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DETERMINISTIC MODELS OF CHANNEL HEADWALL EROSION: INITIATION AND PROPAGATION

FINAL REPORT

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STATEMENT OF THE PROBLEM STUDIED

Channel erosion in small upland watersheds is the general topic of this research effort. Specifically, the effort is directed toward fundamental mechanics of channel erosion resulting from the development of channel (gully) headwalls and scour holes, from channel sidewall failure and from general shear excess. The emphasis is on erosion in cohesive channel materials.

Entry of sediment into our nations waterways is a serious pollution problem, intensified by the chemicals adsorbed on the exchange phase of clay particles in the sediment. The subject of upland erosion from rill and interrill areas has been the subject of intensive investigations, both empirical and physically based. Channel erosion has been studied primarily from an empirical basis, with virtually little physically based information on channel headwall development and propagation and of channel bank failure.

In the research conducted under this project, models are developed which predict channel erosion resulting from shear in gradually varied flow, shearing forces resulting from submerged and partially submerged jets and shearing forces resulting from free jets impinging a plunge pool. These models are linked with a runoff routing algorithm to develop the CHANNEL model. This model predicts general channel erosion resulting from time varying gradually varying flow as well as predicts the development and propagation of channel headwalls.

In addition to the channel model, a model is developed of water movement into and out of channel banks as a result of rising and falling channel water levels. This model is combined with a stress strain model which considers the influence of water content on soil strength properties, and a prediction developed of the location of a failure surface.

SUMMARY OF MOST IMPORTANT RESULTS

CHANNEL Model

Model Description. Early models of development of channel headwalls and scour holes were primarily empirical in nature, with conclusions based on general morphological concepts (Harvey, et al., 1985; Schumm et al., 1984; Piest Grisinger, 1980; Patton and Schumm, 1975). Recent studies have emphasized hydraulics in an attempt to understand the process of channel headwall (Holland and Pickup, 1976; Begin et al., 1980a, 1980b; Stein, 1990). The Stein study is perhaps the most advanced attempt to date to utilize hydraulic principles to understand the development and propagation of scour holes. Stein used principles of a free jet impinging tailwater zone, resulting in an impinging jet in a scour hole zone to predict the rate of growth of a scour hole in a homogenous soil. He further used a model of drawdown upstream of a brink to predict the increased shear immediately upstream of the brink and resulting detachment. Using these two algorithms, Stein predicted whether the scour would result in a migrating head wall or would wash out.

The Stein model identifies prediction methodologies that have merit, identifies the significant processes that need to be modeled and thus sets the stage for significant further development. His model required the specification of an overfall of a given height at time zero, steady flow and homogenous soil. In addition, the model did not include a routing mechanism. The CHANNEL model developed in this research effort attempts to fill these gaps by specifying the development of the scour hole from an assumption of a uniform slope, allows for soils of varying erodibilities and sizes in the horizontal as well as vertical plane, allows for time varying flows, and incorporates a hydraulic routing algorithm. In addition, the CHANNEL model predicts scour hole detachment arising from partially submerged jets and hydraulic jumps which may occur with the scour hole.

The fundamental approach in the model is based on the premise that a critical tractive force exists and that detachment potential is proportional to the excess of shear over critical tractive force (Foster, 1982; Foster and Meyer, 1975; Haan et al., 1991), or:

$$D_{rc} - K(\tau - \tau_c) \tag{1}$$

where D_{re} is detachment potential, K is an erodibility constant, τ is shear on the channel bed and τ_e is critical shear stress.

Based on the shear excess concept, areas within a channel will exist where τ is less than τ_c due to soil or flow properties. When these exist adjacent to areas



Figure 1. Conceptual Model of Scour Hole Development

3

where τ is greater that τ_c , the conditions are favorable for formation of a scour hole and potential headwall. A model of scour hole development has been developed in this project, based on the following concepts (see Figure 1):

- Due to changes in flow or changes in soil properties, a transition occurs between a segment where $\tau < \tau_c$ and $\tau > \tau_c$.
- Detachment starts at the point where τ first exceeds τ_c and ends when sediment load reaches the transport capacity (Figure 1a).
- As detachment continues, a scour hole is formed. After sufficient detachment, a submerged jet or hydraulic jump forms. Submergence results from downstream tailwater controls (Figure 1d).
- After sufficient additional downstream scour, the tailwater depth is lowered below the brink elevation and the jet becomes a free jet. At this point, a head wall begins to form (Figure 1d).
- The headwall may wash out, or move to becoming a near vertical headwall, depending on flow and soil properties.
- Additional detachment, after becoming a free jet, causes undercutting of the headwall. With sufficient undercutting, the headwall becomes unstable and fails. Then a new headwall is formed and the channel headwall moves up the channel where τ is less than τ_c .

Computation procedures in the model follow the flow diagram in Figure 2. Details are given in Barfield et al., (1991). The model first calculates the water surface profile throughout the channel reaches using either a dynamic wave model or a steady state water surface profile analysis. Using the computed water surface profile and initial channel geometry, shear is distributed around the channel walls, soil is detached by the shear excess from equation 1 and the channel geometry modified following procedures in Hirschi and Barfield (1988).

Next, all node points are checked to see if criteria are satisfied for a scour hole. A scour hole is assumed to exist upstream if a segment has an adverse slope and/or a segment upstream passes critical slope after having previously had less than critical slope and the angle between the supercritical slope and the adjacent upstream slope segment exceeds a set value. If a potential scour hole is identified, the beginning and end of the scour hole are identified and routing through the scour hole accomplished by steady state relationships. For subsequent time steps,



Figure 2. Flow Diagram for the CHANNEL Model

the channel is divided into subchannels and entry occurs upstream and downstream from the scour hole, with the scour hole serving as a control section.

