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13. ABSTRACT (Maximum 200 words)

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generally not of any special importance in this context but is intended to provide
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The most complete results were obtained for the blending of surfaces that are
described by either the explicit or implicit boundary representations. The
final report provides a description of the results.

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ANALYSIS OF BLENDING ALGORITHMS IN COMPUTER GRAPHICS: FINAL REPORT

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1. Statement of Problem

An important problem in solid modeling systems is the creation of surfaces that blend together two or more surfaces that describe objects. The purpose of blending is to create a single, more complex object. The blending surface itself is generally not of any special importance in this context but is intended to provide a smooth transition between the surfaces to be blended. As such, an equation describing the blending surface should be easy to determine and the surface should be of low degree because a low degree surface allows points on the surface can be computed fast.

The most commonly occurring surfaces in solid modeling are those whose boundary representations are described by algebraic equations in several variables. These surfaces are called algebraic surfaces. For simplicity of language, when we use the term "surface" in the remainder of this report, we mean an algebraic surface.

This is called a boundary representation. If the equation is solved explicitly for one of the coordinate variables, the representation is called explicit; otherwise the representation is called implicit. If the coordinates of points on the surface are described by a set of parametric equations, then the representation is called parametric.

The smooth transition between surfaces usually means that the blending surface B is to be tangent to the given surfaces, say F and G, usually in a "distinguished" curve. If the surfaces are described by the equations of their boundaries, then we can represent a curve on a surface by the intersection of that surface and an "auxiliary surface". We will usually use the implicit form of representation of surfaces.

The blending surface B satisfies the following conditions:

1. B intersects the surfaces F and G

2. B is tangent to the surface F in a distinguished curve $C1$ that is represented as the intersection of F and a preassigned auxiliary surface H.

3. B is tangent to the surface G in a distinguished curve $C2$ that is represented as the intersection of G and a preassigned auxiliary surface K.

Low degree surfaces are especially desirable since the blending surface is often needed in later calculations involving the determination of hidden surfaces to be removed from a graphical display of the object. These computations involve both the equation of the surface and the equations of the normal lines to the surface at each point. Since the gradient of the surface at a point is a three dimensional vector whose components are polynomials (at least for polynomial equations of surfaces), the degrees of the gradients also affect the complexity of the computation of the blending surface. The degree of an algebraic surface is defined to be the total degree (= sum of powers of variables) of the term of highest total degree in the implicit equation describing the surface. The equation is generally assumed to be irreducible in order to simplify the geometric assumptions and to simplify computations.

Other subsequent computations that can be performed much faster if the blending surface is of low degree involve the computations of roots of simultaneous equations involving the lines normal to a surface at a point and lines representing light rays. All of the later computations in a solid modeling system are simpler and faster if the blending surface has a low degree and all three of the components of the gradient of the blending surface have a low degree.

The purpose of this research is to investigate existing methods for computing blending surfaces, develop new methods, and to evaluate the use of symbolic computation in this area.

2. Description of existing research

There has been a considerable amount of research effort directed toward finding low degree blending surfaces. Some authors look for a geometrically simple method involving the model of a "rolling ball" ([6], [5]). In this model, the blending surface is a portion of the surface that is swept out by

a sphere of constant radius that moves along the two surfaces to be blended. This method often produces a geometrically acceptable blend. However, high degree surfaces often result from this method. For example, the simple case of blending together a right circular cylinder that intersects a plane at a right angle results in a torus, which has degree eight.

An attempt to automate the "rolling ball" method has been made by Mummy as part of a solid modeling system produced by Boeing Computer Services [4]. This method produces blending surfaces that are frequently unsatisfactory for reasons of high computational complexity of later operations and unusual geometric shapes.

Hopcroft and Hoffman [2] have developed a method called the projective method. This method produces a blending surface B whose equation in implicit form is

$$B(x,y,z) = 0$$

where B is a blending surface and is guaranteed to be of the lowest possible degree that meets certain conditions. The surface B blends together two surfaces that are described by the equations

$$F(x,y,z) = 0$$

and

$$G(x,y,z) = 0$$

The name "projective method" is appropriate since the method is based on some relationships between the surfaces in projective space.

