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DYNAMICAL SYSTEMS ANALYSIS OF AN AERODYNAMIC DECELERATOR'S BEHAVIOR DURING THE INITIAL OPENING PROCESS

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A new mathematical model is developed and analyzed to determine the qualitative behavior of an aerodynamic decelerator during the initial opening phase. Currently, all models of canopy opening are valid only once the canopy has begun to inflate and has some assumed shape. The ability to determine the appropriate initial shape would greatly enhance these models.

A set of elastodynamic equations for a simplified canopy model is nonlinearly coupled to Lighthill's model for the relative velocity between a cylinder and the flow past it. Previous work used a linear model for the fluid structure interaction. The new model presented in this paper removes this restriction by using a nonlinear interaction model. The resultant set of nonlinear partial differential equations is expanded in terms of a complete set of eigenfunctions. The result is an infinite set of coupled ordinary differential equations in time. This set is then truncated to obtain various sets of "low-dimensional" models. These models are investigated to determine the stability of the canopy with respect to the fluid's velocity, the line tension and the drag coefficients. Using dynamical system theory, an understanding of the bifurcation process is obtained without having to solve the full system of nonlinear partial differential equations. Hence, it is possible to predict the onset of divergence and flutter as a function of the system parameters in an efficient manner.

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Preface

The work described in this report on the behavior of an aerodynamic decelerator during the initial opening process was undertaken during the period October 1987 to September 1991. The author was the principal investigator and project officer of this research, which was performed in the Engineering Technology Division of the Aero-Mechanical Engineering Directorate at the U.S. Army Natick Research, Development and Engineering Center, Natick, MA 01760.

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List of Symbols

x	Axial location along tube axis
y	Displacement normal to tube axis
t	Time
i	Angle of perpendicular to tube axis with y axis
τ	Dimensionless time
D	Tube diameter
L	Tube length
A	Cross-sectional area of tube
M	Virtual mass of fluid per unit length
m	Mass of tube per unit length
EI	Flexural rigidity of tube
ρ	Fluid density
U	Free stream fluid velocity
v	Relative velocity between cylinder and fluid
F_N	Normal viscous force per unit length
F_L	Longitudinal viscous force per unit length
C_{Dp}	Coefficient of form drag
C_f	Coefficient of friction drag
β	Ratio of virtual mass to total mass
η	Coefficient of viscosity
e	Strain
E	Young's modulus
T_o	Axial tube tension
α, σ	Dimensionless viscoelastic structural damping
κ	Dimensionless nonlinear membrane stiffness
Γ	Dimensionless tension
ζ	Dimensionless length
u	Dimensionless fluid velocity
R	Ratio of coefficient of form drag to coefficient of friction drag
$\mathbb{R}; \mathbb{R}^n$	Real numbers; real n-space

Summary

The ability to predict the behavior of an aerodynamic decelerator during the initial opening phase is crucial to predicting a canopy's behavior when deployed at high speeds. This report develops a mathematical model that may be used to examine the effect of deployment velocity, line tension and drag coefficients on the behavior of a decelerator during the initial phase of the opening.

A nonlinear partial differential equation is derived for the interaction of the decelerator with the external flow field. Using the Galerkin technique a set of low-dimensional models is derived. Each set consists of a finite number of ordinary differential equations in time. The equations are nonlinear and are shown to have three bifurcation parameters, the fluid velocity, the initial tension in the canopy and the ratio of the normal and longitudinal drag coefficients. The lowest order model is examined and it is shown that for the case where the fluid velocity is sufficiently small, the undeflected initial shape is globally stable. For the case were the fluid velocity exceeds a bound depending upon the initial tension, the decelerator's initial undeflected shape is still stable but will be achieved only for a narrow set of initial conditions.

It is recommended that the larger finite dimensional models be examined and that numerical work be performed to compare the finite dimensional predictions with the solutions to the partial differential equations. It is also recommended that an experimental program be initiated to verify that the model's predictions agree with experimental results.

DYNAMICAL SYSTEMS ANALYSIS OF AN AERODYNAMIC DECELERATOR'S BEHAVIOR DURING THE INITIAL OPENING PROCESS

Introduction

The prediction of a circular decelerator's behavior during the initial phase of the opening process is an important technical barrier for the airdrop community. As a canopy is expected to open successfully at higher speeds and lower altitudes, the ability to predict the stability of the initial interaction between the decelerator and the flow field becomes crucial. A stability analysis of a mathematically rigorous model for this process would predict which conditions would result in various types of stable and unstable behavior. If under certain conditions, the unstable behavior caused the parachute to exhibit oscillations of increasing amplitude, a fabric failure might occur before the parachute could open. If the conditions of deployment resulted in a sustained oscillation of bounded amplitude, it is not known how the opening would be affected. The conjecture is that this motion might prevent the failure known as "squiding," where the parachute never begins to inflate. Conditions may also be such that the canopy assumes a stable deflected shape. In any of these cases, the analysis would provide details about the shape of the canopy just prior to the inflation process. Currently, all models of the opening process (Steeves, 1986,1989; Ross, 1970, 1971; Ludtke, 1986; Purvis, 1982; Reddy, 1974; Heinrich and Jamison, 1966) are applicable only after the decelerator has attained some assumed initial shape. Since this assumed shape is determined by the behavior of the decelerator during the initial phase of the opening process, an understanding of a decelerator's behavior just prior to the inflation phase is of fundamental value.

The subject of this report is the development of a mathematical model of a canopy's behavior just after the canopy is extracted. This model requires the development of a partial differential equation (PDE), which governs the interaction of the canopy with the flow field. The derived model could be analyzed using dynamical system theory to determine the stability of the canopy as a function of the values of various physical parameters (e.g., airspeed and line tension).

Literature Review

There are numerous models in the open literature for the interaction of a flexible structure with a flow field; for an overview see Dowell, 1975, 1980; Bisplinghoff and Ashley, 1962. Typically, these models were created to analyze the aerodynamic flutter problem of a structure in a supersonic flow field. While the fluid structure interaction is easier to model using this assumption, most Army airdrops are performed at subsonic speeds. Thus, these models are not directly applicable to the decelerator problem. Work has been published for problems where a pipe either conveys fluid or is in a flow field (Paidoussis and Issid, 1974; Thompson and Lunn, 1981; Paidoussis, 1966). These models were either not applicable, such as those for pipes conveying fluid, or too restrictive such as the linear models of Paidoussis, 1966. Hence, the development of an applicable model was essential.

The application of dynamical system theory to aerodynamic stability problems is not new. For a good mathematical introduction to dynamical systems see Guckenheimer and Holmes, 1983; for an introduction from an engineering viewpoint see Thompson and Stewart, 1986. In fact, the supersonic flutter problem has been examined in some detail (Holmes and Marsden, 1978; Dowell, 1966, 1980; Marsden and Hughes, 1983). The stability of pipes conveying fluid has also been examined by Holmes, 1977. The idea is to use dynamical system techniques to find the "essential generic models" from the full system of PDEs (Holmes, 1977; Holmes and Rand, 1976).

