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MODELLING, DETECTION, AND CLASSIFICATION OF RANDOM UNDERWATER ACOUSTIC TRANSIENTS

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Modelling, Detection, and Classification of Random Underwater Acoustic Transients

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Abstract

Common solutions for the problem of random transient detection are based on the classical optimum detector for random signals in noise. However, such a detector does not explicitly use the non-stationary character of the signal as a priori available information. Reformulation of the optimum detection in the time-frequency plane allows one to exploit this distinguishing signal feature and suppress noise. This is accomplished here by use of the Wigner-Ville signal representation and an optimum signal/noise subspace decomposition that improves the transient signal to noise ratio. The new detection/classification procedure eliminates the subspace where major part of the energy of random noise sample will fall.

1. INTRODUCTION

The problem of random transient detection arises in the areas of underwater surveilance, seismic signal processing, biomedical signal processing, etc. Transients can be described as signals with duration that is short compared to the observation interval. They can be either deterministic finite energy signals of unknown form, or random finite duration non-stationary signals. The problems of detection and classification of finite duration random signals, such as the underwater acoustic transients, have standard solutions based on the classical optimum detectors of random signals in noise[1, 2, 3]. These detectors are not designed to particularly distinguish non-stationary signals from stationary noise background. Here, we shall show that, for the case of Gaussian transient detection in Gaussian noise, it is possible to take advantage of this distinguishing signal feature by reformulating the optimum detector in the time-frequency plane. This formulation is based on the Wigner-Ville signal representation and relies on the generalized singular value decomposition to optimally separate the signal and noise subspaces.

Classical optimum detection of Gaussian random signals in Gaussian noise is reviewed in Section 1. Then, in Section 2, the optimum detection procedure is reformulated in the the time frequency plane. The basic tool used here is the Wigner-Ville signal representation. Finally, in Section 3, the generalized singular value decomposition is used to generate an optimum stationary/non-stationary subspace decomposition. This is the basis for the new detector which then eliminates the subspace where major part of the energy of random noise sample will fall.

2. CLASSICAL OPTIMUM DETECTION

We consider the problem of detecting a zero-mean, non-stationary, Gaussian random signal in the zero-mean, stationary, colored, Gaussian noise background, i.e.:

$$\begin{aligned} H_0 &: \quad r = n \\ H_1 &: \quad r = n + s \end{aligned} \tag{1}$$

where r, n, s are N-dimensional vectors of samples representing the received waveform, random noise, and random signal, respectively. The noise is assumed to be composed of two independent components: colored interference and white noise background. Therefore, the noise has a full-rank $N \times N$ covariance matrix that can be expressed as

$$R_n = \sigma_c^2 R_c + \sigma_w^2 I \tag{2}$$

where σ_c^2, σ_w^2 are the variances of the interference and white noise components, respectively. The signal has $N \times N$ covariance matrix R_s which is possibly rank-defficient. It is further assumed that the signal and noise are uncorrelated and, therefore, statistically independent. Then, the detection problem can be rephrased in terms of the following two multivariate normal models:

$$\begin{aligned} H_0 &: \quad r : N(0, R_n) \\ H_1 &: \quad r : N(0, R_n + R_s) \end{aligned} \tag{3}$$

In words, the decision has to be made whether the received data vector is a realization of a multivariate zero-mean normal distributed random vector with the covariance matrix R_n or $R_n + R_s$.

This problem has a well-known classical solution [1, 3] that will be outlined next. We shall then reformulate this solution in the time-frequency domain, using the Wigner-Ville representation, following the approach in [4].

2.1. Classical formulation

Standard solution to the detection problem above, based on the likelihood ratio, compares the following sufficient test statistic[1, 3]:

$$\eta = r^T \left[R_n^{-1} - (R_n + R_s)^{-1} \right] r \underset{H_0}{\overset{H_1}{\gtrless}} t \tag{4}$$

with a threshold t determined according to a suitable optimization criterion, such as the Neyman-Pearson criterion, for example. It is preferable to 'diagonalize' the computation of this statistic by means of the generalized eigendecomposition of covariance matrices R_n, R_s. Namely, since R_n has full rank, there exist a non-singular $N \times N$ matrix V such that[5]

$$
\begin{aligned}
V^T R_s V &= \Lambda = diag(\lambda_i) \\
V^T R_n V &= \mathrm{I} = diag\{1\} \\
V &= \left[v^{(1)}\ v^{(2)}\ \ldots\ v^{(N)}\right]
\end{aligned}
\tag{5}
$$

where λ_i's are the generalized eigenvalues of R_s with respect to R_n, aranged in a non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_Q > 0$, $Q = rank(R_s) \leq N$, and the columns of V are the corresponding generalized eigenvectors $v^{(k)}$'s.

