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MODEL TO DESCRIBE MELT RATES BY HEAT CONDUCTION

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13. ABSTRACT (Maximum 200 words)

A simplified model that describes solid heating and melting rates was developed. Examples using the model and the software used to complete melting rates are included. The limiting conditions, including equations, are also discussed.

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PREFACE

The work described in this report was authorized under Project No. 1L161102A71A, Research in CW/CB Defense. This work was started and completed in April 1990.

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MODEL TO DESCRIBE MELT RATES BY HEAT CONDUCTION

1. PURPOSE

This report presents a model developed to predict conductive heat transfer through thick solids where melting occurs. Sample calculations will illustrate the use of this model, and an estimating function for the range of applicability will be discussed.

2. INTRODUCTION

Predicting melt rates and thermal profiles in solids is of fundamental interest to the flame and incendiary program. Such analysis allows one to integrate the combustion characteristics of an incendiary with the target heat transfer parameters to predict weapon effectiveness against the target and to optimize the fill composition. This modeling process minimizes the experimental and chemical formulation effort that, in turn, reduces research and development costs.

As an initial effort, a conductive heat transfer model that predicts solid internal temperatures and melt rates was developed. Model simplicity was of significant importance, because it provided a closed analytical solution that allowed for a functional description of the heat transfer process and a basis on which to expand this effort. Future model verification will also require analysis of the test system, which was designed to accommodate controlled test conditions rather than actual target challenges.

3. DESCRIPTION OF MODEL

As depicted in Figure 1, an incendiary mix at temperature T_{mx} of thickness B burns for time t_{rxn}. The mix maintains contact with the target; heat transfer is by conduction. The outer target skin melts at temperature T_m, providing a constant forcing temperature to the remaining solid mass. The surrounding air mass is a poor heat transfer media and is assumed to act as an insulator during the period of interest (1-5 s). The difference between the energy released from the incendiary mix and the heat conducted into the solid is used to melt the solid mass at the liquid interface, also at temperature T_m.

To simplify heat conduction analysis, a target of infinite thickness is assumed. The burn rate is assumed proportional to mix area A which, for this model, is constant. Hence, if ΔH is the energy released per pound of mix,

$$(dQ/dt)mix = \Delta H * A * Kr * \rho_{mx} \quad (1)$$

$$= Constant$$

where

$(dQ/dt)mix$ = Energy release rate from mix (btu/hr)

Kr = Burn rate constant (ft/hr)

A = Area (ft^2)

ΔH = Heat released per # of mix

ρ_{mx} = Mix density ($\#/ft^3$)

Thus,

$$(dQ/dt)mix = Total\ energy\ released/time\ of\ burn \quad (2)$$

$$= A * B * \rho_{mx} * \Delta H / t_{rxn}$$

where

$Kr = B/t_{rxn}$

B = Mix thickness

3.1 Heat Production.

The fraction of the energy rate that is delivered to the target is either the difference of energy produced minus the energy to heat the mix to temperature t_{mx}/t_{rxn} or equation 3:

$$A * B * \rho_{mx} [\Delta H - C_p (T_{mx} - T_0)] / t_{rxn} \quad (3)$$

where

C_p = Mix specific heat

T_0 = Ambient temperature

$T_m \leq T_{mx}$ (For melting to occur)

3.2 Conductive Heat Loss.

For one directional heat transfer into solids of infinite thickness, the following can be shown:[1]

$$(T-T_0)/(T_1-T_0) = 1-erf[x/(4*\alpha*t)]^{1/2} \quad (4)$$

where

$T = T(x,t)$

T_0 = Ambient temperature

T_1 = Surface forcing temperature

α = Thermal Diffusivity ($K/\rho C_p$)

K = Thermal conductivity

t = Time

x = Depth

For thick plates, this equation will give a reasonable estimate of solid internal temperatures. As a rule, where $(T-T_0)/(T_1-T_0) < 0.01$ for the back of the plate, the infinite plate approximation is considered valid. Thus, if the minimum plate thickness D_{min} is greater than or equal to $4(\alpha*t)^{1/2}$, one can assume that this model is accurate. Clearly D_{min} must be greater for either lighter, more conductive materials or for longer burn times.

