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This grant provided partial support for IMA academic year programs on Nonlinr waves (1988-89), Dynamical Systems (1989-90), and Phase Transitions (1990-91). Each program involved approximately 700 researchers. A total of approximately 400 preprints and approximately 25 proceedings volumes of research results were produced. Of particular note were the 8 one week workshops supported by ARO which focused attention on special subject areas in these fields which are of particular interest at the present time.

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NONLINEAR WAVES,
DYNAMICAL SYSTEMS,
& OTHER APPLIED MATHEMATICS PROGRAMS

FINAL REPORT

WILLARD MILLER, JR.

October 4, 1991

U.S. ARMY RESEARCH OFFICE

DAAL03-88-K-0132

INSTITUTE FOR MATHEMATICS AND ITS APPLICATIONS
514 Vincent Hall
University of Minnesota
Minneapolis, Minnesota 55455

A: STATEMENT OF THE PROBLEM STUDIED.

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This grant provided partial support for IMA academic year programs on Nonlinear waves (1988-89), Dynamical Systems (1989-90), and Phase Transitions (1990-91). Each program involved approximately 700 researchers. A total of approximately 400 preprints and approximately 25 proceedings volumes of research results were produced. Of particular note were the 8 one week workshops supported by ARO which focused attention on special subject areas in these fields which are of particular interest at the present time.

B. SUMMARY OF THE MOST IMPORTANT RESULTS.

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IMA NEWSLETTER #126

November 1 - November 15, 1988

1988-89 Program
NONLINEAR WAVES

NEWS AND NOTES

Workshop on APPLICATIONS OF SOLITONS

November 7-11 1988
Organizers: David Kaup and Yuji Kodama

This workshop will be primarily directed towards the applications of solitons in physics, biology and engineering, with a special concentration on nonlinear optics. Particular topics to be discussed will also include scattering theory, fluid mechanics, solitons in biology, electrical engineering and cosmology, and perturbation methods. The emphasis will be on significant physical applications of the mathematical methods from soliton theory.

The lectures will be held in Conference Hall 3-180 on the entry floor of the new Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office.

The detailed Workshop schedule can be found in the pages to follow.

SCHEDULE FOR NOVEMBER 1 - 15

Tuesday, November 1

11:00 am PROBLEM SEMINAR Vincent Hall 570

Abstract: This weekly informal seminar will take up various open problems in integrable systems and nonlinear optics.

Wednesday, November 2

IMA SEMINAR IN { Mathematics / Physics / Vincent Hall 570 }

11:15 am Percy Deift, Courant Institute — *R*-matrices and the *QR* algorithm

Abstract: The *QR* algorithm to compute the eigenvalues of a matrix is the integer time evaluation of a completely integrable Hamiltonian system. Here we show that all the associated commuting integrals generate flows that can be solved explicitly via a factorization procedure on a suitable finite, or infinite dimensional, Lie algebra.

Thursday, November 3

SEMINAR ON DYNAMICS AND MECHANICS

11:15 am Z. Fang, Aerospace Eng.& Mechanics, U. of Minnesota — Bifurcation of resonant surface waves

The Seminar is held in Vincent Hall 570

SPECIAL LECTURE

2:15 pm Philip Rosenau, Technion/Los Alamos Nat. Labs. — From micro-complexity to macro-simplicity and back: Modeling in the large of physical phenomena

Abstract: A considerable part of mathematical physics in the last two centuries was devoted to the study of partial differential equations that describe the action of some canonical physical processes on the continuous level. A modern need for a more realistic physical modelling caused many to attempt to extend the continuous level of description. As a rule all these trials failed. This resulted in anything between confusion and working myth that the gap between the microscopic dynamics and the continuous simplification cannot be bridged. We propose a unifying mathematical approach that enables one to extend the continuum in a natural way that incorporates the essential elements of the micro-cosmos. The resulting quasi-continuum models more faithfully the physical process. It enables us to bridge between the dynamics of a long mass-spring chain and that of the string, random walk and diffusion or the kinetic Boltzmann description and the hydrodynamic, gas-dynamic, equations.

The lecture will be given in Vincent Hall 570

SPECIAL LECTURE

4:15 pm Alan Weiss, AT&T Bell Laboratories — New applications in large deviations

The lecture will be given in Vincent Hall 570

Friday, November 4

SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	Alan Weiss AT&T Bell Laboratories	Applications of large deviations to computers and communication

Abstract. The theory of large deviations is relatively young, yet has proven itself very well suited to analyzing certain problems in computers and communications. After a necessarily brief outline of some relevant portions of the theory, I will outline applications to:
1) Erlang's traffic model
2) Correlations in a multiprocessor model
3) Analysis of "rollback", a parallel simulation paradigm
4) Accelerated simulation of rare events

The SEMINAR will be held in Conference Hall EE/CS 3-180.

LONG TERM PARTICIPANTS SEMINAR

1:15 pm	Brian Coomes IMA	Polynomial flows, polynomial symmetry groups, and the Jacobian conjecture

Abstract: Consider the initial value problem

$$\dot{y}\,(\equiv \frac{dy}{dt}) = V(y), \qquad y(0) = x \in R^n$$

where V is a C^1 vector field on R^n and for each t the solution (flow) ϕ is a polynomial function of the initial condition x. Call such a vector field a *p-f vector field*, and its flow a *polynomial flow*. Notice that p-f vector fields include, but are not restricted to, linear ones. We will discuss properties of p-f vector fields and discuss how to use polynomial symmetry groups to determine whether a vector field has a polynomial flow. We will also present a connection between polynomial flows and an open question from Algebraic Geometry, Keller's Jacobian conjecture.

The SEMINAR is held in Vincent Hall 570

Workshop on

APPLICATIONS OF SOLITONS

November 7-11 1988

Organizers: David Kaup and Yuji Kodama

Monday, November 7

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am	**Yuji Kodama** Ohio State & Nagoya Universities/IMA	Perturbations of solitons

Abstract: We present a general scheme of perturbation method for perturbed soliton systems, based on the normal form theory and the method of multiple scales. By this scheme, we discuss behavior of solitons under the presence of perturbations. Then we show that solitons interact inelastically, and that a new solitary wave as well as radiation tails are generated as a result of the interaction. This interaction property holds commonly for the nonintegrable systems of the dispersive nonlinear wave equations.

10:30 am	**Coffee Break**	Conference Hall Foyer
11:00 am	**Jerry Bona** Pennsylvania State University	Solitons and formation of sand ridges

Abstract: Tidal fluctuations generate trains of large-amplitude internal solitary waves at the edge of continental shelves. These propagate shoreward and interact strongly with the shelf floor. A model for wave-floor interactions predicting the formation of sand ridges is developed and discussed in the context of these large amplitude internal waves.

2:00 pm	**Darryl Holm** Los Alamos National Labs/IMA	Hamiltonian chaos in nonlinear optical polarization dynamics

Abstract: This paper applies Hamiltonian methods to the Stokes representation of the one-beam and the two-beam problems of polarized optical pulses propagating as travelling waves in nonlinear media. We treat these two dynamical systems as follows. First, we use the reduction method of Marsden and Weinstein to map each of the systems to the two-dimensional sphere, S^2. The resulting reduced systems are then analysed from the viewpoints of their stability properties and of bifurcations with symmetry; in particular, several degenerate bifurcations are found and described. We also establish the presence of chaotic dynamics in these systems by demonstrating the existence of Smale horseshoe maps in the two and three dimensional cases, as well as Arnold diffusion in the higher dimensional cases. The method we use to establish this complex dynamics is the Mel'nikov technique, as extended by Holmes and Marsden, and Wiggins for the higher dimensional cases. These results apply to perturbations of homoclinic and heteroclinic orbits of the reduced integrable problems for static, as well as travelling waves, solutions describing either a single optical beam, or two such beams counterpropagating. Thus, we show that these optics problems exhibit complex dynamics and predict the experimental consequences of this dynamics.
This is joint work with M. V. Tratnik and D. David.

3:00 pm	**P. L. Christiansen** Tech. University of Denmark	Stability of sine-Gordon solitons

Abstract: The perturbed sine-Gordon equation is used to model the Josephson junction in linear and annular overlap geometries. Each current singularity in the I-V characteristic corresponds to a soliton dynamic state. The stabilities of the solitons at the front of a singularity (high bias current) will be discussed. A global bifurcation of the travelling wave solution for the annular junction is connected to the switching phenomenon.

Tuesday, November 8

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am	**Mark Ablowitz** Clarkson University	Solitons, nonlinear evolution equations, inverse scattering and cellular automata

Abstract: A discussion and comparison of the method for solving a class of physically significant nonlinear evolution equations in 1 space, 1 time and 2 space, 1 time dimensions will be given. The $\bar{\partial}$ method is used to solve the inverse problem in the latter case. The $\bar{\partial}$ method is an effective technique even for multi-dimensional inverse problems.

A particularly interesting multi-dimensional nonlinear evolution equation is the Davey Stewartson equations. These equations and exact solutions in the strong coupling limit will be discussed. The ideas apply to certain other nonlinear evolution equations as well, such as the Zakharov equations, which arise in plasma physics. Finally, if time permits, aspects of recent work involving solitons which arise in a cellular automaton will be discussed.

10:30 am	**Coffee Break**	Conference Hall Foyer

11:00 am C. David Levermore — University of Arizona — Semiclassical limits of nonlinear wave equations

Abstract: The talk will be a discussion of the semiclassical limits of nonlinear wave equations in both he integrable and nonintegrable cases The integrable cases to be discussed include the Korteweg-deVries and nonlinear Schrödinger equations. Serious gaps in our knowledge exist in ascertaining the existence and computing these limits for nonintegrable cases.

2:00 pm Linn F. Mollenauer — AT&T Bell Laboratories — Solitons in optical fibers: An experimental account

Abstract: Solitons are easily created, and once formed. are quite stable in the one dimensional world of single mode optical fibers. As a consequence, we have been able to carry out a number of fundamental experiments and have found some exciting uses for solitons in fibers. In this talk, I shall describe both experiments and uses.

One use is in the soliton laser, a mode-locked (short pulse) laser whose pulse characteristics are determined by a length of fiber in its optical feedback loop. The soliton laser has in turn been a unique source of precisely controlled ultra short (picosecond and subpicosecond) pulses for further soliton experiments. With its help, for example, we have discovered a steady down shift in optical frequency of the soliton, or "soliton self frequency shift", which can be understood as a Raman self pumping of the soliton. This discovery has important implications for modification of the nonlinear Schrödinger equation. Pulses from the soliton laser have also enabled study of the interaction forces between co-propagating pairs of solitons, on a convenient scale of a few hundred meters path length.

In the latest round of experiments, we circulate a train of 50 ps solitons many times around a closed 42 km loop in which the dissipative loss is exactly canceled by optical gain from the stimulated Raman effect (That is, a separate, cw laser continuously pumps the loop to provide the necessary gain.) To date, we have successfully demonstrated soliton transmission over distances as great as 6000 km, with obvious implications for long distance telecommunications. We are also just now engaged in a study of the interaction forces between co-propagating solitons, but this time, of course, over a vastly greater scale of distance.

4:00 pm IMA Tea (and more!) — Vincent Hall 502 (the IMA Lounge)

Wednesday, November 9

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am John Weiss — UC San Diego — Bäcklund transformations and the Painlevé property

Abstract: For systems with the Painlevé property, Bäcklund transformations can be defined. These appear as specializations (truncations) of certain expansions of the solution about its *singular manifold*. With reference to the Lax pair for a system, the Bäcklund transformations are equivalent to transformations of linear systems developed by Darboux (Bäcklund-Darboux transformations).

For specific systems the Bäcklund-Darboux transformations lead to a reformulation of these systems in terms of the Schwarzian derivative. We find the Bäcklund transformations of these systems and study their periodic fixed points.

The periodic fixed points of the Bäcklund transformations determine a finite dimensional invariant manifold for the flow of the system. The resulting (ordinary) differential equations have a hamiltonian structure and the flow of the (partial) differential system is represented by commuting flows on the finite dimensional manifold.

10:30 am Coffee Break — Conference Hall Foyer

11:00 am A. V. Mikhailov — Landau Institute, Moscow — Polarization domains in nonlinear optics

Abstract: Stable (ground) polarization states have been found in the case of the propagation of electromagnetic waves in opposite directions in a mirror-symmetry medium with a cubic nonlinearity along a fourfold axis if the optical Kerr self-effect is ignored. The evolution of an arbitrary initial polarization gives rise to a "domain structure." Explicit solutions describing domain walls are found.

SHORT TALKS

(20 minute talks with 5 minutes between presentations)

2:00 pm	Alejandro Aceves University of Arizona	Snell's laws at the interface between nonlinear dielectrics
2:25 pm	**Willy Hereman** U. of Wisconsin, Madison	Derivation of the Dym equation and its implicit solitary wave solution
2:50 pm	**David Muraki** Northwestern University	Solitons in weakly birefringent fibers: polarization dynamics
3:15 pm	**P. Rosenau** Technion, Haifa	Evolution and breaking of IM acoustic waves
3:40 pm	**Cherie Schultz** Clarkson University	Trace formula for D-bar limit case of Davey-Stewartson I
4:05 pm	**G.E. Swaters** University of Alberta	Solitary drift-vortex solutions of the Charney-Hasegawa-Mima equation

THE SHORT TALKS WILL BE PRESENTED IN VINCENT HALL 570

Thursday, November 10

The morning talks today are in Conference Hall EE/CS 3-180

9:30 am	**Allan Fordy** Leeds University	Factorisation of energy dependent Schrödinger operators: Miura maps and modified systems

Abstract: We consider the energy dependent Schrödinger operator $L = \sum_{i=0}^{N} \lambda^i(\varepsilon_i\partial^2 + u_i)$, which we have previously shown to be associated with multi-Hamiltonian structures. In this paper we use an unusual form of the Lax approach to derive by a *single construction* the time evolutions of the eigenfunctions of L, the associated Hamiltonian operators and the Hamiltonian functionals.

We then generalise the well known factorisation of standard Lax operators to the case of energy-dependent operators. The simple product of linear factors is replaced by a λ-dependent quadratic form. We thus generalise the resulting construction of Miura maps and modified equations. We show that for some of our systems there exists a sequence of N such modification, the r^{th} modification possesing $(N-r+1)$ Hamiltonian structures.

The above construction is generalised to a super-extension of the above Schrödinger operator giving the super-KdV and super-Harry Dym like equations.

10:30 am	Coffee Break	Conference Hall Foyer
11:00 am	**A. S. Fokas** Clarkson University	$\bar\partial$(DBAR) and RH(Riemann-Hilbert) methods and the integrability of Davey-Stewartson equations

Abstract:

1. We consider an inverse problem associated with the linear equation

$$\mu_{x_0} + \sigma \sum_{\ell=1}^{n} (J_\ell \mu_{x_\ell} + i k_\ell (J_\ell \mu - \mu J_\ell)) = Q\mu, \quad \sigma = \sigma_R + i\sigma_I, \tag{1}$$

where $x_0 \in R$, $x_\ell \in R$, $k_\ell \in C$, $\ell = 1, \ldots, n$, μ is an $N \times N$ matrix J_ℓ, $\ell = 1, \ldots, n$ is a $N \times N$ real, constant diagonal matrix, and the potential Q is an $N \times N$ off-diagonal matrix. Using a $\bar{\partial}$ formalism, the potential Q is reconstructed from appropriate asymptotic data T. For $n \geq 2$ the data T satisfy a nonlinear characterization equation, which turns out to be a necessary and sufficient condition for the existence of a unique map between Q and T. The special case of $\sigma = i$, $n = 1$, $N = 2$ can be used to integrate the DSII equation, a nonlinear evolution equation in two spatial dimensions (x_0, x_1).

2. Equation (1) with $\sigma = 1$, $n = 1$, $N = 2$ is associated with DSI, which was recently shown to have the remarkable property of possessing solitons, exponentially decaying in both spatial dimensions. A RH formalism, which incorporates the above solitons, is used to integrate DSI.

1:30 pm	**Excursion to the Saint Anthony Falls Hydraulic Research Laboratory**	

Make (solitary) waves at the St. Anthony Falls Hydraulic Research Lab of the University of Minnesota. Transportation will be provided. Demonstrations: Experimental and simulation studies on wave phenomena, including hydroacoustics, surges and pressure waves in large pipe systems, wave effects on gas exchange at air water interfaces, surface waves. The demonstrations will conclude about 3:00 pm. For those who wish to stay, there is a colloquium at 3.30 pm: Gas transfer in hydraulic structures. The speaker is John Thene of the St.Anthony Falls Hydraulics Lab.

4:30 pm	**C. David Levermore** University of Arizona	The incompressible Navier-Stokes limit of the classical Boltzman Equation

Abstract: The talk will start by introducing the Boltzman equation. The formal derivations of the compressible Euler equations via the Hilbert expansion and the Navier-Stokes correction via the Chapman-Enskog method are well known. A new expansion will be presented which leads directly to the incompressible Navier-Stokes equation. Moreover, this limit can be made rigorous. We use the recent results of R. DiPerna and P.-L. Lions on the existence of global solutions to the (normalized) classical Boltzman equation. This is joint work with F. Golse and C. Bardos.

This talk will be given in Vincent Hall 570

Friday, November 11

The first three talks today are in Conference Hall EE/CS 3-180

9:30 am	**Alwyn Scott** University of Arizona	Local modes in molecules

Abstract: The quantum stability of local modes of molecular vibration (stationary solitons) is compared for two types of classical nonlinearity: i) **intrinsic** which arises from a nonlinear restoring force (such as the CH stretching modes of benzene), and ii) **Extrinsic** which comes about through coupling to low frequency phonons (as is observed for the CO stretching modes of crystalline acetanilide).

10:30 am	**Coffee Break**	Conference Hall Foyer
11:00 am	**S. V. Manakov** Landau Institute, Moscow	Multidimensional solvable nonlinear evolution equations and the dressing method
2:00 pm	**Fred Ernst** Clarkson University	Interaction of gravitational plane waves

Abstract: This is joint work with I. Hauser on the initial value problem for colliding gravitational plane waves and on a new homogeneous Hilbert problem approach to the initial value problem. This work relates to soliton investigations in other areas but that relationship will not be spelled out in detail in the talk.

3:00 pm	Curtis Menyuk University of Maryland	Transient solitons in stimulated Raman scattering

Abstract. In recent years, solitons or at least soliton-like pulses have been observed experimentally by Druhl, et al. and Wenzel, et al. It is of some interest to know whether these structures can persist permanently or must ultimately vanish. In the present work, it is shown that these soliton-like objects are necessarily impermanent in finite-energy pulses and hence can be properly referred to (at best) as "transient solitons". Kaup has shown that the equations describing stimulated Raman scattering can be solved using spectral transform methods in the limit where collisional attenuation can be neglected. The spectral transform approach has non-standard features in this case, and, in particular, it is shown that the poles in spectral transform space need bear no apparent relationship to solitons.

This talk will be given in Vincent Hall 570

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SOLITONS IN PHYSICS, MATHEMATICS, AND NONLINEAR OPTICS

EDITORS: PETER J. OLVER AND DAVID SATTINGER

IMA VOLUMES IN MATHEMATICS AND ITS APPLICATIONS

PUBL: SPRINGER-VERLAG VOLUME # 25

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IMA NEWSLETTER #148

October 13 - 29, 1989

1989-90 Program
DYNAMICAL SYSTEMS AND THEIR APPLICATIONS

NEWS AND NOTES

Workshop on PATTERNS AND DYNAMICS IN REACTIVE MEDIA

October 16–20, 1989
Organizers: R. Aris, D. Aronson, S.-N. Chow, P. Fife, M. Golubitsky, R. McGehee, G. R. Sell and H. Swinney

The speakers at this workshop will include representatives from chemistry, chemical engineering, and mathematics. The topics from chemistry to be covered are: Bursting, complex chemistry, electrochemical reactors, pattern formation, stability of fronts, stirred tanks dynamics, and tubular dynamics. The mathematical topics are: Bifurcation theory, connection theory, forced oscillations, invariant manifolds, reaction diffusion equations, renormalization methods, singular perturbations, spectral issues, and symmetries.

The lectures will be held in Conference Hall 3-180 on the entry floor of the new Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

The detailed Workshop schedule, and the regular IMA program, can be found in the pages to follow.

Academic Festival
to celebrate
the 60th Birthday of
RUTHERFORD ARIS

October 13 and 14, 1989
University of Minnesota

The faculty of the Department of Chemical Engineering and Materials Science is planning a 60th Birthday Festival for Gus Aris on Friday, October 13, 1989. This will consist of a Symposium during the day and a Banquet that evening. The remodeling of Amundson Hall has just been completed (or should be by mid-October), and there will also be a dedication of the the building on October 13.
Events are scheduled in Amundson Hall and in the Radisson University Hotel, a short block down Washington Avenue. The Department has reserved rooms for out of town visitors at the Radisson, and slightly cheaper rooms arè available at the Day's Inn, about 4 blocks away. The Symposium schedule is contained in this IMA Newsletter; it should be of interest to IMA participants. For more information concerning the Aris Academic Festival, contact

Ms. Teresa Bredahl
Department of Chemical Engineering and Materials Science
University of Minnesota
Minneapolis, MN 55455
612-624-5248

—

SCHEDULE FOR OCTOBER 13 - 29

Friday, October 13

The Aris Symposium talks today are in Amundson Hall B75

9:30 am	**Ted Davis & Lanny Schmidt** University of Minnesota	Welcome and Introduction
9:40 am	**George Gavalas** Cal Tech	Reaction and diffusion in porous solids: A case of intentional pore plugging
10:30 am		Break
11:00 am	**Mort Denn** UC Berkeley	Flow of anisotropic fluids

****

IMA SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	**Avner Friedman** IMA	Mathematical problems in industry in Japan

Abstract: The talk will survey some of the mathematical problems presented to me last summer when I visited the Japanese companies: Hitachi, Fujitsu, Kao, Toyota, and ATR Telecommunications. I will also draw comparisons with similar visits to U.S. industries.

The SEMINAR meets in Vincent Hall 570.

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1:30 pm	Ken Keller University of Minnesota	Introduction
1:30 pm	Harmon Ray University of Wisconsin	Some adventures in polymerization reaction engineering
2:30 pm		Break
3:00 pm	Thomas Clayton Classics, University of Minnesota	Hypertextuality and Shakespear's most artistic success

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Workshop on PATTERNS AND DYNAMICS IN REACTIVE MEDIA

October 16–20, 1989

Organizers: R. Aris, D. Aronson, S.-N. Chow, P. Fife, M. Golubitsky, R. McGehee, G. R. Sell and H. Swinney

Monday, October 16

The scheduled talks today are in Conference Hall EE/CS 3-180

9:00 am	Registration and coffee hour	Reception Room EE/CS 3-176
9:30 am	Martin Feinberg University of Rochester	Some recent results in chemical reaction network theory

Abstract: The aim of chemical reaction network theory is to draw connections between reaction network structure and qualitative properties of the corresponding differential equations. Some recent results will be discussed, in particular those relating to the possibility of multiple steady states in complex isothermal CSTRs, to mechanism discrimination in heterogeneous catalysis, and to the possibility of travelling composition waves on isothermal catalyst surfaces.

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	John Guckenheimer Cornell University	Computational tools for multiparameter bifurcation problems

Abstract: This lecture will discuss the numerical calculation of bifurcation diagrams for multiparameter bifurcation problems. Emphasis will be placed upon global bifurcations accessible only through long time integrations or iterations of dynamical systems. An interactive computational environment for studying these problems will be described. Examples drawn from studies of coupled oscillators and the Hodgkin-Huxley equations will be described.

2:00 pm	Stephen Scott University of Leeds	The influence of self-heating on isothermal chemical oscillators

Abstract: Chemical reactions frequently evolve heat and proceed at rates that are sensitive functions of the local temperature. The effects of coupling self-heating with chemical feedback are investigated through an extension of the cubic autocatalator model for isothermal oscillations. Period-doubling routes to chaos, mixed-mode oscillations and thermal explosions are features of the extended model. Similar responses are also found with a linear, chemical feedback step.

4:00 pm	Vincent Hall 502 (The IMA Lounge)	IMA Tea (and more!)

—

Tuesday, October 17

The scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am	Yannis G. Kevrekides Princeton University	Scaling laws and pattern formation in coupled reaction-diffusion systems

Abstract. Coupled reaction-diffusion equations are known to exhibit a wealth of multiple coexisting stationary solution patterns as the characteristic length of the system grows. We describe and implement a technique which allows us, by studying only stationary solution branches at small system lengths, to predict the complex bifurcation structure occurring at large system lengths without actually computing this structure. This technique is applicable to arbitrary isothermal or nonisothermal systems of coupled reaction-diffusion equations with periodic or no-flux boundary conditions. We use a number of standard literature examples (like the Brussellator, and the Catalano-Eilbeck scheme) to demonstrate the use of this technique. This provides a compact way of describing and comparing stationary pattern formation in a large class of systems, extending beyond coupled reaction-diffusion equations. We demonstrate this by comparing stationary patterns for our test reaction-diffusion systems with patterns in thin film flow. We also discuss some computational aspects of performing these bifurcation calculations, in particular the use of approximate inertial forms.

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	Gregory S. Yablonskii Tubinskii Complex Dept.	A kinetic polynomial: New concept of chemical kinetics

Abstract: We found for linear mechanisms, by methods of graph theory, a structured form of kinetic equation. In the general case a system of pseudo-steady-state equations for catalytic reactions can always be reduced to a polynomial in terms of the steady-state reaction rate, this is called a kinetic polynomial. The coefficients of this polynomial are polynomials in the parameters of the elementary reaction rates. The form of lowest coefficient of the polynomial ensures the thermodynamic validity of this form of kinetic model. This leads to a standard form of the dynamics of reaction and is used for the analysis of hysteresis and bifurcations.

Joint Chemical Engineering Colloquium/IMA Talk

1:15 pm	Dan Luss University of Houston	Spatial and spatiotemporal structures on catalytic surfaces

Abstract: Symmetry breaking can lead to the occurrence of structured solutions in many systems. For example, when a chemical reaction is carried out on heated catalytic ribbons or unheated catalytic disks nonuniform temperature patterns and waves are observed. This can lead to rather intricate dependence of the overall rate on the operation conditions and to a disguise of the intrinsic kinetic behavior.

Simplified mathematical models are used to study the features of the system. They predict rather intricate dynamic behavior and point out the need to gain a better understanding and prediction of mode interaction in diffusion reaction problems.