Definitions

Under certain conditions, i.e. wind load, line tension, the decelerator may attain a specific shape and remain that way until inflation begins. This would indicate stability. Sometimes, when the conditions are changed, a dramatic change in the behavior of the decelerator occurs. The new state may be one in which the decelerator is oscillating steadily with bounded amplitude (flutter, or sustained flutter), one in which the amplitudes of the oscillation grow without bound (divergent flutter), one in which the oscillations are chaotic, or one in which the tube is displaced to a new equilibrium position (divergence). The definitions used here for flutter and divergence are in accordance with nonlinear theory and differ from the definitions of divergence and flutter for linear theory (in which divergence is defined as a zero frequency displacement that grows without bound and flutter is defined as an oscillatory motion whose amplitude grows without bound). This rapid change as a function of the value of a given parameter or parameters is termed a bifurcation.

Dynamical System Theory

The advantage to applying dynamical system theory in the study of these complicated physical problems is that the parameter values for which qualitative changes in decelerator behavior occur may be determined by looking at a set of equations which are much simpler than the original PDE. This simpler set, or reduced set, of equations is easier to analyze, both mathematically and numerically, yet still contains all the information required to describe the bifurcation phenomena. In general, the reduced set of equations is a finite system of nonlinear ordinary differential equations (ODEs) in time. This set of ODEs is called a low dimensional model because it contains a small number (one, two or perhaps four) of ODEs as opposed to the original PDE, which was infinite dimensional. There are a number of techniques for making the reduction (Holmes, 1977; Guckenheimer and Holmes, 1983; Carr, 1981). One technique is to expand each of the PDE's variables as an infinite series of appropriate space functions with time-dependent coefficients, substitute the series into the original PDE and truncate the system to a finite number of ODEs in time. Another technique is to determine if a center manifold exists for the PDE. This would also reduce the PDE to a finite set of ODEs in the variables which were changing slowly in time on the center manifold. The local bifurcation behavior is then described by the set of ODEs which govern the dynamics on this usually low dimensional space (see Marsden and McCracken, 1976; Henry, 1981; Guckenheimer and Holmes, 1983; Carr, 1981).

Scope

The scope of this report is limited to narrow tubes with simple supports on each end. This model was chosen because of the parachute's shape and attachments during the initial extraction phase. Typically, the parachute has one end attached to the payload and is extracted by another parachute attached at the opposite end. While the extracting parachute can not perfectly fix the top of the extracted parachute, films of actual parachute deployments show that the assumption of a simple support is not unreasonable as a first approximation. In addition, prior to opening the canopy appears to assume a tube-like shape. The PDE derived in this report is not restricted to the assumption of simply supported end conditions. Other supports (such as fixed-free) may be used and the appropriate set of ODEs would be derived based upon a slightly different eigenfunction expansion. The tubes are assumed to be viscoelastic and may have a general cross-section. The flow field is parallel to the initial (at rest) axis of the tube and a nonlinear fluid-solid interaction model is derived. The extension of the fluid-solid interaction model to include nonlinear terms, while important for subsonic flows, has not been derived previously because the effect is small for supersonic flows (see Holmes, 1977).

The section titled Physical Model contains the basic assumptions used in developing the PDE that describes the uninflated decelerator and its interaction with the external flow field. This section also develops the extension of the fluid-solid interaction model to include nonlinear terms. A complete PDE is then presented and compared to models already published, which serve as special cases of this more general model.

The section titled Derivation of Low Dimensional Models begins with the PDE derived in the previous section and determines a set of eigenfunctions for the nonlinear problem. Each of the PDE's variables is then expanded as an infinite series (with time-dependent coefficients) in these eigenfunctions. The substitution of these series representations into the original PDE and the use of standard, albeit tedious, calculations yields an infinite system of coupled nonlinear ODEs. Truncation of this system at various levels gives a sequence of low dimensional models, which are examined in the next section.

The Results and Discussion present the models developed and the results of the stability analysis along with phase plane plots and time history plots for the lowest dimensional model. This section examines and compares the results to those found by previous authors for similar systems of equations and also describes the numerical software used.

The Conclusions are the salient findings of the report. The implications of these findings to the initial opening problem of a decelerator are then discussed.

The Recommendations outline a set of theoretical and experimental studies for the future. The course of action presented is based upon the results obtained in the current investigation and number of new approaches and extensions, which the author believes could prove fruitful.

Physical Model

The decelerator is modelled as a long thin tube, with mass per unit length m, which is simply supported at each end. The tube is immersed in a fluid which is flowing with a free stream velocity U parallel to the initial (undeformed) axis of the tube. The tube may also be subject to an initial axial load, T_0, applied at the right support by displacing the right support.

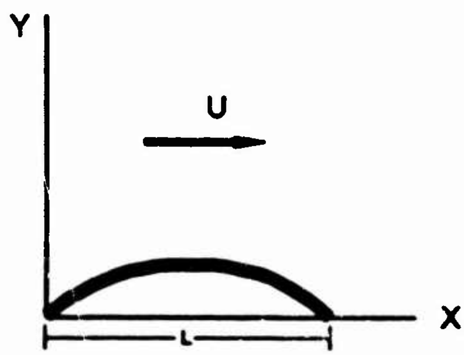

Figure 1. Simply Supported Tube in Axial Fluid Flow.

The boundary conditions are chosen to model the attachment of the decelerator to the payload at one end and to the extraction chute at the other end. The boundary conditions wherein both ends are fixed or one end is fixed with the other free may also be used.

Thus far the geometrical model is the same as that used by Paidoussis, 1966 to construct a linear equation of motion for the flexible cylinder. At this point, the assumptions used to construct the nonlinear model analyzed in this report are described.

The material is assumed to be viscoelastic and to obey the Kelvin-Voight model as was assumed by Holmes, 1977 and Paidoussis and Issid, 1974. Thus, the stress strain law for the material is

$$\sigma = Ee + \eta\dot{e} \,, \tag{1}$$

where σ = *stress*, E = *Young's modulus*, η = *coefficient of viscosity*,

e = *strain, and* $()^{\cdot} \equiv \dfrac{\partial()}{\partial t}$.

The study is also limited to motions which result in small strains in the tube. Under these assumptions (see Holmes, 1977), the axial extension, w(x), induced by the lateral deflection y(x) is given by

$$w(x) = \frac{1}{2}\int_0^L (y'(x))^2 \, dx \tag{2}$$

where $()' \equiv \dfrac{\partial()}{\partial x}$.

Expression 4 and Equation 5 are used to modify the linear model of Paidoussis, 1966. The resulting PDE is structurally nonlinear.

Before writing down the PDE for the motion of the tube, the extension of the fluid-structure interaction model will be derived. Assuming small lateral motions for the tube, Paidoussis, 1966 used the result of Lighthill, 1960 for the resultant relative velocity, v(x,t), between the tube and the flow. V(x,t) then has the form (see Lighthill, 1960)

$$v(x,t) = \dot{y} + U\,y' . \qquad (3)$$

By letting M be the virtual mass of the fluid for a unit length of tube, Paidoussis, 1966 shows that the resultant lateral force on the tube may be expressed as

$$([\frac{\partial}{\partial t} + U\,[\frac{\partial}{\partial x}])\,(M\,v(x,t)) . \qquad (4)$$

In addition, Paidoussis, 1966 assumed that there were viscous forces acting on the tube and using the results of Taylor, 1952 for long inclined tubes with turbulent boundary layers obtained

$$F_N = \frac{1}{2}\rho D U^2 (C_D \sin^2(i) + C_f \sin(i)) \qquad (5)$$

for the viscous normal force per unit length of tube.

