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The validity of a common radiometeorological application of Monin-Obukhov (M-O) similarity theory to potential refractivity (χ), which is a nonlinear combination of θ and q, is determined by whether the properly nondimensionalized χ gradient is a universal function of $z \backslash L$. We develop expressions for the flux of χ (and its scaling parameter, χ_*) in terms of temperature and moisture fluxes, and an M-O similarity expression for the vertical χ gradient. Results show that even if θ and q are accepted as exactly following M-O similarity expressions, when the surface layer is stable, χ does not obey such an expression. That is, when properly nondimensionalized, the vertical gradient of χ does not collapse to a single universal function of z/L. The assumption that χ behaves as a similarity variable is approximately correct for well-mixed surface layers under unstable and near-neutral conditions.

The gradient of χ is an important factor in determining microwave propagation conditions. We demonstrate the error induced in a simple algorithm when χ is assumed to obey M-O similarity theory. An alternative methodology, consistent with the application of similarity theory to θ and q, is then developed without requiring that χ itself satisfy similarity theory.

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POTENTIAL REFRACTIVITY AS A SIMILARITY VARIABLE

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Abstract. The validity of a common radiometeorological application of Monin–Obukhov (M–O) similarity theory to potential refractivity (χ), which is a nonlinear combination of θ and q, is determined by whether the properly nondimensionalized χ gradient is a universal function of z/L. We develop expressions for the flux of χ (and its scaling parameter, χ_*) in terms of temperature and moisture fluxes, and an M–O similarity expression for the vertical χ gradient. Results show that even if θ and q are accepted as exactly following M–O similarity expressions, when the surface layer is stable, χ does not obey such an expression. That is, when properly nondimensionalized, the vertical gradient of χ does not collapse to a single universal function of z/L. The assumption that χ behaves as a similarity variable is approximately correct for well-mixed surface layers under unstable and near-neutral conditions.

The gradient of χ is an important factor in determining microwave propagation conditions. We demonstrate the error induced in a simple algorithm when χ is assumed to obey M–O similarity theory. An alternative methodology, consistent with the application of similarity theory to θ and q, is then developed without requiring that χ itself satisfy similarity theory.

1. Introduction

Microwave propagation near the sea surface depends primarily on the refractive structure of the marine atmospheric boundary layer. The refractivity, or refractive index, of air is related to its density and is a nonlinear function of temperature and humidity. Analysis and modeling of the mean temperature and moisture structures within the surface layer have a rich history, resulting in what are generally referred to as flux-profile relationships based on Monin–Obukhov (M–O) similarity theory (Monin and Obukhov, 1954). Since refractivity is so closely related to these similarity variables, it has been widely assumed that it can be treated as a similarity variable as well (e.g., Anderson and Gossard, 1953; Gossard, 1964; Brocks, 1965; Jeske, 1971; Hitney, 1975; Gossard, 1978; Gossard, 1981; Thompson, 1987; and Paulus, 1989). In particular, these authors all use similarity relationships involving potential refractivity (χ):

$$\chi = \frac{AP_0}{\theta}\left(1 + \frac{Bq}{\epsilon\theta}\right), \tag{1}$$

where, for microwaves with wavelengths greater than 1 cm, A and B are constants, equal to 77.6 °K mb^{-1} and 4810 °K, respectively (Bean and Dutton. 1966), P_0 = 1000 mb, θ is the potential temperature, q is the specific humidity, and $\epsilon = 0.622$.

Potential refractivity, rather than normal refractivity, is generally used because

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it has been felt that as a conservative scalar (like θ and q) it necessarily must also be a good variable for similarity theory applications. We shall demonstrate however, that χ cannot be treated as a true similarity variable in a scheme consistent with the treatment of θ and q. We shall assess the magnitude of the error one makes when assuming that similarity theory applies to χ and show that, for practical purposes, χ can be treated as a similarity variable with little error under unstable and near-neutral conditions. This may explain its widespread usage in this manner. Under stable conditions, however, χ deviates sufficiently from similarity theory to cause significant errors in its application.

