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PARAMETRIC INVESTIGATION OF HOLOGRAPHIC GRATINGS AND OPTICAL PHASE CONJUGATION THROUGH DEGENERATE FOUR WAVE MIXING IN SATURABLE ABSORPTIVE/RESONANT/NONRESONANT SYSTEMS

11

Final Report

Putcha Venkateswarlu

November 26, 1991

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INSTITUTION Alabama A&M University, Normal, AL 35762

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1 STATEMENT OF THE PROBLEM STUDIED

Parametric investigation of volume holographic gratings and optical phase conjugation are studied through coherent and incoherent beam couplings in $BaTiO_3$ and through the study of bistability in phase conjugate resonators. Experiments have been done on the effect of color centers in the formation of resonant systems and holographic grating formation.

2 SUMMARY OF THE IMPORTANT RESULTS

2.1 BEAM COUPLINGS

Beam couplings and self pulsations in self pumped $BaTiO_3$ crystal have been studied using two incoherent beams from a cw Ar⁺ laser at different wavelengths and similarly with two coherent beams.

2.11 Incoherent Self-Pumped Beam Coupling

Eason and Smout^{1,2} carried out an analysis of mutually incoherent beam coupling in BaTiO₃ when they are incident on a single face of the crystal with incident angles of 15° and 16° in the quadrant favourable for self pumping. They reported observation of a loop of light resulting in two phase conjugate outputs. We have reported³ our results on beam couplings in BaTiO₃ self-pumped at 5145, 4880, 4765 and 4580 A by two incoherent beams A_1 and A_2 with variable power ratios. In the first set of experiments, the beams cross in the crystal at 2°, while in the second they cross at the same angle before reaching the crystal. Fig.1 shows the experimental arrangement which is similar to the one used by Eason and Smout¹. With a ratio of about 4:1 in the powers of $A_1(16.5mw)$

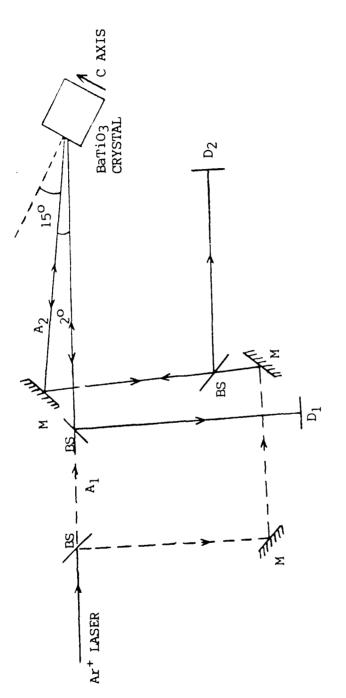
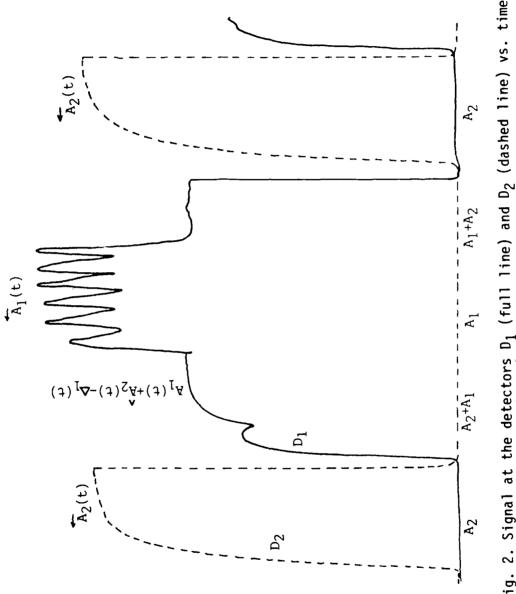


Fig. 1. Experimental arrangement for incoherent beam coupling. Beam A₁ and A₂ are incoherent. Path difference 150 cm. Multimode Ar⁺ laser. M: mirror, BS: beam splitters, D : detectors.

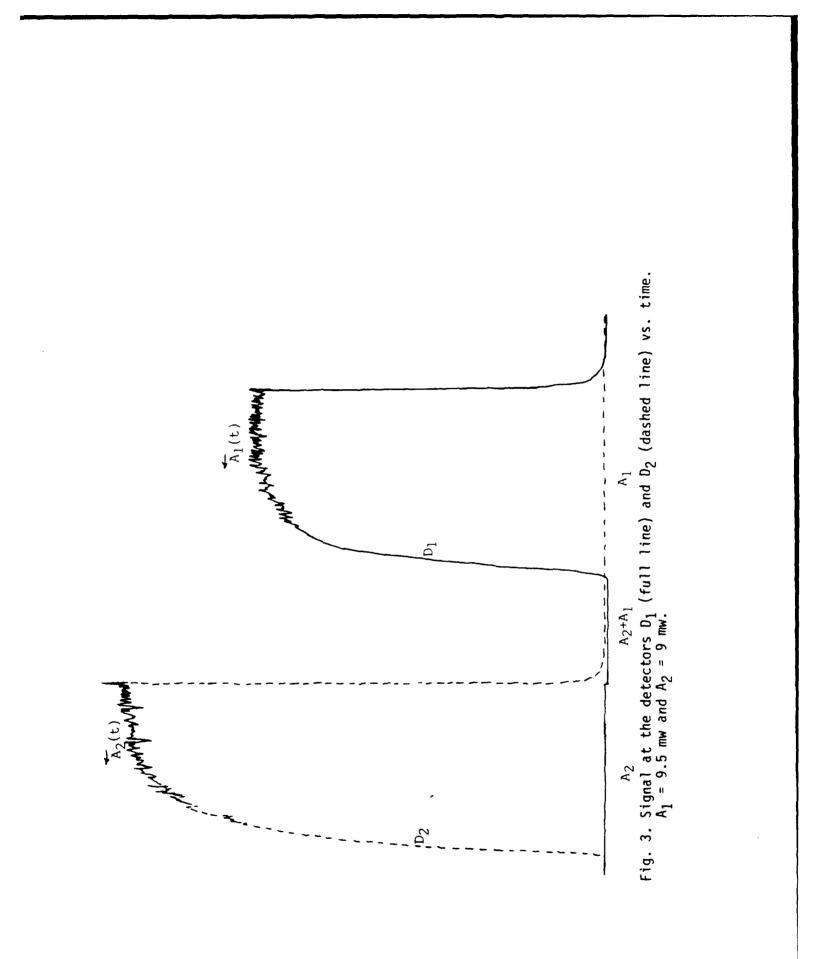
and $A_2(4mw)$, the signal $\overleftarrow{A_1}(t)$ at D_1 shows self oscillations while $\overleftarrow{A_2}(t)$ at D_2 remains steady when individually pumped (Fig. 2). Here $\overleftarrow{A_1}(t)$ and $\overleftarrow{A_2}(t)$ represent phase conjugates of A_1 and A_2 respectively. The frequency of oscillations at D_1 increases with the wavelength of the laser and its power. With simultaneous pumping, the signal at D_1 becomes stable and the one at D_2 goes to zero (see Fig. 2).

With the beam powers nearly equal, in the range, 6-17 mw $\overline{A_1}(t)$ and $\overline{A_2}(t)$ do not coexist and each beam effectively erases the other beam grating (Fig. 3). When the crystal is pumped by A_1 and A_2 simultaneously, it is seen in all the experiments that the signal at D_1 may be expressed as $\overline{A_1}(t) + \widehat{A_2}(t) - \Delta_1(t)$ and that at D_2 as $\overline{A_2}(t) + \widehat{A_1}(t) - \Delta_2(t)$. Here $\overline{A_2}(t)$ and $\overline{A_1}(t)$ are the Bragg diffractions of A_2 and A_1 in the opposite directions of A_1 and A_2 respectively. $\Delta_1(t)$ and $\Delta_2(t)$ are the general erasure effects of the beams A_2 and A_1 in the gratings responsible for the self-pumps of A_1 and A_2 respectively.

