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6. AUTHOR(S)
Drs David Colton & Peter Monk

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
University of Delaware
Dept of Mathematical Sciences
501 Ewing Hall
Newark, DE 19716

8. PERFORMING ORGANIZATION REPORT NUMBER
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This project was concerned with the inverse scattering problem for time harmonic acoustic and electromagnetic waves. A new method has been developed to solve problems of this type based on the theory of Herglotz wave functions and nonlinear optimization methods. Preliminary numerical examples have been given for the case of both acoustic and electromagnetic waves.

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Final Report

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Summary: This research project was concerned with the inverse scattering problem for time harmonic acoustic and electromagnetic waves. A new method has been developed to solve problems of this type based on the theory of Herglotz wave functions and nonlinear optimization methods. Preliminary numerical examples have been given for the case of both acoustic and electromagnetic waves.

Professional Personnel

Dr. David Colton — Professor of Mathematics
University of Delaware
Principal Investigator

Dr. Peter Monk — Associate Professor of Mathematics
University of Delaware
Principal Investigator

Report on Research

The research of AFOSR Grant 89–0284 was concerned with the inverse scattering problem for time–harmonic acoustic and electromagnetic waves. In particular, the goal was to determine the physical and geometric properties of an unknown inhomogeneity from a knowledge of the given incident field and the far field pattern of the scattered field. As with many inverse problems in partial differential equations, the main mathematical difficulties arise from the fact that such a problem is nonlinear and improperly posed (c.f. [1]). The resolution of these difficulties can be obtained through the use of nonlinear optimization methods combined with recent developments in the theory of Herglotz wave functions. The method has already been numerically implemented for various problems in acoustic scattering theory. The aim of this project was to extend the basic theoretical results obtained for the scattering of acoustic waves by an obstacle to the case of acoustic and electromagnetic scattering by an inhomogeneous medium, to extend the results obtained for acoustic scattering by an obstacle to the case of electromagnetic scattering by a possibly imperfect conductor where polarization effects are given particular consideration and, finally, to use these results to derive new numerical methods for solving the inverse scattering problem.

The inverse scattering problem for acoustic waves in an inhomogeneous medium was investigated in [3], [4], [9], [11], [12] and [14]. In [3] and [11] the dual space method of Colton and Monk was shown to be viable for a nonabsorbing medium provided the wave number k is not a transmission eigenvalue or for an absorbing medium for all positive values of k. The set of transmission eigenvalues was shown to be at most a countable set. In [4], [9] and [14] a modified dual space method was derived which avoids the problem of transmission eigenvalues. Of particular note are the numerical reconstructions obtained using this modified method for a non-stratified medium in $\mathbb{R}^2$ [14]. The dual space method and the modified dual space method were numerically compared in [12]. A survey of some of these results was given in [2].

The above results for acoustic waves in an inhomogeneous medium were extended to the case of electromagnetic waves in [7] and [8]. These results were surveyed in [15]. However, in contrast to the case of acoustic waves, numerical experiments are still pending. In [13], sufficient conditions were given for the uniqueness of reconstructing the index of refraction from the electric far field data. This result extends those of Nachman, Sylvester and Ulhmann for the scalar problem to the case of Maxwell's equations. In a slightly different direction, the inverse scattering problem for electromagnetic waves scattered by a perfectly or imperfectly conducting obstacle was considered in [5], [6] and [10] where particularly attention was given to polarization effects. Numerical examples for an infinite cylinder were given in [6] and for axially symmetric obstacles in $\mathbb{R}^3$ in Misici and Zirilli. An inverse problem for the three dimensional vector Helmholtz equation for a perfectly conducting obstacle with incomplete data (to appear).

As mentioned above, numerical examples utilizing the theoretical results of [7] and [8] are still pending. The difficulty lies in the lack of an efficient method for solving the three dimensional Maxwell equations in an inhomogeneous medium. Kirsch and Monk have recently proposed a method which combines the use of finite elements in the inhomogeneous medium with boundary integral equations in the exterior of this region. This approach was implemented for the scalar case in [14].

However, there are serious difficulties in extending the ideas of [14] to the vector case. Research in this direction is promising but still in progress ([16], [17], [18], [19]). A survey of some of this work can be found in [20].

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