2. Methodology

Using Reynolds averaging, the turbulent flux of a scalar quantity, f, at the surface is (Fleagle and Businger, 1980):

$$E_f = \bar{\rho}\overline{w'f'}\,,$$

where $\bar{\rho}$ is the surface layer atmospheric density, w' is the turbulent component of the vertical wind, and f' is the turbulent fluctuation of the scalar f. M–O similarity theory states that in the surface layer, the vertical gradient of such a scalar quantity is independent of the character of the surface (e.g., roughness) and is determined by the density $\bar{\rho}$, the "buoyancy parameter" g/θ_0, a measure of the turbulent shear stress u_*, a measure of the vertical heat flux θ_*, and E_f, the flux of f (see Monin and Yaglom, 1971, Section 7.2, for a discussion of why these particular parameters are relevant). Furthermore, the gradient may be described by

$$\frac{1}{f_0}\frac{\partial f}{\partial z} = F\left(\frac{z}{L}\right), \tag{2}$$

where $F(z/L)$ is a universal function of z/L. The Obukhov length is defined by $L = u_*^2\,\theta_0/gk\theta_*$ where g is the acceleration due to gravity and k is von Kármán's constant. Here f_0 has the dimensions of $\partial f/\partial z$ and based on the dimensions of the available parameters, we may write (Monin and Yaglom, 1971):

$$f_0 = \frac{E_f}{\bar{\rho}u_* L}\,,$$

and Equation (2) becomes

$$\frac{L}{f_*}\frac{\partial f}{\partial z} = F(\zeta)\,, \tag{3a}$$

where $\zeta \equiv z/L$. The scaling parameter, f_*, is defined by

$$f_* = \frac{E_f}{\bar{\rho} u_*}. \tag{3b}$$

In its more standard form, Equation (3a) is usually written

$$\frac{k\alpha z}{f_*}\frac{\partial f}{\partial z} = \frac{k\alpha z}{L} F(\zeta) \equiv \Phi(\zeta), \tag{4}$$

where $\Phi(\zeta)$ is another universal function and α is the ratio of the turbulent diffusivities of the scalar property f to momentum at neutral stability.

In the surface layer, nondimensional expressions for the potential temperature and specific humidity gradients have been derived from similarity theory (Monin and Obukhov, 1954; Businger *et al.*, 1971), and may be written

$$\frac{k\alpha z}{\theta_*}\frac{\partial \theta}{\partial z} = \phi(\zeta), \tag{5a}$$

$$\frac{k\alpha z}{q_*}\frac{\partial q}{\partial z} = \phi(\zeta), \tag{5b}$$

where θ_* and q_* are scaling parameters and $\phi(\zeta)$ is the universal function defined by

$$\phi(\zeta) = 1 + b\frac{z}{L}, \tag{5c}$$

for stable conditions where $b = 7$, and

$$\phi(\zeta) = \left(1 - a\frac{z}{L}\right)^{-1/2}, \tag{5d}$$

for unstable conditions where $a = 16$ (Liu *et al.*, 1979).

When the scalar property f equals potential refractivity χ, we wish to know whether Equation (4) is satisfied. However, to test Equation (4) we need the surface potential refractivity flux, E_χ. We now describe a method for relating the potential refractivity flux to the temperature and moisture fluxes.

If we assume that Equation (1) applies to instantaneous values of χ, and we decompose the dependent variables into means and turbulent perturbations (e.g., $\chi = \bar{\chi} + \chi'$, etc.), then we may write

$$\bar{\chi} + \chi' = \frac{AP_0}{\bar{\theta}}\left(1 + \frac{\theta'}{\bar{\theta}}\right)^{-1} + \frac{ABP_0\bar{q}}{\epsilon\bar{\theta}^2}\left(1 + \frac{q'}{\bar{q}}\right)\left(1 + \frac{\theta'}{\bar{\theta}}\right)^{-2}.$$

Expanding binomially, neglecting second-order and higher-order terms, and subtracting the mean, $\bar{\chi}$, yields

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$$\chi' \cong -\left(\frac{AP_0}{\bar{\theta}^2} + \frac{2ABP_0\bar{q}}{\epsilon\bar{\theta}^3}\right)\theta' + \left(\frac{ABP_0}{\epsilon\bar{\theta}^2}\right)q' .$$