In the second set of experiments, the beams A_1 and A_2 with nearly equal powers (6-17mw) cross before reaching the crystal surface. However, because of the size of the beams the individual filaments generates by them very likely overlap to introduce couplings between the two beams. With $A_1 = A_2 = 11$ mw, the phase conjugates $\overline{A_1}(t)$ and $\overline{A_2}(t)$ of the two beams A_1 and A_2 show oscillations under individual pumping while they are nearly stable under simultaneous pumping (see Fig. 4a). If after simultaneous pumping A_2 is shut off, the signal at D_1



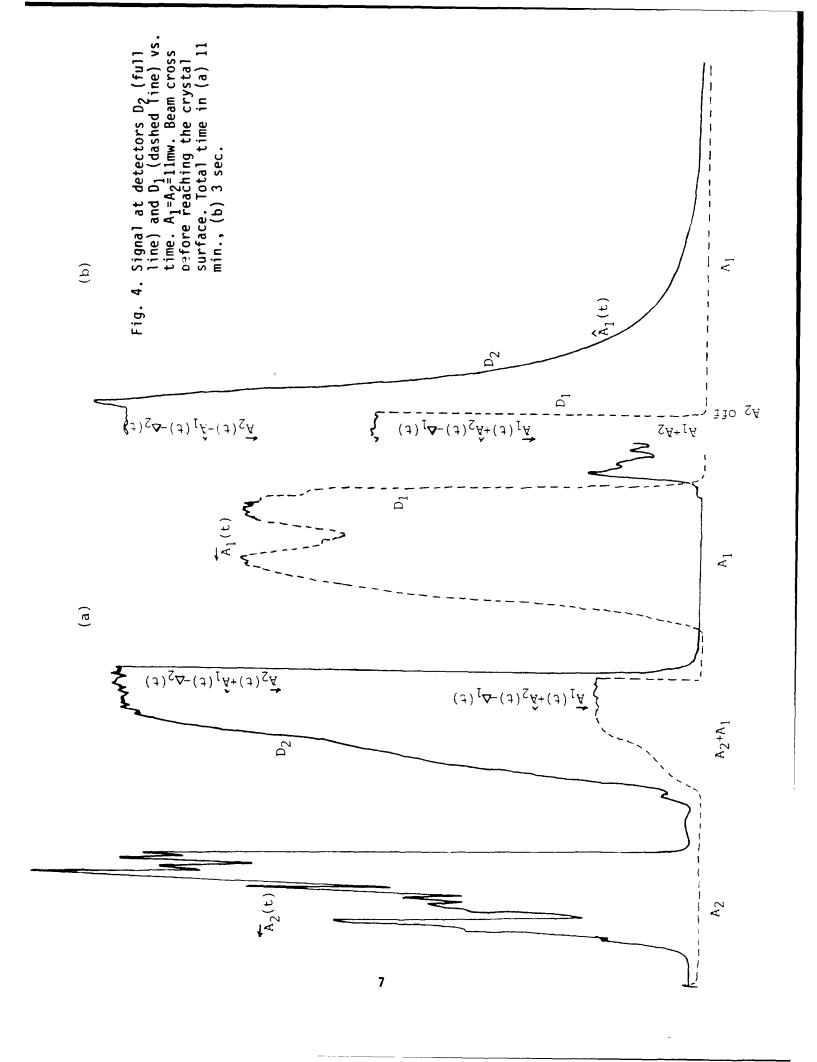




shows a sharp drop to zero while that at D_2 shows an exponential decay (Figs. 4a and 4b). This clearly indicates the existence of the cross coupled signals $\hat{A}_1(t)$ and $\hat{A}_2(t)$ in the directions of $\hat{A}_2(t)$ and $\hat{A}_1(t)$, respectively. As indicated earlier, the signals at D_1 and D_2 under simultaneous pumping may be represented by $[\hat{A}_1(t) + \hat{A}_2(t) - \Delta_1(t)]$ and $[\hat{A}_2(t) + \hat{A}_1(t) - \Delta_1(t)]$ respectively. Thus, when A_2 is shut off, $\overleftarrow{A}_2(t)$ and $\Delta_1(t)$ drop to zero and $\dot{A}_1(t)$ decays exponentially depending on the life of the relevant transient grating (Fig. 4b). The signal at D_1 drops to zero abruptly when A_2 is shut off because A₂ drops to zero, and also probably because $A_1(t) - \Delta_1(t) \cong 0$. The signal at D₁ then grows up and reaches its maximum of $\overleftarrow{A_1}$ as the signal is under the individual pumping of A₁ only (Fig. 4a). Similarly, when A_1 is put off, D_1 shows the decay of $\hat{A}_2(t)$ while D_2 shows a sudden drop to zero as probably $\overline{A}_1(t) \cong \Delta_1(t)$. It is found that $\hat{A}_2(t)$ is less than $A_{1}(t).$

2.12 Coherent Beam Couplings

Venkateswarlu <u>et al</u>⁴ used different configurations to study coherent beam couplings. In the first case which is a parallelogram configuration, two coherent beams $A_1(3.9\text{mw})$ and $A_2(4.4\text{mw})$ from an Ar^+ laser (4580Å) cross (with $\theta = 40^\circ$) in an electrically poled BaTiO₃ crystal (8x8x6 mm³) with the horizontal laser polarization, c axis being in the same plane (Fig. 5). If the crystal is individually exposed to A_1



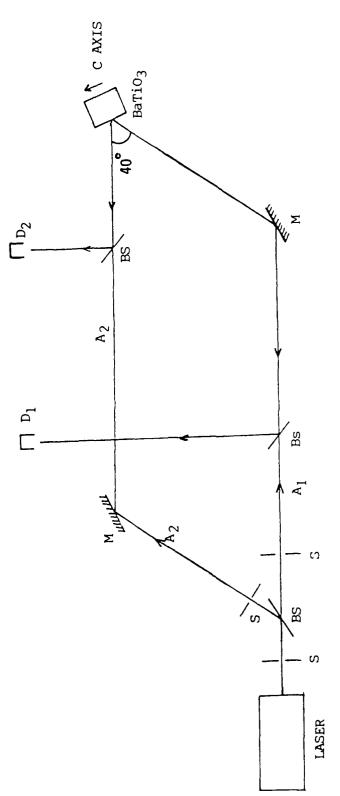


Fig. 5. Parallelogram configuration for phase conjugation: BS: beam Splitter, M: mirror, S: slit, D_1 and D_2 : detectors, A_1 and A_2 : input beams which meet at the center of the crystal, $A_1(t)$ and $A_2(t)$: self-pumped phase conjugates of A_1 and A_2 , $\hat{A}_1(t)$ and $\hat{A}_2(t)$ cross coupled beams of A_2 and A_1 respectively.

(or A_2), one observes self-pumped beam $\overleftarrow{A}_1(t)$ [or $\overleftarrow{A}_2(t)$]. When A_1 is allowed to enter the crystal after the self-pumped phase conjugate signal $\overleftarrow{A}_2(t)$ levels off, there is a sudden increase in the signal at detector D_2 (see Fig. 6). The signals at D_2 and D_1 are represented in the same manner as in the earlier section. One can see from Fig. 6 that the steady state values of $\overleftarrow{A}_2(t)$, $\widehat{A}_1(t)$ and $\Delta_2(t)$ are in the approximate ratio 9:36:7. One can see in a similar manner that $\overleftarrow{A}_1(t)$, $\widehat{A}_2(t)$ and $\Delta_1(t)$ in the ratio 8:175:6. A_1 and A_2 both self pump but $\overleftarrow{A}_1(t) << \overleftarrow{A}_2(t)$ and $\widehat{A}_2(t) > \widehat{A}_1(t)$. These experiments were repeated with 4756Å excitation. The results are similar except that the individual self-pumping of A_1 is better with 4850A than with 4765Å.