3:00 pm	J.L. Hudson University of Virginia	Dynamics of some electrochemical reactions

Abstract: Experiments on a few electrochemical reactions are discussed. Time series of either current or voltage, obtained under potentiostatic or galvanostatic conditions respectively, are presented and characterized. We first show some examples of dynamic behavior such as chaos, quasiperiodicity, and period doubling of tori obtained during the electrodissolution of copper. Some apparent higher order chaos is then discussed.

Finally, we discuss briefly some dynamics of an electrodeposition reaction and the electrocatalytic reduction of hydrogen peroxide.

Wednesday, October 18

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 pm	Christopher Jones University of Maryland	Tracking invariant manifolds with differential forms.

Abstract: To find heteroclinic or homoclinic orbits in a fast-slow system requires understanding how the unstable (or stable) manifold of a critical point behaves as it passes near a slow manifold of the system. In joint work with Nancy Kopell, we use the induced flow on the space of exterior forms to determine the transfer of information that occurs during this passage. Using the product of this analysis and transversality information from the fast flow, the relevant orbits can be determined. For the case of travelling wave solutions of some PDE the information is also relevant to stability.

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	M. I. Vishik Moscow State University	Uniform asymptotics of solutions of evolution equations depending on parameters

Abstract: Trajectories of evolutionary equations (for example reaction diffusion equations, damped hyperbolic equations) regularly or singularly depending on a parameter are considered. The principal term of asymptotic behaviour for all $t \geq 0$ with respect to the parameter is constructed. The asymptotic trajectories are piecewise continuous functions of t and except for the first continuous piece they lie on unstable manifolds of the equilibria. These manifolds correspond to the limit value of the parameter. The estimation of the difference between the trajectories and their asymptotics is uniform with respect to t and the initial data.

2:00 pm	Joel Smoller University of Michigan	Bifurcation from symmetry

Abstract: We give a new general bifurcation theorem in the presence of symmetry and apply it to find asymmetric (non-radial) solutions of

$$\Delta u + f(u) = 0 \quad \text{on} \quad \Omega = n\text{–ball}$$

$$\alpha u - \beta \frac{du}{du} = 0 \quad \text{on} \quad \partial\Omega,$$

having various symmetry properties.

3:00 pm	Ehud Meron Weizmann Institute	Wavefront interactions and their effects on pattern formation in excitable media

Abstract: The nonlinear interactions of solitary wavefronts in excitable media are determined by the manner of recovery to the rest state. The distance between a pair of wavefronts tends to lock at one of countably many possible values in the case of oscillatory recovery, while it increases indefinitely when the recovery is monotonic, We derive these results from the basic reaction diffusion equations and study the implications on pattern formation in one and two space dimensions. In particular we demonstrate how spatiotemporal complexity may arise in one dimension, and discuss possible consequences of the interplay between wavefront-interactions and curvature in two dimensions.

Thursday, October 19

All talks today, except the Colloquium, are in Conference Hall EE/CS 3-180

9:30 am	Yasumasa Nishiura Hiroshima University	Singular limit approach to the existence of curved fronts in phase field model

Abstract: We consider the instability of planar solidification wave advancing in a supercooled liquid in a channel. A phase field model is employed in which surface tension and dynamic cooling effects are automatically incorporated. The interface described by this model has finite thickness of order E, where E is a small microscopic parameter. Using, what is called, the SLEP method, we obtain a bifurcation of curved fronts from planar one for small positive E when the channel width and latent heat paramoters vary. This could be regarded as an onset of more complicated patterns such as dendrite.

10:30 am	Coffee break	Reception Room EE/CS 3-176

11:00 am	Robert Franzosa University of Maine	A Computational approach to connection matrices and the algebraic transition theory

Abstract: A discussion of the connection matrix theory from a computational viewpoint, and recent results about detecting connections via the algebraic transition theory.

2:00 pm	W. W. Farr Worchester Polytechnic University	Rotating chemical waves in the Gray-Scott model

Abstract: We consider a model set of reaction-diffusion equations defined on a circle and investigate the stability of rotating wave solutions formed via Hopf bifurcations which break the circular (O(2)) symmetry. Stable rotating waves have been observed in experiments with the Belousov-Zhabotinskii reaction. Using hybrid numerical/analytical techniques we perform center manifold/normal form reductions to analyze symmetry-breaking Hopf bifurcations, degenerate Hopf bifurcations, and Hopf-Hopf mode interactions. We find that the behavior of this model can be very complex, including two and three frequency motions, but that stable rotating waves exist over broad ranges of parameter values. (Joint work with Martin Golubitsky.)

Joint School of Mathematics Colloquium/IMA talk

3:15 pm	**Harry Swinney** University of Texas at Austin	Experiments on bifurcations in chemical spatiotemporal patterns

Abstract: Experiments have been conducted on several different spatially extended one-and two- dimensional reactors that have been designed for studies of bifurcations in reaction-diffusion systems. The reactors can be maintained indefinitely in well-defined nonequilibrium states. The sequences of bifurcation that are observed as a function of a control parameter (chemical concentration) are in qualitative accord with those found for a simple reaction-diffusion model system with two species. The model provides insight into the observed behavior.

The Colloquium will take place in Vincent Hall 16
Tea will be served at 2:55 pm in Vincent Hall 120.

Friday, October 20

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am	**Jeffrey Derby** University of Minnesota	Dynamics in materials processing: Outstanding problems in melt crystal growth

Abstract: We are entering an era in which chemical engineers and materials scientists are attempting to control structure in solids at the molecular level. This control requires detailed understanding of the dynamics of the processes which are used to produce these materials. This seminar will address some outstanding issues in understanding the growth of large-dimension laser host crystals. We will present recent results from mathematical models of the Czochralski crystal growth process and emphasize current inadequacies in our methods for analysis of the dynamical behavior of these systems.

10:30 am	Coffee break	Reception Room EE/CS 3-176

11:00 am	Jack K. Hale Georgia Institute of Technology/IMA	Effect of domain variation on dynamics

Abstract: For reaction-diffusion systems (regular or irregular) we discuss how variations in the shape of the domain can influence the dynamics. Particular attention will be devoted to the case where the dimension of the perturbed domain is larger than the dimension of the unperturbed one at some points.

2:00 pm	Hiroshi Matano University of Tokyo/IMA	Blow up in nonlinear diffusion equations

Abstract: Much progress has been made in the past few years in the study of profile of solutions at the blow-up time. I will talk about the recent developments in this area. Related problems such as "quenching" will also be discussed.

Special Lecture-Demonstration

3:30 pm	David Lomen University of Arizona	Computer Enhanced Education at the University of Arizona

Abstract: Our overriding principle has been to use computers only when they improve the quality of education, We have identified a number of ways in which computers may be used to advantage for instructional purposes:

1. As an aid to the instructor during class,
2. As an aid to students in problem solving,
3. To encourage and facilitate student exploration,
4. To diagnose the areas in which a student needs remediation before proceeding with a course.

We will elaborate on these points and demonstrate some of the software we have developed. The focus will be on the first two years of college mathematics.

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Tuesday, October 24

SEMINAR IN { Reaction Dynamics / Vincent Hall 570 }

11:15 am	Peter Poláčik Comenius Univ., Bratislava/IMA	Simple and complicated dynamics in scalar semilinear parabolic equations

Abstract: The dynamics of scalar semilinear parabolic equations of second order will be discussed. Several results will be presented showing that while the typical behavior of trajectories is very simple (monotone convergence to equilibria), still one can find equations where complicated dynamics occurs.

SEMINAR IN { Mathematics / Physics / Vincent Hall 570 }

2:15 pm	Debra Lewis Cornell/IMA	Nonlinear stability of homogeneous elastic bodies

Abstract: A homogeneous elastic body (aka pseudo-rigid body) is an elastic body for which the deformation gradient is constant throughout the body. The configuration space of all such bodies can be identified with the matrix group GL(3). We consider the nonlinear stability of rotating relative equilibria within this class of elastic bodies. By means of the reduced energy momentum method, we obtain explicit stability conditions for several classes of materials, including Ciarlet-Geymonet and Mooney-Rivlin materials.

SEMINAR ON MATHEMATICAL VISUALIZATION

3:30 pm	Igor Rivin Wolfram Research	MATHEMATICA and graphics

This talk will be held in Vincent Hall 16

Wednesday, October 25

SEMINAR IN { Reaction Dynamics / Vincent Hall 570 }

11:15 am	Carlos Rocha Lisbon	Attractors for the scalar bistable reaction diffusion equation

Abstract: We consider the problem of classification for the attractors of the equation $u_t = u_{xx}+f(u)$, $x \in [0,1]$ with Neumann boundary conditions and compare with the cases where $f = f(x,u)$ and $f = f(x,u,u_x)$.

IMA POSTDOC SEMINAR

3:10 pm	Mary Lou Zeeman MIT/IMA	An introduction to monotone systems

Abstract: We shall discuss, at an introductory level, some of the ideas, results and techniques of monotone systems - mainly those developed by M.W. Hirsch in his series of papers: Systems of Differential Equations that are Competitive or Cooperative I - VI.

Thursday, October 26

SEMINAR ON INDUSTRIAL PROBLEMS: UNDERGRADUATE COMPONENT

10:10 am	Walter Littman University of Minnesota	Open only to selected undergraduate participants

Background: A group of a half dozen undergraduates are meeting on a regular basis with Professor W. Littman in Vincent Hall 570 in conjunction with the Industrial Problem Seminar. One of the aims of this activity is to bridge the gap between the knowledge acquired in regular course work and the know-how necessary to attack problems as they actually appear in industry. Another aim is to channel the enthusiasm for computing that many students have into productive scientific channels.

SEMINAR IN { Dynamics and Mechanics / Vincent Hall 570 }

11:15 am	Debra Lewis Cornell/IMA	The reduced energy-momentum method: Nonlinear stability of simple mechanical systems

Abstract: A simple mechanical system consists of a canonical phase space, with configuration and momentum variables (q,p), a Lie group G which acts on the phase space by canonical transformations, and a G-invariant Hamiltonian $H(q,p) = V(q)+1/2|p|^2$. Many classical mechanical systems fall into this category. The reduced energy-momentum method is an energy method for the nonlinear stability analysis of such systems. This method has a number of computational advantages, including the following: the energy functional (Smale's amended potential) is defined on the configuration space, rather than the full phase space; the configuration variations are decomposed into infinitesimal group motions and 'internal' variations, which decouple with respect to the energy functional; all momentum constraints are automatically incorporated in the variations, resulting in apparently sharp stability conditions.

SEMINAR ON FLOER HOMOLOGY

2:00 pm	Richard Moeckel, Organizer

Abstract: The goal of this SEMINAR is to understand Floer's approach to the problem of finding periodic orbits of Hamiltonian systems. We will work through several survey papers as well as some of Floer's. The

SEMINAR will meet weekly as long as there is sufficient interest. Details about each session will be provided in due course.

The SEMINAR meets in Vincent Hall 570.

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Friday, October 27

SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	Carl A. Nelson Honeywell	Fundamental problems in the theory of shaped-charge jets

Abstract: A shaped charge is a high-explosive device which implodes a metal-lined cavity and produces a high velocity jet. A brief overview of shaped-charge phenomena will be given. this will be followed by a discussion of the theory of Euler flows and their application to jets, and the complications arising from compressibility, unsteady flow, etc. Finally, some unsolved problems of practical interest will be presented.

The SEMINAR meets in Vincent Hall 570.

SPECIAL DEMONSTRATION

12:30 pm	E.J. Doedel Concordia University, Montreal	Tutorial on AUTO

Abstract: Doedel, the author of AUTO, will give an AUTO tutorial in the IMA Seminar Room (VH 570). AUTO is a state of the art program for tracing bifurcations. For more (advance) information, see Don Aronson. AUTO is available on the Apollo network.

SPECIAL LECTURE

3:15 pm	E.J. Doedel Concordia University, Montreal	Coupled Josephson Junctions: The video!

The SPECIAL LECTURE will be in Vincent Hall 570.

PATTERNS AND DYNAMICS IN REACTIVE MEDIA
EDITORS: HARRY SWINNEY, GUS ARIS AND DON ARONSON
IMA VOLUMES IN MATHEMATICS AND ITS APPLICATIONS
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IMA NEWSLETTER #149

October 30 - November 17, 1989

1989-90 Program
DYNAMICAL SYSTEMS AND THEIR APPLICATIONS

NEWS AND NOTES

Motorola Joins IMA Participating Corporations

Motorola became an IMA Participating Corporation, effective October 16. This brings the number of PC's to 9.

Workshop on DYNAMICAL ISSUES IN COMBUSTION THEORY

November 13–17, 1989
Organizers: S.-N. Chow, P. Fife, M. Golubitsky, Amable Liñán, R. McGehee, G. R. Sell and F. Williams

The aim of the workshop is to cross fertilize research groups working in topics of current interest in combustion dynamics and mathematical methods applicable thereto.

Specific topics to be covered include: detonation instabilities, renormalization methods, complex chemistry, flame front instabilities, dynamics of diffusion flames, pattern formation in combustion, and ignition events.

The lectures will be held in Conference Hall 3-180 on the entry floor of the new Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

The detailed Workshop schedule, and the regular IMA program, can be found in the pages to follow.

SCHEDULE FOR OCT 30 - NOV 17

Tuesday, October 31

IMA SEMINAR IN { Foundations of Dynamical Systems, Vincent Hall 570 }

11:15 am	A.V. Babin Moscow Inst. of Transportation	Asymptotics as $t \to +\infty$ of the solutions of nonlinear evolution equations

Abstract: For solutions $u(t)$ of different types of partial differential equations and systems the finite-dimensional asymptotics $\bar{u}(t)$ are constructed. Local results are applicable to a variety of problems including Navier-Stokes equations, reaction -diffusion systems etc. For systems having a Lyapunov functional global results are obtained. For semilinear damped wave equations global analytic linearization is executed.

SEMINAR IN { Mathematics, Physics, Vincent Hall 570 }

2:15 pm	Richard James University of Minnesota	Structure of domains in ferromagnetic and shape-memory materials

Abstract: We will discuss recent progress on models for predicting domain structure. The general idea is to develop models with no a priori geometric restrictions.

4:15 pm	Vincent Hall 570	IMA Halloween Party

Wednesday, November 1

SEMINAR IN { Reaction Dynamics, Vincent Hall 570 }

11:15 am	A.N. Sharkovsky Ukrainian Academie of Sciences	Chaotic behavior one dimensional maps

Abstract: There are a few notions which characterize chaotic behavior for dynamical systems, (e.g., topological entropy, structure of family of w-limit sets, existence of "good" invariant measures, chaos in sense of Li-Yorke, existence of "a lot of" unstable trajectories etc). We discuss the inter-connections of some of these notions with other notions of topological dynamics in the case of one dimensional maps.

IMA POSTDOC SEMINAR

3:10 pm	Matthew Stafford Loyola University/IMA	Markov partitions for expanding maps of the circle

The POSTDOC SEMINAR will be held in Vincent Hall 570

PDE SEMINAR

4:10 pm	Wei Ming Ni University of Minnesota	The shape of least-energy solutions of a semilinear Neumann problem in pattern formation

The SEMINAR will be held in Vincent Hall 206

Thursday, November 2

SEMINAR IN { Dynamics and Mechanics
Vincent Hall 570

11:15 am	Mary Lou Zeeman MIT/IMA	Competitive Three-Dimensional Lotka-Volterra Systems

Abstract: We study the space of Lotka-Volterra systems modelling three mutually competing species, each of which, in isolation, would exhibit logistic growth. A theorem of M. W. Hirsch says that the compact limit sets of such a system lie on a unique invariant hypersurface. We use this to generalise to three dimensions the geometric analysis of the classical two-dimensional case, thereby defining a simple equivalence relation on the space, whose equivalence classes reflect the dynamic behaviour at the equilibria. We then refine the geometric analysis to investigate the occurrence of Hopf bifurcations within these classes.

SEMINAR ON FLOER HOMOLOGY

2:00 pm	Richard Moeckel, Organizer

Abstract: The goal of this SEMINAR is to understand Floer's approach to the problem of finding periodic orbits of Hamiltonian systems. We will work through several survey papers as well as some of Floer's. The SEMINAR will meet weekly as long as there is sufficient interest. Details about each session will be provided in due course.

The Seminar meets in Vincent Hall 570.

School of Mathematics Colloquium

3:15 pm	Mathew Grayson UCSD and Geometry Supercomputer Group	Curve shortening

The Colloquium will take place in Vincent Hall 16
Tea will be served at 2:55 pm in Vincent Hall 120.

—

Friday, November 3

IMA SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	David K. Misemer 3M	Mathematical modeling of dielectric waveguides for optoelectronic applications

Abstract: Devices that incorporate optical dielectric waveguides in their design now play a significant role in communications and signal processing. Optical fibers used in telephone communications are a familiar example of passive waveguides, but research activity is growing in the area of devices that actively switch or modulate light. The increasing interest in optical waveguides and the increasing complexity of waveguide devices have stimulated the development of mathematical tools for waveguide analysis. One set of important but difficult problems is characterizing propagation in inhomogeneous structures, i.e. waveguide structures that vary in the direction of propagation. After a brief review of the fundamental phenomena and principles governing waveguide behavior, several examples of inhomogeneous structures will be given. the strengths and limitations of one method for calculating inhomogeneous propagation, the Beam Propagation Method, will be discussed.

The SEMINAR meets in Vincent Hall 570.

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Monday, November 6

SEMINAR IN { Reaction Dynamics
Vincent Hall 570

11:15 am	**William Langford** University of Guelph	Pattern formation, bistability and modulated rotating waves in the Taylor-Couette problem

Abstract: We study interactions of stationary and Hopf bifurcations in the presence of O(2) symmetry. This phenomenon occurs in the Taylor-Couette experiment on flow between counterrctating cylinders. Here there is a bicritical point where the onset of instabilities to Taylor vortex flow (a steady-state bifurcation) and spiral vortex flow (a Hopf bifurcation) meet. The nonlinear mode interactions near such bicritical points are analysed, taking careful account of the effects of symmetry in the problem. A Liapunov-Schmidt reduction from the full Navier-Stokes equations for the Taylor-Couette problem has been performed. This analysis predicts a variety of stable flow patterns and transitions, including wavy vortices, bistability, hysteresis, and several quasiperiodic flows including a modulated rotating wave solution, which is however unstable in the Taylor-Couette case. These qualitative predictions are confirmed by recent experimental work.

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Tuesday, November 7

SEMINAR IN { Reaction Dynamics
Vincent Hall 570

11:15 am	**George Sell** University of Minnesota/IMA	Approximation dynamics: Inertial manifolds and hyperbolic sets

Abstract: There are three objectives in this paper. First we present a general theory of approximate inertial manifolds (AIMs) for nonlinear dissipative dynamical systems on infinite dimensional Hilbert spaces. The goal of this theory is to prove the Basic Theorem of Approximation Dynamics, wherein we show that there is a fundamental connection between the order of the approximating manifold and the amount of long-time dynamical information which is preserved by the approximation. The second objective is to present a new general method for the construction of an AIM. Thirdly we show that this construction applies to the Navier-Stokes equations on any bounded region in 2D (and on certain thin 3D regions) as well as to reaction diffusion equations in any space dimension. All these equations have good AIMs which preserve the essential dynamics of the global attractor.

SEMINAR ON MATHEMATICAL VISUALIZATION

3:30 pm	**Mark Phillips** University of Maryland	Complex hyperbolic geometry, interactive computer graphics, and Mathematica

Abstract: Complex hyperbolic geometry is a field which is rich with intriguing examples and yet about which surprisingly little is known. This talk will describe a project which combines interactive 3-dimensional computer graphics with a high-level computational language using Mathematica in an effort to see and understand geometric phenomena in complex hyperbolic space. The talk will include a brief introduction to complex hyperbolic space, and will discuss ways in which computer graphics pictures have led to research results. The techniques of this project could be useful in investigating and developing intuition in other types of geometries as well.

The organizer for the SEMINAR is Charlie Gunn of the Geometry Supercomputer Project.

This talk will be held in Vincent Hall 570

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Wednesday, November 8

SEMINAR IN { Reaction Dynamics
Vincent Hall 570

11:15 am	Yasumasa Nishiura Hiroshima University	Singular limit approach to the dynamics of interface in reaction diffusion systems

Abstract: Sharp transition layers frequently appear in the dissipative systems such as chemical reaction, solidification, morphogenesis, and so on. We introduce what is called the SLEP method which is useful to study systematically the stability and bifurcation of such layered solutions. Then we apply this method to various problems: stability of traveling waves, instability of planar fronts, layer oscillation (breathers), and homoclinic bifurcation.

IMA POSTDOC SEMINAR

3:10 pm	**Maciej Krupa** University of Houston/IMA	Coupled oscillators with symmetry and applications to arrays of Josephson junctions

Abstract: We discuss dynamics of a system of coupled oscillators with S_n symmetry derived in the study of large series arrays of Josephson junctions. The following topics will be included: Primary bifurcation curves. Analysis of period doubling bifurcations. Numerical simulations. Dynamics found by simulation theory, and believed to exist.

PDE SEMINAR

4:10 pm	**M. I. Vishik** Moscow State University	Attractors and semigroups depending on a parameter

The SEMINAR will be held in Vincent Hall 206

Thursday, November 9

SEMINAR ON INDUSTRIAL PROBLEMS: UNDERGRADUATE COMPONENT

10:10 am	**Walter Littman** University of Minnesota	Open only to selected undergraduate participants

Background: A group of a half dozen undergraduates are meeting on a regular basis with Professor Littman in Vincent Hall 570 in conjunction with the Industrial Problem Seminar. One of the aims of this activity is to bridge the gap between the knowledge acquired in regular course work and the know-how necessary to attack problems as they actually appear in industry. Another aim is to channel the enthusiasm for computing that many students have into productive scientific channels.

SEMINAR IN { Dynamics and Mechanics
Vincent Hall 570

11:15 am	**Mary Silber** UC Berkeley/IMA	The Faraday experiment in square geometry

Abstract: In the Faraday experiment a container of fluid is oscillated in the vertical. This produces standing surface waves with frequency half that of the forcing. We analyze the experimental results, obtained by Simonelli and Gollub for a fluid held in a container of square cross–section, in terms of a D_4–equivariant stroboscopic map. The parametric instability is treated as a period–doubling bifurcation of the trivial fixed-point of the map. The small amplitude experimental results are understood in terms of a codimension–three D_4–equivariant bifurcation problem.

SEMINAR ON FLOER HOMOLOGY

2:00 pm — Richard Moeckel, Organizer

Abstract: The goal of this SEMINAR is to understand Floer's approach to the problem of finding periodic orbits of Hamiltonian systems. We will work through several survey papers as well as some of Floer's. The SEMINAR will meet weekly as long as there is sufficient interest. Details about each session will be provided in due course.

The Seminar meets in Vincent Hall 570.

SEMINAR IN { Reaction Dynamics
Vincent Hall 570

3:10 pm	Bruce Clarke University of Alberta	Geometry of chemical processes in reaction network dynamics

Abstract: When the rate constants of a chemical reaction network take all non-negative values, the possible kinds of dynamics can be understood in terms of chemical processes called "extreme currents". These processes have a geometrical interpretation as the vertices of a high dimensional convex "current polytope". When the dynamics has an attractor of dimension k, the important processes usually lie on a face of the current polytope with at most $k+1$ dimensions, and involve at most $k+2$ extreme currents. This principle has been used to eliminate unimportant reactions from complex chemical mechanisms.

Global flows can be understood from the geometry of another polytope called the "kinetic polytope" which is associated with each extreme current. This approach gives a clear picture of how chaos arises in a simple network.

A program allows you to sketch a network with a mouse, view the current polytope, see the unstable regions in parameter space, simulate the dynamics, and determine the important processes.

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Friday, November 10

IMA SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	Jeffrey Saltzman Los Alamos Nat. Lab.	A linear diffusion with a nonlinear boundary condition from rock porosity experiments

Abstract: A series of experiments measuring rock porosity using microacoustical noise has been performed. A simple linear diffusion model along with a nonlinear boundary condition along an outflow boundary is derived. The solution of the boundary value problem is discussed and the effects of the nonlinear outflow condition are compared with a closed system solution.

The SEMINAR meets in Vincent Hall 570.

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Workshop on DYNAMICAL ISSUES IN COMBUSTION THEORY

November 13–17, 1989

Organizers: S.-N. Chow, P. Fife, M. Golubitsky, Amable Liñán, R. McGehee, G. R. Sell and F. Williams

Monday, November 13

The scheduled talks today are in Conference Hall EE/CS 3-180

9:00 am	**Registration and coffee hour**	Reception Room EE/CS 3-176

9:30 am	**J.W. Dold** University of Bristol	Asymptotic laminar triple-flamelet structures in non-premixed turbulence

Abstract: A laminar triple-flamelet in strained and sheared flow is taken to represent the local structure of the end of a turbulent diffusion flame that has been extinguished (usually by the flow) in some region but not in another. The most important feature of such a flamelet is its ability to propagate. This greatly enriches the usual one-dimensional picture of an infinite laminar diffusion flamelet in uniformly strained flow that extinguishes everywhere at some critical strain-rate $\sigma = \sigma_c$. Purely asymptotic arguments, based on a large Zeldovich number $\beta \gg 1$, show that the triple-flamelet reaches an infinite negative propagation speed (propagation of an extinction front) at this strain-rate. The propagation speed only becomes positive (and much slower) below a relatively small strain-rate $\sigma < \sigma_0 = O(\sigma_c/\beta)$. Thus a turbulent diffusion flame that is punctured at some moment, by having $\sigma > \sigma_c$ locally, pops like a balloon—a hole rapidly opens throughout the region in which $\sigma > \sigma_0$ on the diffusion flame.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176

11:00 am	**David Wagner** University of Houston	Detonation and deflagration wave solutions to reacting compressible Navier Stokes equations

Abstract: This lecture concerns the existence and behavior of detonation and deflagration wave solutions to reacting compressible Navier Stokes equations, and various special limits of these solutions (ZND, high activation energy, etc .).

2:00 pm	**D.S. Stewart** University of Illinois, Urbana	Discrete modeling of beds of propellant subjected to strong stimulus

Abstract: This talk describes a theoretical investigation of violent burning processes in beds of propellant when subjected to strong stimulus. The point of view taken will consider the discrete character of the bed and calculate and experimentally characterize the particle-particle interactions within the bed. A principal goal is to find the propagation and failure mechanism for the propagation of a self-sustaining reactive wave. The motivation for the research is to aid in the design of low vulnerability propellant. As far as possible, quantities in the theory and experiment will relate directly to the properties of the propellant that can be changed in the manufacturing process.

The structure of the theory leads to recursion relations which predict certain dynamics and the stability of certain steady states within the context of these recursion relations will be discussed.