Multiplying by $\rho w'$ and performing a Reynolds average gives the flux of χ as

$$E_\chi = \bar{\rho}\overline{w'\chi'} \cong -\left(\frac{AP_0}{\bar{\theta}^2} + \frac{2ABP_0\bar{q}}{\epsilon\bar{\theta}^3}\right)\bar{\rho}\overline{w'\theta'} + \left(\frac{ABP_0}{\epsilon\bar{\theta}^2}\right)\bar{\rho}\overline{w'q'} ,$$

and thus, from Equation (3b), the scaling parameter to be used in Equation (4) is

$$\chi_* \cong -\left(\frac{AP_0}{\bar{\theta}^2} + \frac{2ABP_0\bar{q}}{\epsilon\bar{\theta}^3}\right)\theta_* + \left(\frac{ABP_0}{\epsilon\bar{\theta}^2}\right)q_* . \tag{6}$$

We note that this scaling parameter can also be derived by taking the vertical derivative of Equation (1) and substituting Equation (5) for $\partial\theta/\partial z$ and $\partial q/\partial z$.

To determine whether χ behaves as a similarity variable, one would ideally use observational data to verify Equation (4). Unfortunately, accurate, abundant marine observations of χ are not available so we developed a numerical procedure to test the relationship. This approach has the advantage of eliminating consideration of random observational errors from the analysis. In our procedure, the integrated form of Equation (5) from Liu *et al.* (1979) was used to compute similarity profiles of θ and q at $\Delta z = 0.1$ m intervals from 0 to 40 m above the surface, from which we obtain χ using Equation (1). $\partial\chi/\partial z$ at each level was then calculated using centered finite differences with truncation error of $O(\Delta z^4)$. We calculate χ_* by evaluating Equation (6) using $\bar{\theta}$ and $\bar{q}$ at the reference height, taken in this paper to be 2 m. Then we compute the left-hand-side of Equation (4), with $f = \chi$, at each grid point for a wide range of stability. The results are then plotted as a function of ζ to see if the nondimensionalized χ gradient is a universal function of ζ.

3. Results

Figure 1 shows the characteristic surface-layer profile curvatures of $\Delta\theta$, Δq, and $\Delta\chi$, which were computed from Equations (5) and (1) for both stable and unstable conditions. Difference quantities were chosen to eliminate scale differences between the stable and unstable cases. The unstable profiles in Figure 1a were computed with arbitrarily chosen values of sea surface temperature equal to 18 °C, and reference height values of air temperature equal to 12 °C, wind speed equal to 3.6 m s^{-1}, and relative humidity equal to 80%. The pressure at the reference height was set at 1000 mb and was computed hydrostatically away from the reference height. These profiles show characteristic logarithmic shapes reflecting sharp gradients near the sea surface and nearly constant values up into the well-mixed surface layer. Under stable conditions, such as those found near the Gulf Stream

