ARO 25844.3 -MA

ADAPTIVE CONTROL OF NONLINEAR SYSTEMS WITH APPLICATIONS TO THE CONTROL OF FLEXIBLE ROBOT ARMS

Final Report

by

Professor S. S. Sastry

June 15, 1988 through June 14, 1991

for

U. S. Army Research Office

Contract DAAL03-88-K-0106

ELECTRONICS RESEARCH LABORATORY

College of Engineering
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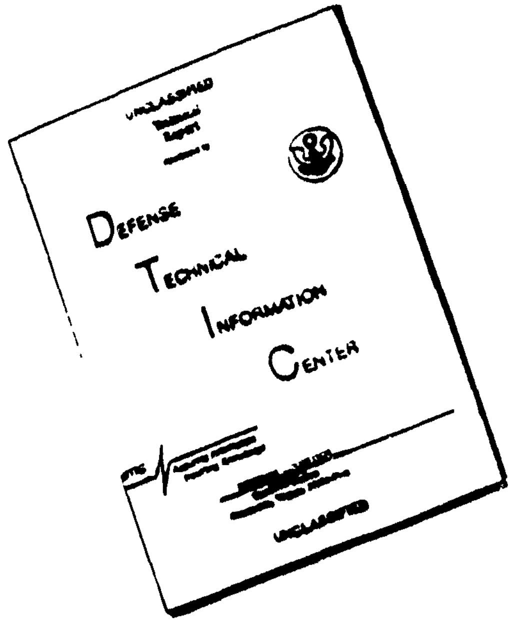

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SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION
Unclassified

1b. RESTRICTIVE MARKINGS

2a. SECURITY CLASSIFICATION AUTHORITY

2b. DECLASSIFICATION/DOWNGRADING SCHEDULE

3. DISTRIBUTION/AVAILABILITY OF REPORT
Approved for public release; distribution unlimited.

4. PERFORMING ORGANIZATION REPORT NUMBER(S)

5. MONITORING ORGANIZATION REPORT NUMBER(S)
ARO 25844.13-MA

6a. NAME OF PERFORMING ORGANIZATION
Electronics Research Lab

6b. OFFICE SYMBOL *(If applicable)*

7a. NAME OF MONITORING ORGANIZATION
U.S. Army Research Office

6c. ADDRESS *(City, State, and ZIP Code)*
253 Cory Hall
University of California
Berkeley, CA 94720

7b. ADDRESS *(City, State, and ZIP Code)*
P.O. Box 12211
Research Triangle Park, NC 27709-2211

8a. NAME OF FUNDING/SPONSORING ORGANIZATION
U.S. Army Research Office

8b. OFFICE SYMBOL *(If applicable)*

9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER
DAAL03-88-K-0106

8c. ADDRESS *(City, State, and ZIP Code)*
P.O. Box 12211
Research Triangle Park, NC 27709-2211

10. SOURCE OF FUNDING NUMBERS

PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO	WORK UNIT ACCESSION NO.

11. TITLE *(Include Security Classification)*
Adaptive Control of Nonlinear Systems with Applications to the Control of Flexible Robot Arms (unclassified)

12. PERSONAL AUTHOR(S)
Professor S. S. Sastry

13a. TYPE OF REPORT
Final

13b. TIME COVERED
FROM 6/15/88 TO 6/14/91

14. DATE OF REPORT *(Year, Month, Day)*
1991 October 28

15. PAGE COUNT
36

16. SUPPLEMENTARY NOTATION The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.

17. COSATI CODES

FIELD	GROUP	SUB-GROUP

18. SUBJECT TERMS *(Continue on reverse if necessary and identify by block number)*
Adaptive Control, Nonlinear Control, CAD tools for control systems design (CACSD(

19. ABSTRACT *(Continue on reverse if necessary and identify by block number)*

See reverse page for Abstracts

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT
☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT. ☐ DTIC USERS

21. ABSTRACT SECURITY CLASSIFICATION
unclassified

22a. NAME OF RESPONSIBLE INDIVIDUAL

22b. TELEPHONE *(Include Area Code)*

22c. OFFICE SYMBOL

On this grant we made major progress in three areas:

1. Adaptive Control of Nonlinear Systems
2. Approximate Linearization (by state feedback) of nonlinear systems
3. Software tools for CAD of nonlinear control

Adaptive Control of Nonlinear Systems: In this work, we extended our previous work on direct adaptive control of Single Input Single Output nonlinear systems to schemes for adaptive identification, indirect adaptive control and also adaptive model matching of Multi Input Multi Output nonlinear systems. We also studied adaptive versions of the nonlinear regulator.

Approximate Linearization (by state feedback) of nonlinear systems: While the full set of conditions for input-output linearization of a nonlinear system by state feedback have been given in the literature, the question of how to proceed when the conditions follow slightly short of being met have not been answered. For example, input-output linearization hinges on a certain set of regularity conditions (existence of relative degree in the SISO case) and minimum phase conditions being met by the plant. If the plant is not regular and is slightly nonminimum phase the techniques of input-output linearization need to be modified. We discussed these techniques in the context of flight control and also other examples, for instance, the ball and beam system. This in turn led to a deeper understanding of the structure of the zero dynamics of a nonlinear system and their structure under perturbation.

CAD tools for nonlinear controller design: We have developed a set of CAD tools for linearization and approximate linearization of nonlinear systems using spline software which operates in real time and is capable of accepting nonlinear system description in numeric, tabular or functional form. A user interface is being written and it is being tried out on several examples.

Final Report for ARO Grant DAAL 88 - K - 0106

Prof. S. S. Sastry

In the years of this grant, of the students supported on the grant I have had two Ph. D. dissertations (S. Behtash and J. Hauser), two M. S. Plan II reports (R. Kadiyala and A. Teel) completed and three Ph. D. dissertations are nearing completion (R. Kadiyala, A. Pradeep and A. Teel, all expect to graduate between January and June 1992). The grant has enabled the PI to switch my focus from *linear adaptive control*, which was the work supported by the previous ARO grant to me, ARO DAAG 85-K-0572, to several areas of *nonlinear and adaptive control* and more recently to the development of CAD tools for nonlinear control systems design, a project which continues with the newest ARO grant DAAL 03-91-G-0171. We have made several trips to the US Army Ordinance Research Center at Picatinny Arsennal, NJ and have set up a good working relationship with the group of Dr. Norman Coleman and a design project in fire control for Apache helicopters which is supported at Integrated Systems Inc., Santa Clara, California.

The work done on this grant has had major impact on the field in two areas: the adaptive linearization of nonlinear systems (publications [2] and [9] of the list below) have begun a new field of research which has advanced the theory of nonlinear control and has important implications for CAD tools for nonlinear control systems design; and the approximate linearization of nonlinear systems (publications [1], [3], [10] and [11]) has opened new lines of investigation in developing a nonlinear control systems design methodology. Finally, we have begun developing software tools for CAD of nonlinear control systems (publications [12, 13]) below. A brief outline of the areas of research findings is now given:

1 Brief Outline of Research Findings

1.1 Adaptive Control of Nonlinear Systems

There has been a great deal of recent progress in the development of basic theory for the input output linearization of a large class of nonlinear systems by state feedback. The chief drawback of these techniques has been that they rely on the exact cancellation of nonlinear terms. When the nonlinearities are not known exactly adaptive control may be used to asymptotically make the cancellation exact. In past work supported by the grant we had developed a direct adaptive control algorithm for this purpose. In work in this time period we have proposed a variety of other schemes, referred to as indirect and semi-indirect adaptive control. In the former scheme, we developed a number of different identification techniques for nonlinear systems and coupled them with the input output linearizing control law using the certainty equivalence principle. We gave conditions for the convergence of the scheme and showed that it had several important advantages over the direct scheme including no need for over-parameterization. The conditions for convergence were, however, far more restrictive than those for the direct scheme. Consequently, we also proposed a semi-indirect scheme which combined several of the attractive features of the direct and indirect schemes and gave a convergence proof. We have been comparing all of these schemes with nonadaptive alternatives such as sliding mode control on several examples such as induction motors. (publications [6], [12])

We also began the study of adaptive control of MIMO adaptive nonlinear systems : in this research, we (joint work with M. Di Benedetto from the Università di Roma) studied two schemes for the adaptive tracking control of MIMO systems with parametric uncertainty in their dynamics. The first approach is an adaptive version of a static feedback law for tracking control based on some results on asymptotic model matching recently proposed by Di Benedetto. This scheme is based on some some results on asymptotic model matching recently proposed by Di Benedetto. This scheme is based on some new techniques for extending the so-called zero dynamics algorithm of Isidori and Byrnes to problems of stable model matching followed by their specialization to tracking. The second scheme is an adaptive version of a dynamic precompensation law of Descusse and Moog for linearization using dynamic state feedback. (publication [9] below).

The schemes are in the spirit of our earlier work on adaptive linearization of nonlinear systems. done on this grant with Isidori and Kokotovic, (publication [1]) which were however confined to the SISO case. These schemes represent the initiation of a full theory of Model Reference Adaptive Control of MIMO nonlinear systems. Thus, in a collection of papers with Isidori, Kokotovic. Kadiyala. Teel and Di Benedetto. we have laid out the rudiments of a theory of parameter adaptive control for nonlinear tracking and regulation. direct and indirect for systems with parameter uncertainty. This result has also generated a large volume of activity in the research community and has also helped us understand adaptive splining for our CAD design package. In future work we will develop a complete theory of Nonlinear Model Reference Adaptive Control. Another area that we will explore is the question of how to identify nonlinear models which are presented in non-symbolic form using techniques from approximation theory. This will also enable us to move away from exclusively splined approximations for our CAD package.

1.2 Structure of Zero Dynamics of Nonlinear Systems

Stability properties of zero dynamics are among the crucial input-output properties of both linear and nonlinear systems. Unstable, or "non-minimum phase", zero dynamics are a major obstacle to input-output linearization and high gain designs. An analysis of the effects of regular perturbations in system equations on zero dynamics shows that, whenever a perturbation increases the system's relative degree. it manifests itself as a singular perturbation of zero dynamics. In this work, conditions are given under which the zero dynamics evolve in two time scales characteristic of a standard singular perturbation form that allows a separate analysis of slow and fast parts of the zero dynamics. The slow part is shown to be identical to the zero dynamics of the unperturbed system, while the fast part, represented by the so called ***boundary layer system***, describes the effects of perturbations. It is remarkable that, as the perturbation parameter ϵ tends to zero, the boundary layer system becomes a linear system. whose stability is easy to analyze. When this system is unstable the perturbed systems is *slightly non-minimum phase* and the exact nonlinearity cancellation or a high gain design should be avoided. (see publications [1] and [10]).

1.3 Approximate Input-Output Linearization by State Feedback

In a collection of papers with Hauser and Kokotović we began a study of how to enlarge the domain of applicability of nonlinear control laws to systems which did not fit all the assumptions for the rigorous application of the theoretical results. Our work was strongly influenced by two important and practical design examples: the dynamical model of V/STOL aircraft and that of a ball and beam (modeling slosh) in fuel tanks on aircraft wings. This work was primarily for SISO systems and has been important in that it has spawned a large effort on the part of the research community at large on approximate linearization. In our own work it has been important in helping us develop the design CAD package In future work we will extend this with researchers from the University of Rome to MIMO systems. The subtleties of the theory in these systems make this a very challenging enterprise. This will be, to our knowledge, the first attempt also to confront MIMO robustness issues for nonlinear systems head on. One report describing a robust version of the Descusse Moog algorithm for dynamic decoupling is under preparation. (publication [11] and one more in preparation).

1.4 CAD and Implementational Tools for Nonlinear Systems

The chief drawback of the recent advances in nonlinear control has been that they have been based on detailed analytical models of the systems to be controlled. These analytical models are required, since the design methodology involves in a fundamental way differentiation of the functions describing the dynamics. The reason that this has been a drawback is that there are in practice a large number of nonlinear systems whose parts are described by tabular means or in some instances from empirical observations. These problems are especially acute in flight control, where the aerodynamic or wind tunnel data is available only at discrete points in the flight envelop or in the instance of fire control for helicopters where a large number of the nonlinear parameters can only be measured empirically. In collaboration with Integrated Systems Incorporated (ISI) and Picatinny Arsenal we have been developing a computer aided design package for spline fitting graphical data and then computing input-output linearizing control laws, approximate linearization control laws and also observers. The package is written in C and involves adaptation on the order of the spline fit as well as the accuracy of the approximate linearization. The CAD tools being developed are fast enough to be real time and the coefficients of the spline fit are recomputed. The primary developer of the software (R. Kadiyala) is also involved in validating the software on the gun control models being developed by ISI and the fire control group of Dr. Coleman at Picatinny Arsenal. We are also actively exchanging ideas and software with a group at University of California, Davis under Prof. Arthur Krener for approximate linearization.

Thus, one of the most important goals on this grant. also continuing forward with the next grant is to develop at least at a conceptual level user friendly tools for nonlinear control. which contain on the one hand recent advances in the theory, but on the other hand also take advantage of recent advances in workstations to provide graphical and symbolic visualization of simulations. Our software has incorporated graphical depiction of our control laws on Sun workstations. This, we believe, is essential to allow for rapid prototyping of new nonlinear and adaptive control laws. The systematic development of the software in C with a good user interface are current topics of research. What has begun as an off-line CAD tool design effort has, owing to the development of computer hardware, become an attractive option for real time control: consequently the real time aspects of the computations are our future priorities. (publications [5] and [13])

1.5 Robust and Adaptive Nonlinear Output Regulation

A new topic of excitement in the nonlinear control design literature has been the development of techniques of output regulation for nonlinear systems. In contrast to the work on input-output linearization by state feedback which is the nonlinear analog of a zero cancelling control law. these methods do not need the underlying system to be non-minimum phase. Our research on this grant has been aimed at understanding the robustness of these control laws to parametric uncertainty. In the instance that the parametric uncertainty is too large, an adaptive scheme is proposed with slowly varying parameter update to achieve asymptotic regulation.

