the blue line ($n_{ECH} D_{ASY} / 2V_{RSV}$). During that time, blue helicopters could rearm and refuel without losing time in combat. While the lengths of combat time and "lull" time change with blue and red force employment options, clearly blue will gain some benefit in helicopter availability from the breaks in combat. We assume that the impact of the lulls in combat is to increase the availability of blue helicopters on the defensive from 33 percent to 40 percent. Subsequent sensitivity studies performed with VFM showed that the outcome of the campaign (red net territorial gain) was not a strong function of helicopter availability.

2. Offensive Employment

The last mission of attack helicopters to consider is their employment by the theater attacker to pursue and destroy withdrawing defenders. In VFM, blue may choose to withdraw a fraction of his defending units w from each line before they are destroyed in ground combat. The total depth D of blue's theater defense is given in Equation 4 below:¹⁹

$$D = D_{LI}(n_{LI} - w) / (1 - w) \quad (4)$$

One can see that increasing w increases the rolling depth of blue's total theater defense. Therefore red's destroying blue AFVEs as they withdraw effectively reduces w and thus the total depth of the theater defense, possibly leading to a red breakthrough.

The effectiveness of red attack helicopters in pursuit of withdrawing blue forces was determined through the use of the JANUS simulation. A withdrawal scenario was constructed in which blue vehicles deployed on a line were withdrawn to the next rearward line (over a distance of five kilometers) while red helicopters and ground forces pursued them. The blue vehicles withdrew fast enough so that they could not be caught by the red ground forces alone, thus all blue casualties were caused by helicopters. The scenario was run for a varying number of red helicopters and blue air defense units. Relations governing blue ground casualties and red helicopter casualties were determined from the data collected. The fraction of withdrawing blue ground vehicles that survive is given as:

$$w_{SURV} = \left[1 - \frac{H_{MAX}}{w\hat{B}_{LI}} \left(0.188 + \frac{0.088}{\rho_{SB}} \right) \right]^w \quad (5)$$

where:

- H_{MAX} = the number of pursuing red helicopters (per 2 km of line) = the maximum number of helicopters deployable (per 2 km of line)
- $w\hat{B}_{LI}$ = the number of blue vehicles withdrawing (per 2 km of line)
- ρ_{SB} = the ratio of blue short-ranged air defense units to ground AFVEs.

The number of red helicopters available for pursuit per two kilometers of defended line is taken to be either the maximum possible number (equal to the maximum AFVE density) or the number of helicopters required to eliminate all of the blue

defenders if they all tried to withdraw, whichever is lower. In that way red tunes his deployment to a certain degree by not pursuing with helicopters that would cause no marginal blue casualties.

The number of red helicopters lost per blue line pursued also is derived from the JANUS data:

$$\frac{dH}{dn} = \frac{\lambda_{ATK}}{2} H_{MAX} \left(1 - \exp \left\{ -\frac{\hat{B}_{LI}}{H_{MAX}} [0.52 + 0.753 \rho_{SB}] \right\} \right) \quad (6)$$

Red is assumed to pursue the blue defenders as long as he has helicopters available to do so. Once his helicopters have been exhausted, he does not pursue any further, and the fraction of blue defenders that survive every line after that point is equal to w . The number of blue lines that red could pursue, given his inventory of helicopters dedicated to the pursuit mission, is calculated as:

$$n_{LP} = \text{int} \left[1 + \left(H_P - H_{MAX} \frac{\lambda_{ATK}}{2} \right) \left(\frac{dH}{dn} \right)^{-1} \right] \quad (7)$$

where H_P is the total number of red attack helicopters dedicated to offensive pursuit and is equal to the product of the total number of red attack helicopters in the theater H_R and the fraction used in pursuit ϕ_{HP} .

If the number of lines pursued is less than the calculated total number of lines in the blue theater defense, the following implications obtain. First, red will run out of helicopters in the middle of the campaign, and thus the blue defensive lines encountered after that point will not suffer the attrition caused by pursuit that was suffered by the previous blue lines. That means that for those lines, w_{SURV} , the final fraction of blue forces on a line that survive combat (including withdrawal) will be higher. Since more blue forces will survive withdrawal after the red helicopters are gone, as follows from Equation 4 above, the true total depth of the blue theater defense will be higher than the depth that would be calculated if we assumed that those forces were eliminated by helicopter pursuit. The extra surviving blue forces will in fact form additional lines of defense, so the total depth of

the theater defense must be adjusted accordingly. The extra surviving blue units can be thought of as forming new lines behind the line that would have been the last line in the theater if the red helicopters had survived the entire campaign. The number of those new lines may be derived from the preceding to be:

$$n_{LP1} = (D/D_L - n_{LP})(w - w_{SURV}) \quad (8)$$

where D/D_L is the pre-adjusted total number of blue lines of defense, calculated using Equation 4. The new lines can be thought of as analogous to the lines of the prepared theater defense n_{LI} and therefore the extra theater depth provided by the lines n_{LP1} may be calculated using Equation 4 if w_{SURV} is replaced by w :

$$\Delta D = D_L(n_{LP1} - w)/(1 - w) \quad (9)$$

The true total theater depth is then given by the sum of D from Equation 4 and ΔD from Equation 9.

If the number of lines n_{LP} that red is able to pursue is greater than the total number of lines in the blue theater defense (D/D_L (Equation 4)), then any helicopters that were initially allocated to pursuit but did not participate in the pursuit of the first line in the theater or were not used to replenish helicopter losses incurred during pursuit of subsequent lines in the theater (i.e. contributed to the pursuit mission in no way whatsoever) are shifted to the defense of red's flanks. The true fraction of red helicopters that is used in pursuit of withdrawing blue defenders becomes:

$$\phi_{HP} = \frac{\lambda_{ATK} \frac{D}{2} + \frac{D}{D_L} \frac{dH}{dn}}{H_p} \quad (10)$$

That fraction is used to calculate the number of red helicopters that are available for flank defense.

Finally, we assume that the availability of red's helicopters participating in the pursuit of withdrawing blue defenders is equal to unity, in that the pursuit missions are of very short duration (on the order of 15 minutes) and are spaced far enough apart for red to be able to rearm and refuel all of his helicopters between missions.

B. Utility and Transport Helicopters

Utility and transport helicopters have a number of potential missions in modern conventional warfare. Their high speed relative to ground forces and their ability to fly over difficult terrain and over or around enemy forces presents great potential via the large advantage in operational and tactical mobility of the forces that they carry. They can transport light forces consisting of infantry, engineers, anti-tank teams, light AFVs and light artillery within the theater to quickly block enemy offensive actions.²⁰ They also can transport those light forces, accompanied by attack helicopters, into the enemy's rear areas and seize important objectives, such as bridges or airbases, or attack enemy command posts, logistics targets, and reserves.²¹ Airmobile units may be inserted into enemy rear areas through breakthroughs created by ground forces,²² through gaps in the enemy line if they exist,²³ or even over enemy forces if enemy air defenses can be suppressed and/or the helicopters are sufficiently "stealthy."²⁴ Heliborne forces may also act as cavalry, screening the front and flanks of friendly ground forces on the move, performing reconnaissance, and providing rear area security against enemy airborne or airmobile insertion.²⁵

Some of the above missions are not applicable to the situation VFM models. First, VFM considers any red breakthrough to be a victory, thus it does not address operations subsequent to a breakthrough. Second, VFM assumes a continuous front, and thus there are no gaps through which airmobile forces could fly. Third, VFM assumes that the red and blue lines are always in contact, thus the forward edges of forces only move freely in friendly territory, where they do not need helicopter reconnaissance or screens. Last, inserting airmobile forces over defended enemy lines would certainly result in heavy casualties, thus making the missions unattractive.²⁶ Furthermore, such missions are difficult to model accurately. All of the airmobile insertion missions described are somewhat similar in purpose to fixed-wing interdiction missions and could be substituted for accordingly. Therefore, the mission of utility and transport helicopters that is included in the VFM code is the rapid transport of light forces, composed of infantry and anti-tank units, to counterconcentrate against an enemy attack.

The counterconcentration mission is performed by blue helicopters carrying infantry forces to a point opposite the red thrust and by red helicopters carrying infantry to points on red's flank opposite the blue counterattack. Blue heliborne infantry is assumed to fly to and deploy on the last line of the theater defense. That is the line at which the blue forces deployed, plus reserves accumulated up to the point in the campaign at which the attacking red forces reach that line, of which heliborne infantry is a portion, are sufficient to stop the red forces driving into blue territory in a single engagement. That relationship is expressed mathematically as:²⁷

$$hB_{OM}(t^*) = R_{OFV}(t^*) \quad (11)$$

where:

- B_{OM} = the strength of the blue reinforced line (AFVEs)
- R_{OFV} = the strength of red's surviving attack force (AFVEs)
- h = the loss exchange ratio on the final line (red/blue)
- t^* = the time of the culmination of red's attack (hrs).

The strength of the blue reinforced line is given by:²⁸

$$B_{OM}(t^*) = \begin{cases} B_{LI} & \text{if } t^* < t_{BST} + t_{BPREP} \\ B_{LI} + B_{RSVA} + \psi_{BOMT} (t^* - t_{BST} - t_{BPREP}) & \text{if } t_{BST} + t_{BPREP} \leq t^* \leq \frac{\lambda_{THR}}{V_{RSV}} \\ E_{LI} + B_{RSVA} + B_{RSV} (1 - \phi_{CA}) & \text{if } t^* > \frac{\lambda_{THR}}{V_{RSV}} \end{cases} \quad (12)$$

where:

- B_{LI} = the initial strength of the line (AFVEs)

B_{RSVA} = the strength of blue airborne forces (AFVEs)
 ψ_{BOMT} = the arrival rate of blue reserves (AFVEs/hr)
 B_{RSV} = the total strength of blue reserves (AFVEs)
 ϕ_{CA} = the fraction of blue reserves employed in counterattack
 λ_{THR} = the width of the theater (km)
 V_{RSV} = the average ground reserve velocity (km/hr).

We assume that blue can deploy his airborne forces as soon as he can react to the red offensive (t_{BST}), that their flight time to the reinforced line is small compared to the movement time of ground forces, and that the airborne forces will be able to fight after they have prepared their defensive positions (t_{BPREP}).

The loss exchange ratio h is determined by the force compositions of the red attackers and the blue defenders on the reinforced line, in a manner similar to that given by Equation 1 for the defensive lines taken before the last battle is fought:

$$h = \frac{1}{2\rho_{MAX}} \left[14.32\beta v\phi_{INF}(t^*) + 11.80\phi_T + 1.45\hat{H}_B V + \frac{0.22\hat{R}_{ARTECH}}{(0.01 + v)} + \frac{693.6}{\hat{R}_{ARTECH}(1+v)} + \mu_B \right] \quad (13)$$

where $\phi_{INF}(t^*)$ represents the sum of the fractions of the red and blue forces that are infantry at the time of the last battle. Because the airborne forces are assumed to be all infantry, ϕ_{INF} will change over time, increasing immediately after the arrival of the airborne infantry, and decreasing as blue reserve armor arrives on the reinforced line. Equation 13 also reflects a number of assumptions regarding the final engagement that cause it to be slightly different from Equation 1. First, as this is the last line, blue does not withdraw to another. Second, the forces in direct contact will be of roughly equal numbers and their force density (AFVEs/2 km) will be roughly equal to ρ_{MAX} . Third, since red will prepare carefully before the battle and advance no further afterwards, the disruption of red attacking echelons may be neglected.²⁹

The expression for red strength at the time of the last battle $R_{OFV}(t^*)$ is simply:³⁰

$$R_{OFVST} - \psi_{ROFVT} t^*$$

(14)

where ψ_{ROFVT} is the red loss rate due to ground casualties,³¹ air attack and ACM casualties, and the diversion of forces to flank defense. The red ground loss rate may be calculated as the product of the casualties per blue line taken (Equation 1) and the ground advance rate in lines per unit time (Equation 3); air attack and ACM will be addressed in Sections III and IV of this appendix. We now turn to the diversion of red forces and the use of helicopters for flank defense.

As he penetrates blue territory, red must allocate forces to his flanks to protect the line of communication to the main body of his forces. The red objective, when defending his flanks, is to deploy just enough force to halt the blue counterattack on the flank defensive line directly adjacent to the line of communication (an area running through the middle of red's penetration into blue territory whose width is governed by the size of the red theater attacking force). Blue may launch a counterattack at any point along red's flanks, so we assume that red defends his flanks with an constant density of forces and deploys his reserves (if he has any) to the point of the blue counterattack. Those predeployed forces are assumed to be deployed in lines, similarly to blue's theater defense, parallel to the axis of the red offensive; the reserves are assumed to be sent to the last line of the flank defense. The blue counterattack develops similarly to the red offensive in that each red flank line is taken in a discrete engagement with blue suffering casualties as a function of the force ratio and the force compositions of both sides. The average counterattack velocity is not optimized like the red average attack velocity. Rather it is assumed that the blue counterattacker must advance relatively quickly in order to succeed before the red theater attacker achieves a breakthrough, and thus the counterattack velocity is predetermined by the user of the code. Complete derivations of the equations governing the red flank density are presented in Biddle, et al.,³² and will not be repeated here except where necessary to illustrate the treatment of red's employment of helicopters to defend his flanks.

Red helicopters used in flank defense are assumed to be held in reserve until the location of the counterattack is known. They are then deployed to the point on red's flanks on the last flank line, directly opposite the blue counterattack. The strength of the last red defended line is then given by:

$$R_{OM}(t^*_{CA}) = \begin{cases} R_{LI} & \text{if } t^*_{CA} < t_{RST} + t_{RPREP} \\ R_{LI} + \frac{R_{RSVA}}{2} + \psi_{ROM}(t) t^*_{CA} & \text{if } t^*_{CA} \geq t_{RST} + t_{RPREP} \end{cases} \quad (15)$$

where:

- R_{LI} = the number of AFVEs per defended line predeployed opposite the counterattack
- ψ_{ROM} = the arrival rate of red ground reserves (AFVEs/hr)
- R_{RSVA} = the strength of red airmobile reserves (AFVEs)
- t_{RST} = the time it takes for red to recognize and react to the blue counterattack (hrs)
- t_{RPREP} = the time it takes arriving red reserves (ground or airmobile) to prepare their positions (hrs)
- t^*_{CA} = the time from the start of the counterattack to the time at which it reaches the last red line of flank defense (hours).

Since we assume that red deploys just enough force -- taking into consideration the availability of red ground or airmobile reserves -- on his flanks to stop the blue counterattack on the last defended line, it holds that

$$h_{CA} R_{OM}(t^*_{CA}) = B_{CA}(t^*_{CA}) \quad (16)$$

where:

- $B_{CA}(t^*_{CA})$ = the surviving strength of the blue counterattack at the last red defended line (AFVEs)
- h_{CA} = the loss exchange ratio on the last red defended line.

The surviving blue counterattack strength is a function of the casualties blue takes over the course of the counterattack before he reaches the last red defended line. Furthermore, those casualties are calculated analogously to the casualties of the red theater attacker as given in Equation 1, except as a function of the red strength predeployed on the red flanks R_{LI} and the force compositions of both the blue counterattacker and the red defender. Since neither are blue helicopters used in counterattack, nor are red helicopters used in defense of lines in front of the last red defended line, the blue counterattack casualty rate is unchanged

by the addition of helicopters. Moreover, since the rate of advance of the blue counterattacker is governed only by his casualty rate, it, also, is not affected by the presence of helicopters.³³

The loss exchange ratio on the last red defended line given below is a function of the force compositions of the blue counterattacker and the red flank defender:

$$h_{CA} = \frac{1}{2\rho_{MAX}} \left[14.32\beta v_{CA} \phi_{INFC A} + 11.80\phi_T + 1.45\hat{H}_R V_{CA} + \frac{0.22\hat{R}_{ARTOM}}{(0.01 + v)} + \frac{693.6}{\hat{B}_{ARTECH}(1 + v)} + \mu_{RS2} \right] \quad (17)$$

where:

v_{CA} = the average attack velocity of the blue counterattacker

$\hat{H}_R$ = the number of red attack helicopters per 2 km of defended line

$\hat{B}_{ARTECH}$ = the number of artillery tubes supporting each blue assault echelon per 2 km of line

$\hat{R}_{ARTOM}$ = the number of artillery tubes supporting the last red defended line per 2 km of line

μ_{RS2} = Blue counterattacking casualties caused by red CAS and ACM per 2 km of line (see Sections III and IV).

Of the variables in Equation 17 $\phi_{INFC A}$, ϕ_T and $\hat{H}_R$ are functions of the time t^*_{CA} that it takes the blue counterattack to reach the last red defended line. The red infantry fraction changes when airmobile forces, being purely infantry, arrive on the line and as red ground reserves, having a fraction of infantry equal to the rest of the red forces, arrive on the line over time after blue begins his counterattack. The number of red attack helicopters available per two kilometers of defended line goes from zero at the start of the counterattack to $\hat{H}_R$ for all t^*_{CA} greater than t_{RST} , the time it takes for red to recognize and react to the blue counterattack, assuming that the flight time of the attack helicopters is negligible compared to the movement time of ground reserves. Therefore, in turn, h_{CA} is a function of R_{LI} and t^*_{CA} .

To finally solve for the red flank strength, we first express t^*_{CA} as a function of R_{LI} by using the relationship between blue's rate of advance, which is a function of blue's casualty rate and thus R_{LI} , the distance between red's flank and his last defended line D_{CA} , and t^*_{CA} :

$$t^*_{CA} = D_{CA} \psi_{ROACA}^{-1} \quad (18)$$

where ψ_{ROACA} is blue's rate of advance, analogous to red's theater rate of advance (Equation 3).

We then use Equation 18 to substitute for t^*_{CA} in Equations 15 and 17, and to calculate the values of $R_{OM}(t^*_{CA})$ and $B_{CA}(t^*_{CA})$ in Equation 16. Substituting for h_{CA} in Equation 16 and solving for R_{LI} yields the strength of each predeployed red line from which the total red flank density and the red loss rate due to diversion of forces to flank defense (Equation 14) are calculated.

Given the preceding derivations of the red loss rates due to ground casualties and diversion of forces for flank defense, and the derivation of the loss rate due to air attack and ACM (Sections III and IV), we can now solve for the time of culmination of the red offensive t^* :

$$t^* = \frac{-Q_{203} + Q_{202} Q_{205} - R_{OFV}}{(Q_{204} + Q_{205}) \psi_{BOMT} - \psi_{ROFVT}} \quad (19)$$

where:

$$Q_{200} = \phi_{INFB} B_{LI} + B_{RSVA} - \psi_{BOMT} \phi_{INFB} (t_{BST} + t_{BPREP})$$

$$Q_{202} = B_{LI} + B_{RSVA} - \psi_{BOMT} (t_{BST} + t_{BPREP})$$

$$Q_{203} = \frac{Q_{200}}{2\rho_{\text{MAX}}} (14.32 \beta V - 11.80)$$

$$Q_{204} = \frac{\phi_{\text{INFB}}}{2\rho_{\text{MAX}}} (14.32 \beta V - 11.80)$$

$$Q_{205} = \frac{1}{2\rho_{\text{MAX}}} \left[\frac{14.32\beta V\phi_{\text{INFR}} + 11.80(2 - \phi_{\text{INFR}}) + 0.22\hat{R}_{\text{ARTECH}}}{1.45\hat{H}_B V + \frac{0.22\hat{R}_{\text{ARTECH}}}{(0.01 + V)} + \frac{693.6}{\hat{R}_{\text{ARTECH}}(1 + V)} + \mu_B \right]$$

ϕ_{INFB} = the fraction of blue maneuver AFVEs that are infantry

ϕ_{INFR} = the fraction of red maneuver AFVEs that are infantry

The total net territorial gain of the red offensive is then given by:

$$G = \psi_{\text{ROA}}(t^*) \quad (20)$$

III. TACTICAL AIRCRAFT AND AIR TRANSPORTS

Tactical aircraft have the qualities of great speed and range that allow them to strike targets out of reach of other weapons on the battlefield and to concentrate firepower more quickly and at more distant locations than other weapons. Air defense units and counterair operations are intended specifically to reduce the effectiveness of enemy tactical air power. The effects of tactical

and air defense were handled in an abstract and highly aggregate manner in the previous versions of VFM.³⁴ In this work, tactical air defense were modeled in greater detail to reflect the actual numbers and effectiveness of specific systems present on both sides. In addition to improving the treatment of tactical air, this work introduces intratheater air transportation to the VFM code. Air transports have the potential to be used to transport ground forces within the theater of operations in response to enemy action, as well as to transport them to the theater.³⁵ This effort allows VFM to be used to estimate the impact of intratheater air transportation upon the outcome of a modern conventional battle.

Tactical air operations, as modeled in VFM, are separated into five general categories: offensive counterair, defensive counterair, air interdiction, close air support (CAS) and suppression of enemy air defense (SEAD). Offensive counterair consists of attacks upon enemy airbases to destroy his capability to conduct air operations,³⁶ and is modeled explicitly in VFM. Defensive counterair consists of the interception of enemy air missions by friendly fighter aircraft, and is addressed as part of the air loss calculations associated with each type of ground attack mission. Air interdiction is generally defined to include any ground attack missions flown against targets not in contact with friendly forces.³⁷ Close air support consists of those missions flown against enemy targets in contact with friendly forces in order to provide firepower in support of the ground battle.³⁸ Both air interdiction and CAS are modeled explicitly in VFM. Air defense suppresses enemy ground based air defenses. In VFM, SEAD is handled indirectly in missions with the intent of reducing friendly losses to enemy ground based air defenses. Other missions that may be flown by tactical aircraft, such as reconnaissance or electronic warfare, are not modeled explicitly in VFM, but are considered implicitly in the derivation of the mathematical relationships governing the interaction of air and ground forces.