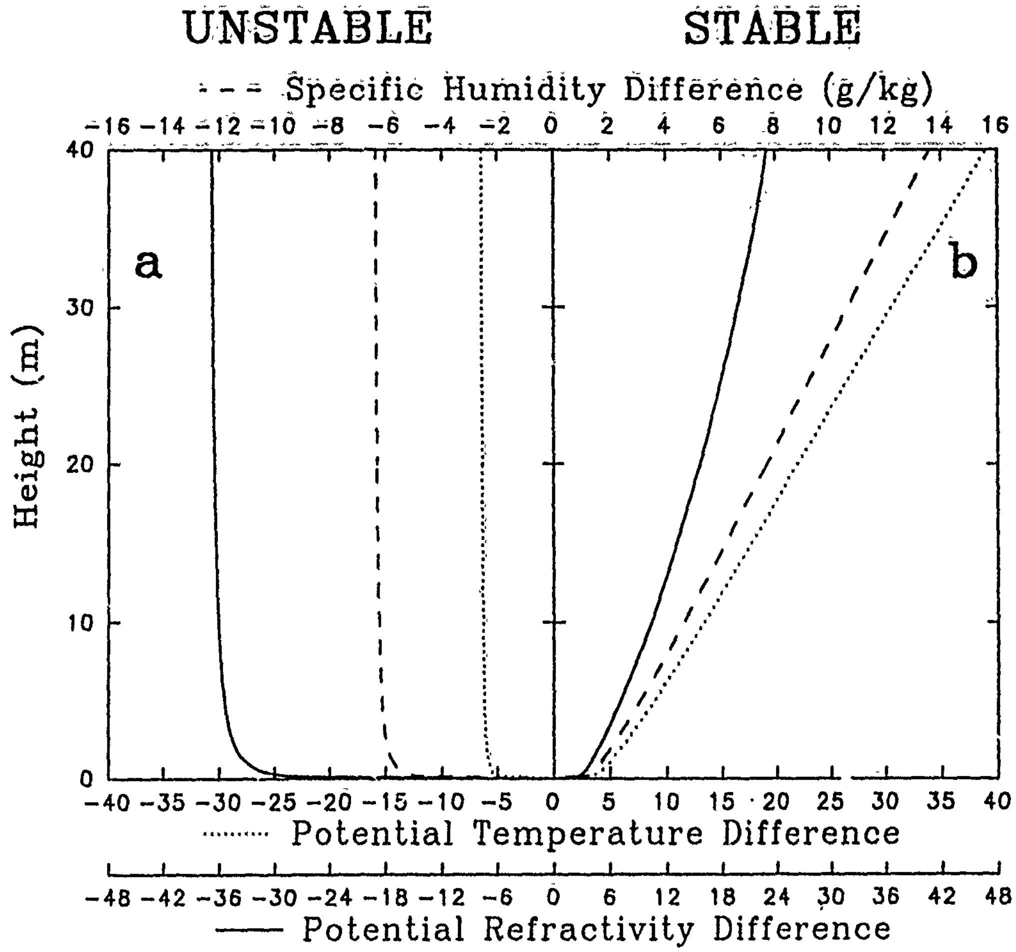

Fig. 1. (a) Profiles under unstable conditions of potential temperature, specific humidity, and potential refractivity difference between the profile at height z and the sea surface. Computed for sea surface temperature equal to 18 °C, and reference height values of air temperature equal to 12 °C, relative humidity equal to 80%, and wind speed equal to 3.6 m s^{-1}. (b) Same as Figure 1a except for stable conditions with air temperature equal to 24 °C.

(Ross, 1981; Hayes, 1981) or in the North Sea (Keller *et al.*, 1989) when warm air advects over cooler water, the θ and q profiles in Figure 1b show surface-based inversions with constant slopes above a shallow interfacial layer; the slope of the χ profile is, however, not constant. These profiles were computed with the same parameters as above except for reference height air temperature equal to 24 °C. Since x is a nonlinear function of θ and q, its vertical gradient is not constant even though the vertical gradients of θ and q are. Under unstable conditions, θ and q are nearly constant with height, and thus so is χ.

Figure 2 shows $\Phi_\chi(\zeta)$ as computed from Equation (4) with $f = \chi$, and $\phi(\zeta)$ from Equation (5) for many different profiles over a wide range of stability. The reference-height air-sea temperature difference was varied from $-1.5°$ to $+6$ °C in 0.25 °C steps while sea surface temperature, wind speed, and relative humidity were fixed at the values cited above. This figure shows that under stable conditions,

