ON THE SUBJECT OF GEOMETRIC SPACING
OF METEOROLOGICAL SENSORS

November 1991

Henry Rachele
Arnold Tunick

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On the Subject of Geometric Spacing of Meteorological Sensors

Henry Rachael and Arnold Tunick

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In an earlier study the authors concluded that to estimate turbulence characteristics such as the friction velocity, the temperature scaling length, and the Obukhov length from discretely measured data, it is desirable and efficient, but not necessary, to have the sensors spaced geometrically with height. In that study the authors accepted the traditional geometric spacing equations and compared results from those equations with a set of more general formulations for a few examples.

For most turbulence studies in the boundary layer, however, one nominally desires single values of the turbulence characteristics through a layer of say 20, 30, or 100 m; that is, one normally assumes that the characteristic lengths are constant with height. In this study the authors present our mean value theorem mathematical development for the case of geometric spacing relative to the geometric mean for a single sublayer, and assume that the values for the total layer can be determined by averaging the sublayer values. Finally, the authors show in tabular form comparisons of $U^*$, $T^*$, $L$, and $R_i$ for four sublayers (1/2 - 2 m), (1 - 4 m), (2 - 8 m), and (4 - 16 m) by using the proposed Rachael/Tunick (RT) formulations and the traditional formulations.
ACKNOWLEDGMENTS

The authors thank the late Louis D. Duncan and Frank V. Hansen of the U.S. Army Atmospheric Sciences Laboratory, White Sands Missile Range, New Mexico, who have shown special interest in this topic, have encouraged the study, and have taken the time to review and comment on the contents.
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8. $U^*, \theta^*, L, \text{ and } R_i$ for the Sublayer $(4 - 16 \text{ m})$ (stable case) .......... 19
1. INTRODUCTION

In an earlier study (Rachele et al., 1991), we concluded that to estimate turbulence characteristics such as the friction velocity, temperature scaling length, and the Obukhov (1946) length from discretely measured data, it is desirable and efficient, but not necessary, to have the sensors spaced geometrically with height. In that study we presented the mathematics necessary to show that geometric spacing is not necessary. The mathematical argument for geometric spacing was not developed or presented. Furthermore, we could not find a reference for it. Even so, we considered the traditional geometric spacing equations and compared results from them with our more general formulations based on the mean value theorem for a few examples and found that for practical applications the differences were not significant.

For most turbulence studies in the boundary layer, one desires single values of the turbulence characteristics through a layer of perhaps 20, 30, or 100 m; that is, one typically assumes that the characteristic lengths are constant with height. A procedure for determining the total layer values is to compute the values in the geometrically spaced sublayers and then average the sublayer values.

In this study we consider the case where the sensors are geometrically spaced. We then present our mathematical development requiring geometric spacing for a single sublayer based on the mean value theorem and assume that the values for the total layer (summation of sublayers) can be determined by averaging the sublayer values.

2. MATHEMATICAL DEVELOPMENT

In the development that follows, we focus on the vertical gradient of windspeed written in similarity form for unstable and stable conditions. Results for temperature are similar. The neutral case results from the reduced form of the unstable or stable case.

2.1 Unstable Formulations

We write the wind gradient for unstable conditions as

\[
\frac{\partial V}{\partial z} = \frac{U^*}{kz} \left( 1 - \frac{\beta z}{L} \right)^{-1/4} = \frac{U^*}{kz} \phi_{\beta} \left( \frac{z}{L} \right), \tag{1}
\]

where

\[
\begin{align*}
V & \quad \text{- windspeed} \\
U^* & \quad \text{- friction velocity} \\
k & \quad \text{- Karman's constant} \\
z & \quad \text{- height}
\end{align*}
\]
\( \beta \) - constant

\( L \) - Obukhov length

Integrating equation (1) results in

\[
V = \frac{U^*}{k} \left\{ \ln \frac{z}{z_o} - \psi_m \left( \frac{z}{L} \right) \right\},
\]

where

\( z_o \) - roughness length,

\( z/L \) - the Monin and Obukhov (1954) scaling ratio

and

\[
\psi_m \left( \frac{z}{L} \right) = 2 \ln \left( \frac{1 + X}{2} \right) + \ln \left( \frac{1 + X^2}{2} \right) - 2 \tan^{-1}X + \frac{\pi}{2}
\]

\[
X = \left( 1 - \beta \frac{z^4}{L} \right)^{1/4}
\]

as shown by Paulson (1970).