4:00 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

—

Tuesday, November 14

The scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am	Rupert Klein Princeton University	On the dynamics of weakly curved detonations

Abstract: Extensive experimental evidence from studies of gaseous explosions and condensed phase combustion calls for a theoretical description of the dynamics of curved shock-reaction waves. Typical radii of curvature of such fronts are large compared to an average reaction zone thickness. This implies weak surface curvature on the reaction zone length scale and allows an approach using the tool of asymptotic expansions.

We first consider diverging waves evolving on the timescale given by Huyghens' principle for a front moving at nearly the Chapman-Jouguet velocity. The ansatz of a quasi steady and quasi-onedimensional reaction zone structure yields an Eigenvalue problem for the front normal speed D parametrized by the local mean surface curvature κ. Its solution provides a relation $D = D(\kappa)$ which determines the evolution of the front given appropriate initial data. We emphasize the crucial influence of the chemical model on the curvature-speed relation and, in particular, address the effect of backward reactions.

The quasisteady theory fails for focussing waves, i.e. for negative curvature. An inherently unsteady wave acceleration occurs and two potential limit situations must be distinguished. The wave either retains its quasi-onedimensional structure but quickly leaves the near-CJ regime, or transverse pressure gradients eventually enforce nonnegligible tangential flow divergence so that multidimensional effects enter the picture, still under near-CJ conditions. For both cases, we outline the modifications needed to maintain asymptotic description.

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	Amable Liñán Madrid	Ignition fronts of diffusion planes
2:00 pm	B. Larrouturou INRIA	On some mathematical results on planar and curved premixed flames

Abstract: We present a set of rigorous mathematical results obtained in collaboration with several colleagues (H. Berestycki, R. Ferretti, J.M. Roquejoffre) concerning the propagation of planar or curved flames in an infinite cylindrical tube.

We first consider planar flames in the framework of the isobaric approximation, and investigate in detail the behaviour of unsteady solutions when the reaction term is initially small but non zero in the fresh gases; in other words, we present a rigorous analysis of the well-known cold boundary difficulty.

Then, we present some error estimates for bounded domain approximations of the planar traveling front solutions; these estimates are of interest for the numerical approximation of these solutions.

Lastly, we consider curved flames in the framework of the constant-density (thermo-diffusive) approximation, in the presence of a non uniform velocity field, and present mathematical results for both cases of a unit Lewis number and of a non unit Lewis number.

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Wednesday, November 15

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 pm	Martine Marion Ecole Centrale de Lyon	Attractors and turbulence for some models of combustion

Abstract: There is a close relationship between the number of degrees of freedom describing fluid flow and the fractal dimension of the Navier-Stokes attractor. Hence studying attractors leads to a new insight in turbulence theory.

The aim of this lecture is to extend this approach to some models of combustion. We consider two problems: a model in incompressible fluid with a Boussinesq approximation and a model in solid combustion. We prove the existence of a universal attractor and derive estimates of its dimension.

·10:30 am	Coffee Break	Reception Room EE/CS 3-176

11:00 am	Bernard J. Matkowsky Northwestern University	Bifurcation, pattern formation and chaos in combustion

Abstract: We employ analytical and numerical methods to describe solutions of both gaseous and solid fuel (condensed phase) combustion.

In gaseous fuel combustion we consider a flame stabilized on a line source of fuel. A basic solution describes a smooth cylindrical flame front, separating burned from unburned gases. By varying critica' parameters of the problem, we construct additional solutions on branches which bifurcate from the basic solution. These solutions exhibit both spatial and temporal patterns, which become more and more complex as the distance from the bifurcation point is increased. They describe e.g. (i) cylindrical flames which oscillate (both sinusoidal and relaxation oscillations) about a mean position given by the stationary cylindrical flame front, (ii) stationary cellular flames, and (iii) oscillatory cellular flames, which describe both traveling waves about, and standing waves about, and standing waves on the cylindrical front.

In condensed phase combustion we consider a reaction fron (so-called solid flame) propagating through a cylindrical sample. Such problems describe the process of self propagating high temperature synthesis (SHS). Ignition of a cylindrical sample at one end, sends a thermal wave through the sample, converting unburned reactants to solid products. We describe various modes of propagation, which have been experimentally observed, as bifurcations from a basic solution corresponding to a uniformly propagating planar front. These include (i) oscillatory combustion - in which a planar front propagates with an oscillatory velocity, (ii) spinning combustion - in which a hot spot (or spots) moves in a helical fashion along the surface of the sample, and (iii) multiple point combustion - in which the hot spot appears, disappears, and reappears repeatedly. Finally we predict additional modes of propagation, including intermittent and chaotic combustion, exhibiting yet more complex behavior, which have not yet been experimentally observed.

2:00 pm	Michael Gorman University of Houston	Periodicities and deterministic chaos in laminar premixed flames

Abstract: Our experiments on flat, laminar premixed flames have been conducted on circular, square, annular and linear burners housed in a glass chamber in which the ambient pressure can be controlled. Some of the dynamic modes of flame front propagation we have found are: For a circular burner, an axial mode in which the entire flame front moves alont the axis perpendicular to the burner surface, a radial mode in which the flame front changes its radial extent, a drumhead mode in which parts of the flame front vibrate perpendicular to the burner surface (with some modes having spatial characteristics similar to those of a circular drumhead), a doubly periodic spiral mode in which the flame front is a point which moves as a rotating spiral and a pinwheel mode in which the flame front becomes 3 luminous spots which rotate around the center of the burner like a pinwheel. In another region of parameter space the flame front physically flutes forming cusps and folds which demark the boundaries of cells. In this talk we will show that both pulsating flames and cellular flames have nonperiodic modes of propagation which are described by deterministic chaos. We will present measurements of the spatial and temporal properties of the periodic and the chaotic states. We will show videotape of all these modes.

3:00 pm	Victor Roytburd Rensselaer Polytechnic Institute	Dynamics of unstable detonations: Numerics and asymptotics

Abstract: In a broad range of regimes traveling detonation waves develop instabilities – pulsating instabilities in case of one-dimensional waves. A highly efficient numerical method has been recently applied for the direct simulation of this phenomenon. We discuss this method involving front tracking and adaptive mesh refinement. We compare the method with other methods on the test problem of unstable detonations. Numerical results reveal a very interesting dynamic behavior which can be interpreted in the framework of an asymptotic theory. The results are obtained in a joint work with A. Bourlioux and A. Majda.

Thursday, November 16

The morning talks today are in Conference Hall EE/CS 3-180

9:30 am	Michael Frankel Indiana U.-Purdue U.	On surface dynamics generated by free boundary problems associated with combustion

Abstract: We introduce a number of free boundary problems whose boundary dynamics mimics that of flame fronts, and generates either cellular-chaotic or oscillatory-spinning behavior. The behavior of free boundaries can be modeled by coordinate-free surface dynamics equations expressed in terms of local geometrical characteristics. The desirability of numerical and rigorous study of these models is discussed.

10:30 am	Coffee break	Reception Room EE/CS 3-176
11:00 am	M. R. Baer Sandia National Labs.	Reactive waves in granulated energetic materials

Abstract: The combustion of granulated energetic materials, such as porous explosives or damaged propellants, involves the coupling of a variety of thermal, chemical and mechanical processes. Accelerated combustion is greatly influenced by the granular microstructure, and combustion-induced compaction preconditions the granular reactant prior to the onset of flame spread. It is now known that this mechanical process is the key that links the modes of multiphase combustion from deflagration to detonation.

Guided by experimental observations, we have developed a multiphase mixture model to describe flame spread and the growth to detonation in granular materials. This description treats each phase as fully compressible and in thermodynamic nonequilibrium. Model closure is achieved using a rate-dependent evolution description of solid volume fraction. Since the treatment of volume fraction as an independent kinematic variable has not been well studied, we focus on dynamic compaction issues.

We will describe our studies of compaction waves and discuss model improvements motivated by recent experiments. Numerical calculations of low-velocity impact experiments reveal details of compaction and combustion waves near the threshold of deflagration-to-detonation transition (DDT).

We then present a study of the mathematical structure of the reactive multiphase equations. A characteristics analysis has revealed the hyperbolic nature of the field equations when dissipative effects are neglected. Sonic conditions appear as singularities in this wave description when characteristics coalesce. Pore collapse during dynamic compaction may play a critical role in producing these sonic flow conditions.

2:00 pm	John D. Buckmaster University of Illinois, Urbana	Activation energy asymptotics and detonation stability

Abstract: Activation energy asymptotics has proven to be of enormous value in unraveling the mysteries of low Mach number combustion. It has been applied to the problem of detonation stability (a high Mach number application) with mixed results. These will be reviewed.

This talk will be held in Conference Hall EE/CS 3-180

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Joint School of Mathematics Colloquium/IMA talk

3:15 pm	**Andrew Majda** Princeton University	Unstable detonations and dynamic homogenization for condensed phases

This talk will be held in Vincent Hall 16.

Tea will be served at 2:55 pm in the Mathematics Commons Room Vincent Hall 120.

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WORKSHOP BANQUET

A buffet dinner will be held at the Campus Club, Coffman Union. Wine-and-cheese starts at 5:45 pm and dinner at 6.30 pm. Details about signing up for the dinner will be disseminated at the Workshop. The banquet speaker is D. R. Kassoy.

approx. 7:30 pm	David R. Kassoy University of Colorado, Boulder	Confessions of the singularly perturbed professor

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Friday, November 17

All scheduled talks today are in Conference Hall EE/CS 3-180

9:30 am	F.A. Williams UC San Diego	Reduced kinetic mechanisms for complex combustion chemistry

Abstract: Recent studies by asymptotic methods are considered, in which complex chemistry involving hundreds of elementary steps is reduced systematically to a few steps that can be used for calculating flame structures. Revisions in structures of premixed flames and diffusion flames, implied by the reduced mechanisms, will be indicated. These results help to clarify the role that has been played by activation-energy asymptotics and to point toward future trends in asymptotic methods for flames.

10:30 am	Coffee break	Reception Room EE/CS 3-176

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IMA SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	J. Allen Cox Honeywell	Applications and modeling of diffractive optical elements

Abstract: Diffractive optics technology provides a new approach to optical design and fabrication using computer-aided design tools and integrated circuit manufacturing methods. With this approach, one creates an efficient holographic element in the form of a surface relief profile etched into an optical substrate. The appropriate interference fringe pattern is created on the surface with high resolution lithography.

An accurate mathematical model of diffractive elements is essential for design development and performance prediction. A rigorous mathematical description is provided by solutions to Maxwell's equations in R^3 using the surface relief profile with complex optical constants as the boundary condition. Several groups have examined this problem in detail for the case of singly or doubly periodic surface patterns.

Applications and modeling of periodic devices will first be reviewed. The need to model more complex contours and to extend the theory to include nonlinear coupling is then discussed with additional examples.

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2:00 pm	A.N. Sharkovsky Ukraine Academie of Science	Space-time chaos and self-stochasticity in deterministic systems

Abstract: The phenomenon, which we call self-stochasticity, can take place in mathematical patterns of deterministic distributed systems. It consists of the fact that the system behavior is described exactly by stochastic laws if time is large asymptotically.

DYNAMICAL ISSUES IN COMBUSTION THEORY
EDITORS: P. FIFE, A. LIÑAN AND F.A. WILLIAMS
IMA VOLUMES IN MATHEMATICS AND ITS APPLICATIONS
PUBL: SPRINGER-VERLAG VOLUME # 35

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DYNAMICAL THEORIES OF TURBULENCE IN FLUID FLOWS
EDITORS: C. FOIAS AND G.R. SELL

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PAPERS RECEIVED TO DATE:

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IMA NEWSLETTER #162

May 25 - June 10, 1990

1989-90 Program
DYNAMICAL SYSTEMS AND THEIR APPLICATIONS

Workshop on DYNAMICAL THEORIES OF TURBULENCE IN FLUID FLOWS

May 29–June 2, 1990
Organizers: S.-N. Chow, C. Foias, M. Golubitsky, R. McGehee, G. R. Sell and R. Temam

The Workshop will treat the following topics: experimental evidence for dynamics in fluids, modeling of fluids with dynamical systems, statistical theory of fluids, modeling with random dynamics (renormalization and decimation), computational fluid dynamics, Lagrangian turbulence, attractors for fluid flows, and approximate inertial manifolds.

Workshop on NONLINEAR PHENOMENA IN ATMOSPHERIC & OCEANIC SCIENCES

June 4–8, 1990
(jointly with the Minnesota Supercomputer Institute)
Organizers: S.-N. Chow, G. F. Carnevale, M. Golubitsky, R McGehee, R. Pierrehumbert and G. R. Sell

The study of the earth's atmosphere and oceans has received tremendous stimulus in the past decade. The development of powerful methods for analyzing nonlinear dynamical systems, numerical simulations at unprecedented resolution, and a wealth of new observations have been contributing factors. This workshop seeks to promote cross-fertilization of ideas between investigators working in these diverse fields.

SCHEDULE FOR MAY 25 - JUN 10

Friday, May 25

SEMINAR IN { Foundations of Dynamics
Vincent Hall 570

11:15 am	Ciprian Foias Indiana University/IMA	Connections between the global attractor of the 2D Navier-Stokes equations and turbulence theory, I

Abstract: A presentation of the inertial range concept in developed turbulence theory in connection with the rigorous mathematical theory of the Navier-Stokes equations and the rigorously established properties of the global attractor of those equations. The relevance of approximate inertial manifolds to this topic will also be discussed.

SEMINAR IN { Foundations of Dynamics
Vincent Hall 570

1:25 pm	Ciprian Foias Indiana University/IMA	Connections between the global attractor of the 2D Navier-Stokes equations and turbulence theory, II

Abstract: See the abstract for the first talk.

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Monday, May 28

Memorial Day is a University holiday. University offices and the main IMA Office will be closed.

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Workshop on DYNAMICAL THEORIES OF TURBULENCE IN FLUID FLOWS

May 29–June 2, 1990

Organizers: S.-N. Chow, C. Foias, M. Golubitsky, R. McGehee, G. R. Sell and R. Temam

Most of the workshop talks will be held in Conference Hall 3-180 on the entry floor of the new Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

Tuesday, May 29

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

10:00 am	Registration and coffee	Reception Room EE/CS 3-176

10:30 am	**R. Temam** Univ. Paris-Sud	Inertial manifolds and multigrid methods

Abstract: In this lecture, two topics will be addressed:

-Inertial Manifolds, and their approximation

-The utilization of inertial manifolds in large scale scientific computing.

Inertial manifolds have proved to be a powerful tool of investigation for dynamical systems and an interesting approach to turbulence since an inertial manifold provides an interaction law between "short" and "long" waves.

When inertial manifolds do not exist (e.g. for reaction-diffusion equations in high space dimension or for damped wave equations), or when inertial manifolds are not known to exist (e.g. for the Navier-Stokes equations), an interesting substitute is the concept of Approximate Inertial Manifold (AIM). An AIM is a smooth finite dimensional manifold which attracts all orbits in a thin neighborhood; we expect it to reproduce the coarse structure of the attractor and to neglect the fine structure. Infinite sequences of AIM have been constructed which produce better and better approximations to the attractor. They can be constructed as graphs above a finite dimensional space or as more complicated analytic sets. The first part of the lecture will consist in a survey of the present status of Approximate Inertial Manifolds.

The construction of inertial manifolds and AIMs is mostly based on the utilization of spectral bases. This a serious restriction for large scale computing since, for all practical purposes, we are then limited to spectral and pseudo-spectral methods. In view of approximating fractal attractors in the context of finite difference methods, we have proposed a methodology similar but different from the Multigrid Methods which are classical in numerical analysis, using the concept of Incremental Unknowns. Inertial Manifolds and Multigrid Methods are two theories which have developed independently in parallel, during the last years. Inertial manifolds are motivated by the understanding of nonlinear problems. Multigrid methods are devoted to the approximation of linear and nonlinear, elliptic and parabolic problems, with an emphasize on linear elliptic problems. Although these theories seem far apart, they have some underlying ideas in common and this seems to be an interesting route for the utilization of inertial manifolds in large scale computing in the context of finite differences.

2:00 pm	**Philip Holmes** Cornell University	Low dimensional projections of a turbulent boundary layer

Abstract: In spite of enormous activity since 1970 in dynamical systems and fluid mechanics, and some notable successes in studies of closed systems such as Taylor-Couette flow and Rayleigh-Bénard convection, the ideas of Foias, Hopf, Landau, Ruelle-Takens et.al. on finite dimensional attractors and bifurations have proved remarkably difficult to apply to the fully developed turbulence characteristic of open flows. However, it now seems that a number of mathematical tools are available, and, while rigorous results are easier to come by in model problems such as the Ginzburg-Landau equation, these tools can guide our analyses of Navier Stokes in technologically important flow geometries.

In this talk I will describe recent efforts by Armbruster, Aubry, Berkooz, Guckenheimer, Lumley, Stone and myself to use the proper orthogonal or Karhunen-Loève decomposition to provide an optimal, low-dimensional subspace on to which to project the Navier-Stokes equations governing flow in the near-wall region of a turbulent boundary layer. The resulting dynamical systems (sets of 10-50 ODEs) inherit symmetries from the physical flow which prove crucial in determining dynamical behavior, including structurally stable heteroclinic cycles which lead to intermittency. I will discuss the constraints and limitations imposed upon the model by the projection process, and relate them to classical issues such as the closure problem in turbulence modelling.

I hope to show that, not only are ideas from dynamical systems useful in the analysis of the turbulence production mechanism, but that the physical problem and modelling process is a wonderful source of mathematical challenges. The lecture may consequently be richer in conjecture than rigor.

3:00 pm	**Peter Constantin** University of Chicago	Dirichlet quotients and the 2-D Navier-Stokes equations
4:00 pm	Vincent Hall 502 (The IMA Lounge)	IMA Tea (and more!)

Wednesday, May 30

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:15 am	Coffee Break	Reception Room EE/CS 3-176
9:30 am	George Sell University of Minnesota	Approximation dynamics: Hyperbolic sets and inertial manifolds

Abstract: There are three objectives in this paper. First we present a general theory of approximate inertial manifolds (AIMs) for nonlinear dissipative dynamical systems on infinite dimensional Hilbert spaces. The goal of this theory is to prove the Basic Theorem of Approximation Dynamics, wherein we show that there is a fundamental connection between the order of the approximating manifold and the amount of long-time dynamical information which is preserved by the approximation. The second objective is to present a new general method for the construction of an AIM. Thirdly we show that this construction applies to the Navier-Stokes equations on any bounded region in 2D (and on certain thin 3D regions) as well as to reaction diffusion equations in any space dimension. All these equations have good AIMs which preserve the essential dynamics of the global attractor.

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	J. D. Gibbon Imperial College, London	On the possibility of soft & hard turbulence in the complex Ginzberg Landau Equation

Abstract: We present analytical methods which predict the occurrence of both soft (weak) and hard (strong) turbulence in the complex Ginzburg Landau (CGL) equation

$$A_t = RA + (1 + i\nu)\Delta A - (1 + i\mu)A|A|^2$$

on a periodic domain $[0,1]^D$ in D spatial dimensions. Hard turbulence is characterized by large fluctuations away from spatial and temporal averages with a cascade of energy to small scales. This form of hard turbulence appears to occur not in $1D$ but only in $2D$ and $3D$ in parameter regions which are bounded by hyperbolic curves in the 2nd & 4th quadrants of the $\mu - \nu$ planes where the system is modulationally unstable ($\epsilon = 1 + \mu\nu < 0$). This region goes out to the dissipationless limit ($\mu, \nu \to \mp\infty, \pm\infty$) where the CGL equation becomes the NLS equation. When $D \geq 2$ this latter equation blows up in finite time & it is clear from our results that this finite time singularity is fundamental in causing strong turbulent behaviour. The CGL equation has an attractor consisting of C^∞ functions for all finite μ & ν when $D = 1$ and 2. When $D = 3$ we have the same regularity in part of the $\mu - \nu$ plane which covers some of the predicted hard turbulent area. The CGL equation also possesses inertial manifolds when $D = 1$ and 2. Our results are based on a new method where we consider an infinity of Lyapunov functionals of rank $2n$

$$F_n = \int \{|\nabla^{n-1} A|^2 + \alpha_n |A|^{2n}\} dx \qquad \alpha_n > 0.$$

For large times and large R, upper bounds exist for the infinite set of $F_n' s$, constructed from the hierarchy of differential inequalities

$$\dot{F}_n \leq (2nR + c_n \|A\|_\infty^2) F_n - b_n F_n^2 / F_{n-1} \qquad c_n, b_n > 0,\ (F_0 = 1)$$

Estimates for the "bottom rung" F_2 give upper bounds for the whole ladder. Long time upper bounds on F_2 and $\|A\|_\infty^2$ (& hence all F_n) are well controlled in the soft region but become much larger in the hard region, whereas spatial and temporal averages remain comparatively small. When the nonlinearity is $A|A|^{2q}$, the critical case $qD = 2$ gives parallel results. (This work was done in conjunction with M. Bartuccelli, P. Constantin, C. Doering & M. Gisselfult).

2:00 pm	**W.G. Pritchard** Penn State University	Rimming flows

Abstract: A small fraction (of the order of 10%) of the volume of a circular cylinder is filled with a very viscous liquid. The cylinder is rotated at a steady speed about its axis, which is carefully set to be nearly horizontal. At very small and at very .arge rotational speeds the flows are found to be steady, but at intermediate speeds these steady flows lose stability to an interesting collection of unsteady motions, the details of which will be described.

3:30 pm	**Akerman Hall 130D**	Tour of Dan Joseph's laboratory

We will show experiments on water lubricated pipelining with bamboo waves and corkscrew waves, two dimensional cusped interfaces which violate Laplace's law, rollers, drafting, kissing and tumbling of fluidized particles, non linear stabilization of fingering instabilities in porous media, drag reduction using riblets. The tour probably will last 30 or 40 minutes.

Thursday, May 31

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:15 am	**Coffee Break**	Reception Room EE/CS 3-176
9:30 am	**Robert Kraichnan** Los Alamos Nat. Labs.	Dynamical stochastic modeling of turbulence

Abstract: Fully developed Navier-Stokes turbulence has attracted advances from some of the exciting methods of contemporary nonlinear dynamics, but it has proved quite resistant. The basic reason is that an irreducibly large number of modes are excited. In this situation, it is unimportant whether the attractor is strange or merely complicated. The difference is undetectable and, moreover, the system shows most of its interesting features long before it has time to explore the attractor.

One tool that allows some progress is dynamical stochastic modeling, by which I mean the exact solution of model dynamical systems that have some relation to the true dynamics. The fact that a system is solved exactly ensures some important consistency properties: The results may be inaccurate or wrong, but they do not blow up and are not ridiculous. Dynamical stochastic models that have intrinsic randomicity have given qualitatively and, in some cases, quantitatively good approximations to major features of fully developed turbulence, including sensitivity to initial conditions, eddy viscosity, spectral energy cascade and vortex intensification.

Most of this talk is devoted to a new kind of dynamical stochastic modeling based on nonlinear mapping of Gaussian fields to dynamically evolving non-Gaussian fields. The resulting approximations are systematic and non-perturbative. They yield intermittency phenomena in a natural way and the predictions to date are excellently supported by simulations. These results suggest that the intermittency of dissipation that is so characteristic of turbulence is essentially independent of Reynolds number and does not arise from a fractal cascade process.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Andrew Majda** Princeton University	The Mathematical theory of renormalization for turbulent transport
2:00 pm	**Basil Nicolaenko** Arizona State University	Homoclinic chaos and vorticity bursts in periodic Navier-Stokes flows

Abstract: For the two-dimensional Kolmogorov flow an enhanced transport phenomenon occurs intermittently that is related to symmetry-breaking heteroclinic orbits. This is the two-dimensional flow of a viscous liquid

induced by a unidirectional force periodic in one of the coordinates. For this flow, the generalized system of small-scale eddies turns out to be unstable to long-wave instabilities.

For the Kolmogorov flow, we find dynamical regimes for *Re* up to $O(150)$ that are characterized by sparsely distributed bursts in time. The most striking feature of this transition is that the bursts generate substantial spatial disorder and drive developed turbulence. The bursts generate a high degree of stochasticity and a large amount of enstrophy, while the flow remains fairly organized in the mean time outside the bursts, with large scale eddies dominating the dynamics. We show that heteroclinic orbits connect some equivariant hyperbolic states under symmetry groups of the equations. The homoclinic cycle is the sum of individual heteroclinic connections. Such equivariant states are associated with large scale vortices. Indeed, the intermittent chaotic (turbulent) behavior of the Kolmogorov flow is deeply connected with its groups of symmetries and related symmetry breakings. Similar results are found for three-dimensional Arnold-Beltrami-Childress flows.

3:00 pm	**G. Minea** Increst, Bucarest	Loss of stability of the globally unique steady-state equilibrium and the bifurcation of closed orbits in a class of Navier-Stokes type dynamical systems

Abstract: We characterize the injective Navier-Stokes type operators $Au+B(u,u)$ in R^3 and also the injective operators whose linearization at each point has the spectrum in the right open half plane, such that for each right hand side f the unique stationary solution of the equation $\dot{u}+Au+B(u,u)=f$ be stable.

It follows that in the class of injective operators the (contrary) case is generic. For an injective operator in generic position, we study the evolution of the attraction basin of the stationary solution, the Hopf bifurcation and the stability of the bifurcating closed orbits.

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Friday, June 1

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:15 am	**Coffee Break**	Reception Room EE/CS 3-176
9:30 am	**Geneviève Raugel** Université Paris-Sud, Orsay	Navier-Stokes equations on thin three-dimensional domains

Abstract: We study the Navier-Stokes (NS) equations on a thin three dimensional domain $\Omega_\varepsilon = Q_2 \times (0,\varepsilon)$, where Q_2 is an adequate bounded domain in $\mathbf{R}^2$ and ε is a small positive parameter. We consider these equations especially with periodic bounded conditions. We show that there are *large sets* $R(\varepsilon)$ in $H^2(\Omega_\varepsilon)$ and $\varphi(\varepsilon)$ in $W^{1,\infty}((0,\infty);L^2(\Omega_\varepsilon))$ such that if the initial data U_0 is in $R(\varepsilon)$ and the forcing term F in $\varphi(\varepsilon)$, then (NS) has a strong solution $U(t)$ which remains in $H^1(\Omega_\varepsilon)$ for all $t \geq 0$ and in $H^2(\Omega_\varepsilon)$ for all $t > 0$. We show that (NS) has a local attractor A_ε in $H^2(\Omega_\varepsilon)$. Under reasonable hypotheses, A_ε is upper semicontinuous at $\varepsilon = 0$.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Edgar Knobloch** UC Berkeley	Finite-dimensional description of doubly diffusive convection

Abstract: Finite-difference techniques show that two-dimensional doubly diffusive convection can, in appropriate circumstances, take the form of chaotic standing oscillations. The behavior is believed to be due to the Shil'nikov mechanism. When the wavelength of the oscillations is small, asymptotic techniques can be used to derive a third order system of ordinary differential equations which can be shown rigorously to describe chaotic oscillations due to this mechanism.