Related research in this area concerns enlarging the domains of attraction for these control laws, since they are originally derived to be local control laws. A key difficulty with the new scheme appears to be very small domains of attraction. In research to date, we have proposed several augmented to the extended scheme to enhance the domains of attraction. (publications [7] and [8])

1.6 Sliding Mode Control of MIMO Nonlinear Systems

The problem of developing precise matching conditions for nonlinear systems which are not linearizable by static state feedback has proved to be a surprisingly hard nut to crack. In early work on the grant we encountered success in developing matching conditions for MIMO systems linearizable by static state feedback. The extension of these results to either dynamically decouplable MIMO systems or other more general systems is not yet complete.

However, our earlier experiments with sliding mode control laws have enabled us to understand solutions to stabilization problems where it may be shown that the underlying control system cannot be stabilized by continuous, state feedback. (publication [14])

2 Scientific Personnel and Degrees awarded

1. S. Behtash — Ph. D. awarded January 1989.
2. J. Hauser — Ph. D. awarded August 1989.
3. R. Kadiyala — M.S. awarded December 1989. (Ph. D. expected Dec 1991)
4. A. Teel — M. S. awarded December 1989. (Ph. D. expected March 1992)
5. A. K. Pradeep — Ph. D. expected December 1991.
6. Prof. P. V. Kokotovic — Visiting Professor, Fall 1988.
7. Prof. M. D. Di Benedetto — Visiting Professor, Fall 1990.
8. Prof. S. S. Sastry

3 List of Manuscripts Submitted or Published

1. "Zero Dynamics of Regularly Perturbed Systems may be Singularly Perturbed," S. Sastry, J. Hauser and P. Kokotovic, Systems and Control Letters, Vol. 13, (1989), pp. 299-314.

2. "Adaptive Control of Linearizable Nonlinear Systems," S. Sastry and A. Isidori, IEEE Trans. on Auto. Control, Vol. 34 (1989), pp. 1123-1131.

3. "Approximate Tracking for Nonlinear Systems with Applications to Flight Control," J. Hauser, Ph. D. dissertation, Univ. of Calif. Berkeley, August 1989.

4. " A Design Example using Q-des and Indirect Adaptive Systems," R. Kadiyala, M.S. Report, Univ. of Calif., Berkeley, Dec 1989.

5. " Topics in Nonlinear Control," A. Teel, M.S. Report, Univ. of Calif., Berkeley, Dec 1990.

6. A. Teel, R. Kadiyala, P. Kokotovic and S. Sastry, "Indirect Techniques for Adaptive Input- Output Linearization", Proceedings of the American Control Conference, San Diego, May 1990; International Journal of Control, Vol. 53(1991), pp. 193-222.

7. A. Teel, " Toward Larger Domains of Attraction for Local Nonlinear Control Schemes", in the Proceedings of the European Control Conference, Grenoble, August 1991.

8. A. Teel, "Robust and Adaptive Nonlinear Output Regulation", in the Proceedings of the European Control Conference, Grenoble, August 1991.

9. M. Di Benedetto and S. Sastry, "Adaptive Control for MIMO Nonlinear Systems, in the Proceedings of the European Control Conference, Grenoble, August 1991 and to appear in the Journal of Mathematical Control, Estimation and Systems.

10. A. Isidori, S. Sastry, P. Kokotovic and C. Byrnes "Singularly Perturbed Zero Dynamics of Nonlinear Systems", to appear in the IEEE Transactions on Automatic Control, 1991.

11. J. Hauser, S. Sastry and P. Kokotovic, "Approximate linearization of nonlinear systems: the ball and beam example", to appear in the IEEE Transactions on Automatic Control, November 1991.

12. R. Kadiyala, "Indirect Adaptive Control of Induction Motors", submitted to the IFAC Conference on Nonlinear Control Design, Bordeaux, France, 1992.

13. R. Kadiyala, "AP-LIN: A Tool Box for Approximate Linearization of Nonlinear Systems", submitted to the IEEE Conference on Computer Aided Control Systems Design, 1992.

14. A. K. Pradeep and S. Sastry, "Generalized Matching Conditions for Perturbed Nonlinear Systems," preprint, October 1991.

Manuscripts, Abstracts, and Reprints

not previously submitted

Approximate Tracking for Nonlinear Systems with Application to Flight Control

by

John Edmond Hauser

Shankar Sastry
Chairman

ABSTRACT

In this dissertation, we embark on a project to make recent theoretical advances in geometric nonlinear control into a *practicable control design methodology*.

The method of input-output linearization by state feedback provides a natural framework to design controllers for systems, such as aircraft, where output tracking rather than stabilization is the control objective. Central notions include relative degree and zero dynamics. Roughly speaking, the relative degree of a system is the dimension of the part of the system that can be input-output linearized and the zero dynamics are the remaining (unobservable) dynamics. Systems with exponentially stable zero dynamics are analogous to minimum phase linear systems and can be controlled to track a rich class of output trajectories with internal stability.

While investigating the use of these methods in the control of the V/STOL Harrier aircraft, we noticed that the small forces produced when generating body moments caused the aircraft to have an unstable zero dynamics, i.e., to be nonminimum phase. However, if this coupling were zero, then the aircraft could be input-output linearized with no zero dynamics. In other words, a small change in a parameter resulted in a significant change in the system structure!

With this observation as the driving force, this dissertation studies the effects of system perturbations on the structure of the system and develops methods for tracking controller design based on approximate systems.

After reviewing the basics of geometric nonlinear control, we show that small regular perturbations in the system can result in singular perturbations in the zero dynamics. We give asymptotic formulas for the resulting fast dynamics.

Next, we develop techniques for tracking control design for systems that do not have a well defined relative degree. Using an approximate system with a well defined relative degree, we design tracking controllers that guarantee approximate tracking for the true system. This approach is shown to be superior to the usual Jacobian linearization method on a simple ball and beam system.

Returning to the aircraft control problem, we use a highly simplified planar VTOL aircraft model to illustrate the (slight) nonminimum phase characteristic of these systems and develop a controller to guarantee approximate tracking. We also develop a formal theory for this class of systems.

Toward Larger Domains of Attraction for Local Nonlinear Control Schemes *

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Abstract

This paper is motivated by the observation that the success of some recent nonlinear control approaches is very sensitive to initial conditions. The discussion in this paper centers around the recently developed nonlinear output regulation theory of [1]. The focus of this paper will be extending the region of attraction for this approach by augmenting the existing scheme.

Keywords. Nonlinear Output Regulation, Center Manifolds, Domains of Attraction.

1 Introduction

The tool box for achieving tracking in nonlinear systems is growing rapidly. One recent addition has been the nonlinear regulator of [1]. This solution allows the control designer the flexibility of using either output feedback or state feedback, permits disturbances to the plant and does not require a well-defined relative degree. However, it has been found, through simulation studies, that this control scheme can be rendered ineffective by the choice of seemingly benign initial conditions. In what follows, we propose a means of augmenting the scheme that is provably convergent (locally) and, in simulations, displays an increased region of effectiveness.

2 Problem Statement

We begin by reviewing the problem statement and solution found in [1]. Consider the nonlinear composite system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + p(x)w \\ \dot{w} &= s(w) \\ e &= h(x) + q(w) \end{aligned} \tag{1}$$

where $x \in U \subset \mathbb{R}^n$ is the state of the plant, $w \in W \subset \mathbb{R}^s$ is the state of an (autonomous) exosystem, $u \in \mathbb{R}^m$ and $e \in \mathbb{R}^k$. As usual, f and the columns of g and p are assumed to be smooth vector fields and $h(x)$ is a smooth mapping on U. Also, s is a smooth vector field and $q(w)$ is a smooth mapping defined on W. Further, it is assumed that $f(0) = 0$, $s(0) = 0$, $h(0) = 0$, $q(0) = 0$ so that, for $u = 0$, the composite system (1) has an equilibrium state $(x, w) = (0, 0)$ which yields zero error.

We focus on the following state feedback regulator problem: Given a nonlinear system of the form (1), find, if possible, a feedback $u = \alpha(x, w)$ such that

1. the equilibrium $x = 0$ of
$$\dot{x} = f(x) + g(x)\alpha(x, 0) \tag{2}$$
is asymptotically stable in the first approximation.

2. there exists a neighborhood $V \subset U \times W$ of (0,0) such that, for each initial condition $(x(0), w(0)) \in V$, the solution of the closed loop system satisfies
$$\lim_{t\to\infty}(h(x(t)) + q(w(t))) = 0$$

For the solution to this problem in [1], the following two hypotheses were made:

(H1) the linear approximation of the plant disconnected from the exosystem is stabilizable.

(H2) the point $w = 0$ is a stable equilibrium of the exosystem, and there is an open neighborhood of the point $w = 0$ in which every point is Poisson stable. In short, this assumption implies that the eigenvalues of the linear approximation of the exosystem lie on the imaginary axis.

The following solution to this problem was then developed:

Theorem 2.1 (Byrnes, Isidori) *Under hypotheses (H1) and (H2), the state feedback regulator problem is solvable if and only if there exist $C^k (k \geq 2)$ mappings $x = \pi(w)$, with $\pi(0) = 0$ and $u = c(w)$, with $c(0) = 0$, both defined in a neighborhood $W^\circ \subset W$ of 0, satisfying the conditions*

$$\begin{aligned} \frac{\partial \pi}{\partial w}s(w) &= f(\pi(w)) + \\ &\quad g(\pi(w))c(w) + p(\pi(w))w \\ h(\pi(w)) + q(w) &= 0 \end{aligned} \tag{3}$$

*Research supported in part by the Army under grant ARO DAAL-88-K0572, and NASA under grant NAG2-243.

...

1. The proof relies on center manifold theory and constructs a state feedback

$$u = \alpha(x,w) = c(w) + K[x - \pi(w)] \quad (4)$$

that is shown to be a solution of the state feedback regulator problem. K is a matrix of feedback gains such that the eigenvalues of the linear approximation of the plant (disconnected from the exosystem) have negative real part. The manifold $x = \pi(w)$ is seen to be an error-zeroing manifold that is rendered invariant by the control $u = c(w)$. Solving the state feedback regulator problem reduces to solving for the the mappings $x = \pi(w)$ and $u = c(w)$.

2. A very useful observation was made in [2] that these mapping could be approximated up to arbitrary order and still achieve approximate tracking. This result also followed from center manifold theory. (The same observation was made in [4] regarding a similar solution to this same problem.) This observation makes actual application of the nonlinear regulator theory more feasible.

3 Augmenting the Solution

In this section, we propose to augment the solution to the state feedback regulator problem given in [1]. We begin by motivating this augmentation with an application example initially studied in [3] and later in [2], [4] and [5].

Consider the well-known ball and beam example. This dynamic system can be modeled by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_1 x_4^2 - G\sin(x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= u \\ y &= h(x) = x_1 \end{aligned} \quad (5)$$

where x_1 is ball position, x_2 is ball velocity, x_3 is the angle of the beam, and x_4 is the beam's angular velocity. (For a derivation of these equations, see [3]. For all of the simulation results mentioned in this paper, the acceleration due to gravity was taken to be the normalized value 1. The magnitude of the signal to track is then relative to this normalized value.)

The task at hand is to cause the ball position x_1 to (at least almost) track a sinusoid produced by the exosystem

$$\begin{aligned} \dot{w}_1 &= -\lambda w_2 \\ \dot{w}_2 &= \lambda w_1 \\ q(w) &= -w_1 \end{aligned} \quad (6)$$

As presented in [2], [4] and [5], approximating the manifold to either first or third order yielded nice approximate tracking results. However, as discovered in [..] the region of attraction to this manifold could be very small. Further, the region of attraction did not seem to improve as the order of the approximation was increased. The simulation results of section 4 spell this out in more detail.

With this in mind, we propose augmenting the solution to the state feedback regulator problem in the following manner. First consider the composite system in expanded form

$$\begin{aligned} \dot{x} &= Ax + Bu + Pw + \phi(x,w,u) \\ \dot{w} &= Sw + \psi(w) \end{aligned} \quad (7)$$

We retain hypothesis (H2) and modify hypothesis (H1) in the following way:

(H1a) the pair (A, B) is controllable.

Now augment the exosystem with the following dynamics

$$\begin{aligned} \dot{y} &= Ay + BK_s y + BM(\epsilon)y \\ \dot{\epsilon} &= 0 \end{aligned} \quad (8)$$

where $y \in Y \subset \mathbb{R}^n$, $\epsilon \in E \subset \mathbb{R}^{mn}$, and $M(\epsilon)$ is an $m \times n$ matrix with smooth entries. We subject the augmented exosystem to the following hypotheses:

(H3) K_s is such that all of the eigenvalues of $(A + BK_s)$ have zero real part.

(H4) $\epsilon(0)$ is chosen sufficiently small and such that all of the eigenvalues of $(A + BK_s + BM(\epsilon(0)))$ have negative real part.

The initial conditions for y will be specified in the following theorem which is analogous to the theorem of [1].

