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SELECTED TOPICS IN THE RHEOLOGY OF FLUIDS

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FINAL REPORT

DANIEL D. JOSEPH

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AUGUST 25, 1991

U.S. ARMY RESEARCH OFFICE

DAALO3-88K-0083

UNIVERSITY OF MINNESOTA

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REPORT DOCUMENTATION PAGE			Form Approved OMB No 0704-0188	
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1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE Aug. 25, 1991		3. REPORT TYPE AND DATES COVERED
4. TITLE AND SUBTITLE Selected Topics in the Rheology of Fluids			5. FUNDING NUMBERS DAAL 03-88-K-0083	
6. AUTHOR(S) Daniel D. Joseph				
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9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) U. S. Army Research Office P. O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSORING/MONITORING AGENCY REPORT NUMBER ARO 25648.26-MA	
11. SUPPLEMENTARY NOTES The view, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy, or decision, unless so designated by other documentation.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT Approved for public release; distribution unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) Problems of flow of viscoelastic liquids were considered emphasizing the issues generated by the fluid's elasticity, which gives rise to hyperbolicity and waves. These studies are summarized in the book FLUID DYNAMICS OF VISCOELASTIC LIQUIDS published by Springer (1990). We did many studies of drag reductions using water lubrication, riblets and polymers. These studies will be summarized in my new book (with Yuriko Renardy): TWO-FLUID DYNAMICS. We started a computational numerical effort aimed at studying the motion of particles in fluids using the Navier-Stokes equations and the particle equations of motion.				
14. SUBJECT TERMS Viscoelastic liquid, Drag reduction, Hyperbolicity, Water lubricated pipelining, Two phase flow			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT UL	

1. Foreword

This is a final report for the grant to Daniel D. Joseph, University of Minnesota on "Selected Topics in the Rheology of Fluids," proposal no. 25648-MA, Funding Document DAA03-88.k-0083. This final report gives a summary of all of the works which were produced under this grant.

2. Research Accomplishments

I was elected to membership in the National Academy of Engineering in February 1990; awarded the G.I. Taylor Medal for distinguished contributions to fluid mechanics by the Society of Engineering Science at the joint meeting of that society and the Society for Rheology in Santa Fe in October, 1990; and was a plenary lecturer for the Society for Rheology at that same meeting. I have been awarded a chair, "Russell Penrose Professor of Aerospace Engineering and Mechanics", by the University of Minnesota. I was elected to the National Academy of Sciences in May 1991.

I worked with graduate students Kangping Chen, Howard Hu and other students not supported on any Army Grant. No post docs were involved. I did some work in May, 1990 with Prof. Salvatore Rionero of the University of Naples and Prof. Paolo Galdi of the University of Ferrara, whom I helped to support.

We do research on the rheology and fluid mechanics of viscous and viscoelastic liquids in four areas: (1) The development of the science for technological applications of fluid dynamics of lubricating one fluid by another. In particular, we have written 9 papers on water lubricated pipelining. Huge reductions in drag can be achieved in transporting crude oil by lubricating the flow with water. We have developed methods to get the results of linear and weakly nonlinear stability for the stability of core-annular flow which we compare with our own experiments. A book, *Two-Fluid Dynamics* by D.D. Joseph and Y. Renardy, is in preparation. (2) We are interested in developing the mathematics to describe the dynamics of viscoelastic liquids. We developed this subject strongly, emphasizing hyperbolic properties of the equations. This work has been collected into a book, *Fluid Dynamics of Viscoelastic Liquids* by D.D. Joseph, published in March 1990 by Springer. (3) We have an active interest in developing theory and instruments for the measurement of material properties of viscoelastic liquids. Recently, we showed how one could determine the extensional viscosity of a viscoelastic liquid by measuring the height to which it climbs on a rotating rod. We are developing a spinning rod tensiometer for molten polymers ($<300^{\circ}\text{C}$). This work is partially supported by Hoechst-Celanese. Our instrument will also be good as a dynamic rheometer, but we need to develop the mathematical theory. (4) We are developing numerical codes to study two-dimensional motion of spherical particles in a fluidized suspension. We move the particles using the particles equations under forces computed from exact numerical solutions of the Navier-Stokes equations. This is like a molecular dynamic simulation except that it is exact; nothing is modeled.