There are two representations that are commonly used with the projective method. Hopcroft and Hoffman develop the blending surface with the representation

$$FG - P^2$$

where P is determined in terms of H and K; in fact, $P = HK$.

Warren [8] used a somewhat different formulation of the projective method. He used the representation

$$FH^2 + GK^2$$

and remarked that this describes blending surfaces of the same lowest degree. Both papers point out unusual situations that can occur if the equations representing the surfaces are not irreducible.

Middleditch and Sears [3] provide a somewhat different method of determining the lowest degree blending surface with other tangency conditions.

The most common formulations of these methods allow the use of a single real parameter to manipulate the surfaces. The resulting blending surface's equation can be replaced by an equation which is a convex combination of the two expressions. This allows a higher degree of freedom when plotting the blending surface.

These two methods of blending always produce a relatively low degree surface that blends the two existing surfaces and that is always tangent to these surfaces in two preassigned curves provided that we define the preassigned curve as the intersection of two surfaces and include the equation of the auxiliary surface in our resulting blending surface equation.

Warren [10] has shown that, subject to some technical considerations, every algebraic blending surface that solves the blending surface problem as formulated is the solution of an equation of the form

$$PFG + QFH^2 + RGK^2 + SH^2K^2 = 0,$$

where P, Q, R, and S are arbitrary polynomials.

It appears that these methods provide a solution to the blending surface problem for implicitly defined surfaces. For example, Hopcroft & Hoffman indicate that any degree four surface that blends together two quadric surfaces must be of their form. However, these blending surfaces need not have

acceptable behavior in the region between the two existing surfaces. For example, there is no obvious reason that would preclude the blending surface obtained by any of these methods from being self-intersecting between the two surfaces F and G, or even if the surfaces are embeddable in three dimensional space. It is also clear that surfaces other than surfaces of revolution can be obtained even if the existing surfaces were surfaces of revolution. See [11] for a discussion of other research in this area and for examples of problems with existing blending methods.

The information provided in intersection curves using auxiliary surfaces sometimes forces a higher degree blending surface than might be necessary for a solution to the original problem. The problem of blending two surfaces of revolution with a common axis can be reduced to a problem in two dimensions which has a simple solution. The solution gives rise to a corresponding blending surface which has a degree of 2, 4, or 6.

There is a simpler method that can be used in this case; we refer to this method as the surface of revolution method. The details are as follows:

1. Translate and rotate the problem if necessary so that the common axis is the positive x-axis and that one of the surfaces is to be blended with the curve of intersection in the plane $x = 0$.

2. The problem can now be reduced to a two dimensional problem.

3. Project the two surfaces to be blended onto the $x-z$ plane. Their traces in the $x-y$ plane are symmetrical about the x-axis.

4. The problem now reduces to the determination of a function $y = h(x)$ such that the two dimensional graph of h passes through the points $(0, f_1)$ and (x_2, f_2) and has slopes of f_{1p} and f_{2p} at these two points, respectively. If we look for the two dimensional curve of lowest degree passing through the two points, we need only consider a cubic curve of the form

$$y = ax^3 + bx^2 + cx + d$$

which gives rise to a surface of revolution of degree 2, 4, or 6, with the lower degrees possible if the leading coefficients a, b, or c being 0.

5. Rotate the two dimensional solution to get a blending surface of revolution.

The surface of revolution and rolling ball methods always lead to a surface of revolution; however, the resulting surface sometimes cannot be embedded in real three space.

Here is an example of the surface of revolution method. If the two surfaces to be blended are cylinders of the same radius and have the same axis, then the two dimensional cubic curve reduces to a straight line since the values of f_1 and f_2 are equal and the slopes are both 0. When this straight line is rotated about the x-axis, the blending surface is a portion of a cylinder which is the simplest possible blend in this situation. Note that the degree of the equation of the cylinder is 2.

