AD A244 563

UNCLASSIFIED MASTER COPY - FOR REPRODUCTION PURPOSES

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1a. REPORT SECURITY CLASSIFICATION Unclassified	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION / DOWNGRADING SCHEDULE	3. DISTRIBUTION / AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S)	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION University of Maryland 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION U. S. Army Research Office
6c. ADDRESS (City, State, and ZIP Code) IPST University of Maryland College Park, Md. 20742	7b. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211
8a. NAME OF FUNDING / SPONSORING ORGANIZATION U. S. Army Research Office 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
8c. ADDRESS (City, State, and ZIP Code) P. O. Box 12211 Research Triangle Park, NC 27709-2211	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (Include Security Classification)
Numerical solution of convection diffusion equation - final report, unclass.

12. PERSONAL AUTHOR(S) R. Bruce Kellogg

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15. PAGE COUNT
final	FROM 5/88 TO 6/91	August 15, 1991	

16. SUPPLEMENTARY NOTATION
The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

17. COSATI CODES		
FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
boundary layers, singular perturbations, viscous compressible flow, fluid dynamics, iterative methods

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

The original proposal contained four parts, and we have divided the final report in the same way. In each part, we briefly describe our major findings.

20. DISTRIBUTION / AVAILABILITY OF ABSTRACT ☐ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified
22a. NAME OF RESPONSIBLE INDIVIDUAL	22b. TELEPHONE (Include Area Code) / 22c. OFFICE SYMBOL

DD FORM 1473, 84 MAR

83 APR edition may be used until exhausted.
All other editions are obsolete.

SECURITY CLASSIFICATION OF THIS PAGE
UNCLASSIFIED

FINAL REPORT

1. ARO PROPOSAL NUMBER: 25395 - MA

2. PERIOD COVERED BY REPORT: May, 1988 - June, 1991

3. TITLE OF PROPOSAL: Numerical solution of convection diffusion equation

4. CONTRACT OR GRANT NUMBER: DAAL03 - 88 - K - 0081

5. NAME OF INSTITUTION: University of Maryland

6. AUTHOR OF REPORT: R. Bruce Kellogg

7. MANUSCRIPTS SUBMITTED OR PUBLISHED:

1. V. Vanaja and R. B. Kellogg, "Iterative methods for a forward-backward beat equation", SIAM J. Numer. Anal. 27(1990), 622-635,

2. H. Han and R. B. Kellogg, "Differentiability properties of solutions of the equation $-\varepsilon^2\Delta u+ru = f(x,y)$ in a square", SIAM J. Math Anal. 21, (1990), 394-408,

3. S. E. Chen and R. B. Kellogg, " An interior discontinuity of a nonlinear elliptic-hyperbolic system", SIAM J. Math Anal. 22, (1991), 602-622.

4. R. B. Kellogg, "Spectral bounds and iterative metbods in convection dominated flow" , pp. 80-102, in *Computational Methods for Boundary and Interior Layers in Several Dimensions*, J. J. H. Miller, editor, Boole Press, Dublin, 1991.

5. R. B. Kellogg, "Iterative methods for convection dominated flow" accepted for publication by Rendiconti del Seminario Mathematico, Universita Polytechnico Milano.

6. S. E. Cben and R. B. Kellogg, "Interior pressure discontinuities in compressible viscous steady state flow", Eighth Army Mathematics Conference, ARO Report 91-1, 511-520.

7. A. K. Aziz and R. B. Kellogg, "On homeomorphisms for an elliptic equation in domains with corners". Submitted to the Transactions of the American Mathematical Society.

8. SCIENTIFIC PERSONNEL SUPPORTED

1. Professor R. Bruce Kellogg
2. Dr. Senhuei Chen was partly supported during his Ph.D work. The Ph.D tbesis of Dr. Chen is substantially the same as [3].

Major Research Findings

The original proposal contained four parts, and we have divided the final report in the same way. In each part, we briefly describe our major findings.

1. Flow directed iterative methods

It was proposed to develop iterative methods for non-symmetric matrices of the type arising from convection-diffusion problems.

Our results in this area are contained in [1,4,5]. [1] deals with an iterative method for a particular problem, and illustrates the usefulness of shaping the iterative method to the problem at hand. Our major result in this area is contained in [4], in which it is shown that for convection diffusion problems, an appropriate cboice of iteration parameters gives work estimates for the solution of the problem tbat are comparable to work estimates for the model elliptic self adjoint problem; that is, optimally over-relaxed SOR applied to the model 5 point formula for the Laplace operator. In [5] we have given a survey of some of our results and conjectures in this area. We are planning some numerical experiments to test these results in the near future.

2. Numerical solution of parabolic problems with interior layers

The proposed work consisted of studying the asymptotic structure of internal layers and moving fronts. Our results in this area have not achieved final form, so we have not published anything on this.

3. Viscous compressible flow

The proposed work in this area is to establish the existence of discontinuous viscous compressible flows, and to determine important properties of these flows.

Our work in this area is contained in [3,6]. Each of these papers contains major findings. In [3] we have established the existence of discontinuous solutions of a nonlinear system of partial differential equations with an elliptic-hyperbolic character. The system is related to, but simpler than the Navier-Stokes equations. The analysis depends on the careful choice of Banach space; the norm in this space must be strong enough to fulfill demands set by the hyperbolic part of tbe system, and weak enough to fulfill demands set by the elliptic part of the system. This work constitutes the Ph.D thesis of Senhuei Chen, who was partly supported by the grant.

In [6] we give a closer study of the linearized viscous compressible flow equations, linearized at a uniform ambient flow. Using the Fourier transform and *Mathematica*, the most important properties of a discontinuous solution are found. These properties include an $r \ln r$ type of singularity in the velocity at the entrance and exit points of the curve of discontinuity. These results are being rewritten for publication in a refereed journal. We have continued this line of investigation, and bave formulated a linearization of the flow equations around an ambient *discontinuous* solution of tbe nonlinear system. Using this we have established the linearized stability of putative discontinuous viscous compressible flows that are close to uniform. These results are also being written up for publication.

4. Corners in singular perturbation problems

The proposed work here is to study the interaction of boundary layers and corner singularities in singularly perturbed elliptic boundary value problems in polygonal domains.

[2] contains our published work in this area. We have actively continued this work, and obtained major new results that will shortly be available. (A copy will be sent to the ARO when it is submitted for publication.) The new work considers the singularly perturbed elliptic boundary value problem

$$\begin{aligned} Lu &:= -\varepsilon^2 \Delta u + qu = f \text{ in } \Omega, \\ u &= g \text{ on } \Gamma = \partial\Omega, \end{aligned}$$

where $\Omega \subset R^2$ is the interior of a polygon Γ. ([2] considers the case of a rectangle.) The study involves the interaction of two distinct phenomena, corner singularities and boundary layers. We obtain an asymptotic expansion of the solution that displays both the corner singularities occuring near the vertices of Γ and the boundary layer behavior that is present when ε is small. The asymptotic expansion can be differentiated to provide an asymptotic expansion for the derivatives of u. As a consequence of the asymptotic expansion, bounds for tbe derivatives of the solution are derived that display both the dependence on ε and the dependence on the distance to a corner or side of Γ. These bounds show that while a small value of ε produces boundary layers along the sides of Γ that decay exponentially as one moves into Ω, the small value of ε also mitigates the effect of the corner singularities. That is, the corner singularity is multiplied by an attenuation factor of the form $e^{-ad_v/\varepsilon}$, where d_v is the distance to the corner. This means, for example, that the pollution effect of the corner singularity on the error in a finite element computation will be mitigated by a small value of ε.