The work on viscoelastic liquids will have an impact on the study of systems of PDE's which are of composite type, partly hyperbolic, partly elliptic. The lubrication studies have big areas of application for energy. Oil companies worldwide are interested. We expect that the rheological instruments we develop will have a large impact on the practice of rheometry; the wave speed meter, the rod climber, and the spinning rod tensiometer. The numerical codes we use to compute the motion of particles in fluids from exact simulations of the Navier-Stokes equations will almost surely enter into the CFD literature as a new and useful implementable procedure.



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I have an ongoing collaboration with Dr. Miles Miller, the director of the fluid mechanics laboratory of the CRDEC in Aberdeen, MD. One of my suggestions helped Dr. Miller to eliminate an instability which had been troubling the Army for years. Our help is acknowledged in Dr. Miller's paper "Elimination of viscous liquid-fill flight instability by means of lower viscosity, immiscible liquid additive." Dr. Miller tells me that the suppression of the viscous liquid fill instability by the addition of small amounts of water has been recently verified in actual flight tests. Dr. Miller and I organized a conference at the Army High Performance Computing Research Center (AHPCRC) at the University of Minnesota on this subject in April, 1991.

3. List of publications

1. "Lubricated pipelining, Part I: The stability of core-annular flow" (with L. Preziosi and K. Chen), *Journal of Fluid Mechanics* **201**, 323-356 (1989).
2. "Lubricated pipelining, Part II" (with H. Hu), *Journal of Fluid Mechanics* **205**, 359-396 (1989).
3. "Anomalous elongational flows and change of type" (with K. Chen), *Journal of Non-Newtonian Fluid Mechanics* **28**, 47-60 (1988).
4. "Stability of periodic arrays of cylinders across the stream by direct simulation of steady flow" (with P. Singh, Ph. Caussignac, A. Fortes, and T.S. Lundgren), *Journal of Fluid Mechanics* **205**, 553-571 (1989).
5. "Change of type and loss of evolution of the White-Metzner model" (with C. Verdier), *Journal of Non-Newtonian Fluid Mechanics* **31**, 325-343 (1989).
6. "Similarity solutions that give rise to hyperbolicity and change of type in steady flows of viscoelastic fluids" (with C. Verdier), *Journal of Non-Newtonian Fluid Mechanics* **31**, 301-323 (1989).
7. "Autoregressive methods for chaos on binary sequences for the Lorenz attractor" (with P. Singh), *Physics Letters A* **135**, 247-253 (1989).
8. "Heat waves" (with L. Preziosi), *Reviews of Modern Physics* **61**, 41-73 (1989).
9. "Application of the singular value decomposition to the numerical decomposition of the numerical computation of the coefficients of amplitude equations and normal forms" (with K. Chen), *Applied Numerical Analysis* **6**, 425-430 (1989/90).
10. "Generalization of the Foscolo-Gibilardo analysis of dynamic waves." *Chemical Engineering Science* **45**(2), 411-414 (1990).
11. "Ensemble averaged and mixture theory equations for incompressible fluid-particle suspensions" (with T.S. Lundgren, and an appendix by R. Jackson and D.A. Saville), *International Journal of Multiphase Flow* **16**(1) 35-42 (1990).
12. "Remarks on inertial radii, persistent normal stresses, secondary motions, and non-elastic extensional viscosities," *Journal of Non-Newtonian Fluid Mechanics* **32**(1), 107-114 (1989).
13. "Stability of core-annular flow in a rotating pipe" (with H. Hu), *Physics of Fluids* **A1**(10), 1677-1685 (1989).
14. "Lubricated pipelining III: Stability of core annular flow in vertical pipe" (with K. Chen and R. Bai), *Journal of Fluid Mechanics* **214**, 251-286 (1990).
15. "Drag reduction in pipes lined with riblets" (with K.N. Liu, C. Christodolou and O. Riccius), *AIAA Journal* **28**(10), 1697-98 (1990).