Paidoussis, 1966 also obtained

$$F_L = \frac{1}{2}\rho D U^2 C_f \cos(i)$$

Where

$$i = \arcsin\left(\frac{v}{U}\right) \qquad (6)$$

for the viscous longitudinal force per unit length of tube. In equations 5 and 6, D is the tube diameter, ρ is the density of the incompressible fluid, C_{Dp} is coefficient of form drag for a cylinder in cross flow and C_f is the coefficient of friction drag. Paidoussis, 1966 implicitly assumes that v << U which implies that sin (i) << $\sin^2$ (i) and cos (i) = 1.

Thus, Paidoussis, 1966 obtains

$$F_N \approx \frac{1}{2}\rho D U^2 \sin(i)$$

and

$$F_L \approx \frac{1}{2}\rho U^2 C_f \ . \qquad (7a,b)$$

Equations 7a and 7b represent the linearization by Paidoussis, 1966.

If no approximation is made in equations 5 and 6, one obtains

$$F_N = \frac{1}{2}\rho U^2 \left(C_D \frac{v^2}{U^2} + C_f \frac{v}{U}\right)$$

and

$$F_L = \frac{1}{2}\rho D U C_f (U^2 - v^2)^{1/2} \ . \qquad (8a,b)$$

Using equations 3, 8a, and 8b the normal drag force may be written as

$$F_N = \frac{1}{2}[C_f U\dot{y} + C_f y' + C_D(\dot{y})^2 + 2C_D\dot{y}y' + C_D U^2 (y')^2] \ . \qquad (9)$$

Also, noting that for v/U << 1, one may make the approximation

$$[1-(v/U)^2]^{1/2} \approx 1 - \frac{1}{2}\frac{v^2}{U^2} \ . \qquad (10)$$

Thus, the longitudinal drag force may be written as

$$F_L = \tfrac{1}{2}\rho D C_f [U^2 - \tfrac{1}{2}(\dot{y})^2 - U\dot{y}y' - \tfrac{1}{2}U^2(y')^2] \ . \tag{11}$$

Define

$$\varepsilon = \frac{V}{U} \ll 1 . \tag{12}$$

Then, assuming that $O(C_D) \sim O(C_f)$ one can easily show that

$$F_N \approx O(\varepsilon + \varepsilon^2)$$
$$\text{and} \quad F_L \approx O(1-\varepsilon^2)^{1/2} \ . \tag{13}$$

Equations 9 and 11 are quite complex and a simple estimate of the relative importance of the nonlinear terms in each leads to a simpler form. Let E_N and E_L be the fractional errors due to neglecting the nonlinear terms in F_N and F_L respectively. Then

$$E_N = \frac{-\varepsilon}{1+\varepsilon}$$
$$\text{and} \quad E_L = \frac{\tfrac{1}{2}\varepsilon^2}{1-\tfrac{1}{2}\varepsilon^2} \ . \tag{14}$$

Then the ratio of these errors is easily seen to be

$$\frac{E_N}{E_L} = \frac{\tfrac{1}{2}\varepsilon^2 - 1}{\tfrac{1}{2}\varepsilon(1+\varepsilon)} \approx \frac{-2}{\varepsilon} \ . \tag{15}$$

Let $\varepsilon = .1$

$$\text{then} \qquad \frac{E_N}{E_L} \approx -20$$

$$\Rightarrow \left| \frac{E_N}{E_L} \right| \gg 1 \ .$$

The estimate given above is actually quite good. Using MACSYMA the actual expansion of E_N / E_D when evaluated gives -18.045. Also, using MACSYMA a Taylor series expansion about zero gives

$$\frac{E_N}{E_L} = \frac{-2}{\varepsilon} + 2 - O(\varepsilon) \ . \qquad (16)$$

Hence, for ε small, equation 16 agrees with both equation 15 and the MACSYMA results. Since the effect of neglecting the nonlinear terms in F_N is 20 times greater than the effect of neglecting the nonlinear terms F_L, F_N is retained in the form given by equation 9 but F_L is simplified to the linear form used by Paidoussis, 1966 and given here as equation 7b.

Using equations 2, 3, 4,7b, and 9, the equation of motion may be written as (see Paidoussis, 1966; Holmes, 1977)

$$\alpha \dot{y}'''' + y'''' + [u^2(1-\tfrac{1}{2}\zeta c_T(\tfrac{1}{2}-x)) - \Gamma - \kappa\int_0^1 (y')^2\, dx - \sigma\int_0^1 y'\dot{y}'\, dx]\, y''$$
$$+ 2\beta^{1/2}u\dot{y}' + \zeta c_T u^2 y' + \tfrac{1}{2}\beta^{1/2}\zeta c_T u\dot{y}$$
$$+ \ddot{y} + \tfrac{1}{2}c_T uR[\zeta\beta\dot{y}^2 + 2\zeta\beta^{1/2}\dot{y}y' + \zeta(y')^2] = 0 \ . \qquad (17)$$

The boundary conditions for equation 17 are

$$y = y'' = 0 \qquad at \qquad x = 0,1 \ . \qquad (18a,b)$$

Equation 17 is in dimensionless form. The nondimensional variables are defined as

$$\bar{x} = x/L, \quad \bar{y} = y/L, \quad \tau = \frac{t}{L^2}\left(\frac{EI}{m + M}\right)^{1/2}, \quad \beta = \frac{M}{m + M},$$

$$\Gamma = \frac{T_o L^2}{EI}, \quad \zeta = \frac{L}{D}, \quad u = UL\left(\frac{M}{EI}\right)^{1/2}, \quad \kappa = \frac{AL}{2}I,$$

$$\alpha = \frac{\eta}{L^2}\left(\frac{I}{E(m + M)}\right)^{1/2}, \quad \sigma = \frac{\eta AL}{(EI(m + M))^{1/2}},$$

$$c_T = \frac{D^2}{M}\rho C_f, \quad R = \frac{C_{D_p}}{C_f}, \qquad (19a\text{-}1)$$

where A is the cross-sectional area of the tube, I is the cross-sectional moment of inertia of the tube, and all other terms are as define previously. In equation 17, the bars have been dropped from the x and y variables and $()^{\cdot} \equiv \frac{\partial()}{\partial\tau}$ for convenience. If only the linear terms in equation 17 are retained, the resulting equation is equivalent to the linear PDE examined by Paidoussis, 1966. If the last nonlinear term on the left hand side (LHS) of equation 17 is eliminated (by taking R = 0), the resulting equation is similar to that derived in Holmes, 1977.