Because the heat flux through the solid at the solid surface is just:

$$-K(\partial T(x,t)/\partial x)\,|_{x=0} \quad (5)$$

After substituting equation 4 into the above expression, one gets equation 6:

$$-K[\partial T(x,t)/\partial x]\,|_{x=0} = K(T_1-T_0)(\pi\alpha t)^{-1/2} \quad (6)$$

---

1 Bird, R.B., Stewart, W.E., and Lightfoot, E.N., Transport Phenomena, p 354, John Wiley and Sons, Inc., New York, NY, 1960.

Or for this model, we get equation 7:

$$-K[\partial T(x,t)/\partial x]\big|_{x=0} = K(T_m - T_0)(\pi\alpha t)^{-1/2} \tag{7}$$

3.3 Melt Rate.

Equation 7 gives the rate of heat conduction per unit area across the solid/liquid interface. Clearly, as time increases, the energy flux decreases, because T(x,t) increases, thus reducing the thermal driving force into the solid. The predicted flux in equation 7 will increase to infinity as time goes to zero. This type of increase happens because an infinite flux is required to achieve an elevated surface temperature in zero time. Consequently, if the rate term is integrated to determine the depth of melt over a time period, a significant overestimate of depth is predicted, even though the rate quickly drops to the correct values. This energy flux expressed as a melt rate is shown in equation 8.

$$\rho_{tg} * A * \Delta H_f * (dx_L/dt) \tag{8}$$

where

ΔH_f = Heat to melt target/# of target

ρ_{tg} = Target density

dx_L/dt = Melt rate

x_L = Liquid melt depth

3.4 Energy Balance.

For the liquid phase, the energy delivery rate to the target minus the energy loss rate into a solid equals the energy accumulation for melting. Hence, one gets equation 9:

$$A * B * \rho_{mx}[\Delta H - C_p(T_{mx} - T_0)]/t_{rxn} - K(T_m - T_0)(\pi\alpha t)^{-1/2} = \rho_{tg} * A * \Delta H_f * (dx_L/dt) \tag{9}$$

Solving for (dx_L/dt) gives us either equation 10

$$(dx_L/dt) = \Big((\rho_{mx}/\rho_{tg})(B/t_{rxn})\ \big\{[\Delta H - C_p(T_{mx} - T_0)]/\Delta H_f\big\}\Big) - \big\{K(T_m - T_0)/[(\pi\alpha)^{1/2}\rho_{tg}\Delta H_f]\big\}t^{-1/2} \tag{10}$$

or equation 11:

$$(dx_L/dt) = A1 - A2 * t^{-1/2} \tag{11}$$

where

$$A1 = \{(\rho_{mx}/\rho_{tg})(Kr)[(\Delta H - C_p T_{mx} - T_0)/\Delta H_f]\}$$

$$A2 = \{K(T_m - T_0)/[(\pi\alpha)^{0.5}\ \rho_{tg}\ \Delta H_f]\}$$

$$Kr = B/t_{rxn}$$

The expression Kr is used to show that experimental rate constants can be entered into the equation to predict melt rates.

The initial melt rates are negative because the conductive flux is greater than the heat generation term. This is because the initial condition requires an elevated surface temperature step function at zero time. However, within a short time, the flux predicted by this equation reaches a more reasonable initial state in which the conduction rate equals the production rate. Typical times for this to occur are 0.001 to 0.01 s. To make this equation more usable for analysis, a time translation of $(A2/A1)^2$ is added to the time t. This translation is the same as stating that at time zero, the melt rate and melt depth are zero. With this modification, equation 11 becomes equation 12:

$$(dx_L/dt) = A1 - A2 * [t + (A2/A1)^2]^{-1/2} \tag{12}$$

where

$$A1 = \{(\rho_{mx}/\rho_{tg})(Kr)[(\Delta H - C_p(T_{mx} - T_0)/\Delta H_f]\}$$

$$A2 = \{K(T_m - T_0)/[(\pi\alpha)^{1/2}\ \rho_{tg}\ \Delta H_f]\}$$

$$Kr = B/t_{rxn}$$

The depth of melt over time period t is now obtained by integration. Thus, one gets equation 13:

$$x_L(t) = A1\int_0^t dt - A2\int_0^t dt/[t + (A2/A1)^2]^{1/2} \tag{13}$$

Solving for the melt depth gives one equation 14:

$$x_L(t) = (A1)(t) - (2)(A2)\left\{[t+(A2/A1)^2]^{1/2} - (A2/A1)\right\} \qquad (14)$$

As can be seen, both (dx_L/dt) and $x_L(t)$ are zero at zero time. The maximum melt rate, having a value of A1, is approached as time goes to infinity.

A more rigorous approach to this analysis is to include an estimated surface temperature forcing function, which goes from ambient to T_m over a time period (e.g., ramp and step), and to solve the heat transfer equation. Unfortunately, this approach leads to a complex functional form that requires computer analysis for equation solution. The main change to this model expected from such a modification is the introduction of a small lag term in the solid conductive energy flux. This lag slows initial solid heating and increases initial melting rates. Therefore, equation 14 should be slightly conservative in estimating melting depths. However, because surface heating proceeds rapidly and approximates a step function, it is expected that these differences will be small; hence, equations 12 and 14 are expected to give good first order approximations of the mixes' target melting performances. Without better knowledge of the actual surface temperature spike, there is little to be gained by a more refined analysis.

3.5 Results and Discussions.

Table 1 provides a summary of expressions used and their definitions. Table 2 summarizes the primary equations described.

Three potential mixes were analyzed for thermal output characteristics and used to predict melt rates. These values were computed using the National Aeronautics and Space Administration [NASA (Washington, DC)] Lewis computer burn simulation.[2] The computed thermal and physical properties of the mix and target materials are shown in Table 3. Computed target melt rates are shown in Table 4. The target materials analyzed were aluminum and iron. Melt rates are shown for the maximum rate possible (infinite time of burn), for the melt rate at 0.5 s, and

2 Gordon, S., and McBride, B.J., Computer Program for Calculation of Complex Equilibrium Compositions, Etc., Report Number SP-273, National Aeronautics and Space Administration, Washington, DC, 1971.

the average melt rate over 0.5 s. The average rate would be that observed by conducting a timed test and measuring the depth of melt. The melt depths and average melt rates were computed using the program found in the appendix.

For an assumed 0.5 s burn duration, maximum melt rates >1 in./s were possible. For aluminum, the minimum target thickness for model validity was 1 in; for iron, the minimum target thickness was 0.5 in. The higher melt rates in aluminum are due to its lower density.

As shown in Figure 2, the melt rates approach the maximum rates very quickly. This rapid approach to constant melt rates results in predicted melt depths, which are very close to those computed using maximum rates. This result is shown in Figures 3 and 4, where the data points were based on computed depths over 0.5 s using equation 14, and the lines were computed using only A1, the maximum melt rate. Although the absolute deviation from computed melt depths and rates using only A1 were not great, the relative error for small values (slower melts) was greater than 500% in some cases. This high error results because the time dependency in the rate constant became important for short times with the slower processes.

4. CONCLUSIONS

An infinite plate conductive heat transfer model was developed in this report. This model provides a basis on which to estimate target heating and melting rates using target material heat transfer characteristics and mix heat generation predictions. Although they are based on an approximate representation of surface heating and are limited to thick plates or short burn times, equations 12 and 14 are expected to correlate well with test results and are easy to use and to relate to the significant target, geometry, and mix parameters. Although equations 12 and 14 should be used when comparing the model to test results for model verification or when computing short burn duration, melting depths and rates, the A1 term can be used in chemical mix optimization studies. If the mix combustion properties are measured or are known, model validation can be accomplished by comparing measured melt depths and burn times to those values predicted using equation 14 with the appropriate experimental rate constant. More accurate finite thickness models should also be developed as these results will more closely match actual data. The finite thickness models will predict more rapid melt rates because a finite reservoir will not remove energy as quickly as an infinite one.