Now, a diagonalizir j transformation can be applied to the received data vector *r* (with dimensionality reduction, since $Q \leq N$)

$$
x = V^T r = \left[x_1\ x_2\ x_3\ \ldots\ x_Q\right]^T \tag{6}
$$

and the detection problem is transformed into a decision which of the following hypothesis is true

$$
\begin{aligned}
H_0 &: \quad x : N(0, \mathrm{I}) \\
H_1 &: \quad x : N(0, \mathrm{I} + \Lambda)
\end{aligned}
\tag{7}
$$

The set of generalized eigenvalues, $\{\lambda_i\}$, represent the distribution of signal-to-noise ratio over the coordinates defined by the generalized eigenvectors, $SNR = trace(\Lambda)$. The test statistic becomes

$$
\eta = x^T \left[\mathrm{I} - (\mathrm{I} + \Lambda)^{-1}\right] x = x^T \left[\Lambda(\mathrm{I} + \Lambda)^{-1}\right] x = x^T \Gamma x \quad \mathop{\gtrless}_{H_0}^{H_1} t \tag{8}
$$

The resulting detector performs the linear transformation, Eq. (6), of the received data, with the result being used in the quadratic form computation, Eq. (8). More explicitly, in terms of the components of vector *x* and the diagonal elements of Γ, Λ, the test statistic is

$$
\eta = \sum_{i=1}^{Q} \frac{\lambda_i}{1 + \lambda_i} x_i^2 = \sum_{i=1}^{Q} \frac{\lambda_i}{1 + \lambda_i} \left[r^T \cdot v^{(i)}\right]^2 \quad \mathop{\gtrless}_{H_0}^{H_1} t \tag{9}
$$

According to Eq. (9), the optimum receiver computes the *Q* squared correlations between the received data and the generalized eigenvectors, followed by their weighted summation. In this sum, the signal components along the coordinates with high signal-to-noise ratio have approximately the same weight ≈ 1, while the components along the coordinates with low signal-to-noise ratio have reduced weight < 1. The test statistic η

is a quadratic form in Gaussian variates with the conditional means and variances

$$
\begin{aligned}
E(\eta \mid H_0) &= trace(\Gamma) = \sum_{i=1}^{Q} \frac{\lambda_i}{1+\lambda_i} \\
E(\eta \mid H_1) &= trace[\Gamma(\mathrm{I}+\Lambda)] = \sum_{i=1}^{Q} \lambda_i \\
Var(\eta \mid H_0) &= 2\, trace(\Gamma^2) = 2\sum_{i=1}^{Q} \left(\frac{\lambda_i}{1+\lambda_i}\right)^2 \\
Var(\eta \mid H_1) &= 2\, trace\left[\Gamma^2(\mathrm{I}+\Lambda)^2\right] = 2\sum_{i=1}^{Q} \lambda_i^2
\end{aligned}
\tag{10}
$$

The probabilities of detection and false alarm, P_D and P_{FA}, are governed by the probability distribution of η and the threshold value. The distribution of the quadratic form η does not have a neat analytical form[2] and neither do the probabilities P_D, P_{FA}. In such a case, however, it is common to use the deflection signal-to-noise ratio as a helpfull gross indicator of the detector performance:

$$
D = \frac{[E(l \mid H_1) - E(l \mid H_0)]^2}{Var(l \mid H_0)} = \frac{\left[\sum_{i=1}^{Q} \frac{\lambda_i^2}{1+\lambda_i}\right]^2}{2\sum_{i=1}^{Q} \frac{\lambda_i^2}{(1+\lambda_i)^2}} \tag{11}
$$

In the high SNR case, most of the generalized eigenvalues are large, $\lambda_i \gg 1$, and

$$
D \approx \frac{1}{2Q}\left[\sum_{i=1}^{Q} \lambda_i\right]^2 = \frac{Q}{2}\,[Mean(SNR_i)]^2 \gg 1 \tag{12}
$$

When the SNR is very low, a situation known as a low energy coherence case or threshold detection, all generalized eigenvalues are small, $\lambda_i \ll 1$, and

$$
D \approx \frac{1}{2}\sum_{i=1}^{Q} \lambda_i^2 = \frac{Q}{2}\, Mean\,(SNR_i^2) \tag{13}
$$

In summary, the classical optimum receiver consists of a bank of correlators, followed by a weighted summer of their squared outputs (Eq. (9)), and the threshold comparator. Its performance is governed by the actual probability distributions of the quadratic form η under the two hypothesis. A gross indicator of the receiver performance is the deflection signal-to-noise ratio, Eq. (11).

2.2. Time-frequency formulation

The optimum detection procedure described in the previous section has an equivalent formulation in the time-frequency domain. This is of interest since the signal to be detected is non-stationary, a problem for which intuition suggests comparison of time-frequency signal representations[4].

The time-frequency domain equivalents of the inner products in the classical optimum receiver, Eq. (9), can be obtained using any bilinear transformation that preserves the inner products. Among such transformations, Wigner-Ville representation[6] has certain advantages that make it the most attractive for our purposes. Namely, it is the only one that has all of the following desirable properties[6, 7]: (1) it is real-valued; (2) it is invariant to time and frequency shifts; (3) it has the same region of support as the signal; (4) it preserves the inner products; (5) it has the proper marginals; (6) among all the bilinear representations satisfying (1)—(5) it is the one best localized in the time-frequency plane.

It should be noted, however, that these properties, while valid in the continuous-time case, are not generally valid for the discrete-time Wigner-Ville representation which is periodic in frequency with period π [6]. All of the above properties are valid only if the discrete-time signal spectrum is non-zero over an interval less than π on its fundamental period 2π [6]. Two practically important such cases are: (a) when the real-valued continous-time signal is oversampled by at least a factor of 2, and (b) when the signal is analytic, i.e. its spectrum vanishes for the 'negative' frequency half of the fundamental period 2π [6]. In the rest of this paper, it will be assumed that the signals of interest satisfy the aforementioned condition.