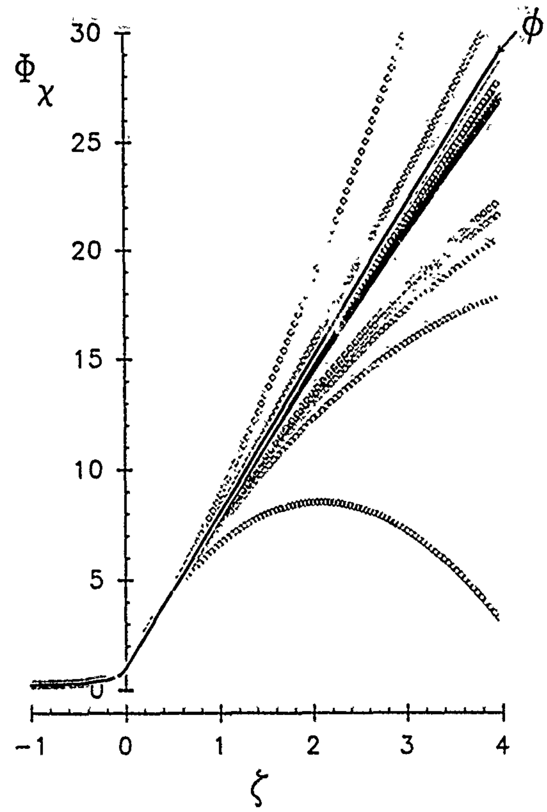

Fig. 2. Profiles of the $\Phi_\chi(\zeta)$ (squares) and $\phi(\zeta)$ (solid line) for near-neutral and stable conditions. The environmental conditions correspond with those in Figure 1 except here the air-sea temperature difference has been varied from −1.5° to +6 °C in 0.25 °C increments. The lines made by superposition of the squares are members of a family of Φ_χ curves.

Φ_χ is clearly not a universal function of ζ; that is, the Φ_χ curves do not fall on top of one another. Φ_χ and ϕ are nearly coincident under unstable and near neutral conditions.

Although χ is a function of two variables, θ and q, which satisfy M–O similarity theory, χ itself does not satisfy the M–O hypothesis [Equation (4)] because of its nonlinear functional dependence. Virtual potential temperature, θ_v, is another nonlinear combination of θ and q to which similarity theory is frequently applied. To investigate whether this assumption is justified, we repeated our experiment, computing θ_v instead of χ. Our results (not shown) verify that θ_v closely follows a universal profile under all of the environmental conditions we investigated.

Applications of χ as a similarity variable are often used to assess the height (δ) of a critical potential refractivity gradient $(\partial\chi/\partial z)_c$ by using Equation (4) with z replaced with δ, $\partial\chi/\partial z$ replaced with $(\partial\chi/\partial z)_c$, and assuming $\Phi_\chi = \phi$ (Anderson and Gossard, 1953; Gossard, 1964; Brocks, 1965; Jeske, 1971; Hitney, 1975; Gossard, 1978; Gossard, 1981; Thompson, 1987; and Paulus, 1989):

