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This paper discusses a generalized Stefan problem which allows for supercooling and superheating and for capillarity in the interface between phases. Simple solutions are obtained indicating the chief differences between this problem and the classical Stefan problem. A weak formulation of the general problem is given.

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Allan Struthers, Ph.D. in Mathematics, May 19, 1991
Jose Matias, currently pursuing a Ph.D.

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Brief Outline of Research Findings

Work continued on the thermodynamics of two–phase continua. The general two–phase Stefan problem with supercooling, superheating, and capillarity, was studied [T19] in collaboration with M. Soner (Carnegie Mellon). Simple solutions – illustrating the chief differences between this problem and the classical Stefan problem – were obtained for the spherically symmetric problem, under the assumption of fast diffusion, with the liquid supercooled at infinity. It is shown that: (i) for $\Omega = \mathbb{R}^3$, a ball of the solid phase of sufficiently small size disappears in finite time, but a sufficiently large ball grows without bound; (ii) for $\Omega = \mathbb{R}^3$ and the solid phase initially situated in a spherical shell of thickness ϵ, the thickness of the solid shell initially increases, but the inner radius of this region decreases to zero in finite time T; the solid ball remaining at time T disappears at a later time or grows without bound according as ϵ is less than or greater than a critical value; in the limit $\epsilon \to 0$ the region occupied by the solid disappears infinitely fast; the problem has no solution for $\epsilon = 0$; (iii) when Ω is the region exterior to a sphere of radius R, with the boundary $r = R$ insulated and with the solid phase initially in a spherical shell of zero thickness at $r = R$, the solid phase grows without bound provided R is sufficiently large. While (ii) and (iii) are of little practical interest, they demonstrate the possibility of growth from a seed of zero volume.

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*Discussed in previous progress reports.