A. Close Air Support

The effect of defensive CAS is expressed in terms of the number of attacking AFVEs killed. In the case of the red theater attacker, an average kill rate (AFVEs/hr) is added to the other red loss rates due to ground combat, air interdiction and diversion of forces to flank defense to produce a total red loss rate. In the case of the blue counterattacker, the number of AFVEs killed per

red flank line taken is included, along with blue casualties due to ground fire, in the determination of red's flank defense requirements.

Offensive CAS, flown either by the red theater attacker or the blue counterattacker, is handled differently. Because of the difficulty in acquiring stationary defending targets from the air, offensive CAS missions serve to add area firepower at the point of attack, rather than to kill individual defending vehicles.³⁹ In VFM, the effect of offensive CAS missions is expressed in terms of artillery tube equivalents, derived from the delivery rate of high explosive carried by the aircraft, and is added to the number of artillery tubes supporting either the red theater attacker or the blue counterattacker at the point of attack.

The calculation of CAS effectiveness for both red and blue begins with the allocation of aircraft to the CAS mission and the allocation of CAS aircraft either to the point of theater attack or the point of counterattack. The total number of CAS aircraft available to each side at the beginning of the red offensive is equal to the number of dedicated CAS aircraft on each side (e.g. A-10s and Su-25s) plus a predetermined fraction of the multi-role "swing" aircraft on each side (e.g. F-16s and MiG-27s) (the fractions may be different for each side). A predetermined fraction of the red and blue CAS aircraft are allocated separately to the point of the theater attack, with the remainder allocated to the point of counterattack (again the fractions may be different for each side). After the aircraft on both sides have been allocated, the total average CAS sortie rates over the length of the red offensive, for each side, at both the point of theater attack and the point of counterattack, are calculated from the aircraft loss rates (Section III.A.2). Once the average CAS sortie rates are known, CAS effectiveness is calculated.

1. Air-Ground Effects

The average number of red AFVEs killed per hour by blue defensive CAS at the point of the theater attack is equal to the product of number of red AFVEs killed per blue line taken and the rate at which red takes blue lines:

$$\Psi_{ROFVTC} = \frac{\mu_{BS}}{D_L} \Psi_{ROA} \quad (21)$$

where μ_{BS} is the number of red attacking AFVEs killed per blue line taken. The quantity μ_{BS} is the product of the rate at which blue CAS aircraft can kill red AFVEs and the time during the taking of the blue line that the red AFVEs are moving and exposed to air attack. We started the assumption earlier that attacking vehicles were vulnerable to defensive CAS only when they were moving and thus visible from the air. The time required for an attacker to take a defender's line (the denominator in Equation 3, for red taking a blue line) includes time during which the attacker is stationary and is preparing to launch subsequent echelons at the line. The time during the taking of a blue defensive line that red attacking AFVEs are moving and vulnerable to defensive CAS is equal to the time required for the attacking echelons to advance from the previously taken defensive line to the new defensive line D_L/V_A , plus the sum of the times it takes each assault echelon to move from the point behind the FLOT that it begins to come under attack from CAS to the previously taken enemy line:

$$t_{EC} = \frac{D_L}{V_A} + \frac{n_{ECH} D_{ASY}}{V_{RSV}} \quad (22)$$

where:

V_A = the assault velocity of the attacker (km/hr).

To define the point at which attacking echelons begin to come under attack from CAS, one must answer the question of where CAS ends and where air interdiction begins. We have defined CAS to include all ground attack missions flown against an enemy echelon moving to contact a friendly force, and air interdiction to include all missions deeper than that. At the point at which one attacking echelon has just been exhausted in ground combat against a defending line, and the next attacking echelon is moving to contact, that next enemy echelon is separated from the defending line by exactly one echelon separation distance; thus, that is the depth of the area under which an attacking echelon will come under attack by CAS.

Having defined the time of exposure, the number of attacking AFVEs killed per defending line taken is the product of the exposure time, the total defensive CAS sortie rate S_{CAS} and the number of attacking AFVEs killed per defensive CAS sortie K_{CAS} :

$$\mu_{BS} = t_{EC} S_{CAS} K_{CAS} \quad (23)$$

The blue CAS sortie rate is calculated from the total number of aircraft available and the losses those aircraft incur, as stated above, except when the calculated blue sortie rate would place more CAS aircraft over the FEBA at the point of the red attack than could effectively be controlled by airspace management. Due to the lethality of ground-based air defenses, CAS missions must be orchestrated so that the aircraft spend a minimum amount of time over the enemy defenses. Because of the ground attack procedures employed to obtain the maximum effectiveness of an attack (to avoid multiple "kills" of the same target), CAS aircraft must be carefully distributed across the area being attacked. Those two factors place a limit on the maximum number of CAS aircraft that may operate over an area at once.⁴⁰ That airspace management constraint is accounted for by placing a cap on the total sortie rate which can be flown over the FEBA. The cap is given by:

$$S_{MAX} = \frac{\rho_{AC} A_{FEBA}}{t_{SD}} \quad (24)$$

where:

ρ_{AC} = the maximum aircraft density over the FEBA (aircraft/km²)
 A_{FEBA} = the area over which CAS aircraft fly (km²)
 t_{SD} = the time spent over the FEBA per defensive CAS mission (hrs).

The area over which CAS aircraft fly is taken in VFM to be the product of the width of the attack or counterattack frontage and the depth to which CAS aircraft fly: the separation distance between attacking echelons. Since the effects of CAS are applied across the attack frontage, the width of the area overflowed by CAS aircraft is clearly equal to the frontage of the attack.

In the case of offensive CAS missions, the time spent over the FEBA is taken to be lower than in the case of defensive CAS, and thus the cap on the total CAS sortie rate is higher. Offensive CAS aircraft are assumed to be providing area fire support by making a single pass over the FEBA and dropping munitions with an area of effect like cluster bombs, napalm or high explosive. They are also assumed not to employ "shoot-look-shoot" tactics to maximize the effectiveness of such munitions

among the aircraft in a single wave, or even between waves. Those two factors serve to increase the number of aircraft that can fly over the FEBA at once. That effect is modeled through the reduction of the time per mission spent over the FEBA for offensive CAS sorties.

The sortie cap on defensive CAS employing PGMs poses a choice for the defending air force. Although PGMs are more effective on a per sortie basis than unguided area munitions, if the defending air force has more aircraft available than it can fly because of the sortie cap on defensive CAS, a way to use more or all of the available aircraft would be to switch from PGMs to area munitions and take advantage of the higher cap on area munition sorties. Switching to area munitions would be the optimal choice if the attacker's casualties expected from the greater number of sorties employing area munitions were greater than the attacker's casualties expected from the lesser number of sorties employing PGMs. In VFM, the defending air force, be it blue versus the red theater attack, or red versus the blue counterattack, is allowed to switch to area munitions when the kill rate expected from sorties flown with area munitions is greater than the kill rate expected from sorties flown with PGMs. The number of artillery tube equivalents per two kilometers of attack frontage produced by flying CAS with area munitions is given by:

$$A_{ECD} = SCASF P_{AC} T_{TEAM} \frac{2}{\lambda_{ATK}} \quad (25)$$

where:

$SCASF$ = the total CAS sortie rate, limited by the number of available aircraft or the sortie cap
 P_{AC} = the payload carried by a CAS aircraft (metric tons)
 T_{TEAM} = the number of artillery tubes equivalent to the delivery of one tonne of munitions per hour.

The casualties incurred by an attacking force per two kilometers of attack frontage in the taking of a single defended line due to those artillery tube equivalents of defensive CAS may be drawn from the blue artillery term in Equation 1:

$$\chi = \frac{0.22 A_{ECD}}{0.01 + v} \frac{B_{LI}}{R_{ECH}} \quad (26)$$

If the number of casualties per defending line taken caused by area munitions χ in Equation 26 is greater than the casualties caused by PGMs μ_{BS} in Equation 23 (in which the sortie rate is at the sortie cap for PGM missions), the defending air force is assumed to fly the area munition mission.

In addition to killing attacking red AFVEs, blue defensive CAS affects the ground battle by changing the force to force ratio at the point of contact between the red assault echelons and the blue defending line. Since defensive CAS is flown to some depth behind the FLOT, some red AFVEs in the assault echelon would be killed before they came within range of the blue line. Those red casualties would serve to reduce the force to force ratio at the point of contact. This effect is modeled by subtracting the casualties inflicted by CAS per red assault echelon per blue line taken μ_{BS}/n_{ECH} from the strength of the red assault echelon, for the purpose of calculating the red casualties incurred due to ground combat. The term R_{ECH} in Equation 1 is replaced by $R_{ECH} - \mu_{BS}/n_{ECH}$ and the integration over the life of the echelon is carried out as in the previous version of VFM.⁴¹ It should be noted that if one assault echelon is able to take more than one defensive line, then the casualties inflicted by CAS per assault echelon per line taken is equal to μ_{BS} .

Now we turn to offensive CAS flown by either the red theater attacker or the blue counterattacker. Offensive CAS missions are always assumed to be flown with area munitions, so the effect of offensive CAS is to add artillery tube equivalents to either the red theater attack or the blue counterattack. The number of tubes added is calculated in exactly the same manner as for defensive CAS that has switched to area munitions. The final number of tubes is given in Equation 25. The extra tubes are added into the artillery term for the attacker's artillery in Equation 1.

2. Aircraft Losses

Losses of CAS aircraft are calculated as a function of the number of engagements over the campaign in which CAS aircraft are used, and the number and effectiveness of the air defense units CAS aircraft face in each engagement. Losses are determined separately for each red and blue CAS mission, i.e. for red CAS in the attack and counterattack sectors and for blue CAS in the attack and counterattack sectors. Once the total number of CAS aircraft lost over the course of the offensive is known, the average sortie rate can be calculated. We assume that all air missions are flown to maintain a constant level of air

support over the length of the red offensive equal to the average. The effectiveness of the CAS mission over the course of the offensive is directly related to the average sortie rate, as described in the previous section. The total number of CAS aircraft lost by a side over the course of the offensive while flying a particular mission is taken to be:

$$L_{CAS} = n' L_{TOT} M_{CLI} L_{CASM} \quad (27)$$

where:

$n' L_{TOT}$ = the estimated number of lines red will take before the culmination of his offensive

M_{CLI} = the number of CAS missions flown per line taken

L_{CASM} = the number of CAS aircraft lost per mission

The estimated number of lines $n' L_{TOT}$ is calculated from the estimated time of culmination t^* and the red rate of advance ψ'_{ROA} calculated in the iteration of the code just previous to the current one:

$$n' L_{TOT} = \psi'_{ROA} \frac{t^*}{D_L} \quad (28)$$

It is necessary to estimate the time of culmination of the red offensive in order to calculate the losses of aircraft, and from that the average effectiveness of all tactical aircraft. Since tacair effectiveness is used to determine the final outcome of the red offensive (i.e. the net territorial gain), we need to know the effects of tacair before we can calculate the outcome of the offensive. Ordinarily, one would use the real time of culmination t^* in the expressions determining tacair losses, add those equations to the existing system of equations governing ground losses, and solve the new system for t^* analytically. Unfortunately, the nature of the existing system of equations and the tacair loss equations is such that the former approach yields a new system of equations that cannot be solved analytically. Therefore we must turn to a numerical approach.

The numerical approach taken here is to use an estimated time of culmination $t^{*'} to calculate quantities, primarily tacair and ACM effectiveness values, needed to determine the outcome of the offensive, that are themselves functions of the real time of culmination of the offensive t^* . After those quantities are determined, the rest of the calculations are carried out, ending in the$

real time of culmination t^* and the real net territorial gain of the offensive. The estimated time $t^{*'}_1$ is then compared to the real time t^* . If the relative difference between them ($(t^{*'}_1 - t^*)/t^*$) is small (as defined by the input to the VFM code), the estimate is accepted and the code proceeds on to its next calculation. If the relative difference is not small, a new estimate is made, the calculations are performed and the new estimate is compared to t^* . That process is repeated until an acceptable estimate is found. In that manner, while we do not solve for a self-consistent t^* analytically, the estimated time is chosen to be close to the real time and the calculated effectiveness of tacair and ACM is close to the real effectiveness.

The initial value of the estimated time $t^{*'}_1$ is calculated from the total depth of the blue defense D and an estimated rate of advance of the red offensive:

$$t^{*'}_1 = D/\psi'_{ROA} \quad (29)$$

where ψ'_{ROA} is calculated as ψ_{ROA} but without taking into account the effects of tacair and ACM.

Returning to Equation 28, the number of CAS missions flown per line taken M_{CLJ} is equal, in the case of defensive CAS, to the time of exposure of the attacking echelons t_{EC} (Equation 22) divided by the time spent over the FEBA in a single defensive CAS mission t_{SD} :

$$M_{CLID} = \frac{t_{EC}}{t_{SD}} \quad (30)$$

In the case of offensive CAS, M_{CLJ} is equal to the time spent by the attacker in combat divided by the time spent over the FEBA in a single offensive CAS mission:

$$M_{CLIO} = \frac{D_L}{v t_{SO}} \quad (31)$$

The number of CAS aircraft lost per mission L_{CASM} is calculated as the product of the fraction of CAS aircraft lost and the number of CAS aircraft flying each CAS mission. We assume that tacair missions are broken down into a number of small

flights of aircraft which undergo attack by air defense units (ADUs) independently. We assume that each ADU that could potentially shoot at the flight (i.e., that comes within range) has a probability of detection of each aircraft in the flight d and a probability of killing a single aircraft given a detection k . This is equivalent to the Lanchester linear law in that the probability of making a detection is proportional to the number of targets.⁴² The fraction of CAS aircraft lost per mission is then given by:

$$L_{CM} = 1 - (1 - F_{LCP})^{(2p)} \quad (32)$$

where p is the number of passes over the FEBA made by the aircraft (the factor of two accounts for both legs of the trip) and F_{LCP} is the fraction of CAS aircraft lost per pass:

$$F_{LCP} = 1 - (1 - k/N_F (1 - (1-d)^{N_F S_{ADC}})) \quad (33)$$

where N_F is the number of aircraft in the flight and S_{ADC} is the number of ADUs to which the flight is exposed. We assume that all of the aircraft in a flight occupy a single point along the front, so the number of ADUs to which a flight would be visible would be equal to the number of ADUs that are within range to either side of the point. We further assume that ADUs are distributed evenly across the front. Thus the number S_{ADC} is equal in the case of blue defenders to the product of the linear density of ADUs and twice the range of an ADU:

$$S_{ADC} = \frac{\phi_{FWD} B \rho_{SB}}{\lambda_{THR} n_{LI}} 2r_S \phi_{SO} \quad (34)$$

where:

- B = the total number of blue AFVEs in the theater
- r_S = the range of a SHORAD (km)
- ϕ_{SO} = the fraction of SHORADs of a given side that are operational at any one moment

ρ_{SB} = the ratio of SHORADs to ground AFVEs.

In the case of the red theater attacker, the number of ADUs to which each blue CAS flight is exposed S_{ADC} is equal to the product of the linear density of SHORADs in the attacking echelon and twice the range of the SHORAD:

$$S_{ADC} = \rho_{MAX} \rho_{SR} 2r_S \phi_{SO} \quad (35)$$

The number of CAS aircraft lost per mission flown is then equal to the product of the fraction lost L_{CM} and the number of aircraft that actually fly over the FEBA during the mission. We make the approximation that the number of aircraft that fly over the FEBA during a mission is equal to the maximum number allowable, due to the airspace management constraints described in the section above. Experience with VFM has shown that approximation to be equal to the actual value in all cases but those in which aircraft inventories are quite low (<250 aircraft flying CAS missions). Thus the number of CAS aircraft lost per mission L_{CASM} is given by:

$$L_{CASM} = L_{CM} \rho_{AC} A_{FEBA} \quad (36)$$

Having completed the derivation of the expression for the total number of CAS aircraft lost over the course of the campaign, we must convert that to an average sortie rate. If we assume that CAS losses occur at a constant rate, then the average sortie rate is simply equal to the product of the individual aircraft sortie rate S_{AC} and the average of the initial N_{CAS} and final numbers of CAS aircraft:

$$S_{CAS} = S_{AC} (N_{CAS} - 0.5L_{CAS}) \quad (37)$$

If the total CAS losses are such that they would cause the number of CAS aircraft present at the end of the campaign to be negative, the side in question actually runs out of CAS aircraft sometime during the offensive, and the average sortie rate is given by:

$$SCAS = \frac{SACNCAS}{2L_{CAS}} \quad (38)$$

B. Air Interdiction

Air interdiction includes attacks against enemy forces moving in the rear, command and control targets, logistical targets and infrastructure such as bridges and railroads.⁴³ In VFM, air interdiction is assumed to cause casualties against and delay the movement of red follow-on echelons, blue reserves moving to counterattack or to counterconcentrate, and red reserves (if present) moving to defend against the counterattack. Casualties caused by air interdiction will serve to delay the movement of units in addition to reducing their strengths. Units in the rear that come under air attack generally will have been moving in columns along roads. In the course of taking casualties, they will tend to become disorganized as they seek cover.⁴⁴ Disabled vehicles will have to be cleared from the road or negotiated by the remainder of the unit. If a unit takes sufficient casualties it may have to stop and wait for replacements before being committed to battle. All of these factors add up to impose some delay upon the movement of units in the rear as a function of the casualties they take from air attack.

Interdiction missions against command and control, logistical and infrastructure targets were not modeled. We did not address command and control targeting because there exists no body of theory from which to derive the appropriate mathematical relations for the models. Developing such a body of theory was beyond the scope of this effort. The effects of command and control losses are accounted for to a certain degree by the delay imposed upon units moving in the rear -- the loss of command and control nodes due to air attack could make it more difficult or time-consuming to reorganize and restart a unit after it comes under attack.

Logistical targeting was not modeled because of the time delay between the execution of the attacks and their effect being felt in combat.⁴⁵ Historically, interdiction campaigns flown by the Allies against German logistical targets in Italy (Operation STRANGLE, 1944) and Normandy were not felt at the front until approximately four to six weeks after they were carried out. It took that long for the increase in demand for supplies at the front caused by ground fighting to place sufficient stress on the logistical net and for its shortcomings to be felt by the defending Germans.⁴⁶ Warsaw Pact regiments typically carry five days of supplies with them into combat, further delaying the impact of logistical targeting.⁴⁷ Previous experience with VFM has shown

that the time of culmination of a single red offensive rarely exceeds 100 hours. The effects of such a logistical targeting campaign would not be felt before the offensive either achieved a breakthrough or came to a halt.

Infrastructure targeting also was not modeled explicitly for a number of reasons. Europe is a rather heavily built up area, with a very extensive road and rail network. If NATO aircraft were to interdict some roads or rail lines, Warsaw Pact echelons could still, after some delay, choose some of the many alternative routes available. Therefore we do not explicitly model targeting against road and rail networks. Even if targeting is assumed to be directed against bridges over the major rivers a Pact offensive would have to cross, enough bridges exist that the effect of the campaign would be merely to delay the crossing of some of the units involved rather than to bring the offensive to a halt. In that regard, from the point of view of the VFM model, the results would not be significantly different than those from a campaign directed against the units themselves, causing casualties and imposing delay.

1. Air-Ground Effects

The modeling of air interdiction in VFM begins with the allocation of aircraft. The number of aircraft available to fly air interdiction mission on either side is equal to the fraction of long-range strike aircraft (e.g. F-111s and Su-24s) devoted to the interdiction mission determined before the beginning of the offensive (the remainder of the long-range strike aircraft are devoted to offensive counterair). All red interdiction aircraft fly missions against blue units moving either to counterattack or to counterconcentrate. Blue interdiction aircraft are divided between the interdiction of the movement of red follow-on echelons and the movement of red reserves responding to the blue counterattack. The aircraft are split proportionally to the number of red AFVEs in the follow-on echelons and the estimated number of red AFVEs in the reserves:

$$I_{BC} = \frac{I_0 R'_{RSV}}{R_{OFVST} + R'_{RSV}} \quad (39)$$

$$I = I_0 - I_{BC} \quad (40)$$

where:

I_O = the number of strike aircraft in the theater
 I_{BC} = the number of aircraft intercepting reserves
 I = the number of aircraft intercepting follow-on echelons
 R'_{RSV} = the estimated strength of the red reserves (AFVEs)
 R_{OFVST} = the strength of the red attacking force (AFVEs).

The estimated strength of the red reserves is calculated as:

$$R'_{RSV} = \psi_{ROM} (t_{CA}^{*'} - t_{RST} - t_{RPREP}) \quad (41)$$

where ψ_{ROM} is the arrival rate red reserves and $t_{CA}^{*'}$ is the same estimated time of culmination of the counterattack used by blue to determine the jump-off time for his counterattack.⁴⁸ The estimated time of culmination of the counterattack is adjusted as the code proceeds through the iterative process that matches the estimated time of culmination $t^{*'}$ with the actual time of culmination t^* . The iterative process and the use of $t^{*'}$ is covered in the section on air losses, III.A.2.

Once aircraft have been allocated to their respective missions, an average total sortie rate is calculated for each mission over the length of the campaign as a function of the losses suffered by each mission. This calculation is described in the next section.