Based on physical reasoning, the ranges of the parameters defined in equations 19a -1 are restricted. One sets the structural damping parameters $\alpha, \sigma > 0$, and fixes $\beta, \kappa > 0$. Also, it is typically assumed (see Holmes, 1977) that viscous damping terms such as c_T and R are fixed at some positive value. This then leaves the nondimensional tension Γ (which may be either positive, negative or zero) and the nondimensional fluid velocity $u \geq 0$, as the parameters which may be varied. Hence, equation 17 would represent a two-parameter family of PDEs with the so-called control parameters (see Holmes, 1977; Holmes and Marsden, 1978; Guckenheimer and Holmes, 1983) $\bar{\mu} = (u, \Gamma) \in \mathbb{R}^2$.

This choice of control parameters is a function of the particular physical problem being analyzed. In the system of equations examined in this report, equations 17 and 18a,b, the ratio of drag coefficients R is a new parameter, which may be varied in conjunction with the previously mentioned tension and velocity parameters. Thus, one may consider equation 17 as a three-parameter family with

$\bar{V} = (u, \Gamma, R) \in \mathbb{R}^3$.

Derivation of Low Dimensional Models

The equilibrium states may be found by setting all time derivatives to zero in equation 17. The resulting ODE is

$$y'''' + [u^2(1-\tfrac{1}{2}\zeta c_T(\tfrac{1}{2}-x))-\Gamma-\kappa\int_0^1 (y')^2\,dx]\,y'' + \zeta c_T u^2 y' + \tfrac{1}{2}c_T u R\zeta (y')^2 = 0 \tag{20}$$

with the boundary conditions given by equations 18a, b.

Following Holmes, 1977, it is observed that for $c_T = 0$, equation 20 reduces to a nonlinear equation examined by Reiss, 1969. This representation will allow one to extract a complete set of eigenfunctions for the equation 20 by solving for the eigenfunctions of the linearized ODE

$$w_j'''' + \lambda_j w_j'' = 0\,, \qquad |w_j'| = \frac{(u^2-\Gamma-\lambda_j)}{\kappa}\,. \tag{21}$$

For simple supports, equations 18a, b, the resulting eigenfunctions are $w_j(x) = a_j \sin(j\pi x)$. Thus, using Galerkin averaging (see Dowell, 1966; Holmes, 1977) and the eigenfunctions from equation 21, let

$$v(x,\tau) = \sum_{j=1}^{n} r_j(\tau)\, w_j(x)\,. \tag{22}$$

Of course, the use of equation 22 assumes that the series approximation for v converges to v as $n \to \infty$.

Now, substitute equation 22 into equation 17 (and note that as expected the boundary conditions given by equations 18a, b are satisfied because equation 22 is a linear combination of functions, each of which satisfies these boundary conditions) to obtain

$$
\begin{aligned}
&\alpha\sum_{j=1}^{n} w_j'''' \dot{r}_j + \sum_{j=1}^{n} w_j'''' r_j + [u^2(1-\tfrac{1}{2}\zeta c_T(\tfrac{1}{2}-x))-\Gamma]\sum_{j=1}^{n} w_j'' r_j \\
&+ 2\beta^{1/2}u\sum_{j=1}^{n} w_j' \dot{r}_j + \zeta c_T u^2\sum_{j=1}^{n} w_j' r_j + \tfrac{1}{2}\beta^{1/2}\zeta c_T u\sum_{j=1}^{n} w_j \dot{r}_j \\
&+ \sum_{j=1}^{n} w_j \ddot{r}_j - [\kappa\int_0^1(\sum_{k=1}^{n} w_k' r_k)^2 dx]\sum_{j=1}^{n} w_j'' r_j \\
&- [\sigma\int_0^1(\sum_{k=1}^{n} w_k' r_k)(\sum_{k=1}^{n} w_k' \dot{r}_k)\,dx]\sum_{j=1}^{n} w_j'' r_j \\
&+ \tfrac{1}{2}c_T uR\zeta\beta(\sum_{j=1}^{n} w_j r_j)^2 + c_T uR\zeta\beta^{1/2}(\sum_{k=1}^{n} w_k \dot{r}_k)(\sum_{j=1}^{n} w_k' r_k) \\
&\qquad + \tfrac{1}{2}c_T\zeta uR(\sum_{k=1}^{n} w_k' r_k)^2 = 0 \quad .
\end{aligned}
\tag{23}
$$

The Galerkin procedure then requires that equation 23 be multiplied by $w_s(x)$ and integrated from x = 0 to x = 1. The resulting equation is then reduced using the standard orthonormality relationships. The result is a set of n second order ODE in the coefficients $r_s(\tau)$.

By truncating the series at n = 1, 2, and 3, three low dimensional models are developed. They are

<u>*for n = 1*</u>:

$$
\begin{aligned}
&\ddot{r}_1 + A_1\dot{r}_1 + b_1 r_1 + \kappa\frac{\pi^4}{2}r_1^3 \\
&\quad + \sigma\frac{\pi^4}{2}r_1^2\dot{r}_1 + \frac{4c_T u\zeta\beta R}{3\pi}\dot{r}_1^2 + \frac{2\pi c_T u\zeta R}{3}r_1^2 = 0 \quad ,
\end{aligned}
\tag{24}
$$

for n = 2:

$$\ddot{r}_1 + A_1\dot{r}_1 + B_1 r_1 + \tfrac{8}{9}\zeta c_T u^2 r_2 - \tfrac{16}{3}\beta^{1/2} u\dot{r}_2$$
$$+ \kappa\pi^4[\tfrac{1}{2}r_1^2 + 2r_2^2]r_1 + \sigma\frac{\pi^4}{2}[r_1\dot{r}_1 + r_2\dot{R}_2]r_1 \tag{25a}$$
$$+ \frac{c_T uR\zeta\beta}{\pi}[\tfrac{4}{3}\dot{r}_1^2 + \tfrac{16}{15}\dot{r}_2^2] + uR\zeta\beta^{1/2}\frac{\pi}{2}[r_1\dot{r}_2 - \dot{r}_1 r_2]$$
$$+ \pi c_T u\zeta R[\tfrac{2}{3}r_1^2 + \tfrac{56}{15}r_2^2] = 0 \quad ,$$

$$\ddot{r}_2 + A_2\dot{r}_2 + B_2 r_2 + \tfrac{32}{9}\zeta c_T u^2 r_1 - \tfrac{16}{3}\beta^{1/2} u\dot{r}_1$$
$$+ \kappa\pi^4[2r_1^2 + 8r_2^2]r_2 + \sigma\frac{\pi^4}{2}[4r_1\dot{r}_1 + 16r_2\dot{R}_2]r_2 \tag{25b}$$
$$+ \frac{c_T uR\zeta\beta}{\pi}[\tfrac{32}{15}\dot{r}_1\dot{r}_2] + uR\zeta\beta^{1/2}\frac{\pi}{2}[r_1\dot{r}_1]$$
$$+ \pi c_T u\zeta R[\tfrac{16}{15}r_1 r_2] = 0 \quad ,$$

for n = 4:

$$\ddot{r}_1 + A_1\dot{r}_1 + B_1 r_1 + \zeta c_T u^2[\tfrac{8}{9}r_2 + \tfrac{16}{225}r_4]$$
$$-2\beta^{1/2}u[\tfrac{8}{3}\dot{r}_2 + \tfrac{16}{15}\dot{r}_4] + \kappa\pi^4[\tfrac{1}{2}r_1^2 + 2r_2^2 + \tfrac{9}{2}r_3^2 + 8r_4^2]r_1$$
$$+ \sigma\frac{\pi^4}{2}[\dot{r}_1 r_1 + 4r_2\dot{r}_2 + 9r_3\dot{r}_3 + 16r_4\dot{r}_4]r_1$$
$$+ c_T\frac{uR\zeta\beta}{\pi}[\tfrac{4}{3}\dot{r}_1^2 + \tfrac{16}{15}\dot{r}_2^2 - \tfrac{8}{15}\dot{r}_1\dot{r}_3 + \tfrac{36}{35}\dot{r}_3^2 - \tfrac{64}{105}\dot{r}_2\dot{r}_4 + \tfrac{64}{63}\dot{r}_4^2]$$
$$+ c_T\frac{\pi}{2}uR\zeta\beta^{1/2}[-2\dot{r}_1 r_2 + r_1\dot{r}_2 - 3\dot{r}_2 r_3 + 2r_2\dot{r}_3 - 4\dot{r}_3 r_4 + 3r_3\dot{r}_4]$$
$$+ \pi c_T u\zeta R[\tfrac{2}{3}r_1^2 + \tfrac{56}{15}r_2^2 - \tfrac{12}{5}r_1 r_3 + \tfrac{306}{35}r_3^2 - \tfrac{608}{105}r_2 r_4 + \tfrac{992}{63}r_4^2] = 0 \; . \tag{26a}$$

$$
\begin{aligned}
&\ddot{r}_2 + A_2\dot{r}_2 + B_2 r_2 + \zeta c_T u^2 [\tfrac{32}{9} r_1 + \tfrac{96}{25} r_3] \\
&-2\beta^{1/2} u [\tfrac{8}{3}\dot{r}_1 + \tfrac{24}{5}\dot{r}_3] + \kappa\pi^4 [2r_1^2 + 8r_2^2 + 18r_3^2 + 32r_4^2] r_2 \\
&\quad + \sigma\frac{\pi^4}{2} [4\dot{r}_1 r_1 + 16 r_2\dot{r}_2 + 36 r_3\dot{r}_3 + 64 r_4\dot{r}_4] r_2 \\
&\quad + c_T\frac{uR\zeta\beta}{\pi} [\tfrac{32}{15}\dot{r}_1\dot{r}_2 + \tfrac{32}{21}\dot{r}_2\dot{r}_3 - \tfrac{64}{105}\dot{r}_1\dot{r}_4 + \tfrac{64}{45}\dot{r}_3\dot{r}_4] \\
&\quad + c_T\frac{\pi}{2} uR\zeta\beta^{1/2} [\dot{r}_1 r_1 - 3\dot{r}_1 r_3 + r_1\dot{r}_3 - 4\dot{r}_2 r_4 + 2\dot{r}_4 r_2] \\
&\quad + \pi c_T u\zeta R [\tfrac{16}{15} r_1 r_2 + \tfrac{48}{7} r_2 r_3 - \tfrac{416}{105} r_1 r_4 + \tfrac{224}{15} r_3 r_4] = 0 \ . \qquad (26b)
\end{aligned}
$$

$$
\begin{aligned}
&\ddot{r}_3 + A_3\dot{r}_3 + B_3 r_3 + \zeta c_T u^2 [\tfrac{216}{25} r_2 + \tfrac{432}{49} r_4] \\
&-2\beta^{1/2} u [\tfrac{24}{5}\dot{r}_2 + \tfrac{48}{7}\dot{r}_4] + \kappa\pi^4 [\tfrac{9}{2} r_1^2 + 18 r_2^2 + \tfrac{81}{2} r_3^2 + 72 r_4^2] r_3 \\
&\quad + \sigma\frac{\pi^4}{2} [9\dot{r}_1 r_1 + 36 r_2\dot{r}_2 + 81 r_3\dot{r}_3 + 144 r_4\dot{r}_4] r_3 \\
&\quad + c_T\frac{uR\zeta\beta}{\pi} [-\tfrac{4}{15}\dot{r}_1^2 + \tfrac{16}{21}\dot{r}_2^2 + \tfrac{72}{35}\dot{r}_1\dot{r}_3 + \tfrac{4}{9}\dot{r}_3^2 + \tfrac{64}{45}\dot{r}_2\dot{r}_4 + \tfrac{64}{165}\dot{r}_4^2] \\
&\quad + c_T\frac{\pi}{2} uR\zeta\beta^{1/2} [2\dot{r}_1 r_2 + \dot{r}_2 r_1 - r_4\dot{r}_1 + \dot{r}_4 r_1] \\
&\quad + \pi c_T u\zeta R [\tfrac{14}{15} r_1^2 - \tfrac{8}{21} r_2^2 + \tfrac{36}{35} r_1 r_3 + 2 r_3^2 + \tfrac{352}{45} r_2 r_4 + \tfrac{736}{165} r_4^2] = 0 \ . \qquad (26c)
\end{aligned}
$$

$$
\begin{aligned}
&\ddot{r}_4 + A_4\dot{r}_4 + B_4 r_4 + \zeta c_T u^2 [\tfrac{256}{225} r_1 + \tfrac{768}{49} r_3] \\
&-2\beta^{1/2} u [\tfrac{16}{15}\dot{r}_1 + \tfrac{48}{7}\dot{r}_3] + \kappa\pi^4 [8 r_1^2 + 32 r_2^2 + 72 r_3^2 + 128 r_4^2] r_4 \\
&\quad + 8\sigma\pi^4 [\dot{r}_1 r_1 + 4 r_2\dot{r}_2 + 9 r_3\dot{r}_3 + 16 r_4\dot{r}_4] r_4 \\
&\quad + c_T\frac{uR\zeta\beta}{\pi} [-\tfrac{64}{105}\dot{r}_1\dot{r}_2 + \tfrac{64}{45}\dot{r}_2\dot{r}_3 + \tfrac{128}{63}\dot{r}_1\dot{r}_4 + \tfrac{128}{165}\dot{r}_3\dot{r}_4] \\
&\quad + c_T\frac{\pi}{2} uR\zeta\beta^{1/2} [r_1\dot{r}_2 + 3\dot{r}_1 r_3 + 2 r_2\dot{r}_2] \\
&\quad + \pi c_T u\zeta R [\tfrac{352}{105} r_1 r_2 - \tfrac{32}{15} r_2 r_3 + \tfrac{64}{63} r_1 r_4 + \tfrac{192}{55} r_3 r_4] = 0 \ . \qquad (26d)
\end{aligned}
$$