Evaluating equation (2) for two geometric heights \( z_1 \) and \( z_2 \), where \( z_2 > z_1 \), we obtain

\[
V_2 - V_1 = \frac{U^*}{k} \left( \ln \frac{z_2}{z_1} - \psi_m \left( \frac{z_2}{L} \right) + \psi_m \left( \frac{z_1}{L} \right) \right).
\]

Dividing equation (5) by \( z_2 - z_1 \) gives

\[
\frac{V_2 - V_1}{z_2 - z_1} = \frac{U^*}{k(z_2 - z_1)} \left( \ln \frac{z_2}{z_1} - \psi_m \left( \frac{z_2}{L} \right) + \psi_m \left( \frac{z_1}{L} \right) \right).
\]

The mean value theorem of calculus states that there exists at least one point \( \xi \), where \( z_1 \leq \xi \leq z_2 \) such that
\[
\frac{V_2 - V_1}{z_2 - z_1} = \frac{\partial V}{\partial z} \bigg|_\xi ,
\]

where

\[
\frac{\partial V}{\partial z} \bigg|_\xi = \frac{U^*}{k\xi} \left(1 - \beta \frac{\xi}{L}\right)^{-1/4} .
\]

Using equations (6), (7), and (8) we find that

\[
\xi = \frac{(z_2 - z_1) \left(1 - \beta \frac{\xi}{L}\right)^{-1/4}}{\ln \frac{z_2}{z_1} - \psi_m \left(\frac{z_2}{L}\right) + \psi_m \left(\frac{z_1}{L}\right)} .
\]

Now suppose that a scaling constant \( \epsilon > 0 \) exists such that

\[
\epsilon \frac{\partial V}{\partial z} \bigg|_\xi = \frac{\partial V}{\partial z} \bigg|_{z_g} ,
\]

where \( z_g \) is the geometric mean of \( z_1 \) and \( z_2 \), that is, \( z_g = (z_1 z_2)^{1/2} \).

Using equation (1), we can write equation (10) as

\[
\epsilon \frac{U^*}{k\xi} \left(1 - \beta \frac{\xi}{L}\right)^{-1/4} = \frac{U^*}{kz_g} \left(1 - \beta \frac{z_g}{L}\right)^{-1/4} .
\]

Solving equation (11) for \( \epsilon \) gives

\[
\epsilon = \frac{\xi}{z_g} \left(1 - \beta \frac{z_g}{L}\right)^{-1/4} \left(1 - \beta \frac{\xi}{L}\right)^{-1/4} .
\]

Substituting \( \xi \) from equation (9) into equation (12) yields
\[
\varepsilon = \frac{(z_2 - z_1) \left(1 - \beta \frac{z_g}{L}\right)^{-1/4}}{z_g \left(\ln \frac{z_2}{z_1} - \Psi_m \left(\frac{z_2}{L}\right) + \Psi_m \left(\frac{z_1}{L}\right)\right)}
\]  

(13)

Combining equations (7), (10), and (13) results in

\[
\frac{\partial V}{\partial z} \bigg|_{z_g} = \frac{(V_2 - V_1) \left(1 - \beta \frac{z_g}{L}\right)^{-1/4}}{z_g \left(\ln \frac{z_2}{z_1} - \Psi_m \left(\frac{z_2}{L}\right) + \Psi_m \left(\frac{z_1}{L}\right)\right)}
\]

(14)

Since

\[
\frac{\partial V}{\partial z} \bigg|_{z_g} = \frac{U^*}{k z_g} \left(1 - \beta \frac{z_g}{L}\right)^{-1/4}
\]

(15)

then

\[
U^* = \frac{k z_g}{\left(1 - \beta \frac{z_g}{L}\right)^{-1/4}} \frac{\partial V}{\partial z} \bigg|_{z_g}
\]

(16)

or by substituting equation (14) into equation (16)

\[
U^* = \frac{k (V_2 - V_1)}{\left(\ln \frac{z_2}{z_1} - \Psi_m \left(\frac{z_2}{L}\right) + \Psi_m \left(\frac{z_1}{L}\right)\right)}
\]

(17)