The Hopcroft & Hoffman method leads to a surface of degree at least 4 because of the appearance of the product of the expressions H and K. The Warren method leads to an expression of degree 4 also. The degrees of the surfaces obtained by any method may be reduced by factoring the equations in certain instances. The irreducibility of the surfaces is an essential feature of the Hopcroft-Hoffman and Warren theories. The irreducibility of any equation describing a surface is difficult to determine using current technology without the use of symbolic computation. Thus the irreducibility of such surfaces and their actual degrees are not obvious.

Consider the "rolling ball" method for blending a particular instance of a cone and a cylinder. The difficulty is easiest to see when we project the surfaces onto the x - y plane. The center of the circle used in the "rolling ball" must lie on both a line through the point $(0,1)$ on the (projected) boundary $y = 1$ perpendicular to $y = 1$ and a line through the point $(2, m(2-a))$ that is perpendicular to the line $y = m(x-a)$. Since m is non-zero, the two lines must intersect. However, the distances from their possible "center" to $(0, 1)$ and $(2, m(2-a))$ need not be equal and hence there may not be a "rolling ball" blending surface matching the surfaces in the desired curves.

There are thus two possible remedies to the possible failure of the "rolling ball" method : relax the conditions of tangency at the fixed curves or allow the rolling ball to intersect one or both of the surfaces at curves "near", but not coincidental with, the desired curves.

It is also not clear that the surfaces produced by the other methods are always embeddable in real three space. In our investigation, we found many cases of non-embeddability of blending surfaces. We also found several cases of blending surfaces having cusps or ther undesirable features.

3. Results Obtained

The most complete results were obtained for the blending of surfaces that are described by either the explicit or implicit boundary representations.

The first problem that we studied was the determination of the relative quality of blending surfaces that are produced by several popular blending surface methods. In order to quantify our results and have a testbed for evaluating alternate methods, we developed a database consisting of many commonly used quadric and higher degree surfaces.

The database entries were selected as follows. All of the standard forms of quadric surfaces were used together with rotations about the principal axes and the origin. This included planes, spheres, ellipsoids, cones, hyperboloids (of one and two sheets), paraboloids, and cylinders. Other surfaces with simple third and fourth degree algebraic equations were also included in the database; these were subjected to rotations and the rotated surfaces were also included in the database.

In order to make sure that no surfaces with an interesting physical shape were excluded, we studied most of the heavy machinery, laboratory equipment, and exposed plumbing valves in Howard University's School of Engineering building. An examination of many physical objects indicated that we had covered all the basic combinations that were needed for most practical situations.

We compared the results of the Hopcroft & Hoffman method with that of the Warren method for determining blending surfaces for pairs of surfaces taken from the database. We compared the resulting blending surfaces purely on the basis of the complexity of the blending surface. The complexity of the blending surface was measured by the degree of surface, the number of terms appearing in the algebraic equation of the surface, the degree of each of the three coordinate functions in the gradient of the surface, and the number of terms in each of the three coordinate functions appearing in the gradient of the surface.

We found that the two methods often produced blending surfaces with considerably different complexity results. There was no clear pattern that one of the two methods produced simpler results in all cases. The total complexity (as measured by the sum of the degrees and number of terms in both the equation of the surface and the coefficient functions of the gradient) for each of these methods was generally within one or two for the two methods. Thus we concluded that the two methods are similar in terms of the complexity of the blending surface produced. The two methods also take approximately the same amount of time to produce a blending surface and thus we consider the two methods to be equivalent in terms of their effectiveness in producing algebraic equations for blending surfaces.

The computations were quite complex and were error-prone if done by hand. We used the easy-to-use, menu-driven symbolic computation software "Derive" on PC's in order to perform the computations. They were done by two students sitting next to one another and checking the results by hand. While this was somewhat inefficient, it was the best available system during much of the contract period. In any event, the results of the machine computations were checked by hand and so there is confidence that the results obtained were correct.