1:25 pm	**V. P. Maslov** MIEM, USSR	On a new type of turbulence for incompressible magnetohydrodynamics

Abstract: The behaviour of charged incompressible liquid in a cylinder or in a torus is considered for large hydrodynamic and magnetic Reynolds numbers Re in the case of special initial conditions rapidly oscillating under the action of perturbing fluctuations $\approx \varepsilon$ (ε is a new small parameter) along the radius. If $Re \gg \frac{1}{\varepsilon^2}$, the problem becomes unstable. In this situation the appearing averaged turbulent flow depends essentially on the amplitude of perturbing fluctuations.

Aerospace Engineering and Mechanics Colloquium

2:30 pm	Jerry Gollub Haverford College	Interfacial Waves: Nonlinear Dynamics and Transport

The COLLOQUIUM meets in Akerman Hall 225

PROBABILITY SEMINAR

3:35 pm	**Andrzej Lasota** Silesian U., Poland/IMA	Stochastically perturbed dynamical systems

The SEMINAR meets in Vincent Hall 207

Saturday, June 2

The scheduled talks today are in Conference Hall EE/CS 3-180

9:15 am	**Coffee Break**	Reception Room EE/CS 3-176
9:30 am	**Yannis G. Kevrekidis** Princeton University	Low-dimensional behavior and some approaches to numerical bifurcation calculations

Abstract: Fluid flow phenomena and PDEs used to model them often exhibit temporally complicated but spatially coherent, low-dimensional dynamic behavior. This suggests that the long-term dynamics of such infinite-dimensional systems may live on a low-dimensional manifold in phase space. Knowing such a manifold would allow the detailed stability and bifurcation analysis of the system dynamics using a small set of ODEs, but with the same accuracy as large-scale, more traditional discretizations. For some systems, the theory of inertial manifolds provides the proof of the existence of such a manifold, and also motivates its numerical approximation. In other cases, especially in complex geometries, a similar model reduction can be obtained using "empirical" eigenfunction expansions (proper orthogonal decomposition). It is also possible to try and understand the qualitative nature of certain bifurcations exploiting empirical models, obtained through signal processing of experimental time-series. We will discuss some illustrative "reduced model" bifurcation calculations using these approaches.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Ciprian Foias** Indiana University/IMA	Some open problems in connecting dynamical systems to fluid dynamics

Abstract: In 1987 I presented at the Boulder AMS workshop on infinite dimensional dynamical systems some old and some new open problems concerning the functional dynamical systems associated to the Navier-Stokes equations and to other related partial differential equations. I shall review those problems and discuss their present status as well as their relevance to conventional turbulence theory.

Noon	**Workshop Picnic**	The picnic will be held in the courtyard between Lind Hall and the EE/CSci Building

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Workshop on
NONLINEAR PHENOMENA
IN
ATMOSPHERIC & OCEANIC SCIENCES

June 4–8, 1990

(jointly with the Minnesota Supercomputer Institute)

Organizers: S.-N. Chow, G. F. Carnevale, M. Golubitsky, R McGehee, R. Pierrehumbert and G. R. Sell

Most of the workshop talks will be held in Conference Hall 3-180 on the entry floor of the new Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

In addition to the scheduled lectures, there will be opportunities during the workshop for a number of informal, impromptu presentations. Participants are encouraged to bring along material they might want to present.

Also, on June 5 a video session will be held at the Minnesota Supercomputer Institute. Participants with interesting visuals, in the form of videotapes or online IRIS or MacintoshII movies, are encouraged to bring them. (An IRIS can read cartridge tapes in standard UNIX tar format. It is also possible to read magnetic reel tapes in UNIX tar format on the IMA Apollo network, and then transfer the files to an IRIS over the campus network.) If you wish to contribute to the video session, please let our staff know in advance, so that we can have the right equipment available.

For those who want to send data along electronically for online movies, we offer anonymous ftp access on a Sun SparcStation. Type

```
ftp 128.101.152.11
```

to connect. Log in as "anonymous" and give your e-mail address as the password. You should now be logged in. Type

```
cd pub
```

to get into the public directory. At this point, you can use the "cd" command to get into a subdirectory. To transfer <file> back to your home machine, type

```
get <file>
```

For more information, type

```
man ftp
```

from a UNIX shell, or use the "help" command while in ftp.

Monday, June 4

Introduction and Statistical Questions

The talks today are in Conference Hall EE/CS 3-180

9:00 am	Registration and coffee	Reception Room EE/CS 3-176
9:30 am	R. T. Pierrehumbert University of Chicago	Introduction and overview
10:15 am	R. T. Pierrehumbert University of Chicago	On the enstrophy cascading subrange in two dimensional turbulence

Abstract: There has been a seemingly endless controversy surrounding the nature of the shortwave (enstrophy cascading) spectrum in two dimensional fluid turbulence. Classical scaling arguments predict a universal k^{-3} energy spectrum (k^{-1} enstrophy spectrum). Some numerical experiments appear to verify this argument, while others yield steeper (typically k^{-2} enstrophy and k^{-4} energy) spectra. The steep spectra are commonly ascribed to the "freezing out" of well separated intense vortices, which interact rarely and hence block further enstrophy cascade. We review the arguments leading to the classical k^{-3} spectrum, and some salient features of numerical experiments leading to steeper spectra.

The question is of more than academic interest. The shape of the spectrum affects predictability decay of large scale atmospheric motions, and determines the level of truncation that can be tolerated in atmospheric models without severely distorting the large scale flow. It is crucial to the prospects for parametrizing enstrophy dissipation by unresolved scales of motion. It is also central to the understanding of "negative viscosity" effects, whereby large eddies maintain themselves by acquiring the energy of small eddies.

The interpretation of the spectra is facilitated by recasting the problem in geometric terms. We show by elementary arguments that, if the fractal (actually correlation) dimension of the support of strong vorticity is d, then the isotropic 2D enstrophy spectrum is $k^{(1-d)}$. Since $0 < d < 2$, the k^{-1} classical spectrum is as steep as it can be and still permit self similar vorticity structures at all scales. Steeper spectra do not have enough power at small scales to permit self-similarity. This is consistent with the appearance of coherent vorticies with preferred scales in such flows.

The chaotic mixing of a passive tracer by a specified time dependent large scale flow provides a convenient model problem for probing the physics of the situation. We present an analysis of mixing and resulting spectra in an example involving vacillating large scale Rossby waves. The chaotic mixing is seen to produce a k^{-1} concentration variance spectrum. Now, potential vorticity is a tracer, though not a passive tracer. In order to determine the extent to which the properties of passive tracer mixing carry over to vorticity mixing, we carried out a series of fully nonlinear integrations with a high resolution spectral model of the barotropic vorticity equation. This shows that if the small scale vorticity is not too strong, it indeed mixes in a fashion qualitatively similar to a passive tracer (i.e. it forms filaments). The fully nonlinear model is also used to probe what happens as the initial small eddy perturbation is made progressively stronger, and ultimately persistent small vortices emerge. An interesting sidelight of this is that the energy lost to the small eddies as they are distorted is immediately gained by the large eddy. This provides a robust mechanism for large eddy maintainence.

The bottom line is that the presence of an energetic large eddy can dominate the enstrophy evolution down to arbitrarily small scales, making nonlocality a central fact of life in 2D turbulence and not a small correction to a basically local picture. Conversely, if the initial condition contains a small eddy sufficiently intense to dominate the strain due to the large eddy, the large eddy will never be able to tear it apart and cascade its enstrophy to smaller scales. Taken together, these properties cast doubt on the existence of a universal spectrum independent of initial conditions and the nature of the large scale forcing.

11:00 am	Coffee Break	Reception Room EE/CS 3-176
11:30 am	Theodore G. Shepherd University of Toronto	Arnol'd stability applied to fluid flow: Successes and failures

Abstract: V.I. Arnol'd's hydrodynamical stability theorems, and their various extensions, provide rigorous bounds on the growth of disturbance norms. These bounds apply to *large*-amplitude disturbances, and are not based in any way on small-amplitude expansions. Because the theory is exact, the choice of a stable basic flow is completely arbitrary, and this arbitrariness may be used to seek the tightest constraint on a given initial condition. For example, given an unstable flow (plus a small disturbance), one may consider this initial condition as a finite-amplitude disturbance to some stable basic flow, and then invoke the nonlinear stability theorem to derive a bound on the saturation amplitude of the instability.

Arnol'd's theorems have, to date, proven quite powerful in the context of two-dimensional Euler flow and three-dimensional baroclinic quasi-geostrophic flow, two systems of great interest in geophysical fluid dynamics. Yet they have failed to achieve any result whatsoever for stratified flows supporting gravity waves; this despite the fact that experience has shown that such flows can indeed be stable. My talk will present the origin of Arnol'd stability theorems, in terms of the symmetries underlying the Hamiltonian structure of the problem, and then discuss the various successes and failures alluded to above.

2:00 pm	William Young Scripps Institution	Statistics of agglomeration
4:00 pm	Vincent Hall 502 (The IMA Lounge)	IMA Tea (and more!)

Tuesday, June 5 (at Minnesota Supercomputer Institute)

2-D Turbulence: Pure and Geostrophic

The talks today will be held in Room 3125 (and 3155) of the Minnesota Supercomputer Center, 1200 Washington Avenue South, Minneapolis. The IMA will provide a Transit Coach to pick people up on Union Street (along side the EE/CS Building). The bus will start boarding at 8:45 am, depart for MSI at 9:00 am, and arrive at MSI by 9:15 am.

The (free) University Campus Bus #13A can be used to reach MSI. It leaves Fraser Hall at 8:35 am (and 9:35, 10:35, etc.), and arrives at MSI at 8:39 am. The last return #13A bus to leave MSI is at 4:40 pm. (Buses leave from MSI at 40 after the hour from 8:40 am to 4:40 pm.)

You can also walk from the IMA to MSI. The distance is approximately one mile.

9:15 am	Coffee Break	
9:30 am	James C. McWilliams NCAR	The search for simplified models of two-dimensional, structured turbulence

Abstract: Two-dimensional, structured turbulence is characterized by the emergence of coherent vortices whose dynamics is nearly conservative when well separated and strongly dissipative during close approaches. By modeling coherent vortices as discrete entities with a small number of internal degrees of freedom it is hoped that one can obtain a reduced description of turbulence providing both physical insight and increased computational efficiency. The simplest model of the nearly conservative motion of well separated coherent vortices is a collection of point vortices, a system whose dynamics is described by a non-integrable Hamiltonian. The chaotic dynamics of point vortices displays many complex phenomena which may be important for structured turbulence; these phenomena include vortex pairing, chaotic scattering, an exponential distribution of times between close approaches, and non-ergodic behavior. Comparison of the dissipative scattering of coherent vortices and the complex but conservative scattering of point vortices is one of several approaches that should yield insight into the construction of a dissipative vortex model with few internal degrees of freedom that can provide an elemental basis for two-dimensional, structured turbulence. (Joint work with Jeffrey B. Weiss.)

10:30 am	Coffee Break

11:00 am	**Bernard Legras** Ecole Normale Supérieure, Paris	Coherent eddies and the elliptical model of two-dimensional vortex dynamics

Abstract: It is now widely recognized that coherent vortices are the dominant feature of a very broad class of incompressible two-dimensional flows with high Reynolds number.

Descriptions based on Fourier modes do not provide a suitable framework for the study of coherent vortices. The Lagrangian formalism of contour dynamics is a well-adapted tool to describe the evolution of nested patches of vorticity but still requires a discretization with many points even for simple vortex shapes. A comparison has been conducted between contour dynamics and pseudo-spectral code has been conducted on the evolution of single vortex disturbed by a shear or a non-axisymmetric deformation. Both methods are used at very high resolution. The result is a remarkable agreement between the two methods for very large times and even when a small number of contours is used to discretize a continuous profile.

In order to obtain a simple representation of the dynamics of large-scale structures within two-dimensional flows, we have developed an approximate model of non-axisymmetric vortices in which each vortex is assumed to consist of a nested stack of elliptical regions of uniform vorticity. In the basic model, the part of the motion preserving the elliptical shape is exactly retained and higher-order deformations are neglected. Hence we obtain a set of coupled nonlinear ordinary differential equations for the aspect ratio, orientation and centroid of the bounding contour of each region. The model admits an Hamiltonian formulation and possesses all the invariants of the original equations.

Numerical tests that will be presented in this seminar and in the joint seminar of D. Dritschel show that the elliptical model is a very good representation of the full equations in finding the equilibrium contour shapes of a multi-contour family of vortices, the linear stability of this family, the equilibrium, non-elliptical shapes of two co-rotating vortex patches, and the interaction between two symmetrical vortex patches, including merging.

We hope to present also new results based on the extension of the model to quasi-geostrophic equations and finite third-order deformations. (Joint work with D. Dritschel.)

Noon	**Lunch Break**	Lunch in the MSI Reception Area provided for registrants
1:00 pm	**Video session**	Participant-supplied visuals, in the form of videotapes or online IRIS or Macintosh II movies, will be shown.
2:30 pm	**David Dritschel** University of Cambridge	To be announced

Abstract: This talk concerns joint work with Bernard Legras and is mentioned in the abstract of Legras.

3:00 pm	**Lee Panetta** Texas A&M University	Stable zonal jets in baroclinic β-plane turbulence
3:30 pm	**Geoffrey Vallis** UC Santa Cruz	Two-dimensional turbulence and beta-plane dynamics

Abstract: Two-dimensional turbulence on the beta-plane, although a simple model, contains many of the dynamical features believed important in determining the large scale structure of the atmosphere and ocean. We will first review the salient features of the classical phenomenology of two-dimensional inertial ranges and the effects of differential rotation, including simple closures. We will then present a review of results from a wide range of direct numerical simulations in order to delineate the broad dependence of flow type on forcing parameters. For most parameter values the energy spectra are markedly steeper than the classical enstrophy inertial range prediction, and although such spectra can be produced, the regime is not robust and the Kolmogorov constant not universal. Long-lived, coherent vortices form in many cases, accompanied by a steep energy spectra and a higher than Gaussian kurtosis. If the forcing is confined to low wavenumbers and is such as to introduce anisotropy, then the anistropy is seen to persist at all wavenumbers, indicative of a spectrally non-local enstrophy 'cascade'.

With the addition of differential rotation, a small number of fairly distinct flow regimes are observed. Coherent vortices weaken and finally disappear as the strength of the the beta-effect increases, concurrent with increased anisotropy and decreased kurtosis. Even in the absence of coherent vortices and with a Gaussian value of the kurtosis, the spectra remain relatively steep, although not always as steep as for the non-rotating cases.

For those simulations that are forced at relatively high wavenumbers, a well resolved, universal and very robust $k^{-5/3}$ energy inertial range is observed, its low wavenumber extent being effectively limited by the beta-effect. If forcing is introduced at two-scales i.e. at both low and high wavenumbers (crudely mimicking the effects of both baroclinic and convective forcing in the atmosphere) then an upscale energy cascade and downscale enstrophy transfer can simultaneously co-exist, with no necessity for a sink mechanism in between.

Since coherent structures appear to be a ubiquitous feature in turbulence, we may present a natural method for their construction which exploits the dynamics of the equations of motion themselves.

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Wednesday, June 6

Dynamical Systems Analysis

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:15 am	**Coffee Break**	Reception Room EE/CS 3-176
9:30 am	**Henry D.I. Abarbanel** UC San Diego	Classifying and predicting for chaos

Abstract: In chaotic systems prediction is limited by the fundamental instabilites of the nonlinear dynamics. These instabilities also cause nonperiodic orbits which preclude using Fourier analysis to classify the dynamics. Using time domain methods relying on phase space information about neighbors, we are able to classify these systems by their invariants and to build predictive models which are as accurate as the instabilities allow. These ideas will be discussed both in general and within the context of geophysical applications.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Tom Warn** McGill University	Quasi-geostrophy - generalizations and limitations

Abstract: A disparity between the frequencies of the physical processes which excite large scale atmospheric and oceanic motion and those of high frequency inertial-gravity waves seem to produce motion which is close to but departs from geostrophy. By formally exploiting the smallness of the departures from geostrophy one is led to the quasi-geostrophic equations to leading order. Attempts to extend these notions to higher order will be reviewed as will the related concepts of the "slow manifold", normal mode initialization, and intermediate models. Inadequacies in our current understanding and certain implications for numerical weather prediction will also be discussed.

2:00 pm	**S. Wiggins** Caltech	Chaotic advection in a Rayleigh-Bénard flow

Abstract: We consider the problem of transport of a passive tracer in the time-dependent flow corresponding to a Rayleigh number R slightly above the R_t at the onset of even oscillatory instability for Rayleigh-Bénard convection rolls. The flow can be modelled by a stream function, which can be split into a time independent part plus a time dependent perturbation. The motion of fluid particles can therefore be regarded as the flow for a near integrable, "one-and a half" degree of freedom Hamiltonian system. In absence of molecular diffusivity, the evolution of a certain region of phase space can thus be viewed as the motion of a dyed part of fluid. The most important objects for a theory of transport are the invariant manifolds for the Poincaré map of the flow homoclinic to fixed points, which physically correspond to the stagnation points. As fluid particles cannot cross invariant lines, these curves constitute a sort of "template" for their motion. We

use segments of stable and unstable manifold to define the time dependent analogue of the roll boundaries. Transport of fluid across a boundary can then be attributed to the way a region bounded by segments of stable and unstable manifold, or "lobe", is evolving under map iterations. This and symmetry considerations allow us to write explicit formulae for describing the tracer transport in terms of iterations of one of these lobes. By using symmetries we derive analytically a lower and upper bound for the first time tracer invades a roll, and a lower bound on the stretching of the interface between dyed and clear fluid. These bounds show that the number of invaded rolls grows linearly in time while the stretching is exponential. These results are independent of the fact that the perturbation is small. When this is the case however, the analytical tools of the Melnikov and subharmonic Melnikov functions are available, so that an approximation to the lobe areas and location and size of the island bands can be determined analytically. In agreement with the experimental and numerical findings of Solomon and Gollub, we show that the amount of fluid transported across a roll boundary grows linearly with the amplitude of the perturbation (which is porportional to $(R - R_t)^{\frac{1}{2}}$) and is independent of wavelength of the roll pattern. Furthermore, the Melnikov analysis shows dependence of transport on the period of the oscillation and this suggests an effect for which no experimental verification is currently available. The presence of molecular diffusivity introduces a (long) time scale into the problem. We discuss the applicability of the theory in this situation, by introducing a simple rule for determining when the effects of diffusivity are negligible, and perform numerical simulations of the flow in this case to provide an example. (Joint work with R. Camassa.)

3:00 pm	J. S. Allen Oregon State University	Chaos in a model of forced quasigeostrophic flow over topography: An application of Melnikov's method

Thursday, June 7

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:15 am	**Coffee Break**	Reception Room EE/CS 3-176
9:30 am	**Lorenzo M. Polvani** MIT	The dynamics of Neptune's Great Dark Spot: Chaotic mixing on a planetary scale

Abstract: The recent encounter of the Voyager 2 spacecraft with the planet Neptune, revealed the presence of a large vortex in its atmosphere, dubbed the Great Dark Spot. Its diameter is roughly 10,000 km, about the size of the Earth. Unlike Jupiter's Great Red Spot, this vortex displays dramatic variations in its shape. We compare the shape oscillations of the Great Dark Spot with those predicted by three simple classical models of atmospheric flows. We find that the oscillations of the Great Dark Spot are well reproduced by our models. Fits to the data yield new improved estimates for the values of the vorticity inside and the shear around the Great Dark Spot. Our models exhibit very large zones of deterministic chaotic Lagrangian trajectories surrounding this vortex, and thus suggest the presence of chaotic mixing on a planetary scale in the atmosphere of Neptune. (Joint work with Jack Wisdom.)

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**W. R. Peltier** University of Toronto	Breaking internal waves over localized topography

Abstract: "Breaking" occurs in the field of internal waves forced by stratified flow over localized topography when the characteristic Froude number exceeds a critical value. This critical value corresponds to the development of a region of reversed streamlines in the flow which, in the long wave limit, first occurs at a height $z_c = 3\lambda_z/4$ where λ_z is the vertical hydrostatic wavelength. Subsequent to the establishment of this critical condition, in complete nonlinear simulations of the evolution of the flow, an intense instability is triggered which, in terms of the wave drag on the obstacle, leads to an increase of this quantity by several hundred percent. The transition is explicable in terms of a linear supercritical instability of the nonlinear wave that has been extensively analysed by solving the associated two dimensional non-separable eigenvalue

problem. The maturation of this instability drives a strong interaction with the mean flow which transform it to a degree and in a way which is strongly suggestive that this interaction is at the heart of the severe downslope windstorms that are observed in nature. The intensity of the "storm" induced by wave breaking appears itself to be limited by a higher order instability of the transformed flow. this sequence of events will be discussed in detail using a video-animation of the phenomenon for illustrative purposes.

2:00 pm	Michael Ghil UCLA	Unstable limit cycles & climatic variability: From ice ages to the greenhouse effect
3:00 pm	Ka Kit Tung University of Washington	Wave saturation and mean flow equilibration in nonlinear baroclinic adjustment

Abstract: It is known that in the absence of heat transporting eddies, the north-south temperature gradient of the atmosphere would be much larger than what is observed. The observed temperature gradient is furthermore approximately constant in the course of a year, despite large differences in solar insolation. Seasonal differences in solar heating instead lead to differences in eddy heat flux. These properties of the atmospheric flow suggest the possibility of simple parameterizations for the eddies and mean flow in climate models.

Stone (1978) proposed a quasi-linear mechanism of baroclinic adjustment to address the overall effect of the eddies, on the meridional temperature gradient. He suggested that as external (radiative) driving is increased, the atmosphere becomes baroclinically unstable. The growing wave transports heat northward, thereby preventing the temperature gradient from becoming supercritical. Increasing external driving merely feeds more energy into the baroclinically unstable wave without increasing the mean temperature gradient.

The simple hypothesis of baroclinic adjustment was later criticized as being inappropriate in nonlinear settings, as supercritical temperature gradients were found from nonlinear model calculations. Different mechanisms for mean flow equilibration were thought to be effective in different parameter regimes.

We shall reexamine the concept of baroclinic adjustment in the context of a fully nonlinear two-layer model on a β-plane. Based on our results, we propose a single, conceptually very simple mechanism of the nonlinear equilibration of waves and the mean flow, which we term *nonlinear baroclinic adjustment*. The relevance of the concept to climate modeling will also be discussed. [Joint work with Priscilla Cehelsky.]

4-4:30 pm	I. M. Navon Florida State University	Finite-time blowup in finite-difference and finite- element models of the shallow water equations

Abstract: Energy-conserving finite-diffeence and finite-element models of the shallow-water (S-W) equations on a limited-area domain, are integrated for extended time periods. Finite-time blow-ups are observed due to nonlinear instabilities near the Nyquist "cut-off" and non-conservation of potential enstrophy in the discretized schemes. An analogy between 3-D flow (in the absence of formal conservation of potential enstrophy in the 2-D energy conserving models) and the S-W equations model yielding a net transfer of rotational energy towards higher wavenumbers is observed in the vicinity of the critical blow-up time, T_c. Discussion and comparison of the finite-difference and finite-element models is presented related to the critical dissipativity required to stabilize them beyond the critical time T_c. (See also Sadourny (1975)). A relation yielding $T_c \sim C(\Delta)Z_0^{-1/2}$, where $C(\Delta)$ is a constant depending on the mesh-size and increasing with a decreasing Δ, is verified for the models.

6:30 pm	**Workshop Dinner**	Dinner at Grandma's Restaurant

Friday, June 8

Dynamo

The talks today are in Conference Hall EE/CS 3-180

9:15 am	Coffee Break	Reception Room EE/CS 3-176

9:30 am	**Paul H. Roberts** UCLA	Dynamo theory

Abstract: Many astrophysical bodies possess magnetic fields. The Earth has one, the geomagnetic field, and it is apparently as old as the Earth. Since the temperature throughout most of the Earth is above the Curie points of all known materials, the Earth is not a permanent magnet. The geomagnetic field must be produced by electric currents flowing mainly within its core, which is a good electrical conductor. Lacking sources, such currents would die out in about 10,000 years. It is now believed that the currents are maintained by electromagnetic induction, as in a commercial dynamo, the moving parts of the geodynamo being motions in the outer core, which is fluid.

The first question in dynamo theory is whether self-excitation is possible in a body having such a low degree of symmetry as the Earth's core. This well-studied "kinematic dynamo problem" defines a nontrivial linear eigenvalue system, which will be discussed after the basic electrodynamic theory has been summarized. The second question is whether motions of sufficient vigor, and which have the "helicity" required for field generation, would arise naturally in the Earth's outer core. This nonlinear "magnetohydrodynamic ("MHD") dynamo problem" raises some interesting questions in rotating magnetoconvection. The third question is whether the required energy source is available to power the dynamo. For bodies such as the solar convection zone, energy is available in abundance, but the geodynamo's needs are less easily met.

During the course of lectures, other topics will be touched on, including fast dynamos, auto-reversing dynamos, chaotic dynamos and numerical dynamos. Dynamos in bodies other than the Earth will be briefly examined.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Bruce Bayly** University of Arizona	Infinitely conducting dynamos and other horrible eigenvalue problems

Abstract: The kinematic dynamo problem in the limit of infinite fluid conductivity leads to an eigenvalue problem of the form

$$\lambda f(T(x)) = A(x)f(x)$$

where T is a nonlinear transformation on a compact region and A is a matrix-valued function of position. Numerical simulations indicate that the "eigenfunctions" are singular over much of the domain, but estimates of the eigenvalues are extremely accurate with quite modest resolution. Some progress has been made in understanding this situation, but it is basically not understood.

Eigenproblems of this type also occur in spectral theory of Schrödinger operators, hydrodynamical stability theory, and the thermodynamic formalism of dynamical systems theory. In the latter case, we suggest a perhaps novel method for calculating quantities like topological entropy and other quasi-thermodynamic functions.