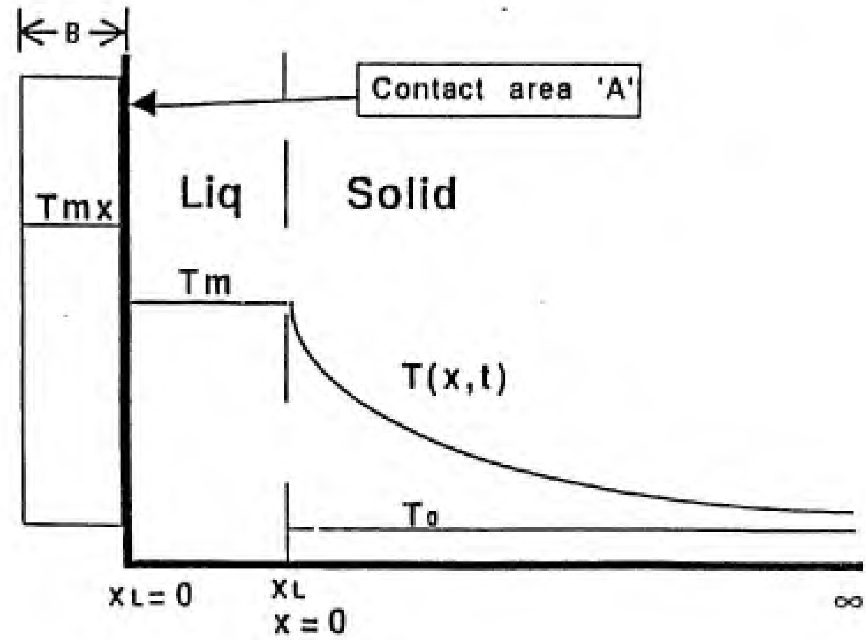

Figure 1. Heat Transfer Model

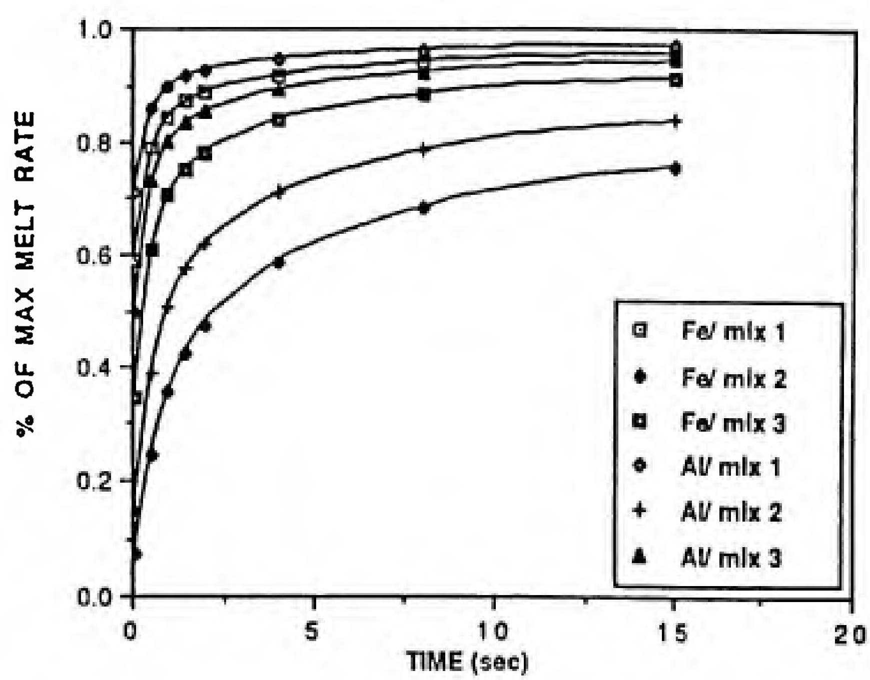

Figure 2. Rate of Approach to Maximum Melt Rates

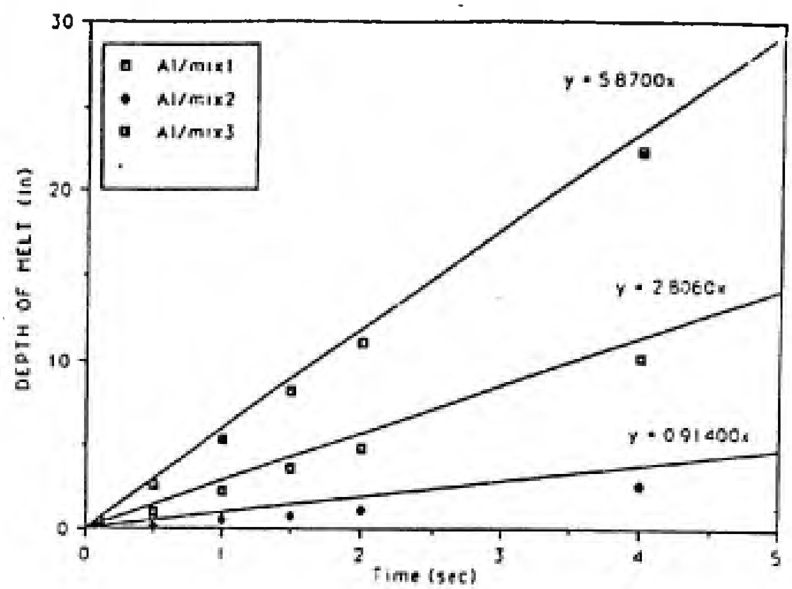

Figure 3. Cumulative Melt Depths

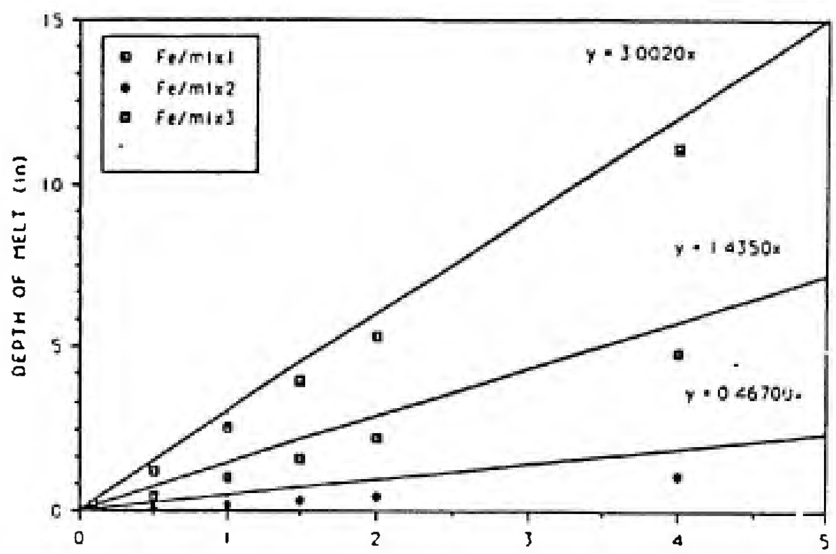

Figure 4. Cumulative Melt Depths

Table 1. Definition of Terms

TERM	DEFINITION
$(dQ/dt)_{mix}$	Energy release rate from mix
$K_{r,B}/t_{rxn}$	Mix burn rate constant
A	Mix area on target
B	Mix thickness over target
D_{min}	Minimum target thickness
ρ_{tg}	Target density
ρ_{mx}	Mix density
ΔH	Heat released per # of mix
Cp	Mix specific heat
T	Temperature
To	Ambient temperature
T_m, T_1	Surface forcing temperature
T_{mx}	Mix flame temperature
x	Depth in soil
x_L	Liquid melt depth
dx_L/dt	Melt rate
t	Time
K	Thermal conductivity
α	Thermal diffusivity $(K/\rho\, C_p)$
A1	$\{(\rho_{mx}/\rho_{tg})\ (K_r)\ (\Delta H C_p(T_{mx} T_o))/\Delta H_f)\}$
A2	$\{K(Tm-To)/((\pi\alpha)^{1/2}\ \rho_{tg}\ \Delta H_f)\}$