In a separate parallelogram experiment, the self pump of the beams A_1 and A_2 , and their beam couplings are studied at different points of entrance on the crystal surface, at different excitation wavelengths and at different powers. The beam crossing angle is 48° and the angle of incidence of A_2 is 20°. If the point of entry is 1.5 mm from the edge nearest to A_2 , A_2 shows systematic oscillations whose frequency increases with wavelength and power while A_1 continues to be steady. If the beams A_1 and A_2 pump simultaneously, the detectors at D_1 and D_2 both show about 16 fold increase on intensities (Fig. 7). Thus, the signal at D_2 under simutaneous pumping represented by $[\tilde{A}_2(t) + \hat{A}_1(t) - \Delta_2(t)]$ is about 16 times more than the signal $\tilde{A}_2(t)$ under individual pumping. Similar observations are made with the signals at D_1 . This is because of

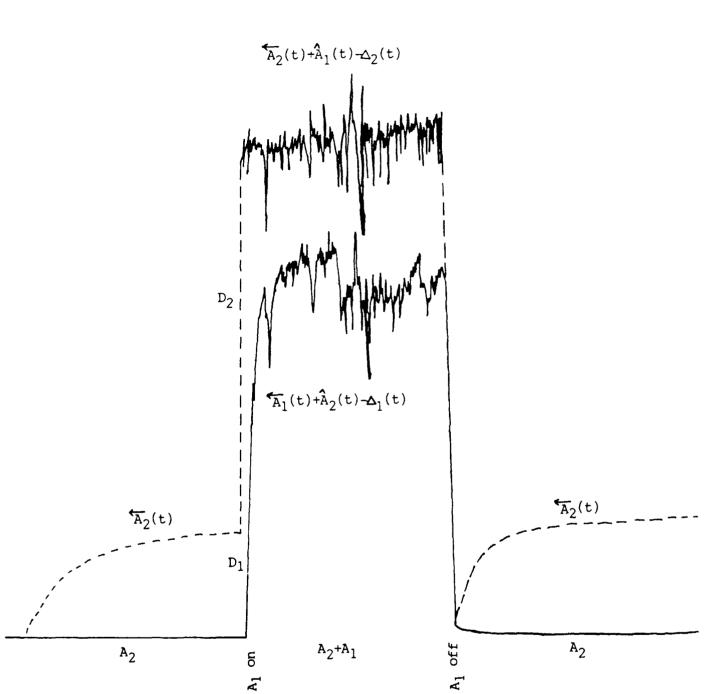


Fig. 6. Self-pumped phase conjugate signals: D_1 and D_2 are signals at detectors D_1 and D_2 in Fig. 5. $\Delta_1(t)$ and $\Delta_2(t)$ are erasure effects on $A_1(t)$ and $A_2(t)$ due to the A_2 and A_1 respectively. Ar⁺ laser (4580 Å) has horizontal polarization, D_1 (full line) and D_2 (dashed line). Total time 5 min.

Fig. 7. Self-pumped phase conjugate signals at detectors D_2 in full line and those at detector D_1 in dashed line. Ar⁺ laser at 4765 Å. Parallelogram configuration with beam crossing angle as 48° Beam A₂ enters at 1.5 mm from the nearest edge with 20° angle if incidence. Under individual pumping the signal at D₂ oscillates but not the one at D₁. When A₁ is also opened, the signal at D₂ increases 16 times, and the signal at D₂ and the signal at D₂ increases 16 times 10 min.

λ_(t)

Λ2

 $A_2^{+A_1}$

A2

i١

Å₂(t)

the significant mutual fanning, A_1 and A_2 Bragg diffract considerably into the reverse directions of each other to D_2 and D_1 respectively making $\hat{A}_2(t) \gg \hat{A}_1(t)$ and $\hat{A}_1(t) \gg \hat{A}_2(t)$. The self pump of A_1 is found to be better with 4580 Å and 4765 Å excitations than other wavelengths.

2.13 Effects Of Relative Beam Intensities On Coherent Beam Couplings In Self-Pumping BaTiO₃

The experimental configuration⁵ and the connected details are shown in Fig. 8. A 15 mw He-Ne cw laser has been utilized in this experiment, using horizontal polarization. After passing through an adjustable slit the beam is split using a variable beam splitter into two separate beams A₁ and A₂. An electrically poled ($8x8x6 \text{ mm}^3$) BaTiO₃ crystal is used. The c axis of the crystal is along the 6 mm edge and is marked in the Fig. 8. The beams A₁ and A₂ cross each other in the middle of the crystal at an angle of 46°. The angle of incidence of the beam A₁ is 20.5° with the normal to the front face, and that of A₂ is 25.5°. The variable beam splitter (VBS) at the dividing point of the beam can vary the intensities of the beam as needed. The two other conventional beam splitters, BS₁ and BS₂, enable us to record signals of self-pumped phase conjugates of A₁ and A₂ at detectors D₁ and D₂, respectively.

The experiments were carried out by Moghbel⁵ using different values of intensities for the beams A_1 and A_2 . It has been found that the observed behaviour of the self-pumped phase conjugate signals $\overline{A_1}$ and

 \tilde{A}_2 , the cross coupled signals \hat{A}_1 and \hat{A}_2 and the mutual erasure effects Δ_1 and Δ_2 on the grating of one beam by the other and the self erasure effects δ_1 and δ_2 of each beam on their phase conjugates \tilde{A}_1 and \tilde{A}_2 depend on the intensities of A_1 and A_2 . For example A_1 showed self-pumped phase conjugation appreciably over a wide range of intensities, from 4.5 mW to 10 mW, and showed negligible self pumping at 2 mW and below, while A_2 did not show any self pumping for $A_2 < 13$ mW.

In region 1 of Fig. 9, both beams were initially kept turned on and the detectors D_1 and D_2 showed steady state signals of 1.1 and 0.6 volts, respectively. Beam A_2 was then turned off and the signals at D_1 and D_2 decayed to zero. No self-pumped phase conjugate signal was noticed at detector D_1 though A_1 was kept on. This shows that when both beams are on, most of the contribution to the signal at D_1 comes from the cross coupling of A_2 which is denoted as \hat{A}_2 . The beam A_2 was turned on after another 2.5 minutes (region 2). The signal at D_1 and D_2 started to grow and leveled off after 8.5 minutes at 1.13 and 0.62 volts, respectively, in a total scale of 4 volts. A_1 was then shut off and the signal at D_2 came to zero showing no self pumping of A_2 , though A_2 was on (region 3).

This shows again that the signal at D_2 , when both beams are on, is due to the cross coupling of A_1 , denoted by \hat{A}_1 . Then A_1 was turned on again (region 4). The growth in the region 4 is similar to that in region 2.

Fig.10 shows a slow recording of the region 1 indicating the decay

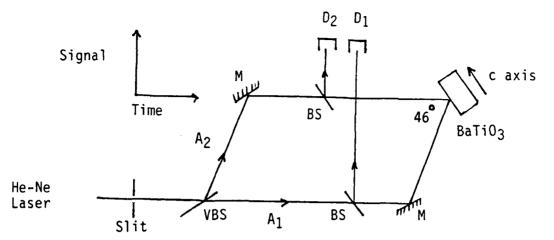


Fig.8: Experimental configuration of beam coupling using parallelogram set up. BS: beam splitter, VBS: variable beam splitter, M: mirror, D: detectors.

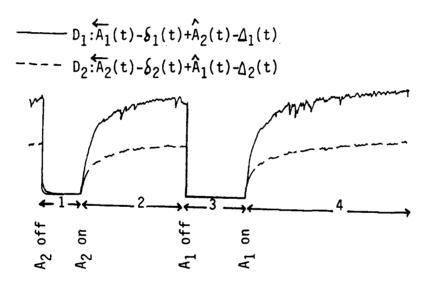


Fig.9: Phase conjugate signal due to coherent beams ${\rm A}_1$ and ${\rm A}_2$ in the above set up at detectors ${\rm D}_1$ and ${\rm D}_2.$ Total time 16 minutes.

of the signals at the detectors D_1 and D_2 . In this part of the experiment, both beams were first kept on until the signals at D_1 and D_2 leveled off. After a few seconds beam A_2 was put off for the remaining time of the 4 minutes of recording. When A_2 is off, the signal at D_1 decays to zero and indicates a small growth in the end because of the self-pumped phase conjugate of A_1 which is denoted as $\overline{A_1}$. The signal at D_2 when A_2 is off, is because of the cross coupling of A_1 and it decays exponentially as its grating dies off. Fig. 11 depicts the recording of the decay of the signals shown in the Fig. 10 at a still slower speed. Fig. 12 shows the decay of the signals at D_1 , D_2 (region 3) when A_1 is turned off.