SPECIAL LECTURE

Noon	**Susan Friedlander** University of Illinois, Chicago	Inverse scattering and a Lax pair formulation for the Euler equation

Abstract: A Lagrangian description is presented for the two and three dimensional flow of an ideal fluid. A Lax pair equation is obtained for the evolution of a quadratic form from which the Lagrangian velocity can be reconstructed. Inverse scattering in 2-D is discussed and a Hamiltonian structure is exhibited.
Joint work with Misha M. Vishik.

NONLINEAR PHENOMENA IN ATMOSPHERIC AND OCEANIC SCIENCES

EDITORS: G.F. CARNEVALE AND R.T. PIERREHUMBERT

IMA VOLUMES IN MATHEMATICS AND ITS APPLICATIONS

PUBL: SPRINGER-VERLAG VOLUME # 40

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IMA NEWSLETTER #163

June 11 - June 30, 1990

Workshop on CHAOTIC PROCESSES IN THE GEOLOGICAL SCIENCES

June 11–15, 1990

(jointly with the Minnesota Supercomputer Institute)

Organizers: S.-N. Chow, M. Golubitsky, R McGehee, G. R. Sell and D. A. Yuen

The goal of the geological sciences is to understand the earth well enough to construct detailed quantitative models of physical and chemical processes within the earth. Recently, concepts from dynamical systems and chaos have been used to derive a better understanding of the behavior of complex geological systems. The purpose of this workshop is to be arena for scientific exchanges between earth scientists and mathematical researchers, especially with experts in dynamical systems.

IMA Summer Program

RADAR AND SONAR

June 18 - June 29, 1990

Organizing Committee: Alberto Grunbaum (chairman), Marvin Bernfeld Richard E. Blahut, Richard Tolimieri

The program goal is to increase the interaction between mathematicians and electrical engineers in universities and mathematical scientists in industry working on significant problems in radar or sonar. The first week of the program is tutorial. The second week will be devoted to presentations of problems by scientists from industry or universities. The problems we are looking for should be of significance to the field of radar or sonar as well as containing mathematical issues.

SCHEDULE FOR JUNE 11 - 30

Workshop on CHAOTIC PROCESSES IN THE GEOLOGICAL SCIENCES

June 11–15, 1990

(jointly with the Minnesota Supercomputer Institute)

Organizers: S.-N. Chow, M. Golubitsky, R McGehee, G. R. Sell and D. A. Yuen

Most of the workshop talks will be held in Conference Hall 3-180 on the entry floor of the new Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

In addition to the scheduled lectures, there will be opportunities during the workshop for a number of informal, impromptu presentations. Participants are encouraged to bring along material they might want to present.

Also, on June 12 a video session will be held at the Minnesota Supercomputer Institute. Participants with interesting visuals, in the form of 16 mm. movies, videotapes or online SUN workstation, IRIS or MacintoshII movies, are encouraged to bring them. (An IRIS or SUN can read cartridge tapes in standard UNIX tar format. It is also possible to read magnetic reel tapes in UNIX tar format on the IMA APOLLO network, and then transfer the files to an IRIS or SUN over the campus network.) If you wish to contribute to the video session, please let our staff know in advance, so that we can have the right equipment available.

For those who want to send data along electronically for online movies, we offer anonymous ftp access on a Sun SparcStation. Type

```
ftp 128.101.152.11
```

to connect. Log in as "anonymous" and give your e-mail address as the password. You should now be logged in. Type

```
cd pub
```

to get into the public directory. At this point, you can use the "cd" command to get into a subdirectory. To transfer <file> back to your home machine, type

```
get <file>
```

Similarly, you can use the "put" to transfer a file to the IMA. For more information, type

```
man ftp
```

from a UNIX shell, or use the "help" command while in ftp.

Monday, June 11

Introduction and Mantle Convection, Mantle Rheology

The talks today are in Conference Hall EE/CS 3-180

9:00 am	Registration and coffee	Reception Room EE/CS 3-176
9:30 am	David A. Yuen University of Minnesota	Nonlinearities and chaotic processes in the geological sciences

Abstract: An introduction will be given on the various types of nonlinearities which arise in geological phenomena and how these give rise to interesting time-dependent phenomena, ranging from chaotic to turbulent regime The different phenomena and equations will be described to give a common background for the workshop We will also discuss the influences of geological material properties on generating interesting nonlinear time-dependent processes, which may or may not have counterparts in other physical disciplines.

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	W.R. Peltier University of Toronto	Mantle rheology and mantle convection

Abstract. The Earth is a radially stratified viscoelastic object which cools on a timescale determined by the efficiency of convective heat transport through its "solid" mantle. The Rayleigh number of this circulation depends upon mantle viscosity which is itself a strong function of temperature because of the thermally activated nature of the solid state creep process that governs mantle "flow". Mantle viscosity may be inferred on the basis of a number of different geophysical observations, most but not all connected to the response of the planet to the 10^5 yr cyclc of glaciation and deglaciation that has been a durable characteristic of hte geological record throughout the past 900,000 yrs of Earth history. The formal problem of inferring mantle viscosity from such data is a classical problem in geophysical inverse theory, although one that is somewhat exotic because of the nature of the data that must be inverted. Considerable progress in the development of suitable theory has recently been achieved that completes the partial analysis presented previously in Peltier (1976). This theory has now been applied to both synthetic and "real" data, allowing for the first time a detailed quantitative analysis of the inherent resolving power of the viscoelastic relaxation data of postglacial rebound and a detailed assessment of the extent to which the radial variation of mantle viscosity is constrained by these data. A Bayesian formulation of this inverse problem has proven to be especially advantageous. The results of these recent analyses will be discussed in detail and their implications with respect to the style of convection in the earth's mantle will be discussed.

2:00 pm	U. R. Christensen Max-Planck Inst.	Three-dimensional mantle convection with variable viscosity: Mantle plumes and toroidal excitation

Abstract: We report numerical calculations for three-dimensional convection with variable viscosity. A hybrid spectral and finite difference method is used. The coupling of modes in the equation of motion, which is caused by lateral viscosity variations, is treated iteratively. solutions for bimodal, hexagonal, square, triangular and spoke patterns are reported for bottom heated convection at infinite Prandtl number. The Rayleigh number is between critical and 10^5 and temperature-induced viscosity contrasts up to 100 are considered. In agreement with results from laboratory experiments we find that temperature-dependent viscosity favours at low Rayleigh number flow patterns like squares or hexagons, where a columnar rising current is surrounded by sheet like descending flow. The dichotomy in geometry between upwelling and sinking flow becomes more pronounced with increasing viscosity contrast. The temperature-dependence of viscosity gives rise to a toroidal velocity component. However, it amounts only to a few percent of the total velocity In contrast, at the earth's surface an approximate equipartitioning of poloidal and toroidal energy is found. We show that with non-Newtonian and depth-dependent rheology the toroidal component at the free surface can become significant, and a pattern reminiscent of plate motion can arise in a free convection model. Although these results are obtained in a parameter range which is not directly applicable to the earth, they support the conclusions that (i) upwelling flow in the mantle is unlikely to be sheet-like and will probably be in the form of columnar plumes, and that (ii) the toroidal motion found at the earth's surface is due to the highly non-linear rheology which leads to the existence of mobile surface plates and is not caused by viscosity variations related to lateral temperature contrasts deeper in the mantle.

3:00 pm	Michael Gurnis University of Michigan	Large-scale mantle convection and dynamic interaction with plates

Abstract: The dynamic coupling of tectonic plates (including non-subducting continental lithosphere and subducting oceanic lithosphere) with thermal convection is being investigated with finite element models. In

a range of cases, the plates are viscous entities (like the background fluid) and are introduced into the system with various rheologies and boundary conditions. The models are meant to explore rather specific parts of the over-all coupling between tectonic plates and convection and to help us understand the relationship between various quantities measured on the Earth's surface and the dynamic processes in the interior.

Particular attention will be given to the interaction between large non-subducting continental lithosphere and the deep mantle and how these interactions effect seismic structure, dynamic topography, the geoid, and sea-level (measured as continental flooding). I will demonstrate that such interactions have a first order influence on the spatial and temporal pattern of continental flooding; I will suggest how the pattern of observed marine deposits on continental platforms can be used to constrain aspects of mantle rheology and mantle flow.

4:00 pm	Vincent Hall 502 (The IMA Lounge)	IMA Tea (and more!)

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Tuesday, June 12 (at Minnesota Supercomputer Institute)

Earthquakes & Turbulence in Thermal Convection, Visualization

The talks today will be held in Room 3125 (and 3155) of the Minnesota Supercomputer Center, 1200 Washington Avenue South, Minneapolis. The IMA will provide a Transit Coach to pick people up on Union Street (along side the EE/CS Building). The bus will start boarding at 8:30 am, depart for MSI at 8:45 am, and arrive at MSI by 9:00 am.

Also, the (free) University Campus Bus #13A can be used to reach MSI. It leaves Fraser Hall at 8:35 am (and 9:35, 10:35, etc.), and arrives at MSI at 8:39 am. The last return #13A bus to leave MSI is at 4:40 pm. (Buses leave from MSI at 40 after the hour from 8:40 am to 4:40 pm.)

You can also walk from the IMA to MSI. The distance is approximately one mile.

9:00 am	J.B. Rundle Livermore National Lab.	Lattice automata models for earthquakes and frictional sliding

Abstract. A variety of phenomena associated with earthquakes and laboratory friction experiments suggest fundamental similarities to nonlinear dynamical systems operating near a critical point. These phenomena include a large scaling regime for number of earthquakes as a function of area, and the observation of hysteresis loops in frictional sliding experiments, suggesting the existence of a long lived metastable state. Important basic questions about friction remain unanswered, including the origin of rate dependence in laboratory studies of friction between clean, dry surfaces.

The physical processes associated with frictional sliding and earthquakes can be illuminated by the use of a simple lattice automaton recently proposed in the literature (JBR, J. Geophys. Res., 93, 6237, 1988). In this model, interactions between lattice points x_i and x_j are specified by means of an operator T_{ij}, also called the stress Green's function. The independent field variable ϕ_i (order parameter) is the reduced slip, or slip deficit $\phi_i = s_i - Vt$, where s_i is the shear slip at x_i, V is the time averaged rate of sliding, and t is time. The force, or shear stress σ_i at each lattice point is given by $\sigma_i = p_i + \Sigma_j T_{ij}\phi_j$, where p_i is the background, or bias stress, at x_i. Dynamical evolution of the system occurs when the stress σ_i increases, through increases in p_i, or as t increases by constant (independent of V) decay time increments δt. When σ_i equals or exceeds a failure level σ_i^F, the slip s_i is increased according to a specified rule. On each time step, s_i is adjusted at all failed lattice points, prior to again incrementing t by δt. The stress at each lattice point decays from the unrelaxed value to its relaxed value over a lifetime Γ, which may be greater or less than δt, the time interval between loadings.

The nature of the model is sufficiently general that it incorporates both the quasistatic Burridge-Knopoff model of a shear fault, as well as the recently proposed Self Organized Criticality model of Bak and colleagues. Numerical results for a specific model simulating laboratory friction experiments will be described (JBR and SR Brown, Phys. Rev. Lett., submitted, 1990). A scaling regime can be seen which grows as V increases, indicating increasing proximity to the dynamical critical point where a spanning cluster of failed points appears. The size of the scaling region can be used to estimate the failure correlation length ξ_d, which

diverges at the critical point. The most interesting result is the appearance of rate dependence, similar in magnitude and sign to the rate and state dependence seen in laboratory sliding friction experiments. Joint work with S.R. Brown.

9:50 am	Lou Howard Florida State University	On turbulent thermal convection
10:40 am	Coffee Break	
11:00 am	A.C. Fowler Oxford University	Convection and Chaos

Abstract: This rather discursive talk will discuss two types of chaotic behaviour exhibited by high Prandtl number convection, that is, "phase chaos" and plumes. In mantle convection, these differing aspects of the motion find their expression in the migration of subduction zones and hotspots, respectively. The analysis of plumes in a single convection cell can be undertaken in the framework of Howard's "bubble" model of convection, using an asymptotic analysis based on a similar method applied to the Lorenz equations. This leads to an approximate Poincaré map, wherein chaotic behaviour arises through the near attainment of trajectories which form homoclinic connections between states of conductive thermal equilibrium. Alternatively, "cellular chaos" at large Rayleigh number can be modelled using a set of ordinary differential equations for variables which describe the size and location of slowly varying convection cells. The differential equations are parametrised using quasi-stationary boundary layer theory. The same method can in principle be extended to three dimensions, and represents a paradigm for the study of time-dependent motions of the earth's lithospheric plates.

11:50 am	Lunch Break	Lunch in the MSI Reception Area provided for registrants
1:30 pm	S. Balachandar University of Illinois, Urbana	Direct numerical simulation of turbulent thermal convection

Abstract: A direct numerical simulation of thermal convection between horizontal plane boundaries has been performed, at $Ra = 9800Ra_c$, where Ra_c is the critical Rayleigh number for the onset of convection. The flow is found to be fully turbulent, and an analysis of the probability distributions for temperature fluctuations indicates that this is within the "hard turbulence" regime, as defined by the Chicago group. Plots of temperature vertical velocity correlation along with flow visualizations indicate the presence of turbulent thermal plumes and these coherent structures are found to make significant contribution to the overall heat transfer. The effect of finite domain on the classical scaling properties, which are based on a semi-infinite domain, is explored. The highly intermittent nature of vorticity and temperature distributions is revealed by their non-Gaussian probability distributions. Under proper scaling a universal probabilistic description exists over the entire convection cell, except for the very narrow sub-layer near the top and bottom boundaries.

2:20 pm	U. Hansen U. zu Köln	Chaotic thermal convection at infinite Prandtl number

Abstract: Numerical simulations of $2-d$ high Rayleigh (Ra) number, base heated convection in large aspect ratio boxes are presented for infinite Prandtl number fluids as applied to the earth's mantle. Typically, flows at $Ra > 10^6$ consist of large-scale cells with intermittent boundary layer instabilities. Due to the instabilities the flow displays strong temporal variations while the existence of the large-scale flow leads to spatial coherent structures. Changes in the large-scale component (appearance or disappearance of convection cells) are found to operate on a time scale much longer than the one associated with the boundary layer instabilities. For Ra exceeding 10^7 we find the heat transfer mechanism changing from one, characterized by mushroom-like plumes to one consisting of disconnected instabilities. The evolution and the fate of the boundary-layer instabilities is determined by the kind of interaction between those instabilites and a 'wind', generated and maintained by the large scale circulation. Collisions of instabilities within the thermal boundary layers lead to

pronounced plumes having the tendency to develop 'pulse-like behavior'. Tilting of the instabilites, resulting from the shear which is exerted by the wind is another typical feature of the multiscale flow. The richness of the dynamical phenomena resulting from the interaction of the different scales is potentially interesting with respect to the observed variety of geodynamical features.

3:10 pm Coffee Break

3:30 pm	Video session	Participant-supplied visuals, in the form of videotapes or online IRIS, MacintoshII or SUN movies, will be shown.

Presentations by Gurnis, Hansen, Jaupart, Spera, Peltier, Yuen, Balachandar, Glatzmaier and others.

MSI Chaos Workshop talk

4:30 pm	Yongmin Kim University of Washington	The University of Washington graphics system processor
6:00 pm	Reception	Grandma's Saloon , near Holiday Inn on West Bank

Reception from 6:00 to 8.00 PM on the ground-floor of Grandma's Saloon. Cash bar; hors d'oeuvres will be served .

Wednesday, June 13

Magma Physics, Soliton Dynamics, Volcanic Transitions

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:00 am	George W. Bergantz University of Washington	Conjugate solidification and melting in open and closed multi-component systems

Abstract: Underplating and intrusion of continental crust by basaltic magma has repeatedly been invoked as a mechanism to provide the needed heat and mass transfer to initiate and sustain magmatism in a variety of tectonic settings. This superposition yields a strongly time dependent and non-linear feedback system with both conductive and convective components. Theoretical and experimental studies have been undertaken which focus on the manner in which transport properties and thermal history are buffered by enthalpy changes associated with simultaneous solidification and melting as a function of the change of melt fraction for naturally ocurring compositions. This is particularly important as the crystallization of basalts and the melting interval for common crustal rocks overlap; this yields strong changes in material properties which govern the heat transfer systematics at the solidification-melting interface.

Experiments: The underplating process was modeled by superposing mixtures of fluid and solid paraffins representing "basalt" and "country rock" in a variety of geometries. The paraffins have similar Stefan numbers, and relationships between density, viscosity, enthalpy, and most importantly, melting interval, as magmas. These paraffins exhibit no nucleation delay. Turbulent convection in the underplated material was not observed: the system exhibited a dynamical and thermal history largely consistent with models based on a coupled conductive solidification of the "basalt" and partial melting of the "country rock".

Numerical modeling: of this conjugate system in the presence of both forced and fully coupled free convection yields a variety of time dependent interactions from melting back of the wall in the case of turbulent forced flow to a short-lived period of natural convection associated with the fully coupled and more geologically realistic case. The governing dynamical quantities are the slope of the solid fraction distribution, the degree of superheat relative to the temperature where the viscosity changes by a factor of approximately 1/e, and the background temperatures of the country rock. The Peclet number associated with interdendritic flow is demonstrably small, and can be neglected. The formulation of the problem in terms of a general dependent variable, the chemical potential, will be discussed.

9:50 am	Claude Jaupart Université Paris 7	Transitions of eruption regimes in silicic volcanoes

Abstract: In silicic volcanoes, eruptions commonly begin with violent explosive phases and evolve towards a regime of dome formation. Dome growth is initially not a stable process and usually leads to an explosive phase. Thus, the volcano may alternate between phases of explosive activity and dome growth. These transitions are characterized by a decrease of gas volume fraction which has usually been attributed to chemical gradients in the volcano chamber. Petrological and geochemical studies suggest that this interpretation may be oversimplified. a critical observation is that the eruption rate decreases as an eruption proceeds and is markedly smaller during dome growth than during explosive activity. We suggest that the transition from explosive activity to dome formation is due to gas loss through permeable conduit walls and is a direct cnsequence of the observed decrease in eruption rate. We derive the equations for the dynamics of a lava/gas mixture rising towards the surface. In the volcanic conduit, the gas content of lava is determined by two processes which act in opposite directions. pressure release leading to gas exsolution and expansion, and gas loss to the country rocks. The eruption rate depends on the pressure which obtains in the volcano chamber. This pressure steadily decreases with time as the chamber empties, implying a decrease of eruption rate. In turn, this decrease acts to increase the amount of gas lost to the country rocks and hence to reduce the gas content of the erupted material. This model therefore predicts that, with time, the eruption should in general undergo transitions from Plinian to Peleean conditions and then to dome formation. These transitions occur as bifurcations in the evolution of gas volume fraction with height in the conduit. We find that very small pressure fluctuations of the order of one bar lead to large changes of gas content at the vent. This sensitivity to tiny pressure changes provides an explanation for observed alternations between Plinian and Peleean phases, as well as between explosive phases and dome formation.
Joint work with Claude J. Allegrè.

10:40 am	Coffee Break	Reception Room EE/CS 3-176
11:10 am	**Herbert E. Huppert** University of Cambridge	Solidification and multicomponent convection within the earth

Abstract: Solidification processes occur principally in three different areas within the earth: as the liquid outer core cools it forms a solid inner core; the magma in a magma chamber crystallizes and may form plutons, magma flowing up a dyke may solidify against the dyke walls and the associated volcanic eruption may cease. the seminar will discuss some of the fundamental processes involved in all these areas. A major driving mechanism, which will be analysed in detail, is that upon solidification almost all multicomponent fluids release fluid of a different composition and density. This density difference can drive strong convective motions which can alter the form and rate of the solidification.

1:30 pm	**Frank J. Spera** UC Santa Barbara	Nonlinear interactions in magmatic and hydrothermal flow driven by two sources of buoyancy

Abstract: In the natural convective flow of both magma and hydrothermal fluids in the crust of the Earth, two sources of buoyancy are generally present (thermal and chemical buoyancy). Because chemical diffusion is generally much slower than the conductive transport of heat, these systems are classic examples of multiply-diffusive convection. Furthermore, due to phase change and the constraints implied by local thermodynamic equilibrium, buoyancy sources may be local and complex in both time and space.

The dynamics of mixing of an initially chemically stratified system (brine-fresh water) or (rhyolitic-basaltic melt) has been studied as a function of Ra, R_ρ and Le for viscous fluids and for porous media flows. Video animations illustrating these sometimes chaotic flows will be presented in an effort to clarify underlying physical mechanisms. Finally, some results from a recent multiphase model (i.e. convection plus solidification) relevant to a simple binary-eutectic system will be shown. The latter model is a continuum one with specific accounting for the relative motion between viscous fluid and crystalline solid in the two-phase mushy region, for latent heat effects, for chemical diffusion in the melt phase and for inertial effects (finite Pr). Complex C-field sometimes develop in these flows; this suggests that geologic flows may be intrinsically unsteady.
Joint work with Curtis Oldenburg and Nina Rosenberg.

2:20 pm	**David J. Stevenson** Caltech	Aspects of the dynamics in two-phase media

Abstract: A large fraction of the Earth is either solid nor liquid but a combination of these phases. I will concentrate on partial melts, the precursor to all igneous activity. After a brief introduction to the underlying theory and a summary of the properties of magmons (non-linear solitary waves) I will shift to recent work concerning the possible instability of partial melts under shear, especially the fascinating and non-linear phenomenon of the possible development of a dendritic drainage network as the origin of macrosegregation.

3:10 pm	Coffee Break	Reception Room EE/CS 3-176
3:30 pm	Grae Worster Northwestern University	Solidification of magma chambers
4:20 pm	Reception	A reception for workshop participants, and sponsored by the University of Minnesota Geology and Geophysics Department, will be held in Pillsbury Hall

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Thursday, June 14

Chaotic Convection in Spherical Systems and Geodynamo

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:15 am	Coffee Break	Reception Room EE/CS 3-176
9:30 am	Pascale Chossat U. of Nice	Bifurcation and dynamics in spherically invariant systems

Abstract. Substantial progress in the study of spherically symmetric systems has recently been made in a joint work of Lauterbach, Melbourne and myself, thanks to the development of symmetry-breaking bifurcation theory. Initiated by Busse in the early 70's and developed by several contributors since then, the bifurcation analysis, when the critical spherical modes span a $2\ell + 1$ dimensional irreducible representation space with $\ell < 5$, is "almost" fully understood. The case $\ell = 5$, i.e., dimension 11, is also understood to a great extent. In this case several new solutions have been found, including time-periodic solutions (rotating waves with low frequency) which bifurcate from the trivial equilibrium and can be asymptotically stable. When a mode interaction between spherical harmonics is assumed, more complicated and quite fascinating dynamics have been observed and studied by Armbruster and myself. The mode interaction $\ell = 1, 2$ already shows a remarkable behavior, characterized by the successive exploration of various types of equilibria (or limit cycles) and following an intermittent-like dynamics. Such a behavior was already noticed in 1986 by Friedrich and Haken in a numerical investigation of the system resulting from a center manifold reduction applied to the Bénard problem in a self-gravitating spherical shell, when the spherical modes with $\ell = 1$ and 2 are both excited at the onset of convection. This dynamics is related to the presence of heteroclinic cycles connecting equilibria belonging or not to the same group orbit of solutions. These invariant sets share the property of being "robust" under small perturbations which keep the system spherically invariant. The dynamics driven by some of these objects is very reminiscent of the Earth's magnetic field behavior in geological time (pole reversals). I will present an overview of these results and of the group-theoretic methods which have been employed. My hope is to show that geophysical problems can be very stimulating for the mathematicians, whose partial answers may lead in return to interesting questions for the geophysicists...

10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	Phillippe Machetel French Space Agency of Toulouse	Influences of chaotic mantle convection on thermal anomalies
2:00 pm	Gary A. Glatzmaier Los Alamos National Lab.	Three-dimensional numerical simulations of chaotic convection in the Earth's mantle

Abstract: Three-dimensional numerical simulations of highly viscous thermal convection in a spherical fluid shell are used to try to understand the basic properties of convection in the Earth's mantle. The anelastic approximation filters out sound waves while accounting for a density stratification. All dependent variables are expanded in spherical harmonics to describe their latitudinal and longitudinal structures and in Chebyshev polynomials to describe their radial structures. A semi-implicit time integration scheme that uses a spectral transform method to compute the nonlinear terms evolves the time-dependent solution. Two cases are presented. One is totally heated from within the model mantle and has a Rayleigh number of 120,000. It initially has a tetrahedral spatial pattern that is periodic in time. After several periods this spatial and temporal patt.... evolves into a chaotic time-dependent structure. The second case is 80% heated from within the mantle and 20% heated from below and has a Rayleigh number of 1,600,000. This case, which is more representative of the Earth's mantle, is strongly chaotic and characterized by long narrow downflows near the outer boundary and cylindrical upflows near the inner boundary. 16 mm. movies of both cases are shown.

3:00 pm	F.H. Busse University of Bayreuth	Theory of the geodynamo and the problem of core-mantle coupling

Abstract: The possibilities for numerical investigations of the dynamo process in rotating spherical shells will be discussed with emphasis on physically feasible self-consistent models. The recent progress of Zhang and Busse (1987, 1988, 1989, 1990) will be reviewed and new developments will be described. Attention will be focussed on the problem of core-mantle coupling in connection with the dynamo process in the presence of lateral variations of the conductivity in the lowermost mantle. Provided the spatially varying component of the conductivity is sufficient large, standing and drifting components of the non-axisymmetric part of the magnetic field can be separated. Possible relationships to observations of secular variation and to decade changes in the length of the day will be discussed.

4:00 pm	Paul H. Roberts UCLA	Magnetoconvection patterns in rotating convection zones

Abstract. In addition to the well-known granulation and supergranulation observed on the solar photosphere, a third scale of motion has been postulated in the solar convection zone (="SCZ"): that of the so-called "giant cells". It is usually supposed that these span the entire thickness of the SCZ, and stretch from pole to pole in a sequence of elongated cells, forming a "cartridge-belt" or "banana cell" pattern round the Sun. Despite observational efforts, conclusive verification of the existence of giant cells is still lacking. Ribes and others have been led, from an analysis of sunspot motion, to believe that convective motions near the solar surface form a pattern that is the antithesis of the cartridge belt: a system of "toroidal" or "doughnut" cells, with edges parallel to the lines of solar latitude. Jones, Galloway and Roberts have recently tried to meet the resulting theoretical challenge.