Theorem 3.1 *Under hypotheses (H1a), (H2), (H3), (H4), the state feedback regulator problem is solvable if there exist $C^k (k \geq 2)$ mappings $x = \psi(w,y,\epsilon)$, with $\psi(0,0,0) = 0$ and $u = d(w,y,\epsilon)$, with $d(0,0,0) = 0$ both defined in a neighborhood $W^\circ \times Y^\circ \times E^\circ \subset W \times Y \times E$ of $0 \times 0 \times 0$, satisfying the conditions*

$$\frac{\partial\psi}{\partial w}s(w) + \frac{\partial\psi}{\partial y}(Ay + BK_s y + BM(\epsilon)y) = f(\psi(w,y,\epsilon)) + g(\psi(w,y,\epsilon))d(w,y,\epsilon) + p(\psi(w,y,\epsilon))w$$

$$h(\psi(w,y,\epsilon)) + q(w) - h(y) = 0 \quad (9)$$

and $y(0)$ is such that $|x(0) - \psi(w(0), y(0), \epsilon(0))|$ is sufficiently small.

Proof The proof follows the proof of theorem 2.1. Accordingly, assume the conditions (9) are satisfied and consider as a possible solution the state feedback

$$\alpha(x,w,y,\epsilon) = d(w,y,\epsilon) + K[x - \psi(w,y,\epsilon)]$$

where all the eigenvalues of $(A + BK)$ have negative real part. The existence of a K such that this is true follows from hypothesis (H1a). We now check that this state feedback is a solution to the state feedback regulator problem. Requirement (i) is satisfied because $\alpha(x,0,0,0) = Kx$. From hypotheses (H2) and (H3), the overall composite system can be transformed into coordinates for which center manifold theory directly applies. Since $\alpha(\psi(w,y,\epsilon),w,y,\epsilon) = d(w,y,\epsilon)$, by construction $x = \psi(w,y,\epsilon)$ is such a manifold in the original coordinates. Also, by (9), the error is given by

$$e(t) = h(x(t)) - h(\psi(w,y,\epsilon)) + h(y(t))$$

It follows from hypotheses (H2), (H4), the choice of K and the triangular structure of the composite system that the point $(x,w,y,\epsilon) = (0,0,0,0)$ is a stable equilibrium for the composite system. So, for sufficiently small $(x(0),w(0),y(0),\epsilon(0))$, the solution $(x(t),w(t),y(t),\epsilon(t))$ remains in an arbitrarily small neighborhood of $(0,0,0,0)$ for all $t \geq 0$. (Notice $x(0)$ and $w(0)$ sufficiently small are provisions from the problem statement, $\epsilon(0)$ sufficiently small follows from hypothesis (H3), and $y(0)$ sufficiently small follows from the choice of $y(0)$ such that $|x(0) - \psi(w(0),y(0),\epsilon(0))|$ is sufficiently small together with $x(0)$, $w(0)$, and $\epsilon(0)$ sufficiently small.) With this stability property, we can apply a property of center manifolds yielding there exist real numbers $M > 0$ and [illegible] such that

$$[illegible]\psi(w,y,\epsilon)(t)|| \leq Me^{-at}||x(0) - \psi(w,y,\epsilon)(0)||$$

[illegible] 0. Finally, from the continuity of h, together with hypothesis (H4) and the fact that $h(0) = 0$, we have $\lim_{t\to\infty} e(t) = 0$. We conclude that this choice of [illegible] solves the state feedback regulator problem.

Remarks.

1. The manifold $x = \psi(w,y,\epsilon)$ is an error-zeroing manifold in the limit as $y \to 0$. This manifold is rendered invariant by the control $u = d(w,y,\epsilon)$.

2. Theoretically, in terms of regions of attraction, we do not gain anything over the result in [1] because we are still dealing with fairly unspecified local neighborhoods of the origin. However, the improved simulation results in some instances are quite striking.

3. The reason for the improved simulation performance is that, with the additional y states we have created an augmented error

   $$e_a(t) = h(x(t)) + q(w(t)) - h(y(t))$$

   for which $x = \psi(w,y,\epsilon)$ is an error-zeroing manifold. By quickly regulating to the manifold $x = \psi(w,y,\epsilon)$, the control steers the system slowly to the original error-zeroing manifold $x = \pi(w)$. Regulating to the manifold $x = \psi(w,y,\epsilon)$ is relatively easy because the system trajectory necessarily starts close to this manifold.

4 Examples and Simulations

In this section we begin by presenting an example that clearly demonstrates the augmented solution proposed here. Then we demonstrate the usefulness of this augmented scheme on the ball and beam example. We provide simulation results that demonstrate this scheme's ability to handle a wider range of initial conditions in the plant.

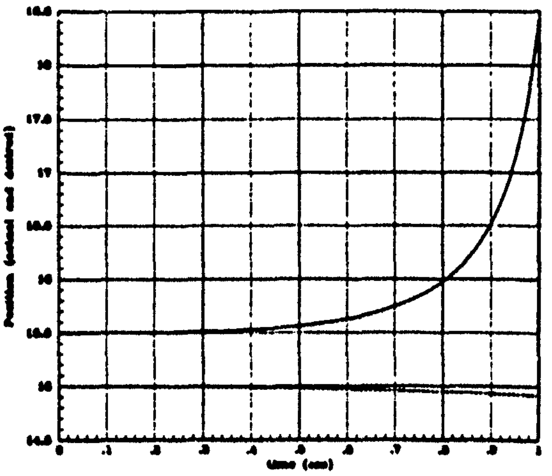

Figure 1: Tracking Results - Standard Nonlinear Regulator for example 4.2. (The dotted line (lower) represents the desired output. The solid line represents the actual output.)

Example 4.1 Consider the system

$$\begin{aligned} \dot{x}_1 &= x_1 + x_2 + x_1^2 + w_1 \\ \dot{x}_2 &= u \\ \dot{w}_1 &= -w_2 \\ \dot{w}_2 &= w_1 \\ e &= h(x) + q(w) = x_1 - w_1 \end{aligned} \tag{10}$$

Augment the exosystem in the following manner:

$$\begin{aligned} \dot{y}_1 &= y_1 + y_2 \\ \dot{y}_2 &= -y_1 - y_2 - \epsilon_1 y_1 - \epsilon_2 y_2 \\ \dot{\epsilon}_1 &= 0 \\ \dot{\epsilon}_2 &= 0 \end{aligned} \tag{11}$$

with ϵ_1, ϵ_2 sufficiently small and $\epsilon_1 > \epsilon_2$. We now solve for the mapping $x = \psi(w,y,\epsilon)$ and $u = c(w,y,\epsilon)$ as specified in theorem 3.1 and find:

$$\begin{aligned} \psi_1(w,y,\epsilon) &= w_1 + y_1 \\ \psi_2(w,y,\epsilon) &= -2w_1 - w_2 - (w_1 + y_1)^2 + y_2 \\ c(w,y,\epsilon) &= 2w_2 - w_1 - 2(w_1 + y_1)[y_1 + y_2 - w_2] \\ &\quad -y_1 - y_2 - \epsilon_1 y_1 - \epsilon_2 y_2 \end{aligned} \tag{12}$$

Finally, we choose $(y_1(0), y_2(0))$ in a neighborhood of the point (y_1^*, y_2^*) given by

$$\begin{aligned} y_1^* &= x_1(0) - w_1(0) \\ y_2^* &= x_2(0) + 2w_1(0) + w_2(0) + x_1(0)^2. \end{aligned}$$

Example 4.2 Again consider the ball and beam system given in (5) and the exosystem (6). Augment the

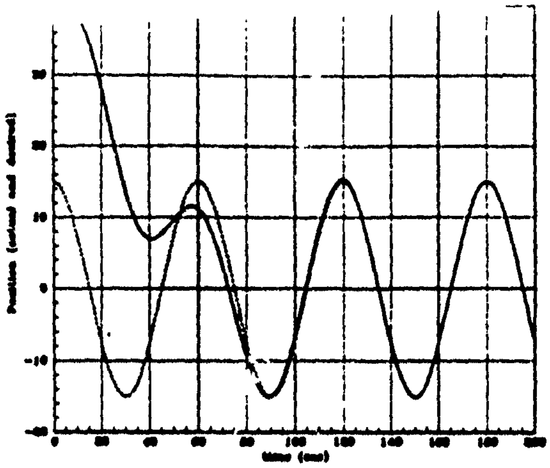

Figure 2: Tracking Results - Augmented Nonlinear Regulator for example 4.2. (The dotted line (lower) represents the original desired output. The solid line represents the actual output.)

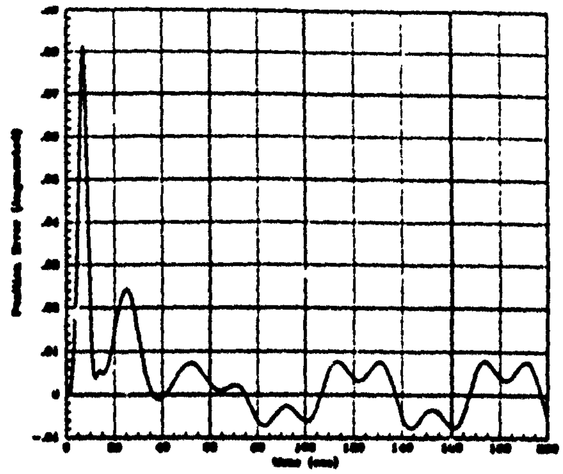

Figure 3: Tracking Error w.r.t. Augmented Trajectory for example 4.2.

osystem in the following manner:

$$\begin{aligned}\dot{y}_1 &= y_2\\ \dot{y}_2 &= -Gy_3\\ \dot{y}_3 &= y_4\\ \dot{y}_4 &= \epsilon_1 y_1 + \epsilon_2 y_2 + \epsilon_3 y_3 + \epsilon_4 y_4\\ \dot{\epsilon}_i &= 0\end{aligned} \quad (13)$$

th ϵ_i sufficiently small and such that the equilibrium point $y = 0$ is asymptotically stable. We calculate a first order approximation to the mappings $= \psi(w, y, \epsilon)$ and $u = c(w, y, \epsilon)$

$$\begin{aligned}(w, y, \epsilon) &= w_1 + y_1\\ (w, y, \epsilon) &= -\lambda w_2 + y_2\\ (w, y, \epsilon) &= \tfrac{1}{G}\lambda^2 w_1 + y_3\\ (w, y, \epsilon) &= -\tfrac{1}{G}\lambda^3 w_2 + y_4\\ (w, y, \epsilon) &= -\tfrac{1}{G}\lambda^4 w_1 + \epsilon_1 y_1 + \epsilon_2 y_2 + \epsilon_3 y_3 + \epsilon_4 y_4\end{aligned}$$

lly, we choose $y(0)$ in a neighborhood of the point given by

$$\begin{aligned}y_1^* &= x_1(0) - w_1(0)\\ y_2^* &= x_2(0) + \lambda w_2(0)\\ y_3^* &= x_3(0) - \tfrac{1}{G}\lambda^2 w_1(0)\\ y_4^* &= x_4(0) + \tfrac{1}{G}\lambda^3 w_2(0)\end{aligned}$$

simulation purposes, in the original exosystem, we t $\lambda = \frac{\pi}{30}$, $w_1(0) = 15$, and $w_2(0) = 0$. Consently, the task is for the ball position, x_1, to track $(\frac{\pi}{10}t)$. (By way of reminder, we continue to use rmalized value $G = 1$.)

now compare the augmented scheme (using first approximations) to the original scheme (using er approximations). Observe that, with $y \equiv 0$, d of theorem 3.1 reduce to π and c of theorem nce, we compare the control laws:

$$\alpha(x, w) = c(w) + K[x - \pi(w)] \quad (14)$$

and

$$\alpha(x, w, y, \epsilon) = d(w, y, \epsilon) + K[x - \psi(w, y, \epsilon)] \quad (15)$$

where K stabilizes the pair (A, B). First consider the control (14). Figure 1 shows the inability of this standard solution to regulate to the desired trajectory from the initial conditions

$$\begin{aligned}x_1(0) &= 15.5\\ x_2(0) &= 0\\ x_3(0) &= 0\\ x_4(0) &= 0\end{aligned}$$

Now consider the control (15). Figure 2 shows the ability of the augmented solution to regulate to the desired trajectory from the initial conditions

$$\begin{aligned}x_1(0) &= 40\\ x_2(0) &= 0\\ x_3(0) &= 0\\ x_4(0) &= 0\end{aligned}$$

Note that the initial error with respect to the original desired output has been increased by a factor of 80. Figure 3 shows the tracking error with respect to the augmented trajectory. Note that the small steady-state tracking error is due to approximating the manifold to first order.

5 Conclusion

In this paper, the nonlinear output regulation theory of [1] was reviewed and applied to the ball and beam example. It was found that for some desired tracking signals, the region of attraction for the error-zeroing manifold was very small. An augmentation to the existing scheme was proposed to handle a larger range of initial conditions. The exosystem was augmented in such a way that a new manifold could be calculated which passed arbitrarily close to the initial conditions of the plant and asymptotically decayed to the original error-zeroing manifold. This augmented scheme was demonstrated in simulations using the ball and beam example.

Acknowledgements The author wishes to thank Professor Shankar Sastry and Professor Alberto Isidori for suggesting research in this area.

References

[1] C. Byrnes and A. Isidori. Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, 35, No.2:131–140, 1990.

[2] B. Castillo. Almost tracking through singular points: via the nonlinear regulator theory. Preprint, University of Roma, La Sapienza, 1990.

[3] J. Hauser, S. Sastry, and P. Kokotovic. Nonlinear control via approximate input-output linearization: the ball and beam example. *IEEE Transactions on Automatic Control*, 1991. to appear.

[4] Jie Huang and Wilson J. Rugh. An approximate method for the nonlinear servomechanism problem. Technical Report JHU/ECE 90/08.1, 1990.

[5] Andrew R. Teel and Shankar Sastry. Applications of nonlinear output regulation. In *NSF-UC-NASA Workshop on Nonlinear Control: Abstracts*, April 1990.