The effectiveness of red interdiction attacks against blue reserves is assessed in terms of the rate at which those reserves are killed and the degree to which the movement rate of the survivors is reduced through delay. The kill rate produced by red interdiction is simply the product of the total average sortie rate and the number of blue AFVEs killed per red interdiction sortie:

$$\delta_4 = k_{INT} S_{INT} \quad (42)$$

The delay imposed upon blue units that survive the air attacks is expressed in terms of t_D , the AFVE-hours of delay imposed per hour by red air interdiction, where one AFVE-hour of delay imposes a delay of one hour on one AFVE. The rate at

which red imposes delay upon blue is merely the product of the red interdiction kill rate and the delay in AFVE-hours caused per casualty, due to interdiction. Since blue reserves are assumed to be randomly distributed across the width of the theater λ_{THR} and to travel at the same average velocity $\bar{v}$, their arrival rate is given by:

$$\psi_{BOMT} = \frac{B_{RSV} \bar{v}}{\lambda_{THR}} \quad (43)$$

The average velocity $\bar{v}$ is equal to the average distance traveled $\frac{\lambda_{THR}}{2}$ divided by the average time it takes to travel. The time it takes the average reserve AFVE to cover the average distance is equal to the time in the absence of interdiction $\frac{\lambda_{THR}}{v_{RSV}}$ plus the delay imposed by interdiction upon the average AFVE:

$$\bar{v} = \frac{\lambda_{THR}}{2v_{RSV}} + \frac{t_D}{B_{RSV}} \quad (44)$$

where:

t_D = the total amount of delay in AFVE-hrs imposed upon blue reserves
 B_{RSV} = the total number of blue AFVEs in reserve.

Since the average distance traveled is equal to $\frac{\lambda_{THR}}{2}$, the average velocity $\bar{v}$ becomes:

$$\bar{v} = \frac{\lambda_{THR}}{\frac{\lambda_{THR}}{v_{RSV}} + \frac{2t_D}{B_{RSV}}} \quad (45)$$

and, taking the reserve loss rate caused by red interdiction into account, the arrival rate becomes:

$$\psi_{BOMT} = \frac{\frac{B_{RSV} V_{RSV}}{\lambda_{THR}} - \delta_4}{1 + \frac{2 V_{RSV} t_D}{\lambda_{THR} B_{RSV}}} \quad (46)$$

where the numerator in Equation 46 is the undelayed arrival rate of AFVEs that survive air attack and the denominator may be thought of as a delay factor. The term t_D in the denominator poses a problem in that, in order to know the total amount of delay imposed upon blue reserves, one must know the length of the war. We do not know the length of the war before the calculation is complete, but if we examine the delay factor, we see that the quantity $\frac{\lambda_{THR}}{V_{RSV}}$ is equal to the time it takes for all blue reserves to arrive in the absence of interdiction. The total amount of delay imposed, t_D , divided by the undelayed time is equal to t_{DR} , the rate at which red interdiction imposes delay upon the blue reserves. The factor of two in the expression arises from the fact that the average number of blue AFVEs that are targets for delay at any given point in the campaign is equal to half the total starting number of blue reserve AFVEs (the final number of blue reserve AFVEs is taken to be zero, so the average is merely half the initial number). Thus the final expression for the arrival rate of blue reserve AFVEs is given by:

$$\psi_{BOMT} = \frac{\frac{B_{RSV} V_{RSV}}{\lambda_{THR}} - \delta_4}{1 + \frac{2 t_{DR}}{B_{RSV}}} \quad (47)$$

Blue reserves that arrive are divided between counterattack and counterconcentration according to the fraction of counterattack being chosen by blue for that particular iteration of the model.

Blue interdiction of red follow-on echelons is assessed in terms of the number of red AFVEs killed in the time it takes red to take a blue line, and the extra time required to take the line due to delay. The number of red AFVEs killed per blue line taken is equal to the product of the rate at which blue interdiction aircraft kill red vehicles and the time t_{EXP} during which the red follow-on echelons come under air attack in the taking of the blue line:

$$\Psi_{RLII} = S_{INT} K_{INT} t_{EXP}$$

(48)

We assume that interdiction attacks are directed against vehicles moving in columns along roads. We assume, as we did with CAS, that vehicles that are stationary are under cover and may not be attacked. We further assume, in the case of air interdiction, that units that have deployed for battle, namely red assault echelons moving to contact a blue line, are spread out to a degree that they are no longer optimal targets for interdiction missions.⁴⁹ Therefore, the time at which red follow-on echelons are exposed to attack by air interdiction is the time immediately after a blue line is taken, during which the echelons that did not participate in the taking of the old line stage forward to the jumping off point for the assault on the new line.

In order for the attacks on the follow-on echelons to be carried out, blue must realize that they are staging forward and vulnerable to attack, he must plan the strikes in detail to minimize losses to air defenses, and the aircraft must fly to and attack the target. All of these actions impose a time lag t_{REAC} between the point at which the red follow-on echelons begin to stage forward and the time at which they come under attack, thus creating a window of vulnerability in which the red echelons may suffer air attack. If the distance the echelons stage forward was short enough, it is possible that blue would not have time to carry out any attacks upon them; their window of vulnerability would be closed.

This window of vulnerability poses an interesting choice for red. He may want to take advantage of the time lag by stopping and starting his echelons as they stage forward to their next assault positions. This would reduce his vulnerability to air interdiction by forcing blue to reacquire the echelons, and replan the strikes against them, possibly to the point that they would not come under attack at all. However, it also would slow down the red advance, as red would have to take the time to go from road-bound columns to intermediate assembly areas and back to road-bound columns every time he stopped. This time required to change formations is currently assessed when calculating the red rate of advance (Equation 3) as t_{OPREP} , the time it takes to prepare a red echelon for an assault. If red decided to stop and start while staging his echelons forward, t_{OPREP} would be added to the total time required to take a blue line for every time he stopped and started. The expression for the time t_{EXP} that the window of vulnerability is open is thus:

$$t_{EXP} = \frac{D_L + n_{ECH} D_{ASY}}{v_{RSV}} - n_J t_{REAC} \quad (49)$$

where n_J is the number of "jumps" or stops and starts red makes while staging forward.

As stated earlier, blue air interdiction also serves to delay red and increase the time it takes for him to take a blue line. The amount of delay (in AFVE-hours) is the product of the number of red AFVEs killed and the delay imposed per AFVE killed. The amount of extra time it takes red to take a blue line is equal to the amount of delay in AFVE-hours t_{DECH} divided by the average number of red AFVEs in the attacking force R_{OFVST} plus any delay imposed by red's "jumping" to avoid air attack, yielding a new expression for the red rate of advance, replacing Equation 3:

$$\psi_{ROA} = \frac{D_L}{\frac{D_L}{v} + n_{ECH} \left(n_J t_{OPREP} + \frac{n_{ECH} D_{ASY}}{2v_{RSV}} \right) + \frac{2t_{DECH}}{R_{OFVST}}} \quad (50)$$

In addition to flying air interdiction against red follow-on echelons, blue also flies against red reserves moving to defend against a counterattack. As was the case with the attacks upon the follow-on echelons, interdiction serves to kill red AFVEs in the reserve and delay the movement of the survivors. The net effect upon the reserve arrival rate is calculated in exactly the same manner as the effect of red interdiction upon the blue reserve arrival rate is calculated:

$$\psi_{ROMO} = \frac{\frac{\psi_{ROM} - S_{INT} K_{INT}}{t}}{1 + \frac{2t_{DR}}{R_{RSV}}} \quad (51)$$

where R_{RSV} , the size of the red reserve, is calculated by using the estimated time of culmination of the counterattack as given in Equation 29, and ψ_{ROMO} is the red reserve arrival rate in the absence of blue air interdiction.

2. Air Losses

The calculation of aircraft losses in the case of air interdiction (and offensive counterair) missions is made more complex by the fact that aircraft flying those missions take losses not just from point defenses at the target like CAS aircraft, but also from the SAM belt between the FEBA and the target and from enemy fighters flying defensive counterair missions. Furthermore, those enemy fighters also take losses from friendly attack aircraft and escorts. Lastly, aircraft losses are generally functions of time due to the attrition that occurs over the course of the campaign.

We assume that interdiction aircraft suffer losses from two sources: air defense and enemy defensive counterair missions. We assume that loss rates due to air defense are proportional to the total rate at which air interdiction missions are flown and that the fractional losses suffered in each mission are equal and constant over time. We assume that losses due to enemy defensive counterair missions are proportional to the rate at which individual aircraft sorties are flown and to the number of enemy fighters flying defensive counterair. The assumption of proportionality to the individual sortie rate may be justified if we assume that interdiction missions will be flown across the FEBA in a few large strikes to minimize exposure to air defense at the FEBA and maximize the effectiveness of friendly SEAD.⁵⁰ The rate at which the strikes are flown is the rate at which friendly interdiction aircraft are exposed to enemy fighters. The individual aircraft sortie rate, rather than the total sortie rate, will govern the rate at which the large strikes will be flown, as the presence of more aircraft (which would increase the total sortie rate, but not the individual) would serve to increase the size of the strikes, rather than their frequency. An increase of the individual aircraft sortie rate would allow the same large strikes to be flown at a higher rate.

The proportionality of air interdiction losses to the number of enemy fighters dedicated to defensive counterair missions follows if one assumes that detection of the penetrating friendly aircraft is automatic and that the sortie rates of enemy fighters are high enough that the rate at which engagements between friendly strikes and enemy fighters occur is limited only by the rate at which friendly strikes are flown.

The preceding assumptions yield the following expression for the loss rate of interdiction aircraft:

$$\frac{dI}{dt} = L_{IAD} S_{AC} I - K_{IF} S_{AC} F \quad (52)$$

where:

- I = the number of friendly aircraft flying interdiction missions
- L_{IAD} = the loss rate suffered per sortie by interdiction aircraft due to air defense (hr^{-1})
- K_{IF} = the number of friendly interdiction aircraft killed per enemy fighter per engagement
- F = the number of enemy fighters dedicated to defensive counterair missions against interdiction.

In order to solve Equation 52 we need a relation governing the loss rate of enemy fighters. We assume that friendly attack aircraft and escorts inflict losses upon enemy interceptors in the same manner interceptors inflict losses upon attack aircraft:

$$\frac{dF}{dt} = K_{FI} S_{AC} I \quad (53)$$

where K_{FI} is the number of interceptors killed per attack aircraft per engagement. Friendly escorts serve to increase losses inflicted upon enemy fighters by increasing K_{FI} :

$$K'_{FI} = K_{FI} \frac{I_O + E}{I_O} \quad (54)$$

where E is the total number of friendly fighters dedicated to escort missions and I_O is the total number of friendly strike aircraft in the theater.

The allocation of strike aircraft was described in the previous section. Fighter aircraft are allocated either to escort or interception missions before the beginning of the run of the code. Fighters allocated to interception missions are divided between enemy interdiction, counterattack interdiction (in the case of red only), and offensive counterair missions, proportionally to the number of enemy strike aircraft allocated to each of those missions. Fighters allocated to escort missions serve to increase the loss coefficient K_{FI} as shown in Equation 54.

Once the aircraft have been allocated, the differential equations (52) and (53) are solved to yield the number of friendly interdiction aircraft surviving in the theater as a function of time $I(t)$. Boundary conditions are the initial loss rates of friendly interdiction aircraft and enemy fighters. That solution is then integrated over the length of the offensive to yield an average number of interdiction aircraft $\bar{I}$:

$$\begin{aligned} \frac{1}{t^*} \int_0^{t^*} I(t) dt = & \frac{S_{AC}}{t^*} \left(\frac{C_3}{L_{IAD} (1 - \exp(-L_{IAD} t^*))} \right) \\ & + \frac{D_{LI}}{L_1 (\exp(L_1 t^*) - 1)} \\ & + \frac{D_2}{L_2 (\exp(L_2 t^*) - 1)} \end{aligned} \quad (55)$$

where:

$$L_1 = \frac{-L_{IAD} + (L_{IAD}^2 + 4K_{IF} K'_{FI} S_{AC}^2)^{\frac{1}{2}}}{2} \quad (56)$$

$$L_2 = \frac{-L_{IAD} - (L_{IAD}^2 + 4K_{IF} K'_{FI} S_{AC}^2)^{\frac{1}{2}}}{2} \quad (57)$$

$$D_1 = \frac{-K_{IF} S_{AC} C_1}{L_1 + L_{IAD}} \quad (58)$$

$$D_2 = \frac{-K_{IF} S_{AC} C_2}{L_2 + L_{IAD}} \quad (59)$$

$$C_1 = \frac{K'_{FI} S_{AC} I + L_2 F}{L_2 - L_1} \quad (60)$$

$$C_2 = F - C_1 \quad (61)$$

$$C_3 = I - D_1 - D_2 \quad (62)$$

Equation 55 is valid in the case that both friendly interdiction aircraft and enemy interceptors survive the campaign. If not, the solution to the differential equations must be broken down into two regions, one in which both strike aircraft and interceptors exist (in which Equation 55 is valid) and one in which either strike aircraft or fighters exist. If, in the second region, only strike aircraft exist, the interceptor term in Equation 52 is eliminated and Equation 52 is solved for $I(t)$, beginning with the number of strike aircraft surviving at the time the interceptors were exhausted. If only fighters exist, the number of strike aircraft is defined to be uniformly zero until the time of the culmination of the offensive t^* . In either case, $I(t)$ is integrated over time to yield an average number of friendly interdiction aircraft available $\bar{I}$.

After calculating the average number of aircraft available in the theater for interdiction (and counterattack interdiction, in the case of blue) missions, the average sortie rate for interdiction missions is simply:

$$S_{INT} = S_{AC} \bar{I} \quad (63)$$

Having solved the loss equations to determine the average interdiction sortie rate, we must now turn to the air defense loss coefficient L_{IAD} in Equation 52. The coefficient L_{IAD} represents the loss rate per sortie suffered by interdiction aircraft due to air defenses. Air defenses against interdiction (and offensive counterair) missions are modeled in VFM as comprising three parts:

the SHORADs at the FLOT, the SAM belt, and the point defense SHORADs at the target. Attacking aircraft pass through the defenses in series on the way in to the target and on the way back out. The fraction of aircraft lost over a mission is given by:

$$L_{IAD} = 1 - (1 - L_F)^2 (1 - L_B)^2 (1 - L_T) \quad (64)$$

where:

- L_F = the fraction of aircraft lost per pass across the FLOT
- L_B = the fraction of aircraft lost per pass through the SAM belt
- L_T = the fraction of aircraft lost at the target.

The fractions of aircraft lost per mission are calculated in a manner similar to that used to calculate the fraction of CAS aircraft lost per mission in Section III.A.2. Interdiction missions are assumed to pass through the FLOT in large strikes in order to minimize exposure to the large number of ADUs there. They are then assumed to split into small flights, such as CAS missions, and proceed through the SAM belt to the targets. The aircraft are assumed to return via the same path; thus the squared terms in Equation 64.

Equation 33 is used to calculate the fraction of aircraft lost per trip across the FLOT the same way the fraction of aircraft lost per pass was calculated for CAS, except that the number of aircraft in the flight and the number of ADUs to which the flight is exposed are different (probabilities of detection d and kill k are the same as those used in Equation 33):

$$L_F = 1 - (1 - k/N_{FS} (1 - (1 - d)^{N_{FS} S_{ADFR}}))) \quad (65)$$

We assume that interdiction aircraft fly across the FLOT in large strikes, and that the size of the strikes N_{FS} (replacing the quantity N_F in Equation 33) is equal to a predetermined fraction of the total number of interdiction aircraft in the theater. We also assume that the aircraft fly over the FLOT at points outside the attack sector, as that is where most of the ADUs organic to ground

units would be found. Furthermore, we assume that off-axis artillery units on both sides fire barrages in order to suppress the enemy ADUs that are present in the strike corridor.

The number of effective ADUs to which blue interdiction strikes are exposed S_{ADFR} (replacing the quantity S_{ADC} in Equation 33) is equal to the number of red ADUs organic to red ground units in the corridor, minus those that are suppressed by blue off-axis artillery or electronic warfare (EW) aircraft, and is calculated as follows:

$$S_{ADFR} = \left(\rho_{MIN} \rho_{SR}^{2r} - \sigma_B \right) \phi_{SO} - \epsilon_{FB} \quad (66)$$

where:

- ρ_{MIN} = the red off-axis force density (AFVEs/km)
- σ_B = the number of red ADUs in the corridor that are suppressed by blue off-axis artillery
- ϵ_{FB} = the number of unsuppressed, active ADUs that are suppressed by blue EW aircraft.

The number of red ADUs in the corridor that are suppressed by blue off-axis artillery is given by:

$$\sigma_B = K_{SA} \frac{40 + 2r \phi_{SFW} \rho_{ARTB}}{\lambda_{THR}} \min(2, n_L) \quad (67)$$

where:

- ρ_{ARTB} = the ratio of blue artillery to ground AFVEs in the theater
- K_{SA} = the number of ADUs suppressed by each tube of firing artillery.

To arrive at Equation 67, we assume that all of the off-axis blue artillery within range of the corridor fires a two hour barrage aimed at suppressing the red ADUs and that the firing batteries know the general locations of the ADUs in the corridor. The corridor was assumed to have a width equal to twice the range of a SHORAD. The distances from the sides of the corridor

which blue artillery (including rocket launchers) is eligible to fire was taken to be equal to the average range of an artillery piece (assumed to be 20 km). The depth from which the artillery is eligible to fire included all artillery pieces supporting the first two lines (approximately 10 km) of the blue theater defense. We calculated the number of ADUs suppressed by each artillery piece during the barrage K_{SA} from estimates of how well the locations of the ADUs would likely be known, the number of rounds each piece could fire during the barrage, the suppressive radius of each round fired, and the probability that any ADU within the radius would be suppressed.

The number of active red ADUs that are suppressed by blue EW aircraft was taken to be equal to the minimum of a predetermined fraction ϕ_{EW} of the blue aircraft in the strike or the number of active red radar-homing SAMs in the corridor:

$$\epsilon_{FB} = \min \left\{ \phi_{EW} N_{FS}, \phi_{RS} (\rho_{MIN} \rho_{SR}^{2r_S} - \sigma_B) \phi_{SO} \right\} \quad (68)$$

where ϕ_{RS} is the fraction of red SHORADs that are radar-homing SAMs. The fraction of aircraft ϕ_{EW} is assumed to be equal to the product of the ratio of EW aircraft to strike aircraft in the theater, the number of anti-radiation missiles (ARMs) each EW carries, and the cumulative lethal and non-lethal probability of suppression of a radar-homing SAM by an ARM.

If we now turn to the case of red aircraft penetrating the blue FLOT, the number of blue ADUs in the corridor eligible to shoot at the red aircraft is equal to the number of blue SHORADs organic to the blue ground units in the corridor, minus those that are suppressed by red off-axis artillery and red EW aircraft:

$$S_{ADFB} = \left(\frac{\phi_{FWD} B \rho_{SB}^{2r_S}}{\lambda_{THR}} - \sigma_R \right) \phi_{SO} - \epsilon_{FR} \quad (69)$$

where:

σ_R = the number of blue ADUs in the corridor that are suppressed by red off-axis artillery

ϵ_{FR} = the number of unsuppressed, active ADUs that are suppressed by red EW aircraft.

The number of blue ADUs in the corridor that are suppressed by red off-axis artillery σ_R is calculated in the same manner as was σ_B :

$$\sigma_R = K_{SA} (40 + 2r_s) \rho_{MIN} \rho_{ARTR} \quad (70)$$

where ρ_{ARTR} is the ratio of red artillery to ground AFVEs in the theater.

The number of active blue ADUs in the corridor that are suppressed by red EW aircraft ϵ_{FR} is calculated the same as ϵ_{FB} for blue.

Having derived the expression for the fraction of aircraft lost in the penetration of the FLOT, we now address the penetration of the SAM belt. We assume that the SAMs on both sides are distributed randomly in a belt across the width of the theater. Interdiction and offensive counterair targets are assumed to lie at the far end of the belts. Interdiction missions are assumed to be flown in straight lines from the FLOT to the target and back out. The width of the penetration corridor is taken to be equal to twice the effective range of a SAM; effective range is used in place of actual range as aircraft are assumed to fly low enough that terrain masking limits the lines of sight from the SAMs to the aircraft. While the corridors to the targets are straight, aircraft are assumed to be able to maneuver within the corridor to avoid some of the SAMs.

Aircraft losses incurred while penetrating the SAM belt are calculated from Equation 33 similarly to the way they were for CAS missions and for interdiction missions penetrating the FLOT. Losses incurred while penetrating the belt are calculated exactly the same way for red as they are for blue. The loss equation for penetration of the SAM belt is:

$$L_B = 1 - (1 - k/N_F (1 - (1-d)^{N_F S_{ADB}})) \quad (71)$$

Interdiction missions that penetrate the FLOT in large strikes are assumed to split up into smaller flights to fly through the SAM belt to their targets. The number of SAMs that are available to fire upon the flights S_{ADB} is equal to the number of SAMs in

the penetration corridor minus those that are avoided through mission planning and those that are suppressed by friendly EW aircraft:

$$S_{ADB} = \frac{\rho_{SAM}(R,B)2r_H}{\lambda_{THR}} - \epsilon_S - \Pi \quad (72)$$

where:

- ρ_{SAM} = the ratio of SAMs to ground AFVEs in the theater
- (R,B) = the number of red or blue AFVEs in the theater (the defending side)
- r_H = the effective range of a SAM
- ϵ_S = the number of SAMs that are suppressed by EW aircraft
- Π = the number of SAMs in a corridor avoided through mission planning.