16. "Climbing constants, quadratic correction of Trouton's viscosity, wave speed and delayed die swell for M1" (with H. Hu, O. Riccius, K. Chen, M. Arney), *Journal of Non-Newtonian Fluid Mechanics* **35**, 287-307 (1990).
17. "Two-dimensional cusped interfaces" (with John Nelson, M. Renardy, and Y. Renardy) *Journal of Fluid Mechanics* **223**, 383-409 (1991).
18. "Lubricated pipelining: Stability of core-annular flow. Part IV: Ginzburg-Landau equations" (with K. Chen), *Journal of Fluid Mechanics* **227**, 587-615 (1991).
19. "Stability of core-annular flow with a small viscosity ratio" (with H. Hu and T. S. Lundgren), *Physics of Fluids* **2**(11) 1945-1954 (1990).
20. "Fluid dynamics of two miscible liquids with diffusion and gradient stresses" *European Journal of Mechanics B/Fluids* **9**(6), 565-596 (1990).
21. "Vortex rings of one fluid in another in free fall" (with N. Baumann, P. Mohr, and Y. Renardy). *Physics of Fluids* (to appear 1991).
22. *Fluid Dynamics of Viscoelastic Liquids*. Springer Applied Math Series (number 84), 1990.
23. "Remarks on long wave and lubrication theories for core-annular flow" (with K. Chen), to appear in *Physics of Fluids* (1991).
24. "Kelvin-Helmholtz mechanism for side branching in the displacement of light with heavy fluid under gravity" (with T.Y.J. Liao and J.-C. Saut), submitted to *European Journal of Mechanics B/Fluids*.
25. "Numerical simulation of viscoelastic flow past a cylinder" (with H. Hu), *Journal of Non-Newtonian Fluid Mechanics* **37**, 347-377 (1990).
26. "Interfacial tension between miscible liquids" (with H. Hu), AHPARC preprint 91-58.
27. "Non-solenoidal velocity effects and Korteweg Stresses in simple mixtures of incompressible liquids" (with H. Hu), AHPARC preprint 91-03 (1991).

4. Abstracts of Results

1. "Lubricated pipelining, Part I: The stability of core-annular flow" (with L. Preziosi and K. Chen), *Journal of Fluid Mechanics* **201**, 323-356 (1989).

The stability of core-annular flow (CAF) in pipes is analyzed using the linear theory of stability. Attention is confined to the potentially stable case of lubricated pipelining with the less viscous liquid, say water, in the annulus. The effects of surface tension and density are included, but gravity is excluded. We find upper and lower branches of the neutral curve in a Reynolds number (R) vs. wavenumber (α) plane. A window of parameters is identified in which CAF is stable to small disturbances. When R is below the lower critical value, CAF is destabilized by surface tension and long waves break up into slugs and bubbles. The sizes of slugs and bubbles of oil in water observed by Charles, Govier and Hodgson [1961] are given by the wavelength of the fastest growing long wave. This long-wave instability is a capillary instability, modified by shear, which reduces to Rayleigh's instability in the appropriate limit. At higher R , the capillary instability is stabilized by shear. At yet higher R , above the upper critical value, the flow is unstable to generally shorter waves which leads to emulsification, water droplets in oil. The theory agrees with experiments. The analysis seems to be applicable to the design of lubricated pipelines; for example, there is an optimum viscosity ratio for stability, greater

stability can be obtained by using heavy liquid as a lubricant when the flow is unstable to capillary modes on the lower branch and by using light liquids when the flow is unstable to emulsifying disturbances on the upper branch.

2. "Lubricated pipelining, Part II" (with H. Hu), *Journal of Fluid Mechanics* **205**, 359-396 (1989).