Robust and Adaptive Nonlinear Output Regulation *

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Abstract

The object of this paper is to prove the stability of an adaptive control scheme designed to asymptotically achieve output regulation for a class of nonlinear systems. The solution proposed in [1] to the nonlinear output regulation problem is reviewed and the robustness of the solution to parametric uncertainty is analyzed. A standard adaptive scheme is then applied to the problem and slowly-varying results are employed to achieve asymptotic output regulation.

Keywords. Nonlinear Output Regulation, Adaptive Control, Center Manifold, Slowly-varying.

1 Introduction

The task at hand is to analyze and account for parameter uncertainty in the nonlinear output regulation problem. Recent work by Isidori and Byrnes [1] has produced necessary and sufficient conditions for the solvability of both the state feedback and output feedback regulator problem for a class of nonlinear systems. In their work, the signals to track are restricted to those that can be considered as the output of a Poisson stable exosystem. Their analysis is based on the local properties of center manifolds. Using the work in [1] as a point of reference, this paper will proceed to examine the same problem in the presence of parameter uncertainty. In section 2, we review the nonlinear regulator theory and the solution developed in [1]. In section 3 we introduce parametric uncertainty to the problem. In section 4 we lay the ground work for our adaptive scheme by reviewing slowly-varying theory for nonlinear systems. Finally, our adaptive scheme is developed in section 5.

2 Nonlinear Regulator Theory

The subsequent discussion follows closely that of [1]. The class of systems that will be examined is those of the form

$$\begin{aligned} \dot{x} &= f(x,\theta^*) + g(x,\theta^*)u + p(x,\theta^*)w \\ y &= h(x) \end{aligned} \tag{1}$$

*Research supported in part by the Army under grant ARO DAAL-88-K0572, and NASA under grant NAG2-243.

where w is the state of an (autonomous) exosystem

$$\dot{w} = s(w,\theta^*) \tag{2}$$

For this system, we will begin by considering $\theta^* \in \mathbb{R}^p$ as a vector of *known* parameters in order to review nonlinear regulator theory in the absence of uncertainty. The control objective is to have the output track a reference signal that is the output of the exosystem and given by $-q(w(t))$. The plant (1) is assumed to have m inputs and o outputs. The state x of the plant is defined on a neighborhood U of the origin in $\mathbb{R}^n$. The state w of the exosystem is defined on a neighborhood W of the origin in $\mathbb{R}^s$. Further, f and the columns of g and p are assumed to be smooth vector fields and $h(x)$ is a smooth mapping on U. Also, s is a smooth vector field and $q(w)$ is a smooth mapping defined on W. The composite system is then

$$\begin{aligned} \dot{x} &= f(x,\theta^*) + g(x,\theta^*)u + p(x,\theta^*)w \\ \dot{w} &= s(w,\theta^*) \\ e &= h(x) + q(w) \end{aligned} \tag{3}$$

Finally, it is assumed that $f(0,\cdot) = 0$, $s(0,\cdot) = 0$, $h(0) = 0$, $q(0) = 0$ so that, for $u = 0$, the composite system (3) has an equilibrium state $(x,w) = (0,0)$ which yields zero error, independent of the value of θ^*. For the state feedback regulator problem, we seek a state feedback of the form

$$u = \alpha(x,w,\theta^*)$$

such that the closed loop system

$$\begin{aligned} \dot{x} &= f(x,\theta^*) + g(x,\theta^*)\alpha(x,w,\theta^*) + p(x,\theta^*)w \\ \dot{w} &= s(x,\theta^*) \\ e &= h(x) + q(w) \end{aligned} \tag{4}$$

exhibits some stability property and $\lim_{t\to\infty} e(t) = 0$. Following [1], we state the nonlinear state feedback regulator problem formally.

State Feedback Regulator Problem. Given a nonlinear system of the form (3), find, if possible, a feedback $u = \alpha(x,w,\theta^*)$ such that

1. the equilibrium $x = 0$ of

$$x = f(x,\theta^*) + g(x,\theta^*)\alpha(x,0,\theta^*) \tag{5}$$

is asymptotically stable in the first approximation. i.e.

$$\sigma(\frac{d}{dx}[f(x,\theta^*)+g(x,\theta^*)\alpha(x,0,\theta^*)]|_{x=0}) \subset \mathbb{C}^*_-$$

2. there exists a neighborhood $V \subset U \times W$ of (0,0) such that, for each initial condition $(x(0), w(0)) \in V$, the solution of the closed loop system satisfies

$$\lim_{t\to\infty}(h(x(t))+q(w(t)))=0$$

Under the following two hypotheses, statements concerning the existence of a solution to the state feedback regulator problem can be formulated:

(H1) the linear approximation of (5) is stabilizable.

(H2) the point $w=0$ is a stable equilibrium of the exosystem, and there is an open neighborhood of the point $w=0$ in which every point is Poisson stable. In short, this assumption implies that the eigenvalues of the linear approximation of the exosystem lie on the imaginary axis.

Byrnes and Isidori state necessary and sufficient conditions for the solution of the state feedback regulator problem.

Theorem 2.1 (Byrnes and Isidori)
Under hypotheses (H1) and (H2), the state feedback regulator problem is solvable if and only if there exist $C^k (k \geq 2)$ mappings $x=\pi(w,\theta^*)$, with $\pi(0,\theta^*)=0$ and $u=c(w,\theta^*)$, with $c(0,\theta^*)=0$, both defined in a neighborhood $W^* \subset W$ of 0, satisfying the conditions

$$\begin{aligned} \frac{\partial\pi}{\partial w}s(w,\theta^*) &= f(\pi,\theta^*)+p(\pi,\theta^*)w \\ &\quad g(\pi,\theta^*)c(w,\theta^*) \\ h(\pi(w,\theta^*))+q(w) &= 0 \end{aligned} \tag{6}$$

Remark. The proof relies on center manifold theory and constructs a state feedback

$$u=\alpha(x,w,\theta^*)=c(w,\theta^*)+K^*[x-\pi(w,\theta^*)] \tag{7}$$

that is shown to be a solution of the state feedback regulator problem. K^* is a matrix of feedback gains such that the eigenvalues of the linear approximation of (5) have negative real part.

3 Parametric Uncertainty

To proceed with the discussion, θ^* is now considered as a vector of unknown parameters. Define the matrices

$$A^* = \left[\frac{\partial f}{\partial x}\right]_{x=0,\theta^*} \quad B^* = g(0,\theta^*)$$

The following structural assumptions are now made

A 1 *The pair (A^*, B^*) is stabilizable for all θ^* in a neighborhood of θ^*.*

A 2 *For all θ^* in a ball around θ^*, $w=0$ is a stable equilibrium of the exosystem, and there is an open neighborhood of the point $w=0$ in which every point is Poisson stable.*

Before attempting to handle the uncertainties of the plant and exosystem with adaptation, the question of robustness is addressed. In this setting, a control is formulated based on a model of the composite system, given by

$$\begin{aligned} \dot{x} &= f(x,\theta^\circ)+g(x,\theta^\circ)u+p(x,\theta^\circ)w \\ \dot{w} &= s(w,\theta^\circ) \\ e &= h(x)+q(w) \end{aligned} \tag{8}$$

where θ° is a *fixed* estimate of θ^*. Assume the following:

A 3 *The estimate θ° lies in ball around θ^*. This ball is atleast such that the gains chosen to asymptotically stabilize the linear approximation of the model (disconnected from the exosystem) also asymptotically stabilize the linear approximation of the actual plant (disconnected from the exosystem.)*

A 4 *For (8), the conditions analogous to (6) are satisfied by the C^k mappings $x=\pi(w,\theta^\circ)$, $u=c(w,\theta^\circ)$, for all θ° allowed by assumption A3.*

A 5 *A certainty equivalence feedback law of the form*

$$u=\alpha(x,w,\theta^\circ)=c(w,\theta^\circ)+K^\circ[x-\pi(w,\theta^\circ)] \tag{9}$$

is applied to the actual composite system (3), where K° is a matrix of gains that stabilizes the pair (A°, B°) defined by

$$A^\circ = \left[\frac{\partial f}{\partial x}\right]_{x=0,\theta^\circ} \quad B^\circ = g(0,\theta^\circ).$$

The stability of the composite system with (9) as input is now examined.

Theorem 3.1 (Bounded error manifold)
Under the assumptions (A1-A5), the composite system (3) with (9) as input has a center manifold at $(0,0,\theta^*)$, the graph of a mapping

$$x=\Psi(w,\theta^\circ,\theta^*)$$

defined in a neighborhood $W^* \times Y \subset W \times \mathbb{R}^p$ of $(0,\theta^*)$, satisfying the condition

$$\begin{aligned} \frac{\partial\Psi}{\partial w}s(w,\theta^*) &= f(\Psi,\theta^*)+p(\Psi,\theta^*)w \\ &\quad g(\Psi,\theta^*)\left[c(w,\theta^\circ)+K^\circ(\Psi-\pi)\right] \end{aligned} \tag{10}$$

As a consequence, the solution (9) to the state feedback regulator problem based on (8) yields bounded tracking error when applied to (3).

Sketch of Proof. First defined $\phi=\theta^\circ-\theta^*$ and replace θ° by $\phi+\theta^*$. Next augment the exosystem with $\dot{\phi}=0$. From the triangular structure and the assumptions concerning the eigenvalues of the plant disconnected from the exosystem, it follows that the closed loop composite system can be transformed into coordinates in which center manifold theory directly applies. In the original coordinates, and replacing ϕ by $\theta^\circ-\theta^*$, this manifold is the graph of mapping $x=\Psi(w,\theta^\circ,\theta^*)$

satisfying the condition (10). (see [2] for details of center manifold theory.)

Finally, by assumption the point $(z, w) = (0, 0)$ is a stable equilibrium of the closed loop composite system. Under this condition, for sufficiently small $(z(0), w(0))$, bounded tracking follows from center manifold theory and the continuity of h. □

Remarks:

1. The manifold $\Psi(w, \theta^*, \theta^*)$ is conceptual and will not need to be calculated.

2. The preceding argument extends naturally to the output feedback regulator problem also described in [1].

4 Slowly-Varying Parameters

The question of robustness is now addressed, under the added assumption that the parameters are allowed to vary slowly. Consider the state defined by $z = x - \Psi(w, \theta^*, \theta^*)$ for fixed θ^* belonging to a compact set Γ.

The dynamics of the state z are then

$$\begin{aligned} \dot{z} &= \dot{x} - \dot{\Psi}(w, \theta^*, \theta^*) \\ &= f(x, \theta^*) + g(x, \theta^*)\alpha(x, w, \theta^*) + p(x, \theta^*)w \\ &\quad - f(\Psi, \theta^*) - g(\Psi, \theta^*)\alpha(\Psi, w, \theta^*) - p(\Psi, \theta^*)w \\ &= F(z + \Psi, \Psi, \theta^*, \theta^*) \end{aligned} \tag{11}$$

From Theorem 3.1, for every $\theta^* \in \Gamma$, the equilibrium point $z = 0$ of (11) is uniformly asymptotically stable, uniformly in the parameter θ^*. Due to this property that is uniform in the parameters, and the differentiability of Ψ, the system meets the requirements of the following useful lemma formulated by Hoppensteadt [3] and recently restated by Khalil, Kokotovic [4].

Lemma 4.1 (Hoppensteadt) *There exists a Lyapunov function $W(z, \theta^*, \theta^*)$ such that*

$$\begin{aligned} \kappa_1(|z|) \le W(z, \theta^*, \theta^*) &\le \kappa_2(|z|) \\ W_z(z, \theta^*, \theta^*)F(z + \Psi, \Psi, \theta^*, \theta^*) &\le -\kappa_3(|z|) \\ |W_{\theta^*}(z, \theta^*, \theta^*)| &\le c_1 \\ |W_z(z, \theta^*, \theta^*)| &\le c_2 \end{aligned} \tag{12}$$

for all $z \in B_r = \{z \in R^n : |z| \le r\}$ and $(\theta^*, \theta^*) \in \Gamma \times \Gamma$, where $\kappa_1(.)$, $\kappa_2(.)$, $\kappa_3(.)$ are strictly increasing functions and c_1 and c_2 are nonnegative constants.

With this Lyapunov function in hand, the slowly-varying analysis proceeds in the following way. Allow θ^* to vary. The dynamics of the state z are now

$$\begin{aligned} \dot{z} &= \dot{x} - \tfrac{\partial\Psi}{\partial w}s(w, \theta^*) - \tfrac{\partial\Psi}{\partial\theta^*}\dot{\theta}^* \\ &= F(z + \Psi, \Psi, \theta^*, \theta^*) - \tfrac{\partial\Psi}{\partial\theta^*}\dot{\theta}^* \end{aligned} \tag{13}$$

Consider now the Lyapunov function of Lemma 4.1 and take its derivative along the trajectories of the system (13).

$$\begin{aligned} \dot{W} &= W_z\dot{z} + W_{\theta^*}\dot{\theta}^* \\ &= W_zF(z + \Psi, \Psi, \theta^*, \theta^*) + [W_{\theta^*} - W_z \tfrac{\partial\Psi}{\partial\theta^*} \ldots \\ \dot{W} &\le -\kappa_3(|z|) + d_1|\dot{\theta}^*| \\ &\le -\kappa(W) + d_1|\dot{\theta}^*| \end{aligned}$$

where $\kappa = \kappa_3 \circ \kappa_2^{-1}$ and $d_1 = c_1 + c_3 sup_\Gamma(\frac{\partial\Psi}{\partial\theta^*} \ldots$ show that z is stable for small $|z(t_0)|$ and suffi… small $|\dot{\theta}^*|$ observe that the set $D = \{W \le \kappa_1(q)\}$ … invariant set under the condition

$$|\dot{\theta}^*| \le \kappa(\kappa_1(q))/d_1$$

If $|z(t_0)| \le \kappa_2^{-1}(\kappa_1(q))$ for any $q \le r$, then, from … $W(t_0) \in D$. Since D is invariant, (12) implies … $|z(t)| \le q$, $\forall t \ge t_0$. In addition, if $\dot{\theta}^*, \to 0$ as $t \to$ … then $|z(t)| \to 0$ as $t \to \infty$ since $W(z(t), \theta^*(t), \theta^* \ldots$ as $t \to \infty$. Note that we can incorporate plant parameter variations about θ^* into variations of θ^*. An example of such a variation might be a slowly-varying exosystem.