For signals of interest that have a finite discrete-time support region T of length $N = 2K+1$, i.e.$x(k) = y(k) = 0, \quad k \notin \mathrm{T}$, the discrete Wigner-Ville representation is given by

$$W_x(k,l) = 2 \sum_{m=-K}^{K} x(k+m)\, x^*(k-m)\, e^{-j\frac{2\pi}{M}lm} \tag{14}$$

where $k \in \mathrm{T}$, and $l \in \Omega$ represents $M \geq N$ frequency domain sampling points.

In the absence of aliasing, the following discrete-time discrete-frequency version of the Moyal's formula is obtained

$$\frac{1}{2M} \sum_{l\in\Omega} \sum_{k\in\mathrm{T}} W_{x_1x_2}(k,l)\, W^*_{x_3x_4}(k,l) = \left(x_1^H \cdot x_3\right)\left(x_2^H \cdot x_4\right)^* \tag{15}$$

For example, this form of the Moyal's formula is applicable to real-valued signals over-sampled by at least a factor of two, and to analytic signals. This is the equation that allows easy reformulation of the classical optimum detection procedure in the time-frequency plane. However, it is necessary to consider the Wigner-Ville representation of random signals[8] first.

Let x be a realization of a harmonizable zero-mean complex-valued discrete-time random process $\mathcal{X}$ with finite discrete-time region of support T of length $N = 2K+1$, e.g. finite length random transient. Then, for $k \in \mathrm{T}$

$$R_x(k+m, k-m) = 0, \qquad |m| > K \tag{16}$$

and the following expectation exists[8]

$$\begin{aligned}\mathcal{W}_{\mathcal{X}}(k,l) &= E\{W_x(k,l)\} = 2 \sum_{m=-K}^{K} E\{x(k+m)\, x^*(k-m)\}\, e^{-j\frac{2\pi}{M}lm} \\ &= 2 \sum_{m=-K}^{K} R_x(k+m, k-m)\, e^{-j\frac{2\pi}{M}lm}, \qquad \begin{matrix} k \in \mathrm{T} \\ l \in \Omega \end{matrix}\end{aligned} \tag{17}$$

and is called the Wigner-Ville spectrum of the process $\mathcal{X}$. Here, T, Ω are the discrete-time and discrete-frequency support regions, respectively.

Infinite duration random processes are usually observed over finite intervals, and their covariances may be known or estimated only on a grid of limited extent. In such a case, it is possible to obtain an approximation of the sampled Wigner-Ville spectrum, smeared one-dimensionally in the direction of frequency, by means of the discrete Wigner-Ville spectrum, Eq. (17).

For example, let $\mathcal{X}$ be a harmonizable infinite duration discrete-time random process observed on the interval $-2K \leq k \leq 2K$, with spectral support less then π and the covariance $R_x(p,q)$ known on the grid $-2K \leq p,q \leq 2K$. Then, $R_x(k+m, k-m)$ is completely known on the grid $-K \leq k, m \leq K$ and the sampled approximation of the Wigner-Ville spectrum can be computed as

$$\begin{aligned}\widetilde{\mathcal{W}_{\mathcal{X}}}(k,l) &= 2 \sum_{m=-\infty}^{\infty} g(m)\, R_x(k+m, k-m)\, e^{-j\frac{2\pi}{M}m} \\ &= 2 \sum_{m=-K}^{K} R_x(k+m, k-m)\, e^{-j\frac{2\pi}{M}lm}, \qquad l \in \Omega, \quad -K \leq k \leq K\end{aligned} \tag{18}$$

where $g(m)$ is the rectangular truncating window with $G(l)$ as its discrete Fourier transform. If the process $\mathcal{X}$ is also stationary, since there is no aliasing in the time-frequency plane, the Wigner-Ville spectrum equals the ordinary power spectrum, and its sampled smeared approximation is

$$\widetilde{\mathcal{W}_{\mathcal{X}}}(k,l) = G(l) \, * \, S_x(l) = \widetilde{S_x}(l), \qquad \begin{matrix} l \in \Omega \\ |k| \leq K \end{matrix} \tag{19}$$

In words, the finite-record approximation of the Wigner-Ville spectrum of an adequately oversampled stationary process is equal to the finite-record approximation of its ordinary power spectral density.

Now, the detection problem, Eq. (1), can be rephrased in the time-frequency plane as the choice between the hypotheses

$$\begin{aligned} H_0 &: \quad W_r = W_n \\ H_1 &: \quad W_r = W_n + W_s \end{aligned} \tag{20}$$

where W_n, W_s are the discrete Wigner-Ville representations of the particular noise and signal realizations.

We are ready now to transform the time-domain correlator based optimum receiver, Eq. (9), into the time-frequency domain correlator based receiver. The inner products in Eq. (9) can be replaced, according to the discrete Moyal's formula, Eq. (15), by

$$\left[r^T \cdot v^{(i)}\right]^2 = \frac{1}{2M} \sum_{l \in \Omega} \sum_{k \in \mathrm{T}} W_r(k,l)\, W_i(k,l) \tag{21}$$

where $W_r(k,l)$ is the Wigner-Ville representation of the received data vector and $W_i(k,l)$ are the Wigner-Ville representations of the generalized eigenvectors $v^{(i)}$. Using this result in Eq. (9), we get

$$\eta = \sum_{l \in \Omega} \sum_{k \in \mathrm{T}} W_r(k,l)\, B(k,l) \underset{H_0}{\overset{H_1}{\gtrless}} t \tag{22}$$

where $B(k,l)$ is the weighted sum

$$B(k,l) = \frac{1}{2M} \sum_{i=1}^{Q} \frac{\lambda_i}{1+\lambda_i} W_i(k,l) \tag{23}$$

which is fixed by the generalized eigendecomposition, Eq. (5), does not depend on the data vector r, and can be precomputed. If the slices of W_r and B for the fixed time-index k are labeled $w_k^T = [W_r(k,0) \;\; W_r(k,1) \;\; \ldots \;\; W_r(k,M-1)]$ and $b_k^T = [B(k,0) \;\; B(k,1) \;\; \ldots \;\; B(k,M-1)]$, respectively, the optimum test has a more explicit form

$$\eta = \sum_{k \in \mathrm{T}} b_k^T \cdot w_k \underset{H_0}{\overset{H_1}{\gtrless}} t \tag{24}$$