Using the water surface profile calculated through the scour hole, the depth of tailwater relative to the brink is determined. If the tailwater is above the brink, but below critical depth, then the jet is assumed to be partially submerged. If the tailwater is below the brink, the jet is assumed to be free.

If the jet is submerged or partially submerged, the angle of the jet relative to the most upstream section of the scour hole is calculated. If the angle is greater than a set value, δ_c , then the shear is calculated by a modification of the impinging jet theory of Beltaos (1976). If the angle is less than δ_c , the shear distribution is calculated by a modification of the wall jet theory of Rajaratnam (1972, 1981).

If the jet is a free jet, the point and angle at which the jet penetrates the downstream water surface is calculated by projectile physics for subcritical flow conditions and by Hager's equations (1983, 1984) for supercritical flow. Both methods require the depth of flow on the brink as input. Relationships from Delleur et al (1956) and Rajaratnam et al. (1976) are utilized to calculate the brink depth if flow is subcritical and relationships from Hager (1983, 1984) are used for supercritical Given the trajectory, the angle of impact can also be calculated.

Once the point and angle of jet entry to the free water surface is determined, the jet angle is assumed to remain constant and the point of interception of the with the scour hole calculated. The actual stagnation point is determined by Beltaos (1976) relationship as a function of impingement angle and flow path length. After the point of interception is determined, the actual longitudinal shear distribution is calculated by Beltaos (1976) and Beltaos and Rajaratnam (1973) impinging jet theory for both the impinging zone and for the free wall region.

Detachment potential is calculated in the scour hole by the shear excess concept and corrected for transport capacity. Transport capacity is determined from the calculated shear using the Yalin (1963) equation as modified by Foster (1982).

As detachment continues and the headwall is undercut, a point of failure of the headwall is reached. After failure, a new headwall is formed, and the scour hole reformed, using the procedures discussed above. **Importance of Model to Channel Erosion Modeling.** As discussed above, the CHANNEL model improves on previous algorithms in the following areas:

- By specifying the development of the scour hole from an assumption of a uniform slope, the model allows the prediction of scour hole development, not just shear due to a predetermined scour hole location.
- The model allows for soils of varying erodibilities and soil properties in the horizontal as well as vertical plane rather than homogenous soils only.
- The model allows for time varying flows, thus the effects of shifts in hydrologic regimes can be addressed.
- The model incorporates a hydraulic routing algorithm
- The model predicts scour hole detachment arising from partially submerged jets and hydraulic jumps which may occur within the scour hole.
- The model predicts not only scour hole depth changes with time, but predicts total scour hole shape.

Channel Bank Erosion Model

Model Description. Previous studies of channel bank failure have tended to be morphological in nature or based on static estimates of the slope stability safety factor (Bradford and Priest, 1980; Thorne, 1982; Little, Throne and Murphey, 1982). Three mechanisms of channel bank failure have been identified: (1) circular arc failure with both deep seated and shallow circles: (2) slab or plane failure; and (3) creation of overhanging banks through the removal of material at the toe of the bank by formation of a 'popout' or by excess shear forces, and failure of the remaining cantilever. The models that have been developed are primarily static and would assume constant stress strain contours in a channel bank.

Stress strain contours in a channel bank tend to be dynamic in nature, particularly where water levels are fluctuating widely with resulting dynamic changes in the water content of the channel walls. The channel bank model developed in this research project is an attempt to fill that void.

Since the soil moisture content has a significant impact on soil stress strain relationships, a major emphasis in the research is the development of a model of

moisture movement into and out of a channel bank as a consequence of dynamic changes in depth of flow in the channel. A two dimensional finite element model of saturated-unsaturated flow has been developed which predicts the location of the free water surface in the channel wall, the presence of seepage surfaces, and the moisture content above the free water surface.

The stress-strain model is a finite element solution to the constitutive relationships which relate load and deformation or stress and strain within the soil matrix. The stress-strain relationship being modelled includes both elastic and plastic behavior as shown in Figure 3. The material initially deforms elastically when loaded to point A. If the load is increased from point A to point B and then removed (point C), a portion of the total strain will remain unrecovered, indicating plastic behavior.

The moisture content predicted with the moisture movement submodel is utilized with empirical relationships to alter stress strain properties of the soil. These properties are used as input to the finite element model and contours of stress and strain predicted. The failure surface will be assumed to occur along a line of maximum stress.

At the present time the model development has proceeded to developing predictions of the stress strain contours so that a potential failure surface can be empirically determined at any time step. It is anticipated that further work will be conducted and that a safety factor will be calculated as a function of time, using the method of slices and the identified failure surface.

Importance of Model to Channel Erosion Research. As stated above, other research on channel bank failure has been morphological or a static prediction of slope stability safety factors. This modeling effort allows for dynamic prediction of stress strain relations in response to changing water content in the channel wall.



Figure 3. Typical stress-strain for material under uniaxial tension (Desai and Siriwardane, 1984).

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Rohlf, R. A., B. J. Barfield and G. K. Felton. 1991. Channel bank failure model: Part 2. Model testing and sensitivity analysis. Technical Report No. 185, KY Water Resources Research Institute, University of Kentucky, Lexington, KY.

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Jeffrey C. McBurnie Asst. Professor Department of Agricultural Engineering University of Maine Oreno, ME. Former Graduate Assistant, University of Kentucky Worked on the project as a graduate assistant, has completed all degree requirements except dissertation. Collected data on shear distribution on a plunging impinging jet.

Steve R. Humel Former Graduate Asst. Department of Agricultural Engineering University of Kentucky Lexington, KY

Worked as graduate assistant on the project. Completed a non thesis masters degree.

REPORT OF INVENTIONS

There were no inventions or patents during the course of this project.

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