ORIENTATION GUIDANCE AND CONTROL FOR MARINE VEHICLES IN THE HORIZONTAL PLANE

by

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June, 1991

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**Title**: Orientation Guidance and Control for Marine Vehicles in the Horizontal Plane

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**Type of Report**: M.S. Thesis

**Date of Report**: June 1991

**Page Count**: 49

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Orientation Guidance and Control for Marine Vehicles in the Horizontal Plane

by

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Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN MECHANICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
June 1991

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ABSTRACT

A pure pursuit guidance law and a heading autopilot are coupled in order to provide path control of submersibles or surface ships in the horizontal plane. Proper design of the combined scheme allows for accurate path keeping during straight line motion. The simulation results are extended to cover cases of step changes in the desired path. The scheme provides a viable alternative to cross track error autopilots.
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ACKNOWLEDGEMENT

The continuous support and patient guidance from my advisor, professor Fotis A. Papoulias motivated me very much to work on this research area. His unfailing attention and concern contributed significantly to the success of my research.
I. INTRODUCTION

In any operational scenario of an underwater vehicle there exists a triple-nested sequence of mission accomplishment operations: Path planning, navigation, guidance, and autopilot design. The path planner takes information from charted obstacles and friendly or hostile environments and generates a smooth plan for the vehicle to follow. A certain level of feedback exists in this operation through the use of sonar beams in order to replan a path when uncharted objects are encountered or when the mission requirements have changed. Based on the desired vehicle positions and orientations at certain points, several classes of smooth paths containing sets of straight line segments, and circular arcs or cubic splines can be obtained [1]. Once a smooth path is generated, the navigator provides through a selected guidance law the appropriate vehicle heading commands which are in turn delivered by the autopilot. Line of sight guidance using a discrete series of way points was studied by Lienard in [2] using sliding mode heaving, propulsion, and depth keeping autopilots. The scheme demonstrated excellent stability and robustness characteristics, although the actual vehicle response was found to lag significantly the commanded straight line paths. The guidance and autopilot functions can be combined when the lateral deviation off the desired path is directly incorporated into the control law design. This leads to the development of a cross track error autopilot. Such schemes have been studied by Chism [3] and Hawkinson [4] for the single input and multiple input
case, respectively. Cross track error autopilots provide more accurate path keeping response but they must be designed more carefully since they tend to be more dependent than heading controllers on an accurate description of the vehicle hydrodynamic characteristics. The main reason for this is the increase in the system dimensionality by one. Underwater vehicles operate in changing environments over a wide range of operating speeds and, therefore, a certain degree of uncertainty exists in the vehicle dynamic modeling. Cross track error autopilots also require accurate positional information updates at the same rate as heading and heading rate.

For these reasons, in this work we go back to the case of a heading autopilot coupled with an orientation guidance law. The two main tasks on which we will concentrate are as follows: First, we must develop a way of establishing stability of the combined autopilot/guidance scheme for straight line commanded motions. Second, the actual vehicle response characteristics must be made to resemble the desired cross track error response with smooth transitions between consecutive straight line segment and with minimal path overshoot. A linear full state feedback control law is used to adjust the heading of the vehicle to any desired value, and a pure pursuit guidance law is used to provide the commanded heading for straight line motion. In this scheme the vehicle commanded heading equals the line of sight angle between the vehicle center and a target point moving on the desired path at a constant lookahead distance from the vehicle. This parallels the case studied in [5] except that in our case the existence of lateral and rotational dynamics add more complications to the problem. All computations are performed for the Swimmer Delivery Vehicle [6] for which a set of hydrodynamic
characteristics and geometric properties is available. Problem formulation and equations of motion are presented in Section II. The analysis procedure is outlined in Section III, and simulation results are presented in the Section IV. Finally, conclusions and recommendations for further research are given in Section V.
II. DEVELOPMENT OF THE MATHEMATICAL MODEL

A. EQUATIONS OF MOTION

In a moving coordinate frame fixed at the vehicle center, Newton’s equations of motion for a rigid body in the horizontal plane are

\[
m(\ddot{v} + ur + x_G \dot{r} - y_G \dot{r}^2) = Y,
\]

(2.1)

\[
I_z \ddot{r} + m x_G (\ddot{v} + ur) - my_G v r = N,
\]

(2.2)

where

\[v = \text{sway(lateral) velocity},\]
\[r = \text{yaw(angular) velocity},\]
\[u = \text{forward(surge) speed},\]
\[Y = \text{sway force},\]
\[N = \text{yaw moment},\]
\[m = \text{vehicle mass},\]
\[I_z = \text{vehicle mass moment of inertia},\]
\[(x_0, y_0) = \text{coordinates of center of gravity}.
\]

