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FINAL REPORT

by

William A. Gardner

November 30, 1991

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A. STATEMENT OF THE PROBLEM STUDIED

We have developed, analyzed, and simulated signal processing algorithms to estimate with high resolution the directions of arrival (DOAs) of temporally and spectrally overlapping signals impinging on an antenna array and to estimate the signal waveforms themselves by spatial filtering, with emphasis on algorithms that, by exploiting cyclostationarity, can operate properly when (1) little or no prior knowledge of the signal environment is available, (2) known or measurable keying rates or carrier frequencies can be used to classify signals as being signals of interest (SOIs) or signals not of interest (SNOIs), and the SOIs constitute only a subset of the set of all received signals, and (3) physical or economic constraints force the number of deployed antennas in the array to be small, perhaps smaller than the total number of signals arriving at the array. These three conditions can arise in surveillance, intelligence, and reconnaissance applications or when the array size is limited as it is for hand-held or truck-mounted arrays; however, they are not typically addressed by conventional direction-finding and signal extraction methods.

We have also studied the related problem of estimating time-difference-of-arrival (TDOA) of radio waves impinging on a pair of antennas, typically on separate platforms, for the purpose of passively locating the source of a communications or telemetry signal in the presence of interfering signals and noise, and we have developed, analyzed, and simulated a new class of signal-selective algorithms that are highly tolerant to interference and noise. These new algorithms exhibit their signal selectivity regardless of the extent of temporal and spectral overlap among received signals, and regardless of proximity between interfering emitters and emitters of interest. It is only required that the signal of interest have a known or measurable carrier frequency or keying rate that is distinct from those of all interfering signals. Yet the computational complexity of these algorithms is comparable to that of conventional generalized crosscorrelation algorithms.

B. SUMMARY OF RESEARCH FINDINGS

1. Overview of Results on Direction Finding and Signal Extraction

a. Introduction

Broadly, one of the questions we have attempted to answer in this research project is "How can data from multiple receiving antennas in a sensor array be used to describe the spatial and temporal characteristics of the signals in a radio-signal environment?" In particular, the signals of interest are man-made communication signals which typically exhibit a statistical property called *cyclostationarity* or, equivalently, *spectral correlation* [5, 3, 6]. Unlike stationary signals, cyclostationary signals are correlated with frequency-shifted as well as time-shifted versions of themselves, a property that enables some signal processing algorithms to estimate parameters of such signals even when they are severely masked by interference and noise.

To this end, we have developed and analyzed signal processing algorithms to estimate the directions of arrival (DOAs) of the signals and to estimate the signal waveforms themselves by spatial filtering, with emphasis on algorithms that, by exploiting cyclostationarity, can operate properly when (1) little or no prior knowledge of the signal environment is available, (2) sufficient prior knowledge is available to classify signals as being signals of interest (SNOIs), and the SOIs constitute only a subset of the set of all received signals, and (3) physical or economic constraints force the number of deployed antennas in the array to be small, perhaps smaller than the number of signals arriving at the array. It is explained in this report that, in some situations, the sufficient prior knowledge required in condition (2) can be very modest (e.g., a keying rate or carrier frequency), and can even be replaced with estimates obtained from the data. Even though these three conditions can arise in surveillance, intelligence,

and reconnaissance applications or when the array size is limited as it is for hand-held or truck-mounted arrays, they are not addressed by conventional direction-finding (DF) methods or signal extraction (i.e., spatio-temporal filtering) methods. Together, the estimated DOAs, the various intermediate parameters found in the process of estimating the DOAs, and the extracted signal waveforms can form a reasonably complete description of the spatial locations and temporal characteristics of the signals in the received environment.

b. **Research Findings**

Within the broad scope of DF and signal extraction (or simply copy), our primary effort has been directed toward estimating the directions of arrival of cyclostationary SOIs having sufficiently high carrier frequencies and narrow bandwidths that the narrowband approximation is valid (i.e., that the response of the sensor array over the frequency band of interest is approximately constant).

In the doctoral dissertation [8] and the survey article [13], we unify a wide range of algorithms for narrowband DF by interpreting them in terms of spatial filtering instead of the less physically motivated abstraction of subspace fitting. Therein we also provide an overview of the DF methods developed in this research project and explain how they are able to perform properly in many applications in which conventional methods perform poorly or fail completely. These explanations are simplified by postponing discussion of the related problems of estimating the number of SOIs present and, for the cyclostationarity-exploiting methods, estimating one or more cycle frequencies of the SOIs. However, in practice these quantities must be estimated prior to processing the data with one of the DF algorithms; subsequent discussion considers the problems of estimating these quantities and of accommodating other departures from ideality, such as the need to adaptively adjust array calibration data to correct

for effects of perturbation in the sensor position and age-induced and temperature-induced drift in component characteristics. This order of presentation parallels the order in which we addressed the corresponding problems in our effort and proceeds from the more easily understood to the more difficult.