The embeddability or non-embeddability of a blending surface was often easy to see from a graph of the blending surface. The symbolic computation software Theorist, which works on Macintosh computers, had an adequate graphical package, which did not abort when non-real results of computations arose. Instead, it did not plot points that corresponded to complex values of the variable z for values of x and y within the designated rectangular region for which the graph was plotted. (This package

required the explicit representation of the surface so that it was necessary to perform a preliminary step of solving for z.) It was easy to detect regions of large change in the surface from these graphs.

The Theorist software was a menu-driven system that was extremely easy to use.

The next problem considered was the extent to which geometric properties of surfaces to be blended were inherited by the blending surfaces obtained by any of the common methods. The geometric properties that was most tractable was that of symmetry.

Consider the case of two surfaces of revolution having a common axis of symmetry. If the two distinguished curves on the surfaces are described as intersections with auxiliary surfaces that are symmetrical with respect to the axis of symmetry, then we should expect that a good blending surface method would produce a blending surface that is also symmetrical about the same axis of revolution.

On this problem, the Hopcroft & Hoffman and Warren methods produced unsatisfactory results unless the simplest formulations of the auxiliary surfaces was used to find the distinguished curves on the surfaces to be blended. Thus curves that would occur naturally as the intersection of two surfaces need not give a simple symmetric blending surface directly unless a preliminary simplification of the problem occurred before the blending algorithm was applied.

These geometric deficiencies suggested that we consider a reformulation of the problem. The reformulation that was most successful was the removal of dependency on auxiliary surfaces to determine the distinguished curve by determining the intersection with the surface to be blended. Instead, we used the symmetry of the problem to determine the distinguished curves by a method with only a single degree of freedom.

The surfaces developed by this method were frequently of lower degree than that of the other methods. In this work, we used the symbolic computation software Mathematica because of the ease of programming loops to repeat calculations with different parameters and because of the superior graphical abilities. This work was done on several NeXT computers, with the final extensive computations done using the facilities of COMSERC, which is the Army's high performance computing center at Howard University.

As a first attempt to study just how well-behaved such methods are, we considered the relationship between the Hopcroft & Hoffman and Warren formulations.

We determined the simplest blending surfaces possible in a variety of situations. The first step was to analyze the equations of the blending surfaces that were created by the Hopcroft & Hoffman method and compare them with the ones that were created by the Warren method. We considered blending pairs of surfaces of degrees 2, 3, and 4. For a set of 40 surfaces, we found that the Hopcroft & Hoffman method produced a surface of higher degree in 9 of the cases; in each of the other cases, the degrees of the blending surfaces were the same. The Hopcroft & Hoffman method produced more complex gradients in the sense that the total of the degrees of the three components were higher in 21 cases than that computed by the Warren method; in 13 cases, the Warren method produced a higher total; and the remaining cases had the same totals. These results are suggestive but not conclusive.

3.1. An Example: Blending a cone and a cylinder

Our work can best be presented by giving an example in detail. Additional information can be found in the papers referenced. We will fix one of the surfaces to be the cylinder whose equation is $y^2 + z^2 = 1$. The other surface will be a cone whose equation is $y^2 + z^2 = [m(x - a)]^2$. The cone is obtained by revolving the line $y = m(x - a)$ about the x-axis; its equation is described by two parameters m and a which represent the slope of the line $y = m(x - a)$ and its x- intercept.

