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13. ABSTRACT *(Maximum 200 words)*

The "sliding method" has been introduced, and the "method of moving planes" has been greatly extended and improved. These methods have been applied to a number of problems for fully nonlinear second order elliptic equations: (i) to treat symmetry and monotonicity of solutions in domains – even with nonsmooth boundaries. (ii) prove existence, uniqueness, monotonicity of traveling waves in a cylinder, for problems arising in combustion theory and in biology, including extensions to higher dimension of work of Kolmogoroff, Petrovsky, Piskounoff Separate work studied flame propagation as a limit problem leading to a free boundary problem. Regularity of the free boundary has been studied by methods which should prove useful in other problems.

Work on statistical mechanics, on deriving hydrodynamic scaling limit equations of Ginzburg-Landau. A rigorous proof was given of the Euler equations for conservation laws from Hamiltonian systems.

The question of existence of "breathers", existence of time-periodic solutions of hyperbolic equations on the line, was studied. Nonexistence of certain solutions depending analytically on a parameter was shown. Formal solutions and various estimates were established. Variational methods have been used for nonlinear equations. heat flow methods have been used in the study of harmonic maps.

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TECHNIQUES IN LINEAR AND NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

FINAL TECHNICAL REPORT

PROFESSOR LOUIS NIRENBERG

OCTOBER 21, 1991

U.S. ARMY RESEARCH OFFICE

DAAL03-88-K-0047

NEW YORK UNIVERSITY
COURANT INSTITUTE OF MATHEMATICAL SCIENCES

A. Problems Studied: A variety of problems were studied:

(i) Travelling waves in a cylinder for models arising in combustion theory and in biology

(ii) Improved forms of the maximum princple

(iii) Regularity in a class of free boundary problems

(iv) Statistical mechanics - on hydrodynamic scaling limit equations of Ginzburg-Landau

(v) Existence or nonexistence of "breathers": time periodic solutions of a semilinear wave equation on an infinite line

(vi) Variational methods in connection with vibration problems and minimal surfaces

B. Summary of the most important results

The series of papers [1–4] by Nirenberg and collaborators are primarily concerned with the study of travelling waves in a cylinder for semilinear second order elliptic equations. These equations occur in models of combustion — flame propagation — and in biology. A basic tool is the maximum principle. The authors have investigated under what conditions the maximum principle holds in general bounded domains — even with nonsmooth boundaries.

The "method of moving planes" has been used to prove symmetry and a monotonicity of solutions of second order nonlinear elliptic equations. Berestycki and Nirenberg have greatly improved the method and simplified its use. In addition they have introduced a new method for proving monotonicity and uniqueness, the "sliding method" and have made this a very effective technique. They have extended it to fully nonlinear elliptic equations.

In their work on travelling waves they prove existence of solutions, monotonicity along the cylinder and uniqueness (also of the travelling speed in many cases; the speed is also

unknown). In particular they have extended to higher dimension the classical work of Kolmogoroff, Petrovsky and Piskounoff concerned with planar travelling waves.

In [6] they studied the limit behaviour of the solutions as they approach a "flame front" which is a free boundary. They proved a number of results concerning how the limit is approached, and studied the regularity of the free boundary. They have carried this out in a rather general framework in order that the results and techniques be applicable to other problems. Li worked on problems related to the papers [1] and [3]. In particular he extended some of the results of [3]. Using the extensions he obtained more general results than those of Craig and Sternberg (Symmetry of solitary waves, Comm. Partial Diff. Eqs. 13 (1988) 603–633).

Yau has worked on some basic problems in statistical mechanics – on deriving hydrodynamic scaling limit equations of Ginzburg-Landau. This involved derivation of macroscopic equations for conserved quantities of microscopic dynamics. He proved that the hydrodynamic equation of a Ginzburg-Landau model is equivalent to the equation governing the minimal entropy production. (He has studied the rate of change of the relative entropy: his work goes beyond previous work — he studies entropy relative to local rather than global equilibrium.) He solved a long standing problem: a rigorous proof of the Euler equation for conservation laws from Hamiltonian systems, with negligible random noise.