Expanding the force \(Y\) and moment \(N\) in added mass, damping, and drag terms, equations (2.1) and (2.2) are written as...
\[ m(\dot{v} + ur + x_G\dot{r} - y_Gr^2) = \frac{\rho}{2} l^4 Y_G\dot{r} + \frac{\rho}{2} l^3 (Y_G\dot{v} + Y_Gur) \]
\[\quad + \frac{\rho}{2} l^2 Y_Guv - \frac{\rho}{2} \int_{C_D} h(x) \frac{(v + xr)^3}{v + xr} dx + \frac{\rho}{2} l^2 y_gu^2 \delta, \quad (2.3)\]

\[ I_G^2 + mx_G(\dot{v} + ur) - my_G\dot{r} = \frac{\rho}{2} l^3 N_G\dot{r} + \frac{\rho}{2} l^3 (N_G\dot{v} + N_Gur) \]
\[\quad + \frac{\rho}{2} l^3 N_Guv - \frac{\rho}{2} \int_{C_D} h(x) \frac{(v + xr)^3}{v + xr} dxd + \frac{\rho}{2} l^3 y_gu^2 \delta, \quad (2.4)\]

where

\[ \rho = \text{water density}, \]
\[ l = \text{vehicle length}, \]
\[ \delta = \text{rudder angle}, \]
\[ h(x) = \text{vehicle height distribution}, \]
\[ C_D = \text{drag coefficient}. \]

The inertial position of the vehicle \((x,y)\) and its heading angle \(\psi\) (see Figure 1) are given by

\[ \dot{\psi} = r, \quad (2.5) \]
\[ \dot{x} = u \cos \psi - v \sin \psi, \quad (2.6) \]
\[ \dot{y} = u \sin \psi + v \cos \psi. \quad (2.7) \]
Figure 1 Top view of the vehicle
B. STATE SPACE EQUATIONS

Choosing \((\psi, v, r)\) as the state vector, the linearized state space equations (2.3), (2.4), and (2.5) are written as

\[
\begin{align*}
\dot{\psi} & = r, \\
\dot{v} & = a_{11}uv + a_{12}ur + b_1u^2\delta, \\
\dot{r} & = a_{21}uv + a_{22}ur + b_2u^2\delta.
\end{align*}
\]

The coefficients in equations (8), (9), and (10) are given by

\[
\begin{align*}
a_{11} & = \frac{(I_z - 0.5pl^2N_y)(0.5pl^2y_y) - (mX_G - 0.5pl^4y_y)(0.5pl^3N_y)}{(I_z - 0.5pl^2N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}, \\
a_{12} & = \frac{(I_z - 0.5pl^2N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}{(I_z - 0.5pl^2N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}, \\
a_{21} & = \frac{(mX_G - 0.5pl^4y_y)(0.5pl^2N_y) - (mX_G - 0.5pl^4N_y)(mX_G - 0.5pl^2y_y)}{(I_z - 0.5pl^2N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}, \\
a_{22} & = \frac{(mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}{(I_z - 0.5pl^2N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}, \\
b_1 & = \frac{(I_z - 0.5pl^2N_y)(0.5pl^2y_y) - (mX_G - 0.5pl^4y_y)(0.5pl^3N_y)}{(I_z - 0.5pl^2N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)}.
\end{align*}
\]
\[ b_2 = \frac{(mX_G - 0.5pl^4y_y)(0.5pl^3N_y) - (mX_G - 0.5pl^4N_y)(0.5pl^2y_y)}{(I_r - 0.5pl^3N_y)(m - 0.5pl^3y_y) - (mX_G - 0.5pl^4y_y)(mX_G - 0.5pl^4N_y)} \]

where

\[ y_y, y_y, y_y, y_y, y_y = \text{lateral hydrodynamic coefficients} \]

\[ N_y, N_y, N_y, N_y, N_y = \text{yaw hydrodynamic coefficients} \]

Equations (2.8), (2.9), and (2.10) describe the dynamics of the system with respect to small deviations around a nominal direction \( \psi = 0 \).

C. PATH KEEPING DEVELOPMENT

1. Heading control

A linear full state feedback control law is of the form

\[ \delta = k_1\psi + k_2v + k_3r \quad (2.11) \]

where \( k_1, k_2 \) and \( k_3 \) are the three gains.

From equation (2.8), (2.9), (2.10) and (2.11), the closed loop system is

\[ \dot{\psi} = r \quad (2.12) \]

\[ \dot{\psi} = b_1u^2k_1\psi + (a_{11}u + b_1u^2k_2)v + (a_{12}u + b_1u^2k_3)r \quad (2.13) \]
\[ i = b_2 u^2 k_1 v + (a_{11} u + b_2 u^2 k_2) v + (a_{22} u + b_2 u^2 k_3) r \]  

(2.14)

The characteristic equation of (2.12), (2.13) and (2.14) is

\[
\begin{vmatrix}
0 - \lambda & 0 & 1 \\
b_1 u^2 k_1 & a_{11} u + b_1 u^2 k_2 - \lambda & a_{12} u + b_2 u^2 k_3 \\
b_2 u^2 k_2 & a_{21} u + b_2 u^2 k_2 & a_{22} u + b_2 u^2 k_3 - \lambda \\
\end{vmatrix} = 0
\]

\[
\lambda[(a_{11} u + b_1 u^2 k_2 - \lambda)(a_{22} u + b_2 u^2 k_3 - \lambda) - (a_{12} u + b_1 u^2 k_3)(a_{21} u + b_2 u^2 k_2)] - b_1 u^2 k_1 (a_{12} u + b_1 u^2 k_3) + b_2 u^2 k_1 (a_{11} u + b_2 u^2 k_2 - \lambda)
\]

\[= 0\]

\[
\lambda[(a_{11} a_{22} u^2 + a_{12} b_2 u^3 k_3 - \lambda a_{11} u + b_1 a_{22} u^3 k_2 + b_1 b_2 u^4 k_3 - \lambda b_1 u^2 k_2 - \lambda a_{22} u)
\]