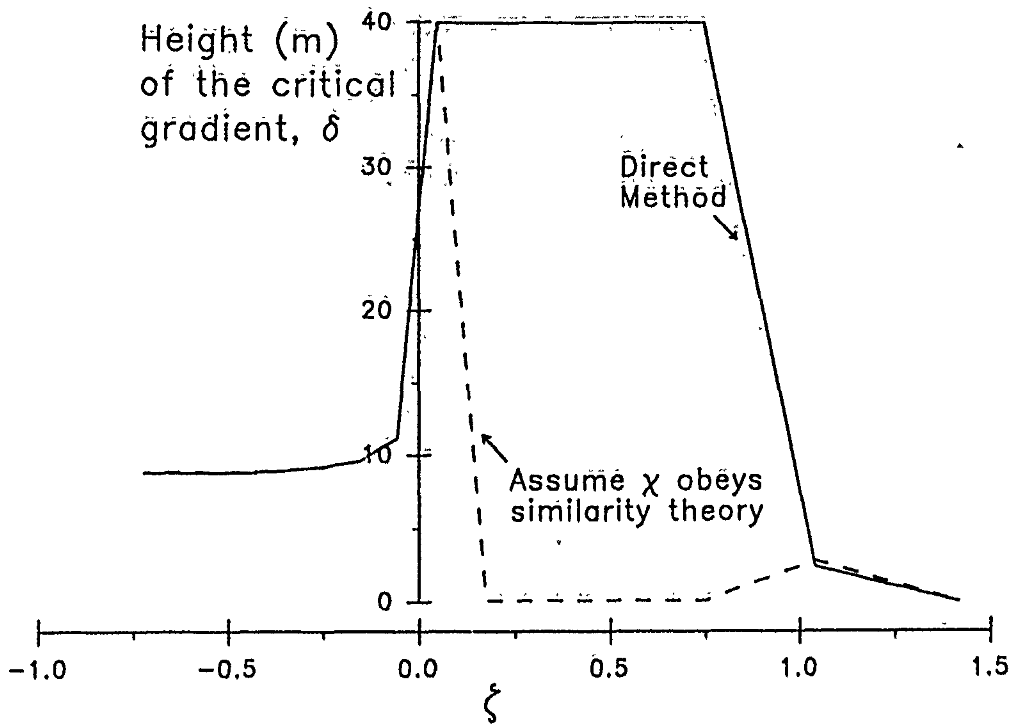

Fig. 3. Height, δ, of the critical gradient $(\partial\chi/\partial z)_c$ versus stability (ζ), calculated using the direct method (solid line) and assuming χ follows M–O similarity theory (dashed line). The two different techniques are described in the text. Computed values of δ greater than 40 m have been set to 40 m and values of δ less than 0 m have been set to 0 m.

$$\frac{k\alpha\delta}{\chi_*}\left(\frac{\partial\chi}{\partial z}\right)_c = \phi\left(\frac{\delta}{L}\right). \tag{7}$$

The height δ, usually called the evaporation duct height, is an important factor in determining shipboard radio and radar propagation conditions. Generally, χ_* is evaluated from the integrated form of Equation (4) using bulk parameter values at a reference height, and then Equation (7) is solved for δ. Figure 3 is a plot of δ versus stability (ζ) for a subset of the cases presented in Figure 2. The dashed line represents the solution to Equation (7) and the solid line (labeled 'direct method') represents the solution to the integrated form of Equation (5), with the computation of θ, q, χ, and $\partial\chi/\partial z$ directly at each profile level, and a searching algorithm to find the height where the χ gradient reaches its critical value. Thus, the direct method only assumes that θ and q satisfy M–O similarity theory – not that χ does. In Figure 3, the direct method shows that nowhere in the profile, from about $0.1 < \zeta < 0.8$, does the χ gradient reach its critical value. (In the figure, the value of δ has been set to 40 m when calculated values exceed 40 m, and set to zero when negative values for δ are calculated.) However, the solutions to Equation (7), which do assume that χ follows similarity theory, give erroneously low values for δ in these cases. Other cases (not shown) show that application of Equation (7) on the stable side under different environmental conditions can also lead to erroneously large values of δ; thus even the sign of the error that one makes when treating χ in this manner is not consistent. This sign change can also be inferred from Figure 2 because Φ_χ can be either smaller or larger than ϕ for the same stability.

In addition, we found the magnitude of $(\partial\chi/\partial\tilde{z})_c$ to have up to 10% variation with height (not shown). This variation was taken into account in the direct method but, for reasons of computational simplicity, a constant value of $(\partial\chi/\partial\tilde{z})_c = -0.131\ \mathrm{m}^{-1}$ was used when solving Equation (7). The use of a constant value for $(\partial\chi/\partial z)_c$ in Equation (7) is consistent with operational Navy computer codes (e.g., Paulus, 1989) and is a small additional source of error in the estimation of δ.

4. Concluding Remarks

We have shown that even if θ and q are accepted as exactly following M–O similarity expressions, when the surface layer is stable, χ, a nonlinear combination of θ and q, does not obey such an expression. That is, when properly non-dimensionalized, the χ gradient does not collapse to a single universal function of z/L. For practical purposes, we found θ_v, another nonlinear combination of θ and q, to obey similarity theory. It is the functional form of χ that is important in determining whether or not it obeys similarity theory. When the surface layer is unstable or near-neutral, χ closely follows similarity theory. It was, therefore, necessary to perform the full range of numerical calculations that we present here, rather than try to make a judgement about χ as a M–O variable based solely on its functional form. The advantage of our numerical methodology is that the results in Figure 2 can be interpreted as systematic errors due to inappropriate application of M–O similarity theory to χ. This source of error needs to be considered in addition to any error due to measurement scatter. Ordinary refractivity is even less suitable as a similarity variable due to its additional functional dependence on pressure.

The methodology and algorithms embodied in Hitney (1975) and Paulus (1989), which assume that χ obeys similarity theory, have been used in military operational computer codes for over ten years. Results from these codes are known to be erroneous under certain circumstances, but the corrections (Paulus, 1985) have not addressed the problem we discuss here. The straightforward alternative to treating χ as an M–O similarity variable is to apply similarity theory to θ and q, and then to compute refractivity as we do in our direct method.

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