The experimental results in the Figs. 9-12 can be understood on the basis that the detectors D_1 and D_2 show signals $[\overline{A}_1(t) - \delta_1(t)]$ and $[\overline{A}_2(t) - \delta_2(t)]$, respectively, when the crystal is individually pumped by the beams A_1 and A_2 . Here $\overline{A}_1(t)$ and $\overline{A}_2(t)$ are self-pumped phase conjugates of A_1 and A_2 , respectively and $\delta_1(t)$ and $\delta_2(t)$ represent the individual self erasure effects of the beams on the gratings responsible for the self-pumping process. Under simultaneous pumping $(A_1 + A_2)$ by both the beams, the detectors D_1 and D_2 show signals $[\overline{A}_1(t) - \delta_1(t) + \widehat{A}_2(t) - \Delta_1(t)]$ and $[\overline{A}_2(t) - \delta_2(t) + \widehat{A}_1(t) - \Delta_2(t)]$ where $\widehat{A}_2(t)$ and $\widehat{A}_1(t)$ are cross coupled signals because of A_2 and A_1 getting Bragg diffracted from the gratings formed by fanning into the directions opposite of the beams A_1 and A_2 , respectively. $\Delta_1(t)$ and $\Delta_2(t)$ represent the erasure effects of the

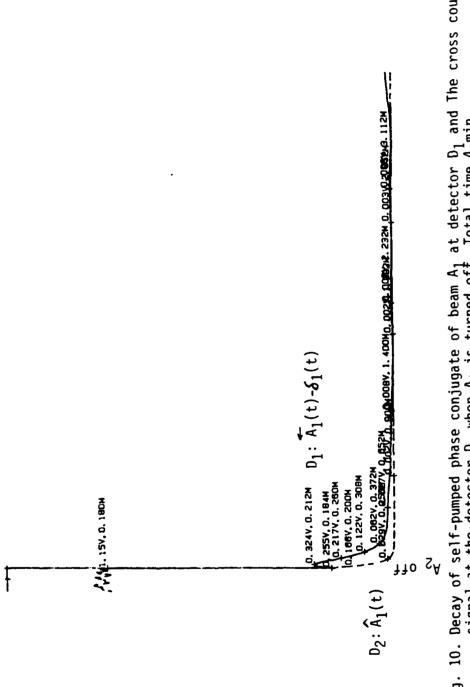
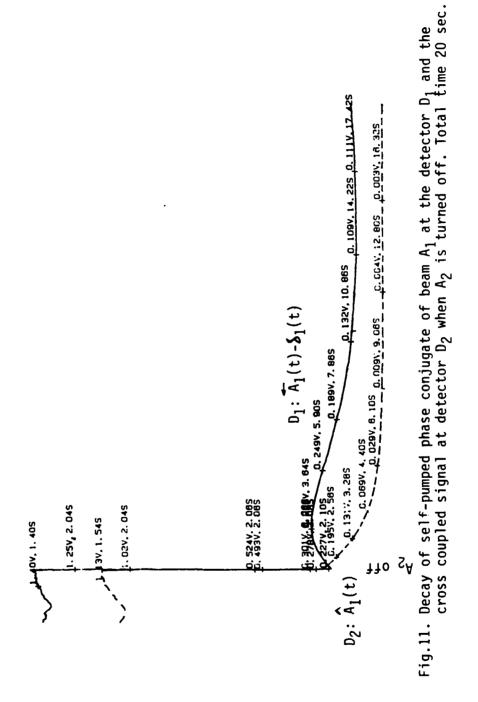
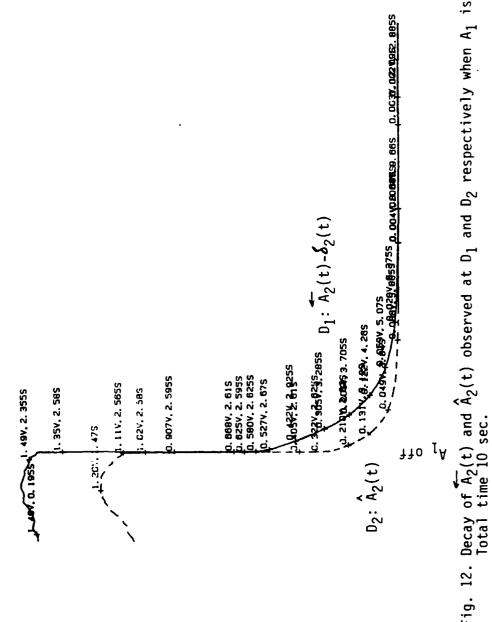


Fig. 10. Decay of self-pumped phase conjugate of beam A_1 at detector D_1 and The cross coupled signal at the detector D_2 when A_2 is turned off. Total time 4 min.







beams A_2 and A_1 on the gratings responsible for the phase conjugates $\overline{A}_1(t)$ and $\overline{A}_2(t)$ respectively.

The absence of the signals at D_1 and D_2 when the crystal is individually pumped shows that $A_1(t)$ - $\delta_1(t) \approx 0$ and also $A_2(t)$ - $\delta_2(t) \approx 0$. It also shows that for both A_1 and A_2 , the threshold values of $\gamma L = 2.34$ is not crossed to show self-pumped phase conjugates⁶.

In Fig.11 the decay of the signal at D_1 , when A_2 is put off, essentially represents the decay of $[\overline{A}_1(t) - \delta_1(t)]$ while that at D_2 similarly represents the decay of $\widehat{A}_1(t)$ as $\overline{A}_2(t)$ and $\Delta_1(t)$ are essentially zero when A_2 is put off. However, the hump in the decay in the signal at D_1 may be understood on the basis that the negative effect of $\Delta_1(t)$ on $\overline{A}_1(t)$ drops down abruptly when A_2 is off enabling $\overline{A}_1(t)$ to increase a little, but the self erasure $\delta_1(t)$ and the decay of the grating take over with time and kill $\overline{A}_1(t)$.

Figs. 13-15 show the results of an experiment with $A_1 = A_2 = 7.5$ mW. The beams meet on the front surface of the crystal, but because of their finite beam waists, the two beams continue to overlap inside the crystal also. Here A_1 self-pumps but A_2 does not. The signal at D_1 which essentially represents $\overline{A}_1(t)$ under individual pumping is nearly smooth, except for minor pulsations, while the signals at D_1 and D_2 show stronger pulsations under simultaneous pumping (see Figs. 13 and 14). In both the figures the signals at D_1 and D_2 under simultaneous pumping are shown

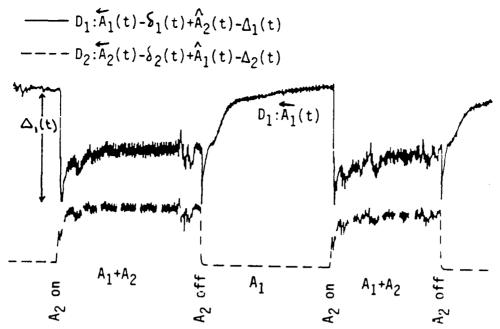


Fig. 13. Phase conjugate signals at D_1 and D_2 . Strong pulsations occur under simultaneous pumping of A_1+A_2 . Total time 15 minutes.

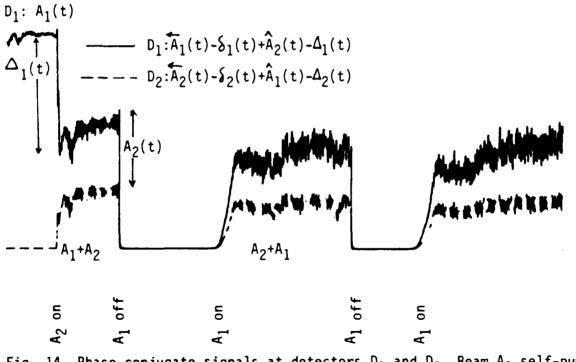


Fig. 14. Phase conjugate signals at detectors ${\rm D}_1$ and ${\rm D}_2.$ Beam ${\rm A}_1$ self-pumped but ${\rm A}_2$ is not. Total time 15 minutes.

as $[\tilde{A}_1(t) + \hat{A}_2(t) - \Delta_1(t)]$ and $[\tilde{A}_2(t) + \hat{A}_1(t) - \Delta_2(t)]$, respectively (neglecting self erasure). The decay of the signals when A₁ is off and when A₂ is off are shown respectively in Figs. 15a and 15b. The signals at D₁ and D₂ when A₁ is off represent essentially the decay of $\hat{A}_2(t)$ and $[\tilde{A}_2(t) - \delta_2(t)]$ respectively. When A₂ is off they represent $\tilde{A}_1(t)$ and $\hat{A}_1(t)$, neglecting $\delta_1(t)$ compared to $\tilde{A}_1(t)$. D₁ shows the growth of $\tilde{A}_1(t)$ while D₂ shows the decay of $\hat{A}_1(t)$.