Corollary 4.1 *Under the assumptions (A1-A5), … sufficiently small initial conditions $(x(0), w(0))$, … stability of the composite system (3) under the input … is robust to plant and exosystem parameter variations that are sufficiently slow and stay in a neighborhood … the nominal parameter value θ^*.*

Proof. Follows immediately from the previous lemma and discussion. □

The previous discussion is now applied to a general indirect adaptive control scheme. Consider the composite adaptive system,

$$\begin{aligned} \dot{x} &= f(x, \theta^*) + g(x, \theta^*)\alpha(x, w, \theta^*) + p(x, \theta^*)w \\ \dot{w} &= s(w, \theta^*) \\ \dot{\theta}^* &= G(x, w, \theta^*, t) \\ e &= h(x) + q(w) \end{aligned} \tag{15}$$

Corollary 4.2 *Under the assumptions (A1-A5), for sufficiently small initial conditions $(x(0), w(0))$, the stability of the composite system (3) under the input (9) is robust to parameter variations in the control law that are sufficiently slow and stay in a neighborhood of the nominal plant parameter value θ^*. Namely, the stability of (15) is achieved if $sup_{t \ge t_0}|G(x, w, \theta^*, t)|$ is sufficiently small.*

Remark. Because the parameter update law is a function of x and θ^*, some additional analysis will be required to guarantee a sufficiently small bound on $sup_{t \ge t_0}|G(x, w, \theta^*, t)|$.

Corollary 4.3 *Under assumptions (A1-A5), for (15), for sufficiently small initial conditions $(x(0), w(0))$ and $sup_{t \ge t_0}|G(x, w, \theta^*, t)|$ sufficiently small, if θ^* converges to some $\bar{\theta}$ then x converges to $\Psi(w, \bar{\theta}, \theta^*)$ and the steady state error, $e(t)$, of system (15) is bounded and given by*

$$h(\Psi(w, \bar{\theta}, \theta^*)) - h(\pi(w, \theta^*))$$

Proof. This is the case of $\dot{\theta}^* \to 0$ as $t \to \infty$ and $r = 0$ so that $z(t) \to 0$ as $t \to \infty$. By definition of z, x converges to $\Psi(w, \bar{\theta}, \theta^*)$. Then by the continuity of π and the stability of the composite system, the steady state error is bounded and given by $h(\Psi(w, \bar{\theta}, \theta^*)) - h(\pi(w, \theta^*))$. □

Corollary 4.4 *Under assumptions (A1-A5), for (15), for sufficiently small initial conditions $(x(0), w(0))$ and $\sup_{t \geq t_0} |G(x, w, \theta^*, t)|$ sufficiently small, if θ^* converges to θ^* then*

$$lim_{t \to \infty} e(t) = 0$$

Proof. Here x converges to $\Psi(w, \theta^*, \theta^*)$. Observe that $\Psi(w, \theta^*, \theta^*)$ satisfies the same partial differential equation as $\pi(w, \theta^*)$ since $\Psi(w, \theta^*, \theta^*)$ is the manifold made invariant by the input (9) with $\theta^* = \theta^*$. Thus, from the properties of center manifolds, x converges to the $\pi(w, \theta^*)$ of Theorem 2.1. Then Theorem 2.1 implies that $lim_{t \to \infty} e(t) = 0$. □

Remark. Typically, it is not possible to guarantee correct parameter convergence a priori without additional assumptions.

5 Adaptive Nonlinear Output Regulation

The last result of the previous section suggested that if an identifier could be constructed that guaranteed θ^* converges to θ^* then asymptotic tracking would be guaranteed as well. However, as is known in the adaptive literature, guaranteeing parameter convergence a priori requires additional assumptions. Rather than take that approach here, a specific identifier will be suggested that will result in asymptotic tracking. This identifier is formulated in the mind-set of indirect adaptive control. Namely, an identifier is constructed to estimate plant parameters and then these parameters are used in a certainty equivalence control law. The identifier used here is analogous to the observer-based identifier found in [5].

Consider again the composite system (3) where θ^* is considered a constant but unknown parameter vector. The following standard assumption for adaptive systems is made.

A 6 *The vector fields $f(x, \theta^*)$ and $s(w, \theta^*)$ and the columns of $g(x, \theta^*)$ and $p(x, \theta^*)$ have the following linear parameter dependence:*

$$\begin{aligned} f(x, \theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* f_i(x) \\ g_j(x, \theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* g_{i,j}(x) \\ p_j(x, \theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* p_{i,j}(x) \\ s(w, \theta^*) &= \textstyle\sum_{i=1}^{p} \theta_i^* s_i(w) \end{aligned}$$

where θ_i^*, $i = 1, \ldots, p$ are unknown parameters, which appear linearly, and the smooth vector fields $f_i(x)$, $g_{i,j}(x)$, $p_{i,j}(x)$, $s_i(w)$ are known.

Regressors are formed as

$$\begin{aligned} \chi_x^T(x, w, u) &= [f_1(x) + g_{1,j}(x)u_j + p_{1,k}(x)w_k, \ldots, \\ &\qquad f_p(x) + g_{p,j}(x)u_j + p_{p,k}(x)w_k] \\ \chi_w^T(w) &= [s_1(w), \ldots, s_p(w)] \end{aligned}$$

where summation over j, k is implied. Consequently, $\chi_x^T(x, w, u) \in \mathbb{R}^{n \times p}$ and $\chi_w^T(w) \in \mathbb{R}^{s \times p}$ contain all of the nonlinearities of the system. Now the composite system can be written as

$$\begin{aligned} \dot{x} &= \chi_x^T(x, w, u)\theta^* \\ \dot{w} &= \chi_w^T(w)\theta^*. \end{aligned}$$

In what follows, the conventional notation for estimates of unknown parameters, $\hat{\theta}$ will replace the previously used θ^*. To estimate the unknown parameters, the following identifier system is used.

$$\begin{aligned} \dot{\hat{x}} &= \Omega_x(\hat{x} - x) + \chi_x^T(x, w, u)\hat{\theta} \\ \dot{\hat{w}} &= \Omega_w(\hat{w} - w) + \chi_w^T(w)\hat{\theta} \\ \dot{\hat{\theta}} &= -\rho\chi_x(x, w, u)P_x(\hat{x} - x) - \rho\chi_w(w)P_w(\hat{w} - w) \end{aligned} \quad (16)$$

Here $\Omega_x \in \mathbb{R}^{n \times n}$, $\Omega_w \in \mathbb{R}^{s \times s}$ are Hurwitz matrices and $P_x \in \mathbb{R}^{n \times n}$, $P_w \in \mathbb{R}^{s \times s}$ are positive definite symmetric solutions to the Lyapunov equations

$$\begin{aligned} \Omega_x^T P_x + P_x\Omega_x &= -I_{n \times n} \\ \Omega_w^T P_w + P_w\Omega_w &= -I_{s \times s}. \end{aligned}$$

Finally, ρ is a small positive constant. Now, define $\epsilon_x = \hat{x} - x$, $\epsilon_w = \hat{w} - w$, and $\phi = \hat{\theta} - \theta^*$. Then the identifier error system becomes

$$\begin{aligned} \dot{\epsilon}_x &= \Omega_x\epsilon_x + \chi_x^T(x, w, u)\phi \\ \dot{\epsilon}_w &= \Omega_w\epsilon_w + \chi_w^T(w)\phi \\ \dot{\phi} &= -\rho\chi_x(x, w, u)P_x\epsilon_x - \rho\chi_w(w)P_w\epsilon_w. \end{aligned} \quad (17)$$

Theorem 5.1 *Under the assumptions (A1-A6), for sufficiently small initial conditions $(x(0), w(0))$, for the composite system (3) under (adaptive) input (9) $\exists\rho > 0$ of the identifier (16) such that*

1. $\phi \in L_\infty$,
2. $\epsilon_x, \epsilon_w \in L_\infty \cap L_2$,
3. $(x, w) \in L_\infty$,
4. $\dot{\epsilon}_x, \dot{\epsilon}_w \in L_\infty$,
5. $\lim_{t \to \infty} \epsilon_x(t) = \lim_{t \to \infty} \epsilon_w(t) = 0$,
6. $\lim_{t \to \infty} e(t) = h(x(t)) + q(w(t)) = 0$.

Proof. Consider the Lyapunov function

$$V(\epsilon_x, \epsilon_w, \phi) = \rho\epsilon_x^T P_x\epsilon_x + \rho\epsilon_w^T P_w\epsilon_w + \phi^T\phi \quad (18)$$

Taking the derivative of V along the trajectories of (17) yields

$$\dot{V} = -\rho\epsilon_x^T\epsilon_x - \rho\epsilon_w^T\epsilon_w \leq 0$$

Hence $0 \leq V(t) \leq V(0)$ for all $t \geq 0$, so that $V, \phi, \epsilon_z, \epsilon_w \in L_\infty$. Since V is a positive, monotonically decreasing function, the limit $V(\infty)$ is well-defined and

$$-\int_0^\infty \dot{V}\, d\tau = \rho \int_0^\infty (\epsilon_z^T \epsilon_z + \epsilon_w^T \epsilon_w)\, d\tau < \infty$$

so that $\epsilon_z, \epsilon_w \in L_2$.

It is now shown that ρ can be chosen so that the analysis of section 4 holds. This will imply that z remains bounded. Consider the parameter update law

$$\dot{\hat{\theta}} = \dot{\phi} = -\rho \chi_z(z, w, u) P_z \epsilon_z - \rho \chi_w(w) P_w \epsilon_w$$

Since χ_z, χ_w are smooth, w is bounded and $\epsilon_z, \epsilon_w \in L_\infty$, it follows that $\exists m > 0$ and a strictly increasing function $\kappa_4(.)$ such that

$$|\dot{\hat{\theta}}| \leq \rho(m + \kappa_4(|z|))$$

for all $z \in B_r = \{z \in \mathbb{R}^n : |z| \leq r\}$. Then for the Lyapunov function of section (4), equation (14) becomes

$$\begin{aligned} \dot{W} &\leq -\kappa_3(|z|) + d_1 |\dot{\hat{\theta}}| \\ &\leq -\kappa_3(|z|) + d_1 \rho (m + \kappa_4(|z|)) \\ &\leq -\kappa(W) + \rho d_1 \kappa_5(W) + \rho d_1 m \end{aligned}$$

where $\kappa = \kappa_3 \circ \kappa_2^{-1}$ and $\kappa_5 = \kappa_4 \circ \kappa_1^{-1}$. Now pick ρ_o sufficiently small such that $(\kappa - \rho_o d_1 \kappa_5)(.)$ is a strictly increasing function of W. Define $\kappa_6 = (\kappa - \rho_o d_1 \kappa_5)$. Then

$$\dot{W} \leq -\kappa_6(W) + \rho d_1 m$$

for all $\rho \leq \rho_o$. Now observe that the set $D = \{W \leq \kappa_1(q)\}$ for any $q \leq r$ is an invariant set if

$$\rho \leq \kappa_6(\kappa_1(q))/(d_1 m) \equiv \rho_1$$

Hence, if ρ is chosen such that $\rho \leq min\{\rho_o, \rho_1\}$ then D is an invariant set. Finally, if $|z(t_0)| \leq \kappa_2^{-1}(\kappa_1(q))$, then from (12) $W(t_0) \in D$. Since D is invariant (12) implies that $|z(t)| \leq q$ for all $t \geq t_0$.

Because z is bounded and w is bounded by assumption, x is bounded. Since x, w are bounded, $\chi_z(x, w, u)$, $\chi_w(w)$ are bounded. This implies $\dot{\epsilon}_z$, $\dot{\epsilon}_w$ are bounded. Since $\epsilon_z, \dot{\epsilon}_z, \epsilon_w, \dot{\epsilon}_w \in L_\infty$ and $\epsilon_z, \epsilon_w \in L_2$, $\lim_{t\to\infty} \epsilon_z = \lim_{t\to\infty} \epsilon_w = 0$.