\[-\lambda b_2 u^2 k_3 - \lambda^2 - a_{12} a_{21} u^2 - a_{12} b_2 u^3 k_2 - a_{21} b_2 u^3 k_3 - b_1 b_2 u^4 k_2 k_3]

\[-b_1 a_{21} u^3 k_1 - b_1 b_2 u^4 k_1 k_2 + a_{11} b_2 u^3 k_1 + b_2 u^4 k_1 k_2 - \lambda b_2 u^2 k_1]
\]

\[= 0 \]

\[
\lambda^3 - (a_{11} u + b_1 u^2 k_2 + a_{22} u + b_2 u^2 k_3) \lambda^2 + (a_{11} a_{22} u^2 + a_{11} b_2 u^3 k_3 + b_1 a_{22} u^3 k_2)
\]

\[-a_{12} a_{21} u^2 - a_{12} b_2 u^3 k_2 - a_{21} b_1 u^3 k_3 - b_2 u^2 k_1) \lambda + b_2 a_{11} u^3 k_1 - b_1 a_{21} u^3 k_1
\]

\[= 0 \]  

(2.15)

2. Desired characteristic equation

The 3rd order ITAE polynomial is defined by

\[
\lambda^3 + 1.75 \omega_0 \lambda^2 + 2.15 \omega_0^2 \lambda - \omega_0^3 = 0
\]  

(2.16)
\[ \omega_0 = \frac{10u_1}{t_c} \]

where

\[ t_c = \text{settling time (dimensionless)} \]

From equations (2.15) and (12.6) we get

\[ -a_{11}u - b_1 u^2 k_2 - a_{22}u - b_2 u^2 k_3 = 1.75\omega_0 \]  
(2.17)

\[ a_{11}a_{22}u^2 + a_{11}b_2 u^3 k_3 + b_1 a_{22}u^2 k_2 - a_{12}a_{21}u^2 \]
\[-a_{12}b_2 u^3 k_2 - a_{21}b_1 u^3 k_3 - b_2 u^2 k_1 = 2.15\omega_0^2 \]  
(2.18)

\[ b_2 a_{11} u^3 k_1 - b_1 a_{21} u^3 k_1 = \omega_0^3 \]  
(2.19)

The system of equations (2.17), (2.18) and (2.19) can be solved for the three gains

\[ k_1 = \frac{\omega_0^3}{b_2 a_{11} u^3 - b_1 a_{21} u^3} \]

\[ k_2 = \frac{u^4(-1.75\omega_0^2 - a_{11} - a_{22})(a_{11}b_2 - a_{21}b_1) - b_2 u^2(2.15\omega_0^2 - a_{11}a_{22}u^2 + a_{12}a_{21}u^2 + b_2 u^2 k_1)}{b_1 u^2(a_{11} - a_{21}) - b_2 u^2(a_{22}b_1 - a_{12}b_2)} \]

\[ k_3 = \frac{b_2 u^2[2.15\omega_0^2 - u^2(a_{11}a_{22} - a_{12}a_{21}) + b_2 u^2 k_1] - u^3(-1.75\omega_0^2 - a_{11} - a_{22})u(a_{22}b_1 - a_{12}b_2)}{b_1 u^2(a_{11}b_2 - a_{21}b_1) - b_2 u^2(a_{22}b_1 - a_{12}b_2)} \]
3. Pure pursuit navigation

For a pursuit navigation

\[ \psi_c = \sigma \]  \hspace{1cm} (2.20)

where

\[ \psi_c = \text{commanded heading} \]

Referring to Figure 1, the line of sight angle \( \sigma \) is defined by

\[ \sigma = -\tan^{-1} \frac{y}{x_d} \]  \hspace{1cm} (2.21)

where \( x_d \) is the vehicle lookahead distance, and the control law (2.11) becomes

\[ \delta = k_1(\psi - \psi_c) + k_2 v + k_3 r \]  \hspace{1cm} (2.22)

Using equations (2.20), (2.21) and (2.22) we get

\[ \delta = k_1(\psi + \tan^{-1}\frac{y}{x_d}) + k_2 v + k_3 r \]  \hspace{1cm} (2.23)

The linearized equation (2.23) becomes

\[ \delta = k_1(\psi + \frac{y}{x_d}) + k_2 v + k_3 r \]  \hspace{1cm} (2.24)

The linearized equation for the lateral deviation \( y \) is obtained from (2.7) as

\[ \dot{y} = u \psi + v \]  \hspace{1cm} (2.25)
Now the complete state vector is $\psi$, $v$, $r$ and $y$, and the state space equations are written as

$$\dot{\psi} = r$$  \hspace{1cm} (2.26)

$$\dot{v} = b_1 u^2 k_1 \psi + (a_{11} u + b_1 u^2 k_2) v + (a_{12} u + b_1 u^2 k_2) r + b_1 u^2 k_1 \frac{1}{x_d}$$ \hspace{1cm} (2.27)

$$\dot{r} = b_2 u^2 k_1 \psi + (a_{21} u + b_2 u^2 k_2) v + (a_{22} u + b_2 u^2 k_2) r + b_2 u^2 k_1 \frac{1}{x_d}$$ \hspace{1cm} (2.28)

$$\dot{y} = u \psi + v$$ \hspace{1cm} (2.29)

and the characteristic equation is

$$\begin{vmatrix}
0 - \lambda & 0 & 1 & 0 \\
b_1 u^2 k_1 & a_{11} u + b_1 u^2 k_2 - \lambda & a_{12} u + b_1 u^2 k_3 & b_1 u^2 k_1 \frac{1}{x_d} \\
b_2 u^2 k_2 & a_{21} u + b_2 u^2 k_2 & a_{22} u + b_2 u^2 k_2 - \lambda & b_2 u^2 k_1 \frac{1}{x_d} \\
u & 1 & 0 & 0 - \lambda
\end{vmatrix} = 0$$  \hspace{1cm} (2.30)
III. COMPUTER SIMULATION