In this paper, we study the linearized stability of three symmetric arrangements of two liquids in core-annular Poiseuille flow in round pipes. Deferring to one important application, we say oil and water when we mean more viscous and less viscous liquids. The three arrangements are (i) oil is in the core and water on the wall, (ii) water is in the core and oil is outside and (iii) three layers, oil inside and outside with water in between. The arrangement in (iii) is our model for lubricated pipelining when the pipe walls are hydrophobic and it has not been studied before. The arrangement in (ii) was studied by Hickox [1971] who treated the problem as a perturbation of long waves, effectively suppressing surface tension and other essential effects which are necessary to explain the flows observed, say, in recent experiments of W. L. Olbricht and R. W. Aul. The arrangement in (i) was studied in Part 1 of this paper (Preziosi, Chen and Joseph [1987]). We have confirmed and extended their pseudo-spectral calculation by introducing a more efficient finite-element code. We have calculated neutral curves, growth rates, maximum growth rate, wavenumbers for maximum growth and the various terms which enter into the analysis of the equation for the evolution of the energy of a small disturbance. The energy analysis allows us to identify the three competing mechanisms underway: interfacial tension, interfacial friction and Reynolds stress. Many results are presented.

3. "Anomalous elongational flows and change of type" (with K. Chen), *Journal of Non-Newtonian Fluid Mechanics* **28**, 47-60 (1988).

Anomalous effects on elongational flows at high rates of elongation reported by Ferguson et al. [1] are here treated as a change of type. Analysis predicts that the vorticity near the drum is hyperbolic, elliptic away from the drum, under the supercritical conditions in the experiment when the shear-wave speeds have the values which we measured with our wave-speed meter.

4. "Stability of periodic arrays of cylinders across the stream by direct simulation of steady flow" (with P. Singh, Ph. Caussignac, A. Fortes, and T.S. Lundgren), *Journal of Fluid Mechanics* **205**, 553-571 (1989).

We treat the problem of the stability of an infinite horizontal array of cylinders, spaced periodically, by a direct numerical simulation of the Navier-Stokes equations for steady flow at Reynolds numbers less than or equal to 100. We find that the only stable configuration for the array is one with equal spacing between cylinders and all cylinders lying on a line perpendicular to the flow. The array is found to be stable under displacements of the cylinders perpendicular and parallel to the array. We say a perturbation is stable when it gives rise to a force which acts to restore the original stable configuration. Our results are consistent with experiments in which spheres were confined by the sidewalls of a fluidized bed to move in two dimensions. As a secondary issue we consider the variation with parameters of the length and width of wakes behind cylinders.

5. "Change of type and loss of evolution of the White-Metzner model" (with C. Verdier), *Journal of Non-Newtonian Fluid Mechanics* **31**, 325-343 (1989).

In this paper a mathematical study of the White-Metzner model is presented. This model gives rise to systems of first order nonlinear (not quasilinear) partial differential

equations. The unsteady case is studied first to determine if the Cauchy problem is well-posed, stable to short waves. Then we study two-dimensional steady flow by classifying the stream function equation for type. The roots of a polynomial have to be analyzed. We show how to use the roots to factorize an operator, we compute the roots and carry out the factorization globally, using a symbol manipulator, and locally, using perturbation analysis. The additional nonlinearities are found to add more hyperbolicity.

6. "Similarity solutions that give rise to hyperbolicity and change of type in steady flows of viscoelastic fluids" (with C. Verdier), *Journal of Non-Newtonian Fluid Mechanics* **31**, 301-323 (1989).

Similarity solutions have proved to be a very useful tool for the study of flows of viscoelastic fluids since they allow us to check numerical computations against them. We compute here hyperbolic regions of the vorticity for an upper convected Maxwell model using the similarity solution of Phan-Thien for flow between rotating disks and using the similarity solution of Menon for an accelerated flow in cylindrical coordinates. We show that the extra tension becomes enormous at the edge of disks of ordinary rheometers under operating conditions.

7. "Autoregressive methods for chaos on binary sequences for the Lorenz attractor" (with P. Singh), *Physics Letters A* **135**, 247-253 (1989).

A binary sequence is defined for the Lorenz attractor. This binary sequence contains some information about the original system. To extract this information we have used autoregressive methods from the theory of signal processing. The binary sequences and the associated methods could also be used to estimate the system characteristics when one does not have access to all the variables involved in the underlying process; this is usually the case in an experimental study. We introduce an autocorrelation function for binary sequences, a one-step predictor and associated power spectra and a macroscopic approximation of the largest Lyapunov exponent.