In equations 24, 25, and 26a-d the coefficients A_s and B_s are defined as

$$A_s = \pi^4 s^4 [\alpha + \tfrac{1}{2}\beta^{1/2}\zeta c_T u]$$

$$B_s = \pi^4 s^4 + [\Gamma - u^2]\pi^2 s^2 \quad . \tag{27a,b}$$

Any one of the systems given by equations 24, 25a,b or 26a-d may be written as a first order ODE on $\mathbb{R}^{2n}$. For example equation 24 may be put in the form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_\mu \boldsymbol{x} + \boldsymbol{N}(\boldsymbol{x}) \quad ; \quad \boldsymbol{x}(0) = \boldsymbol{x}_o \in \mathbb{R}^2 \ , \tag{28a-e}$$

where

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ \dot{x}_1 \end{pmatrix}, \quad \boldsymbol{A}_\mu = \begin{pmatrix} 0 & 1 \\ -B_1 & -A_1 \end{pmatrix}$$

and

$$\boldsymbol{N}(\boldsymbol{x}) = \begin{pmatrix} 0 \\ \frac{\kappa\pi^4}{2} x_1^3 + \frac{\sigma\pi^4}{2} x_1^2 x_2 + \frac{4 c_T u \zeta \beta R}{3\pi} x_2^2 + \frac{2\pi c_T u \zeta R}{3} x_1^2 \end{pmatrix} .$$

Clearly, for a given set of initial conditions, equations 28a-e may be numerically integrated using standard techniques. In fact, all of the low dimensional models may be put in the form of equations 28a-e with the only change being that the entries in equation 28d become matrices of order n x n . Holmes, 1977 points out that the accuracy with which the numerical solutions to any particular low dimensional model agree with the solution to the PDE is dependent upon the speed of convergence as the number of modes goes to infinity. Dowell, 1966 showed that for similar systems n = 4 or 6 was a good approximation. This (see Holmes, 1977) is probably due to a concentration of the energy at the low frequencies. Following Holmes, 1977, as the current report is primarily concerned with the qualitative behavior of solutions, the assumption is made that the convergence is rapid enough for the low dimensional models to be physically reasonable. This assumption will have to be examined later in this research, either analytically or experimentally.

Results and Discussion

This section describes the use of dynamical system theory to investigate the bifurcation behavior of the low dimensional model given by equation 28a-e. It also describes the software which is used to provide the phase plane plots and compute the stable and unstable manifolds associated with the equilibrium points.

As stated previously, one may consider equation 28a-e as a three-parameter family with $\overline{V} = (u, \Gamma, R) \in \mathbf{R}^3$. Equation 28a-e has equilibrium points given by

$$x_2 = 0 \ ,$$

$$\frac{\kappa\pi^4}{2} x_1^3 + \frac{2\pi c_T u \zeta R}{3} x_1^2 + B_1 x_1 = 0 \quad . \tag{29}$$

For R = 0, the results agree with those obtained by Holmes, 1977. They are reproduced here for comparison. If R = 0 and $B_1 > 0$, (0,0) is a unique fixed point and it is a sink, see figure 2, which means that the undeflected shape is stable. If R = 0 and $B_1 < 0$, there exist three equilibrium points given by (0,0) and $(\pm\sqrt{-2B_1/\kappa\pi^4}\ , 0)$. The tube is now unstable since the center fixed point is a saddle and the other two are sinks as shown in figure 3 (see Holmes, 1977). Note also that The change in sign of B_1 from positive to negative takes place when $\Gamma - u^2 = -\pi^2$, (this is the Euler buckling mode).

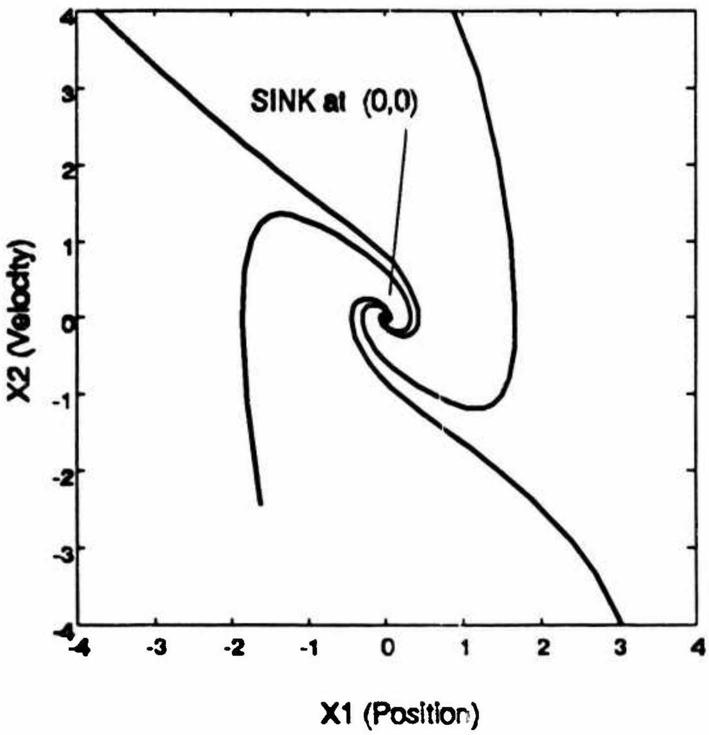

Figure 2. Holmes' One Mode Model with B1 > 0, R = 0.

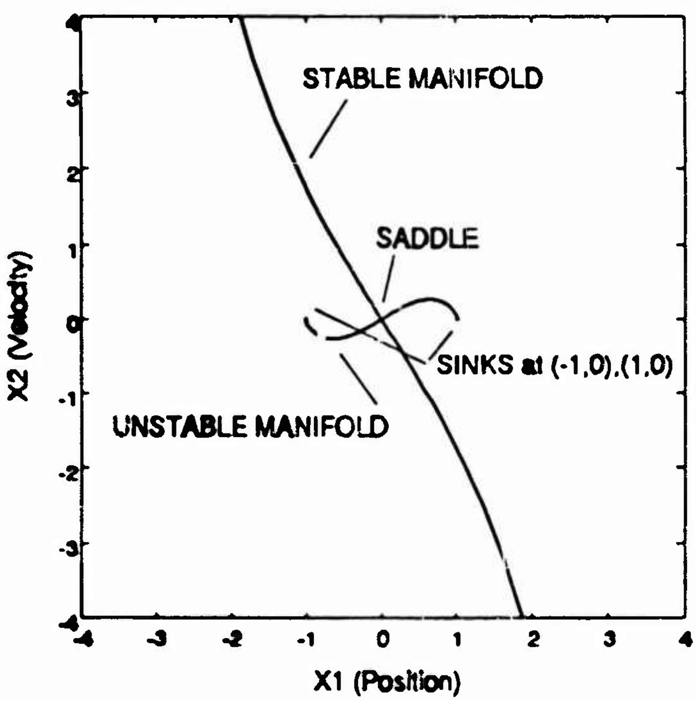

Figure 3. Holmes' One Mode Model with B1 < 0, R = 0.