With $A_1 = 5 \text{ mw}$ and $A_2 = 9 \text{ mw}$, A_1 under individual pumping shows minor pulsations in its self pump $\overleftarrow{A_1}$. A_2 does not self pump. However under simultaneous pumping of $A_1 + A_2$, the detectors D_1 and D_2 show strong coherent oscillations in self pumping (See Fig. 16).

When $A_1 = 0.16-0.4$ mW and $A_2 = 13-14$ mW it is found that A_1 does not self-pump while A_2 does under respective individual excitations. This is the only set where $\overline{A_2}(t)$ has been observed when the crystal was pumped by A_2 . However, the signal $[\overline{A_2}(t) - \delta_2(t)]$ was oscillatory, i.e., it was growing and decaying (Fig. 17) at the rate of about 3 times in 15 minutes. However, under simultaneous pumping the detectors D_1 and D_2 , both show signals which are oscillatory, at about 5 times in 15 minutes, the signal at D_2 being stronger than the one at D_1 (Fig.18). There are minor pulsations superimposed on the signal. Under simultaneous pumping the signals at D_1 and D_2 represent $[\overline{A_1}(\cdot) - \delta_1(t) + \overline{A_2}(t) - \Delta_1(t)]$ and $[\overline{A_2}(t) - \delta_2(t) + \overline{A_1}(t) - \Delta_2(t)]$, respectively.



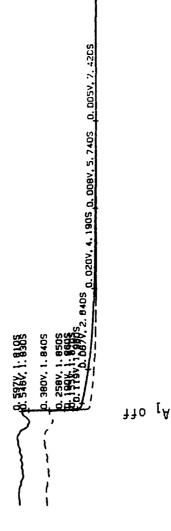


Fig. 15b. Growth of $\overline{A}_1(t)$ at detector D_1 and decay of $\widehat{A}_1(t)$ at detector D_2 , when beam A_2 is turned off. Total time $60 \text{ sec. } D_1$ (full line) and D_2 (dashed line).

"B. 201. 0024 21. 741. 0024 31. 3020 0014 39. 0094 0022 42. 205 ---D-822V. 50. 585 0. 767V. 41. 2864 667V. 27. 2450. 6804, 35. 915 $D_1: \overleftarrow{A}_1(t)$ 3. 498V, 8. 64S Nose .

¹¹o SA

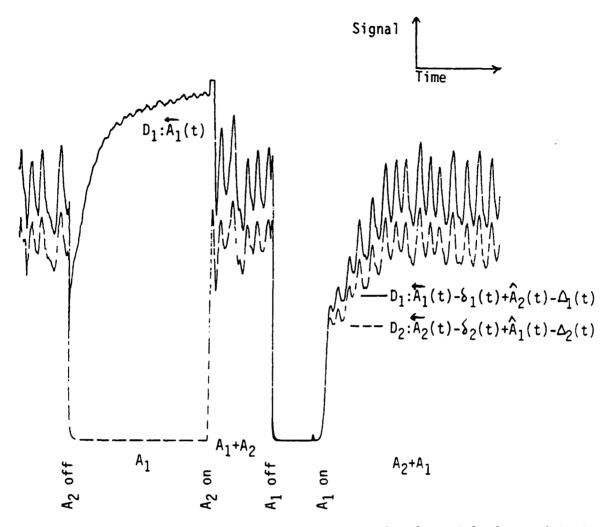


Fig. 16. Phase conjugate signals due to the beams A_1 , A_2 and A_1+A_2 at detectors D_1 and D_2 . Signal due to the A_1+A_2 shows oscillations. Total time 10 minutes.

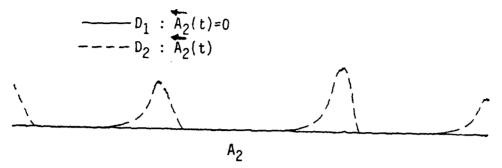


Fig. 17. Self-pumped phase conjugate signal of A_2 at the detector D_2 . Oscillations are 3 per 15 minutes. A_1 does not self pump.

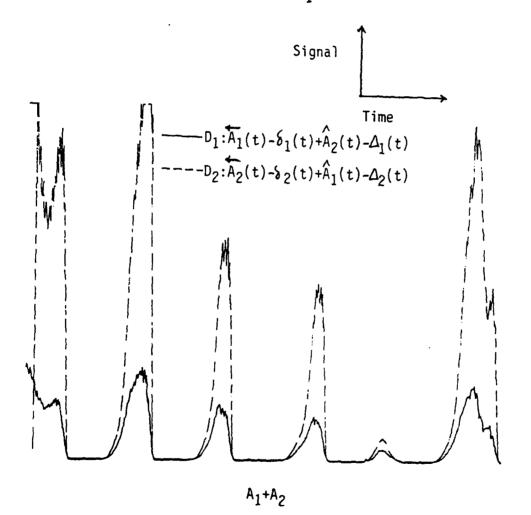


Fig. 18. Oscillatory phase conjugate signals at the detectors ${\rm D}_1$ and ${\rm D}_2$ when beam ${\rm A}_1$ is turned on. Oscilletions are 5 per 15 minutes.

It is seen that A_1 barely self pumps at 2 mW and does not self pump below that value, while A_2 self pumps only at 13 mW and above. If these are taken to represent the relative intensities of the beams A_1 and A_2 , when they just cross over the threshold value of $\gamma L = 2.34$ for self pumping, the coupling parameter γ per unit length for A_1 comes out to be about 6.5 times larger than that for A_2 as the interaction length L is nearly the same for both. This is because A_1 enters the crystal in a favourable quadrant for developing its own self-pumped phase conjugate while A_2 enters in an unfavourable quadrant⁷.

2.2 Beam Couplings In Self-Pumping, Transmission And Reflections

Venkateswarlu, et al⁸ used the experimental configuration shown in Fig. 19 to study beam coupling in transmission and reflection in a BaTiO₃ crystal (8x8x6 mm³), using a He-Ne laser (6328Å) along with an isolator. Two coherent beams A₁ (3.25 mw) and A₂ (3.0 mw) meet in the crystal with horizontal polarization. The crossing angle is 5° with A₁ making 78° with the c axis horizontal (Fig. 19). The point of entry is 2 mm from the nearest edge to A₁. Under individual pumping by A₁ or A₂, one sees the self-pumped beams $\overline{A_1}(t)$ or $\overline{A_2}(t)$ at the corresponding detectors. Also seen are transmitted beams (A₁T₁, A₁T₂) or (A₂T₁, A₂T₂). Reflected beams (A₁R₁, A₁R₂) or (A₂R₁, A₂R₂) are seen even