A. PROGRAMMING

1. Program in MATRIX.X

The MATRIX.X software is in VAX/VMS in the Mechanical Engineering Department. The program in MATRIX.X is written to find gains and poles of the system from the given inputs, forward speed ($u$), settling time ($t_c$) and vehicle lookahead distance ($x_d$). This is used to compute the eigenvalues of the complete system (Equation (2.30)).

The four eigenvalues of the system are computed for the given values of $u$, $t_c$, and $x_d$. For stable vehicle response, all four must be negative (or have negative real parts). If at least one is positive, the vehicle response will be unstable and convergence to the straight line path is not to be expected. In such a case, the parameters (in particular the lookahead distance must be changed) so that the vehicle is stable. A listing of this program is presented in Appendix A.

2. Program in FORTRAN

The first program in FORTRAN, is written to find distance along body fixed axis ($x-y$) from inputs, heading angle ($\psi$), perpendicular distance from vehicle to route ($y$), yaw rate ($r$), forward speed ($u$), settling time ($t_c$) and vehicle lookahead distance ($x_d$). A modified version of this simulation program is presented in Appendix B. The modified version is used to control the vehicle position in a general (X-Y) inertial system. The desired vehicle path is discretized into a series of straight line segments and the same
lookahead distance $x_d$ is used to regulate the vehicle deviation of each segment. Details of this modification are presented in the next paragraph.

3. **Graphics**

The GRAFSTAT graphic package is used to produce 2 dimensional graphs by using data from the simulation programs.

B. **DETAILS**

1. **Poles and gains**

Calculate the system poles and gains for a given forward speed $u$ and various combinations of $t_c$ and $x_d$. Select $t_c$ and $x_d$ such that appropriate (sufficiently negative) poles and gains for the system are produced. This is verified by repeated simulations from step 2 that follows.

2. **Distance in x-y axis**

Using the values of $t_c$ and $x_d$ from the previous step, the system response can be simulated. Unlike the control law design, the simulation is based on the full nonlinear equations of motion for the vehicle, (3), (4), (5) and (7). Typically, the initial conditions consist of nonzero values of the lateral deviation $y$ and heading $\psi$.

3. **Distance in X-Y axis**

A similar procedure is used to simulate the vehicle response in a general path in the X-Y plane, as shown in Figure 2. The perpendicular distance $y$ from the vehicle to the desired route in the X-Y plane is used to compute the commanded heading angle.
The difference $\psi - \alpha$, where $\alpha$ is the angle of the route with respect to the X-axis, is used instead of the heading $\psi$ in the control law. The above computations are performed by appropriate coordinate rotations between the two coordinate systems.

When transiting from one straight line path to the next, the same lookahead distance $x_d$ is used for both segments. The vehicle switches to the next segment when it gets within a specified distance from the terminal way point. This distance is measured along the x-axis and for given $t_c$ and $x_d$, and it should increase as the angle $\alpha$ for the next segment increases. Too high or too low values of this turning distance result in path overshoot and undesirable oscillatory response. The optimum turning distance that allows for the smooth transition between consecutive straight line paths is established with the aid of the simulation program from step 2 as follows.

For a fixed initial heading $\psi$, the initial deviation $y$ is varied until the vehicle response is smooth and sufficiently fast with no path overshoot. The process is repeated for different initial conditions in $\psi$ and a curve in $y$ versus $\psi$ is constructed. This is shown in Figure 3 and is the desirable turning distance versus turning angle curve. The actual curve is approximated by two straight lines which are used to initiate the turn in the simulation program. A copy of this simulation program is included in Appendix B.
Figure 2 Angles and axes
Figure 3 Turning distance and turning angle
IV. SIMULATION RESULTS

The vehicle parameters used in the simulation are:

\[
\begin{align*}
x_u &= -0.0076 & y_r &= 0.0012 \\
x_{rr} &= 0.0040 & y_v &= -0.0550 \\
x_{rr} &= 0.0200 & y_r &= 0.0300 \\
x_{r\delta} &= -0.0010 & y_c &= -0.1000 \\
x_{r\delta} &= 0.0530 & y_\delta &= 0.0270 \\
x_{\delta\delta} &= 0.00173 & c_{Dv} &= 0.3500 \\
x_{s8} &= -0.1000 & W &= 12000 \text{ lb.} \\
n_r &= -0.0034 & l &= 17.4 \text{ ft.} \\
n_v &= 0.0012 & \rho &= 1.94 \text{ slug/ft}^3. \\
n_r &= -0.0160 & g &= 32.2 \text{ ft./sec.} \\
n_v &= -0.0074 & I_s &= 10000 \text{ ft}^4. \\
n_\delta &= -0.0130 & v &= 0.000847 \text{ ft}^2/\text{sec.}
\end{align*}
\]