8. "Heat waves" (with L. Preziosi), *Reviews of Modern Physics* **61**, 41-73 (1989).

The concept of transmission of heat by waves is reviewed and interpreted. The notion of an effective thermal conductivity, an effective heat capacity, and relaxation functions for heat and energy is introduced along lines used recently to describe the elastic response of viscous liquids. An annotated bibliography of the literature on heat waves, from the beginning until now, gives a complete or nearly complete survey of the history of this subject.

9. "Application of the singular value decomposition to the numerical decomposition of the numerical computation of the coefficients of amplitude equations and normal forms" (with K. Chen), *Applied Numerical Analysis* **6**, 425-430 (1989/90).

The Fredholm alternative is a standard procedure by which one generates the coefficients of amplitude equations and normal forms. The alternative requires that the inhomogeneous terms in the underlying system of differential equations, which contain the unknown coefficients, be orthogonal to the independent eigenvectors spanning the null space of the adjoint system of differential equations. The numerical computation of the adjoint eigenvectors and the application to solvability is frequently difficult and inefficient. Typically the underlying system of the inhomogeneous differential equation is discretized and solved as an inhomogeneous matrix-valued eigenvalue problem. We find that the solvability conditions which lead to values of the unknown coefficients are conveniently and economically computed by application of the singular value decomposition directly to the matrix formulation, avoiding completely the computation of an adjoint system of differential equations.

10. "Generalization of the Foscolo-Gibilaro analysis of dynamic waves." *Chemical Engineering Science* 45(2), 411-414 (1990).

A new expression for the particle phase pressure in a fluidized bed generalizing the one used by Foscolo and Gibilaro is derived. In the new theory, uniform fluidization is always unstable.

11. "Ensemble averaged and mixture theory equations for incompressible fluid-particle suspensions" (with T.S. Lundgren, and an appendix by R. Jackson and D.A. Saville), *International Journal of Multiphase Flow* 16(1) 35-42 (1990).

Two-fluid equations for flowing composites of solid particles in a liquid have been given by mixture theory and by ensemble averaging. The mixture theory equations are postulated and the ensemble averaged equations are derived. The ensemble averaged equations give rise to an explicit form for interaction terms. The equations obtained by the two methods are different. The expression for the stress in a Newtonian fluid with rigid particles is the point at issue. Jackson and Saville, in an appendix to this paper, introduce a method of soft spatial averaging which yields the same result as in the case of ensemble averages.

12. "Remarks on inertial radii, persistent normal stresses, secondary motions, and non-elastic extensional viscosities," *Journal of Non-Newtonian Fluid Mechanics* 32(1), 107-114 (1989).

In this note I discuss some consequences of the balance of inertia and normal stresses in nearly steady slow motions. I argue that the fluid's elasticity cannot be determined from its extensional viscosity. A formula is given for the extensional viscosity at quadratic order in the stretch rate and show that one and the same intensity factor for the extensional viscosity arises universally as the intensity factor for the non-Newtonian contribution to the vorticity. It is argued that the appearance of intense vortices and relatively large extensional viscosities are linked in nearly steady slow flow.

13. "Stability of core-annular flow in a rotating pipe" (with H. Hu), *Physics of Fluids* A1(10), 1677-1685 (1989).

The linear stability of core-annular flow in rotating pipes is analyzed. Attention is focused on the effects of rotating the pipe and the difference in density of the two fluids. Both axisymmetric and nonaxisymmetric disturbances are considered. Major effects of the viscosity ratio, interfacial tension, radius ratio, and Reynolds number are included. It is found that for two fluids of equal density the rotation of the pipe stabilizes the axisymmetric ($n=0$) modes of disturbances and destabilizes the nonaxisymmetric modes. Except for small R , where the axisymmetric capillary instability is dominant, the first azimuthal mode of disturbance $|n|=1$ is the most unstable. When the heavier fluid is outside centripetal acceleration of the fluid in the rotating pipe is stabilizing; there exists a critical rotating speed above which the flow is stabilized against capillary instability for certain range of small R . When the lighter fluid is outside the flow is always unstable.