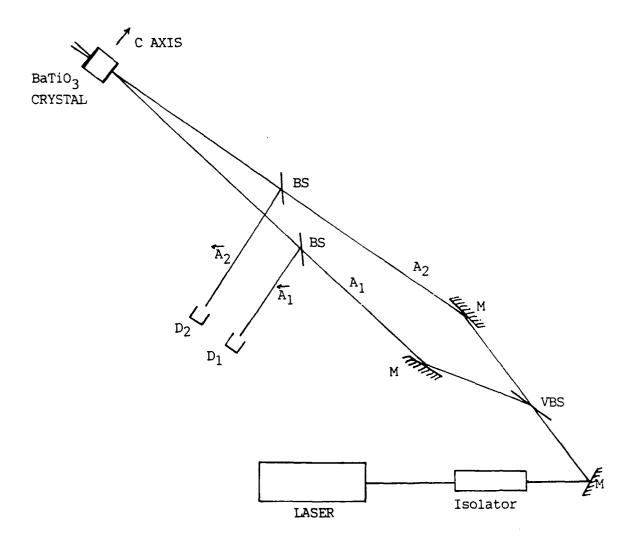
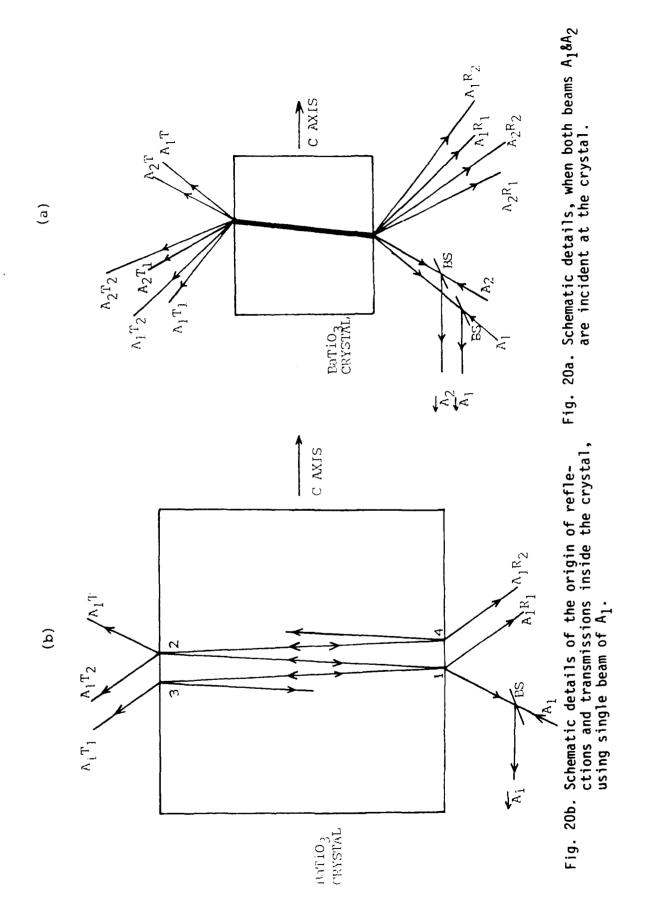


Fig. 19. Experimental arrangement of beam coupling involving a narrow crossing angle using He-Ne laser. BS: beam splitter, VBS: variable beam splitter, M: mirror and D: detector.

when self-pumping is not developed, while the transmitted beams A_1T_1 and A_1T_2 appear only when self-pumping is present.

Fig. 20a shows the different reflected and transmitted beams of A₁ and A₂, while Fig. 20b shows the development of the two transmitted beams and two of the reflected beams for the beam A1 under consideration. It is seen that if A_1R_1 is retroreflected by a mirror, it goes out with A_1T_1 and similarly A_2R_2 gets retroreflected along A_1T_2 . Retroreflections of A_1T_1 and A_1T_2 will similarly emerge out along A_1R_1 and A_1R_2 respectively, These are more separated than expected, because of the slight deviation of the front and back surfaces of the crystal from parallelism . It is seen that instead of running parallel, A_1R_1 and A_1R_2 diverge while A_1T_1 and A_1T_2 first come together and cross very near the surface of the crystal and then diverge. The transmitted beams A_1T_1 and A_1T_2 arise essentially from the self-pumped beams as seen in the figure. Under simultaneous pumping A_1 and A_2 are mutually Bragg diffracted partially at the gratings formed due to fanning, and emerge as A_1 and A_2 in the directions of \overline{A}_2 and \overline{A}_1 respectively. It is seen from the present experiments that under simultaneous pumping, A_1 and A_2 get cross coupled also in transmission and reflection in the same manner as in self-pumped beams.

The development of the signals \overline{A}_1 , A_1R_1 , A_1T_1 , A_1R_2 , A_1T_2 (Fig. 20b) are recorded in different sets of experiments. When A_1 is turned on,



 A_1R_1 and A_1R_2 shoot up immediately, $\overline{A_1}$, A_1T_1 , A_1T_2 grow with time and stabilize while A_1R_1 and A_1R_2 decrease and stabilize. One can see from Fig. 20b that A_1R_1 is a specular reflection which takes place at a denser surface and therefore is out of phase by π with A₁ but in phase with $\overline{A}_1(t)$. The phase conjugate of A_1 gets reflected at the point 1 (Fig. 20b), goes in the direction 1->3 and partly gets phase conjugated to get back to and then goes out in the direction of A_1R_1 . Thus it is in phase with A_1 , but out of phase with A_1R_1 explaining how the intensity of the specular reflection decreases as the self-pumped phase conjugation increases, and this is in agreement with Pepper's observation⁹. The beam A₁ gets internally reflected at the point 2, and comes to the point 4 and then emerges as A_1R_2 . As the reflection at the point 2 is the one into a denser medium, the beam A_1R_2 is in phase with A_1 . However, part of the internally reflected beam 2->4 gets self-pumped and reverses its direction and goes out in the direction of A_1T_2 . This process is responsible for the decrease in intensity of A_1R_2 like that of A_1R_1 as the self-pumped phase conjugation increases.

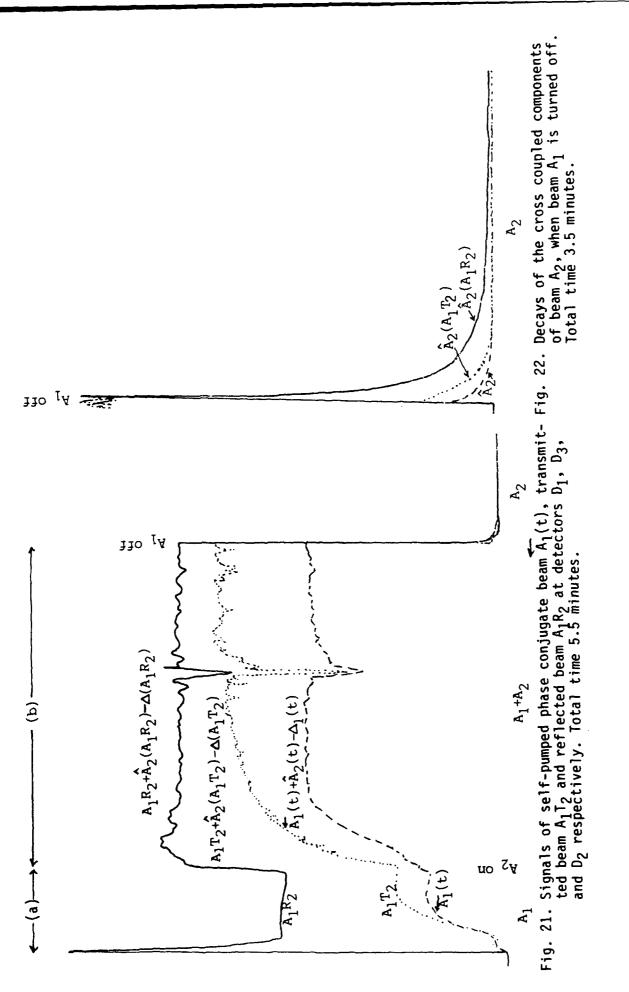
Fig.21 shows in the region a, the self-pumped signal $\overline{A}_1(t)$, the transmitted signal A_1T_2 and the reflected signal A_1R_2 when the beam A_1 only is turned on. One can see that while A_1T_2 grows up with $\overline{A}_1(t)$, A_1R_2 decreases as can be seen in the region a of Fig.21. When A_2 is also turned on all the three signals increase in intensity. When A_1 is put off all the

three decay instead of abruptly coming to zero indicating that the decay is of the cross coupled components of beam A_2 in all the three signals (Figs. 21, 22).

Under simultaneous pumping, the signals at the points A_1T_2 and A_1R_2 are represented in Fig. 21 by $[A_1T_2 + \hat{A}_2(A_1T_2) - \Delta(A_1T_2)]$ and $[A_1R_2 + \hat{A}_2(A_1R_2) + \Delta(A_1R_2)]$ respectively where $\hat{A}_2(A_1T_2)$ and $A_2(A_1R_2)$ represent the cross coupled signals from A_2 at A_1T_2 and A_1R_2 respectively and $\Delta(A_1T_2)$ and $\Delta(A_1R_2)$ represent the erasure effects at these points respectively. The decay curves in Fig.22 are those of the cross coupled signals $\hat{A}_2(t)$, $\hat{A}_2(A_1T_2)$ and $\hat{A}_2(A_1R_2)$ respectively, $\hat{A}_2(t)$ being the cross coupled signal at detector D_1 .