The simulation begins by setting \( \psi = 5 \) degrees, \( x_0 = 2 \) vehicle lengths, \( t_c = 5 \), \( r = 0 \), \( v = 0 \) ft./sec, \( u = 5 \) ft./sec, and \( y = 1 \). When the vehicle moves to a distance \( x = 20 \) then the simulation stops. The route of vehicle and the rudder angle (\( \delta \)) that vehicle used during simulation are shown in Figure 4. The heading angle (\( \psi \)) and commanded heading (\( \psi_c \)) are shown in Figure 5. Yaw velocity and sway velocity are shown in Figure 6.
The values for $t_c$ and $x_d$ were selected based on the results of the previous chapter. From the figures it can be seen that the vehicle response is very fast with limited overshoot. This, of course, depends heavily on the initial conditions of the simulation. The actual heading angle converges rather rapidly, after the initial transients have died out, to the commanded heading angle.

The second series of simulations was performed in order to assess the capability of the control and guidance law to change course and keep the new path. Simulation parameters were $x_d = 2$, $t_c = 5$ and $u = 5$ as before. Initial conditions for the simulations were $\psi = 0$, $r = 0$ and $v = 0$ with the initial vehicle position at $(X_0,Y_0) = (5,0)$ in the global reference frame. The first straight line segment is determined by the way points $(5,0)$ and $(25,0)$ and the second by $(25,0)$ and $(67.89,20)$. For this route the corresponding course change is 25 degrees. The results of this simulation are presented in Figure 7 where along with the actual vehicle path, a side path at distance of 1 vehicle length off the desired path is shown. This corresponds to an arbitrary "safety path band" for the vehicle. The turning distance was fixed at 2 vehicle lengths throughout the simulations. From Figure 7 it can be seen that the vehicle turns to the new course smoothly with no path overshoot. When the second route changes to $(25,0)$ and $(41.78,20)$ which corresponds to 50 degrees course change, the results of Figure 8 show that a path overshoot occurs although it is yet not serious enough according to the artificial safety criterion described above. However, when the second route changes to $(25,0)$ and $(30.36,20)$ which corresponds to 75 degrees course change, significant vehicle oscillatory response and side path overshoot occurs, as demonstrated in Figure 9.
The above simulations demonstrate the need for adjustable turning distance; although path accuracy is obtained regardless of the value of the turning distance, the transient response during course change is not always within some predetermined safety bounds. For this reason we employ the built-in turning distance versus turning angle relationship shown in Figure 3 and repeat the simulations for the aforementioned three course changes. The results are shown in Figures 10, 11 and 12, where it can be seen that the vehicle response is now satisfactory for both course keeping and course changing.

Finally, the last simulation test was performed in order to establish the capabilities of the scheme to follow a general path in the horizontal plane. For demonstration purposes the path was assumed to consist of the way points (0,10), (5,10), (25,0), (35,20), (50,20), (70,10), (50,-10) and (30,-10). Straight line segments were assumed as the desired paths between consecutive way points. The same simulation parameters were used as in the previous runs. The results of this simulation are presented in Figures 13 for adjustable turning distance, and 14 for the fixed turning distance case. It can be clearly seen that when the turning distance is function of turning angle, the scheme achieves excellent path keeping characteristics with smooth course changes and minimal path overshoot.
Figure 4  Pursuit navigation
Figure 5  Pursuit navigation
Figure 6  Pursuit navigation
TURNING ANGLE 25 DEGREES
TURNING DISTANCE CONSTANT

Figure 7 Path control
TURNING ANGLE 50 DEGREES
TURNING DISTANCE CONSTANT

Figure 8 Path control
TURNING ANGLE 75 DEGREES
TURNING DISTANCE CONSTANT

Figure 9  Path control
TURNING ANGLE 25 DEGREES

TURNING DISTANCE FUNCTION OF TURNING ANGLE

Figure 10 Path control
TURNING ANGLE 50 DEGREES

TURNING DISTANCE FUNCTION OF TURNING ANGLE

Figure 11  Path control
TURNING ANGLE 75 DEGREES

TURNING DISTANCE FUNCTION OF TURNING ANGLE

Figure 12  Path control
Figure 13 Pure pursuit navigation
TURNING DISTANCE CONSTANT

Figure 14 Pure pursuit navigation
V. SUMMARY AND CONCLUSIONS

The principal conclusions of this work can be summarized as follows:

1. Orientation control law can be used in order to provide accurate vehicle path keeping when combined with an appropriate guidance scheme.

2. Pure pursuit guidance was found to work very well for the autonomous underwater vehicle case and its simplicity make it a very attractive alternative to cross track error schemes.

3. A built-in turning distance versus course change relationship can be utilized to initiate the turn at the appropriate time in order to avoid path overshoot and achieve smooth path transitions.

4. It is expected that the added robustness that heading schemes naturally enjoy will aid in maintaining stability in cases where incomplete and inaccurate vehicle dynamic descriptions are available.

Some recommendations for further research are as follows:

1. Comparative studies must be performed with other orientation guidance schemes such as proportional navigation and also with velocity guidance laws in order to ensure that the best technique is ultimately employed.
2. Similar studies must be performed in the vertical plane. Combined with the horizontal plane techniques developed in this work and with propulsion control they could be utilized to provide accurate trajectory following in 3-D space.
LIST OF REFERENCES


APPENDIX A.