The number of SAMs in a corridor suppressed by EW aircraft is equal to a precalculated fraction ϕ_{EW} of the number of aircraft in a flight (see Equation 68):

$$\epsilon_S = \phi_{EW} N_F \quad (73)$$

The number of SAMs in a corridor that are avoided through mission planning is a function of the number of SAMs in the corridor and the effectiveness of the plan in routing the flight around the lethal areas of the SAMs. We derived the function Π by plotting circles of fixed diameter, representing the lethal radii of SAMs, at random locations in an area of fixed width and depth and then determining the minimum number of encounters with a SAM that would occur as a flight moved from the front of the area to the back if it were completely free to move about the area and had perfect knowledge of the locations and lethal radii of all SAMs. We did this over a range of SAM densities and fitted the resulting data to a curve representing perfect mission planning. We then assumed that mission plans that were less than perfect would miss fewer SAMs and that a strike conducted without mission planning would encounter SAMs, as indicated in Equation 72 in the absence of Π . The expression for Π is given below:

$$\Pi = K_{\pi} \left(0.8 + \frac{0.7 \rho_{SAM} (R, B) 2r_H}{\lambda_{THR}} \right) \quad (74)$$

where K_{π} is a coefficient of mission planning quality with a value between one (perfect mission planning) and zero (no mission planning).

Having treated aircraft losses caused by the SAM belt, we turn to losses at the target. Aircraft losses at the target are assumed to be caused by ADUs accompanying the columns of vehicles moving in the rear. We assume that blue and red vehicles in the rear move as battalion-sized units and that each flight of interdiction aircraft attacks one unit. Furthermore, we assume that the attacks are coordinated so that no two flights attack the same unit. The aircraft are assumed to make a number of passes over the column while attacking it and to take losses during each pass.

The fraction of interdiction aircraft lost per pass over the target is calculated as in Equation 33, with the number of ADUs available to fire upon the flight S_{ADT} equal to the number of functional ADUs in the column of vehicles. The total fraction of aircraft lost over the target L_T is then calculated as a function of the number of passes made p :

$$L_T = 1 - (1 - k/N_F (1 - (1 - d)^{N_F S_{ADT} p})) \quad (75)$$

where:

$$S_{ADT} = N_{COL} \rho_{SB} \phi_{SO} \quad (76)$$

and N_{COL} is the number of vehicles in a column.

C. Offensive Counterair

Offensive counterair missions are those flown over enemy territory in order to reduce his ability to conduct air operations.⁵¹ In VFM, we assume that offensive counterair missions are flown by long-range strike aircraft against enemy

airbases and that the effect of those missions is to reduce the enemy individual aircraft sortie rate for all missions except offensive counterair. The reason for excluding friendly offensive counterair from the effects of enemy offensive counterair is that we assume that offensive counterair missions would be given priority over other missions employing the same aircraft (particularly air interdiction).⁵² In fact, it is U.S. Air Force doctrine to give priority to missions dedicated to achieving air superiority.⁵³

1. Air Effects

Aircraft are allocated to offensive counterair as described in Section III.B above: long-range strike aircraft are divided between air interdiction and offensive counterair missions before the beginning of the offensive. The average total sortie rates for red and blue counterair missions are calculated from the losses those aircraft take over the length of the campaign (Section III.C.2). Once that is done, the effect of the missions upon friendly individual aircraft sortie rates is calculated as a function of the total enemy offensive counterair sortie rate:

$$S_{AC} = S_{ACO} (1 - K_{OCA} S_{OCA}) \quad (77)$$

where:

S_{OCA} = the total average enemy offensive counterair sortie rate (aircraft/hr)
 K_{OCA} = a coefficient of effectiveness
 S_{ACO} = the initial friendly individual aircraft sortie rate (hr^{-1}).

2. Aircraft Losses

Aircraft losses suffered during offensive counterair missions are calculated in a manner very similar to that used to calculate losses suffered during interdiction missions. Just as with interdiction, offensive counterair missions are assumed to take losses due to enemy defensive counterair and enemy air defenses. The overall loss equations that determine the number of offensive counterair aircraft surviving in the theater as a function of time is the same as the one used for interdiction, i.e., Equation 52.

The values of the variables in Equation 52 as used for offensive counterair are different from those used to calculate interdiction losses in the following ways: different numbers of friendly aircraft will be flying offensive counterair missions and escorting those missions, different numbers of enemy fighters will be flying defensive counterair missions, and, lastly, the air defenses over the targets of offensive counterair missions (airbases) are different than those over the targets of interdiction missions (columns of vehicles).

The allocation of strike aircraft to offensive counterair is described in the previous section. Enemy interceptors are allocated against friendly offensive counterair missions according to the proportion of friendly strike aircraft flying offensive counterair as opposed to interdiction. Friendly escorts serve to increase the loss rates of enemy interceptors in general, as shown in Equation 54.

We assume that offensive counterair missions are flown in the same way that air interdiction missions are flown. Aircraft penetrate the FLOT in large strikes, split up into small flights and fly through the SAM belt to the target, make some number of passes over the target and return via the same path. Aircraft are assumed to suffer losses in each leg of the mission, in and out. Offensive counterair losses to air defense are determined in exactly the same way they are determined for air interdiction (see Equations 52-76) except that the air defenses over the targets of counterair missions are different than those over the targets of interdiction missions.

The calculation of the fraction of offensive counterair aircraft lost over an airbase target follows the same approach as the calculation of the fraction of interdiction aircraft lost over a column of vehicles (Equations 75 and 76). The fraction lost over an airbase L_A replaces the fraction lost over the target L_T in Equation 64 and is given by:

$$L_A = 1 - (1 - k/N_F(1 - (1 - d)^{N_F S_{ADAB,p}})) \quad (78)$$

where S_{ADAB} is the number of ADUs at the airbase eligible to fire at the attacking flight of aircraft:

$$S_{ADAB} = N_{ADAB} - \epsilon_A \quad (79)$$

where N_{ADAB} is the number of ADUs at the airbase, and ϵ_A is the number of those ADUs that are suppressed by friendly EW aircraft:

$$\epsilon_A = \min \left\{ \phi_{EW} N_F, \phi_{RS} N_{ADAB} \right\} \quad (80)$$

The number of ADUs that are suppressed is taken to be the minimum of the predetermined fraction of aircraft in the flight ϕ_{EW} and the number of ADUs at the airbase that are radar homing SAMs (see Equation 68).

Having calculated the fraction of aircraft lost over the target L_A , the total fraction of offensive counterair aircraft lost per mission due to air defense L_{IAD} is calculated as given in Equation 64. The values of all other variables in Equation 64 are exactly the same as those used to calculate the losses of air interdiction aircraft to air defenses. The value for L_{IAD} is then used in Equation 52 along with the values of the other variables calculated, as described in this section, to determine the total average offensive counterair sortie rate over the length of the campaign.

D . Air Transport

Intratheater air transport could be used in a future conflict to move reserves to defensive positions more quickly than they could move over the ground themselves. In VFM, we have installed an elementary model of intratheater air transportation in order to be able to estimate its effect upon the outcome of a conflict.

We assume that the transport aircraft carry red and blue reserve units from their locations at the beginning of the offensive to counterconcentrate against the blue counterattack and the red theater attack, respectively. Air transportation thus serves to increase the reserve arrival rate above and beyond that which could be obtained through ground movement alone. The air transportation rate available for each side (in AFVEs/hr) is the product of the individual aircraft sortie rate of that side S_{AC} , the total number of air transports available n_{AT} , and the carrying capacity (in AFVEs) of an individual transport C_{AT} :

$$\psi_{AT} = S_{AC} \eta_{AT} C_{AT} \quad (81)$$

where the individual aircraft sortie rate is the same as the sortie rate for all other aircraft on that side⁵⁴ and the capacity of the transport in AFVEs is simply the carrying capacity of the aircraft in metric tons divided by the "average" weight of an AFVE. The average weight of an AFVE is equal to the numerically weighted average weight of the tanks and infantry fighting vehicles on each side.

In the case of blue, the number of units in reserve is determined before the beginning of the red offensive. Those units normally move over the ground either to counterattack or to counterconcentrate. Given the availability of air transportation, we assume that those units that are intended for counterconcentration and that could be moved by air, would be moved by air; the remainder are assumed to move along the ground as before. Thus the number of units that move along the ground is equal to the total number of reserves minus the number that could be transported by air over the course of the campaign, and therefore the ground reserve arrival rate (before suffering the effects of air interdiction) is equal to the arrival rate in the absence of air transportation ψ_{BOMTO} minus the fraction of reserves that are transported by air over the course of the campaign:

$$\psi_{BOMTG} = \psi_{BOMTO} \left(1 - \frac{B_{AT}}{B_{RSV}(1 - \phi_{CA})} \right) \quad (82)$$

where the number of reserves transported by air B_{AT} is equal to the product of the air transportation rate ψ_{AT} and the estimated length of the war t^* . It should be noted that the number of AFVEs that can be transported by air can never be greater than the total number of reserve AFVEs. After the air reinforcement rate is calculated, the ground reinforcement rate is reduced as necessary by air interdiction. The total reinforcement rate is then equal to the sum of the modified ground reinforcement rate and the air reinforcement rate.

A similar procedure is followed to determine the red air reinforcement rate. In the absence of air transportation, the red reinforcement rate is assumed to be fixed by the constrictive nature of the road net down his line of communication to his forward

echelons. Red's available air transportation rate is calculated in exactly the same manner as blue's and is added on to red's net reinforcement rate after blue interdiction of the red reserves moving along the ground is assessed.

Finally, it should be noted that air transportation rates available to blue and red can only be reduced through the reduction in the individual aircraft sortie rate caused by enemy offensive counterair missions. Transport aircraft are assumed to land a distance from the actual fighting and thus not to be vulnerable to enemy air defenses.

IV. ADVANCED CONVENTIONAL MUNITIONS

Advanced conventional munitions were treated in the original version of VFM.⁵⁵ The approach used there, and followed here with little deviation, was to treat long-range ACM analogously to air interdiction and to treat short-range ACM as a form of range-limited CAS. The aim of this work was to enable the impact of ACM upon the outcome of the conventional battle to be linked to the actual number of munitions present in the theater on both sides, and to incorporate ACM into the new modeling of the effects of tacair.

A. Long-Range ACM

Long-range ACM is used by red to interdict the movements of blue reserves and by blue to interdict the movements of red follow-on echelons. The effects of red long-range ACM are assessed by adding the kill rate of blue AFVEs produced by it to the kill rate produced by air interdiction in Equation 30. The kill rate of red long-range ACM is determined simply by dividing the total number of red missiles in the theater by the estimated time of culmination of the red offensive t^* and multiplying by the number of blue AFVEs killed per missile fired.

Long-range ACM is used by blue to interdict the movement of red follow-on echelons. The effects of blue long-range ACM are assessed by adding the red vehicles killed by ACM per blue line taken to the number of vehicles killed by air interdiction. The impact of the kills on the red rate of advance are calculated in exactly the same way as before, except the number of kills is equal to the sum of those due to air interdiction and long-range ACM. The number of red vehicles killed per blue line

taken is determined by dividing the number of blue missiles in the theater by the estimated number of lines red will take.⁵⁶ The estimated number of lines n_l' is calculated as in Equation 28.

B. Short-Range ACM

Short-range ACM is used by red and blue to kill AFVEs at the point of attack, and by red to kill blue vehicles as they withdraw from defensive lines, similarly to the manner in which red uses attack helicopters to pursue withdrawing blue vehicles. The effect of blue ACM is assessed in terms of the number of red AFVEs killed per blue line taken μ_{BS} . That total is simply added to the number of AFVEs killed by CAS per line taken (Equation 21) and the net effects of that accounted for in exactly the same way as is done for CAS:

$$\mu_{BS} = t_{EC} K_{CAS} S + \delta_{IB} \frac{\lambda_{ATK}}{2} \quad (83)$$

where δ_{IB} , the number of red AFVEs killed by short-range ACM per blue line taken per two kilometers of attack frontage, is equal to:

$$\delta_{IB} = \frac{\hat{B}_{ARTLI}^n n_{ACMTB} K_{SAB}}{n'_{LITOT}} \quad (84)$$

where:

n_{ACMTB} = the number of short-range ACM rounds per tube of blue artillery

K_{SAB} = the number of red AFVEs killed per blue ACM round.

While ACM may be highly effective against vehicles, due to the nature of the warhead seekers and the warheads themselves, it is not effective against dismounted infantry. Therefore, if the red attack velocity is slow enough, his infantry will be dismounted and thus immune to ACM. We assume that the fraction of red infantry AFVEs that is mounted is proportional to

the attack velocity V , and that once the velocity is equal to 3 km/hr, 100 percent of red's infantry is mounted. That implies that the portion of the red force that is immune to ACM ϕ_{IACM} is equal to:

$$\phi_{IACM} = \frac{\phi_{INFR}}{2} \left(1 - \frac{V}{3} \right) \quad (85)$$

where ϕ_{INFR} is the fraction of the red force that is infantry. The factor of two arises from the fact that infantry AFVEs comprise an infantry squad plus its fighting vehicle. The vehicle remains vulnerable to ACM. The immunity of the dismounted infantry serves to place a cap on the number of red AFVEs in an echelon that may be killed by ACM. Even if blue had enough ACM to kill all of the red AFVEs in an echelon, those consisting of dismounted infantry would not be affected. At velocities greater than 3 km/hr, all of the red force is vulnerable to ACM.

Red uses short-range ACM in the same manner as blue to defend against blue's counterattack. Casualties per red flank line taken caused by ACM are added to casualties caused by CAS and the effects are treated the same way. Counterattacking blue infantry is also taken to be immune to ACM if blue's counterattack velocity is low enough.

On the attack, red uses ACM to kill withdrawing blue AFVEs and also to kill stationary blue vehicles defending lines. The first thing red does is to allocate his short-range ACM between the attack of withdrawing blue vehicles and the attack of blue vehicles defending in place. We assume that red gives priority to attacking withdrawing vehicles, as ACM is more effective against vehicles moving in the open than it is against stationary vehicles under cover.⁵⁷ The number of withdrawing blue vehicles per two kilometers of attack frontage that can be killed by red ACM K_{BW} is governed by the number of artillery tubes supporting the red attack $\hat{R}_{ARTECH}$, their rate of fire, the length of time it takes blue to withdraw D_L/V_A , and the effectiveness of the ACM:

$$K_{BW} = \hat{R}_{ARTECH} \frac{D}{60} \frac{L}{V_A} K_{SAR} \quad (86)$$

where K_{SAR} is the number of withdrawing blue vehicles killed per round of red ACM fired. The factor of 60 arises from the assumption that the sustained rate of fire of red artillery is equal to one round per minute.⁵⁸ Red's killing of withdrawing vehicles with ACM has the same effect as his killing them by pursuing with attack helicopters; it reduces the fraction of blue forces that actually survive withdrawal w_{SURV} below the fraction that intend to withdraw, w . The fraction of blue withdrawing vehicles that survive ACM attack is given by:

$$w_{SURV} = w \left(1 - \frac{\lambda_{ATK}}{2} \frac{K_{BW}}{B_{LI}} \right) \quad (87)$$

In the event that red pursues withdrawing blue defenders with helicopters at the same time he attacks them with ACM, w in Equation 87 above is replaced by w_{SURV} as calculated in Equation 5 to account for the helicopters. If K_{BW} is greater than the number of blue vehicles that actually withdraw $\frac{2wB_{LI}}{\lambda_{THR}}$, then we assume that red conserves his ACM and fires just enough rounds to kill all of the withdrawing vehicles; thus $K_{BW} = \frac{2wB_{LI}}{\lambda_{THR}}$ and $w_{SURV} = 0$.

The number of rounds that red uses per line he takes A_{PL} is equal to the total number of withdrawing blue vehicles killed, divided by the number of kills per round of ACM:

$$A_{PL} = \frac{\lambda_{ATK}}{2} \frac{K_{BW}}{K_{SAR}} \quad (88)$$

The number of ACM rounds red uses to kill withdrawing blue vehicles over the course of the campaign is equal to the product of the number of rounds used per line and the number of lines he takes. This poses a problem analogous to the one posed by red's use of attack helicopters in pursuit: he may run out of ACM rounds before the point of culmination of his offensive and, furthermore, the number of rounds of ACM he fires directly affects the rolling depth (number of lines) of defense available to blue. We solve the problem by calculating the number of lines n_{LIA} over which red can use ACM:

$$n_{LIA} = \frac{n_{ACM}}{A_{PL}} \frac{R_{OFVST}}{R} \quad (89)$$

where n_{ACM} is the number of ACM rounds possessed by red. The factor R_{OFVST}/R accounts for the fact that we assume that red ACM rounds are distributed evenly among all red units. ACM rounds with off-axis units would not be usable against withdrawing blue vehicles in the attack sector. The number of lines over which red can use ACM is analogous to the number of lines over which he can pursue withdrawing defenders with attack helicopters (Equation 7). The effect of red running out of ACM before the culmination of his offensive is handled in exactly the same way as the effect of his running out of attack helicopters. The depth of blue's defense is broken down into zones over which, because of the exhaustion of red's attack helicopters and/or ACM, different values of w_{SURV} obtain. Values of w_{SURV} greater than the value produced by the combination of pursuit by helicopters and use of ACM increase the number of blue vehicles that survive withdrawal. Those extra vehicles serve to increase the rolling depth of blue's defense. For example, it can be shown that the extra depth available to blue due to the exhaustion of both red's attack helicopters and his short-range ACM (assuming, for example again, that red runs out of helicopters first) is given by:

$$\Delta D = \frac{n_{LP2} - w}{1 - w} \quad (90)$$

where:

$$n_{LP2} = (n_{LIA} - n_{LP}) (w'_{SURV} - w_{SURV}) + (n_{LIAH} - n_{LIA}) (w - w'_{SURV}) \quad (91)$$

where:

w'_{SURV} = the fraction of blue defenders that survive withdrawal in the face of ACM, but not in the face of helicopters

n_{LIAH} = the total number of blue lines that would exist if red ACM and helicopters lasted the entire campaign.

The other cases, in which red runs out of helicopters, or ACM, but not both, are handled in a similar manner. If red runs out of neither attack helicopters nor ACM, then the total depth of the blue defense is given by Equation 4, replacing w with w_{SURV} .

If red does not use all of his ACM to kill withdrawing blue vehicles, then he uses the remainder to attack defending blue vehicles and to attack counterattacking blue vehicles on his flanks. Defending blue casualties caused by red ACM are accounted for in the force ratio term in Equation 1 by replacing $\hat{B}_{LI}$ with $\hat{B}_{LI} - \delta_{1R}$ where δ_{1R} is the number of defending blue vehicles killed by ACM per two kilometers of attack frontage, per blue line taken. The quantity δ_{1R} is calculated in exactly the same manner as δ_{1B} in Equation 84, except that the number of ACM rounds per blue artillery tube n_{ACMTB} is replaced by the number of ACM rounds per red artillery tube remaining after ACM has been allocated to the attack of withdrawing defenders n_{ACMTR} and the number of vehicles killed per blue round of ACM K_{SAB} is replaced by the number of vehicles killed per red round of ACM K_{SAR} :

$$\delta_{1R} = \frac{K_A \hat{R}_{ARTECH} n_{ACMTR} K_{SAR}}{n'_{LITOT}} \quad (92)$$

where K_A is the ratio of the effectiveness of ACM against stationary vehicles to its effectiveness against vehicles moving in the open. The number of red ACM rounds per artillery tube remaining after red has set aside those to be used to attack withdrawing blue vehicles is equal to the number red has initially available to his forces in the attack sector minus those used for attacking withdrawing vehicles, divided by the number of tubes in the attack sector:

$$n_{ATR} = \frac{\frac{n_{ACM} R_{OFVST}}{R} - n_{LITOT} A_{PL}}{R_{OFVST} \rho_{ARTR}} \quad (93)$$

where ρ_{ARTR} is the ratio of red artillery to ground AFVEs in the theater.

Since the defending blue AFVEs are assumed to be stationary, all of the blue infantry is assumed to be dismounted and thus invulnerable to red ACM. This invulnerability sets a minimum value on the quantity $\hat{B}_{LI} - \delta_{IR}$ equal to the number of AFVEs of blue dismounted infantry per two kilometers of defensive line $\frac{\hat{B}_{LI} \phi_{INEB}}{2}$. As before, the factor of ϕ arises from the fact that an infantry AFVE consists of an infantry squad plus its fighting vehicle.

In the case of counterattack defense, the number of counterattacking blue casualties per two kilometers of red flank line taken μ_{RS2} is calculated much the same way as the number of red attacking casualties per two kilometers of blue line taken μ_{BS} was calculated in Equation 84, and the number of red defending casualties per two kilometers of defensive line is calculated much the same way as the number of blue defending casualties were in 92. The difference in both cases is that the number of defended lines in the counterattack is different (and usually less) than the number of defended lines in the attack sector. Therefore, blue and red casualties in the counterattack, per two kilometers of line, are calculated using Equations 84 and 92, respectively, except that the estimated number of lines n_{LITOT} is replaced by the number of defended lines on the flank n_{LICA} , where:

$$n_{LICA} = 1 + \frac{\lambda_{ATK}}{3D_L} \quad (94)$$

Casualties caused by ACM are combined with the casualties caused by CAS in the terms μ_{RS2} and μ_{BS} for the purpose of calculating red's requirements for flank defense.