2.3 Phase Conjugate Resonators And Bistabilities

Beam fanning or asymmetric fanning of beams, plays an important role in a number of optical wave mixing experiments and in self-pumped phase conjugate oscillator. Optical bistable oscillations have been observed in some of these experiments¹⁰⁻¹¹. One such geometry reported by Kwong and Yariv¹², involved the use of a single domain barium titanate crystal, and a set of two mirrors, to form a ring passive phase conjugator (RPPC), which may be called the primary oscillator that results in the appearance of a phase conjugate reflection of the pump beam from a multimode Ar^+ laser (514.5nm). The input beam and its conjugate act as pumping beams for oscillation between the crystal and an auxiliary mirror. The oscillation is sustained by the primary oscillation in the ring.



It was found that two such auxiliary resonators cannot co-exist, the result being a bistable mode of oscillation. The power of primary oscillation in the main ring resonator drops below threshold as the oscillations in one of the auxiliary resonators build up. This blocks the development of oscillation in the other auxiliary resonator. We¹³ reported other configurations that also support bistable oscillations. A single crystal, single domain barium titanate $(8x8x6 \text{ mm}^3)$ is used in the RPPC, which is pumped by a single mode Ar⁺ laser (488 nm, 30 mw, 3 mm diameter). Two mirrors M_1 and M_2 and the crystal form the ring resonator (Fig. 23a). A second set of mirrors M_3 , M_4 , M_5 and the crystal form a unidirectional ring resonator (UDRR). The crystal is placed in one of the arms of the triangle of the UDRR. The two resonators have been observed to oscillate in a bistable mode. The oscillations are set up initially in one of the ring resonators (Fig. 23b). The shutter in the other resonator is opened after the oscillating beam in the first resonator reaches a steady state. No oscillations have been observed to develop in the second resonator. The oscillations in the second resonator develop as soon as the shutter in the first resonator is closed and remain unaffected even after the shutter in the first resonator is opened (Fig. 23b). The oscillation can be switched from one resonator to the other in a bistable mode of operation.

Bistable oscillations have been observed between the unidirectional ring resonator (UDRR) and the auxiliary resonator M_5 C (Fig. 24a). Bistable oscillations have been observed also between a linear passive phase conjugate resonator (LPPCR) formed by mirrors M_3 and M_4 and the RPPC. Fig. 24b shows the experimental results. The LPPCR was turned on initially. After the oscillating beam reached its peak power, the

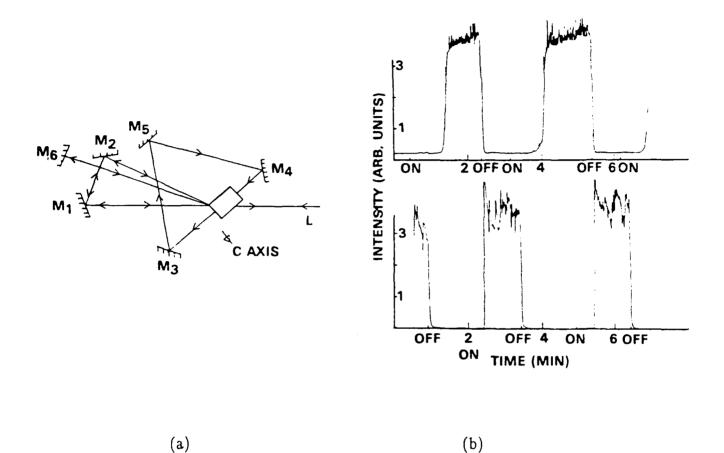


Fig. 23(a). Experimental arrangement for the study of bistable oscillations in a RPPC (M_1 and M_2 and the crystal) and a UDDR (mirror $M_3M_4M_5$). Mirror M_6 and the crystal form the auxiliary resonator.

(b). Bistability in the ring configuration. Upper trace, detector output of the oscillating beam in UDRR. On and OFF on the time axis indicate the times at which the shutters in the two resonators were turned on and turned off. (after Ref. 14).

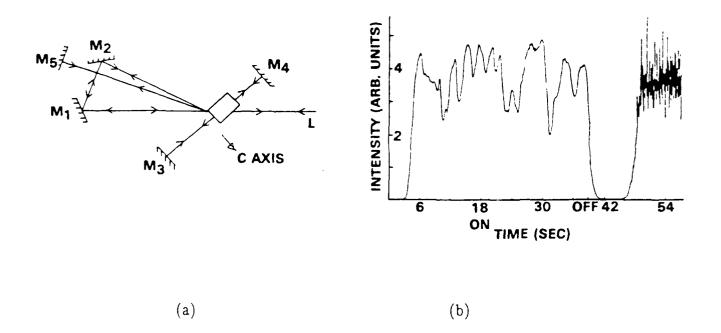


Fig. 24(.). Experimental arrangement for the study of bistable oscillations in a RPPC (M_{2} and M_{2} and the crystal) and in a LPPCR (mirror M_{3} and M_{4}). Mirror M_{5} and the crystal form the auxiliary resonator.(after Ref. 14)

(b). Bistability in RPPC and LPPR. The LPPCR was turned on initially. After the oscillating beam reached its peak power, the shutter in the RPPC was turned on at 18 sec. The two resonators were on simultaneously for 20 sec. before the shutter in the LPPCR was closed. The oscillating beam in the RPPC developed after the shutter was closed (after ref. 14).

shutter in the RPPC was opened, but the RPPC did not oscillate. The oscillations in the RPPC developed only after the LPPCR was closed. Two different LPPCR's are found to oscillate simultaneously and similarly two UDRR's are found to operate simultaneously. These observations prompted us to make further studies on the parametric changes in different parts of experiments¹⁴. In one experiment, the RPPC and the auxiliary resonators M_3 C and M_4 C were formed, using a multimode Ar^+ laser (Fig. 25). It is found that the auxiliary resonators can be made to run, either in a bistable mode, or in a coexisting mode, by changing the relative reflectivity of the mirrors or by inserting suitable attenuation in the oscillators. Either of the two oscillators can be made to continue to operate even after the ring is turned off.

Fig. 26 shows the recording of the signals at the detectors D_1 , D_2 and D_3 which measure the oscillations in the ring, the combined phase conjugate signal and the semilinear resonator M_3 C respectively. Other semilinear resonators are blocked in this experiment.

One can see that the oscillations in the RPPC and the M_3 C are bistable, to the extent that, if the signal intensity increases in one, it decreases in the other simultaneously. A few such points aa', bb' and cc' are marked in the figure. One can also see that the semilinear resonator continues to operate, and even gets stabilized, when the RPPC is cut off. It was found that the ring (RPPC) and four semilinear oscillators could be sustained simultaneously (Fig. 25). It was also observed that any two of these semilinear oscillators could be made to exhibit bistable oscillations without the RPPC. Fig. 27 shows such bistable oscillations between the

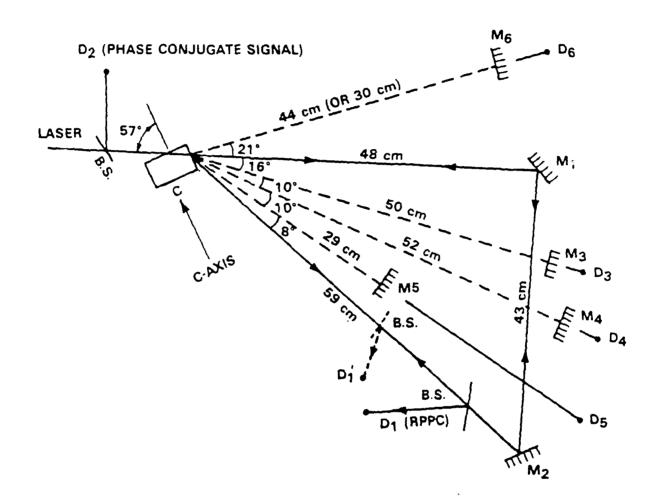
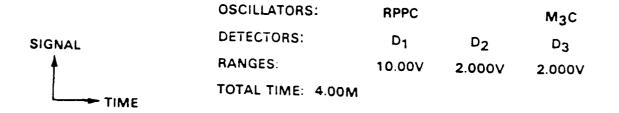


Fig. 25. Experimental set-up for observing phase conjugate resonators and bistabilities B.S.- Beam Splitters, C-BaTiO₃ Crystal, M-mirror, D-Detectors, RPPC-Signal from Ring Passive Phase Conjugator.