For the case where R > 0 , which is the model derived in this report, different results are obtained. Let R > 0 , then equation 29 gives

$$x_2 = 0 \;,$$

and

$$x_1 = 0 \qquad (30)$$

or

$$x_1 = -\frac{2c_T u\zeta R}{3\kappa\pi^3} \pm \sqrt{\frac{4c_T^2u^2\zeta^2R^2}{9\kappa^2\pi^6} - \frac{2B_1}{\kappa\pi^4}} \;.$$

Equation 30 shows that for $B_1 > 0$ there may not exist a unique fixed point at $(x_1, x_2) = (0,0)$. If $B_1 > 0$ and

$$u^2 < \frac{\pi^4[\Gamma + \pi^2]}{\dfrac{2c_T^2\zeta^2R^2}{9\kappa} + \pi^4} \;, \qquad (31)$$

then there is a unique fixed point at (0,0). The phase plane plot for this case is shown in figure 4. Note, that all points in phase space are attracted to (0,0). Thus, the undeflected shape is stable provided that the three bifurcation parameters satisfy equation 31. Physically, this means that as the fluid velocity, u, is increased, the in-plane load must be increased to maintain the stability of the undeflected shape.

If

$$u^2 > \frac{\pi^4[\Gamma + \pi^2]}{\dfrac{2c_T^2\zeta^2R^2}{9\kappa} + \pi^4} \;, \qquad (32)$$

then there are three fixed points, regardless of the sign of B_1. The phase plane plot for this case is shown in figure 5.

While the undeflected position, (0,0), is still stable, the basin of attraction, which is bounded by the stable manifold, is very narrow. Thus any small initial deviation may move the tube off its undeflected position.

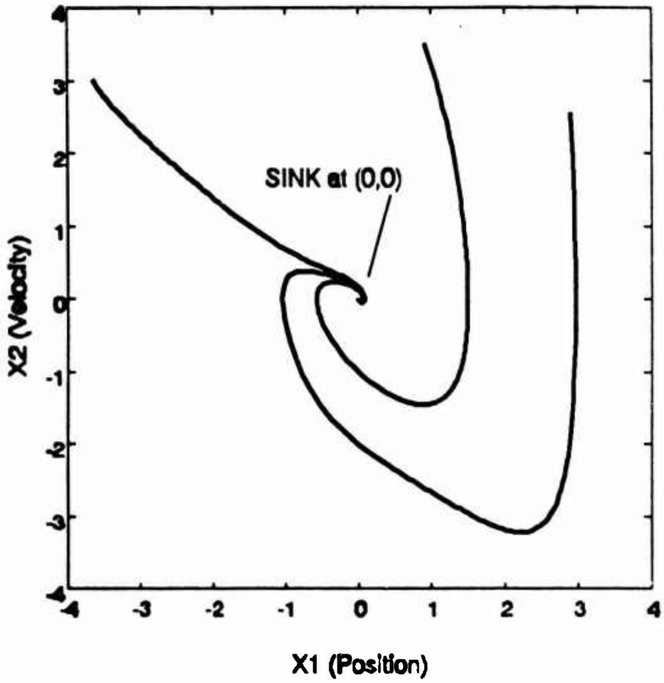

Figure 4. New One Mode Model with B1 > 0 and Eq. 31 Satisfied.

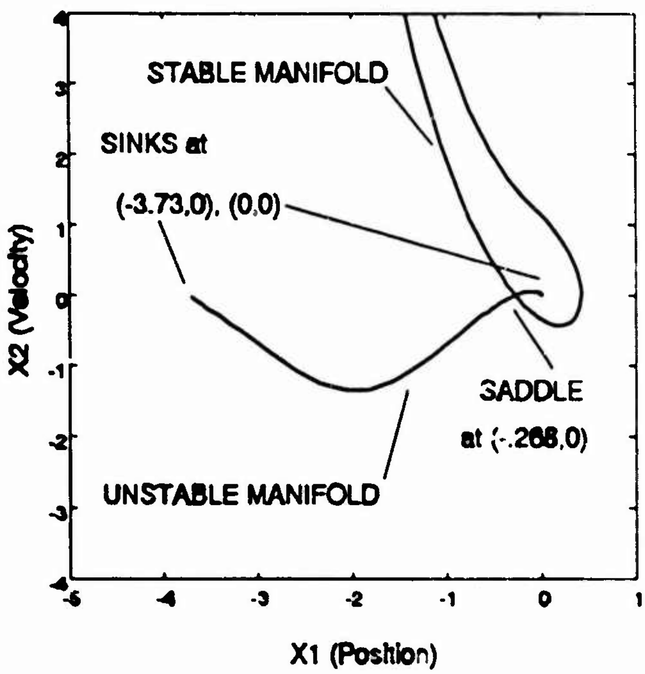

Figure 5. New One Mode Model with B1 > 0 and Eq. 32 Satisfied.

If

$$u^2 = \frac{\pi^4[\Gamma + \pi^2]}{\frac{2c_T^2\zeta^2R^2}{9\kappa} + \pi^4} \quad , \tag{33}$$

then there are two distinct fixed points at (0,0) and at $(-\frac{2c_T u\zeta R}{3\kappa\pi^3}, 0)$. The fixed point at (0,0) is a sink and the other is not hyperbolic.

At this point, the dynamical system software used to examine the low dimensional model is described. The specific software is "kaos(TM); Dynamical System Toolkit with Interactive Graphic Interface" and was developed by J. Guckenheimer and S. Kim. The package is a collection of numerical and graphical routines which are quite helpful in computing equilibrium points, stable and unstable manifolds and other dynamical system phenomena, such as limit cycles. The program runs on a SUN SPARC(TM) workstation and is easy to compile and easy to use. (For a detailed description of the package see J. Guckenheimer and S. Kim; 1990.)Figure 6 is an example of the display produced on screen for the user.

The program allows the users to examine the behavior of their own system of equations in a interactive mode. The graphical output is supplemented by data written to a file. These data may include locations in phase space of equilibrium points, types of fixed points (i.e., saddles, sinks, sources) and eigenvalues and eigendirections for the stable and unstable manifolds. The toolkit also computes time series and allows users to construct Poincare maps. The user may also choose a particular integration algorithm.

The program was obtained free of charge by anonymous ftp. The package contained detailed instructions and appropriate Makefiles to enable one to build a working version on their machine in a very short period of time. For someone well versed in dynamical system theory, this toolkit can greatly reduce the routine calculations one previously had to perform and can provide visualization capability with little or no effort.