14. "Lubricated pipelining III: Stability of core annular flow in vertical pipe" (with K. Chen and R. Bai). *Journal of Fluid Mechanics* 214, 251-286 (1990).

The stability of core-annular flow (CAF) in pipes is analyzed using the linear theory of stability. Attention is confined to the potentially stable case of lubricated pipelining with the less viscous liquid, say water, in the annulus. The effects of surface tension and density are included, but gravity is excluded. We find upper and lower branches of the

neutral curve in a Reynolds number (R) vs. wavenumber (α) plane. A window of parameters is identified in which CAF is stable to small disturbances. When R is below the lower critical value, CAF is destabilized by surface tension and long waves break up into slugs and bubbles. The sizes of slugs and bubbles of oil in water observed by Charles, Govier and Hodgson [1961] are given by the wavelength of the fastest growing long wave. This long-wave instability is a capillary instability, modified by shear, which reduces to Rayleigh's instability in the appropriate limit. At higher R , the capillary instability is stabilized by shear. At yet higher R , above the upper critical value, the flow is unstable to generally shorter waves which leads to emulsification, water droplets in oil. The theory agrees with experiments. The analysis seems to be applicable to the design of lubricated pipelines; for example, there is an optimum viscosity ratio for stability, greater stability can be obtained by using heavy liquid as a lubricant when the flow is unstable to capillary modes on the lower branch and by using light liquids when the flow is unstable to emulsifying disturbances on the upper branch.

15. "Drag reduction in pipes lined with riblets" (with K.N. Liu, C. Christodolou and O. Riccius). *AIAA Journal* 28(10), 1697-98 (1990).

In the present paper, experiments are reported establishing a maximum drag reduction of 5-7% in fully developed turbulent flow of water through 25.4- and 50.8-mm-diameter pipes lined with a film of grooved equilateral triangles of base 0.11mm. The maximum reduction occurs when the height of the riblets is 11 to 16 wall units. This correlates well with the Taylor microscale of the fluctuating velocity gradient.

16. "Climbing constant, second-order correction of Trouton's viscosity, wave speed and delayed die swell for M1" (with H. Hu, O. Riccius, K. Chen, M. Arney), *Journal of Non-Newtonian Fluid Mechanics* 35, 287-307 (1990).

Measurements of the wave speed c in M1 imply a fast time $\lambda = \mu/\rho c^2$ of relaxation. This and the delayed die-swell measurements suggest that M1 is not very elastic. Extensive and very reliable values of the climbing constants show that M1 has weak normal stresses at the level of STP. The climbing constants plus back-extrapolated values of $N_1/\dot{\gamma}^2$ taken from measurements at two temperatures by Binding, and Walters and Prud'homme lead to values of the two coefficients in the second-order regime of slow flow of M1. This gives us all the constants in the Roscoe formula for the quadratic correction of Trouton's viscosity and also allows us to compute the normal stress ratio $-N_2/N_1 = 0.11$, independent of temperature.

17. "Two-dimensional cusped interfaces" (with John Nelson, M. Renardy, and Y. Renardy) *Journal of Fluid Mechanics* 223, 383-409 (1991).

Two-dimensional cusped interfaces are line singularities of curvature. We create such cusps by rotating a cylinder half immersed in liquid. A liquid film is dragged out of the reservoir on one side and is plunged in at the other, where it forms a cusp at finite speeds, if the conditions are right. Both Newtonian and non-Newtonian fluids form cusps, but the transition from a rounded interface to a cusp is gradual in Newtonian liquids and sudden in non-Newtonian liquids. We present an asymptotic analysis near the cusp tip for the case of zero surface tension, and we make some remarks about the effects of a small surface tension. We also present the results of numerical simulations showing the development of a cusp. In those simulations, the fluid is filling an initially rectangular domain with a free surface on top. The fluid enters from both sides and is sucked out through a hole in the bottom.

18. "Lubricated pipelining: Stability of core-annular flow. Part IV: Ginzburg-Landau equations" (with K. Chen), *Journal of Fluid Mechanics* 227, 587-615 (1991).