i. DF Methods for Cyclostationary Signals

In this research project, we have developed and analyzed the Cyclic MUSIC methods and the Cyclic Least Squares (CLS) methods for estimating the DOAs of cyclostationary signals. We derive the methods, explain their capabilities, and analyze their performance (both analytically and using computer simulations) in [12, 7, 13, 8, 18, 14, 10, 2, 4]. Briefly, the forms of the Cyclic MUSIC methods resemble the form of the conventional MUSIC methods, whereas the forms of the CLS methods resemble the form of the conventional maximum likelihood method for unknown signals in white Gaussian noise. Both types of methods estimate the DOAs of the signals having a specified cycle frequency: after estimating the corresponding cyclic autocorrelation matrix from the received data, each method finds the DOAs that “best” describe the spatial characteristics represented by the matrix, where the quality of the solution is determined differently by each method.

We show in the cited papers that, unlike conventional methods, these methods can operate properly when (1) the number of all signals is greater than the number of sensors, provided that the number of SOIs having the cycle frequency of interest is less than the number of sensors, (2) SNOIs are arbitrarily closely spaced to SOIs, and (3) the spatial characteristics of the noise are arbitrary and unknown. Furthermore, by estimating the DOAs of only the SOIs having a particular cycle frequency, these DF methods reduce or eliminate the need for

post-processing steps that would otherwise be needed to classify the DOAs according to signal type.

Although the Cyclic MUSIC and CLS methods are treated as if the cycle frequencies and numbers of signals having each of these cycle frequencies were known, the methods described in the next two sections can estimate these parameters and thus allow the DF methods to be applied if these parameters are unknown. In this sense, the benefits of signal-selective DF that accrue from exploiting the cyclostationarity of the received signals can be enjoyed even when essentially nothing is known about the environment beforehand.

ii. Cycle Frequency Estimation

Cyclostationary signals are correlated with frequency-shifted versions of themselves for certain values of the frequency-shift parameter. Equivalently, the lag-product waveforms of these signals contain finite-strength additive sine waves having frequencies equal to the values of the frequency-shift parameter, which are called *cycle frequencies*. Thus, one means of estimating the cycle frequencies is to estimate the frequencies of the sine waves present in lag-product waveforms computed from finite records of received data. We demonstrate this simple technique in [8, 17, 7], in which the resulting cycle frequency estimates are then used by Cyclic MUSIC (and by CLS in [7]) to estimate the corresponding DOAs. We show that these estimated DOAs have approximately the same mean-squared error (MSE) as those obtained using the exact values of the cycle frequencies.

iii. Estimating the Number of Signals Present

Conventional DF methods estimate the DOAs of all signals present and thus require an estimate of their number, which is typically obtained by analyzing the eigenvalues of the autocorrelation matrix of the data. In contrast, the cyclostationarity-exploiting DF methods operate only on a subset of the signals

and thus require an estimate of the number of signals in each subset. Unlike the statistics of the singular values of the autocorrelation matrix, those of the cyclic autocorrelation matrix are not well understood. Also, the results in the literature on canonical correlation analysis in general (also called common factor analysis) are meager in comparison with those on principal component analysis (also called factor analysis); the former are applicable to the signal-selective DF problem, whereas the latter are applicable to the conventional DF problem. Consequently, the Cyclic Correlation Significance Test (CCST) which we present in [8, 16] is only an initial attempt at solving this difficult problem by using the singular values of the cyclic autocorrelation matrix. Among its noteworthy features is the fact that it uses a penalty function (similar in form to that used by the conventional AIC, MDL, and EDC methods) instead of a subjectively chosen threshold. Alternatively, we show in [18, 8, 12, 7] that the CLS objective function, after maximization over the DOAs for each possible number of cyclostationary signals, can be used to estimate the number of cyclostationary signals, even in the presence of perfectly correlated multipath. However, this method requires a subjectively chosen threshold and has not been extensively tested.

iv. Performance Bounds

In addition to developing and analyzing specific DF algorithms, we have also investigated the Cramer-Rao Lower Bound (CRLB) on the covariance of unbiased parameter estimates of cyclostationary Gaussian signals. Apart from being of theoretical interest (almost all relevant work has dealt with stationary Gaussian signals), the CRLB for this problem can be used to gauge the efficiency of the cyclostationarity-exploiting methods and to indicate the potential for such methods to exhibit far better performance than that of methods for stationary signals. We show in [8, 11, 15] that the CRLB for two closely spaced

cyclostationary signals having different cycle frequencies can be several orders of magnitude less than that for two closely spaced stationary signals. Computer simulations therein show that the Cyclic MUSIC and CLS methods yield DOA estimates in some environments with MSE that is comparable to the CRLB for cyclostationary signals and much less than that for stationary signals, which indicates that the CRLB for cyclostationary signals is not overly optimistic in its prediction of potentially huge performance gains due to exploitation of cyclostationarity.