We need to determine a curve on each of these surfaces that the blending surface will pass through and that the surface will be tangent to the surface in this curve. We will choose these curves by selecting as two auxiliary surfaces the planes $x = 0$ and $x = 2$. The curve of interest on the first surface is thus described by the pair of equations

$$y^2 + z^2 - 1 = 0$$

$$x = 0$$

and the corresponding curve on the second surface is described by the pair

$$y^2 + z^2 - [m(x - a)]^2 = 0$$

$$x - 2 = 0$$

The Hopcroft & Hoffman method provides a blending surface whose equation is

$$FG - H^2 = 0$$

where

$$F = F(X, Y, Z) = y^2 + z^2 - 1,$$
$$G = G(x, y, z) = y^2 + z^2 - [m(x - a)]^2$$

and

$$H = H(x, y, z) = x(x - 2)$$

In implicit form, the equation is

$$(y^2 + z^2 - 1)(y^2 + z^2 - [m(x - a)]^2) - (x(x - 2))^2$$

This blending surface has degree 4. It contains 17 terms. The three components of its gradient have degrees 3, 3, and 3, and they contain 9, 6, and 6 terms respectively.

The Warren method produces a blending surface whose equation is

$$FS_1^2 + GS_2^2 = 0$$

where F and G are as before. Here S_1 and S_2 represent the left hand side of the equations of the auxiliary surfaces $x = 0$ and $x - 2 = 0$, respectively. In implicit form, the equation is

$$(y^2 + z^2 - 1)(x)^2 + (y^2 + z^2 - [m(x - a)]^2)(x-2)^2$$

This blending surface has degree 4. It contains 11 terms. The three components of its gradient have degrees 3, 3, and 3, and they contain 8, 2, and 2 terms respectively.

The method based on surfaces of revolution requires more work than does either of the previous methods. We must solve a system of four equations in order to determine the coefficients of a cubic polynomial $Ax^3 + Bx^2 + Cx + D = 0$ which matches the surfaces at appropriate points after projecting the surface onto the x - y plane. The projection of the blending surface onto the x - y plane must intersect the line $y = 1$ at the point $(0, 1)$ and must intersect the line $y = m(x - a)$ at the point $(2, m(2 - a))$. Each of these points of intersection must be a point of tangency.

This leads to the system

$$-A + B - C + D = 1$$
$$3A - 2B + C = 0$$
$$A + B + C + D = m(1 - a)$$
$$3A + 2B + C = m$$

which has the solutions

$$A = (1 + ma - a)/4$$
$$B = (4m - 3 - 3ma)/4$$
$$C = 0$$
$$D = 1$$

The corresponding blending surface of revolution is obtained by revolving the line $Ax^3 + Bx^2 + Cx + D = 0$ around the x- axis and its equation in implicit form is

$$y^2 + z^2 = [Ax^3 + +Bx^2 + Cx + D]^2$$

This equation has degree 6 and has a total of 12 terms. The three components of its gradient have degree 6, 1, and 1, and have 9, 1, and 1 terms, respectively.

In one special case, the solution is simpler. If the parameters m and a describing the cone have the relationship $m = (a - 1)/a$, then the leading coefficient of A drops out and the degree of the blending surface becomes 4 instead of 6, since we are revolving a parabola instead of a cubic.

The gradient also simplifies in this case. The degrees of the components are 3, 1, and 1, and the number of terms are 5, 1, and 1.

There is a problem with the "rolling ball" method, at least as we might use it in this example. The difficulty is easiest to see when we project the surfaces onto the x - y plane. The center of the circle used in the "rolling ball" must lie on both a line through the point $(0,1)$ on the (projected) boundary $y = 1$ perpendicular to $y = 1$ and a line through the point $(2, m(2-a))$ that is perpendicular to the line $y = m(x-a)$. Since m is non-zero, the two lines must intersect. However, the distances from their possible"center" to $(0, 1)$ and $(2, m(2-a))$ need not be equal and hence there may not be a "rolling ball" blending surface matching the surfaces in the desired curves.

The "rolling ball" method has a solution in this case only if the distances agree. This is possible only if the distance of that is the radius of the rolling ball, which we denote by r, satisfies the equation

$$r = \frac{|r + 1\ ma|}{(m^2 + 1)^{\frac{1}{2}}}$$

which represents the distance from the point $(0, r+1)$ to the line $y = m(x-a)$ being equal to the distance from $(0, r+1)$ to $(0, 1)$, namely, r. This equation is rarely satisfied for m and a.