Accordingly, the time-frequency domain equivalent of the classical optimum receiver computes the slice by slice cross-correlation of the received data Wigner-Ville representation with the precomputed weighted sum of Wigner-Ville representations of the covariance matrix generalized eigenvectors. An alternative, expression for the optimum test is obtained by introducing the matrix notation $W_r = [W_r(k,l)]$, $B = [B(k,l)]$

$$\eta = trace\left\{B\, W_r^T\right\} = trace\left\{B^T\, W_r\right\} \underset{H_0}{\overset{H_1}{\gtrless}} t \tag{25}$$

In the reference [4], a similar reformulation of the continuous-time optimum detector has been presented for the case of white noise only.

The test statistic conditional expectations remain the same as those of the classical receiver, as given by Eqs. (10). It is interesting to relate those results with the Wigner-Ville spectra of signal and noise. Taking the conditional expectations of Eq. (25), using the Eqs. (17) and (18), and combining the results with Eqs. (10) we get

$$
\begin{aligned}
E(\eta \mid H_0) &= trace\left(B\,\widetilde{\mathcal{W}_{\mathcal{N}}^{T}}\right) = \sum_{i=1}^{Q} \frac{\lambda_i}{1+\lambda_i} \\
E(\eta \mid H_1) &= trace\left[B\left(\widetilde{\mathcal{W}_{\mathcal{N}}^{T}} + \mathcal{W}_{\mathcal{S}}^{T}\right)\right] = \sum_{i=1}^{Q} \lambda_i \\
Var(\eta \mid H_0) &= 2\,trace\left[\left(B\,\widetilde{\mathcal{W}_{\mathcal{N}}^{T}}\right)^2\right] = 2\sum_{i=1}^{Q}\left(\frac{\lambda_i}{1+\lambda_i}\right)^2 \\
Var(\eta \mid H_1) &= 2\,trace\left\{\left[B\left(\widetilde{\mathcal{W}_{\mathcal{N}}^{T}} + \mathcal{W}_{\mathcal{S}}^{T}\right)\right]^2\right\} = 2\sum_{i=1}^{Q}\lambda_i^2
\end{aligned}
\qquad (26)
$$

where $\widetilde{\mathcal{W}_{\mathcal{N}}}$ is the finite-record approximation of the Wigner-Ville spectrum of noise, as discussed in connection with Eq. (18), and $\mathcal{W}_{\mathcal{S}}$ is the discrete Wigner-Ville spectrum of a finite duration non-stationary signal, as in Eq. (17). The deflection signal-to-noise ratio is the same as before and given by Eqs. (11) — (13).

We see that an alternative time-frequency domain realization of the optimum receiver is possible. It matches optimality with physical interpretation provided by the use of the Wigner-Ville representation, which is particularly relevant for the detection of non-stationary signals. However, the receiver performance remains the same. The real significance of this time-frequency formulation is that it will allow incorporation of additional information that may bring improvement in performance, as will be shown in the next section.

3. SUBSPACE BASED DETECTION

Our objective is to use a priori available information that signal, as opposed to the noise, is non-stationary in order to suppress that background noise and thereby improve the deflection signal-to-noise ratio and detector performance. How do we accomplish this is discussed next.

3.1. Optimum signal/noise subspace decomposition

The Wigner-Ville spectrum of the stationary random signal and its finite record approximation, Eq. (19), are simple outer products of the power spectral density as a function of frequency and a constant function of time. Consequently, the matrix of samples of the finite record approximation of the Wigner-Ville spectrum of stationary noise, $\widetilde{\mathcal{W}_{\mathcal{N}}}$, is a unit-rank matrix. On the other hand, the discrete Wigner-Ville spectrum of the random transient, Eq. (17), represented by the matrix $\mathcal{W}_{\mathcal{S}}$, has rank that is generally much higher than one. This distinction can be exploited to suppress stationary noise and improve the detector performance.