v . **Signal Extraction**

In addition to their primary function of estimating the DOAs of the SOIs, the CLS method and one of the Cyclic MUSIC methods can also compute the coefficients of spatial filters that attenuate noise and SNOIs and enhance SOIs. In [8, 12, 7, 18], we show that the signal-to-interference-and-noise ratio (SINR) of the signal waveforms estimated by CLS converge to the maximum attainable SINR if the signals are uncorrelated. However, the multidimensional search and array calibration data needed by CLS can be prohibitively expensive. In contrast, the Phase-SCORE Cyclic MUSIC method [8, 12] is based on the Phase-SCORE method for blind adaptive signal extraction [1, 9]. Phase-SCORE requires neither a multidimensional search nor any array calibration data, albeit at the expense of lower output SINR than that of CLS in some cases.

c . **Conclusions**

In this research project, we have developed and analyzed methods that exploit cyclostationarity, which is exhibited by most communications signals, to estimate the directions of arrival of signals arriving at a sensor array without requiring many of the prohibitive assumptions or potentially costly prior knowledge required by conventional direction-finding methods. The inherently

signal-selective methods reduce or eliminate the need to perform post-processing to classify the direction estimates as corresponding to desired or undesired signals. Furthermore, when signals are closely spaced and have different cycle frequencies, the methods we have developed can substantially outperform conventional methods and the Cramer-Rao Lower Bound for stationary signals, even if the cycle frequencies are unknown. In addition to performing DF, some of the signal-selective methods can also extract high-quality estimates of the signal waveforms from the received signal environment. Thus, the methods that we have developed and studied in this project can provide a reasonably complete description of the spatial locations and temporal characteristics of received signals in a much wider variety of received environments, especially those for which little or no prior knowledge is available, than can be accommodated by previously developed methods. Further work is needed on the analytical characterization of performance of the new algorithms and in the pursuit of even better-performing algorithms.

2. Overview of Results on Source Location by Time-Difference-of-Arrival Estimation

a. Introduction

The problem of passively locating the source of a propagating radio signal received at multiple platforms has a number of applications including direction finding for navigation by land, air, or sea, tracking of moving emitters for surveillance (e.g., for research, law enforcement, or national security), monitoring and locating illegal and/or hostile communicators (e.g., contraband, violators of communications regulations, or enemies in a battlefield), military reconnaissance and intelligence, and so on. Unlike active source-location systems, like Radar, Sonar, and Lidar, which transmit signals and then process the received reflections from the objects to be located, passive systems simply process whatever signals are emitted from the object to be located.

In many of these passive source-location applications, the receivers in the source-location system are subject to a variety of types of interference and noise, including natural and manmade signals other than the signal of interest (SOI). These signals not of interest (SNOI) can severely degrade the performance of conventional source-location systems when they are present at the same time and also occupy the same spectral band as the SOI. This can be particularly problematic when the SOI is weak relative to the SNOI and/or noise. The problem is further exacerbated when the locations of the sources of the SNOI are unknown and/or close to that of the SOI.

One of the objectives of this research project was to study a new class of multiple-platform passive source-location methods that exploit inherent signal properties, called *cyclostationarity*, that are characteristic of radio signals used for communications and telemetry to obtain substantial tolerance to all types of interference and noise (except possibly for some interfering signals that are

intentionally designed to be of the same type, e.g., in communication networks). Like most conventional methods that require some degree of tolerance to noise, the new methods are based on crosscorrelation of time-shifted measurements of data from multiple receivers. However, unlike conventional methods,¹ the new methods crosscorrelate frequency-shifted as well as time-shifted versions of the received data in order to exploit the unique cyclostationarity properties of the SOI.

Within the general class of source-location methods that are based on crosscorrelation measurements, there are two distinct subclasses: There are those methods that are designed for an array of closely spaced antenna elements on a single platform (e.g., a ship, aircraft, satellite, ground vehicle, or fixed ground station), for which the element spacing is typically less than half a wavelength for all signals received and phase-alignment methods for beam/null steering are employed; and there are those methods that are designed for two or three widely spaced antenna elements, each often (but not always) on a separate platform, where time-difference measurements are used to obtain location information. Whereas the former class of methods (often called *direction finding methods*) exploits phase differences (less than π radians in order to avoid ambiguities), from element to element, of relatively narrowband signals (or wideband signals decomposed into narrowband components) to estimate angle of arrival, the latter class of methods (often called *time-difference location methods*) exploits relative time differences (the larger the better since root-mean-squared-error of location is approximately inversely proportional to the separation between platforms) of preferably wideband signals from platform to platform to estimate location (both

¹ Although cross-ambiguity methods, which compensate for frequency-difference-of-arrival due to Doppler effects, do crosscorrelate frequency-shifted data, the frequency shifts are relatively small since they correspond to Doppler shifts, whereas the frequency shifts used in the new methods are much larger than typical Doppler shifts and accomplish an entirely different task.

angle of arrival and range). In practice, however, both methods can be applied to all bandwidths used in typical communications and telemetry systems.