A significant parameter of rotating magnetoconvection is the Elsasser number, $\Lambda = \sigma B^2/2\Omega\rho$, where B is the (predominantly zonal) magnetic field, Ω is the angular velocity, ρ is the density and σ is the (turbulent) conductivity. Intuitively, one expects that, when $\Lambda \ll 1$, Coriolis forces dominate Lorentz forces and, because of the Proudman-Taylor theorem, cells will be elongated along the axis of rotation in cartridge-belt style; when $\Lambda \gg 1$, the magnetic field is decisive and the convective cells align themselves with B in toroidal style. Because of the large variation of ρ across the SCZ, Λ is plausibly small at depth, but large near the solar surface.

Jones, Galloway and Roberts have initially aimed at determining the preferred pattern of convection in an apparently simple system intended to mimic regions near the solar equator: a plane horizontal layer of compressible conducting fluid is rotating abour a horizontal axis perpendicular to an ambient uniform horizontal magnetic field. Jones, Roberts and Galloway (1990) analysed the linear stability problem for the onset of convection, and demonstrated that the convection was necessarily time-dependent, a result that held for all Λ, even the non-magnetic case $\Lambda = 0$. Apart from this interesting difference, the model behaved much the same as the corresponding incompressible (Boussinesq) model, for which convection at onset is direct (i.e. time independent). In particular, even for large variations in the local value of Λ across the SCZ, the convection pattern is the same at all depths, and shows no sign of becoming toroidal. To simulate the large superadiabatic gradient in the upper convection zone, we also examined a two-zone model in which the

Rayleigh number, R, in the upper zone is ten times that in the lower zone. Two kinds of convective modes were discovered, one highly concentrated in the upper zone, and one predominant in the lower zone. The latter tends toward the banana cells; the former to the doughnuts.

Jones and Roberts (1991) studied the one-zone model (modified by vertical side walls parallel to the applied B) at finite amplitudes, and showed that the Lorentz force created a large "geostrophic" flow parallel to B and a function of height only In certain circumstances (stress-free side walls) increase of R leads to symmetry-breaking. One mode is concentrated near the surface and moves in one direction, the other is concentrated near the base of the convection zone and moves in the opposite direction. The unfolding of this symmetry breaking through the action of compressibility has been investigated. Some conjectures about the relationship of these modes to the Ribes observations will be made.

6:00 pm	Workshop Dinner	Peking Garden

Dinner is arranged for the Peking Garden located at 2324 University Ave. S.E. (across the street from the Days Inn Hotel). Phone 623-3989. A variety of Chinese food types will be served. A set price of $10.50 per person includes appetizer, fried rice, entrees and tea. More details will be supplied during the Workshop.

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Friday, June 15

The talks today are in Conference Hall EE/CS 3-180

9:15 am	Coffee Break	Reception Room EE/CS 3-176
9:30 am	Donald Turcotte Cornell University	On the route to chaos in the crust and mantle
10:30 am	Coffee Break	Reception Room EE/CS 3-176
11:00 am	Stephen Morris UC Berkeley	Rheological control of solid-state transformations occurring by nucleation and growth

Abstract. A solid transforming by nucleation and growth consists of grains of new phase growing in a matrix of parent phase. The grain radius is seen to increase by four or five orders of magnitude as neighbouring grains grow and merge. It is argued here that growth therefore involves flow in the matrix unless the two phases have very nearly the same density. What follows is a theoretical study of the coupling of flow and growth-rate when the parent and product phases have the same composition. The standard metallurgical treatments of this problem assume growth is controlled completely by interfacial kinetics. The object of the present work is to show why the coupling of flow and growth-rate is important and to show how it can be observed in experiments.

The coupling of flow and growth-rate is studied initially for an isolated spherical grain by taking the interfacial kinetics to be infinitely fast. The timescale for growth is then determined by the rheology of the matrix and by the amount ($P - P'$) by which the applied pressure P exceeds the equilibrium pressure P', i.e., the pressure P' at which the two phases are in equilibrium across a plane interface. Detailed analysis shows that rheologically-controlled growth has a characteristic signature: when the excess pressure ($P - P'$) is independent of time, the volume of an isolated spherical grain grows exponentially with time. This result is general and holds for any matrix in which the deviatoric stress depends on the strain-history and not simply on the instantaneous strain.(For example, it applies when the deviatoric stress is proportional to the strain–rate.)

By contrast, previous studies of kinetic control have shown the volume of an isolated spherical grain to grow algebraically with time. It should therefore be possible to distinguish the two types of control in high-pressure experiments. An unambiguous test is possible by experiments involving single grains, so that the nucleation-rate is not a factor in interpreting measurements of the volume fraction.

Recent experiments by Rubie et al(1990) are compatible with the existence of rheological control, but the results are slightly ambiguous. the authors inferred that only a single grain was present, but were not sure. This work is motivated by a problem in geophysics. The pressure in the earth is nearly hydrostatic, and phase transitions therefore occur on level surfaces if the temperature is horizontally-uniform and the transition is in thermodynamic equilibrium. However the conditions for thermodynamic equilibrium are poorly understood. The essential problem is one of scaling· for application to the mantle, measurements taken in the laboratory must be extrapolated to dilatation-rates 10-12 orders of magnitude slower. If kinetics alone control the growth, this extrapolation leads to the well-known conclusion that significant vertical deflexions of the transition-layer are unlikely at the high temperatures typical of the deep mantle.

By contrast, it is shown here that the existence of rheological control means that laboratory experiments are compatible with significant vertical deflexions of the transition-layer if the strain-rate in the matrix depends strongly on the deviatoric stress. The problem is analysed for two particular choices of matrix. The first example is a power-law matrix in which the strain-rate varies as the n-th power of the deviatoric stress. It is shown that if the matrix is a linear solid(i.e. $n = 1$), rheological control will cause no major deflexion of the transition-layer. In contrast, major deflexions are possible if the strain-rate depends strongly on the deviatoric stress(i.e., $n >> 1$). This behaviour is clarified by the second example which studies the limiting case of an elastic, perfectly-plastic solid with yield stress $2m$ and elastic rigidity G. This example allows us to find the additional dissipation produced if mantle convection occurs through the transition-layer. The volume-integrated caused by phase changes is found to exceed that caused by deformation outside the transition-layer if the yield stress of the matrix satisfies $2m > G/3000$. (The corresponding excess pressure is about 0.2 GPa.)

The additional dissipation may have two effects on the large-scale flow. First, it reduces the turnover speed of the cell and is therefore likely to destabilise the horizontal boundary-layers, and enhance mixing. Secondly, layering may result if sufficient dissipation is added.

2:00 pm	**Alain Vincent** C.E.R.F.A.C.S., Toulouse	The spatial structure and statistical properties of homogeneous turbulence

This talk is cosponsored by the Army High Performance Computing Research Center (AHPCRC).

Abstract: A direct numerical simulation is used to obtain a statistically stationary three-dimensional homogeneous and isotropic turbulent field at a Reynolds number around 1000 (R_λ around 150). The energy spectrum displays an inertial subrange. The velocity derivative distribution, found to be strongly non-Gaussian, is found to be close to but not exponential. The n-th order moments of this distribution, as well as the velocity structure functions, do not scale with n, as predicted by intermittency models. Visualization of the flow structures confirms directly the previous finding that the vorticity is organized in very elongated thin tubes. the width of these tubes is of the order of a few dissipation scales, while their overall length can assume the global scale of the flow.

3:00 pm	**Akerman Hall 130D**	Tour of Dan Joseph's laboratory

We will show experiments on water lubricated pipelining with bamboo waves and corkscrew waves, two dimensional cusped interfaces which violate Laplace's law, rollers, drafting, kissing and tumbling of fluidized particles, non linear stabilization of fingering instabilities in porous media, drag reduction using riblets. The tour probably will last 30 or 40 minutes.

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3:30 pm	**A. Vanderbauwhede** University of Ghent	Periodic solutions in reversible systems

Abstract: Reversible systems are systems of ode's such that if $x(t)$ is a solution, then also $R \times (-t)$ is a solution, where R is a linear involution ($R^2 = I$). Assuming that the eigenvalues +1 and −1 of R have the same multiplicity symmetric periodic orbits will generically appear in one-parameter families. We show how such families originate from equilibria and disappear in homoclinic orbits. Moreover, in certain regions of phase space the set of symmetric periodic orbits can show a cascade of branchings, with solutions along or

branching family being subharmonic to the solution along the primary branch. We describe these branchings and give a method to study them.

This SPECIAL LECTURE will be held in Vincent Hall 570

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CHAOTIC PROCESSES IN THE GEOLOGICAL SCIENCES
EDITED BY DAVID YUEN

CONTENTS

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IMA NEWSLETTER #178

May 12 - May 29, 1991

1990-91 Program
PHASE TRANSITIONS AND FREE BOUNDARIES

NEWS AND NOTES

IMA Workshop: DEGENERATE DIFFUSIONS

May 13 - 18, 1991

Organizers: W.-M. Ni, L.A. Peletier, J.-L. Vazquez

The emphasis in this workshop will be on current and new problems in nonlinear diffusion equations involving free boundaries or sharp interfaces. Such problems include (i) higher order degenerate diffusion equations, such as the porous media equation, but of order 4, 6 or higher, (ii) backward/forward diffusion equations involving a regularization higher order term, (iii) strongly degenerate diffusion equations, of second as well as of higher order and (or) systems of degenerate equations.

The motivation for studying these problems is both mathematical and practical. Equations (i) arise in models for semiconductor fabrication and for behavior of viscous drops, a typical example of (ii) is the Cahn-Hilliard equation arising in the study of phase transitions. Equations of type (iii) arise in astronomy and also in phase transitions and degenerate systems arise in models in population dynamics, and in population biology.

In addition, some effort will be made to survey the state of the art of the more classical subjects on the theory of degenerate diffusion equations.

The study of degenerate diffusion equations is actively pursued in many places. The objective of this workshop is to provide some focus in this endeavor, and by inviting scientists and engineers as well as mathematicians, to keep it firmly linked to concrete problems.

The last two days of the workshop will be a celebration of Jim Serrin's sixty fifth birthday.

Minisymposium:
NONLINEAR DIFFUSION EQUATIONS & THEIR EQUILIBRIUM STATES

May 24 - 25, 1991

Cosponsored by the National Science Foundation & the IMA

Organizers: W.-M. Ni, L.A. Peletier, J. Serrin

Many problems in science and engineering require the analysis of mathematical models involving nonlinear diffusion equations, for example, equations of the form

$$u_t = \Delta u + f(u).$$

These can be studied either in bounded or unbounded domains under various different conditions on the boundary. Mathematically this leads to questions about existence, uniqueness and qualitative properties of the solutions. Depending on the diffusion operator or the source function f, free boundaries may occur.

Recently, research has focused on questions such as global existence (blow-up/extinction) and the rôle of self-similar solutions in the description of short- and large time behaviour of solutions, as well as detailed studies of the rich structure of the associated equilibrium problems. This involves ground states as well as nonlinear global bifurcation problems, in which the growth of the source term plays a critical rôle.

The present meeting is meant to offer an opportunity for the participants to learn of recent results and to interact with scientists and engineers

This is the third in a series of meetings on nonlinear partial- and ordinary differential equations related to diffusion equations. The first one of these was held in 1986, in Berkeley, and the second one in 1989, in Gregynog (Wales).

SCHEDULE FOR MAY 12 - MAY 29

IMA Workshop:
DEGENERATE DIFFUSIONS

May 13 - 18, 1991

Organizers: W.-M. Ni, L.A. Peletier, J.-L. Vazquez

Most of the program talks will be held in Conference Hall 3-180 on the entry floor of the Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

Monday, May 13

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:00 am	Registration and coffee	Reception Room EE/CS 3-176

9:30 am	**Welcome and Orientation**	Conference Hall EE/CS 3-180
9:40 am	**Charles M. Elliott** University of Sussex	Parabolic obstacle problems and phase transformations

Abstract. We shall be concerned with the mathematical and numerical analysis of models of phase transformations based on the Ginzburg-Landau energy functional

1 $$E(u) = \int_\Omega \left[\frac{\gamma}{2}|\nabla u|^2 + \psi(u)\right] dx$$

where $\gamma > 0, \Omega \subset \mathbf{R}^m$ and $\psi(\cdot) : \mathbf{R}^m \rightarrow R$. The homogeneous energy $\psi(\cdot)$ is non-convex with at least two wells. The dynamic problem is given by the relaxation.

2 $$\frac{du}{dt} + DE(u) = 0$$

where D is an appropriate functional derivative. The scalar Cahn-Allen and Cahn-Hilliard equations

3 $$a)\ u_t - \gamma\Delta u + \psi'(u) = 0, \qquad b)\ u_t - \Delta(-\gamma\Delta u + \psi'(u)) = 0$$

are typical examples. One form for $\psi(\ )$ arising in the modelling of phase separation in a binary mixture is

4 $$\psi(u) = \frac{kT_c}{2} - kT\log_e 2 - \frac{1}{2}kT_c u^2 + \frac{1}{2}kT[(1-u)\log_e(1-u) + (1+u)\log_e(1+u)]$$

where k in Boltzmann's constant, T is absolute temperature and T_c is the critical temperature. Here $|u| \leq 1$ and the values $u = \pm 1$ correspond to the pure states of each component of the mixture. In the deep quench limit of $T/T_c \rightarrow 0$, it is appropriate to replace $\psi(\cdot)$ by

$$\psi(u) = \frac{1}{2} = \frac{1}{2}kT_c(1-u^2) + I_{[-1,1]}(u)$$

where $I_{[-1,1]}(u)$ is the indicator function of the interval $[-1, 1]$.
In this talk we will show that the degenerate diffusion equation (3b) - (4) has a solution which converges in the deep quench limit to the solution of a parabolic double obstacle problem. We shall also consider the Cahn-Hilliard model for phase separation in a mixture with M components. Aspects of similar versions of the Cahn-Allen equation (3a) and the phase field equations will be discussed. Numerical simulations of the motion of phase boundaries using these models will be presented. Various parts of this work were performed in collaboration with J.F. Blowey, S. Luckhaus, Xinfu Chen and M. Copetti.

10:40 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Paul Fife** University of Utah	Qualitative properties of Cahn-Hilliard dynamics

Abstract: Some important dynamical issues surrounding the Cahn-Hilliard equation will be delineated, and recent results addressing those issues will be presented. The dynamics of this equation is characterized by multiple time scales. Emphasis will be on short-time and intermediate-time, rather than long-time, phenomena.

Numerical Analysis Seminar

1:25 pm	**Tsutomu Ikeda** Ryukoku University	Numerical approach to interfacial dynamics

The SEMINAR meets in Vincent Hall 570

2:00 pm	**Michiel Bertsch** U di Roma II	A parabolic equation with a nonlinear dependence on the gradient

Abstract. We consider a one-dimensional nonlinear diffusion equation with vanishing diffusion coefficient for infinite gradients In this lecture we shall explain the occurrence of discontinuous solutions, and we shall study the behaviour of solutions near their discontinuities. In addition we shall point out some delicate questions related to modelling aspects and well-posedness.

SEMINAR IN { Mathematics / Physics / Ford Hall 150 }

2:30 pm	**A.V. Turbiner** ITEP, Moscow/CERN, Geneva	Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. I

Abstract. Quasi-exactly solvable problems are a new kind of spectral problems occupying an intermediate place between exactly solvable and non-solvable. The main property of them is the knowledge of a finite number of eigenstates. The simplest example is the potential

$$V = a^2x^6 + 2abx^4 + [b^2 - (2k+3)a]x^2$$

in which the first $N = [k/2] + 1$ eigenstates can be found algebraically. All one dimensional quasi-exactly solvable problems and all known exactly solvable ones have the same hidden symmetry group $SL(2, R)$. Quasi-exactly solvability is related to a new realization of hidden symmetry based on equivalence between the Schroedinger equation and a spectral problem for quadratic elements of the universal enveloping algebra in a representation in terms of differential operators of the first order. Using compact algebras leads to exactly solvable problems without separability of variables living on non-trivial manifolds.

This phenomenon has many connections with different branches of mathematics: spectral theory (a certain hint on classification of spectral problems), group representations, differential geometry, Riemannian geometry, theory of orthogonal polynomials (especially multi-dimensional polynomials). The most interesting results stem from special orbits of non-compact groups, like the conic of $SO(2,1)$, for which, in fact, there is no mathematical theory.

4:00 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

Tuesday, May 14

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Sigurd Angenent** University of Wisconsin/IMA	Anisotropic motion by curvature
10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Robert V. Kohn** Courant Institute	Some open problems involving phase boundary motion by curvature related laws

Abstract: Two fundamentally different approaches are evolving for the mathematical description of phase boundary motion. One, based on viscosity solutions of Hamilton-Jacobi equations, has led to successful computations and mathematical theorems of existence and uniqueness. The other, based on singular limits of reaction diffusion equations, has had less spectacular success but seems in certain ways more flexible and closer to the physics. I will summarize these two approaches, then discuss a variety of open questions. Issues to be addressed include the following: A) Are there initial data for which the HJ and RD models of motion by mean curvature give different motion laws? B) Is the "slow motion manifold" program of Hale/Fusco

and Carr/Pego applicable to motion by mean curvature in several dimensions? C) What is the analogue of motion by mean curvature when there are three or more phases? D) How should one handle the motion of a curve or interface by mean curvature, when it is fixed at part of its boundary? E) In what sense should one interpret the "scaling law" that interfacial area decays as $t^{1/2}$ in a system evolving under motion by mean curvature?

Solid Mechanics Seminar

1:00 pm	**G.I. Barenblatt** USSR Academy of Science/IMA	Similarity and some relevant problems in fracture

The SEMINAR is in Akerman Hall 227

2:20 pm	**Nina N. Uraltseva** Lenigrad/Emory University	Evolution of nonparametric hypersurfaces with speed depending on the mean curvature

Abstract: In this joint work with Vladimir Oliker we consider an evolution which starts as a flow of graphs propagating in space with normal speed proportional to the mean curvature of the current hypersurface. G. Huisken has shown that if the boundary of the domain over which these graphs are considered satisfies the condition of J. Serrin (that is, it is convex "in the mean") then the corresponding initial boundary value problem with Dirichlet boundary data and smooth initial data admits a solution which exists for all time and smooth. Here, we consider the case of arbitrary domains with smooth boundaries not necessarily satisfying the condition of Serrin. Numerical experiments suggest that even if the flow starts with smooth initial data and homogeneous Dirichlet boundary data, singularities may develop in finite time at the boundary of the domain. However, we show that after a sufficiently large time the solution becomes again smooth and assumes the boundary values in classical sense. We also give sufficient conditions that guarantee existence of a solution which is smooth for all time $t \geq 0$. In addition, we give sharp estimates of the rate at which solutions tend to zero as $t \rightarrow \infty$.

3:30 pm **Contributed Papers**

Participants are invited to present contributed papers. Twenty minute time slots have been set aside for these talks. It is hoped that the speakers will leave five minutes of the time for discussion. Please contact the organizers if you wish to give a contributed paper.

3:30 pm	**Xinfu Chen** University of Minnesota	Phase field equations in the singular limit of sharp interface problems
3:50 pm	**Joost Hulshof** University of Leiden/IMA	The dipole solution for the porous media equation in N dimensions
4:10 pm		Fifteen minute break
4:25 pm	**Doug Meade** Purdue University	On the qualitative analysis for an epidemic model with directed diffusion
4:45 pm	**Marek Fila** University of Bratislava/IMA	Boundedness of global solutions of a degenerate diffusion equation
5:05 pm	**Amy Novick-Cohen** Technion	Sivashinsky type equations: one-dimensional steady states

AHPCRC Colloquium

3:30 pm	**Irene Martinez Gamba** Purdue University/IMA	Boundary behavior of solutions to equations modeling semiconductor devices

Abstract: We are going to discuss asymptotic singularities for the Drift-Diffusion two-dimensional model of a typical metal-oxide semiconductor (MOS), socks and boundary layer formation for one-dimensional hydrodynamic models.

The COLLOQUIUM is held in the AHPCRC Seminar Room 170
1100 South Washington Ave., Minneapolis

Wednesday, May 15

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Miguel Herrero** U. Complutense de Madrid	Some results on blow-up for semilinear parabolic problems

Abstract: We shall consider the problem

$$(1) \qquad u_t = u_{xx} + f(u), \quad \text{where } f(u) - u^p (p > 1) \text{ or } f(u) = 0^u$$

$$(2) \qquad u(x,0) = u_0(x) \geq 0$$

let $u(x,t)$ be a solution of (1), (2) which blows up at, say, $x - 0$ and $t = T < +\infty$. A description of the possible blow-up profiles will be given and conditions under which some of these profiles occur will be discussed.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Victor Galaktionov** Keldysh Inst., USSR/IMA	On a blow-up set and monotonicity in time for a degenerate quasilinear heat equation

Abstract: The Cauchy problem for a quasilinear parabolic equation of nonlinear heat conduction type with a source is considered. Sharp estimates of the structure of blow-up sets are proved. It is shown that properties of monotone in time behavior of solutions at a fixed spatial point depends on the relationship between the nonlinear heat operator and the source and the behavior of the initial function in some neighborhood of this point. In particular, it is shown that arbitrary solution is strictly monotone in time at any point which is far enough from the initial support.

These results are proved by the method of intersection comparison either with the explicit nonimvariant solution having the same blow-up time or with continuous set of stationary solutions.

2:00 pm	**G.I. Barenblatt** USSR Acad. of Science	Mathematical model of capillary inbibition
3:30 pm	**Juan Luis Vazquez** U. Autonoma, Madrid/IMA	The Barenblatt equation of elasto-plastic filtration

Abstract: In this work with S. Kamin and L. Peletier we consider the equation

$$u_t + \gamma|u_t| = \Delta u\,, \quad 0 < |\gamma| < 1 \qquad (E)$$

in $Q = \{(x,t) : x \in \mathbf{R}^N, t > 0\}$. The equation arises in the theory of an elastic fluid in an elasto-plastic porous medium, under the assumption that the porous medium is irreversibly deformable, and was studied by Barenblatt and his collaborators since the '50s. On the other hand, we may write the equation in the form $u_t = \max\{L_1(u), L_2(u)\}$ if $\gamma < 0$, $u_t = \min\{L_1(u), L_2(u)\}$ if $\gamma > 0$ where $L_1(u) = \Delta u/(1-\gamma)$ and $L_2(u) = \Delta u/(1+\gamma)$. In this way we see that it is a particular case of the so-called parabolic Bellman equations of dynamic programming.

We construct for every $\gamma \in (-1,1)$ a selfsimilar solution of the form $B(x,t) = t^{-\alpha/2} f(\eta)$, with $\eta = x/t^{1/2}$, exhibiting a singularity at the origin ($x = 0, t = 0$). This solution is unique up to a multiplicative constant. We show that when $\gamma \neq 0$ the similarity exponent α is *anomalous*, i.e., different from the usual $\alpha = N$ of the heat equation. More specifically, we prove that α is a strictly increasing function of γ, with $\alpha(0) = N$. Thus, B is a fundamental solution only if $\gamma = 0$. It is *very singular* if $\gamma > 0$, in the sense that the mass of the initial singularity is infinite, while for $\gamma < 0$ it is a *mildly singular* solution (i.e., the mass of the initial singularity is 0).

Our main result consists in proving that these selfsimilar solutions represent the asymptotic profiles to which every solution of (E) tends for large times, under the assumption that their initial data are nonnegative, continuous and fast decay as $x \to \infty$.

Thursday, May 16

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

SEMINAR ON INDUSTRIAL PROBLEMS: UNDERGRADUATE COMPONENT

9:00 am	**Walter Littman** University of Minnesota	Open only to selected undergraduate participants

Background: A group of a half dozen undergraduates is meeting on a regular basis with Professor W. Littman in Vincent Hall 570 in conjunction with the Industrial Problems Seminar. One of the aims of this activity is to bridge the gap between the knowledge acquired in regular course work and the know-how necessary to attack problems as they actually appear in industry. Another aim is to channel the enthusiasm for computing that many students have into productive scientific areas.

9:30 am	**Rui-Tao Dong** MSRI	Eigenfunctions and nodal sets on Riemannian manifolds

Abstract: I will discuss a BMO estimate of an eigenfunction of the Beltrami-Laplace operator, the Hausdorff measure of the nodal set of the eigenfunction and the connection between them.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Chang-Shou Lin** Nat'l Taiwan University/IMA	Uniqueness of solutions minimizing $\int_\Omega \lvert\nabla u\rvert^2/(\int_\Omega u^{p+1})^{\frac{2}{p+1}}$ in $\mathbf{R}^2$
2:00 pm	**Emmanuele DiBenedetto** Northwestern University	Analyticity of solutions to the fast diffusion equation

SEMINAR IN { Mathematics / Physics / Vincent Hall 570 }

2:30 pm	**A.V. Turbiner** ITEP, Moscow/CERN, Geneva	Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. II

School of Mathematics Colloquium

3:30 pm	**Hiroshi Matano** University of Tokyo	A diffusion equation associated with equimeasurable rearrangement

Abstract: The theory of equimeasurable rearrangement – or symmetrization – has many applications in the calculus of variations, particularly in studying the spatial structure of global minimizers of certain energy functionals. The conventional theory of rearrangement, however, is of little use when it comes to local minimizers rather than the global ones. To fill the gap, one needs to establish a theory that would give an appropriate homotopy between a function u and its rearrangement u^*. The study of the nature of such "homotopy" leads us to investigating a certain diffusion equation under multiple integral constraints. We shall discuss the long-time behavior of solutions and stability of equilibria.

The COLLOQUIUM is held in Vincent Hall 16

Tea will be served at 3:10 pm in Vincent Hall 120

Friday, May 17

Celebration in Honor of Professor James Serrin's 65th Birthday
Sponsored by the School of Mathematics and the IMA

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**S. Antman** University of Maryland	Nonlinear nonlocal problems of fluid-solid interactions

Abstract. A fluid flowing past a deformable boundary influences the shape of the boundary and, conversely, the shape of the boundary influences the flow It is shown how such problems for steady (potential) flows (including cavitational flows) past nonlinearly elastic shells can be accurately formulated and globally analyzed. The need to determine the shape of the boundary introduces technical complications into the study of the flow that do not arise for problems with rigid boundaries.