Finally, the convergence of the tracking error is proved. Return to the Lyapunov function of (18). The nontrivial trajectories corresponding to $\dot{V} = 0$ are given by the set

$$\begin{aligned} S = \{&(\epsilon_z, \epsilon_w, \phi) : \\ &\epsilon_z = 0, \epsilon_w = 0, \chi_z^T(x, w, u)\phi = 0, \chi_w^T(w)\phi = 0\} \end{aligned}$$

From the definition of ϕ, trajectories in this set are such that

$$\begin{aligned} \chi_z^T(x, w, u)\hat{\theta} &= \chi_z^T(x, w, u)\theta^* \\ \chi_w^T(w)\hat{\theta} &= \chi_w^T(w)\theta^* \end{aligned} \qquad (19)$$

From Theorem 3.1, $\Psi(w, \hat{\theta}, \theta^*)$ satisfies the condition

$$\frac{\partial \Psi}{\partial w} \chi_w^T(w)\theta^* = \chi_z^T(\Psi(w, \hat{\theta}, \theta^*), w, u)\theta^* \qquad (20)$$

Further, by assumption (A5), $\pi(w, \hat{\theta})$ satisfies th… dition

$$\frac{\partial \pi}{\partial w} \chi_w^T(w)\hat{\theta} = \chi_z^T(\pi(w, \hat{\theta}), w, u)\hat{\theta}$$

From (19), $\pi(w, \hat{\theta})$ also satisfies

$$\frac{\partial \pi}{\partial w} \chi_w^T(w)\theta^* = \chi_z^T(\pi(w, \hat{\theta}), w, u)\theta^*$$

Now, $\lim_{t\to\infty} \epsilon_z = \lim_{t\to\infty} \epsilon_w = 0$ implies $\lim_{t\to\infty}$ … 0. So from Corollary 4.3, x converges to $\Psi(w,$ … Now since $\pi(w, \hat{\theta})$ satisfies the same manifold equ… as $\Psi(w, \hat{\theta}, \theta^*)$, the properties of center manifolds … that x converges to $\pi(w, \hat{\theta})$. From assumption (… $q(w) = -h(\pi(w, \hat{\theta}))$. Then, from the continuity of …

$$\begin{aligned} \lim_{t\to\infty} e(t) &= \lim_{t\to\infty} h(x(t)) + q(w(t)) \\ &= \lim_{t\to\infty} h(x(t)) - h(\pi(w, \hat{\theta})) = \ldots \end{aligned}$$

□

6 Conclusion

This paper has analyzed the dynamics of a system w… parameter uncertainties in the setting of nonlinear r… ulation. For small initial conditions, the nonlinear r… ulator solutions were shown to be robust to param… eter uncertainties and to slowly-varying paramete… The adaptive nonlinear regulator solution was cast int… this slowly-varying framework. It was shown then th… there exists an identifier with sufficiently small gain… that, in conjunction with a certainty equivalence control law, yielded zero error tracking in the limit.

Acknowledgements The author wishes to thank Professor Shankar Sastry and Professor Alberto Isidori for suggesting research in the area of adaptive nonlinear output regulation.

References

[1] C. Byrnes and A. Isidori. Output regulation of nonlinear systems. *IEEE Transactions on Automatic Control*, 35, No.2:131-140, 1990.

[2] J Carr. *Applications of Centre Manifold Theory* Springer Verlag, 1981.

[3] F.C. Hoppensteadt. Singular perturbations on the infinite interval. *Trans. of the American Math. Soc.*, 123:521-535, 1966.

[4] H.K. Khalil and P.V. Kokotovic. On stability properties of nonlinear systems with slowly-varying inputs. *IEEE Transactions on Automatic Control*, 36, No. 2.229, 1991.

[5] A.R. Teel, R.R. Kadiyala, P.V. Kokotovic, and S S Sastry. Indirect techniques for adaptive input output linearization of nonlinear systems *International Journal of Control*, 53, No 1.193-222, 1991

Adaptive Tracking for MIMO Nonlinear Systems

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Abstract

This paper discusses two schemes for the tracking con- [illegible] MIMO systems with parametric uncertainty in [illegible] dynamics. The first approach is an adaptive ver- [illegible] a static feedback law for tracking control based [illegible] results on asymptotic model matching recently [illegible]ed in ([DB90b], [DB90a]). The second approach [illegible] adaptive version of a dynamic precompensation [illegible] DM87]).

1 Introduction

[illegible]nt years, there has been a great deal of research [illegible] in the adaptive control of nonlinear systems. [illegible] research has been primarily focused on SISO systems and some notable contributions are ([KKM89], [TKMK89], [SI89], [PP89]).

In this paper, we consider a general MIMO nonlinear system P of the form

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where $x(t) \in X$, an open connected subset of $\mathbf{R}^n$, $u(t) \in \mathbf{R}^m$, $y(t) \in \mathbf{R}^p$. Further we will assume that f and the columns of g, namely g_i, are analytic vector fields on $\mathbf{R}^n$ and the functions h_i are real analytic functions on $\mathbf{R}^n$

Our results on nonadaptive tracking using static feedback for general MIMO nonlinear systems is a by product of the results of ([DB90b], [CDB90]) but have not appeared in the literature, to our knowledge. The results on adaptive asymptotic tracking by dynamic precompensation may be viewed as being in the spirit of ([SI89]). We also discuss schemes for adaptive tracking using static state feedback and the general problem of adaptive model reference control for MIMO nonlinear systems. No proofs are given in this paper; they are however available in ([DBS91]).

2 SISO Adaptive Linearization

We recall some results from ([SI89]) to allow for a better understanding of the differences with respect to the MIMO situation illustrated in Section 3.

Consider the system (1) with one input and one output. Let x_0 be an equilibrium point of the undriven system, that is $f(x_0) = 0$ and that the output is zero at x_0, i.e. $h(x_0) = 0$. We will assume that the system (1) has *strict relative degree* γ at x_0 ([Isi89]). One can then choose a new set of coordinates given by $\xi_1 = h(x), \xi_2 = L_f h(x), \ldots, \xi_\gamma = L_f^{\gamma-1} h(x)$ and $\eta \in \mathbf{R}^{n-\gamma}$ such that $d\eta_i\, g \equiv 0$. In these coordinates, the system (1) takes a 'normal form' which reveals the *zero dynamics* to be of the following form

$$\dot{\eta} = q(0, \eta) \tag{2}$$

with $\eta \in \mathbf{R}^{n-\gamma}$. The zero dynamics is said to be *exponentially attractive* to a large ball in X if the following equation holds:

$$\eta^T q(0, \eta) \leq -\alpha|\eta|^2 \ \text{ for } |\eta| \geq R \tag{3}$$

The zero dynamics satisfies a *conic continuity condition* in ξ uniformly in η if

$$|q(\xi, \eta) - q(0, \eta)| \leq k|\xi| \tag{4}$$

It may be verified using a converse Lyapunov argument (as in ([SI89])) that asymptotic tracking with bounded states can be obtained if the system is *exponentially attractive with conic continuity*.

The preceding result has been critically examined in the literature but it has not been appreciated that the condition of (4) is not a global Lipschitz condition on the function q.

Now, for adaptive tracking, assume that the vector fields f, g in (1) and the function h in (1) are unknown but may be parameterized linearly by parameters $\theta^* \in \mathbf{R}^l$ in the form

$$\begin{aligned} f(x, \theta^*) &= \textstyle\sum_{i=1}^{l} \theta_i^* f_i(x) \\ g(x, \theta^*) &= \textstyle\sum_{i=1}^{l} \theta_i^* g_i(x) \\ h(x, \theta^*) &= \textstyle\sum_{i=1}^{l} \theta_i^* h_i(x) \end{aligned} \tag{5}$$

* Research supported in part by a McKay lectureship held at Berkeley in September 1990

† Research supported in part by ARO under grant DAAL-88K0572 and NASA under grant NAG 2-243

known functions of x. In the equation (5) above, it follows that, if some of the θ_i^* are known, they are replaced by their values. The linearizing control laws of the previous section are replaced by their estimates depending on the current estimate $\hat{\theta}(t)$ of θ^* in accordance with a heuristic known as the *certainty equivalence principle*. Thus if the "true" system is *known to have relative degree* γ then the control law is given by

$$u = \frac{1}{\widehat{L_g L_f^{\gamma-1} h(x)}}(-\widehat{L_f^\gamma h(x)} + \hat{v}) \tag{6}$$

Here $\widehat{L_g L_f^{\gamma-1} h(x)}, \widehat{L_f^\gamma h(x)}$ stand for the estimates of $L_g L_f^{\gamma-1} h(x), L_f^\gamma h(x)$ derived by first expressing these function in terms of the known vector fields f_i, g_j and known functions h_k and multilinear products of the form $\theta_i \ldots \theta_m$ and then replacing the multilinear product by an estimate of the form $\widehat{\theta_i \ldots \theta_m}$. We define the multilinear product as a new parameter and estimate it. $\hat{v}$ stands for the estimate of the tracking control law given by

$$v = y_M^\gamma + \alpha_1(y_M^{\gamma-1} - \widehat{L_f^{\gamma-1} h}) + \cdots + \alpha_\gamma(y_M - \hat{h})$$

Note that all the $\widehat{L_f^i h(x)}$ are multilinear functions of θ. Consequently, if one defines $\Theta \in \mathbb{R}^k$ to be the vector of all multilinear products of the θ_i up to terms of degree γ, it follows that the control law of (6) is affine in Θ. Defining the parameter error in Θ to be $\Phi := \Theta - \Theta^*$ and the output error to be $e = y - y_M$, an easy calculation yields that

$$e^\gamma + \alpha_1 e^{\gamma-1} + \cdots + \alpha_\gamma e = \Phi^T W(x, \hat{\Theta}) \tag{7}$$

for some appropriately defined $W(x, \hat{\Theta}) \in \mathbb{R}^k$. Define the (model) transfer function

$$M(s) = \frac{1}{s^\gamma + \alpha_1 s^{\gamma-1} + \cdots + \alpha_\gamma} \tag{8}$$

and an augmented error e_1 to be

$$e_1 = e + (\Theta^T(t)M(s)W(x, \hat{\Theta}) - M(s)\hat{\Theta}^T(t)W(x, \hat{\Theta})) \tag{9}$$

Combining (7) with (9) yields

$$e_1 = \Phi^T M(s)W(x, \hat{\Theta}) \tag{10}$$

It is convenient to denote the filtered regressor $W_1(x, \Theta)$ by

$$W_1(x, \hat{\Theta}) := M(s)W(x, \hat{\Theta})$$

Theorem 2.1 Adaptive Tracking
Consider the system of (1) with the vector fields f, g and function h parameterized as in (5). Assume the system can be globally converted into normal coordinates on X. Further assume that the zero dynamics of the system are exponentially attractive in θ ... of (4) and satisfy the conic continuity conditions of (4). Also, assume that the regressor $W(x,\theta)$ has bounded derivatives in both its arguments. Then given a bounded trajectory y_M with first $\gamma - 1$ derivatives all bounded it follows that the control law of (6) with the parameter update law

$$\dot{\hat{\Theta}} = \dot{\Phi} = -\frac{W_1 e_1}{1 + W_1^T W_1} \tag{11}$$

yields bounded tracking, i.e. $y(t) \to y_M(t)$ with all the states x bounded, provided that the state trajectory is confined to X.

Remarks:

1. The parameter update law is specified for $\hat{\Theta}$. This neglects the multilinear dependence of terms inside the vector. However, this is necessitated by the lack of a systematic theory of nonlinear parameter estimation or identification.

2. Given the form of the linear error equation, there is a large choice available to us for parameter update laws. We choose the *normalized gradient type algorithm* of (11) here for reasons of brevity but we hasten to add that several other normalized algorithms (such as the *normalized least squares*) will do as well (see ([SB89]).

3 MIMO Systems

3.1 Tracking by Static State Feedback

Consider the plant P to be a square, nonlinear plant of the form (1). It is useful to derive the tracking results as a special case of model matching results, consequently we consider a model M of the same form as the plant with state $z \in X_M$ open $\subset \mathbb{R}^{n_M}$ and with f, g, h replaced by f_M, g_M, h_M. We will need to assume that the f, g, f_M, g_M are *analytic* vector fields and that h, h_M are analytic functions. Define $G(x) := \text{span}\{g_1(x), \ldots, g_m(x)\}$ (over the ring of analytic functions) and assume that the dimension of the distribution G is m for all $x \in X$. The notation $y_M(t)$ is used to mean the output of the model starting from state z_0 at 0 if there is no need to highlight the dependence on the initial state.
An extended system Σ^E is associated with the plant and model as follows:

$$\begin{aligned} \dot{x}^E &= f^E(x^E) + \hat{g}(x^E)u + \hat{p}(x^E)v \\ y^E &= h^E(x^E) \end{aligned} \tag{12}$$

with state $(x^E)^T := (x^T, z^T) \in X \times X_M$, inputs u, v and

$$f^E(x^E) = \begin{bmatrix} f(x) \\ f_M(z) \end{bmatrix}, \hat{g}(x^E) = \begin{bmatrix} g(x) \\ 0 \end{bmatrix},$$

$$\hat{p}(x^E) = \begin{bmatrix} 0 \\ g_M(z) \end{bmatrix}, h^E(x^E) = h(x) - h_M(z)$$

Further define

$$g^E(x^E) = [\hat{g}(x^E)\ \hat{p}(x^E)]$$

Also, define the dynamical system with state x^E, input u and output y^E described by the triple $(f^E, \hat{g}, h^E)$ to be $\hat{\Sigma}$. Now, consider a point $x_0^E = (x_0, z_0)$ which is an equilibrium point of f^E and also produces zero output for the system $\hat{\Sigma}$, i.e.

$$f^E(x_0^E) = 0 \quad h^E(x_0^E) = 0$$

Now, assume that

Assumption A1 : (Regularity of $\hat{\Sigma}$)

x_0^E is a regular point for the zero dynamics algorithm applied to $\hat{\Sigma}$ (regular in the sense of ([Isi89], page 304). Actually, the assumption A1 is a sufficient condition in order to apply the zero dynamics algorithm to the system $\hat{\Sigma}$ around x_0^E. Let $\hat{M}_k$ denote the submanifold defined at step k of the algorithm and $\hat{M}^*$ denote the zero dynamics manifold obtained at the conclusion of the algorithm; there further exists a unique smooth control $u_0 : \hat{M}^* \to \mathbf{R}^m$ so as to make $\hat{M}^*$ invariant, i.e. $f^E(x^E) + \hat{g}(x^E)u_0$ is tangent to $\hat{M}^*$. The vector field $f^E(x^E) + \hat{g}(x^E)u_0$ restricted to $\hat{M}^*$ is referred to as the zero dynamics of $\hat{\Sigma}$. It can also be shown that $\hat{M}^*$ can be expressed in a neighborhood of x_0^E as

$$\hat{M}^* = \{x^E \in X \times X_M : \hat{H}^*(x^E) = 0\}$$

for some function $\hat{H}^*$. The following theorem uses the procedure of the zero dynamics algorithm to solve the model matching problem as follows ([DB90a], [CDB90]):