The generalized singular value decomposition of the signal Wigner-Ville spectrum with respect to the noise Wigner-Ville spectrum provides a necessary tool to accomplish this. Assuming that $\widetilde{\mathcal{W}_{\mathcal{N}}}$ and $\mathcal{W}_{\mathcal{S}}$ are $N \times M$, $M \leq N$ matrices, then there exist a non-singular $M \times M$ matrix X and orthogonal matrices $U_s(N \times N)$, $U_n(N \times N)$ such that[5, 9]

$$
\begin{aligned}
&\mathcal{W}_{\mathcal{S}} = U_s D_s X^{-1} \\
&\widetilde{\mathcal{W}_{\mathcal{N}}} = U_n D_n X^{-1} \\
&D_s(N \times M) = diag(\alpha_i), \quad 0 \leq \alpha_1 \leq \alpha_2 \leq \ \dots \ \leq \alpha_M \\
&D_n(N \times M) = diag(\beta_i), \quad \beta_1 \geq \ \dots \ \geq \beta_q > \beta_{q+1} = \ \dots = \beta_M = 0, \\
&\alpha_i^2 + \beta_i^2 = 1, \qquad q = rank(\mathcal{W}_{\mathcal{N}})
\end{aligned}
\tag{27}
$$

where $x^{(i)}$, the columns of X, are the generalized singular vectors of $\widetilde{\mathcal{W}_{\mathcal{N}}}$ and $\mathcal{W}_{\mathcal{S}}$ satisfying

$$
\beta_i^2\, \mathcal{W}_{\mathcal{S}}^T \mathcal{W}_{\mathcal{S}}\, x^{(i)} = \alpha_i^2\, \mathcal{W}_{\mathcal{N}}^T \mathcal{W}_{\mathcal{N}}\, x^{(i)} \tag{28}
$$

Note that for the stationary noise $rank\left(\widetilde{\mathcal{W}_{\mathcal{N}}}\right) = q = 1$ and, therefore, only $\beta_1 \neq 0$.

Let's introduce the following matrix partitions

$$
\begin{aligned}
&X = \begin{bmatrix} x^{(1)} & X_1 \end{bmatrix}, \quad U_s = \begin{bmatrix} u_s^{(1)} & U_{s1} \end{bmatrix}, \\
&U_n = \begin{bmatrix} u_n^{(1)} & U_{n1} \end{bmatrix}, \quad D_s = \begin{bmatrix} \alpha_1 & 0 \\ 0 & D_1 \end{bmatrix}
\end{aligned}
\tag{29}
$$

where

$$
\begin{aligned}
&X_1 = X_1(\ M \times \ M - 1), \\
&D_1 = D_1(N - 1 \ \times \ M - 1) = diag(\alpha_2, \dots, \alpha_M) \\
&U_{s1} = U_{s1}(N \ \times \ N - 1), \\
&U_{n1} = U_{n1}(N \ \times N \ - 1)
\end{aligned}
\tag{30}
$$

Then,

$$
\begin{aligned}
&\mathcal{W}_{\mathcal{S}} X = U_s D_s = [\alpha_1 u_s^{(1)} \quad U_{s1} D_1] \\
&\widetilde{\mathcal{W}_{\mathcal{N}}} X = U_n D_n = [\beta_1 u_n^{(1)} \qquad O_1]
\end{aligned}
\tag{31}
$$

where O_1 is a $N \ \times M - 1$ zero matrix. More specifically,

$$
\begin{aligned}
&\mathcal{W}_{\mathcal{S}} X_1 = U_{s1} D_1 \\
&\widetilde{\mathcal{W}_{\mathcal{N}}} X_1 = O_1
\end{aligned}
\tag{32}
$$

i.e., X_1—transformation maps the Wigner-Ville spectrum of stationary colored noise into an all-zero matrix O_1. At the same time, since $\alpha_1 \leq \alpha_i$ is small, the signal Wigner-Ville spectrum is mapped with the minimum loss of signal power.

We can interpret these results as saying that all of the noise energy, together with a minimal fraction of signal energy, lie along the first generalized singular vector $x^{(1)}$, and there is no noise energy in the subspace determined by the remaining generalized singular vectors that constitute the columns of X_1[9]. Therefore, we can suppress noise by projecting the Wigner-Ville spectrum onto the subspace spanned by the columns of

X_1. To accomplish this, we need the subspace projection matrix that can be computed by the following stable procedure.

Quite often, the matrix X^{-1} is ill-conditioned and its inversion produces inaccurate generalized singular vectors. Since we are mainly interested here in the subspaces spanned by the generalized singular vectors, numerically more stable procedure[5], adequate for our purposes, is to use the *QR* decomposition of this matrix

$$X^{-T} = QR, \quad X^{-1} = R^T Q^T \tag{33}$$

Here R is an upper triangular matrix, and Q is an orthogonal matrix whose first column is the unit vector in the direction of the first generalized singular vector while the remaining columns define an orthonormal basis for the subspace spanned by the remaining generalized singular vectors $x^{(i)}$, $i = 2, 3, \ldots, M$. The generalized singular value decomposition, Eq. (31), now takes the form

$$\begin{aligned} \mathcal{W}_S Q &= U_s D_s\, R^T \\ \widetilde{\mathcal{W}_N} Q &= U_n D_n\, R^T \end{aligned} \tag{34}$$

If we use the partition

$$Q = [q^{(1)} \quad Q_1] \tag{35}$$

where $q^{(1)}$ is the first column of Q while Q_1 contains the remaining ones, then

$$P \equiv Q_1 Q_1^T \tag{36}$$

is the projection matrix onto the subspace spanned by the columns of X_1. Then, the projection of the noise Wigner-Ville spectrum onto this subspace equals zero,

$$\widetilde{\mathcal{W}_N} P = 0 \tag{37}$$

Therefore, using this projection operator the Wigner-Ville spectrum of the stationary noise is anihilated while that of the transient signal is barely affected.

3.2. Detection with noise subspace elimination

The generalized singular value decomposition of $\mathcal{W}_N$ and $\mathcal{W}_S$, Eq (27), thus generates an optimum stationary/non-stationary subspace decomposition, Eq (31), that will be the basis of the new detection scheme discussed next.