Nonlinear stability of core-annular flow near points of the neutral curves at which perfect core-annular flow loses stability is studied using Ginzburg-Landau equations. Most of the core-annular flows are always unstable. Therefore the set of core-annular flows having critical Reynolds numbers is small, so that the set of flows for which our analysis applies is small. An efficient and accurate algorithm for computing all the coefficients of the Ginzburg-Landau equation is implemented. The nonlinear flows seen in the experiments do not appear to be modulations of monochromatic waves, and we see no evidence for soliton-like structures. We explore the bifurcation structure of finite-amplitude monochromatic waves at criticality. The bifurcation theory is consistent with observations in some of the flow cases to which it applies and is not inconsistent in the other cases.

19. "Stability of core-annular flow with a small viscosity ratio" (with H. Hu and T. S. Lundgren), *Physics of Fluids* 2(11) 1945-1954 (1990).

It is known that the stability problem for core-annular flow of very viscous crude oil and water is singular, the water annulus appears to be inviscid with boundary layers at the pipe wall and at the interface. In the present paper, this singular problem is treated by the method of matched asymptotic expansions using $\epsilon = m/R\alpha$ as a small parameter. There are two cases of instability corresponding to different positions of the critical point in the annulus. One case is when the critical point is far away from the interface, the other is when the critical point is close to the interface within a distance of order $\epsilon^{1/3}$. In both cases, the equations for the eigenvalues are derived, and the explicit forms for the neutral curves are given. The stability problem is also treated by the modified finite element code used by Hu and Joseph [*J. Fluid Mech.* 205, 359 (1989); *Phys. Fluids A* 1, 1659 (1989)], taking into account the boundary layers at the pipe wall and at the interface. The results of the two methods agree where they overlap, but the finite element technique goes further.

20. "Fluid dynamics of two miscible liquids with diffusion and gradient stresses" *European Journal of Mechanics B/Fluids* 9(6), 565-596 (1990).

The density of incompressible fluids can vary with concentration ϕ and temperature, but not with pressure. The velocity field \mathbf{u} , of such incompressible fluids is not in general solenoidal, $\text{div } \mathbf{u} \neq 0$. A conservation form for the left hand side of the diffusion equation which differs from the usual substantial derivative of ϕ by the addition of $\phi \text{ div } \mathbf{u}$ is implied by requiring that the mass per unit, total volume of one liquid in a material volume is conserved in the absence of diffusion. The possibility that stresses are induced by gradients of concentration and density in slow diffusion of incompressible miscible liquids, as in the theory of Korteweg [1901], is considered. Such stresses could be important in regions of high gradients, giving rise to effects which can mimic surface tension. The small but interesting history of thought about interfacial tension between miscible liquids is collected here. The presence of a sharp interface in the case of slow diffusion in rising bubbles and falling drops has been documented in many experiments and in the experiments reported here. The shape of such interfaces can scarcely be distinguished from the shapes of bubbles and drops of immiscible liquids with surface tension. The usual description of interface problems for miscible liquids with classical interface conditions but with zero interfacial tensions misses out on slow diffusion on the one hand and gradient stresses on the other. The usual description of diffusion with $\text{div } \mathbf{u} = 0$ is also inexact, though it is a good approximation in some cases.

21. "Vortex rings of one fluid in another in free fall" (with N. Baumann, P. Mohr, and Y. Renardy). *Physics of Fluids* (to appear 1991).

We present and interpret experiments in which vortex rings of one immiscible liquid are created in another from drops falling from rest under gravity. These rings are associated with circulations generated by viscosity and, unlike classical vortex rings which occur in miscible liquids at high Reynolds numbers. Since the rings do not diffuse, they are well-defined. Nonetheless, there are many similarities in the dynamics of formation and flow of miscible and immiscible rings. We identify parameters which appear to correlate our observations and show photographs of some of the more interesting things we have seen.

22. *Fluid Dynamics of Viscoelastic Liquids*. Springer Applied Math Series (number 84), 1990.
23. "Remarks on long wave and lubrication theories for core-annular flow" (with K. Chen), to appear in *Physics of Fluids* (1991).