Theorem 3.1 Stable Model Matching
Consider the system of (12) and assume that there exists an x_0^E such that

1. *A1 holds,*
2. *$\hat{\Sigma}$ is minimum phase at x_0^E, and*
3. *span $\{\hat{p}(x^E)\} \subset T_{x^E}\hat{M}_k +$ span $\{\hat{g}(x^E)\}$ in a neighborhood of x_0^E in $\hat{M}_k$, for all $k \geq 0$.*

Then, there exist neighborhoods U of x_0 and U_M of z_0 and an integer ν such that the compensator Q defined by

$$\begin{aligned} \dot{\chi} &= a(\chi, x) + b(\chi, x)v \\ u &= c(\chi, x) + d(\chi, x)v \end{aligned} \qquad (13)$$

for appropriately defined analytic a, b, c, d and $\chi \in \mathbf{R}^\nu$ and a function $F : U \times U_M \to \mathbf{R}^\nu$ and $L \in \mathbf{R}_+$ such that

a) If $v(t) \equiv 0$ then the point $(x_0, \chi_0 := F(x_0, z_0))$ is an asymptotically stable equilibrium point of the closed loop $P \circ Q$, i.e. of the system

$$\begin{aligned} \dot{x} &= f(x) + g(x)c(x, \chi) \\ \chi &= a(\chi, x) \end{aligned}$$

b) If $|v(t)| < L$ for all $t \geq 0$ then

$$\lim_{t\to\infty} y^{P\circ Q}(x, F(x, z), t) - y_M(z, t) = 0$$

for all $(x, z) \in U \times U_M$

Remarks:

1. In view of the propositions of ([Isi89], Appendix B.2) the fulfillment of (a) above guarantees that, given $\epsilon > 0$, there exist δ, K such that if $|(x(0), \chi(0))| < \delta$ and $|v(.)| < K$, then $|(x(t), \chi(t))| < \epsilon$ for all $t \geq 0$.

2. The proof of the preceding theorem in the above mentioned references is constructive and the compensator may be shown to be of the form

$$\begin{aligned} \dot{\chi} &= f_M(\chi) + g_M(\chi)v \\ u &= u(\chi, x, v) \end{aligned} \qquad (14)$$

initialized at $\chi_0 = z_0$, i.e. $\chi_0 = F(x_0, z_0) = z_0$. As a consequence, we have that $\chi(t) \equiv z(t)$ and one may define the control law in terms of x^E alone rather than x, χ, z as

$$u(x^E, v) := u^*(x^E, v) + M^{-1}(x^E)K\hat{H}^*(x^E) \qquad (15)$$

where $M(x^E) \in \mathbf{R}^{m\times m} := d\hat{H}^*(x^E)\hat{g}(x^E)$ and $u^*(x^E, v) := u_0(x^E) + u_1(x^E)v$ is the unique solution u of the equation

$$d\hat{H}^*(x^E)(f^E(x^E) + \hat{g}(x^E)u + \hat{p}(x^E)v) = 0$$

so that

$$u_0(x^E) = M^{-1}(x^E)d\hat{H}^*(x^E)f^E(x^E)$$

and

$$u_1(x^E) = M^{-1}(x^E)d\hat{H}^*(x^E)\hat{p}(x^E)$$

Further $K \in \mathbf{R}^{m\times m}$ is chosen to stabilize part of the system dynamics as specified below.

Let x_0 (respectively x_0^E) be an equilibrium point of P (respectively $\hat{\Sigma}$) such that $h(x_0) = 0$ (respectively $h(x_0^E) = 0$). Then, the following assumptions are equivalent.

Assumption A1' : (Strong Regularity of $\hat{\Sigma}$)

$\hat{\Sigma}$ is right-invertible and $(x_0^E, y^E \equiv 0)$ is a strongly regular pair for $\hat{\Sigma}$ (strongly regular in the sense of [DBG90]).

Assumption A2 : (Strong Regularity of P)

P is right-invertible and $(x_0, y \equiv 0)$ is a strongly regular pair for P.

Weaker assumptions than A2, e.g. the regularity hypothesis of [Isi89]), p. 302 are also sufficient for our purposes. The implications of these assumptions are discussed in ([DBS91]). It is shown in ([DB90b]) that if the hypothesis A1 of Theorem 3.1 is replaced by the assumption A2 above, then one can construct a local change of coordinates $(\xi, \eta, z') = \Psi(x, z)$ with $z' = z - z_0, \Psi(x_0, z_0) = 0$ and $\xi = \hat{H}^*(x, z)$ such that the plant with the controller of equation (14) has the form

$$\begin{aligned} \dot{\xi} &= A\xi + q_1(\xi, \eta, z') + p_1(\xi, \eta, z')v \\ \dot{z}' &= f_M(z' + z_0) + g_M(z' + z_0)v \\ \eta &= \psi(\xi, \eta, z') + \phi(\xi, \eta, z')v \end{aligned} \qquad (16)$$

The dependence of the matrices W_2, W_3 on the data, and for that matter on W_1 above, is involved. The equations (24) are affine in Φ as a consequence of the linear parameterization of the control law by the unknown parameter Θ.

We are not as yet able to give a stability proof for a parameter update law derived on the basis of a composite Lyapunov function involving the system of equation (24) and an equation for $\dot{\Phi}$. However, there is one important special case for which an adaptive scheme can be derived and this is the case when the function $q_1 \equiv 0$. It can be shown from Theorem 4.2 on page 272 of [Isi89] that this assumption is satisfied if the plant P can be linearized by static state feedback. This condition is slightly weaker than the condition that P has vector relative degree (in which case it can be both linearized and decoupled by static state feedback)

Theorem 3.3 Static State Feedback Adaptive Tracking
Consider the system of (21) and the model of (19) with the assumption that $q_1(\xi, \eta, z') \equiv 0$ Assume that A2 above holds and that P is exponentially attractive on X. Also assume that the vector fields $\psi(\xi, \eta, z'), \phi(\xi, \eta, z'), W_3(\xi, \eta, \Phi, v, \hat{\Theta})$ are Lipschitz continuous in their variables on $X \times X_M \times \mathbb{R}^m \times \mathbb{R}^k$. Further, assume that W_2 has bounded derivatives with respect to their arguments.
Then, under reparameterization with the control law of (23), assume that the system can be expressed as in equation (24).
Then there exists a choice of parameter update law for Φ such that the control law of (23) yields asymptotic tracking, with bounded states when $\sup_{t \geq 0}(|y_M(t)|, \ldots, |y_M^{\mu-1}(t)|) < \delta_1$ and $|x_0|, |\Phi(0)| < \delta_2$ and the trajectory of $x^E \in X \times X_M$. The proof of the theorem constructs the parameter update law.

3.3 Adaptive Tracking by Dynamic State Feedback

We now turn our attention to tracking by dynamic state feedback. Several algorithms have been proposed in the literature for this problem and we now recall the one of ([DM87]). We change notation slightly to refer to the process P as Σ_0. Set $k = 0$ and $x^e = x$.

Step 1 Let r_i be the relative degree of the i th output of Σ_k. Define the decoupling matrix $A(x)$ to have its ij th entry

$$a_{ij}(x) = L_{g_j} L_f^{r_i - 1} h_i(x)$$

and denote its normal or generic rank by s_k. If $s_k = m$, stop.

Step 2 If $s_k < m$, assume that the first s_k rows of $A(x)$ are linearly independent at each point of an open, dense subset of X (this can always be achieved by a permutation of the components of the output). Apply the static state feedback

$$u = \alpha_k(x) + \beta_k(x) v \tag{25}$$

with α_k, β_k analytic functions of x such t... coupling matrix of Σ_k with the control law ... the form

$$A_1(x) = \begin{bmatrix} I_{s_k \times s_k} & 0 \\ M(x) & 0 \end{bmatrix}$$

Step 3 *There exist q_k columns of $A_1(x)$ (wi... of generality the first q_k) with two or more n... ements* Put an integrator in series with q_k corr... ing input channels, i.e. define the dynamic e... of Σ_k composed with (25) as

$$\dot{\zeta}_i = v_i$$

for $i = 1, \ldots, q_k$. Let Σ_{k+1} be the new system ob... by composing Σ_k with (25) and (26) and return t... 1 with $k \leftarrow k + 1$ and $x^e \leftarrow \{x^e\} \cup \{\zeta_i\}$

If the original system is right-invertible, then the p... dure converges in a finite number of steps to a syst... denoted Σ^e, having vector relative degree $(r_1^e, \ldots$

Let (f^e, g^e, h^e) be the triple characterizing Σ^e, ... (x, ζ) its state, u^e its input and y^e its output ... struct a local change of coordinates $\phi(x^e) = (\xi, \eta) \ldots$ $\xi = \text{col}(\xi_i)$ by setting

$$\begin{aligned} \xi_i &= \text{col}\ (h_i^e(x^e), L_{f^e} h_i^e(x^e), \ldots, L_{f^e}^{r_i^e - 1} h_i^e(x^e)) \\ &:= \text{col}\ (\xi_1^i, \xi_2^i, \ldots, \xi_{r_i^e}^i) \end{aligned}$$

and using some complementary coordinates η The Σ^e takes the standard form ([Isi89], pg. 240):

$$\begin{aligned} \dot{\eta} &= q(\xi, \eta) + p(\xi, \eta) u^e \\ \dot{\xi}_1^i &= \xi_2^i \\ &\vdots \\ \dot{\xi}_{r_i^e - 1}^i &= \xi_{r_i^e}^i \\ \dot{\xi}_{r_i^e}^i &= b_i^e(\xi, \eta) + \textstyle\sum_{j=1}^m a_{ij}^e(\xi, \eta) u_j^e \\ y_i^e &= \xi_1^i \end{aligned} \tag{27}$$

for $i = 1, \ldots, m$ and $a_{ij}^e(\xi, \eta) = L_{g_j^e} L_{f^e}^{r_i^e - 1} h_i^e(\phi^{-1}(\xi, \eta))$ for $1 \leq i, j \leq m$ and $b_i^e(\xi, \eta) = L_{f^e}^{r_i^e} h_i^e(\phi^{-1}(\xi, \eta))$ for $1 \leq i \leq m$ At this point, asymptotic tracking may be obtained by applying the following control law:

$$u^e = (A^e)^{-1}(-b^e + \begin{bmatrix} y_{M1}^{r_1^e} + \alpha_{11}(y_{M1}^{r_1^e - 1} - \xi_{r_1^e - 1}^1) + \cdots + \alpha_{1 r_1^e}(y_{M1} - \xi_1^1) \\ \cdots \\ y_{Mm}^{r_m^e} + \alpha_{m1}(y_{Mm}^{r_m^e - 1} - \xi_{r_m^e - 1}^m) + \cdots + \alpha_{m r_m^e}(y_{Mm} - \xi_1^m) \end{bmatrix}$$

where the polynomials $s^{r_i^e} + \alpha_{i1} s^{r_i^e - 1} + \cdots + \alpha_{i r_i^e}$ are all Hurwitz

Prior Information for Adaptive Control
One assumes that the true system is right invertible Now, the variables A^e, b^e, ξ_i^j are all functions of the unknown parameter θ^*. The functions α_k, β_k are functions of θ^*. To estimate these one needs knowledge of the relative degrees of the system Σ_k at every step in the procedure above and in particular, the vector relative degree of the system Σ^e namely $(r_1^e, \ldots, r_m^e)$ Also, the integers s_k, q_k representing the rank of th...

where the matrix A is rendered Hurwitz by appropriate choice of K in (15). The states ξ contain in particular the output errors as some of their entries. Also the functions q_1 and p_1 satisfy some extra conditions, namely

$$q_1(0,\eta,z') \equiv 0 \quad \frac{\partial q_1}{\partial \xi}(0,0,0) = 0$$

and

$$p_1(0,\eta,z') \equiv 0$$

Further, the dynamical system

$$\begin{aligned} \dot{z}' &= f_M(z'+z_0) \\ \dot{\eta} &= \psi(0,\eta,z') \end{aligned} \tag{17}$$

represents the zero dynamics of $\hat{\Sigma}$ and the system

$$\dot{\eta} = \psi(0,\eta,0) \tag{18}$$

represents the zero dynamics of P. The zero dynamics manifold of $\hat{\Sigma}$ is now given by

$$\hat{M}^* = \{(\xi,\eta,z') \mid \xi = 0\}$$

The form (17) of the zero dynamics of the system $\hat{\Sigma}$ shows that it is minimum phase if the zero dynamics of P and the undriven model dynamics are asymptotically stable. In fact, the decomposition (16) can be used to extend the proof of Theorem 3.1 to cover the case where, instead of assuming the asymptotic stability of the zero dynamics of $\hat{\Sigma}$, one assumes that the variables z' are bounded by a sufficiently small constant and that the zero dynamics of P is asymptotically stable. This can then be usefully applied to solve trajectory tracking as a special case of the model matching problem in which the desired trajectory y_M is generated by a model consisting of chains of integrators driven by the appropriate derivatives of the y_{Mi}. More precisely, define μ_i to be the essential order of the i th output of the plant y_i as defined in ([GM89]). Then define the model to be matched to have state $z = \mathrm{col}(z_i, i = 1, \ldots, m)$ with dynamics

$$\begin{aligned} \dot{z}_{i1} &= z_{i2} \\ \dot{z}_{i2} &= z_{i3} \\ &\cdots \\ \dot{z}_{i\mu_i} &= v_i \\ y_{Mi} &= z_{i1} \end{aligned} \tag{19}$$

The model satisfies the third hypothesis of Theorem 3.1 by the definition of the μ_i and corresponds to $y_{Mi}^{(\mu_i)} = v_i$. Define $\mu = \max_i \mu_i$.