Rather than correlating the received data Wigner-Ville representation with a reference, as in Eq. (25), we propose to first use the noise suppressing projection onto the subspace spanned by the columns of X_1. It is easy to show that

$$\begin{aligned} H_0: &\quad E(W_r Q_1) = \widetilde{\mathcal{W}_N} Q_1 = O_1 \\ H_1: &\quad E(W_r Q_1) = \widetilde{\mathcal{W}_N} Q_1 + \mathcal{W}_S Q_1 \equiv Y \end{aligned} \tag{38}$$

Therefore, we propose to compute the test statistic

$$\begin{aligned} \eta &= trace\left\{(BQ_1)(W_r Q_1)^T\right\} \\ &= trace\left\{B_1 W_r^T\right\} \underset{H_0}{\overset{H_1}{\gtrless}} t_1 \end{aligned} \tag{39}$$

where t_1 is the threshold determined by a suitable criterion, while using P, the projection matrix onto the subspace spanned by the columns of X_1 (Eq. (36)), we have

$$B_1 \equiv B\left(Q_1 Q_1^T\right) = BP \tag{40}$$

The conditional means can be expressed as

$$\begin{aligned} E(\eta|H_0) &= trace\left[B_1 \mathcal{W}_{\mathcal{N}}^T\right] = 0 \\ E(\eta|H_1) &= trace\left[B_1(\mathcal{W}_{\mathcal{N}} + \mathcal{W}_S)^T\right] = trace\left(B_1 \mathcal{W}_S^T\right) \end{aligned} \tag{41}$$

However, the conditional variances cannot be brought into a neat form, but, intuitively we expect them to be reduced with respect to the classical case due to the elimination of noise subspace by the projection onto the X_1—subspace. This point is verified by Monte-Carlo simulations discussed next.

3.3. An example

As an illustration, we consider a case where signal and noise are specified according to Fig. 1. The signal autocovariance and Wigner-Ville spectrum are given in Figs. 1.a and 1.b, respectively. Power spectra of the signal, noise, and signal plus noise are given in Figs. 1.c, 1.d, and 1.e, while the Wigner-Ville spectrum of the noise is shown in Fig. 1.f. Narrowband interference is five times stronger than the white noise component. Overall signal to noise ratio is -7.8dB, while signal to white noise ratio is 0dB. Eigenspectra of the signal and noise covariance matrices are shown in Figs. 2.a and 2.b. Rank of the signal covariance is 14, while the noise covariance has full rank 32. Singular value spectra of the signal and noise Wigner-Ville spectra (WVS) are presented in Figs. 2.c and 2.d. Rank of the signal WVS is 31 and that of the noise WVS is 1. Generalized singular values of signal and noise WVS's and generalized eigenvalues of signal and noise covariances are shown in Figs. 2.e and 2.f. Finally, the corresponding WVS matching function B, Eq. (23), and the WVS subspace matching function B_1, Eq. (40), are displayed in Figs. 3.a and 3.b. Alternatively, the effect of projecting (filtering) the signal and noise WVS's onto the no-noise subspace is shown in Figs. 3.c and 3.d.

In order to verify the conjecture made at the end of the previous section, the operation of both the classical and the new detector on the sample vectors of signal and noise defined above was simulated with 500 Monte-Carlo runs. While this number of simulations is insufficient for accurate estimates of the probabilities of detection and false alarm, it was sufficient to estimate the deflection signal to noise ratios of two detectors. The estimated deflection signal to noise ratio gain obtained in this manner was 1.9 (or about 3dB), confirming our conjecture that the new detector provides additional noise suppression. Of course, this is far from being a definite proof of superiority of the new detector under all conditions. A more comprehensive set of simulations is necessary for more definite statement of such kind. It should be emphasized, however, that the noise suppression procedure described here is non-parametric, relies only on noise stationarity feature, and therefore should be quite robust providing gain under various different signal/noise scenarios.

4. SUMMARY

A new procedure for detection of Gaussian transients in stationary colored Gaussian noise was presented that exploits the signal non-stationarity as its distinguishing feature. The classical optimum detector for Gaussian random signal was first reformulated in the time-frequency domain using the Wigner-Ville representation. In this domain the distinction between non-stationary and stationary processes is easy to exploit. The generalized singular value decomposition of the signal and noise Wigner-Ville spectra generated the optimum signal/noise subspace decomposition. By an appropriate subspace mapping, the noise subspace was eliminated providing an extra degree of noise suppression. This point was verified by simulations for a particular set of conditions. Although the new procedure appears to be robust with respect to changing signal/noise situations, a more comprehensive set of simulations is needed for a definite conclusion. The approach advanced here can easily be extended to the optimal linear-quadratic detector for problems involving certain classes of non-Gaussian background noise[10].

