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New polyhedral methods have been developed for the solution of a class of programming problems of importance in VLSI design. There methods have made possible an order-of-magnitude increase in the size of problems that can be successfully solved.

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Final Report on AFOSR Grant #90-0273

Polyhedral Methods for the Max-Cut Problem

Principal Investigator: Robert E. Bixby, Rice University

The max-cut problem is an NP-complete problem and has several important applications, among them the Ising spin-glass problem and VLSI design. Our research has concentrated on spin-glass problems, but applies equally to VLSI problems. Indeed, the latter problems are probably of more interest, but it is also much more difficult to gain direct access to real problem instances.

Our basic approach is one based on polyhedral methods. In the typical polyhedral framework, one would study a problem of the form

$$\begin{array}{ll} \max & c^T x \\ \text{s.t.} & x \in P \end{array}$$

where c is a given "weight function" defined on the edges of some underlying graph, and P is the convex hull of the incidence vectors of cuts in that graph. Our approach differs from the standard one in that we first apply a heuristic procedure to generate a good initial cut, and then transform the given objective function to a new objective function such that the initial cut is optimal if and only if the empty cut is optimal with respect to the new objective function. The effect of this transformation is not only that we can make use of a good initial guess, unlike the standard approach, but also that, since the empty set becomes the focus of the optimization, we are in effect optimizing over the positive hull of P, the so-called cut cone, rather than P. The effect is that amount of time spent on separation is four to five times less than seems to be required for separation over P.

Prior to our work, the best available results for the Ising problem were obtained by Barahona, Grötschel, Jünger and Reinelt. They managed to treat instances with up to 1800 edges. With our methods, we can now treat problems with up to approximately 15000 edges, an improvement of one order of magnitude. Details of the approach we have used and computational results are described in the attached thesis of Sanjay Saigal "Optimizing over the Cut Cone: A New Polyhedral Algorithm for the Maximum-Weight Cut Problem." This thesis is the principal result of this grant.

It should be mentioned in connection with the above work that the principal computational step in our algorithm is the solution of a sequence of appropriately generated linear programming problems. These LP's are highly degenerate and large, the biggest having approximately 15000 rows, 45000 columns and 500,000 constraint-matrix nonzeros. For the most difficult max-cut instances it was necessary to solve over 1000 LP's of this type. Total computation time was on the order of 2 hours on a Cray Y-MP. Improving LP techniques for solving the linear programs that arise in integer programming was also an important consideration in our research.