V. BARRIERS AND MINEFIELDS

In an attack against a given defensive force at a given velocity, the addition of barriers will directly increase the attacker's casualties by attrition of vehicles. Barriers also will increase the effectiveness of the defender's fire by forcing the attacker to stop or to canalize his attack into killing zones.⁵⁹ The attacker, however, may choose to mitigate the effects of the barriers by reducing his attack velocity.⁶⁰ That would give him more time to deploy his engineering assets to try to remove the barriers and carefully to pick his way through the defense. More effective artillery barrages and smoke screens could be used to support both activities.⁶¹ Since VFM allows the attacker to make a casualty-velocity trade-off, barrier defenses are represented by a numerical

factor β which increases the sensitivity of the attacker's casualties to his velocity. In Equation 1, the expression for the number of attacking casualties suffered per defending line taken, the barrier defense factor β is applied to the loss term arising from the presence of infantry. The magnitude of that term is proportional to the velocity of the attacking force. In order to represent the attacker's ability to trade velocity for casualties when assaulting a defense aided by barriers, the barrier factor β must be applied to that term.

In the previous version of VFM, minefields and barriers were treated parametrically. The factor β was changed over a range of values representing best estimates of the effectiveness of barriers in increasing an attacker's casualties. Such estimates typically ranged from a factor of 1.5 to 3.0.⁶² In this treatment we aim to relate the effectiveness of barrier defenses to the size of the engineering assets available to the defender and the amount of time before the beginning of the attacker's offensive he has to employ them.

We calculate the increase in the attacker's casualties as a function of the number of engineering squad-hours and bulldozer-hours applied to a given defended area before the start of the red offensive. The following equations relating squad-hours and bulldozer-hours to the effectiveness of barrier defenses were derived from a statistical analysis of an AMSAA study of combat engineering effectiveness.⁶³ In addition to barriers and minefields that were constructed and laid by engineering squads, the AMSAA study included the effects of using bulldozers to dig-in friendly vehicles and reduce their vulnerability to enemy fire. We include those effects here. The expression for β is given by:

$$\beta = 1 + 0.047 \ln \left[\frac{40\theta}{\lambda_{THR} D} + 1 \right] + 0.166 \ln \left[\frac{9 \xi n_{LI}}{n_{LITOT} B \phi_{FWD}} + 1 \right] \quad (95)$$

where θ is the effective number of engineering squad-hours and ξ is the number of bulldozer-hours available before the start of the offensive. The other factors in Equation 95 are used to scale the total squad and bulldozer-hours available and the total defended area in the theater to the assets used and area covered in the AMSAA study. The term $\frac{40}{\lambda_{THR} D}$ relates the total defended

area in the theater to the area covered by the AMSAA study (40 km²). The term $\frac{9 n_{LI}}{n_{LITOT} B \phi_{FWD}}$ relates the total number of vehicular positions to be dug in the theater to the number dug in the study (nine). The term n_{LITOT}/n_{LI} accounts for the fact that the defending vehicles will withdraw during the campaign and positions must be prepared for them on subsequent defensive lines as well as the initial lines on which they are deployed.

The effective number of blue engineering squad-hours available before the beginning of the offensive θ is equal to the gross number of squad-hours adjusted by red offensive engineering. The gross number of engineering squad-hours available to the blue defender before the start of the offensive is taken to be the product of the number of engineering squads available for work at the front and the number of hours before the beginning of the offensive in which they may begin to prepare defenses. We assume that the number of engineering squads deployed across the theater is equal to the sum of those squads organic to units deployed forward and those squads present in higher (corps or army) level units, all of which are assumed to be allocated to preparing defenses at the front. Thus the gross number of engineering squad-hours is given by:

$$\theta_0 = B \rho_{EB} (\phi_{FWD} \phi_{EU} + 1 - \phi_{EU}) t_E \quad (96)$$

where:

- ρ_{EB} = the ratio of blue engineering squads to ground AFVEs in the theater
- ϕ_{EU} = the fraction of engineering assets organic to divisions (as opposed to corps or armies)
- t_E = the time before the offensive available for engineering work (hours).

The number of bulldozer-hours available to the defender is simply taken to be the product of the number of engineering squad-hours available and the ratio of bulldozers to engineering squads in the theater.

Equation 95 gives the basic relationship between engineering assets and barrier defense effectiveness used in VFM. Another way the defender can increase the effectiveness of his defensive barriers is to use artillery scatterable mines (FASCAM). We model the effects of the employment of FASCAM by relating the number of mines that are laid using FASCAM to the number

of mines laid by an engineering squad.⁶⁴ The effectiveness of the FASCAM rounds is translated into equivalent gross engineering squad-hours and added to θ_0 (Equation 96). We assume that all blue artillery tubes supporting forward deployed units can fire all of their FASCAM over the course of the offensive. Thus the number of equivalent gross engineering squad-hours provided by FASCAM θ_F is given by:

$$\theta_F = 0.01\phi_{FWD}B\rho_{ARTB}n_{FTB} \quad (97)$$

where n_{FTB} is the number of FASCAM rounds per blue artillery tube.

Now that we have addressed the use of defensive engineering assets to prepare barriers and minefields, we will address the use of offensive engineering assets to counter barriers and minefields. Here offensive engineering primarily includes operations like minefield clearing conducted by combat engineers, mine ploughs or the use of explosive charges.⁶⁵ Our approach was to assume that offensive engineering efforts (expressed in effective engineering squad-hours) could directly reduce the effects of defensive engineering efforts. In other words, the effective amount of engineering effort available to the defender per unit area defended is taken to be equal to the total amount of defending effort per unit area minus the effective amount of offensive effort per unit area.

To calculate the effectiveness of offensive engineering, we used an area equal to that used in the AMSAA study (40 km²) and a time equal to that required by the red attacker to take a blue defensive line. The net effectiveness of the effort provided by blue engineering squads is equal to the gross number of blue engineering squad hours θ_0 minus an offensive engineering term:

$$\theta = \theta_0 - \rho_{MAX} \rho_{ER} \eta \left(\frac{D_L}{V} - \frac{D_L}{V_A} \right) \frac{\lambda_{THR} D}{40} \quad (98)$$

where:

ρ_{ER} = the ratio of red engineering squads to ground AFVEs in the theater

η = the ratio of the effectiveness of offensive engineering to the effectiveness of defensive engineering.

The offensive engineering term in Equation 98 is equal to the product of the number of red engineering squads employed per 40 km² of blue defensive position (the depth of the blue position is taken to be equal to the separation distance between the blue lines D_{L1} , so the width is simply 40 km²/ D_{L1}), the time they have to complete their tasks before the ground force engagement takes place, the number of 40 km² areas in the blue defense (to account for the fact that θ is normalized to the number of 40 km² areas in Equation 95), and a factor η accounting for the difference in effectiveness between offensive and defensive engineering. In general, because the attacker does not have to clear entire minefields, but only paths through them, offensive or mine clearing efforts take fewer man-hours than defensive or mine laying efforts. Thus η has some value greater than one. We derived a value for η from an AMSAA analysis of mine warfare.⁶⁶

The last thing we considered in our treatment of mine warfare was the use of FASCAM by the attacker to lay a minefield behind the defensive line he is attacking in order to cause casualties among defending vehicles when they withdraw from combat. We assume that the red artillery supporting an attack will fire a number of FASCAM rounds equal to the fraction of the total number of rounds fired, which is equivalent to the initial fraction of FASCAM rounds possessed by the artillery. Those rounds will produce a minefield that will cause casualties among withdrawing blue vehicles as a function of the minefield's linear density (mines per meter of line). To calculate the casualties suffered by the withdrawing vehicles, we performed a statistical analysis of data taken from an IDA study of landmine effectiveness and assumed that the vehicles withdrew in columns of three when crossing the field.⁶⁷ The number of mines scattered by red per two kilometers of attack frontage is given by:

$$\hat{n}_M = 9 \hat{R}_{ARTECH} \phi_{AFR} 60 \left(\frac{D_L}{v} - \frac{D}{v_A} \right) \quad (99)$$

where ϕ_{AFR} is the fraction of red artillery ammunition that is FASCAM. The factor of nine arises from the fact that each FASCAM round contains nine mines. The factor of 60 comes from the assumed rate of fire of the artillery (one round per minute). The fraction of withdrawing blue vehicles that survive the FASCAM field is given by:

$$\phi_{WF} = 1 - 2.0 \ln \left(\frac{\hat{n}_M}{2000} + 1 \right) \quad (100)$$

The fraction of withdrawing blue vehicles that survive the FASCAM field is applied to the fraction of blue vehicles on a defensive line that attempt to withdraw w to produce an effective fraction of blue vehicles that withdraw. This effective fraction is used in place of w when calculating the total number of blue lines in the theater and the effects of helicopter pursuit and the use of short-range ACM by the attacker.

VI. OPTIMIZATION ALGORITHM

As did the original VFM code described in IDA P-2380, the extended version developed here identifies the best blue and red force employment choices, and the associated red net territorial gain, by sampling blue strategy vectors from the overall strategy space at regular intervals.⁶⁸ A strategy vector consists of a unique blue forward fraction, reserve fraction, withdrawal fraction and number of predeployed defensive lines. Red then chooses the assault velocity that yields the largest territorial gain for each blue strategy vector. Blue then selects the vector that produces the lowest red net territorial gain.

This systematic sampling algorithm, of course, produces only an estimate of the true optimum. Moreover, it does not guarantee that this estimate is not an approximation of a local, rather than a global, optimum (although the larger the number of points sampled in the initial iteration, the lower the likelihood of the former). A survey of the nonlinear optimization theory literature, however, did not uncover an algorithm which could both guarantee a global optimum and apply to the equations used here. Branch and bound techniques, for example, require that the function to be optimized be separable into piecewise convex functions to guarantee a global optimum.⁶⁹ Generalized lagrange multipliers, on the other hand, require that the problem be expressible as a constrained maximization or minimization, and that all resulting constraint functions be linear.⁷⁰ Alternatively, penalty function methods require that the function to be optimized be convex and continuous.⁷¹ The net territorial gain function used here, however, is neither separable into piecewise convex functions, expressible as a constrained maximization or minimization with linear constraint functions, nor convex.⁷² As such, we have opted for the simple, systematic sampling algorithm described in greater detail in IDA P-2380, given its simplicity, conceptual clarity and relatively short run times.

VII. LIMITATIONS

Equation 100 completes the extension of the theory originally developed in IDA P-2380. As with that initial formulation, this extension is not without limitations, however, and must be used with discrimination. For example, the class of phenomena addressed by these equations is limited to high intensity land warfare between sophisticated opponents. As a result, these equations cannot be used to determine counter-infiltration or administrative/logistical requirements that might limit a combatant's ability to reduce forces to very low levels in a theater or war. Likewise, while these equations may be of some utility with respect to high intensity conflicts outside the European theater (e.g., the Middle East, Southwest Asia or Korea), they are not appropriate to consideration of low intensity conflict, and their applicability to combat between low-sophistication opponents is unclear.

The equations above employ a number of simplifying assumptions to streamline the analysis -- while many of these could be relaxed in further work, they must be taken into account for appropriate use of the existing formulation. These include the inability of forward defensive forces to displace laterally for counterconcentration; the inability of blue to optimize his counterattack velocity and his limited range of choice of counterattack frontage; the summary treatment of red's off-axis force requirement to defend against blue cross-border invasion; the restrictions on red's range of choice with respect to attack frontages and the employment of forces in flank defense; and the absence of a requirement for residual red forces to exploit fully an accomplished breakthrough.

An unavoidable limitation of our approach is the absence of empirical data regarding the performance of some of the weapon types represented by the equations. Where possible, quasi-empirical testing was conducted using the LLNL JANUS model. For several of the weapon types involved in this extension, however, JANUS representation was either unavailable or unsatisfactory (e.g., tacair and air defense, ACM and minefields and barriers). Where simulation testing was thus unavailable, we have relied on a priori deduction on the basis of existing theoretical and operational literature. While it may prove possible to develop alternative means of either ex post facto or ex ante testing for the contentions described above, to do so here is beyond the scope of the present inquiry. Of course, a priori deductive modeling can be difficult even for a small number of simple phenomena; it is even more difficult when one addresses a complex interaction like that between tacair and air defense systems.

There also were a number of potentially significant issues which, due to time and resource constraints, received limited attention here. The current treatment of defensive withdrawal allows the attacker to destroy withdrawing AFVEs relatively easily with either attack helicopters or ACM; it may overestimate the vulnerability of withdrawing units by not considering all of the means a defender could have to protect himself (e.g., more aggressive use of cover, or use of defensive helicopters or fixed wing air in an anti-helicopter role). Moreover, we simulated helicopter pursuit with JANUS but we had to model ACM attacks deductively. The result of these theoretical decisions is that the defender's force employment choices can be severely restricted (taking heavy casualties in withdrawal is clearly undesirable), and in particular, his overall defensive depth is limited (withdrawal is one of the primary sources of defensive depth) and net territorial gain is probably overestimated. Perhaps most important for our purposes here, blue's ability to mitigate potentially adverse consequences of changes in the weapon balance is reduced -- thus the sensitivity of net territorial gain to changes in weapon mix is probably overestimated.

In addition, artillery fire against ground force AFVEs and air defense systems is modeled explicitly, but counterbattery fire -- and consequent suppression or loss of artillery systems -- is not. The equations do not address the effects of variations in natural terrain or weapon quality (as opposed to weapon class); they treat logistical and command systems only implicitly. All of the preceding are issues that would benefit from more detailed consideration than was possible here. The extent of formal proof for deduced properties is quite limited in the discussion above; a more thorough mathematical argument would thus be a valuable addition. The effects of intangibles such as morale or training are not considered here, nor are the potential roles of organizational or social variables. We also do not consider here the possibility of variations in the fundamental war aims of the two sides. That is, it is assumed throughout that the underlying objective of the theater invader is to seize and assert political control over the territory of the invaded (and/or to annihilate the opposing armed forces as an essential means to this end). Yet it is possible that at very low force levels -- where seizure and control of territory (or annihilation of opposing forces) can become very difficult -- that an aggressor would choose instead what Archer Jones has termed a "raiding strategy," wherein opposing forces are avoided and military power is used for coercive purposes, to threaten with destruction economic and political assets which are vulnerable, but which an aggressor could not hold against counterattack.⁷³ An unconventional aim of this sort, while not without historical precedent, and while potentially worthy of further analysis, nevertheless lies beyond the scope of the theory developed here.

Finally, it should be noted that less attention was devoted to the equations' behavior in some regions of the potential independent variable space than in others. In particular, time and resource constraints kept us from exploring all of the permutations of independent variables associated with the new weapons systems. As a result we selected values that were, to some degree, based upon a priori arguments. Given the focus of the study on the implications of low force levels, the consequences of very high force levels received limited consideration. While we might expect, for example, diminishing marginal return effects with respect to various phenomena addressed in the equations (e.g., reserve arrival rates as a function of theater force size; or planning and command response times as a function of the size of committed forces), these effects were not given explicit attention. Where the implications of large force size were clearly crucial -- as in the case of the attacker's local AFVE concentration -- phenomena that are probably best represented by diminishing marginal return relationships are instead approximated by imposed ceilings.

Similarly, the nature of the functional forms which emerged from the JANUS testing display particular sensitivity for certain values of certain independent variables. In particular, for very low levels of attacker artillery R_{ARTECH} , Equation 1 will tend to predict unreliably high attacker casualties. To a lesser degree, very low values for ϕ_{INF} , the combined attacker and defender infantry fractions, will tend to inflate casualties. An exponential form for either variable would eliminate this oversensitivity, but the tests conducted did not provide sufficient variance in these parameters to estimate an exponential form with sufficient confidence.

ENDNOTES

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Table B-1. Variable Definitions^{1, 2}

A_{ECD}	Artillery equivalent of defensive CAS missions employing area munitions (tubes), A_{ECD} : $R \rightarrow [0, \infty)$
A_{FEBA}	Area over which CAS aircraft fly (kilometer ²), A_{FEBA} : $R \rightarrow [0, \infty)$
A_{PL}	Total number of red short-range ACM fired against withdrawing blue vehicles per blue line taken, A_{PL} : $R \rightarrow [0, \infty)$
B	Blue maneuver forces in theater (AFVEs), B : $R \rightarrow [0, \infty)$ (#)
B_{AT}	Number of blue AFVEs transported to the last defended line by air, B_{AT} : $R \rightarrow [0, \infty)$
B_{ART}	Blue artillery in theater (tubes), B_{ART} : $R \rightarrow [0, \infty)$ (#)
B_{ARTECH}	Blue artillery supporting a single counterattack echelon (tubes), B_{ARTECH} : $R \rightarrow (0, \infty)$
$\hat{B}_{ARTECH}$	Blue artillery supporting a single counterattack echelon (tubes, scaled to two-kilometer benchmark frontage), $\hat{B}_{ARTECH}$: $R \rightarrow (0, \infty)$
B_{ARTLI}	Blue artillery supporting a single defensive line (tubes), B_{ARTLI} : $R \rightarrow [0, \infty)$
$\hat{B}_{ARTLI}$	Blue artillery supporting a single defensive line (tubes, scaled to two-kilometer benchmark frontage), $\hat{B}_{ARTLI}$: $R \rightarrow [0, \infty)$

¹ Exogenous independent variables are denoted by (#); endogenous independent variables by (\$); constants by (\$); the dependent variable is G. All other variables given are endogenous instrumental variables.

² This table includes all of the variables of the VFM model, some of which are not used in this appendix, but are explained in IDA P-2380.

Table B-1 (continued)

$B_{CA}^{(t_{CA})}$	Blue maneuver forces assigned to counterattack surviving at time t_{CA} (AFVEs), $B_{CA}(t_{CA}): R \rightarrow [0, \infty)$
B_{CAECH}	Blue single assault echelon maneuver force initial strength (AFVEs), $B_{CAECH}: R \rightarrow [0, \infty)$
$\hat{B}_{CAECH}$	Blue single assault echelon maneuver force initial strength (AFVEs, scaled to two-kilometer benchmark frontage), $\hat{B}_{CAECH}: R \rightarrow [0, \infty)$
$B_{CAST}^{(t)}$	Blue maneuver forces available for counterattack at time counterattack begins (AFVEs), $B_{CAST}^{(t)}: R \rightarrow [0, \infty)$
B_{FWD}	Blue maneuver forces allocated to forward positions (AFVEs), $B_{FWD}: R \rightarrow [0, \infty)$
B_{JJ}	Blue maneuver forces defending a single defensive line, (AFVEs), $B_{JJ}: R \rightarrow (0, \infty)$
$\hat{B}_{JJ}$	Blue maneuver forces defending a single defensive line (AFVEs, scaled to two-kilometer benchmark frontage), $\hat{B}_{JJ}: R \rightarrow (0, \infty)$
B_{RSV}	Blue maneuver forces allocated to theater reserve (AFVEs), $B_{RSV}: R \rightarrow [0, \infty)$
B_{RSVA}	Blue immobile reserves (AFVEs), $B_{RSVA}: R \rightarrow [0, \infty)$ (#)
$B_{OM}^{(t)}$	Blue maneuver forces defending final defensive line at time t (AFVEs), $B_{OM}^{(t)}: R \rightarrow [0, \infty)$
C	Red casualties required to take a single blue defensive line (AFVEs), C: $R \rightarrow (0, \infty)$
C_1	Instrumental quantity used to calculate $\frac{dI}{dt}$

Table B-1 (continued)

C_2	Instrumental quantity used to calculate $\frac{dl}{dx}$
C_3	Instrumental quantity used to calculate $\frac{dl}{dx}$
C_{AT}	Carrying capacity of an air transport aircraft of a given side (metric tons), C_{AT} : $R \rightarrow [0, \infty)$ (\$)
C_{CA}	Blue casualties required to take a single red flank defensive line (AFVEs), C_{CA} : $R \rightarrow (0, \infty)$
D	Overall depth of the blue defended zone (kilometers), D: $R \rightarrow (0, \infty)$
d	Probability that a given ADU will detect a given CAS aircraft, per pass over the target (dimensionless), d: $R \rightarrow [0, 1]$ (\$)
D_1	Instrumental quantity used to calculate $\frac{dl}{dx}$
D_2	Instrumental quantity used to calculate $\frac{dl}{dx}$
D_{ASY}	Depth of an assembly area (kilometers), D_{ASY} : $R \rightarrow (0, \infty)$ (\$)
D_{CA}	Depth of Red's flank defense against blue counterattack (kilometers), D_{CA} : $R \rightarrow (0, \infty)$
D_{LI}	Depth of a single defensive line (kilometers), D_{LI} : $R \rightarrow (0, \infty)$ (\$)
E	Number of friendly fighters dedicated to escort missions, E: $R \rightarrow [0, \infty)$ (#)

Table B-1 (continued)