A word about the graphics is in order. All of the graphs were done using Mathematica on a NeXT computer, with the same viewpoint and the same range in the x and y directions. The implicit equations obtained for the blending surfaces were solved for z and the graphs of those explicit equations were plotted. An arbitrary choice was made in the case of multiple solutions for z. Thus jagged edges in a graph indicate only that the graph is near the limits of its range rather than any discontinuity. All of the graphs are included at the end of this report.

Figures 1 through 12 show the nature of some of the blending surfaces for some values of the parameters m and a. As indicated earlier, the graphs were obtained using Mathematica on a NeXT computer and required the representation of the graphs in explicit form. Thus they may only represent portions of graphs.

Figures 1 through 4 are computed using the Hopcroft & Hoffman technique. Note that some of the the surfaces appear to have dents and bulges, which indicate that the surfaces are not surfaces of revolution.

Figures 5 through 8 are computed using the surface of revoution method. Not the apparent discontinuity; this means that the surface is not embeddable in real three dimensional space. The graphs in figures 9 through 12, which were computed using the rolling ball method, show other difficulties.

3.2. Proof that these results cannot be obtained by existing methods

At this point, we addressed the issue of whether this new approach provided new blending surfaces. This is a difficult question to answer because the existing methods (Hopcroft & Hoffman, Warren, Middlediteh & Sears, Rossignac & Rechiqua, Mummy, etc.) often produce surfaces that could be produced by one of the other methods by varying a parameter, which can be a constant parameter or can itself be an algebraic function of three variables.

In order to show that a particular surface (which could be a blending surface or, in fact could be any arbitrary surface) is not achievable by the Warren or Hopcroft & Hoffman methods, it is necessary to show that the surface does not belong to a certain ideal. The ideal is another means of describing the polynomials that are used in Warren's general representation [10]. The ideal consists of polynomials in several variables and is generated by the surfaces to be blended and the auxiliary surfaces in a specific combination. The generators are generally quite complex and it is very difficult to determine if a given polynomial belongs to this ideal.

An equivalent formulation is that we must show that the polynomials that we have developed cannot be written as a linear combination of these polynomials, where the coefficients are themselves allowed to be polynomials in the variables x, y, and z. The computations needed to show that a particular polynomial is not in an ideal are quite complicated and require an efficient algorithm. The complexity of the equations makes the computation impossible to perform by hand.

The method that we use for the computation is the method suggested in [1] and [10]: finding a Grobner basis for the ideal. A Grobner basis for the ideal is a finite set of polynomials organized so that an operation called "reduction", when performed on a particular polynomial, will always terminate. The reduction algorithm produces a result of 0 if and only if the polynomial is in the ideal. (Reduction is basically a degree-reducing method that uses a clever ordering scheme.)

The resolution of this problem was to determine a basis for this ideal which was amenable to computation. The theoretical method of determining a Grobner basis was essential here. Once such a basis was computed, a straightforward algorithm could determine the membership of the given polynomial in the ideal.

However, the computations to determine a Grobner basis were often quite messy and were somewhat error-prone. We used the symbolic computation software Macsyma, which is the oldest commonly available such software. One of the libraries included with Macsyma included functions to use as input a set of generators for an arbitrary ideal and to produce as output a Grobner basis for the ideal and to easily determine if a given polynomial belonged to the ideal generated by this basis.

The software was quite successful in showing that, in the case of surfaces of revolution, the new method produced simple, low degree blending surfaces that cannot be obtained by any of the existing methods.

Implementing an algorithm to compute the Grobner basis of an ideal generated by a set of polynomials and the reduction algorithm is somewhat tricky. Instead, we used the grobner() and grob_reduce() functions included in the Macsyma distribution in the share directory. The results are summarized as follows.

Theorem. The ideal described by Warren does not include all possible implicitly defined blending surfaces. In fact, it includes neither the surface of revolution blend or the rolling ball blend for the cases of a cone and a cylinder or two cylinders.