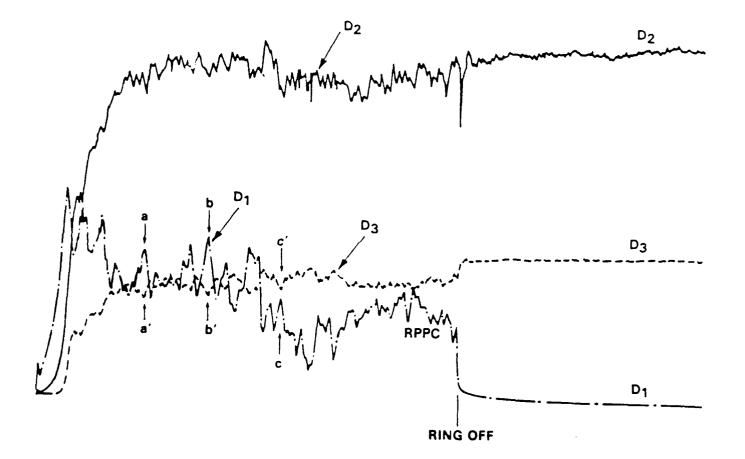


Fig. 26 Signals from the detector D_1 , D_2 , and D_3 representing the ring oscillator (RPPC), semilinear oscillator M_3C , and the total phase conjugation using a multimode Ar⁺ laser. Ring is put off after 2.3 minutes. The RPPC and MC, are bistable. aa', bb', cc' represents a few points of bistability.

oscillators M₄ C and M₅ C.

In another experiment, the RPPC and an auxiliary semilinear oscillator outside the ring are made to show bistable oscillations (Fig. 28). Here the RPPC dominates over the semilinear oscillator, in the sense that, when both the shutters are open, only the RPPC oscillates and the other dies down.

It is clear, from the above experiments that, by suitably adjusting the relative powers, and by introducing suitable attenuations, one can make a set of resonators to operate simultaneously, or can operate any two of them in a bistable mode. The bistable mode could be such that, if one operates the other does not operate at all, or alternatively, if the signal in one increases, the other decreases. It may be mentioned that, Smout^{1,2} reported bistable behaviour and Eason between the self-pumped phase conjugation of two incoherent beams in BaTiO₃, which was explained on the basis of the partial erasure of the grating of one beam by fanning of the other. Similarly the partial erasure of the grating of one resonator by the fanning of the signal output from the other resonator appears to be responsible for the bistability of the resonators observed in the present experiments. Kwong et al¹⁰ observed bistability and hysteresis in a photo refractive passive conjugator by controlling the input power of an erase beam to the conjugator. Yariv $et al^{15}$ used the bistable oscillations of two auxiliary oscillators as a thresholding device, in the experimental demonstration of an all associative holographic memory. It is expected that it would be possible to observe hysteresis in any one of the above resonators and this "switch on and switch off" operation in one resonator can be controlled by changing the reflectivity/power in the

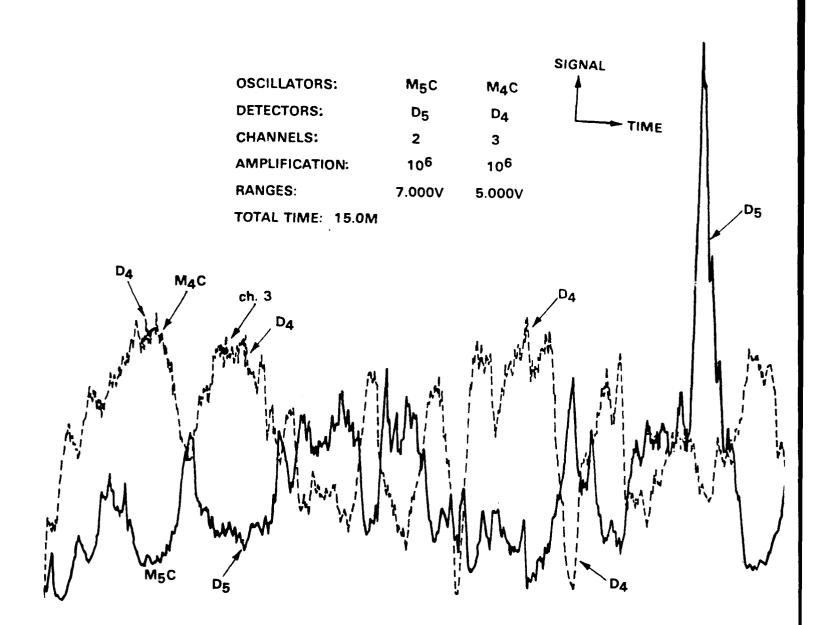


Fig. 27 The signals from the oscillators M_4C and M_5C at the detectors D_4 and D_5 respectively (see Fig. 25). They show bistable oscillations. Ch. 2 and Ch. 3 represent channel numbers of the signal plotter.

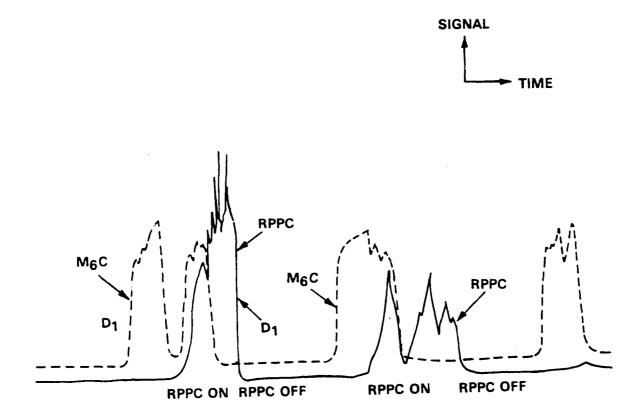


Fig. 28 The bistable signals from the RPPC and the semilinear oscillator. M_6C goes off when the RPPC is put on, and it again comes up when the RPPC is put off. Total time 6 minutes.

other resonator, using a variable beam splitter, or a variable beam reflector as we suggested earlier¹⁶. Experiments have been carried out by us in this direction with slightly different configurations. Results obtained are as anticipated, and they along with the results discussed here have been published in a review article in the SPIE Advent technology series¹⁷.

2.4 Effect Of Color Centers On The Development Of Resonant Systems And Holographic Grating Formation

Alkali halide crystals LiF and NaF, and alkaline earth fluorides BaF_2 , MgF_2 and SrF_2 have been irradiated with γ -rays to develop color centers in these crystals. BaF_2 and MgF_2 did not develop any color while LiF, NaF, SrF_2 and LaF₃ have become colored indicating the presence of color centers. Their absorption spectra have been recorded. The Electron Paramagnetic Resonance (EPR) spectra of the samples of all the above crystals are recorded. Further work is needed to quantitatively analyse the results and identify the centers. Preliminary work on degenerate four wave mixing at wavelengths near to the absorption region has shown indication of holographic grating formation in these crystals. Resonant absorption appears to be the reason for the formation of holographic gratings in these systems. Further detailed work is needed in this connection. LaF₃ doped with Nd³⁺ and LaF₃ doped with Eu³⁺ have become highly colored when exposed to γ -rays indicating the formation of color center complexes with the rare earth ions.

LiNbO₃ was irradiated with γ -rays. There was only a slight development of color. EPR spectrum was recorded but detailed analysis is to be carried out. It is found that the efficiency of holographic formation has increased, but further quantitative work is needed in this connection.

LiF crystal when exposed to γ -rays first became light green and after a day it became yellow suggesting formation of two different types of color centers.

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- Masters Thesis by Mehdi Moghbel. "Coherent Beam Coupling in BaTiO₃ and the Effect of Self-Pumping in Reflection and Transmission". Alabama A&M University, 1989.
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 P. Venkateswarlu, M. Dokhanian, P. Chandrasekhar, M.C. George and H. Jagannath, SPIE Advent Technology Series, Vol. AT2 (1990).

4 SCIENTIFIC PERSONNEL

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