Theorem 3.2 MIMO Asymptotic Tracking

Assume that A2 above holds and that P is minimum phase at x_0. Then, there exist constants δ_1, δ_2 and a compensator Q of the form

$$u = c(x, y_M, y_M, \ldots, y_M^{(\mu-1)}) + d(x, y_M, \dot{y}_M, \ldots, y_M^{(\mu-1)})v \tag{20}$$

such that

1. *If $y_M(t) \equiv 0$ the closed loop system PoQ is asymptotically stable with equilibrium point x_0.*
2. *When $\sup_{t>0}(|y_M(t)|, \ldots, |y_M^{\mu-1}(t)|) < \delta_1$ and $|x_0| < \delta_2$ then*

$$\lim_{t\to\infty} y^{PoQ}(t) = y_M(t)$$

3.2 Adaptive Static State Feedback Tracking

In this section we consider models of the form of equation (1) with the added feature that the dynamics of the plant depend on certain unknown parameters $\theta^* \in \mathbf{R}^l$, i.e.

$$\begin{aligned} \dot{x} &= f(x,\theta^*) + g(x,\theta^*)u \\ y &= h(x,\theta^*) \end{aligned} \tag{21}$$

The assumption A2 of the previous section is assumed to hold for the true value of the plant parameter. Carrying forward the dependence on θ through the derivation of the tracking control law will yield the manifold $\hat{H}^*(x^E,\theta^*)$ and the control law of (15), namely

$$u(x^E, v, \theta^*) := u^*(x^E, v, \theta^*) + M^{-1}(x^E,\theta^*)K\hat{H}^*(x^E,\theta^*) \tag{22}$$

The *prior information needed for adaptive control* is as follows: one assumes that at each step of the zero-dynamics algorithm modified as described above for stable model matching, the manifold $\hat{M}_k$ described as the zero set of the functions $\hat{H}_k(x^E,\theta)$ satisfies the condition that

$$d\hat{H}_k(x^E,\theta)\hat{g}(x^E,\theta)$$

has a left null space of constant dimension as a function of θ. This is a sort of *regularity hypothesis* on the plant as a function of θ. Note that the model is assumed to be known and independent of θ. Since, the parameter θ^* is assumed unknown, we will replace it by its estimate at time t, denoted $\hat{\theta}(t)$. Further, we will assume that the control law can be linearly parameterized as

$$u(x^E, v, \theta^*) = \bar{u}(x^E, v) + W_1(x^E, v, \theta^*)\Theta^*$$

for an appropriately defined matrix $W_1(x^E, v) \in \mathbf{R}^{m\times k}$ and parameter vector $\Theta^* \in \mathbf{R}^k$. Actually both $\bar{u}$ and W_1 are affine in v. As a consequence the *adaptive model matching control law* is given by

$$u(x^E, v, \hat{\Theta}(t)) := \bar{u}(x^E, v) + W_1(x^E, v, \hat{\Theta})\hat{\Theta}(t) \tag{23}$$

Denoting the parameter error $\Phi = \hat{\Theta}(t) - \Theta^* \in \mathbf{R}^k$ we may use the control law of (23) in the system of (21) to yield the following modification of (16) (note the non-existence of p_1 in particular, caused by the special choice of model).

$$\begin{aligned} \dot{\xi} &= A\xi + q_1(\xi,\eta,z') + W_2(\xi,\eta,z',v,\hat{\Theta})\Phi \\ \dot{z}' &= f_M(z'+z_0) + g_M(z'+z_0)v \\ \dot{\eta} &= \psi(\xi,\eta,z') + \phi(\xi,\eta,z')v + W_3(\xi,\eta,z',v,\hat{\Theta})\Phi \end{aligned} \tag{24}$$

...upling matrices and the number of integrators to ...fied at each step in the algorithm described above are assumed known and are independent of θ. Also, the ...ations of the integrators at each step are assumed a- all.

From this information it is possible to compute α_k, β_k as a function of θ. As in the SISO case, we will assume that

$$(A^e)^{-1}b^e, (A^e)^{-1}, \xi_i^j, \alpha_k, \beta_k$$

depend linearly on a new parameterization Θ of the unknown parameters.

The adaptive control law follows by replacing Θ by $\hat{\Theta}$ so that by using the certainty equivalence control. As a consequence, the normal form equation (27) are modifed to have regressor vectors $w_j^i(x^e, \hat{\Theta})$ possibly at every entry corresponding to the mismatch between $\hat{\Theta}$ and Θ^*. Thus the error equations for the tracking errors $e_i = y_i - y_{Mi}$ are given by

$$\begin{aligned} e_i = \; & M_1^i(s)w_1^i(x^e, \hat{\Theta})\Phi + \cdots + \\ & M_{r_i^e-1}^i(s)w_{r_i^e-1}^i(x^e, \hat{\Theta})\Phi + M_{r_i^e}^i(s)w_{r_i^e}^i(x^e, \hat{\Theta})\Phi \end{aligned}$$

where

$$\begin{aligned} M_1^i &= \frac{s^{r_i^e-1} + \alpha_{i1}s^{r_i^e-2} + \cdots + \alpha_{i(r_i^e-1)}}{s^{r_i^e} + \alpha_{i1}s^{r_i^e-1} + \cdots + \alpha_{ir_i^e}} \\ &\;\;\vdots \\ M_{r_i^e}^i &= \frac{1}{s^{r_i^e} + \alpha_{i1}s^{r_i^e-1} + \cdots + \alpha_{ir_i^e}} \end{aligned} \qquad (28)$$

Note that all the transfer functions M_j^i are proper, stable transfer functions. We define the augmented error to be

$$\begin{aligned} e_{1i} = \; & e_i + (M_1^i(s)w_1^i)\hat{\Theta}(t) - M_1^i(s)(w_1^i\hat{\Theta}(t)) \\ & + \cdots + (M_{r_i^e}^i(s)w_{r_i^e}^i)\hat{\Theta}(t) - M_{r_i^e}^i(s)(w_{r_i^e}^i\hat{\Theta}(t)) \end{aligned}$$

It is easy to see that the augmented error is of the form

$$e_{1i} = W_i(x^e, \hat{\Theta})\Phi \qquad (29)$$

where

$W_i(x^e, \hat{\Theta}) = M_1^i(s)w_1^i(x^e, \hat{\Theta}) + \cdots + M_{r_i^e}^i(s)w_{r_i^e}^i(x^e, \hat{\Theta})$ is a filtered regressor. Note the resemblance of this error equation to that in the SISO case.

Under the same hypothesis as in Theorem 2.1 the same conclusions hold. There is however one difference in the proof from the SISO case, namely that the zero dynamics are indeed driven by the input u^e in the MIMO case. As a consequence, as in the case of the proof of Theorem 3.3 we need to insist that the initial conditions of the states x^e, the initial parameter error $\Phi(0)$ and the tracking output y_M and their appropriate derivatives are small enough.

References

[CDB90] R. Castro and M.D. Di Benedetto. Asymptotic nonlinear model matching. In *Proceedings of the 29th IEEE CDC, Honolulu*, pages 3400-3403, 1990

[DB90a] M. D. Di Benedetto. Nonlinear strong and asymptotic model matching with stability. Preprint, Università di Roma, 'La Sapienza', 1990.

[DB90b] M.D. Di Benedetto. Nonlinear strong model matching. *IEEE Transactions on Automatic Control*, 35:1351-1355, 1990.

[DBG90] M. D. Di Benedetto and J. W. Grizzle. Intrinsic notions of regularity for local inversion, output nulling and dynamic extension of nonsquare systems. *Control Theory and Advanced Technology*, 6:357-381, 1990.

[DBS91] M. D. Di Benedetto and S. S Sastry. Adaptive tracking for MIMO nonlinear systems. in preparation, 1991.

[DM87] J. Descusse and C. H. Moog. Dynamic decoupling for right invertible nonlinear systems. *Systems and Control Letters*, 8:345-349, 1987.

[GM89] A. Glumineau and C. H. Moog. Essential orders and the nonlinear decoupling problem. *International Journal of Control*, 50:1825-1834, 1989.

[Isi89] A. Isidori. *Nonlinear Control Systems*. Springer-Verlag, Berlin, 1989.

[KKM89] L. Kanellakopoulos, P. Kokotovic, and R. Marino. Robustness of adaptive nonlinear control under an extended matching condition. In *Proceedings IFAC, NOLCOS, Capri*, pages 192 -197, 1989.

[PP89] J. Pomet and L. Praly. Adaptive nonlinear regulation: Equation error from the Lyapunov equation. In *Proceedings 28th IEEE CDC, Tampa*, pages 1008 - 1013, 1989.

[SB89] S. Sastry and M. Bodson. *Adaptive Systems: Stability, Convergence and Robustness*. Prentice Hall, Englewood Cliffs, New Jersey, 1989.

[SI89] S.S. Sastry and A Isidori. Adaptive control of linearizable systems. *IEEE Transactions on Automatic Control*, 34:1123 - 1131, 1989.

[TKMK89] D. Taylor, P. Kokotovic, R. Marino, and L. Kanellakopoulos. Adaptive regulation of nonlinear systems with unmodeled dynamics. *IEEE Trans. Automat. Control*, 34.405 - 412, 1989.

Adaptive Linearization and Model Reference Control of a Class of MIMO Nonlinear Systems

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May 21, 1991

Abstract

This paper discusses two schemes for the adaptive control of classes of MIMO nonlinear systems with parametric uncertainty in their dynamics. First, the problem of tracking a reference trajectory is considered and an adaptive version of the input-output decoupling algorithm of [DM87] for general right invertible MIMO systems is proposed. Then on the basis of some results of [DB90a], [DB90b] on asymptotic model matching, a scheme is presented for Model Reference Adaptive Control and a solution is given for input-output linearizable systems. Moreover, the non-adaptive model matching results are extended to yield a solution to the problem of tracking by static state feedback.

1 Introduction

In recent years there has been a great deal of research effort in the adaptive control of nonlinear systems. This research has been primarily focused on SISO systems for which there exist, broadly speaking, three types of approaches: those relying on the existence of *certain matching or structural conditions* for the location of the unknown parameters (see for example [KKM89], [TKMK89] and [KKM91]), the second relying on certain assumptions on *the type of the nonlinearities* in the plant (see for example, [SI89], [NA88], [KTKS91])

*Research supported in part by a McKay lecturership held at Berkeley in September 1990.
†Research supported in part by ARO under grant DAAL-88-K-0106 and NASA under grant NAG 2-243

SINGULARLY PERTURBED ZERO DYNAMICS OF NONLINEAR SYSTEMS

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Abstract

Stability properties of zero dynamics are among the crucial input-output properties of both linear and nonlinear systems. Unstable, or "non-minimum phase", zero dynamics are a major obstacle to input-output linearization and high gain designs. An analysis of the effects of regular perturbations in system equations on zero dynamics shows that, whenever a perturbation decreases the system's relative degree, it manifests itself as a singular perturbation of zero dynamics. Conditions are given under which the zero dynamics evolve in two timescales characteristic of a standard singular perturbation form that allows a separate analysis of slow and fast parts of the zero dynamics. The slow part is shown to be identical to the zero dynamics of the unperturbed system, while the fast part, represented by the so called *boundary layer system*, describes the effects of perturbations.

*Research supported in part by MURST, AFOSR 88-0309 and NSF DMS-9008223
†Research supported in part by NASA NAG 2-243 and ARO DAAL 88-K-0106
‡Research supported in part by NSF ECS-88181166 and AFOSR 90-0011
§Research supported in part by MURST, AFOSR 88-0309 and NSF DMS-9008223

Indirect Adaptive Nonlinear Control of Induction Motors *

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NOLCOSS 1992

Abstract

An indirect adaptive control law based on certainty equivalence is designed for a model of the induction motor with the assumption that the magnetic subsystem is linear. This nonlinear control law renders the induction motor system input-output linear and also achieves input-output decoupling. In addition, we find for the specific case of the induction motor we are able to prove parameter convergence and asymptotic tracking of a reference trajectory using the indirect adaptive controller. This result differs from the generic case where we can only show asymptotic tracking. The indirect adaptive control methodology also does not suffer from the drawback of overparameterization as in the direct adaptive control technique. Simulations are also given comparing nonadaptive, direct adaptive, and indirect adaptive nonlinear controllers.

*Research supported in part by the Army under grants DAAL-88-K-0106 and DAAL-91-G-0191, and NASA under grant NAG2-234

AP_ LIN: A Tool Box for Approximate Linearization of Nonlinear Systems *

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CACSD 1992

Abstract

A toolbox for nonlinear control system design is presented. This package contains modules to approximate systems to polynomials systems of arbitrary order and then render them input-output linear or input-state linear with arbitrary order error terms. We also discuss possibilities for real-time control.

*Research supported in part by the Army under grants DAAL-88-K-0106 and DAAL-91-G-0191, and NASA under grant NAG2-234

Generalized Matching Conditions For Sliding Mode Control Of Perturbed Nonlinear Systems

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October 23, 1991

In this paper we present matching conditions for output regulation for four major classes of perturbed nonlinear systems controlled via the method of exact linearization utilizing the sliding mode control methodology. The systems considered are single input single output (SISO) systems with perturbed zero dynamics, multiple input multiple output (MIMO) systems with well defined vector relative degree, left invertible MIMO systems decoupled using the zero dynamics algorithm, right invertible MIMO systems decoupled using the dynamic extension method.

*Research supported in part by ARO-DAAL-K-0106