1=17.425;
inquire xd_1
inquire tc
inquire u
xd=xd_1*1;
a11=-0.045380;
a12=-0.351190;
aa21=-0.002795;
aa22=-0.095680;
bb1= 0.011432;
bb2=-0.004273;
OMEGA=(10.0*U)/(TC*L);
AD1=1.75*OMEGA;
AD2=2.15*OMEGA**2;
AD3=OMEGA**3;
A1=BB1*U*U;
B1=BB2*U*U;
C1=-AD1-(AA11+AA22)*U;
A2=(BB1*AA22-BB2*AA12)*U**3;
B2=(BB2*AA11-BB1*AA21)*U**3;
K1=AD3/((BB2*AA11-BB1*AA21)*U**3);
C2=AD2-(AA11*AA22-AA12*AA21)*U**2+BB2*U*U*K1;
K2=(C1*B2-C2*B1)/(A1*B2-A2*B1);
K3=(C2*A1-C1*A2)/(A1*B2-A2*B1);
a11=0;
a12=0;
a13=1;
a14=0;
a21=bb1*u*u*k1;
a22=aa11*u+bb1*u*u*k2;
a23=aa12*u+bb1*u*u*k3;
a24=bb1*u*u*k1/xd;
a31=bb2*u*u*k1;
a32=aa21*u+bb2*u*u*k2;
a33=aa22*u+bb2*u*u*k3;
a34=bb2*u*u*k1/xd;
a41=u;
a42=1;
a43=0;
a44=0;
a=[a11,a12,a13,a14,a21,a22,a23,a24;
a31,a32,a33,a34,a41,a42,a43,a44];
eig(a)
APPENDIX B.

PROGRAM SUB.FOR

PROUTTICHAI SUWANDEE
NAVAL POSTGRADUATE SCHOOL
MARCH 1991

AUV LINE-OF-SIGHT NAVIGATION AND CONTROL
VARIABLE GAINS INTERNALLY COMPUTED

REAL L,MASS,NRDOT,NVDOT,SR,NV,NDR
REAL IZ,NU,LLL,NSL,K1,K2,K3
DIMENSION X(9),HH(9),VEC1(9),VEC2(9),TT(1000),YY(6,1000),
*ALPHA(10),XZ(10),YZ(10)

LONGITUDINAL HYDRODYNAMIC COEFFICIENTS

PARAMETER(XRR=4.E-3,XUDOT=-7.6E-3,XVR=2.E-2,XRDR=1.E-3,
1
XVV=5.3E-2,XVDR=1.7E-3,XDRDR=-1.E-2)

LATERAL HYDRODYNAMIC COEFFICIENTS

PARAMETER(YRDOT=1.2E-3,YVDOT=-5.5E-2,YR=3.E-2,YV=-1.E-1,
1
YDR=2.7E-2,CDY=3.5E-1)

YAW HYDRODYNAMIC COEFFICIENTS

PARAMETER(NRDOT=-3.4E-3,NVDOT=1.2E-3,NR=-1.6E-2,NV=-7.4E-3,
1
NDR=-1.3E-2)

MASS CHARACTERISTICS OF THE FLOODED VEHICLE

PARAMETER(WEIGHT=12000.,XG = 0.,IZ=10000.,L=17.4,RHO=1.94,
1
G=32.2,NU=8.47E-4)

OPEN (10,FILE='SUB.IN',STATUS='OLD')
OPEN (11,FILE='SUB.OUT',STATUS='OLD')

READ (10,*) TARGET
READ (10,*) TSIM,DELTA,IPRINT
READ (10,*) PSI,R
READ (10,*) U
READ (10,*) TC,VC

NUMBER OF POSITION OF ROUTE AND POSITION OF VEHICLE

READ (10,*) N,XO,YO
C
C POSITION OF ROUTE IN X-Y AXIS
C
DO 30 I=1,N
READ (10,*) XZ(I),YZ(I)
30 CONTINUE
TARGET=TARGET*L
TWOPI =8.0*ATAN(1.0)
PI =0.5*TWOPI
DO 999 M=1,N-1
PSI=0
YTURN=0
XYTURN=0
PSI=PSI*PI/180.0
C
C MOVE ORIGINE OF AXIS TO THE FIRST POINT OF THE ROUTE
C
XF=XZ(M+1)-XZ(M)
IF (XF.EQ.0) XF=0.0000001
YF=YZ(M+1)-YZ(M)
XO=XO-XZ(M)
IF (XO.EQ.0) XO=0.0000001
YO=YO-YZ(M)
C
C ANGLE OF POSITION OF VEHICLE
C
ALPHAO=ATAN(ABS(YO/XO))
IF ((YO.GT.0).AND.(XO.LT.0)) ALPHAO=PI-ALPHAO
IF ((YO.LE.0).AND.(XO.GT.0)) ALPHAO=2*PI-ALPHAO
IF ((YO.LE.0).AND.(XO.LT.0)) ALPHAO=PI+ALPHAO
C
C ANGLE OF ROUTE
C
ALPHA1=ATAN(ABS(YF/XF))
IF ((YF.GT.0).AND.(XF.LT.0)) ALPHA1=PI-ALPHA1
IF ((YF.LE.0).AND.(XF.GT.0)) ALPHA1=2*PI-ALPHA1
IF ((YF.LE.0).AND.(XF.LT.0)) ALPHA1=PI+ALPHA1
C
C ANGLE BETWEEN VEHICLE AND ROUTE
C
BETA=(ALPHAO-ALPHA1)
C
C DISTANCE FROM ORIGINE TO VEHICLE
C
RT=(XO**2+YO**2)**0.5
C
C PROJECTED DISTANCE FROM ORIGINE TO VEHICLE ON THE ROUTE
C
XS=RT*COS(BETA)
XC=(XS)*COS(ALPHA1)
YC=(XS)*SIN(ALPHA1)
PERPENDICULAR DISTANCE OF VEHICLE TO X-AXIS