To evaluate equation (17), we require an estimate of the Obukhov (1946) length L. We do this by using a basic definition of L, that is,
\[ L = \frac{U^* \theta^*}{kg \theta^*}, \]  

where \( \theta^* \) is the potential temperature at a reference height and where \( \theta^* \) is the temperature scaling length in the potential temperature gradient expression for unstable conditions, that is,

\[ \frac{\partial \theta}{\partial z} = \frac{\theta^*}{kz} \left( 1 - \gamma \frac{z}{L} \right)^{-1/2}. \]  

Using the same mathematical development for \( \theta^* \) as was used for \( U^* \), we obtained

\[ \theta^* = \frac{k (\theta_2 - \theta_1)}{(\ln \frac{z_2}{z_1} - \psi_H \left( \frac{z_2}{L} \right) + \psi_H \left( \frac{z_1}{L} \right))}, \]  

where

\[ \psi_H \left( \frac{z}{L} \right) = 2 \ln \left( \frac{1 + \gamma}{2} \right) \]  

\[ y = \left( 1 - \gamma \frac{z}{L} \right)^{1/2}. \]  

Equations (17), (19), and (20) are solved iteratively to obtain \( U^* \), \( \theta^* \), and \( L \).

For neutral conditions, that is, \( L = \infty \), we also use these three equations, but equations (17) and (20) reduce to

\[ U^* = \frac{k_2 (V_2 - V_1)}{\ln \frac{z_2}{z_1}} \]  

\[ \theta^* = \frac{k_2 (\theta_2 - \theta_1)}{\ln \frac{z_2}{z_1}}. \]
We conclude this section with a brief discussion of the Richardson (1920) number that is one measure of stability. By definition the Richardson number can be written as

\[ R_i = \frac{g}{\theta} \frac{\partial \theta / \partial z}{(\partial \psi / \partial z)^2} . \]  

(25)

Substituting equations (1) and (19) into equation (25) gives

\[ R_i = \frac{g}{\theta} \frac{\theta^*}{kz} \left(1 - \frac{\gamma z}{L}\right)^{-1/2} \frac{U^*}{\sqrt{U^*}} \left[1 - \frac{\beta z}{L}\right]^{-1/4} \]  

(26)

If \( \gamma = \beta \), equation (26) reduces to

\[ R_i = \frac{g}{\theta} \frac{k \theta^* z}{U^*} . \]  

(27)

However, from equation (18)

\[ \frac{g}{\theta} \frac{k \theta^*}{U^*} = \frac{1}{L} . \]  

(28)

Therefore, equation (28) simplifies to

\[ R_i = \frac{z}{L} . \]  

(29)

On the other hand, if we substitute equations (17) and (20) into equation (27) we obtain

\[ R_i = \frac{g z}{\theta} \left(\frac{\theta_2 - \theta_1}{1 - \frac{\psi_m (z_2/L)}{\ln z_2/z_1} + \psi_m (z_1/L)}\right) \]  

\[ \frac{k(V_2 - V_1)^2}{1 - \frac{\psi_H (z_2/L)}{ln z_2/z_1} + \psi_H (z_1/L)} \]  

(30)

Equation (30) allows one to compute \( R_i \) for any \( z_1 \leq z \leq z_2 \) given measured or estimated values of \( \theta \) and \( V \).
Finally, using equations (29) and (30) we have an implicit expression for \( L \), that is,

\[
\frac{1}{L} = \frac{q}{\theta z_g} \left\{ \frac{(\theta_2 - \theta_1)}{k(V_2 - V_1)^2} \left( \ln \frac{Z_2}{Z_1} - \psi_m \left( \frac{Z_2}{L} \right) + \psi_m \left( \frac{Z_1}{L} \right) \right) \right\} . \tag{31}
\]

\[2.2 \text{ Stable Formulations}\]

The stable case tracks the development of the unstable case, except that some equations take a different form. As an example, equation (1) takes the form (Dyer, 1974; Hicks, 1976; and Hansen, 1980)

\[
\frac{\partial V}{\partial z} = \frac{U^*}{kz} \left( 1 + 5 \frac{Z}{L} \right) = U^* \phi_m \left( \frac{Z}{L} \right) . \tag{32}
\]

Equation (2) retains the form

\[
V = \frac{U^*}{k} \left( \ln \frac{Z}{Z_0} - \psi_m \left( \frac{Z}{L} \right) \right) , \tag{33}
\]

where

\[
\psi_m \left( \frac{Z}{L} \right) = -5 \frac{Z}{L} . \tag{34}
\]