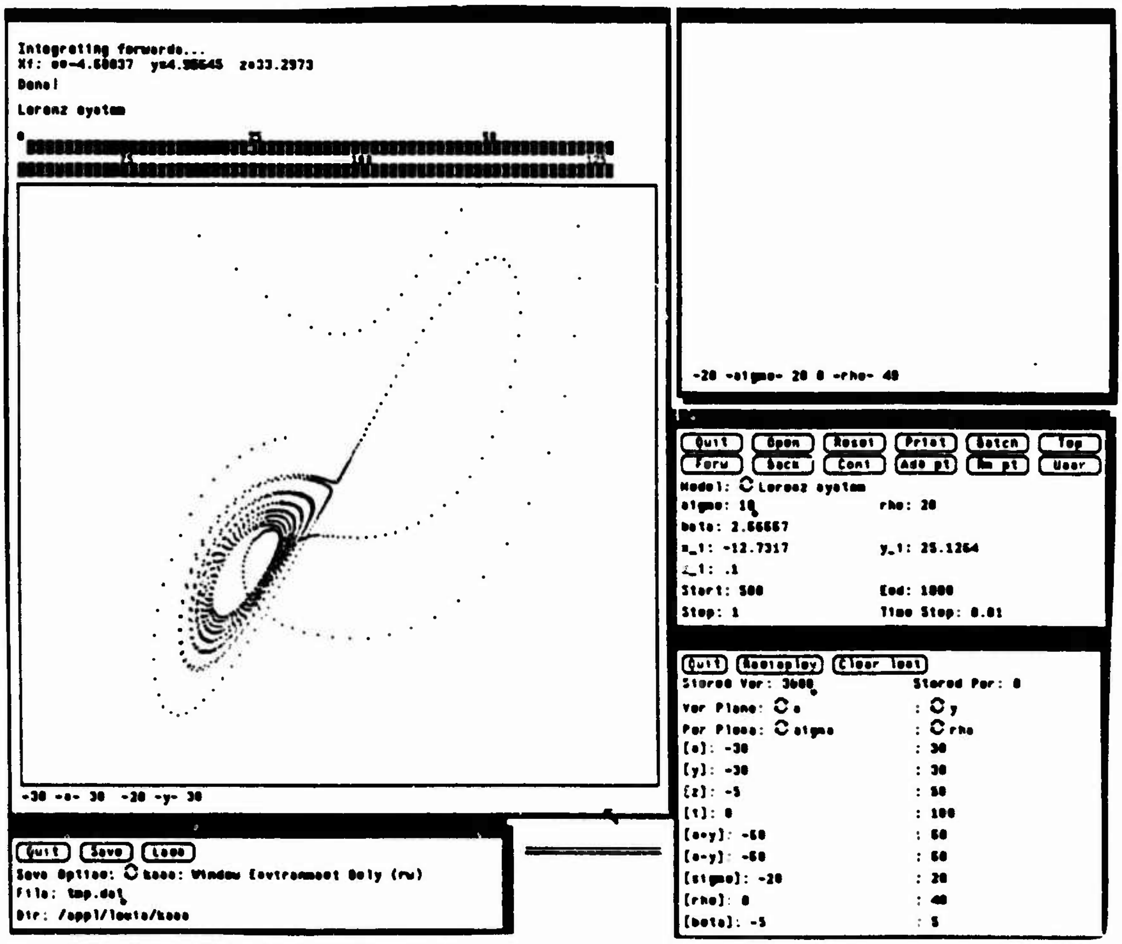

Figure 6. Typical SUN Display of KAOS.

Concluding Remarks

A new nonlinear partial differential equation is derived to model the behavior of a simply supported tube immersed in a viscous fluid flowing parallel to the support axis. The new model extends the linear fluid-structure interaction model used by Holmes, 1977 and Paidoussis, 1966. Using this model, a sequence of low dimensional models is constructed. Each finite dimensional model is a set of time-dependent nonlinear ODEs.

Analysis of the smallest low dimensional model shows that the extension of the linear fluid-structure interaction model to include nonlinear effects has a stabilizing effect. Previously (see Holmes, 1977), the undeflected shape could be made to become unstable by a suitable choice of parameters. In the model examined in this report, while the basin of attraction for the sink characterizing the undeflected shape is small, the equilibrium point is stable. It is also shown that a simple relationship exists, equations 31 and 32, relating the fluid velocity to the in-line tension required for global stability of the initial, undeflected shape.

Recommendations

The results presented in this report are preliminary. Only the smallest low-dimensional model was examined and it is well known (see Holmes, 1977) that the larger models, which allow modal coupling, can present more interesting behavior, such as flutter. It is therefore recommended that the two-mode and four-mode models be analyzed.

The low-dimensional models were derived using the assumption that the series representation for each variable converged to the true solution sufficiently rapidly for a small number of terms to yield good results. It is recommended that the actual PDE be examined using center manifold theory to produce a low-dimensional model and that it be compared to the set of models produced in this report. The PDE should also be directly integrated for specific parameter values and the results compared with the predictions of the low-dimensional models.

Finally, the value of any model is determined by its ability to predict. Therefore, experimental work should be performed to verify that the qualitative behavior predicted by the model agrees with the behavior observed in the laboratory.

This document reports research undertaken at the US Army Natick Research, Development and Engineering Center and has been assigned No. NATICK/TR-92/017 in the series of reports approved for publication.

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SUPPLEMENTARY

INFORMATION

16 Jan 92

ERRATA

TITLE OF TECHNICAL REPORT: Dynamical Systems Analysis of an Aerodynamic Decelerator's Behavior during the Initial Opening Process

REPORT NUMBER: NATICK/TR-92/017

REPORT DATE: December 1991

Please substitute the attached page 11 for the page 11 originally included in the above report.

Thank you,

Louis J. Piscitelle

U.S. Army Natick Research, Development and Engineering Center
Aero-Mechanical Engineering Directorate
Natick, MA 01760-5017

REPLACE PAGE 11 WITH THIS PAGE

Equation 17 is in dimensionless form. The nondimensional variables are defined as

$$\bar{x} = x/L, \qquad \bar{y} = y/L, \qquad \tau = \frac{t}{L^2}\left(\frac{EI}{m + M}\right)^{1/2}, \qquad \beta = \frac{M}{m + M},$$

$$\Gamma = \frac{T_o L^2}{EI}, \qquad \zeta = \frac{L}{D}, \qquad u = UL\left(\frac{M}{EI}\right)^{1/2}, \qquad \kappa = \frac{AL^2}{2I},$$

$$\alpha = \frac{\eta}{L^2}\left(\frac{I}{E(m + M)}\right)^{1/2}, \qquad \sigma = \frac{\eta A}{(EI(m + M))^{1/2}},$$

$$c_T = \frac{D^2}{M}\rho C_f, \qquad R = \frac{C_{D_p}}{C_f}, \qquad (19a\text{-}1)$$

where A is the cross-sectional area of the tube, I is the cross-sectional moment of inertia of the tube, and all other terms are as define previously. In equation 17, the bars have been dropped from the x and y variables and $()^{\cdot} \equiv \frac{\partial()}{\partial\tau}$ for convenience. If only the linear terms in equation 17 are retained, the resulting equation is equivalent to the linear PDE examined by Paidoussis, 1966. If the last nonlinear term on the left hand side (LHS) of equation 17 is eliminated (by taking R = 0), the resulting equation is similar to that derived in Holmes, 1977.

Based on physical reasoning, the ranges of the parameters defined in equations 19a -1 are restricted. One sets the structural damping parameters $\alpha, \sigma > 0$, and fixes $\beta, \kappa > 0$. Also, it is typically assumed (see Holmes, 1977) that viscous damping terms such as c_T and R are fixed at some positive value. This then leaves the nondimensional tension Γ (which may be either positive, negative or zero) and the nondimensional fluid velocity $u \geq 0$, as the parameters which may be varied. Hence, equation 17 would represent a two-parameter family of PDEs with the so-called control parameters (see Holmes, 1977; Holmes and Marsden, 1978; Guckenheimer and Holmes, 1983) $\bar{\mu} = (u, \Gamma) \in \mathbb{R}^2$.