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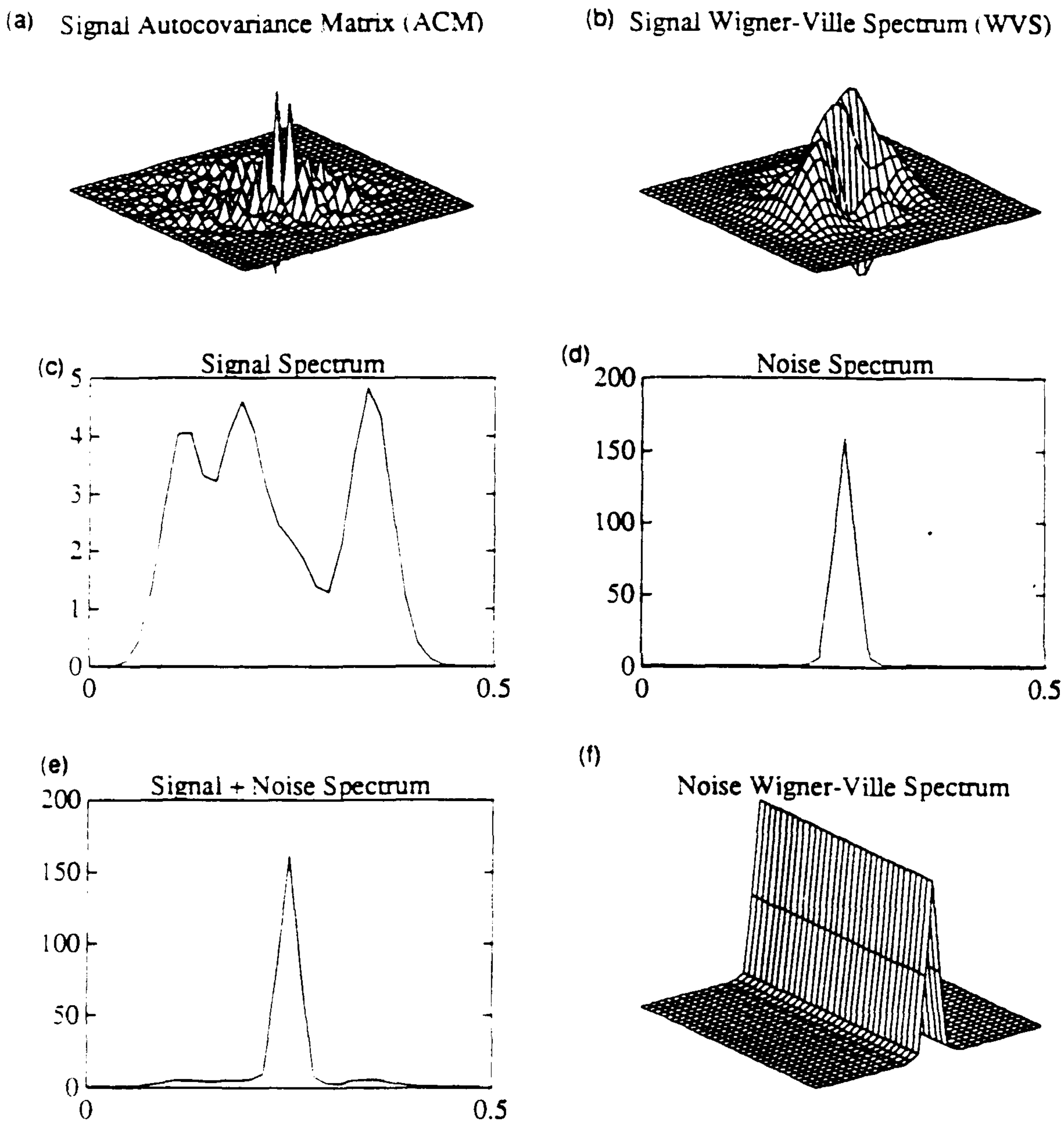
(a) Signal Autocovariance Matrix (ACM)
(b) Signal Wigner-Ville Spectrum (WVS)
(c) Signal Spectrum
5
4
3
2
1
0
0
0.5
(d) Noise Spectrum
200
150
100
50
0
0
0.5
(e) Signal + Noise Spectrum
200
150
100
50
0
0
0.5
(f) Noise Wigner-Ville Spectrum

Figure 1.