As with the unstable case, when we integrate equation (33) for two geometric heights \( z_1 \) and \( z_2 \), where \( z_2 > z_1 \), we obtain

\[
V_2 - V_1 = \frac{U^*}{k} \left( \ln \frac{Z_2}{Z_1} - \psi_m \left( \frac{Z_2}{L} \right) + \psi_m \left( \frac{Z_1}{L} \right) \right) . \tag{35}
\]

Dividing equation (35) by \( z_2 - z_1 \), gives an equation of the same form as equation (6), that is,

\[
\frac{V_2 - V_1}{z_2 - z_1} = \frac{U^*}{k(z_2 - z_1)} \left( \ln \frac{Z_2}{Z_1} - \psi_m \left( \frac{Z_2}{L} \right) + \psi_m \left( \frac{Z_1}{L} \right) \right) . \tag{36}
\]
Again, as for the unstable case, we employ the mean value theorem to obtain

$$\frac{V_2 - V_1}{z_2 - z_1} = \frac{\partial V}{\partial z}\bigg|_{\xi} ,$$

(37)

but, where,

$$\frac{\partial V}{\partial z}\bigg|_{\xi} = \frac{U^*}{k\xi} \left( 1 + 5 \frac{z}{L} \right) .$$

(38)

From equations (36), (37), and (38) we obtain

$$\xi = \frac{(z_2 - z_1) \left( 1 + 5 \frac{\xi}{L} \right)}{\ln \frac{z_2}{z_1} - \Psi_m \left( \frac{z_2}{L} \right) + \Psi_m \left( \frac{z_1}{L} \right)} .$$

(39)

Assuming that an $\epsilon$ exists such that

$$\epsilon = \frac{\partial V}{\partial z}\bigg|_{z_1} ,$$

(40)

we obtain, by use of equation (32),

$$\epsilon = \frac{U^*}{k\xi} \left( 1 + 5 \frac{\xi}{L} \right) = \frac{U^*}{kz_g} \left( 1 + 5 \frac{z_g}{L} \right) .$$

(41)

Solving equation (41) for $\epsilon$ gives

$$\epsilon = \frac{\xi}{z_g} \left( \frac{1 + 5 \frac{z_g}{L}}{1 + 5 \frac{\xi}{L}} \right) .$$

(42)
Substituting $\xi$ from equation (39) into equation (42) results in

$$e = \frac{(z_2 - z_1) \left( 1 + 5 \frac{z_2}{L} \right)}{z_g \left( \ln \frac{z_2}{z_1} - \psi_m \left( \frac{z_2}{L} \right) + \psi_m \left( \frac{z_1}{L} \right) \right)} \quad (43)$$

Combining equations (37), (40), and (43) results in

$$\frac{\partial V}{\partial z} \bigg|_{z_g} = \frac{(V_2 - V_1) \left( 1 + 5 \frac{z_2}{L} \right)}{z_g \left( \ln \frac{z_2}{z_1} - \psi_m \left( \frac{z_2}{L} \right) + \psi_m \left( \frac{z_1}{L} \right) \right)} \quad (44)$$

Since

$$\frac{\partial V}{\partial z} \bigg|_{z_g} = \frac{U^*}{kz_g} \left( 1 + 5 \frac{z_2}{L} \right) \quad (45)$$

then

$$U^* = \frac{kz_g}{\left( 1 + 5 \frac{z_2}{L} \right)} \frac{\partial V}{\partial z} \bigg|_{z_g} \quad (46)$$

or by substituting equation (44) into equation (46), we obtain

$$U^* = \frac{k(V_2 - V_1)}{\left( \ln \frac{z_2}{z_1} - \psi_m \left( \frac{z_2}{L} \right) + \psi_m \left( \frac{z_1}{L} \right) \right)} \quad (47)$$

which is of the same form as the unstable case. However, $\psi_m \left( \frac{z}{L} \right)$ stable differs from $\psi_m \left( \frac{z}{L} \right)$ unstable.
To obtain the Obukhov length $L$ for stable conditions, we again use the basic form of $L$ as

$$ L = \frac{U^* \theta^*}{k \theta^*}, \quad (48) $$

where

$$ \theta^* = \frac{k (\theta_2 - \theta_1)}{\left( \ln \frac{Z_2}{Z_1} - \psi_{\theta} \left( \frac{Z_2}{L} \right) + \psi_{\theta} \left( \frac{Z_1}{L} \right) \right)} \quad , \quad (49) $$