$G(t)$	Red net territorial gain at time t (kilometers), $G: R \rightarrow [0, \infty)$
$G_{CA}(t_{CA})$	Ground gained by blue counterattack at time t_{CA} (kilometers), $G_{CA}(t_{CA}): R \rightarrow [0, \infty)$
H_B	Number of blue attack helicopters in the theater, $H_B: R \rightarrow [0, \infty)$ (#)
$\hat{H}_B$	Number of blue attack helicopters deployed on a single defensive line (scaled to two-kilometer benchmark frontage), $\hat{H}_B: R \rightarrow (0, \hat{H}_{MAX}]$
$\hat{H}_{MAX}$	Maximum number of attack helicopters deployable on a defensive line (scaled to two-kilometer benchmark frontage), $\hat{H}_{MAX}: R \rightarrow (0, \infty)$ (#)
H_P	Total number of red attack helicopters dedicated to pursuit of withdrawing blue AFVEs, $H_P: R \rightarrow [0, H_R)$
H_R	Total number of red attack helicopters in the theater, $H_R: R \rightarrow [0, \infty)$ (#)
$\hat{H}_R$	Number of red attack helicopters deployed on a red defensive line (scaled to two-kilometer benchmark frontage), $\hat{H}_{MAX}: R \rightarrow [0, \infty)$
h	Loss exchange ratio in combat on final blue defensive line (dimensionless: [Red AFVEs lost/blue AFVEs lost]), $h: R \rightarrow [0, \infty)$
h_{CA}	Loss exchange ratio in counteroffensive combat on final red defensive line (dimensionless: [blue AFVEs lost/red AFVEs lost]), $h_{CA}: R \rightarrow [0, \infty)$
I	Number of blue aircraft flying interdiction missions against red follow-on echelons, $I: R \rightarrow [0, \infty)$ (#)
$\bar{I}$	Average number of aircraft flying interdiction missions over the course of the offensive for a given side, $\bar{I}: R \rightarrow [0, \infty)$

Table B-1 (continued)

I_{BC}	Number of blue aircraft flying interdiction missions against red reserves, $I_{BC}: R \rightarrow [0, \infty)$ (#)
I_O	Number of blue strike aircraft in the theater, $I_O: R \rightarrow [0, \infty)$ (#)
k	Probability that a given ADU, having detected an aircraft, can kill it (dimensionless), $k: R \rightarrow [0, 1]$ (\$)
k_0	Constant, minimum attack frontage (kilometers) (\$)
k_1	Constant, related to C (dimensionless) (\$)
k_3	Constant, related to C (dimensionless) (\$)
k_4	Constant, related to C (dimensionless) (\$)
k_5	Constant, related to α (dimensionless) (\$)
k_6	Constant, related to C (dimensionless) (\$)
k_8	Constant, (fractional increase in attacker casualties due to entropy effect of depth per kilometer advanced through defended territory) (\$)
k_{ACMC}	Constant, ratio of ACM and CAS lethality vs stationary targets to ACM and CAS lethality vs moving targets (dimensionless) (\$)
k_{AE}	Constant, number of offensive echelons whose organic artillery is available to support a single assault by the lead echelon (dimensionless) (\$)
K_{BW}	Number of withdrawing blue AFVEs killed by red short-range ACM, per blue line taken (scaled to two-kilometer benchmark frontage), $K_{BW}: R \rightarrow [0, \hat{B}_L)$

Table B-1 (continued)

K_{CAS}	Red AFVEs killed per blue CAS sortie, $K_{CAS}: R \rightarrow [0, \infty)$ (\$)
k_{DEF}	Constant, red AFVEs required to defend against one reserve blue AFVE away from the point of attack (dimensionless) (\$)
K'_{FI}	Number of enemy interceptors killed per friendly interdiction aircraft per engagement, for a given side, modified by the presence of friendly escort aircraft, $K'_{FI}: R \rightarrow [0, \infty)$
K_{FI}	Number of enemy interceptors killed per friendly interdiction aircraft per engagement, for a given side, $K_{FI}: R \rightarrow [0, \infty)$ (\$)
K_{IF}	Number of friendly interdiction aircraft killed per enemy fighter per engagement, for a given side, $K_{IF}: R \rightarrow [0, \infty)$ (\$)
K_{INT}	Number of AFVEs killed per interdiction sortie of a given side, $K_{INT}: R \rightarrow [0, \infty)$ (\$)
K_{OCA}	Coefficient of effectiveness of offensive counterair missions of a given side (fraction of enemy individual aircraft sortie rate lost per friendly offensive counterair sortie flown per hour), $K_{OCA}: R \rightarrow [0, \infty)$ (\$)
k_{PIN}	Constant, red AFVEs required to pin one forward blue AFVE away from the point of attack (dimensionless) (\$)
k_{SA}	Number of ADUs suppressed by each tube of artillery firing into an air interdiction corridor, $k_{SA}: R \rightarrow [0, \infty)$ (\$)
k_{SAB}	Red AFVEs killed per blue short-range ACM round fired, $k_{SAB}: R \rightarrow [0, \infty)$ (\$)
k_{SAR}	Number of withdrawing blue AFVEs killed per round of red short-range ACM fired, $k_{SAR}: R \rightarrow [0, \infty)$ (\$)

Table B-1 (continued)

k_λ	Constant, increase in attack frontage per offensive AFVE (kilometers/AFVE) (\$)
K_π	Coefficient of interdiction mission planning effectiveness, $K_\pi: \lambda \rightarrow [0,1]$ (\$)
L_1	Instrumental quantity used to calculate $\frac{dl}{d\lambda}$
L_2	Instrumental quantity used to calculate $\frac{dl}{d\lambda}$
L_A	Fraction of an offensive counterair flight lost per pass over an enemy air base, $L_A: R \rightarrow [0,\infty)$
L_B	Fraction of strike aircraft lost per pass through the SAM belt, $L_B: R \rightarrow [0,1]$
L_{CAS}	Total CAS aircraft lost by a side over the length of the offensive, $L_{CAS}: R \rightarrow [0, N_{CAS}]$
L_{CASM}	Number of CAS aircraft lost by a side per CAS mission flown, $L_{CASM}: R \rightarrow [0,\infty)$
L_{CM}	Fraction of CAS aircraft flying a single CAS mission lost by a side, per mission, $L_{CM}: R \rightarrow [0,1]$
L_{CP}	Fraction of CAS aircraft flying a single CAS mission lost by a side, per pass over the target, $L_{CP}: R \rightarrow [0,1]$
L_F	Fraction of strike aircraft lost per pass across the FLOT, $L_F: R \rightarrow [0,1]$
L_{IAD}	Loss rate suffered per sortie by interdiction aircraft due to air defense (aircraft per hour), $L_{IAD}: R \rightarrow [0,\infty)$
L_T	Fraction of strike aircraft lost per pass over the target, $L_T: R \rightarrow [0,1]$

Table B-1 (continued)

M_{CLI}	Number of CAS missions flown by a side per blue line taken, $M_{CLI}: R \rightarrow [0, \infty)$
M_{CLID}	Number of defensive CAS missions flown per blue line taken, $M_{CLID}: R \rightarrow [0, \infty)$
M_{CLIO}	Number of offensive CAS missions flown per blue line taken, $M_{CLIO}: R \rightarrow [0, \infty)$
n_{ACM}	Number of short-range ACM rounds possessed by red in the theater, $n_{ACM}: I \rightarrow [0, \infty) (\#)$
n_{ACMTB}	Number of short-range ACM rounds per tube of blue artillery in the theater, $n_{ACMTB}: R \rightarrow [0, \infty) (\#)$
n_{ACMTR}	Number of rounds of short-range ACM per red artillery tube in the theater, remaining after red allocates ACM to attacking withdrawing blue vehicles, $n_{ACMTR}: R \rightarrow [0, \infty)$
n_{ADAB}	Average number of ADUs defending an air base of a given side, $n_{ADAB}: R \rightarrow [0, \infty) (\#)$
n_{ASLT}	Number of successive assaults completed successfully by a single offensive echelon prior to reference assault (dimensionless), $n_{ASLT}: R \rightarrow (0, \infty)$
N_{ASLT}	Total number of successive assaults completed successfully by a single offensive echelon (dimensionless), $N_{ASLT}: R \rightarrow (0, \infty)$
N_{AT}	Number of air transport aircraft, of a given side, in the theater, $N_{AT}: I \rightarrow [0, \infty) (\#)$
N_{CAS}	Number of aircraft flying CAS missions on a given side, $N_{CAS}: R \rightarrow [0, \infty) (\#)$

Table B-1 (continued)

N_{COL}	Average number of vehicles in a column of blue reserves moving in the rear, $N_{COL} : R \rightarrow (0, \infty)$ (#)
n_{ECH}	Number of offensive echelons required to take a single defensive line (dimensionless), $n_{ECH} : R \rightarrow (0, \infty)$
n_{ECHCA}	Constant, estimated number of blue echelons required to take a single red defensive line; related to t_{JO} determination (dimensionless), $n_{ECHCA} : R \rightarrow (0, \infty)$ (\$)
N_F	Number of aircraft in a flight, $N_F : I \rightarrow \{1, \infty\}$ (#)
N_{FS}	Number of aircraft in a flight for strike missions, $N_{FS} : R \rightarrow \{1, \infty\}$
N_{FTB}	Number of FASCAM rounds per tube of blue artillery in the theater, $N_{FTB} : I \rightarrow \{0, \infty\}$ (#)
n_J	Number of times follow-on red echelons stop while traveling between their former staging area and their new staging area after the taking of a blue line, $n_J : I \rightarrow \{1, \infty\}$ (#)
n_{LI}	Number of predeployed blue defensive lines (dimensionless), $n_{LI} : R \rightarrow \{1, \infty\}$ (##)
n_{LLA}	Number of blue lines over which red can fire short-range ACM at withdrawing vehicles, constrained by supply of ammunition, $n_{LLA} : R \rightarrow \{n_{LITOT}\}$
n_{LIAH}	Number of blue defensive lines that would exist if red helicopters and ACM lasted the entire campaign, $n_{LIAH} : R \rightarrow \{n_{LI}, \infty\}$
n_{LICA}	Number of predeployed red flank defense lines (dimensionless), $n_{LICA} : R \rightarrow \{1, \infty\}$

Table B-1 (continued)

n_{LTOT}	Total blue defensive lines, predeployed and subsequently occupied (dimensionless), n_{LTOT} : $R \rightarrow [1, \infty)$
n'_{LTOT}	Estimated number of lines the red offensive will take, n'_{LTOT} : $R \rightarrow [1, \infty)$
n_{LP}	Number of blue defensive lines, after which red engages, he can pursue with helicopters, n_{LP} : $I \rightarrow [0, n_{LTOT}]$
n_{LPi}	Number of blue defensive lines formed by extra AFVEs that survive withdrawal after the exhaustion of pursuing red attack helicopters, n_{LPi} : $R \rightarrow (0, \infty)$
n_{LP2}	Number of blue defensive lines formed by extra AFVEs that survive withdrawal after the exhaustion of pursuing red attack helicopters and short-range ACM, n_{LP2} : $R \rightarrow (0, \infty)$
$\hat{n}_M$	Number of FASCAM mines scattered behind withdrawing blue vehicles by red, per blue line taken (scaled to two-kilometer benchmark frontage), $\hat{n}_M$: $R \rightarrow [0, \infty)$
p	Number of passes over a target made by a given air mission, p : $I \rightarrow [1, \infty)$ (#)
P_{AC}	Payload of area munitions carried by a CAS aircraft (metric tons), P_{AC} : $R \rightarrow [0, \infty)$ (\$)
Q_1	Instrumental quantity, related to red casualties per average assault
Q_2	Instrumental quantity, related to red casualties per average assault
Q_{10}	Instrumental quantity, related to C_{CA}

Table B-1 (continued)

Q_{200}	Instrumental quantity, related to time of culmination of red attack	
Q_{202}	Instrumental quantity, related to time of culmination of red attack	
Q_{203}	Instrumental quantity, related to time of culmination of red attack	
Q_{204}	Instrumental quantity, related to time of culmination of red attack	
Q_{205}	Instrumental quantity, related to time of culmination of red attack	
R	Red maneuver forces in theater (AFVEs), R: $R \rightarrow (0, \infty)$ (#)	
R_{ART}	Red artillery in theater (tubes), R_{ART} : $R \rightarrow [0, \infty)$ (#)	
R_{ARTECH}	Red artillery supporting a single assault echelon (AFVEs), R_{ARTECH} : $R \rightarrow [0, \infty)$	
$\hat{R}_{ARTECH}$	Red artillery supporting a single assault echelon (tubes, scaled to two-kilometer benchmark frontage), $\hat{R}_{ARTECH}$: $R \rightarrow (0, \infty)$	
R_{ARTLI}	Red artillery supporting a single defensive line (tubes), R_{ARTLI} : $R \rightarrow [0, \infty)$	
$\hat{R}_{ARTLI}$	Red artillery supporting a single defensive line (tubes, scaled to two-kilometer benchmark frontage), $\hat{R}_{ARTLI}$: $R \rightarrow [0, \infty)$	
$\hat{R}_{ARTOM}$	Red artillery supporting the last red defensive line (tubes, scaled to two-kilometer benchmark frontage), $\hat{R}_{ARTOM}$: $R \rightarrow (0, \infty)$	
R_{BPT}	Red residual maneuver strength at which a single assault echelon will break off an attack (AFVEs), R_{BPT} : $R \rightarrow [0, \rho_{MAX}]$ (#)	

Table B-1 (continued)

R_{ECH}	Red single assault echelon maneuver force initial strength (AFVEs), R_{ECH} : $R \rightarrow [0, \infty)$
$\hat{R}_{ECH}$	Red single assault echelon maneuver force strength (AFVEs, scaled to two-kilometer benchmark frontage), $\hat{R}_{ECH}$: $R \rightarrow (0, \infty)$
r_H	Range of a belt SAM of a given side (kilometers), r_H : $R \rightarrow [0, \infty)$ (\$)
R_{LI}	Red maneuver forces defending a single defensive line (AFVEs), R_{LI} : $R \rightarrow$
$\hat{R}_{LI}$	Red maneuver forces defending a single defensive line (AFVEs, scaled to two-kilometer benchmark frontage), $\hat{R}_{LI}$: $R \rightarrow [0, \infty)$
R_{RSV}	Red contingency reserve for augmenting flank defenses (AFVEs), R_{RSV} : $R \rightarrow [0, \infty)$
$\hat{R}_{RSV}$	Estimated strength of red reserves (AFVEs), $\hat{R}_{RSV}$: $R \rightarrow [0, \infty)$
R_{RSVA}	Red airmobile reserves (AFVEs), R_{RSVA} : $R \rightarrow [0, \infty)$ (#)
$R_{OFV}(t)$	Red maneuver forces assigned to offensive use surviving at time t (AFVEs), $R_{OFV}(t)$: $R \rightarrow [0, \infty)$
R_{OFVST}	Red maneuver forces available for offensive use at time theater offensive begins (AFVEs), R_{OFVST} : $R \rightarrow (0, \infty)$
$R_{OM}(t)$	Red maneuver forces defending final defensive line at time t (AFVEs), $R_{OM}(t)$: $R \rightarrow [0, \infty)$
r_S	Range of a SHORAD of a given side (kilometer), r_S : $R \rightarrow [0, \infty)$ (\$)

Table B-1 (continued)

S_{AC}	Sortie rate of an individual aircraft of a given side (sorties per hour), S_{AC} : $R \rightarrow [0, \infty)$
S_{ACO}	Sortie rate of individual aircraft on a given side in the absence of enemy offensive counterair (sorties per hour), S_{ACO} : $R \rightarrow (0, \infty)$ (\$)
S_{ADAB}	Number of ADUs at an air base available to fire at a counterair flight, after enemy suppression, S_{ADAB} : $R \rightarrow [0, \infty)$
S_{ADB}	Number of ADUs in the SAM belt of a given side that are available to fire on interdiction flights, S_{ADB} : $R \rightarrow [0, \infty)$
S_{ADC}	Number of ADUs to which a CAS flight is exposed, S_{AC} : $R \rightarrow [0, \infty)$
S_{ADFB}	Number of effective ADUs to which a red interdiction mission is exposed per pass over the FLOT, S_{ADFB} : $R \rightarrow [0, \infty)$
S_{ADFR}	Number of effective ADUs to which a blue interdiction mission is exposed per pass over the FLOT, S_{ADFR} : $R \rightarrow [0, \infty)$
S_{ADT}	Number of ADUs at the target of an interdiction mission available to fire upon a single flight, S_{ADT} : $R \rightarrow [0, \infty)$
S_{CAS}	Blue CAS sortie rate (sorties per hour), S_{CAS} : $R \rightarrow [0, \infty)$
S_{CASF}	Total blue CAS sortie rate, limited by available aircraft or airspace management limits (sorties per hour), S_{CASF} : $R \rightarrow [0, \infty)$
S_{INT}	Average total sortie rate for interdiction aircraft for a given side (sorties per hour), S_{INT} : $R \rightarrow [0, \infty)$
S_{MAXB}	Maximum total blue CAS sortie rate imposed by airspace management limits (sorties per hour), S_{MAXB} : $R \rightarrow [0, \infty)$

Table B-1 (continued)

S_{OCA}	Total average offensive counterair sortie rate of a given side (sorties per hour), $S_{OCA}: R \rightarrow [0, \infty)$
t	Time (hours, measured from initiation of theater offensive), $t: R \rightarrow [0, \infty)$
$\bar{t}$	Average time required for blue reserve AFVEs to reach a defensive position, $\bar{t}: R \rightarrow [0, \infty)$
t^*	Time red culminating point is reached (hours, measured from initiation of theater offensive), $t^*: R \rightarrow (0, \infty)$
$t^{*'}$	Estimated time of culmination of the red offensive (hours), $t^{*'}: R \rightarrow (0, \infty)$
$t_1^{*'}$	First estimate of time of culmination of red offensive (hours), $t_1^{*'}: R \rightarrow (0, \infty)$
t_{CA}^*	Time blue counterattack reaches culminating point (hours, measured from blue counterattack jump-off), $t_{CA}^*: R \rightarrow (0, \infty)$
$t_{CA}^{*'}$	Estimated time of culmination of the blue counterattack, $t_{CA}^{*'}: R \rightarrow (0, \infty)$
t_{BPREP}	Time required to prepare blue reinforcement positions for combat (hours, measured from blue reinforcement arrival), $t_{BPREP}: R \rightarrow [0, \infty)$ (#)
t_{BRKB}	Estimated time that blue will break through red flank rear defense line if not halted (hours, measured from blue counterattack jump-off), $t_{BRKB}: R \rightarrow [0, \infty)$
t_{BRKR}	Estimated time that red will break through blue theater rear defense line if not halted (hours, measured from initiation of theater offensive), $t_{BRKR}: R \rightarrow [0, \infty)$

Table B-1 (continued)

t_{BST}	Time that blue begins to move reserves toward point of attack (hours, measured from initiation of theater offensive), $t_{BST}: R \rightarrow (-\infty, \infty)$ (#)
t_{CA}	Time (hours, measured from blue counterattack jump-off), $t_{CA}: R \rightarrow [0, \infty)$
t_D	Delay imposed on blue reserve movement by red interdiction (AFVE-hours), $t_D: R \rightarrow [0, \infty)$
t_{DECH}	Total delay imposed by blue interdiction aircraft on red follow-on echelons per blue line taken (AFVE-hours), $t_{DECH}: R \rightarrow [0, \infty)$
t_{DR}	Rate at which red interdiction aircraft imposed delay on blue reserve AFVEs moving in the rear area (AFVE-hours per hour), $t_{DR}: R \rightarrow [0, \infty)$
t_E	Time available for blue to conduct engineering before the start of the red offensive (hours), $t_E: R \rightarrow [0, \infty)$ (\$)
T_{EAM}	Number of artillery tubes equivalent to the delivery of one metric ton of area munitions per hour (tubes/(metric tons per hour)), $t_{EAM}: R \rightarrow [0, \infty)$ (\$)
t_{EC}	Time of exposure of red echelons to blue CAS during the taking of single blue line (hours), $t_{EC}: R \rightarrow [0, \infty)$
t_{EXP}	Time of exposure of red follow-on echelons to blue interdiction during the taking of a single blue line (hours), $t_{EXP}: R \rightarrow [0, \infty)$
t_{JO}	Time blue counterattack jumps off (hours, measured from initiation of theater offensive), $t_{JO}: R \rightarrow [0, \infty)$

Table B-1 (continued)

t_{MV}	Time required for uncommitted assault echelon to complete approach march (hours, measured from completion of preparation), t_{MV} : $R \rightarrow [0, \infty)$
t_{OPREP}	Time required to prepare uncommitted assault echelon to begin approach march (hours, measured from termination of preceding echelon's assault), t_{OPREP} : $R \rightarrow (-\infty, \infty)$ (#)
t_{REAC}	Reaction time between blue's detection of the movement of follow-on red echelons and his launching of interdiction missions against them (hours), t_{REAC} : $R \rightarrow [0, \infty)$ (\$)
t_{RPREP}	Time required to prepare red flank reinforcement positions for combat (hours, measured from red reinforcement arrival), t_{RPREP} : $R \rightarrow [0, \infty)$ (#)
t_{RST}	Time that red begins to move reserves toward point of counterattack (hours, measured from initiation of blue counterattack), t_{RST} : $R \rightarrow (-\infty, \infty)$ (#)
t_{SD}	Time spent over the FEBA by a single defensive CAS mission (hours), t_{SD} : $R \rightarrow [0, \infty)$ (\$)
t_{SO}	Time spent over the FEBA by a single offensive or area munition CAS mission (hours), t_{SO} : $R \rightarrow [0, \infty)$ (\$)
V	Assault velocity (kilometers per hour), V : $R \rightarrow (0, \infty)$ (##)
$\bar{V}$	Average speed of blue reserves in the rear area (kilometers per hour), $\bar{V}$: $R \rightarrow (0, \infty)$ (##)
V_A	Tactical assault velocity of attacking echelons (kilometers per hour), V_A : $R \rightarrow (0, \infty)$ (#)

Table B-1 (continued)