Kichenessamy works on the problem of breathers, i.e., time periodic solutions for the equation

$$u_{tt} - u_{xx} + f(x,u) = 0\ , \qquad -\infty < x < \infty\ .$$

This problem has attracted much attention. He proved that solutions depending analytically on a parameter in a certain way do not exist, in general. In addition he has studied formal solutions and has also obtained a priori estimates for solutions if they exist.

F. H. Lin studied area minimizing sets in hyperbolic space. He showed that if Γ is a compact smooth $(n-1)$-dimensional submanifold of R^{n+k-1} then there is a complete area minimizing locally rectifiable n dimensional current T in hyperbolic space $\boldsymbol{H}^{n+k}$ which

is asymptotic to Γ at infinity and he studies more precisely the asymptotic approach at infinity. The proof requires various analytic estimates and techniques from geometric measure theory.

J. Grotowski (a student) studied the heat flow approach to harmonic maps between manifolds: between spheres, and from the ball B^3 in R^3 into the sphere S^2. Under certain conditions he proved the existence of global (i.e., all time > 0) of smooth solutions of the flow, converging to harmonic maps. Under other conditions he established that solutions blow up in finite time.

I. Birindelli (a student) is working on linear and nonlinear differential equations: (a) She has proved the existence of time periodic solutions for a class of systems of ordinary differential equations (b) She has found a very general extension of the Hopf lemma for elliptic equations to domains with corners.

L. Sadun studied the spectrum and scattering theory of the Landau Hamiltonian on noncompact surfaces, with constant magnetic field. In some cases, a continuous spectrum appears, while not in others.

F. Yi works in minimal surface theory. Among other things he proved that if M is a complete immersed minimal surface in R^3 with finite total curvature, and if its Gauss map to S^2 (given by the normal) omits 3 points on S^2 then the total curvature of $M \leq -20\pi$.

S. Wu has been working on the solvability of the Dirichlet problem for elliptic systems in Lipschitz domains, with data in L^p. She has reduced the problem to one for pseudo-differential operators, and proved the existence of a fundamental solution for them.

C. List of Publications and preprints.

[1] H. Berestycki, L. Nirenberg, Monotonicity, symmetry and antisymmetry of solutions of semilinear elliptic equations, J. Geometry and Physics 5 (1988) 237–275.

[2] H. Berestycki, B. Larrouturou, L. Nirenberg, A nonlinear elliptic problem describing the propagation of a curved premixed flame, Proc. NATO Advanced Res. Workshop on Math. Modelling in Combustion and Related Topics.

[3] H. Berestycki, L. Nirenberg. Some qualitative properties of solutions of semilinear elliptic equations in cylindrical domains, Analysis etc., ed. P. Rabinowitz et al., Academic Press (1990) 115–164.

[4] ————, Travelling front solutions of semilinear equations in n dimensions, Frontiers in pure and applied Mathematics, ed. R. Dautray, North-Holland (1991) 31–41.

[5] ————-, On the method of moving planes and the sliding method, to appear.

[6] H. Berestycki, L. A. Caffarelli, L. Nirenberg, Uniform estimates for regularization of free boundary problems, Analysis and partial differential equations, ed. C. Sadowsky, Marcel Dekker (1990) 567–619.

[7] Lin, Fanghua, Asymptotic behavior of area-minimizing currents in hyperbolic space, Comm. Pure Appl. Math. 42 (1989) 229–242.

[8] H. T. Yau, Minimal entropy production and hydrodynamics of a Ginzburg-Landau model, Letters of Math. Phys. 22 (1991) 63.

[9] S. Kichenessamy, Breather solutions of the nonlinear wave equation, Comm. Pure Appl. Math. 44 (1991) 789–818.

[10] Y. Fang, On the Gauss map of complete minimal surfaces with finite total curvature, preprint.

D. Participating personnel

L. Nirenberg; H. T. Yau; S. Kichenessamy; F. H. Lin, L. Sadun; Wu, Sigue; Yi, Fang.

Also these graduate students:

Li, Cong Ming, received Ph.D. in 1989.

J. Grotowski, received Ph.D. in 1990.

I. Birindelli, working on Ph.D. thesis.