and

$$ \psi_{\theta} \left( \frac{Z}{L} \right) = - \frac{5Z}{L} . \quad (50) $$

3. TRADITIONAL FORMULATIONS AND APPROACH

The traditional approach (see Hansen, 1980) starts with the basic form of the Richardson number, that is,

$$ R_i = \frac{g}{\theta} \frac{\partial \theta}{\partial z} \left( \frac{\partial V}{\partial z} \right)^2 \quad , \quad (51) $$

which, as shown in section 2, reduces to

$$ R_i = \frac{g}{\theta} \frac{k \theta^*}{U^*^2} z \quad . \quad (52) $$

Using the following approximations for $\theta^*$ and $U^*$

$$ \theta^* = \frac{k (\theta_2 - \theta_1) \left( 1 - \beta \frac{Z_2}{L} \right)^{1/2}}{\ln \frac{Z_2}{Z_1}} \quad (53) $$

and

$$ U^* = \frac{k (V_2 - V_1) \left( 1 - \beta \frac{Z_2}{L} \right)^{1/4}}{\ln \frac{Z_2}{Z_1}} \quad (54) $$
equation (52) becomes

\[ R_i = \frac{g z (\theta_2 - \theta_1)}{\theta (V_2 - V_1)^2} \ln \frac{z_2}{z_1}. \]  (55)

For \( z = z_g \) equation (55) becomes

\[ R_{ig} = \frac{g (\theta_2 - \theta_1)}{\theta z_g (V_2 - V_1)^2} z_g \ln \frac{z_2}{z_1}. \]  (56)

4. COMPARISONS

In this section we present comparisons of \( U^*, \theta^*, L, \) and \( R_i \) for four sublayers, (1/2 - 2 m), (1 - 4 m), (2 - 8 m), and (4 - 16 m), using the proposed Rachele/Tunick (RT) formulations and the traditional formulations.

The values of \( V \) and \( \theta \) required to evaluate the equations were obtained from profiles we constructed assuming that \( U^* = 0.4 \text{ m/s}, \theta^* = -0.5 \text{ °C}, L = -24.747 \text{ m}, \) and \( z_o = 0.01 \text{ m} \) for the unstable case, and \( U^* = 0.4 \text{ m/s}, \theta^* = +0.5 \text{ °C}, L = -24.747 \text{ m}, \) and \( z_o = 0.01 \text{ m} \) for the stable case.

Results are presented in tables 1 through 8.

**TABLE 1.** \( U^*, \theta^*, L, \) and \( R_i \) FOR THE SUBLAYER (1/2 - 2 m) (unstable case)

<table>
<thead>
<tr>
<th></th>
<th>Rachele/Tunick (RT)</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^* ) (cm/s)</td>
<td>40.0000</td>
<td>39.84015</td>
</tr>
<tr>
<td>( \theta^* ) (°C)</td>
<td>-0.49999</td>
<td>-.496729</td>
</tr>
<tr>
<td>( L ) (m)</td>
<td>-24.74775</td>
<td>-24.76738</td>
</tr>
<tr>
<td>( R_i )</td>
<td>-4.04077 x 10^{-2}</td>
<td>-4.037568 x 10^{-2}</td>
</tr>
</tbody>
</table>

**TABLE 2.** \( U^*, \theta^*, L, \) and \( R_i \) FOR THE SUBLAYER (1 - 4 m) (unstable case)

<table>
<thead>
<tr>
<th></th>
<th>Rachele/Tunick (RT)</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^* ) (cm/s)</td>
<td>40.0000</td>
<td>39.879406</td>
</tr>
<tr>
<td>( \theta^* ) (°C)</td>
<td>-0.49999</td>
<td>-.4984686</td>
</tr>
<tr>
<td>( L ) (m)</td>
<td>-24.74775</td>
<td>-24.67877</td>
</tr>
<tr>
<td>( R_i )</td>
<td>-8.08154 x 10^{-2}</td>
<td>-8.104131 x 10^{-2}</td>
</tr>
</tbody>
</table>
### TABLE 3. \( u^* \), \( \theta^* \), L, and \( R_i \) FOR THE SUBLAYER (2 – 8 m) (unstable case)