We compare the linearization of nonlinear amplitude equations based on lubrication theory with the long wave expansion of the full problem. We find that the two approaches are basically different; the lubrication-based theories always require additional restrictions on parameters. In the problem associated with core-annular flows of oil lubricated by water, the lubrication theory misses out on shear stabilization of capillary instability. A hybrid theory in which lubrication theory is used in the water film but not in the core captures the stabilizing effects of shear when the film is very small. We show that the results of lubrication theory are not robust and the stabilizing effects of the inertia of the core and annulus may reverse for larger, but still small, thicknesses.

24. "Kelvin-Helmholtz mechanism for side branching in the displacement of light with heavy fluid under gravity" (with T.Y.J. Liao and J.-C. Saut), submitted to *European Journal of Mechanics B/Fluids*.

The problem of stability of smooth fingering motions which may develop from the Rayleigh-Taylor instability when the initial data is analytic is considered. A second-order ordinary linear differential equation with time-dependent coefficients is derived for the evolution of a small wavy perturbation of the interface in a local approximation when the waves are short. It is possible to have a stable, but Hadamard unstable, perturbation if the time-dependent coefficients satisfy certain growth conditions. The strongest Hadamard instabilities occur as Kelvin-Helmholtz instabilities associated with a velocity difference at the sides of falling fingers.

25. "Numerical simulation of viscoelastic flow past a cylinder" (with H. Hu), *Journal of Non-Newtonian Fluid Mechanics* 37, 347-377 (1990).

The flow of an upper-convected Maxwell fluid past a circular cylinder is simulated numerically using the algorithm SIMPLER, which is based on a finite volume discretization on a staggered grid of the governing equations and an iterative solution to the nonlinearly coupled equations. The effect of the viscoelasticity of the fluid on the flow is examined. The drag force on the cylinder and the heat transfer from the cylinder to the surrounding fluid are calculated and compared with those obtained in experiments. One feature seen in the experiments is the existence of a critical velocity across which the variation of the drag and heat transfer changes from a characteristic Newtonian variation to a flat response. This feature is interpreted as a change of type when the critical speed U_c of flow becomes greater than the material wave speed $c = \sqrt{\eta/\lambda\rho}$. The numerical computation is completely consistent with this interpretation. Using an Einstein-type

formula for the relaxation time $\lambda = A\phi$, with A independent of ϕ , we find that the experiments follow a scaling law $U_c\phi^{1/2} = \text{constant}$, where ϕ is the mass fraction of polymers in water in the regime of extreme dilution, the drag reduction range, consistent with the interpretation that $U_c = c$.

26. "Interfacial tension between miscible liquids" (with H. Hu), AHPCRC preprint 91-58.

We study some basic problems of fluid dynamics of two incompressible miscible liquids modeled as a simple mixture in which the volume of the mixture does not change on mixing. In general, the expansion $\Delta = \text{div } u$ in these problems does not vanish. The velocity in such a mixture can be decomposed into a solenoidal and an expansion part. The expansion velocity is induced by diffusion which is proportional to the gradient of the volume fraction in a simple mixture. The expansion can be large at certain times and places. We have carried out an analysis of transient or dynamic interfacial tension for the problem of smoothing of an initial discontinuity of composition across a plane and spherical surface. We find no jump in the mean normal stress across a plane layer but there is a jump proportional to the curvature across the spherical surface. The dynamic tension at the spherical interface decays as $t^{-1/2}$, it has two terms, one term arises from the Korteweg stress and it gives rise to a stress opposing the internal pressure as in the case of equilibrium tension if the Korteweg coefficient has the appropriate sign. The other term arises from the expansion velocity and is proportional to the rate of change of viscosity with volume fraction. This term has the wrong sign for interfacial tension in the case of glycerin and water solutions but has the right sign when the light fluid is more viscous.

27. "Non-solenoidal velocity effects and Korteweg Stresses in simple mixtures of incompressible liquids" (with H. Hu), AHPCRC preprint 91-03 (1991).

abstract same as above

5. Participating Personnel

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