Proof. The result of the Macsyma function grob_reduce(), when applied to the two types of blending polynomials, does not produce 0 and the result cannot be transformed to 0 by the reduction operation.

> Note 1: The proof is somewhat disquieting in that it is extremely difficult to verify by hand computation. However, the software had been tested by others on several different computers and was tested by us using the examples provided in [10]. Thus we have confidence in the computations.

Note 2: The reason for our polynomials not being representable in Warren's form is of course that we reformulated the problem by not using the auxiliary surfaces.

Note 3: The ideals, Grobner bases, and the results of the reductions do not change if we replace the planes by appropriate spheres in our auxiliary surfaces.

Note 4: We have developed methods that produce new blending surfaces for implicitly defined algebraic surfaces. These methods are often lower in complexity than some of the existing methods.

3.3. Summary of Techniques Used

The major techniques used in this research were symbolic computation (with the software packages Derive, Theorist, Mathematica, and Macsyma), symmetry, reformulation of the problem to reduce complexity by simplifying the role of auxilliary surfaces, and Grobner bases (to determine membership in ideals).

The most useful symbolic computation packages were Mathematica because of its excellent graphics and Macsymabecause of the completeness of its libraries.

3.4. Other results from the use of symbolic computation

We studied another problem: determination of relationships among the coefficients of parametric representations of surfaces that indicated particular geometric shapes. For example, we considered the problem of determining the nature of algebraic surfaces of degree two and three from particular parametric representations. We were unable to solve the general problem at the time because of a limited amount of physical memory devoted to Macsyma computations. Partial results have been obtained in a few simple examples and the work is continuing on a system with more memory.

4. FUTURE RESEARCH

Our surface of revolution method and the classical rolling ball method always provides surfaces of revolution when the original surfaces are also surfaces of revolution. The equations obtained in the other methods do not always describe surfaces of revolution. When they do, this is not clear without simplification of the expressions involved. This is not needed in our method for surfaces of revolution. Embeddability in real three dimensional space remains a serious problem.

In several methods, blending surfaces may be parameterized by replacing one or more of the polynomials in the general polynomial ideal formulation by a constant that can be varied to change the shape of the blend. Perhaps this can remove some of the problems of bulges, dents, cusps, lack of symmetry, or embeddability problems.

Many questions remain to be answered about the characterization of general blending surfaces. Incorporation of information about the geometry of the surfaces to be blended, such as being surfaces of revolution, having any types of symmetry, etc, may significantly change the resulting blending surface.

We will expand our limited set of experiments (40 pairs of surfaces) to a more general discussion of how the existing methods work on low degree surfaces. This work will be continued using Mathematica on an Alliant supercomputer. We need the high performance because there are many possible combinations of surface types (8 of degree 2, 19 of degree 3, and 27 of degree 4 [7]). We will also consider the use of geometric features of the surfaces to be blended such as being surfaces of revolution, having other symmetry, or being ruled surfaces.

REFERENCES:

1. Buchberger, B., A Theoretical Basis for the Reduction of Polynomials to Canonical Form, ACM SIGSAM Bulletin, vol 10, no. 3, 19-29 (1976).

2. Hoffman, C. M. and J. E. Hopcroft., Quadratic Blending Surfaces, Computer Aided Design, 8, no. 6, 301-306, July/August 1986.

3. Middleditch A. and K. Sears, Blend Surfaces for set Theoretic Volume Modelling Systems, SIAM Conference on Applied Geometry, Albany, New York, 1987.

4. Mummy, M., SIAM Conference on Applied Geometry, Albany, New York, 1987.

5. Rockwood, A. and J. Owen, Blending Surfaces in Solid Modeling, Proc. SIAM Conference on Geometric Modeling and Robotics, Albany, New York, July, 1985.