Table 2. Equation Summary

PARAMETER	EQUATION
$(T-T_o)/(T_1-T_o)$	$1-erf(x/(4*\alpha*t)^{1/2}$
D_{min}	$4(\alpha*t)^{1/2}$
(dx_L/dt)	$A1-A2*(t+)A2/A1)^2)^{-1/2}$
$x_L(t)$	$A1t-2\ \ A2((t+(A2/A1^2)^{1/2}-(A2/A1)$
$(dx_L/dt)_{max}$	A1

Table 3. Values of Input Terms

SYMBOL	UNITS	MIX 1	MIX 2	MIX 3	Fe TARGET	Al TARGET
T_{mx}	deg F	6020	2780	6200	-	-
t_{rxn}	Hrs	1.388^{-4}	1.388^{-4}	1.388^{-4}	-	-
ΔH	Btu/#	2052	660	2736	-	-
B	Ft	.01	.01	.01	-	-
ρ	$\#/Ft^3$	343	166	123	490.8	169.2
Tm	deg F	-	-	-	2786	1217
ΔH_f	Btu/#	-	-	-	114.7	170.1
K	Btu/HrFt°F	-	-	-	40.8	128.3
Cp	Btu/#°F	-	-	-	0.108	0.23
α	Ft^2/Hr	-	-	-	0.7697	3.30

Table 4. Computed Melt Rates

MELT RATE (In/Sec)	MIX 1		MIX 2		MIX 3	
Max @ ∞ sec.	3.00	5.90	0.47	0.91	1.44	2.81
Max @ .5 sec.	2.65	5.43	0.18	0.51	1.09	2.37
Mean @ .5 sec.	2.37	5.04	0.11	0.40	0.88	2.04
TARGET	Fe	Al	Fe	Al	Fe	Al

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APPENDIX

COMPUTER LISTING FOR COMPUTING MELT DEPTHS AND RATES

This basic computer listing was prepared to compute melt depths and rates using equations 12 and 14 in the main text. The software is compatible with Mac Basic.

There are three input variables: A1, A2, and t. The user should enter '1' to enter/change A1, enter '2' to enter/-change A2, and enter '3' to enter/change t. Entering '4' will compute melt depth and melt rate in inches and inches per second, respectively.

The input units on t are seconds. The input units on A1 are feet per hour. The input units on A2 are feet per hour$^{\frac{1}{2}}$. Table 1 in the main text provides the required equations to compute A1 and A2. The output variable XBAR is the melt depth over the time period t. The output variable X'BAR is the average melt rate over the time period t.

```
PRINT"enter time as sec, units out= in, in/sec"
LBLs:
PRINT" compute XBAR and X'BAR using"
PRINT"      A1, A2, and t"
PRINT "To change: A1=1, A2=2, t=3, compute=4"
LBL0:
INPUT"Enter number, press return"; X
ON X GOTO LBL1, LBL2, LBL3, LBL4
LBL1:
INPUT"A1="; A1
GOTO LBL0
LBL2:
INPUT"A2="; A2
GOTO LBL0
LBL3:
INPUT"t="; t
t=t/3600
GOTO LBL0
LBL4:
XBAR=(A1*t)+A2*2*((A2/A1)-((A2/A1)^2+t)^.5)
XBAR=XBAR*12
DXBAR=XBAR/t/3600
PRINT "XBAR=";XBAR;" X'BAR=";DXBAR
INPUT "CONTINUE Y/N"; c$
IF c$="Y" OR c$="y" THEN GOTO LBLs
STOP
```

END

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