<table>
<thead>
<tr>
<th></th>
<th>Rachele/Tunick (RT)</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^* ) (cm/s)</td>
<td>40.0000</td>
<td>39.980267</td>
</tr>
<tr>
<td>( \theta^* ) (°C)</td>
<td>-0.50000</td>
<td>-.5019645</td>
</tr>
<tr>
<td>L (m)</td>
<td>-24.74775</td>
<td>-24.588487</td>
</tr>
<tr>
<td>( R_i )</td>
<td>-0.1616308</td>
<td>-.1626777</td>
</tr>
</tbody>
</table>

### TABLE 4. \( u^* \), \( \theta^* \), L, and \( R_i \) FOR THE SUBLAYER (4 – 16 m) (unstable case)

<table>
<thead>
<tr>
<th></th>
<th>Rachele/Tunick (RT)</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^* ) (cm/s)</td>
<td>39.99999</td>
<td>40.102282</td>
</tr>
<tr>
<td>( \theta^* ) (°C)</td>
<td>-.50000</td>
<td>-.50595617</td>
</tr>
<tr>
<td>L (m)</td>
<td>-24.74775</td>
<td>-24.509748</td>
</tr>
<tr>
<td>( R_i )</td>
<td>-.3232616</td>
<td>-.3264007</td>
</tr>
</tbody>
</table>

### TABLE 5. \( u^* \), \( \theta^* \), L, and \( R_i \) FOR THE SUBLAYER (1/2 – 2 m) (stable case)

<table>
<thead>
<tr>
<th></th>
<th>Rachele/Tunick (RT)</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>( U^* ) (cm/s)</td>
<td>40.00000</td>
<td>40.634</td>
</tr>
<tr>
<td>( \theta^* ) (°C)</td>
<td>0.50000</td>
<td>0.50793</td>
</tr>
<tr>
<td>L (m)</td>
<td>24.74775</td>
<td>25.05282</td>
</tr>
<tr>
<td>( R_i )</td>
<td>0.04041</td>
<td>0.03327</td>
</tr>
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</table>

### TABLE 6. \( u^* \), \( \theta^* \), L, and \( R_i \) FOR THE SUBLAYER (1 – 4 m) (stable case)

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<tr>
<th></th>
<th>Rachele/Tunick (RT)</th>
<th>Traditional</th>
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<tbody>
<tr>
<td>( U^* ) (cm/s)</td>
<td>40.00000</td>
<td>41.332</td>
</tr>
<tr>
<td>( \theta^* ) (°C)</td>
<td>0.50000</td>
<td>0.51665</td>
</tr>
<tr>
<td>L (m)</td>
<td>24.74775</td>
<td>25.58276</td>
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<tr>
<td>( R_i )</td>
<td>0.08081</td>
<td>0.05621</td>
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TABLE 7. \( U^*, \theta^*, L, \) and \( R_i \) FOR THE SUBLAYER (2 - 8 m) (stable case)

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<td>( U^* ) (cm/s)</td>
<td>40.00000</td>
<td>42.823</td>
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<td>( \theta^* ) (°C)</td>
<td>0.50000</td>
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<td>( L ) (m)</td>
<td>24.74776</td>
<td>26.63664</td>
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<tr>
<td>( R_i )</td>
<td>0.16163</td>
<td>0.08576</td>
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TABLE 8. \( U^*, \theta^*, L, \) and \( R_i \) FOR THE SUBLAYER (4 - 16 m) (stable case)

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<tr>
<td>( U^* ) (cm/s)</td>
<td>40.00000</td>
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<td>( \theta^* ) (°C)</td>
<td>0.50000</td>
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<tr>
<td>( L ) (m)</td>
<td>24.74777</td>
<td>28.87072</td>
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<tr>
<td>( R_i )</td>
<td>0.32326</td>
<td>0.11616</td>
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5. SUMMARY AND CONCLUSIONS

This study was performed to develop formulations that can be used to compute estimates of the height derivatives at the geometric mean height of a sublayer (layer between any two successive heights) and the similarity scaling constants for the sublayers using the mean value theorem of calculus. As such, we require that discrete vertical wind and temperature data be obtained from sensors that are geometrically spaced with height.

A prior study (Rachele et al., 1991) showed that the height derivative at the geometric mean height for similarity functional forms was not generally equal to the difference derivative of the discrete data.

Values of the scaling constants, Obukhov length, and vertical profiles of wind and temperature obtained from the formulations developed in this report were compared with values obtained from traditional equations showing minor differences for the unstable case but somewhat larger differences for the stable case, especially when considering \( R_i \).
LITERATURE CITED


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