6. Rossignac, J. and A. Rechiqua, Constant-Radius Blending in Solid Modeling, Comput. Mech. Engr., 65-73, July, 1984.

7. Salmon, G., "A Treatise on the Analytic Geometry of Three Dimensions", Longmans, Green and Co., New York, 1915.

8. Warren, J. PhD. Thesis, Rice University, 1988.

9. Warren, J., Blending Algebraic Surfaces, ACM Trans Graph vol 8, no. 4, (Oct 1989) 263 - 278.

10. Winkler, F., Solutions of Equations `· Polynomial Ideals and Grobner Bases, Lecture Notes, Conference on Computers and Mathematics, Stanford University, Palo Alto, California, August 2-5, 1986.

11. Woodark, J. R., Blends in Geometric Modeling, in The Mathematics of Surfaces vol II, (R. R. Martin , ed.) Clarendon Press, Oxford, England, 1987.

5. Problems Encountered

Much of the research was slowed by computer hardware and software problems. The primary workstation was difficult to use for a period of many months because of a bad disk drive and controller board. After several replacements, the system is still not functioning at full efficiency because of intermittant printer glitches; these seem to be caused by a problem in the serial port. I am only able to print Postscript files; ordinary listings of source code are not possible at this time. Lack of availability of the primary equipment was the main reason for the use of the Derive and Theorist packages for symbolic manipulation.

6. Patents and Inventions
None

7. Publications

1. *Actual Complexity of Parallel Evaluation of low Degree Polynomials* Parallel Computing 13(1990), 73-83. (with O. Michael Atogi and Razeyah R. Stephen)

2. *SANAC: A Tool for Measuring Ada Coupling*, Proceedings of the Eighth Annual National Conference on Ada Technology, March, 1990, Atlanta, GA, 546-559. (with K. S. Bagley, I. L. Brown, D. E. Butler, and P. W. Spann)

3. *Ada Exceptions and Fault-Tolerance*, Proceedings of the Eighth Annual National Conference on Ada Technology, March, 1990, Atlanta, GA, 338-343.

4. *A Model of a User Interface*, Proceedings of the Human Computer Interaction Conference (HCI International'89), Boston, MA, Sept. 18-22, 1989, 18-19. (with D. M. Coleman)

5. *The Efficiency of Unix Inter-Process Communication*, login, (with H. M. Crawley and D. N. Underwood)

6. *Software Metrics and Software Maintenance* Software Maintenance, vol 2 (1990), 133-142.

7. *A Advanced Operating Systems Project with Concurrency* SIGCSE Bulletin, vol 23, no. 3 (1990), 39-44.

8. *N-version Programming Using the Ada Tasking Model* Proceedings of the Ninth Annual National Conference on Ada Technology, March 4-7, 1991, Washington, DC, 135-141.

9. *An Ada Experiment on the Intel Hypercube* To appear in: Proceedings of the Tenth Annual National Conference on Ada Technology, , Washington, DC (with D. M. Coleman and L. P. Tan)

Publications Submitted:

1. *Blending Surfaces of Revolution*,

2. *User Interface Systems* (with D. M. Coleman, K. S. Bagley, R. E. Bennett, and S. C. Daniels)

3. *On the Economics of User Interface Management Systems*

4. *Parallel Efficiency can be Greater than Unity*

Presentations Given

      Annual National Conference on Ada technology, March, 1991 - presented by Ronald J. Leach

      Graduate Research Symposium, March, 1991 - presented by Erica N. Anderson

      Proceeding of the National Technical Association, March, 1991 - presented by Barbara P. Clarke

Personnel Supported During this Period:

      Ronald J. Leach   (Principal Investigator)

Students Supported:

Charles Alleyne
Tonia R. Nicholson
Satish Chavan
Barbara P. Clarke
Erica N. Anderson

Degrees Awarded

Charles Alleyne:	MS in Computer Science, December, 1990
Satish Chavan:	MS in Computer Science, August, 1991
Barbara P. Clarke:	MS in Computer Science, August, 1991
Erica N. Anderson:	MS in Computer Science, to be awarded December, 1991

APPENDIX

Graphs of certain blending surfaces