V_{CA}	Counterattack assault velocity (kilometers per hour), $V_{CA}: R \rightarrow (0, \infty)$ (#)
V_{RSV}	Reserve road march velocity (kilometers per hour), $V_{RSV}: R \rightarrow (0, \infty)$ (#)
w	Fraction of maneuver forces defending a given line to be withdrawn (dimensionless), $w: R \rightarrow [0, 1]$ (##)
w_{SURV}	Fraction of maneuver forces defending a given line that survive withdrawal (dimensionless) $w_{SURV}: R \rightarrow [0, 1]$
w_{SURVF}	Fraction of withdrawing blue vehicles that survive the red FASCAM field, $w_{SURVF}: R \rightarrow [0, 1]$
w'_{SURV}	Fraction of withdrawing blue AFVEs that survive withdrawal in the face of ACM but not attack helicopters, $w'_{SURV}: R \rightarrow [0, 1]$
$\bar{a}$	Instrumental quantity, related to C_{CA}
$\bar{b}$	Instrumental quantity, related to C_{CA}
$\bar{c}$	Instrumental quantity, related to C_{CA}
α	Scalar multiple representing the decrease in attacker casualties in a given assault as a result of early termination of defensive fire upon withdrawal, relative to a fight to the finish under otherwise identical circumstances (dimensionless), $\alpha: R \rightarrow (0, 1]$
β	Scalar multiple representing the increased slope of the attacker's casualty-velocity tradeoff frontier as a result of the availability to the defender of additional barrier preparation labor not organic to the defending maneuver units themselves (dimensionless), $\beta: R \rightarrow [1, \infty)$
γ	Scalar multiple representing the increase in attacker casualties in a given assault as a result of entropy induced by the lead echelon's advance through defended depth prior to the assault in question, relative to an attack conducted with perfect coherence under otherwise identical circumstances (dimensionless), $\gamma: R \rightarrow [1, \infty)$

Table B-1 (continued)

δ_{1B}	Number of red AFVEs killed by blue short-range ACM per blue line taken (scaled to two-kilometer benchmark frontage), $\delta_{1B}: R \rightarrow [0, \infty)$
δ_{2B}	Blue CAS contribution (red AFVE kills per assault per two kilometers), $\delta_{2B}: R \rightarrow [0, (\hat{R}_{ECH} - \delta_{1B})] \text{ (#)}$
δ_{1R}	Number of defending blue AFVEs killed by red short-range ACM per blue line taken (scaled to two-kilometer benchmark frontage), $\delta_{1R}: R \rightarrow [0, \hat{B}_{LI}]$
δ_{2R}	Red CAS contribution (blue AFVE kills per assault per two kilometers), $\delta_{2R}: R \rightarrow [0, (\hat{B}_{LI} - \delta_{1R})] \text{ (#)}$
δ_3	Blue BAI long-range ACM contribution (red AFVE kills per hour), $\delta_3: R \rightarrow [0, R_{OFVST}] \text{ (#)}$
δ_4	Red BAI long-range ACM contribution (blue AFVE kills per hour), $\delta_4: R \rightarrow [0, B_{RSV}] \text{ (#)}$
δ_B	Number of red AFVEs killed by blue short-range ACM per blue line taken (scaled to two-kilometer benchmark frontage), $\delta_B: R \rightarrow [0, \infty)$
ϵ_A	Number of enemy ADUs eligible to fire of a single friendly offensive counterair flight that are suppressed by friendly EW aircraft, $\epsilon_A: R \rightarrow [0, 1)$
ϵ_{FB}	Number of unsuppressed active red ADUs, at the FLOT, in an interdiction corridor, suppressed by blue EW aircraft, $\epsilon_{FB}: R \rightarrow [0, \infty)$
ϵ_{FR}	Number of unsuppressed, active blue ADUs at the FLOT in an interdiction corridor, suppressed by red EW aircraft, $\epsilon_{FR}: R \rightarrow [0, \infty)$
ϵ_S	Number of SAMs in the belt of a given side that are suppressed by EW aircraft accompanying an enemy interdiction flight, $\epsilon_S: R \rightarrow [0, \infty)$

Table B-1 (continued)

η	Ratio of the effectiveness of offensive counterengineering efforts to defensive engineering efforts (dimensionless), $\eta: \mathbf{R} \rightarrow [0, \infty)$ (\$)
θ	The effective number of engineering squad-hours available to blue before the start of the red offensive, $\theta: \mathbf{R} \rightarrow [0, \infty]$
θ_F	Equivalent gross engineering squad-hours provided by blue use of FASCAM (squad-hours), $\theta_F: \mathbf{R} \rightarrow [0, \infty)$
θ_O	Total number of engineering squad-hours available to blue before the start of the red offensive, in the absence of red counterengineering, $\theta_O: \mathbf{R} \rightarrow [0, \infty)$
λ_{ATK}	Length of red theater attack frontage (kilometers), $\lambda_{ATK}: \mathbf{R} \rightarrow [0, \lambda_{THR})$ (##)
λ_{CA}	Length of blue counterattack frontage (kilometers), $\lambda_{CA}: \mathbf{R} \rightarrow (0, \infty)$ (##)
λ_{LOC}	Length of red frontage required as clear channel to resupply and reinforce assault elements (kilometers), $\lambda_{LOC}: \mathbf{R} \rightarrow (0, \lambda)$
λ_{THR}	Length of theater (kilometers), $\lambda_{THR}: \mathbf{R} \rightarrow (0, \infty)$ (#)
μ_{BS}	Contribution of blue CAS and short-range ACM on defense (red AFVE kills per red assault per two kilometers), $\mu_{BS}: \mathbf{R} \rightarrow [0, \hat{R}_{ECH}]$
μ_{BS2}	Contribution of blue CAS and short-range ACM on offense (red AFVE kills per blue assault per two kilometers), $\mu_{BS2}: \mathbf{R} \rightarrow [0, \hat{R}_{IJ}]$
μ_{RS}	Contribution of red CAS and short-range ACM on offense (blue AFVE kills per red assault per two kilometers), $\mu_{RS}: \mathbf{R} \rightarrow [0, \hat{B}_{IJ}]$
μ_{RS2}	Contribution of red CAS and short-range ACM on defense (blue AFVE kills per blue assault per two kilometers), $\mu_{RS2}: \mathbf{R} \rightarrow [0, \hat{B}_{ECH}]$

Table B-1 (continued)

ξ	Number of bulldozer-hours available to blue before the start of the red offensive, ξ : $R \rightarrow [0, \infty)$
π	Number of belt SAMs in an interdiction corridor available to fire on a single interdiction flight that are avoided through enemy mission planning, π : $R \rightarrow [0, \infty)$
ρ_{AC}	Maximum aircraft density over the FEBA (aircraft per kilometer ²), ρ_{AC} : $R \rightarrow (0, \infty)$ (\$)
ρ_{ARTB}	Ratio of blue artillery tubes to maneuver AFVEs in the theater, ϵ_{ARTB} : $R \rightarrow [0, \infty)$ (#)
ρ_{ARTR}	Ratio of red artillery tubes to maneuver AFVEs, ρ_{ARTR} : $R \rightarrow [0, \infty)$ (#)
ρ_{EB}	Ratio of blue engineering squads to maneuver AFVEs in the theater, ρ_{EB} : $R \rightarrow [0, \infty)$ (#)
ρ_{ER}	Ratio of red engineering squads to maneuver AFVEs in the theater, ρ_{ER} : $R \rightarrow [0, \infty)$ (#)
ρ_{FLK}	Density of red flank defense (AFVEs per kilometer), ρ_{FLK} : $R \rightarrow [0, \infty)$
ρ_{MAX}	Maximum maneuver force density for single assault echelon at point of attack (AFVEs per kilometer), ρ_{MAX} : $R \rightarrow (0, \infty)$ (#)
ρ_{MIN}	Minimum maneuver force density required by red away from point of attack (AFVEs per kilometer), ρ_{MIN} : $R \rightarrow [0, \infty)$
ρ_{SAM}	Ratio of SAMs in the belt of a given side to maneuver AFVEs, ρ_{SAM} : $R \rightarrow [0, \infty)$ (#)
ρ_{SB}	Ratio of blue SHORADs to maneuver AFVEs, ρ_{SB} : $R \rightarrow [0, \infty)$ (#)
ρ_{SR}	Ratio of red SHORADs to maneuver AFVEs, ρ_{SR} : $R \rightarrow [0, \infty)$ (#)

Table B-1 (continued)

σ_B	Number of red ADUs at the FLOT in an interdiction corridor suppressed by blue off-axis artillery, $\sigma_B: R \rightarrow [0, \infty)$
σ_R	Number of blue ADUs at the FLOT in an interdiction corridor suppressed by red off-axis artillery, $\sigma_R: R \rightarrow [0, \infty)$
ϕ_{AFR}	Fraction of red artillery ammunition in the theater that is FASCAM, $\phi_{AFR}: R \rightarrow [0, 1]$ (#)
ϕ_{CA}	Fraction of blue reserve AFVEs allocated to counterattack (dimensionless), $\phi_{CA}: R \rightarrow [0, 1]$ (##)
ϕ_{EU}	Fraction of engineering assets organic to divisions (as opposed to corps or armies), $\theta_{EU}: R \rightarrow [0, 1]$ (#)
ϕ_{EW}	Ratio of ADUs in an interdiction corridor that are suppressed by electronic warfare, to the number of aircraft in a single interdiction flight, $\phi_{EW}: R \rightarrow [0, \infty)$ (#)
ϕ_{FWD}	Fraction of blue AFVEs deployed forward (dimensionless), $\phi_{FWD}: R \rightarrow [0, 1]$ (##)
ϕ_{HP}	Fraction of red attack helicopters used to pursue withdrawing blue AFVEs, $\phi_{HP}: R \rightarrow [0, 1]$ (#)
ϕ_{LACM}	Fraction of an attacking echelon that is immune to ACM by being dismounted infantry, $\phi_{LACM}: R \rightarrow [0, \phi_i/2]$
ϕ_{INF}	Sum of fraction of red and fraction of blue maneuver AFVEs that are infantry (dimensionless), $\phi_{INF}: R \rightarrow [0, 2]$ (#)
ϕ_{INFB}	Fraction of blue maneuver AFVEs that are infantry (dimensionless), $\phi_{INFB}: R \rightarrow [0, 1]$ (#)
ϕ_{INFCA}	Sum of fraction of counterattacking blue and defending red AFVEs that are infantry (dimensionless), $\phi_{INFCA}: R \rightarrow [0, 2]$
ϕ_{INFR}	Fraction of red maneuver AFVEs that are infantry (dimensionless), $\phi_{INFR}: R \rightarrow [0, 1]$ (#)

Table B-1 (continued)

ϕ_{RS}	Fraction of SHORADs of a given side that are radar-homing SAMs, ϕ_{RS} : $R \rightarrow [0,1]$ (#)
ϕ_{SO}	Fraction of SHORADs of a given side that are operational at any given moment, ϕ_{SO} : $R \rightarrow [0,1]$ (\$)
ϕ_T	Sum of fraction of red and fraction of blue maneuver AFVEs that are tanks (dimensionless), ϕ_T : $R \rightarrow [0,2]$
ϕ_{WF}	Fraction of withdrawing blue AFVEs that survive the red FASCAM field, ϕ_{WF} : $R \rightarrow [0,1]$
χ	Casualties incurred by attacking forces in the taking of a single defended line due to CAS missions employing area munitions (AFVEs, scaled to two-kilometer benchmark frontage), χ : $R \rightarrow [0,\infty)$
ψ_{AT}	Reserve arrival rate produced by air transportation for a given side (AFVEs per hour), ψ_{AT} : $R \rightarrow [0,\infty)$
ψ_{BCAGCA}	Rate of change in blue counterattack force due to losses (AFVEs per kilometer of penetration), ψ_{BCAGCA} : $R \rightarrow (-\infty, 0]$
$\psi_{BCAST}(t)$	Rate of buildup of blue reserves for counterattack at time t (AFVEs per hour), $\psi_{BCAST}(t)$: $R \rightarrow (0,\infty)$
ψ_{BCAT}	Rate of change in blue counterattack force due to losses (AFVEs per hour), ψ_{BCAT} : $R \rightarrow (-\infty, 0]$
$\psi_{BOMT}(t)$	Rate of arrival of blue reserves on final line (AFVEs per hour), $\psi_{BOMT}(t)$: $R \rightarrow (0,\infty)$
ψ_{BOMTG}	Rate of arrival on the last defended line of blue reserve AFVEs moving on the ground (AFVEs per hour), ψ_{BOMTG} : $R \rightarrow [0,\infty)$
ψ_{BOMTO}	Rate of arrival on the last defended line of blue reserve AFVEs in the absence of air transportation (AFVEs per hour), ψ_{BOMTO} : $R \rightarrow \{0,\infty\}$
ψ_{RLII}	Number of red AFVEs killed by blue interdiction aircraft per blue line taken, ψ_{RLII} : $R \rightarrow [0,\infty)$

Table B-1 (continued)

ψ_{ROA}	Rate of red theater advance (km per hour), ψ_{ROA} : $R \rightarrow (0, \infty)$
ψ'_{ROA}	Estimated rate of red theater advance (km per hour), ψ'_{ROA} : $R \rightarrow (0, \infty)$
ψ_{ROACA}	Rate of blue counterattack advance (km per hour), ψ_{ROACA} : $R \rightarrow (0, \infty)$
ψ_{ROFVT}	Rate of change of available red attack forces (AFVEs per hour), ψ_{ROFVT} : $R \rightarrow (-\infty, 0]$
ψ_{ROFVTC}	Rate of change of available red attack forces due to blue CAS (AFVEs per hour), ψ_{ROFVTC} : $R \rightarrow (-\infty, 0]$
ψ_{ROFVG}	Rate of change in available red attack forces (AFVEs per kilometer of penetration), ψ_{ROFVG} : $R \rightarrow (-\infty, 0]$
$\psi_{ROM}(t_{CA})$	Rate of arrival of red reserves on final line at time t_{CA} (AFVEs per hour), $\psi_{ROM}(t_{CA})$: $R \rightarrow [0, \infty)$
ψ_{ROMO}	Red reserve arrival rate in the absence of blue air interdiction (AFVEs per hour), ψ_{ROMO} : $R \rightarrow [0, \infty)$ (#)

Appendix C

TESTS FOR VALIDITY

Mr. Stephen Biddle and Mr. David G. Gray

TESTS FOR VALIDITY

A. INTRODUCTION

Appendix B describes the VFM modifications undertaken to expand the range of weapon types explicitly considered by the model. Where possible, the equations in Appendix B were derived as the result of a process of hypothesis and test using the JANUS model (for a detailed description and motivation of this process, see IDA P-2380, Appendix D). For several of the weapon types addressed here, this was not possible in that the available version of the JANUS model could not support an examination of, for example, interdiction tacair or combat engineering. For military helicopters, however, JANUS was both suitable and essential. In particular, JANUS experiments were conducted for the use of attack helicopters in two roles: tactical defense and offensive pursuit. This appendix describes these experiments, and the results of the associated statistical analysis.

B. EXPERIMENTAL DESIGN AND PROCEDURE

The JANUS experiments were designed to examine two contentions relevant to the incorporation of attack helicopters into the theory described in Appendix C of IDA P-2380. If these contentions were false, the experiments were designed to disprove them. If not, the experiments would provide constant parameters that would then be incorporated both into the theory and into the VFM model. These contentions are:

- (1) that the slope of the casualty-velocity curve increases as the number of defending attack helicopters increases;
- (2) that the fraction of defenders who successfully withdraw decreases as the number of attack helicopters in pursuit increases and would increase as the number of defending air defense teams increases.

To test these contentions, a modified complete factorial design was employed. Two series of JANUS experiments were conducted. To test contention (1), 37 separate JANUS engagements were fought under seven unique scenarios comprising two different weapon mixes for attackers at four different assault velocities.¹ The defender weapon mix and the terrain file upon which the battle was fought were identical to those used in the bulk of the previous JANUS experiments.² To test contention (2), 36 separate JANUS engagements were fought under six unique scenarios comprising three different helicopter allocations for attackers and four different levels of defending air defense.³ To maintain consistency with the withdrawal runs conducted for P-2380, the basic scenario of the earlier runs was retained, with the addition of helicopters and air defense units.⁴

Casualty data from these engagements were then fit to several candidate functional forms consistent with those presented in P-2380. Falsification criteria consisted either of coefficient values outside the bounds implied by contentions (1) and (2) or of coefficient values that were inconsistent with the criteria presented in Appendix D of P-2380. In the event that the data failed to falsify, these coefficient values provided fitted constants for use in the modified VFM model.

In the series of JANUS experiments that examined the impact of attack helicopters used in the tactical defensive, the experimental procedure employed in the earlier study was retained. For each scenario (each JANUS scenario being a unique combination of force levels, weapon mixes, velocity and terrain), a variety of assault configurations were examined subject to the proviso that each configuration met the required velocity and represented a plausible use of forces in the context of known military doctrines. The lowest casualty configuration was accepted as the efficient assault for that velocity. Defenders were deployed along standard doctrinal lines.⁵ Defensive deployment was held constant across scenarios with a given defensive force composition and terrain sample. Although JANUS can be run as an interactive game, all experimental runs were conducted as closed simulations; i.e., movement and engagement orders, dismount points and artillery preparations were determined as scenario conditions and not altered during the course of experimental runs for the given scenario.

Velocity was defined as the distance to be covered by the assault (measured in kilometers from the jump-off point of the initial assault wave to the objective line), divided by the time required to cover the given distance and defeat the defenders on the objective.⁶ Attacker casualties were assessed as AFVEs lost prior to engagement termination, where termination was determined by the defeat of defenders on the position. "Defeat" was defined as the destruction of 60 percent or more of the

defending AFVEs. Elapsed time was measured from initiation of preparatory artillery fire to the arrival on the objective line of the first assault wave for which the defender defeat criterion had been met. Given that JANUS is a stochastic simulation, velocity by this definition can vary for individual runs within a scenario. Four broad classes of *attempted* velocities were considered, however: a slow case, in which available infantry were dismounted following 60 minutes of preparatory artillery by the accompanying artillery complement and four minutes of smoke preparation; a moderate case, in which infantry were dismounted following 12 minutes of preparatory artillery and four minutes of smoke; a fast case, in which a mounted assault followed four minutes of smoke and a two-volley suppressive artillery barrage; and a very fast case, representing a hasty attack in which a mounted assault proceeded directly from the march with only the support of suppressive artillery that could be brought to bear during the advance itself.⁷

In the series of JANUS experiments that examined the pursuit mission, the experimental procedure was somewhat different than that developed for P-2380. Whereas in the earlier study the engagement was terminated by the defender's withdrawal, in this series of JANUS experiments the engagement was allowed to continue as the attack helicopters flew their pursuit mission. Withdrawing vehicles were engaged by the pursuing helicopters, and the helicopters were engaged by air defense units and by defending vehicles in over-watch positions. Defender casualties were assessed as AFVEs lost after the commencement of withdrawal.⁸ Attacker casualties were assessed in terms of individual helicopters lost during the pursuit mission

C. RESULTS AND STATISTICAL ANALYSIS

The data on the defensive use of helicopters were compiled and analyzed using SAS on a VAX 8600. The data recorded during the pursuit scenarios were compiled separately and analyzed using Minitab on a WIN TurboAT personal computer.

The SAS procedure used for this study was PROC NLIN, a technique used by SAS to fit non-linear regression models by least squares.⁹ PROC NLIN uses the Gauss-Newton iterative process to estimate parameter values.¹⁰ With this method, initial values for the constants are estimated from the data. A linear approximation of the non-linear function is then generated

using these initial parameter estimates and the partial derivatives of the non-linear function with respect to these parameters. The principle of least squares is used to find, from the linear approximation, corrections to the initial estimates for the constants. These corrections are used to find new estimates for the parameter values, which in turn are used to generate a second linear approximation to the non-linear function. This second linear approximation is used to generate new corrections to the parameter estimates. The process is repeated until the error sum of squares is minimized.¹¹

1. Statistical Fits

The statistical analysis produced the following form for the casualty equation:

$$\text{casualties} (= C) = (B/R) * [(K3 * \text{inf} * \text{vel}) + (k2 * \text{helo} * \text{vel}) + (k4 * \text{tank}) + ((k1 * \text{Ba}) / (\text{vel} + .01)) + (K6 / (Ra * (\text{vel} + 1)))].$$

where:

inf	=	infa + infd
infa	=	fraction of attacker AFVEs that are infantry
infd	=	fraction of defender AFVEs that are infantry
tank	=	(2 - inf)
helo	=	number of defending attack helicopters
B	=	blue maneuver force strength (AFVEs)
R	=	red maneuver force strength (AFVEs)
vel	=	assault velocity (kilometers per hour)
Ba	=	blue artillery strength (tubes)
Ra	=	red artillery strength (tubes).

The treatment presented earlier in this Appendix and in Appendix D of P-2380 implies the following falsification criteria:

$k3 < 0$ (implying that casualties do not increase with velocity),

- $k1 < = 0$ (implying that the slope of the casualty-velocity curve does not decrease as the defender's artillery fraction increases),
- $k6 < = 0$ (implying that the slope of the casualty-velocity curve does not increase as the attacker's artillery fraction increases),
- $k2 < = 0$ (implying that the slope of the casualty-velocity curve does not increase as the defender's number of attack helicopters increases).¹²

SAS produced the following statistics regarding the constants $k3$, $k2$, $k1$, $k6$, and $k4$:

parameter	estimate	std.err	t-ratio ¹³
$k3$	14.32	0.42	34.10
$k1$	0.22	0.26	0.85
$k6$	693.63	103.92	6.67
$k4$	11.80	2.23	5.29
$k2$	1.45	0.35	4.14

sum of squares (corrected total): 206623.6
 error sum of squares: 58378.1
 degrees of freedom: 314
 adjusted R2:¹⁴ 0.708

Statistically, all fitted parameter estimates fall outside the falsification range identified above, with confidence in excess of the .01 level for $k2$, $k3$, $k4$, and $k6$; and with confidence in excess of the .4 level for $k1$. Thus, the observed experimental data tend to corroborate the hypothesized relationship.