10:15 am	**Coffee Break**	Reception Room EE/CS 3-176
10:45 am	**J. Nohel** University of Wisconsin, Madison	Quadratic systems of ODE's occurring in shear flows of non-Newtonian fluids

Abstract: After a brief discussion of known results and open problems for quadratic systems of ODE's in the context of Hilbert's sixteenth problem, we discuss qualitative results for two problems: (1) pressure-driven flows that are approximated by a one-parameter family of a system of two quadratic ODE's; (2) piston-driven flow that can be approximated by a system of four coupled quadratic ODE's. From the point of view of qualitative theory of ODE's, the most difficult issue is to rule out existence of periodic and homoclinic orbits in both problems. Problem (1) is solved completely by phase plane analysis that can be used to explain interesting phenomena in pressure-driven shear flows under cyclic loading and unloading. Problem (2) arose as an approximation for a mathematical model for a recent experiment in which nearly periodic disturbances were observed in order to maintain a fixed volumetric flow rate. The experiment is viewed as a feedback control problem: Find the driving pressure gradient (i.e., the feedback) subject to the constraint that the volumetric flow rate is fixed. The approximating system of four quadratic ODE's is globally well-posed in time. We present numerical and some qualitative results. Since none of the standard two dimensional theory is applicable, the global analysis presents formidable open problems that are of independent interest.

1:30 pm	**N. Trudinger** Australian National University	On viscosity solutions of uniformly elliptic fully nonlinear elliptic equations

Abstract: We consider the uniqueness and continuous differentiability of weak solutions (in the viscosity sense) of fully nonlinear elliptic equations satisfying natural structure conditions. The resultant derivative estimates extend earlier estimates of ours for classical solutions. As a special case we see that Lipschitz solutions of quasilinear elliptic equations are classical if the coefficients are sufficiently smooth.

2:15 pm	**Coffee Break**	Reception Room EE/CS 3-176
2:30 pm	**K. McLeod** University of Wisconsin, Milwaukee	Long-time behaviour of solutions of quasi-linear parabolic equations

Abstract: We will discuss some ongoing work (joint with A. Milani) for equations of the form

$$u_t - \sum_{i,j} a_{ij}(\nabla u) D^2_{ij} u = f,$$

where the right-hand side is in the Sobolev space H^s, s large.

3:15 pm	**Coffee Break**	Reception Room EE/CS 3-176V
3:45 pm	**R. Redheffer** UCLA	Some diverse but related nonlinear problems

Abstract: A variety of topics pertaining to nonlinear partial differential equations of the 2nd order are seen to be related, in that they can be discussed by similar methods and many of them have some connection with the work of James Serrin. The talk is, for the most part, expository.

Saturday, May 18

Celebration in Honor of Professor James Serrin's 65th Birthday
Sponsored by the School of Mathematics and the IMA

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am **P. Rabinowitz** — Multibump solutions of a semilinear elliptic equation on $\mathbf{R}^n$
University of Wisconsin, Madison

Abstract: For a family of semilinear elliptic equations of which

$$* \qquad -\Delta u + u = a(x)|u|^{s-1}u$$

is a special case, we will discuss the existence of k bump solutions for all $k > 1$. In (*), $a(x)$ is positive and periodic in the components of x and $1 < s < \frac{n+2}{n-2}$ if $n > 2$.

10:15 am **Coffee Break** — Reception Room EE/CS 3-176

10:45 am **Giovanni P. Galdi** — Exterior stationary solutions to the Navier-Stokes equations with zero velocity at infinity
University Degli Studi di Ferrara

Abstract: We analyze the asymptotic structure of solutions to the Navier-Stokes equations past a three-dimensional body, subject to zero velocity at infinity and such that

$$\int_\Omega \nabla v : \nabla v < \infty,$$

where v is the velocity field (D-solution). Specifically, we show that if the data are small compared to the viscosity, every D-solution presents at large distances the same behaviour of the Stokes fundamental tensor and, in particular, it behaves like $0(1/|x|)$. Furthermore, we show that *unless the data satisfy certain compatibility conditions* v does not belong to Lebesgue spaces L^q, for all $g \in (1,3]$ and, as a byproduct, that the kinetic energy of the fluid is infinite. On the other hand, such conditions are proved to be also sufficient to ensure the existence of solutions in these L^q-spaces. One of the basic results is obtained by adapting to the case at hand a well-known technique introduced by J. Serrin in 1963 within the context of uniqueness of unsteady flows.

1:30 pm **J. Smoller** — Non singular static solutions of Einstein-Yang/Mills equations
University of Michigan

Abstract: The only static solution to the vacuum Einstein equations is the celebrated Schwarzschild solution which is singular at $r = 0$. Similarly the pure Yang/Mills equations on $\mathbf{R}^4$ have no static, regular solutions, and if one couples Einstein's equations to Maxwell's equations, the only static solution is the Reissner-Nordström metric which is again singular at the origin. Finally, for any gauge group G, in $(2+1)$-space-time dimensions, the Einstein-Yang/Mills (EYM) equations likewise admit no static regular solutions. We prove that the contrary holds in $(3+1)$-space-time dimensions. In fact, with $SU(2)$ gauge group we prove that the EYM equations admit non-singular static solutions whose metric is asymptomatically flat (i.e., Minkowskian). Thus for non-abelian gauge fields, the Yang/Mills repulsive force can balance gravitational attraction, and prevent the formation of singularities in space-time.

2:15 pm **Coffee Break** — Reception Room EE/CS 3-176

2:30 pm **P. Pucci** — Precise damping conditions for global asymptotic stability of second order variational systems
University Degli Studi Modena

Abstract: We discuss global asymptotic stability of the rest state for Euler-Lagrange systems of ordinary differential equations of the form

$$[\nabla G(u')]' + \delta(r)\nabla G(u') + f(r, u) = 0.$$

The most important conditions are that G be strictly convex in $\mathbf{R}^N$, $\nabla G(0) = 0$, and $(f(r,u), u) > 0$ for $u \neq 0$ and r sufficiently large. We obtain necessary and sufficient conditions for global asymptotic stability under general assumptions given in terms of the damping coefficient $\delta(r) \geq 0$. Some of the results also extend to general variational systems

$$\{g(r)\mathcal{F}_p(r, u, u')\}' = g(r)\mathcal{F}_u(r, u, u')$$

and even to quasi-variational systems in which the damping coefficient is allowed to depend on u'. The stability theorems generalize and include a number of previous results due to *Levin & Nohel, Smith, Salvadori, Artstein & Infante* and *Ballieu & Peiffer*. The main tool is a variational identity which supplies a useful new family of Liapunov functions.

3:15 pm	**Coffee Break**	Vincent Hall 502
3:45 pm	**W. Ziemer** Indiana University	The Dirichlet problem for functions of least gradient

Abstract: Let $\Omega \subset R^n$ be a bounded Lipschitz domain and let $g: \partial\Omega \to R^1$ be a continuous function. We consider the problem

$$\inf\left\{\|\nabla u\|(\Omega) : u \in BV(\Omega) \cap C^0(\overline{\Omega}),\ u = g \text{ on } \partial\Omega\right\}.$$

Here, $BV(\Omega)$ is the space of functions whose partial derivatives are measures and $\|\nabla u\|(\Omega)$ denotes the total variation of the vector-valued measure ∇u evaluated on Ω. We show that a solution $u \in BV(\Omega) \cap C^0(\overline{\Omega})$ exists provided $\partial\Omega$ satisfies two conditions, namely, that $\partial\Omega$ has non-negative mean curvature (in a weak sense) and that $\partial\Omega$ is not locally area-minimizing. Furthermore, if either condition fails, we show that there exists boundary data g for which the problem has no solution.

The existence of a solution is established by actually constructing each of its superlevel sets in such a way that they reflect the appropriate boundary conditions and that they are area-minimizing. Each superlevel set is a solution to a variational problem which involves the minimization of the perimeter of sets.

It is also shown that the solution is unique. Solutions can have regularity in the interior no better than that at the boundary. However, it is shown that if $g \in C^{0,\alpha}$, then the solution is $C^{0,\alpha/2}$ in the interior. These results are optimal.

Finally, we consider the problem

$$\inf\left\{\|\nabla u\|(\Omega) : u \in BV(\Omega),\ u = g \text{ on } \partial\Omega\right\}.$$

Under the added assumption that Ω satisfies an exterior ball condition at each of its boundary points, it is shown that the unique minimizer of the first problem is also the unique minimizer of the second. In particular, the solution of the second problem is necessarily continuous in $\overline{\Omega}$.

Buffet Dinner in Honor of James Serrin's 65th Birthday

6:00 pm	Marquette Place Apartments, 1314 Marquette Avenue South, Minneapolis MN, 35th floor Party Room

Cocktails are at 6:00 pm, dinner at 7:00. Reservations required.

Monday, May 20

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

11:15 am Alberto Tesei — Conservation laws with source
Università Di Roma "La Sapienza"

Abstract: We study qualitative properties of solutions of the Cauchy problem

$$\begin{cases} u_t + u^{m-1}u_x = \lambda u^p & \text{in } (0,\infty) \times \mathbf{R} \\ u = u_0 & \text{in } \{0\} \times \mathbf{R} \end{cases}$$

where $m > 1, p > 1, \lambda = \pm 1$.

The meeting today is joint with the PDE Seminar

Numerical Analysis Seminar

1:25 pm **Elena Zampieri** — Finite element preconditioning for spectral approximations and application to linear elasticity
University of Minnesota

The SEMINAR meets in Vincent Hall 570

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

2:30 pm **G.I. Barenblatt** — Evolution of the turbulent burst
USSR Acad. of Sciences/IMA

Real Analysis Seminar

3:00 pm **Luis Caffarelli** — Monge-Ampere equations and the mapping problem
Institute for Advanced Study

The SEMINAR meets in Vincent Hall 311

Tuesday, May 21

SEMINAR IN { Mathematics / Physics / Vincent Hall 570

9:30 am **A.V. Turbiner** — Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. III
ITEP, Moscow/CERN, Geneva

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

11:15 am **Robert Kersner** — The disappearance of free boundaries
Hungarian Academy of Sciences/IMA

Abstract: This talk concerns joint works with P. DeMottoni and Sh. Kamin.

We investigate two models in which free boundaries disappear in finite time. The first model is the equation

$$u_t = \Delta u^m + f(x,t,u)$$

where $m > 1$ is a constant and the function f resembles the reaction term in Fisher's equation. The second model is

$$\rho(x)u_t = \Delta u^m$$

with $\rho(x)$ a positive function which decays to zero as $|x|$ tends to infinity.

AEM Fluid Mechanics Special Seminars

12:20 pm	G.I. Barenblatt USSR Acad. of Sciences/IMA	Turbulence in Stratified Fluids. I

The SEMINAR is held in Conference Hall EE/CS 3-180

IMA Postdoc Seminar

2:30 pm	**David Dobson** IMA	The time-harmonic Maxwell equations in a doubly periodic structure

Abstract. Consider the diffraction of a beam of particles in $\mathbf{R}^3$ when the dielectric coefficient is a constant ϵ_1 above a surface S and a (different) constant ϵ_2 below S, and the magnetic permeability is constant μ throughout $\mathbf{R}^3$. S is assumed to be a doubly periodic surface, say $z = f(x,y)$ with $f(x+mL_1, y+nL_2) = f(x,y)$ for all integers m, n. The existence and uniqueness of a solution satisfying a "radiation condition" at infinity is reduced to a system of Fredholm equations. Thus, for all but a discrete set of ϵ's there exists a unique solution. Joint work with Avner Friedman.

The Seminar meets in Vincent Hall 570

Special Lecture

2:30 pm	**André Avez** Paris	Symplectic geometry and quantum mechanics

The Lecture meets in Vincent Hall 314

Special Math/Physics Lecture

3:35 pm	**Abraham A. Ungar** North Dakota State University	Group-like structure underlying the c-ball in real inner product spaces

Abstract: A group-like structure underlying the c-ball V_c,

$$V_c = \{\sigma \in V_\infty : \|\sigma\| < c\}, \quad c = \text{ const. } > 0,$$

in an abstract real inner product space $V_\infty = (V_\infty, +, \cdot)$ is presented. The binary operation $+$ in V_∞ is no longer closed in V_c. Employing ideas from special relativity theory we define in terms of $+$ and $\cdot$ a binary operation $\oplus$ in V_c such that the pair $(V_c, \oplus)$ forms a group-like object which shares remarkable analogies with the group $(V_\infty, +)$ to which it reduces when $c \to \infty$. Applications to abstract group theory are demonstrated.

The LECTURE meets in Vincent Hall 570

Real Analysis Seminar

4:00 pm	**Luis Caffarelli** Institute for Advanced Study	Monge-Ampere equations and the mapping problem

The SEMINAR meets in Vincent Hall 314

Wednesday, May 22

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Roberta Dal Passo** IAC. Rome	Entropy solutions of a nonlinear parabolic equation: Uniqueness

Abstract. As in the case of first order conservation laws. certain parabolic equations, with vanishing diffusion coefficient for infinite gradients, admit weak solutions which may develop shocks. This phenomenon causes the nonuniqueness of the solution in distributional sense. Therefore it is necessary to impose some additional condition to determine uniquely a solution. We shall present such a condition, which is related to Oleinik's entropy condition. and which turns out to be necessary and sufficient.

THEORETICAL PHYSICS INSTITUTE SEMINAR

12:20 pm	Joseph Fehribach U. Alabama. Huntsville/IMA	Mullins-Sekerka stability analysis for melting-freezing waves in helium

Abstract. Helium, unlike any other material, exhibits the phenomenon of oscillatory melting-freezing waves at its solid-melt interface. This presentation will review the derivation of a moving boundary problem for this physical system, and will discuss the Mullins-Sekerka instability for this problem.

The SEMINAR is held in 435 Physics

Real Analysis Seminar

3:35 pm	**Umberto Mosco** University of Rome/IMA	Energy decay for degenerate Dirichlet forms of subelliptic type

The SEMINAR meets in Vincent Hall 113

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Thursday, May 23

SEMINAR IN { Mathematics / Physics / Vincent Hall 570 }

10:00 am	**A.V. Turbiner** ITEP, Moscow/CERN, Geneva	Quasi-exactly solvable problems: a new way from quantum mechanics to quantum field theory. IV

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Irene Martinez Gamba** Purdue University/IMA	Boundary layer formation for viscosity approximations in transonic flow

Abstract: The boundary layer formation for viscosity approximations in one-dimensional transonic flow models in a bounded interval with a source or a force-collision term is considered. The energy equation is replaced by a pressure-density relationship. The values of the density are imposed at the endpoints.

It can be shown that the viscosity approximation converges to a weak solution satisfying the jump and classical entropy condition.

The boundary layer has a condition that determines the possible range of discontinuities for the density.

These models are used in steady-state hydrodynamic modelling of semiconductor devices and also in steady state nozzle flow where the source term accounts by geometrical effects.

Annual IMA Picnic

3:00 pm		Picnic at Como Dome Shelter in Como Park, St. Paul

School of Mathematics Colloquium

3:30 pm	**Luis Caffarelli** Institute for Advanced Study	The propagation of perturbations through solutions of 2nd order PDE's

The COLLOQUIUM is held in Vincent Hall 16
Tea will be served at 3:10 pm in Vincent Hall 120

IMA Minisymposium: NONLINEAR DIFFUSION EQUATIONS & THEIR EQUILIBRIUM STATES

May 24 - 25, 1991

Cosponsored by the National Science Foundation & the IMA

Organizers: W.-M. Ni. L.A. Peletier, J. Serrin

Most of the program talks will be held in Conference Hall 3-180 on the entry floor of the Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

Friday, May 24

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Man Kam Kwong** Argonne National Laboratory	Uniqueness and multiplicity results for nonlinear boundary value problems

Abstract: We give a survey of recent development of uniqueness and multiplicity results for nonlinear boundary value problems that arise in the study of semilinear elliptic equations. Of particular interest are uniqueness results for the positive radial solutions of equations of the form $\Delta u + u^p \pm u = 0$ in a sphere, and multiplicity results for one-dimensional equations of the form $u'' + f(u) = 0$, with $f(u)$ convex in u.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176

SEMINAR ON INDUSTRIAL PROBLEMS

11:15 am	**Erich Wimmer** Cray Research	Large scale numerical simulations for industrial chemical research

Abstract: Large-scale numerical simulations are rapidly becoming an integral part of industrial chemical research in areas such as pharmaceuticals, agrochemicals, polymeric materials, and catalysis. The success of this exciting development hinges on the ability to calculate and accurately predict atomic-scale properties for systems which are large enough to capture the complexity of technologically relevant compounds and materials. In addition, these simulations have to be performed within a reasonable timeframe to have an impact on the industrial research activity. These three requirements, accuracy, complexity, and speed present a tremendous challenge for the theoretical formulation, the algorithmic choices, and the computational implementation. In this talk, the concepts, capabilities and present limitations of large-scale numerical simulation methods are reviewed. The discussion will include highly accurate ab initio and density functional methods, semiempirical quantum mechanical methods, and more approximative force-field approaches, which are especially useful in the study of the dynamics of proteins and DNA. The talk will conclude with an outlook on future theoretical approaches, computational implementations, and next-generation computer hardware.

1:30 pm	**Frank Merle** University of Paris VI	Existence and stability of a solution of the heat equation with k blow-up points
2:30 pm	**Yi Li** University of Rochester	Symmetry property of positive solutions of semilinear elliptic equations in the entire space

Abstract: In this talk I will present some of the recent results on the symmetry of positive solutions of semilinear elliptic equations in the entire space via the "moving plane" method and analysis of positive solutions at infinity of semilinear elliptic in $\mathbf{R}^n$, $n \geq 2$. For example, the equation of the form $\Delta u + f(u) = 0$ will be discussed.

3:30 pm	Coffee Break	Reception Room EE/CS 3-176
4:00 pm	Mariette C. Knaap Leiden University/IMA	Neumann boundary value problems involving critical Sobolev exponents

Abstract: It is well known that the Dirichlet problem

$$\begin{cases} -\Delta u = u^{\frac{N+2}{N-2}} + \lambda u & \text{in } \Omega \subset \mathbf{R}^N \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

does not admit any non-trivial solution for $\lambda \leq 0$, if the domain Ω is a bounded star-shaped domain in $\mathbf{R}^N$, $N \geq 3$. We replace the boundary condition $u = 0$ by the Neumann boundary condition

$$\frac{\partial u}{\partial n} = 0 \quad \text{on } \partial\Omega,$$

where n is the outward pointing normal on $\partial\Omega$, and address the question of existence of a solution when λ takes values in $\mathbf{R}$.

Saturday, May 25

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**William C. Troy** University of Pittsburgh	Similarity solutions of a model for doping in semiconductors

Abstract: We (Peletier & Troy) investigate a mathematical model for the infiltration of dopant into semiconductors. The model consists of a coupled system of diffusion equations. Because of the invariance properties of the model it is natural to look for solutions in self similar form. The resulting problem is the fourth order boundary value problem

$$\left.\begin{aligned} (f'g - fg')' + \frac{1}{2}xf' &= 0 \\ g'' + \frac{1}{2}pxg' + \frac{1}{2}qxf' &= 0 \end{aligned}\right\} \quad \text{for} \quad 0 < x < \infty$$

with boundary conditions

$$f(0) = 1,\ g(0) = 1 \ \text{ and } \ f(\infty) = 0,\ g(\infty) = 1.$$

We use a two dimensional topological shooting argument to prove the existence of at least one solution of this problem.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:15 am	**Henghui Zou** University of Minnesota	Existence and non-existence results for ground states of quasilinear elliptic equations

Abstract: We (Serrin & Zou) consider existence and non-existence of *radial ground states* for the quasilinear elliptic equation

$$\Delta u + u^p - |\nabla u|^q = 0, \qquad \text{in} \mathbf{R}^n, \tag{I}$$

where $p > 0$, $q > 0$ and $n \geq 3$. By using shooting methods, general Sobolev and Poincare inequalities, and Pohozaev type identities, we establish existence if any one of the following conditions holds:

$$\text{(i) } p > n^*; \qquad \text{(ii) } p = n^*,\ q \leq n_p;$$

Abstract: Consider the following two-phase Stefan problem with an enthalpy dependent source:

$$\begin{cases} \frac{\partial u}{\partial t} - \Delta\beta(u) + f(u) + b(x,t) = 0 & \text{in } Q = \Omega \times (0,T), \Omega \subset \mathbf{R}^n \\ \beta(u) = g(x,t) & \text{on } \sum = \partial\Omega \times (0,T) \\ u(x,0) = u_0(x) & \text{in } \Omega \end{cases}$$

where $u \equiv u(x,t)$ is the enthalpy, $\beta(r)$ is an nondecreasing Lipschitz continuous function with linear growth and $\beta(0) = 0$. Under some assumptions, the space and time regularities of the weak solution will be discussed respectively in Nikolskii space (or BV space) and space $C^\alpha([0,T]; L^1(r))$. For some case, error estimates in $L^\infty(0,T; L^1(r))$ will be deduced for the discrete-time approximation of the enthalpy. The relevant work was recently presented by E. Magenes, C. Verdi and A. Visitin (SIAM, J. Numer. Anal. V. 26, No. 6, pp 1425–1438, 1989).

AEM Fluid Mechanics Special Seminars

12:20 pm	G.I. Barenblatt USSR Acad. of Sciences/IMA	Turbulence in Stratified Fluids. I

The SEMINAR is held in Conference Hall EE/CS 3-180

IMA Postdoc Seminar

2:30 pm	Juan Redondo University of Michigan/IMA	Regularity for level sets of solutions of a penalized problem

Abstract: We study critical level sets of solutions to a penalized obstacle problem and obtain uniform $C^{1,\alpha}$ estimates under some density hypothesis. As a consequence, in the limit we obtain the regularity of the free boundary.

The SEMINAR meets in Vincent Hall 570

Wednesday, May 29

Industrial Postdocs Seminar

1 - 4 pm   Ninth Meeting

The format of the seminar is:
1) Presentation of projects and problems from industry (3M and Honeywell) on which the industrial postdocs are working.
2) Informal suggestions and discussion among the participants.

The seminar is directed by Avner Friedman and Walter Littman. Those wishing to participate should contact A. Friedman.

The SEMINAR meets in Vincent Hall 570

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IMA NEWSLETTER #175

March 9 - March 31, 1991

1990-91 Program
PHASE TRANSITIONS AND FREE BOUNDARIES

NEWS AND NOTES

IMA Workshop: FREE BOUNDARIES IN VISCOUS FLOWS

March 11–15, 1991
Organizers: R. Brown, S. Davis, S. Kistler

Viscous flows interact with liquid/fluid interfaces and solidification fronts in a wide variety of technologies including the processing of coatings, polymers, semiconductors single crystals and other advanced materials. This workshop addresses the mathematical treatment of the dynamical and instability phenomena in such flows. Special topics include solidification with flows, wetting lines, and the instabilities of drops and films. Presentations will focus on the development of asymptotic and numerical strategies for solution of these problems. Applications of analysis of complex flow problems will be high-lighted.

Most of the program talks will be held in Conference Hall 3-180 on the entry floor of the Electrical Engineering/Computer Science Building. This building is located on the corner of Washington Avenue and Union Street, a block from the IMA Main Office. The conference hall is on the Ethernet and has a projection system for display of computer output.

LECTURES IN SUPERCONDUCTIVITY

Allen Goldman and John Ockendon will discuss macroscopic models of superconductivity and their relationship to free boundary problems in a lecture series at 1:25 pm, March 26 - 28, held in the Conference Hall EE/CS 3-180. Professor Goldman will distribute notes for his talks on the physics background of the models.

SCHEDULE FOR MARCH 9 - MARCH 31

Monday, March 11

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:00 am	Registration and coffee	Reception Room EE/CS 3-176
9·30 am	Welcome and Orientation	Conference Hall EE/CS 3-180
9:40 am	**Gerald W. Young** The University of Akron	Mathematical description of viscous free surface flows

Abstract: Viscous free surface flows are of importance to a wide variety of coating and material processing technologies. Designing, developing, and advancing these techniques requires quantitative understanding of the transport processes at hand, as well as the location, dynamics, and stability of the interface. Empirical investigations of these systems may be hindered by the presence of an unusually large number of independent processing parameters, or by difficulties in observing the complex and coupled interactions between the flows and the free surfaces. Further, there are situations where either the fundamental physics is poorly understood or only understood in isolation from one another. Hence, mathematical modeling is called upon to play an ever increasing role in addressing these issues. Forwarding the understanding of the science and assisting in multi-parameter optimization of the technology are key objectives.

This presentation primarily focuses on the formulation, and to a lesser extend the analysis and solution, of models simulating the interaction between viscous flows and free surfaces. Considerations in the modeling of liquid-fluid and liquid-solid interfacial system are discussed. A description of contact-line problems is also examined Examples illustrating film flow, spreading, surface-tension-driven flow, and solidification phenomena are presented. Analysis of the models will concentrate on solution methodologies based upon asymptotic expansion techniques.

10:40 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**Robert A. Brown** MIT	Mathematical description of viscous free surface flows

Abstract: Viscous free surface flows are described as the class of free- and moving- boundary problems in which a viscous flow must be computed in conjunction with the determination of the shape of either a liquid/fluid meniscus or a melt/solid phase boundary. The mathematical descriptions of these problems are stated with special emphasis on the conditions on the free boundaries and contact curves between these surfaces and other phases. Prototypical models of material processes with both melt/gas interfaces and solidification surfaces are used as examples.

The primary features of numerical algorithms for the solution of viscous free-surface flow problems are emphasized in the context of these models. The important issues emphasized are the spatial discretization of the flow equations, the effective representation of the interface and a numerical iteration scheme for simultaneous determination of the interface shape and the flow. These features are addressed fully in the talks by others at this meeting.

Numerical Analysis Seminar

1:25 pm	**Bernardo Cockburn** University of Minnesota	Discontinuous Galerkin methods for 2D gas dynamics

The SEMINAR meets in Vincent Hall 570

2:00 pm	**Josef Bemelmans** Saarbrücken	Free-boundary value problems for the stationary Navier-Stokes equations

Abstract: The motion of an incompressible viscous fluid that occupies a given domain Ω whose boundary Σ is governed by surface tension or self-attraction is described by the following free-boundary problem for the Navier-Stokes equations:

$$(N)\qquad \left.\begin{aligned} -\nu\Delta v + Dp + (v\cdot D)v &= \rho \\ \operatorname{div} v &= 0 \end{aligned}\right\} \text{ in } \Omega$$

$$(m)\qquad v\cdot n = 0,\ t_k\cdot T(v,p)\cdot n = 0 \text{ on } \Sigma,\ k = 1.2$$

$$(H)\qquad 2KH = n\cdot T(v,p)\cdot n \text{ on } \Sigma$$

or

$$(G)\qquad \int_\Omega \frac{g}{|x-y|}dy = n\cdot T(v,p)\cdot n \text{ on } \Sigma.$$

Condition (H) states that the normal component of the stress vector is proportional to the m ean curvature H of Σ; if the fluid body is held together by self-attraction instead of surface tension, then condition (G) is imposed.

We discuss existence, uniqueness, and regularity of solutions to the boundary value problems (N), (m), (H) and (N), (m), (G). For the latter one hard implicit function theorems are a basic tool. The solutions to these problems can be considered to be generalizations of classical equilibrium figures of rotating liquids. This connection leads to interesting open questions.