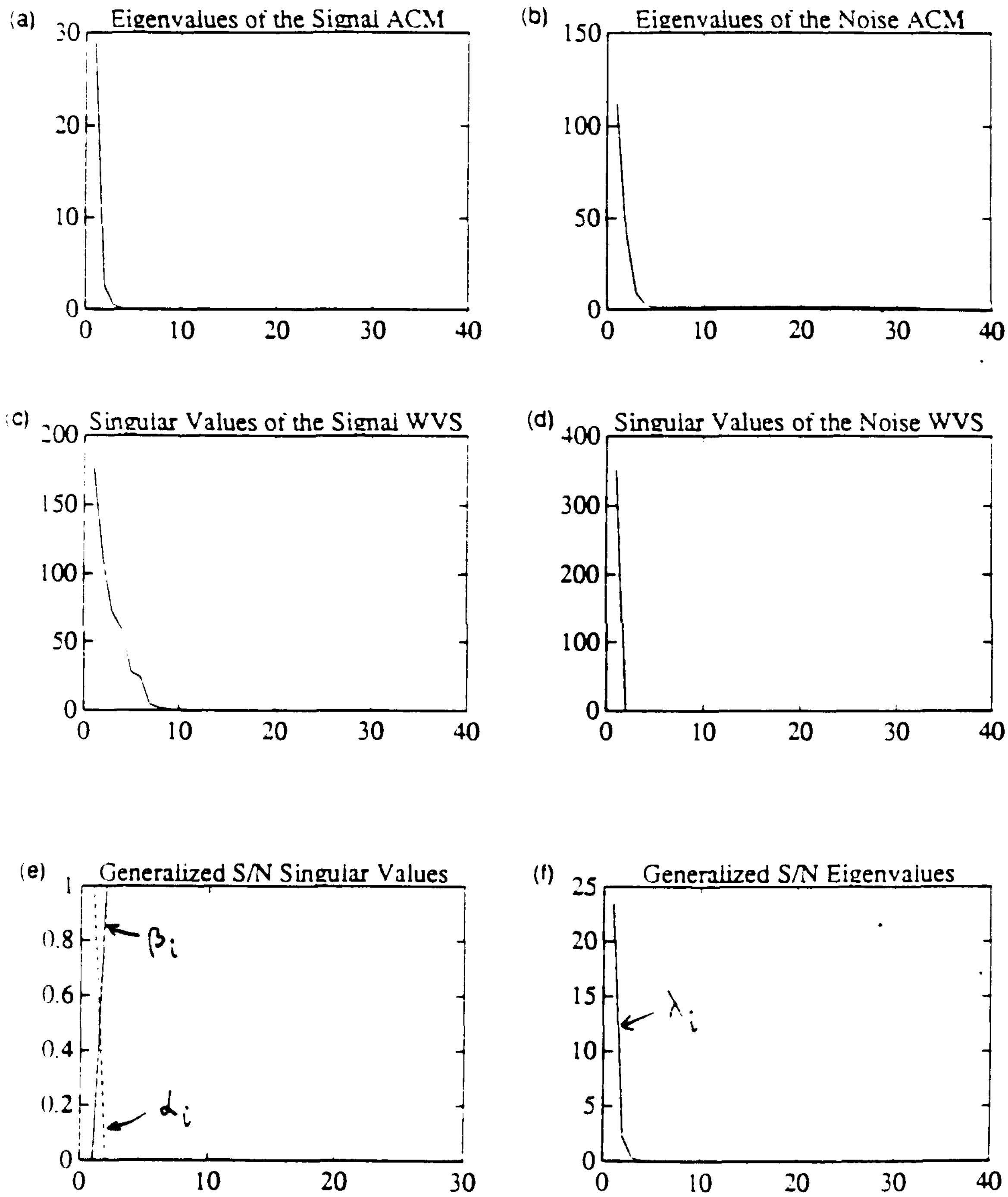
(a)
Eigenvalues of the Signal ACM
(b)
Eigenvalues of the Noise ACM
(c)
Singular Values of the Signal WVS
(d)
Singular Values of the Noise WVS
(e)
Generalized S/N Singular Values
β_i
α_i
(f)
Generalized S/N Eigenvalues
λ_i

Figure 2.

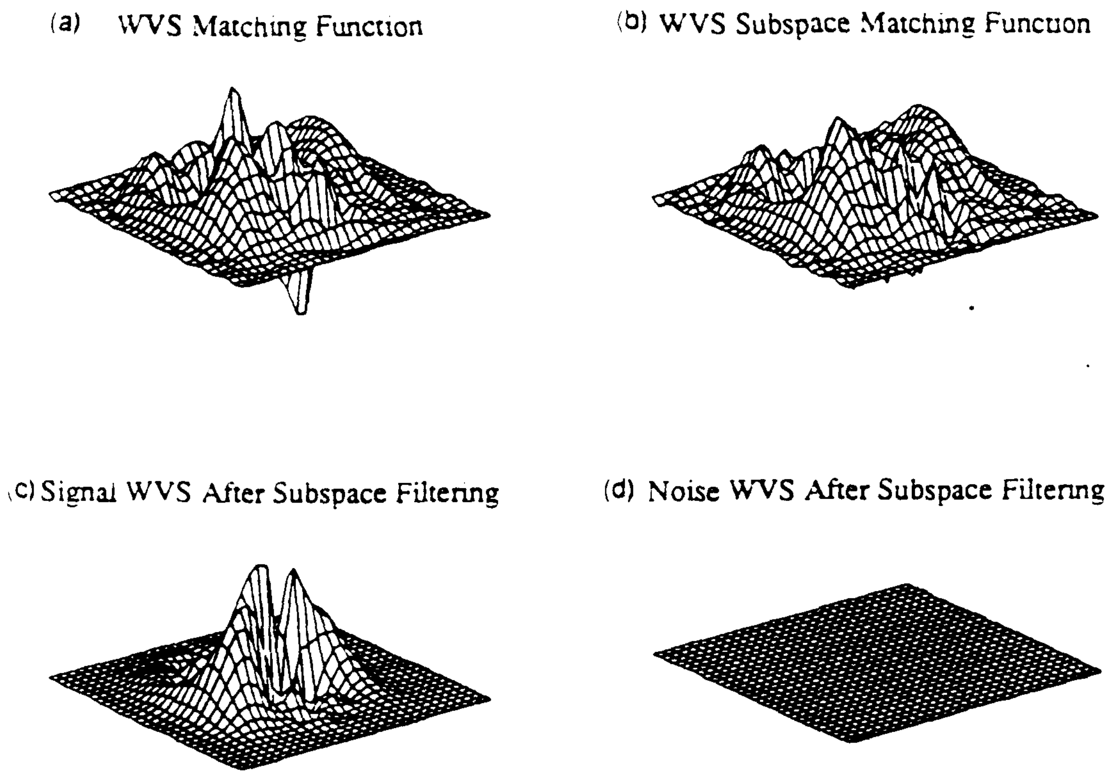
(a) WVS Matching Function
(b) WVS Subspace Matching Function
(c) Signal WVS After Subspace Filtering
(d) Noise WVS After Subspace Filtering

Figure 3.