In Figures C-1 and C-2, experimental results are plotted against casualty curves predicted by the functional form and coefficients given above. In both figures, data and curves are shown for two scenarios that differ only in the presence (or

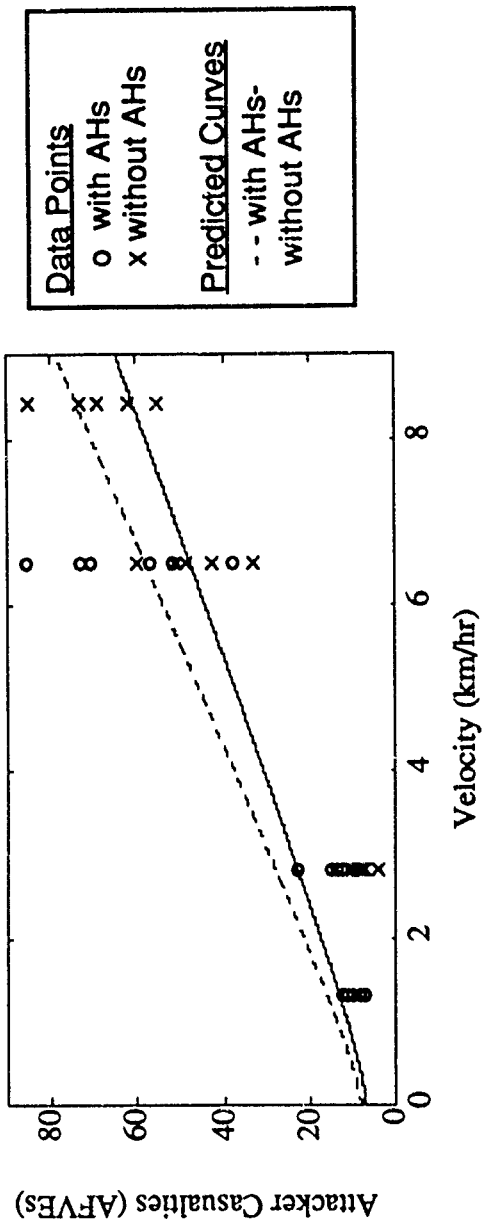


Figure C-1. Experimental Results: Effect of AHs on Infantry-Heavy Attacker

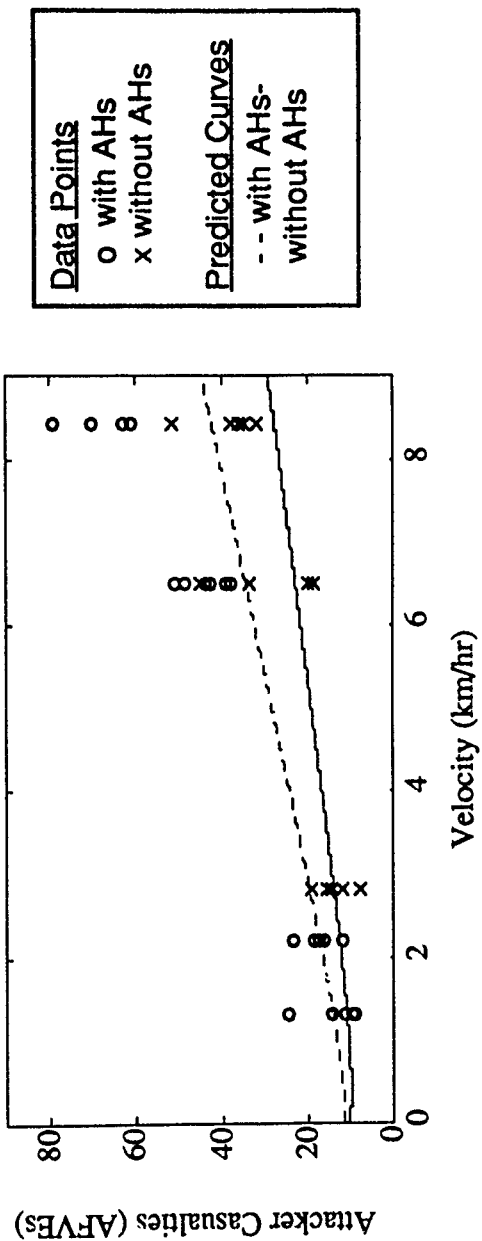


Figure C-2. Experimental Results: Effect of AHs on a Tank-Heavy Attacker

absence) of defending attack helicopters. Figure C-1 provides experimental results and predicted casualties for an infantry-heavy attack assaulting a balanced defender. Figure C-2 provides experimental results and predicted casualties for a tank-heavy attacker assaulting a balanced defender.

As stated above, the functional form that describes the interaction between casualties and withdrawal remains

$$\alpha = 1 - (w^k k^{10})$$

where:

α = fraction of fight-to-the-finish attacker casualties suffered when the defender withdraws a fraction of his strength, w

w = fraction of defending AFVE strength that a defender attempts to withdraw.

However, the assumption that all withdrawing vehicles successfully withdraw has been relaxed. The following functional form describes the relationship between the number of defending vehicles that successfully withdraw and the number that attempt to withdraw:

$$wdsur = 1.0 - ((k20 * hpur) / wdnit) \cdot ((k21 * hpur) / ad)$$

where:

$wdsur$ = fraction of withdrawn vehicles that successfully complete withdrawal

$hpur$ = number of attack helicopters in pursuit

$wdnit$ = number of vehicles that attempt to withdraw

ad = number of air defense teams defending against pursuing attack helicopters.¹⁵

The treatment earlier in this Appendix implies two falsification criteria:

- 1) $k20 < 0$;¹⁶
- 2) $k21 < 0$.¹⁷

Minitab generated the following statistics:

parameter	estimate	std. error	T-ratio
k20	0.188	0.032	5.95
k21	0.088	0.007	11.76

sum of squares (corrected total): 3.26
error sum of squares: 0.80
degrees of freedom: 47
adjusted R²: 0.756

The fitted parameter estimates fall outside the falsification range identified above, with confidence in excess of the .01 level for both k20 and k21. Thus, the observed experimental data tend to corroborate the hypothesized relationship.

In Figures C-3 and C-4, experimental results are plotted against curves predicted by the functional form described above. Figure C-3 depicts the fraction of successful withdrawals as a function of the number of pursuing helicopters, while the number of withdrawing vehicles and the number of defending air defense units are held constant at 19 vehicles and two air defense units, respectively. Figure C-4 shows the impact of increasing the number of air defense units as the number of withdrawing vehicles and the number of pursuing AHs are held constant at 19 vehicles and 16 AHs.

The incorporation of the pursuit mission into the VFM model also necessitated the development of a helicopter loss equation, which was used to estimate the total number of helicopters required to pursue multiple lines of withdrawing vehicles. This loss equation is based upon the JANUS experiments used to validate contention 2. The functional form is

$$hlos = 1.0 - \exp [- (k22 * wdinit / hpur) - (k23 * ad / hpur)]$$

where:

hlos = fraction of attack helicopters that are killed
hpur = number of attack helicopters in pursuit

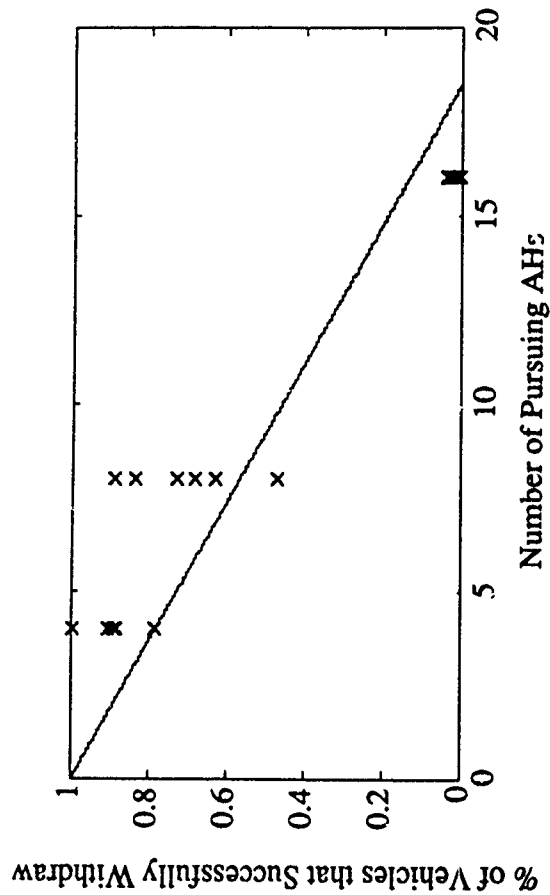


Figure C-3. Experimental Results: Effect of Helicopter Pursuit upon the Success of Withdrawal (number of ADUs held constant)

wdinit = number of vehicles that attempt to withdraw

ad = number of air defense teams defending against pursuing attack helicopters.

For this functional form, Minitab generated the following statistics:

parameter	estimate	std. error	T-ratio
k22	0.52	0.031	16.82
k23	0.75	0.043	17.64

sum of squares (corrected total): 282.01
error sum of squares: 20.16
degrees of freedom: 83
adjusted R²: 0.929

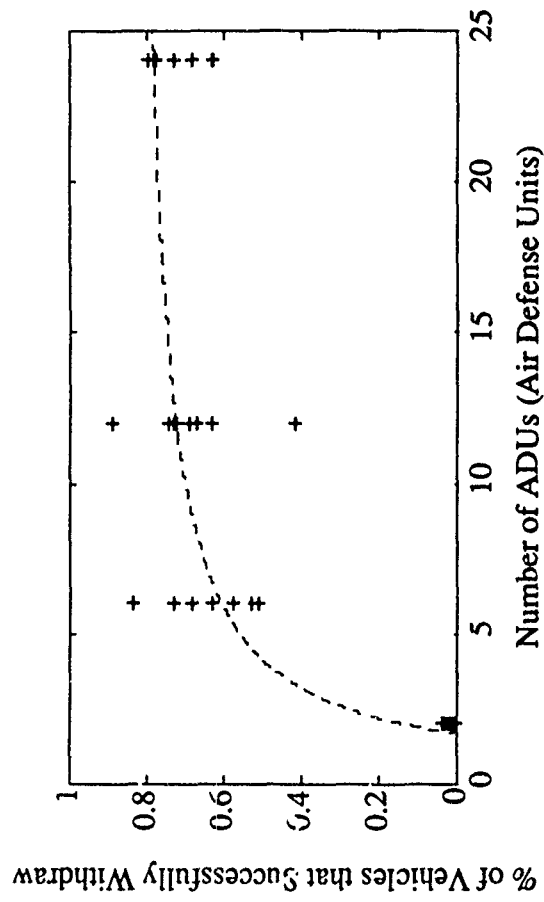


Figure C-4. Experimental Results: Effect of ADUs on the Helicopter Pursuit Mission (number of AHs held constant)

In Figures C-5 and C-6, experimental results are plotted against the helicopter attrition equation described above. Figure C-5 displays the relationship between helicopter attrition and the number of pursuing helicopters, with the number of withdrawing vehicles and the number of air defense units held constant at 19 vehicles and two air defense units, respectively. Figure C-6 depicts the impact upon attrition of increasing numbers of air defense units, with the number of withdrawing vehicles and the number of pursuing helicopters held constant at 19 vehicles and 16 AHs, respectively.

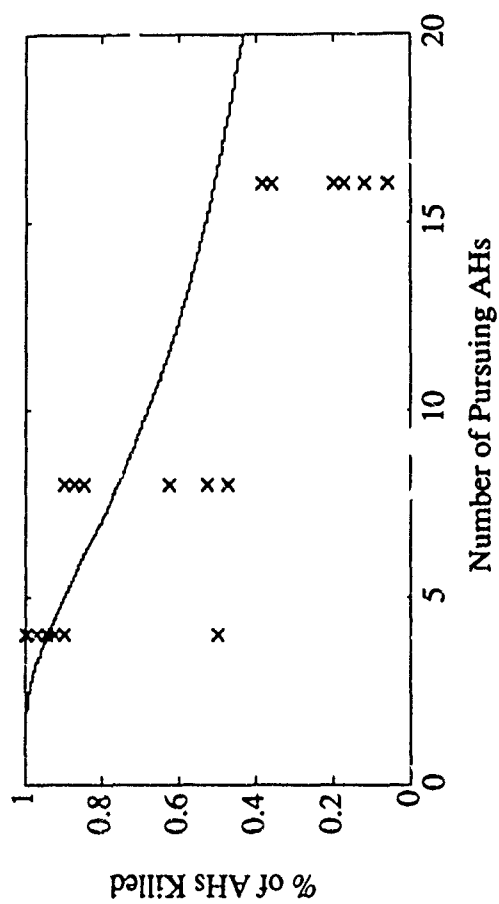


Figure C-5. Experimental Results: Relationship between AH attrition and Number of Pursuing AHs (number of ADUs held constant)

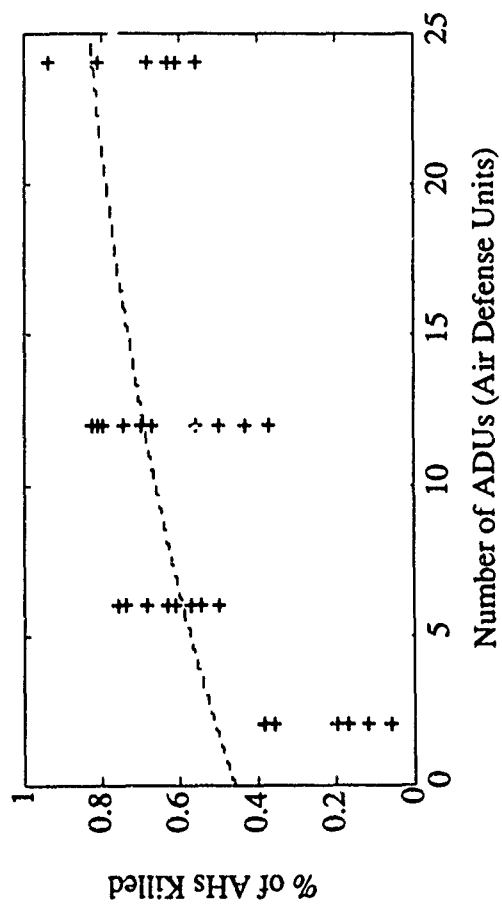


Figure C-6. Experimental Results: Effect of ADUs on Helicopter Attrition (number of pursuing AHs held constant)

ENDNOTES

- 1 Given constraints on available resources, it was decided to consider only the extreme cases of an infantry-heavy attacker and an armor-heavy attacker. Moreover, not all assault velocities were feasible alternatives for both attacker weapon mixes. For example, at high assault velocities, an infantry-heavy attacker could not successfully take a defensive line. See note 14, Appendix D, P-2380.
- 2 A prior examination of the offensive use of attack helicopters concluded that they would rarely be used against a stationary tactical defender. This conclusion was examined during the initial stages of JANUS testing. Several scenarios were constructed in which attack helicopters flew offensive missions in support of an assault upon a static defender. The outcome of these scenarios largely corroborated our initial conclusion that use of attack helicopters on such a mission did little to aid a tactical attacker. As a result, in the bulk of the JANUS engagements, attack helicopters were not employed by the attacker, and the two attacker weapon mixes examined in this study were identical to those used in the JANUS runs conducted for P-2380. As the earlier study had already examined the issue of variations in a defender's weapon mix given those attacker weapon mixes, it was decided that further consideration of variations in the defender's weapon mix was a secondary issue.
- 3 The JANUS scenarios that were run were those in which the attacker's helicopter allocation was varied. As will be seen, it was discovered that, as the number of attack helicopters in pursuit increases, both the fraction of defenders who successfully withdraw and the fraction of helicopters that are killed decrease. Consequently, an intelligent attack would always allocate the maximum possible number of helicopters to the pursuit mission. Given available resources, it was therefore decided to examine variations in air defense while holding helicopter allocation constant at the maximum value.
- 4 As the air defense units only engaged helicopters, and as the attack helicopters only engaged defending vehicles after they had begun to withdraw, the addition of these units did not change the interaction between defending units and attacking ground forces. Therefore, the statistical results for withdrawal presented in Appendix D of IDA P-2380 are still valid. The JANUS experiments conducted for this study, however, do eliminate the assumption that defending units that attempt to withdraw automatically do so successfully.
- 5 See, for example, Headquarters, Department of the Army, *FM 71-2, The Tank and Mechanized Infantry Battalion Task Force* (Washington, D.C.: USGPO, 30 June 1977), and Headquarters, Department of the Army, *FM 71-1, The Tank and Mechanized Infantry Company Team* (Washington, D.C.: USGPO, 22 November 1988).
- 6 Attacker jump-off points and objective lines were held constant across scenarios conducted on a given terrain sample.
- 7 As noted above, not all of these proved feasible for all scenarios.

- 8 To isolate the interaction among withdrawing units, helicopters and air defense units, defending units were opposed by non-firing attacking ground units. As the attack helicopters did not initiate their pursuit until after the defenders had begun to withdraw, no defenders were killed prior to withdrawal.
- 9 Although the final functional form is itself linear, the majority of functional forms that were explored by this study were non-linear. For this reason, it was necessary to use PROC NLIN.
- 10 The description given in the text, and the following, more mathematical explanation, are drawn from "SAS User's Guide: Statistics, Version 5 Edition," (Cary, N.C.: SAS Institute Inc., 1985), pp. 584 - 586.
For a general non-linear function $Y = F(B_0, B_1, B_2, \dots, B_k, X_0, X_2, \dots, X_j)$, in which $B_0 \dots B_k$ are the k parameters and $X_0 \dots X_j$ are the j independent variables, initial values for the parameters $B_0 \dots B_k$ are estimated from the data. Call these initial estimates $B_0' \dots B_k'$. The function Y is then approximated by the following Taylor series expansion of Y using $B_0' \dots B_k'$:
$$Y = F(B_0' \dots B_k', X_0 \dots X_k) + D_0(B_0 - B_0') + D_1(B_1 - B_1') + \dots + D_k(B_k - B_k')$$
where D_k , the partial derivative of Y with respect to the parameter B_k , is evaluated for $B_0 = B_0', B_1 = B_1', \dots, B_k = B_k'$. This approximation of Y is a linear function of the k variables $(B_0 - B_0'), (B_1 - B_1'), \dots, (B_k - B_k')$. Using the principle of least-squares, values $d_0 \dots d_k$ can be estimated for $(B_0 - B_1') \dots (B_k - B_k')$. These values, $d_0 \dots d_k$, represent corrections to the initial estimates for the parameters $B_0 \dots B_k$. SAS then calculates a second approximation to $B_0 \dots B_k$, namely $B_0'' = B_0' + d_0, B_1'' = B_1' + d_1, \dots, B_k'' = B_k' + d_k$. A new Taylor series expansion of Y is generated using $B_0'' \dots B_k''$ in place of $B_0' \dots B_k'$. This expansion of Y is used to estimate corrections to $B_0'' \dots B_k''$. These corrections, $d_1' \dots d_k'$, are used to calculate $B_0''' \dots B_k'''$. This iterating process is repeated until the error sum of squares for the i th iteration meets the criterion given in the following footnote.
- 11 For PROC NLIN, the iterating process is terminated if, for the i th iteration, the following condition has been met:
$$(SSE_{i-1} - SSE_i) / (SSE_i + 10^{-6}) < 10^{-8}.$$
- 12 This criterion derives from contention (1), which implies that if the partial derivative of C with respect to vel and $helo$ (i.e., $d_2C/d(vel)d(helo)$) were not monotonically positive, the result would tend to disconfirm. The form of this partial derivative is
$$d_2C/d(vel)d(helo) = k_2.$$
If k_2 is less than or equal to zero, the partial derivative will thus be negative, and imply disconfirmation.
- 13 This is the approximate T-ratio, calculated as $estimate/std.error$.
- 14 The meaning of the R^2 value is somewhat ambiguous for functional forms that have no constant term. The given R^2 is therefore illustrative, but not mathematically precise. The R^2 was calculated as $1 - (\text{error sum of squares}/\text{total sum of squares})$. The sum of squares has been corrected to account for the lack of a constant, and is identical to the true total sum of squares for the data.
- 15 One SAM or one AAA mounting is scored as one air defense team.
- 16 This criterion derives from contention (2), which implies that if the partial derivative of $wdsur$ with respect to $hpur$ were positive, the result would tend to disconfirm. The form of this derivative is
$$d(wdsur)/d(hpur) = - (k_{20}/wdinit) - (k_{21}/ad).$$
If either k_{20} or k_{21} (but not both) were negative, the partial derivative will thus be positive (since both $wdinit$ and ad are uniformly positive), and imply disconfirmation.

- 17 This criterion derives from contention (2), which implies that if the partial derivative of w_{dsur} with respect to ad were negative, the result would tend to disconfirm. The form of this derivative is
- $$d(w_{dsur})/d(ad) = (k_{21} * h_{pur})/(ad^2).$$
- If k_{21} were negative, the partial derivative will thus be negative (since both h_{pur} and ad are uniformly positive), and imply disconfirmation.