3:00 pm	**Elizabeth N. Dussan** Schlumberger Doll Research Center	Wetting lines
4:00 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

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Tuesday, March 12

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Marc K. Smith** The Johns Hopkins University	Longwave interfacial instabilities in viscous liquid films

Abstract: A thin film of viscous liquid is susceptible to many different kinds of instabilities. One well-known mode of instability displays a wave-like deformation of the interface of the liquid film with a wavelength that is much larger than the thickness of the film. We shall examine this instability and show how the dynamics of the instability mechanism can be explained by carefully interpreting the mathematical solution of the instability problem obtained by using a standard regular perturbation technique.

Several different geometries will be considered. The most fundamental is the isothermal thin film on an inclined plane. Here, we explain how the small effects of liquid inertia lead to the unstable interfacial motion. Next, we consider both heating and cooling the layer from below and show the effects of buoyancy forces and direct liquid expansion. For an annular film in a vertical pipe, we show how the unstable behavior of the film is modified by a large lubrication pressure that appears because of the rigid pipe wall. Finally, a horizontal layer driven by thermocapillary is considered to demonstrate the effect surface-tension gradients have on the instability.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**David S. Dandy** Sandia Nat. Labs.	Finite-difference and finite-volume techniques for intermediate Reynolds number flows

Abstract: Two approximation techniques for solving finite Reynolds number (low Mach number) free-surface flows in two and three dimensions will be compared and contrasted. The first, finite-difference, will be applied

to the buoyancy-driven motion of an axisymmetric deformable bubble. In this treatment, the interface corresponds to a coordinate surface in a boundary-fitted system, and its position is treated as unknown. The governing set of partial differential equations for the coordinate mapping and the flow are approximated by second-order accurate, centered finite-differences, resulting in a set of coupled nonlinear algebraic equations which is then solved using Newton's method. Several sparse matrix algorithms have been applied to the linear Newton system, and these will be described.

The second approximation technique, finite-volume, will be employed on three, three-dimensional nonisothermal flows, a sphere in linear shear, a prolate spheroid in uniform flow at angle of attack, and flow in a curved tube. Three-dimensional, body-fitted curvilinear coordinates yield cell-centered volume elements having four, five, or six sides. The conservation equations, written in integral form, are approximated within each volume element. The volume and surface integral approximations give rise to a set of coupled nonlinear algebraic equations. Due to computer memory limitations, the system of equations is currently solved using a predictor-corrector algorithm that has been optimized for fast convergence, but work is underway to apply Newton's method to the system of equations.

2:00 pm	**Lee-Wing Ho** Nektonics, Inc., Cambridge MA	Imposition of surface-tension boundary conditions in finite simulation of three-dimensional viscous free-surface flows

Abstract: Over the last two decades the finite element method has been applied successfully to the simulation of many free-surface fluid flow problems of fundamental and practical importance. The success of the method can be attributed to the use of variational formulation and domain decomposition techniques. While the former allows simple treatment of variable properties as well as natural imposition of Neumann type boundary conditions, the latter enables the modelling of complex and time-dependent flow geometry in both two and three space dimensions.

In this study, a new variational form is proposed for the imposition of general surface-tension boundary conditions for three-dimensional incompressible viscous free-surface flows. This new form has the following important advantages: first, it provides a consistent treatment for variable surface tension; second, it automatically generates (in a weak sense) natural conditions for C^1-continuity of the free-surface geometry; and lastly, it is entirely surface-intrinsic and thus well-suited for finite element discretization. In the present study this new variational form is discretized spatially using spectral element (high-order or p-type finite element) method and temporally using semi-implicit time-stepping schemes; the resulting system of algebraic equations are inverted using preconditioned conjugate gradient iteration. Several examples are given as a demonstration of the effectiveness of the approach.

3:00 pm	**Coffee Break**	Reception Room EE/CS 3-176
3:30 pm		**Contributed Papers**

Participants are invited to present contributed papers. Twenty minute time slots have been set aside for these talks. It is hoped that the speakers will leave five minutes of the time for discussion. Please contact the organizers if you wish to give a contributed paper.

3:30 pm	**Ulrich Hornung** U. der Bundeswehr München	**Some examples of capillary surfaces**
3:50 pm	**Vladislav Pukhnachov** USSR Acad. Sci., Novosibirsk	**Anomalous Marangoni effect in its manifestation in thin liquid layers**
4:10 pm	**Dan Joseph** University of Minnesota	**Expansion velocity and Korteweg stresses in simple mixtures of incompressible fluids**

Wednesday, March 13

Unless otherwise stated, the talks today are in Conference Hall EE/CS 3-180

9:30 am **Stephan F. Kistler** — Free boundaries in viscous flows
3M

Abstract. In the computational analysis of viscous free-surface flows, coordinate representation and coupling between flow and free-surface calculations are major challenges. Substantial changes in macroscopic flow geometry can be accompanied by the formation of highly curved interfaces in the immediate vicinity of apparent singularities, such as those that arise near cusp-like corners and dynamic wetting lines. After a brief review of previously published Galerkin finite-element techniques for solving viscous free-surface flows, this presentation focuses on a generalized algebraic mesh generation scheme that can track major changes in the macroscopic flow domain, yet simultaneously permits extensive mesh refinement near singularities. Several case studies of coating flow with dynamic wetting lines illustrate the capabilities of the algorithm. Computed predictions are shown to agree with experimental measurements and, where available, with results from matched asymptotic expansions.

10:30 am **Coffee Break** — Reception Room EE/CS 3-176

11:00 am **Jonathan J. L. Higdon** — High order boundary integral methods for Stokes flow
University of Illinois, Urbana

Abstract: We consider the development of high order implementations of the boundary integral method for Stokes flow. Practical issues of discretization, quadrature and matrix inversion are discussed and a general approach for three dimensional flows is presented. For free surface flows, the general algorithm may be supplemented with time stepping algorithms for transient problems or a Newton Raphson scheme for equilibrium configurations.

2:00 pm **Ole Hassager** — Methods for solving free surface problems
DTH Lyngby DENMARK

Abstract: The talk will focus on the application of the finite element method for the solution of viscous free surface problems. It will be illustrated how models for interfacial tension and contact angles may be implemented in a natural way in the variational basis for the finite element method. Time dependent flows and moving meshes will be described. Results will be given for specific flow problems such as the motion of drops in capillaries, and coating flows. Problems associated with moving contact lines will be discussed.

3:00 pm **Ridgway Scott** — Asymptotic and numerical methods for approximating flows with a free boundary
University of Houston

Abstract: The phenomenon which motivates the current work is the flow of a liquid constrained only partly by a container, that is, in which a part of the boundary of the domain filled by the liquid is an interface with another liquid of much smaller density. One model for the behavior of such liquids is based on the assumption that the surface tension between the two liquids is proportional to the curvature of the free surface. This model has been studied extensively in recently years, both experimentally, theoretically, and computationally. We will describe both asymptotic and numerical methods for approximating the solution. The asymptotic methods are based on the so-called lubrication approximation, but one in which inertial terms are included. The numerical techniques are based on finite element approximations of the Navier-Stokes equations in two dimensions. Both the asymptotic and numerical methods show remarkable accuracy. A further objective of the talk is to describe a framework for the analysis of convergence properties of the computational techniques being used. We will describe for a model problem how the numerical approximation of the free boundary affects the approximation of the other variables of the problem and *vice-versa*.

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Thursday, March 14

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**S. R. Coriell** NIST	Interaction between fluid flow and morphological stability

Abstract: During freezing of a liquid phase the crystal-melt interface is subject to morphological instability leading to cellular and dendritic growth. Fluid flow in the melt can alter the conditions for the onset of morphological instability Classic hydrodynamic instabilities can be altered by the presence of a crystal-melt interface (free boundary). The interaction between fluid flow and the crystal-melt interface is an active area of research. Examples of these interactions based primarily on linear stability analyses of the coupled system will be discussed. For example, if a lighter solute is rejected during directional solidification vertically upwards, both convective and morphological instabilities can occur.

10:30 am	**Coffee Break**	Reception Room EE/CS 3-176
11:00 am	**M. Grae Worster** Northwestern University	Interfaces with mushy layers

Abstract. Solidification fronts in binary alloys are prone to morphological instabilities that prevent the front from being planar in many situations. Stability analyses can determine the onset of morphological instability and small, finite-amplitude analyses can follow the evolution of the interface under weakly supercritical conditions. Expensive and time-consuming numerical simulations can predict the formation of deep cells and some of the cell-splitting phenomena that begin to deform the interface on smaller and smaller lengthscales. However, once the solid-liquid interface has become highly convoluted, perhaps dendritic, it is no longer practical to attempt the prediction of its precise morphology. Instead, the study of mushy layers seeks to provide an averaged description of the resultant two-phase medium (solid dendrites and interstitial liquid) on lengthscales larger than the fine-scale structure of the solid-liquid interface. The problem is thus converted from a two-phase problem (solid and liquid) with a single, complicated interface into a three-phase problem (solid, mush and liquid) with two geometrically simpler interfaces. However, this simplification carries the price of having to determine appropriate boundary conditions for the interfaces of an averaged medium. The seminar will comprise a general discussion of the theory of mushy layers with particular attention paid to the thermodynamic interfacial conditions and the ways in which these modify or are modified by natural convection in the melt and through the interstices of the mushy layer.

2:00 pm	**Jeffrey J. Derby** University of Minnesota	Macroscopic interactions of viscous flow, heat transfer, and solidification interfaces in melt crystal growth systems

Abstract: We are entering an era in which chemical engineers and materials scientists are attempting to influence the properties of advanced materials by control at the molecular level. In solidification systems, this control requires detailed understanding of the fundamentals of morphological stability and the effects of macroscopic transport in the processing systems which are used to produce these materials. This seminar will address some issues in understanding the growth of large-dimension laser host crystals. We will detail the development of and present recent results from mathematical models of the Bridgman and Czochralski crystal growth processes.

3:00 pm	**Michael M. Chen** University of Illinois, Urbana	Generic structure of flow and temperature fields in welding and high energy beam processing

Abstract: Thermocapillary convection plays a dominant role in the fluid flow and heat transfer of a number of materials processing techniques, especially welding and other processing modalities involving melting due to concentrated energy beams, such as laser and electron beam processing and plasma processing. Typically, these processes produce a pool of molten material with intense temperature gradients along the surface, and it is the surface tension gradient associated with the surface temperature variation which drives the flow. The dimensionless parameter for thermocapillary convection is the Marangoni number, which can be viewed as the Peclet number based on the elementary thermocapillary velocity scale. For most of the processes cited above, the Marangoni number is of the order of 10^4 or more. Thus the flow is expected to possess singular regions, including thermal and viscous boundary layers and a unique thermocapillary singularity at the corners. These singular regions render numerical computations extremely challenging. In addition, common numerical approximations. such as upwind differencing, can be particularly damaging because of the phenomenon's strong dependence on accurate temperature gradients in these singular regions. Failure to appreciate these difficulties often leads to inaccurate or erroneous numerical results and hence incorrect conclusions concerning processing. In this paper the physics of thermocapillary convection under common materials processing conditions will be described. The structure of temperature and velocity distributions, the scalings of the boundary layers and corner singular regions, and their known implications on material processing will be discussed. In addition, common pitfalls for numerical computations of such flows will be reviewed.

4:00 pm	**Akerman Hall 130D**	Tour of Dan Joseph's laboratory

We will show experiments on water lubricated pipelining with bamboo waves and corkscrew waves, two dimensional cusped interfaces which violate Laplace's law, rollers, drafting, kissing and tumbling of fluidized particles, non linear stabilization of fingering instabilities in porous media. The tour will probably last 30 or 40 minutes. **Please sign up for this tour in the Reception Room, EE/CS 3-176.**

5:45 pm	**Workshop Reception** & Buffet Dinner	Campus Club, 4th floor, The Terrace, Coffman Union

Reservations required. The reception begins at 5:45 (wine and cheese) followed by dinner at 6:30.

Friday, March 15

Unless otherwise indicated, the talks today are in Conference Hall EE/CS 3-180

9:30 am	**Saleh Tanveer** Ohio State University	Asymptotic techniques in Hele-Shaw flows

Abstract: The motion of a less viscous fluid displacing a more viscous fluid in a Hele-Shaw cell under the action of a pressure gradient has been the subject of intensive research over the last few years because of the mathematical analogies to two phase flow in a porous medium, dendritic crystal growth, directional solidification and electrochemical growth. In this talk, we will describe some of the analytical techniques that have proved useful in answering questions on steady state solutions, their linear stability and more recently in the complete time evolution problem for small surface tension at the level of formal asymptotics. The key role of asymptotics beyond all orders will be delineated.

10:30 am	**Coffee Break**	Vincent Hall 502

11:00 am	**L. E. Scriven** University of Minnesota	Theory of viscous free surface flows: Computing steady states, stability and sensitivity

Abstract: Viscous flows of the sort in precision coating operations are made nonlinear by their free surfaces. Inertial and non-Newtonian behavior can contribute as well. Successfully managed, such flows though contorted are steady and two-dimensional apart from edge effects. But they are prey to barring, ribbing and other unwanted instabilities. These are influenced not only by the flow parameters but also by the coater configuration. Sensitivities to persistent external disturbances are likewise important in practice.

Accordingly we have in a series of researches, most notably those carried out by K.N. Christodoulou, solved Navier-Stokes-type systems by Galerkin's method of weighted residuals and representation in terms of finite element basis functions on subdomains, the latter generated with algebraic or elliptic mesh design equations and related to a cartesian computational domain by isoparametric or subparametric mapping that accommodates free surfaces.

Steady states we find from the resulting nonlinear system $A(x)x = b$ by Newton iteration or variants, with initialization by first-order continuation and optimization of parameter step size. Responses to small

transitory distrubances we find from linear stability theory, which leads to a large, sparse, asymmetric, generalized eigenproblem

$$Jx = \lambda Mx$$

in which J and M depend on system parameters and M is singular. We solve for leading modes - eigenvalues of algebraically largest real parts, and their eigenvectors - by a flexible method assembled from the iterative Arnoldi algorithm with Schur-Wielandt deflation, initialization that incorporates rational acceleration to eliminate infinite eigenvalues resulting from algebraic constraints, complex or real shift of eigenvalue as appropriate, and approximately exponential preconditioning by rational transformation suitable to the singular behavior.

Responses of stable states to persistent periodic disturbances we find from linear frequency response theory, which leads to a doubly large, sparse linear system

$$\begin{bmatrix} j & -\omega M \\ \omega M & j \end{bmatrix} \begin{bmatrix} Re & (z) \\ IM & (z) \end{bmatrix} = \begin{bmatrix} -F \\ 0 \end{bmatrix}$$

in which ω is the forcing frequency and F the sensitivity vector with respect to the forcing parameter. This we solve efficiently for amplification versus frequency. When nonlinear transient response is needed we employ the Gresho-Lee-Sani algorithm or in some circumstances the DASSL algorithm.

For parsimonious probing of parameter space we trace steady states marked by turning points (fold tracking) and other critical features such as sensitivity thresholds, ordinary and Hopf bifurcations. To do this we solve by Newton-like methods and continuation the steady-state equations augmented with a set that uniquely characterizes the critical feature. No less complete a theory is needed to optimize configuration and operation of many other processes as well.

References

S.F. Kistler & L. E. Scriven, Coating flow theory by finite element and asymptotic analysis of the Navier-Stokes system, Int. J. Num. Meth. Fluids 4 207 (1984)

K.N. Christodoulou & L.E. Scriven, Finding leading modes of a viscous free surface flow: an asymmetric generalized eigenproblem, J. Sci. Comp. 3 355-406 (1988)

S.F. Kistler & L. E. Scriven, The teapot effect: sheet-forming flows with deflection, wetting and hysteresis, J.Fl. Mech., in press (1991); K. N. Christodoulou & L.E. Scriven Operability limits of free surface flow systems by solving by solving nonlinear eigenvalue problems, Int. J. Num. Meth. Fluids (to appear).

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

2:00 pm	**José Francisco Rodrigues** Univ. Lisbon & CMAF	The two-phase steady-state Stefan problem in strips

Abstract: In this joint work with B. Faltzman, we show the steady-state free boundary of a Stefan problem of continuous casting type, in two dimensions, is given by a Lipschitz wave. This implies the existence of a classical solution in an almost everywhere sense. We establish the stability of this wave with respect to the length L of the domain and by letting $L \to +\infty$, we cover the case of an infinite strip. We obtain an error of order $1/\sqrt{2}$ for the difference, in the C^1-norm, of the temperatures and of the free boundaries associated with the finite and infinite strips, respectively.

Monday, March 18

SEMINAR IN { Free Boundary Problems / Vincent Hall 570

11:15 am	**J. Thomas Beale** Duke University	Exact solitary water waves with capillary ripples at infinity

transitory distrubances we find from linear stability theory, which leads to a large, sparse, asymmetric, generalized eigenproblem

$$Jx = \lambda Mx$$

in which J and M depend on system parameters and M is singular. We solve for leading modes - eigenvalues of algebraically largest real parts, and their eigenvectors - by a flexible method assembled from the iterative Arnoldi algorithm with Schur-Wielandt deflation, initialization that incorporates rational acceleration to eliminate infinite eigenvalues resulting from algebraic constraints, complex or real shift of eigenvalue as appropriate, and approximately exponential preconditioning by rational transformation suitable to the singular behavior.

Responses of stable states to persistent periodic disturbances we find from linear frequency response theory, which leads to a doubly large, sparse linear system

$$\begin{bmatrix} j & -\omega M \\ \omega M & j \end{bmatrix} \begin{bmatrix} Re & (z) \\ IM & (z) \end{bmatrix} = \begin{bmatrix} -F \\ 0 \end{bmatrix}$$

in which ω is the forcing frequency and F the sensitivity vector with respect to the forcing parameter. This we solve efficiently for amplification versus frequency. When nonlinear transient response is needed we employ the Gresho-Lee-Sani algorithm or in some circumstances the DASSL algorithm.

For parsimonious probing of parameter space we trace steady states marked by turning points (fold tracking) and other critical features such as sensitivity thresholds, ordinary and Hopf bifurcations. To do this we solve by Newton-like methods and continuation the steady-state equations augmented with a set that uniquely characterizes the critical feature. No less complete a theory is needed to optimize configuration and operation of many other processes as well.

References

S.F. Kistler & L. E. Scriven, Coating flow theory by finite element and asymptotic analysis of the Navier-Stokes system, Int. J. Num. Meth. Fluids 4 207 (1984)

K.N. Christodoulou & L.E. Scriven, Finding leading modes of a viscous free surface flow: an asymmetric generalized eigenproblem, J. Sci. Comp. 3 355-406 (1988)

S.F. Kistler & L. E. Scriven, The teapot effect: sheet-forming flows with deflection, wetting and hysteresis, J.Fl. Mech., in press (1991); K. N. Christodoulou & L.E. Scriven Operability limits of free surface flow systems by solving by solving nonlinear eigenvalue problems, Int. J. Num. Meth. Fluids (to appear).

SEMINAR IN { Free Boundary Problems
Vincent Hall 570

2:00 pm	**José Francisco Rodrigues** Univ. Lisbon & CMAF	The two-phase steady-state Stefan problem in strips

Abstract: In this joint work with B. Faltzman, we show the steady-state free boundary of a Stefan problem of continuous casting type, in two dimensions, is given by a Lipschitz wave. This implies the existence of a classical solution in an almost everywhere sense. We establish the stability of this wave with respect to the length L of the domain and by letting $L \to +\infty$, we cover the case of an infinite strip. We obtain an error of order $1/\sqrt{2}$ for the difference, in the C^1-norm, of the temperatures and of the free boundaries associated with the finite and infinite strips, respectively.

Monday, March 18

SEMINAR IN { Free Boundary Problems
Vincent Hall 570

11:15 am	**J. Thomas Beale** Duke University	Exact solitary water waves with capillary ripples at infinity

For the non-linear equation (1) with $m > 1$ we give a simple proof of the uniqueness of the solution of first-type with compact support in the space of the variables $\zeta_i = x_i/t^{\alpha}(i = 1, \dots, n)$. Such self-similar solutions are possible only when $\alpha = \alpha_* \equiv [n(m-1)+2]^{-1}$ and they are exhausted by the well-known Barenblatt solutions.

A more complicated situation arises for first-type solutions corresponding to initial data with noncompact (in fact, conical) support. We demand here some regularity on the self-similar solution at the origin but allow it to have power growth when $|\zeta| \equiv |x|/t^{\alpha} \to \infty$. It turns out that these solutions do not exist if $\alpha \leq \alpha_*$. The question of their existence for $\alpha > \alpha_*$ remains open. However, the solvability of the corresponding linearized problem and the investigation of asymptotic properties of the solution as $x\zeta \to 0$ and $\zeta \to \infty$ allows hope for a positive answer to this question.

As an example of the second type of self-similar solutions we study the Cauchy problem for the Boussinesq equation in the filtration theory. This case corresponds to $n = 2$ and $m = 2$ in equation (1). Here the initial function is $u_0 = r^2 g_0(\theta)$ (r and θ are polar coordinates in the plane). The non-negative function g_0 equals zero outside the interval $[-\delta, \delta]$, where $\delta \in (0, \pi)$. In this case the free boundary consists of two rays, $\theta = \pm\lambda(t)$, where $\lambda(0) = \delta$. Our guess is that as t increases the function $\lambda(t)$ increases up to the value π and at a later time the solution blows up. This hypothesis is confirmed by numerical simulation of the problem.

Wednesday, March 20

SEMINAR IN { Open Problems in Applied Mathematics / Vincent Hall 570 }

4:45-5:30 pm **Hans van Duijn**
Delft University of Technology

Background: This is a series of discussions on open problems in applied and industrial mathematics. IMA visitors, especially post-docs, are encouraged to come.

Thursday, March 21

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am **Wei-Ming Ni** — Spike layers in singular perturbation theory
University of Minnesota

Abstract: I shall try to give a brief survey of recent progress on spike layer solutions in singular perturbation problems

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Monday, March 25

Numerical Analysis Seminar

1:25 pm **Alfio Quarteroni** — Domain decomposition for partial differential equations
University of Minnesota

The SEMINAR meets in Vincent Hall 570

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

2:30 pm **Viorel Barbu** — Controlling the free boundary of the obstacle problem in a variable domain
University of Cincinnati

Abstract: One proves that the free boundary of the variational inequality

$$\Delta y = f \text{ in } \{y > 0\}$$
$$\Delta y \leq f \text{ in } \Omega_u = \{(x,y) \in R^2; 0 < x < 1, u(x) < y < A\}$$
$$y \geq 0 \text{ in } \Omega_u$$
$$y = 0 \text{ in } \{y = 1\}, y = 1 \text{ in } \{y = u(x)\}, y_x = 0 \text{ in } x = 0, 1$$

in the variable domain Ω_u, $u \in H^3(0,1), u < A$ is approximately controllable.
A similar result remains true for the corresponding parabolic problem.

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Tuesday, March 26

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Mitchell Luskin** University of Minnesota	Numerical modeling of the microstructure of crystals with symmetry-related variants

Abstract: A continuum theory for crystals which predicts the existence of stable, symmetry-related martensitic variants, phase boundaries, and fine-scale twinning has mathematical features very unlike those usually considered in continuum mechanics. In particular, solutions to the appropriate variational problems must be described in terms of a material microstructure rather than in terms of a deformation. The numerical computation of approximate solutions to the continuum models is difficult because the bulk energy is nonconvex and has many local minima and because the solution is highly oscillatory. For these reasons, we have developed new concepts to interpret the numerical approximation of this microstructure .

We will give computational results and numerical analysis for continuum models for crystals with symmetry-related (martensitic) variants. These models relate the experimentally observed microstructure in shape memory alloys such as CuZn, CuAlNi, NiAl, and InTl to the numerical minimization of bulk energy.

LECTURES IN { Superconductivity / Conference Hall EE/CS 3-180 }

1:25 pm	**Allen Goldman** University of Minnesota	Macroscopic theories of superconductivity: London and Ginzburg-Landau models, I

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Wednesday, March 27

LECTURES IN { Superconductivity / Conference Hall EE/CS 3-180 }

1:25 pm	**Allen Goldman** University of Minnesota	Macroscopic theories of superconductivity: London and Ginzburg-Landau models, II
2:45 pm	**Vincent Hall 502** (The IMA Lounge)	IMA Tea (and more!)

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Thursday, March 28

SEMINAR ON INDUSTRIAL PROBLEMS: UNDERGRADUATE COMPONENT

9:00 am	**Walter Littman** University of Minnesota	Open only to selected undergraduate participants

Background: A group of a half dozen undergraduates is meeting on a regular basis with Professor W. Littman in Vincent Hall 570 in conjunction with the Industrial Problems Seminar. One of the aims of this activity is to bridge the gap between the knowledge acquired in regular course work and the know-how necessary to attack problems as they actually appear in industry. Another aim is to channel the enthusiasm for computing that many students have into productive scientific areas.

SEMINAR IN { Free Boundary Problems / Vincent Hall 570 }

11:15 am	**Hans van Duijn** Delft Univ. of Technology/IMA	A free boundary problem involving a cusp

Abstract: We consider the behaviour of the interface between fresh and salt water in a porous medium (reservoir). The salt water is below the interface (w.r.t. the direction of gravity) and is stagnant. The fresh water is above the interface and moves towards the wells which are present in the reservoir. We give a description of the corresponding flow problem leading to a weak variational formulation involving a parameter Q which is related to the strength of the wells. We show that Q is a critical parameter in the following sense: there exists $Q' > 0$ such that for $Q < Q'$ a smooth interface exists which is monotone w.r.t. Q. For $Q = Q'$ a free boundary with one or more singularities (cusps) will occur at a positive distance from the wells. The global analysis for the problem (uniqueness, monotonicity and existence) is three dimensional. The local cusp analysis is two dimensional.

This is joint work with H.W. Alt from Bonn University.

LECTURES IN { Superconductivity / Conference Hall EE/CS 3-180 }

1:25 pm	**J. R. Ockendon** University of Oxford	Macroscopic superconductivity modelling

C. PUBLICATIONS OF POST DOCS (NOT PREVIOUSLY REPORTED).

D. Lewis

[1] D. Lewis, T. Ratiu, J.C. Simo, and J.E. Marsden, *The Heavy Top: A Geometric Treatment*, IMA Preprint Series.

D. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT.

D. SCIENTIFIC PERSONNEL SUPPORTED BY THIS PROJECT:

John Albert
Haitao Fan
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Maciej Krupa
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