YPOS = RT * SIN(BETA)

TOTAL DISTANCE ON THE ROUTE

TXD = (XF**2 + YF**2)**0.5
XD = ABS(TXD - XS)

HEADING ANGLE

IF (M.EQ.1) PSI = PSI - ALPHA1
IF (PSI.GT.PI) PSI = PSI - 2*PI
IF (PSI.LT.-PI) PSI = PSI + 2*PI
IF (M.EQ.N-1) GO TO 65

NEXT HEADING ANGLE

DUMY = YZ(M+2) - YZ(M+1)
DUMX = XZ(M+2) - XZ(M+1)
IF (DUMX.EQ.0) DUMX = 0.0000001
ALP2 = ATAN(ABS(DUMY/DUMX))
IF ((DUMY.GT.0).AND.(DUMX.LT.0)) ALP2 = PI - ALP2
IF ((DUMY.LE.0).AND.(DUMX.GT.0)) ALP2 = -PI - ALP2
IF ((DUMY.LE.0).AND.(DUMX.LT.0)) ALP2 = PI + ALP2
PSI1 = PSI1 - ALPHA1
PSI1 = PSI1 - 2*PI
PSI1 = PSI1 + 2*PI
PSI1 = PSI1 * 180/PI

TURNING DISTANCE

IF(ABS(PSI1).LE.45) YTURN = ABS(PSI1/25)
IF(ABS(PSI1).GT.45) YTURN = (ABS(PSI1) - 45)*0.3/5 + 1.8
XYTURN = YTURN / ABS(SIN(PSI1*PI/180)) - 0.2
IF(PSI1.EQ.0) XYTURN = 0.0

65
UC = U
OMEGA = (10.0*U)/(TC*L)
AD1 = 1.75*OMEGA
AD2 = 2.15*OMEGA**2
AD3 = OMEGA**3

PISIM = TSIM/DELTA
ISIM = PISIM
ECHO = 1.0/DELTA
IECHO = IPRNT*20
YAW = 0.0
SWAY = 0.0
V = 0.0
DR = 0.0
R = R*PI/180.0
DEFINE THE LENGTH X AND HEIGHT HH TERMS FOR THE INTEGRATION

\[ X(1) = -105.9/12. \]
\[ X(2) = -99.3/12. \]
\[ X(3) = -87.3/12. \]
\[ X(4) = -66.3/12. \]
\[ X(5) = 72.7/12. \]
\[ X(6) = 83.2/12. \]
\[ X(7) = 91.2/12. \]
\[ X(8) = 99.2/12. \]
\[ X(9) = 103.2/12. \]

\[ HH(1) = 0.00/12. \]
\[ HH(2) = 8.24/12. \]
\[ HH(3) = 19.76/12. \]
\[ HH(4) = 29.36/12. \]
\[ HH(5) = 31.85/12. \]
\[ HH(6) = 27.84/12. \]
\[ HH(7) = 21.44/12. \]
\[ HH(8) = 12.00/12. \]
\[ HH(9) = 0.00/12. \]

MASS = WEIGHT/G

\[ P1 = \text{MASS} - 0.5 \times \text{RHO} \times L \times X \times U \times D \]
\[ P3 = \text{MASS} - 0.5 \times \text{RHO} \times L \times Y \times V \times D \]
\[ P4 = \text{MASS} \times X \times G - 0.5 \times \text{RHO} \times L \times Y \times R \times D \]
\[ P5 = 12 \times 0.5 \times \text{RHO} \times L \times 5 \times N \times D \]
\[ P6 = \text{MASS} \times X \times G - 0.5 \times \text{RHO} \times L \times Y \times N \times D \]
\[ D = P5 \times P3 - P4 \times P6 \]

\[ \text{AA11} = (P5 \times 0.5 \times \text{RHO} \times L \times L \times Y \times V - P4 \times 0.5 \times \text{RHO} \times L \times 3 \times N \times V) / D \]
\[ \text{AA12} = (P5 \times (-\text{MASS} + 0.5 \times \text{RHO} \times L \times 3 \times Y \times R) - P4 \times (-\text{MASS} \times X \times G + 0.5 \times \text{RHO} \times L \times 4 \times N \times R)) / D \]
\[ \text{AA21} = (P3 \times 0.5 \times \text{RHO} \times L \times 3 \times N \times V - P6 \times 0.5 \times \text{RHO} \times L \times L \times Y \times V) / D \]
\[ \text{AA22} = (P3 \times (-\text{MASS} \times X \times G + 0.5 \times \text{RHO} \times L \times 4 \times N \times R) - P6 \times (-\text{MASS} \times X \times G + 0.5 \times \text{RHO} \times L \times 3 \times Y \times R)) / D \]

\[ \text{BB1} = (P5 \times 0.5 \times \text{RHO} \times L \times 2 \times Y \times D \times R - P4 \times 0.5 \times \text{RHO} \times L \times 3 \times N \times D \times R) / D \]
\[ \text{BB2} = (P3 \times 0.5 \times \text{RHO} \times L \times 3 \times N \times D \times R - P6 \times 0.5 \times \text{RHO} \times L \times 2 \times Y \times D \times R) / D \]

\[ \text{A1} = \text{BB1} \times U \times U \]
\[ \text{B1} = \text{BB2} \times U \times U \]
\[ \text{C1} = -\text{AD1} \times (\text{AA11} + \text{AA22}) \times U \]
\[ \text{A2} = -\text{AA12} \times \text{BB2} + \text{AA22} \times \text{BB1} \times U \times U \times 3 \]
\[ \text{B2} = -\text{AA21} \times \text{BB1} + \text{AA11} \times \text{BB2} \times U \times U \times 3 \]
\[ \text{K1} = \text{AD3} / (\text{BB2} \times \text{AA11} - \text{BB1} \times \text{AA21}) \times U \times U \times 3 \]
\[ \text{C2} = \text{AD2} \times (-\text{AA12} \times \text{AA21} + \text{AA11} \times \text{AA22}) \times U \times U \times 2 + \text{BB2} \times U \times U \times K1 \]
\[ \text{K2} = (\text{C1} \times \text{B2} - \text{C2} \times \text{B1}) / (\text{A1} \times \text{B2} - \text{A2} \times \text{B1}) \]
\[ \text{K3} = (\text{C2} \times \text{A1} - \text{C1} \times \text{A2}) / (\text{A1} \times \text{B2} - \text{A2} \times \text{B1}) \]
J = 0
IJ=0
JE=0
OFF=0

DRHAT=0.0
DRBAR=0.0

SIMULATION BEGINS

DO 100 I=1,ISIM
   CALCULATE THE DRAG FORCE, INTEGRATE THE DRAG OVER THE VEHICLE
   DO 600 K=1,9
      UCF=V+X(K)*R
      SGN=1.0
      IF (UCF.LT.0.0) SGN=-1.0
      VEC1(K)=HH(K)*UCF*UCF*SGN
      VEC2(K)=HH(K)*UCF*UCF*SGN*X(K)
   600 CONTINUE
   CALL TRAP(9,VEC1,X,SWAY)
   CALL TRAP(9,VEC2,X,YAW)
   SWAY=-0.5*RHO*CDY*SWAY
   YAW=-0.5*RHO*CDY*YAW

   FORCE EQUATIONS

      FP2 = -MASS*U*R+0.5*RHO*L**3*YR*U*R+0.5*RHO*L*L*(
               YV*U*V+YDR*U*U*DR)+SWAY
      1
      FP3 = -MASS*XG*U*R+0.5*RHO*L**4*NR*U*R+0.5*RHO*L**3*
               (NV*U*V+NDR*U*U*DR)+YAW
      1
      VDOT = (P5*FP2-P4*FP3)/(P5*P3-P4*P6)
      RDOT = (P6*FP2-P3*FP3)/(P4*P6-P3*P5)
      PSIDOT=R
      YDOT =U*SIN(PSI)+V*COS(PSI)+VC
      XDOT =U*COS(PSI)-V*SIN(PSI)

   FIRST ORDER INTEGRATION

      PSI = PSI + DELTA*PSIDOT
      V = V + DELTA*VDOT
      R = R + DELTA*RDOT
      XPOS=XPOS+DELTA*XDOT
      YPOS=YPOS+DELTA*YDOT

      YCTE=YPOS
      XAWAY=(XPOS-XD*L)

   IF ((XAWAY).GE.-(XYTURN*L)) OFF=1
RUDDER INPUT CALCULATION

YA = ABS(YPOS)
HDM = ATAN(YPOS)/(-TARGET))
DR = K1*(PSI-HDM)+K2*V+K3*R

IF (DR.GT. 0.4) DR = 0.4
IF (DR.LE.-0.4) DR = -0.4

PRINT RESULTS

JE = JE + 1
IF (JE.NE.IECHO) GO TO 99
WRITE (*,*) 'XAWAY =', XAWAY/L
JE = 0

99 J = J + 1
IF (J.NE.IPRNT) GO TO 100
IJ = IJ + 1
TIME = I * DELTA
XP = XPOS/L
YP = YPOS/L
XI = XZ(M) + XC + XP*COS(-ALPHA1) + YP*SIN(-ALPHA1)
YI = YZ(M) + YC + YP*COS(-ALPHA1) - XP*SIN(-ALPHA1)
WRITE (11,*) XI, YI
J = 0
IF (OFF.EQ.1) GO TO 500

100 CONTINUE

500 PSI = PS11
XO = XI
YO = YI

999 CONTINUE
STOP
END

SUBROUTINE TRAP(N,A,B,OUT)

NUMERICAL INTEGRATION ROUTINE USING THE TRAPEZOIDAL RULE

DIMENSION A(1), B(1)
N1 = N - 1
OUT = 0.0
DO 1 I = 1, N1
     OUT1 = 0.5*(A(I) + A(I+1))*(B(I+1) - B(I))
     OUT = OUT + OUT1
1 CONTINUE
RETURN
END
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