The requirements on accuracy and spatial resolution capabilities of array-based methods become more stringent as the distance between each source to be located and the reception platform increases, since this decreases differences between angles of arrival at the array. In contrast, the requirements on accuracy and temporal resolution capabilities of time-difference-of-arrival (TDOA)-based methods become less stringent as the separation between reception platforms increases, since this increases differences between times of arrival.

The need for high resolution arises primarily when (relatively) closely spaced multiple sources give rise to multiple received signals that cannot be separated by preprocessing (prior to processing for location). For instance, TDOAs of multiple signals that are not separated by more than the widths of their crosscorrelation peaks (whose locations on the time-delay axis correspond to the TDOAs) usually cannot be resolved by conventional TDOA-based methods. To minimize this problem, the distance between platforms is typically made as large as is practically feasible so that the magnitudes of the TDOAs (which are proportional to the distance between platforms) will be as large as possible, thereby minimizing the overlap of adjacent peaks (whose widths depend only on the signal bandwidth—being inversely proportional—not on the distance between platforms).

The best-performing array-based methods attempt to circumvent this resolution problem by locating signal sources in terms of the locations of nulls that are intentionally formed in the spatial reception pattern of the antenna array by linearly combining the output signals from the array elements. But this conceptual distinction is blurred by the fact that most of these null-steering

algorithms also admit interpretations in terms of beam steering. Although the array-based methods offer the advantage of high spatial resolution (with respect to the spatial extent of the source-location system—the array size), and the ability to simultaneously locate a number of signals up to one less than the number of elements in the array, their complexity is typically much higher than that of the time-difference-of-arrival-based methods because of the need for measurement, storage, and usage of large amounts of array calibration data (the recently proposed ESPRIT method is an exception because of its special array design), and because of the use of computationally intensive algorithms: the best performing algorithms require singular value decomposition of crosscorrelation matrices of possibly high dimension, or require the solution of a multidimensional optimization problem.

The new cyclostationarity-exploiting TDOA-based methods studied in this project alter this tradeoff between highly sophisticated high-resolution array-based methods and the relatively simple TDOA-based methods that require widely separated multiple platforms by eliminating the resolution-limitation problem for TDOA. That is, because the spectral correlations used make the new methods signal selective (in spite of temporal and spectral overlap between the SOI and SNOI), they often do not need to resolve multiple crosscorrelation peaks along the time-delay axis. As a result of this novel signal selectivity, the requirements on separation of platforms can be eased considerably. Also, the problem of deciding which peak among multiple crosscorrelation peaks is associated with the SOI, and which with the SNOI, is eliminated. The only requirement of the new methods is that they must know a carrier frequency, or keying rate, or possibly some other *cycle frequency* associated with the cyclostationarity of the random SOI, although such cycle frequencies can be estimated using the same data to be

used for TDOA estimation. The cycle frequencies are the frequency-shifts used to obtain signal selectivity.

Unfortunately, when multiple SOIs sharing the same cycle frequencies arrive at each receiver, the TDOA-resolution problem remains. In this most difficult situation, although the high-resolution array based methods can be used, an attractive alternative that avoids the need for calibrating the array and can provide more accurate estimates is to use an array on each of two (or more) platforms to separate the multiple SOI by blind-adaptive spatial filtering using our new spectral-correlation-restoring (SCORE) algorithms (which require no calibration), and then apply TDOA-based methods to the separated signals.

Motivated by the potential utility of the cyclostationarity-exploiting TDOA-based approach to emitter location, we have developed various algorithms to perform signal-selective TDOA estimation and we have evaluated their performance. The results obtained are very encouraging.

b. Research Findings

In this research project, our primary effort has been directed toward applying the least-squares approach to the problem of exploiting second-order cyclostationarity for obtaining signal-selective methods of TDOA estimation. In the first part [1] of a two-part paper, we have derived a number of methods and corresponding algorithms by performing least-squares fitting on measurements of cyclic correlations or cyclic spectra (spectral correlations). In [2], we have shown that some of these algorithms are partial implementations of the weak-signal maximum-likelihood TDOA estimator for signals in white Gaussian noise. In our other work in this project, we have evaluated the RMSE performance of a number of new algorithms in various signal and noise/interference environments by simulation, generalized several of the algorithms for multiple platform-pairs,

compared and contrasted the performance characteristics of algorithms that use only cyclic autocorrelation measurements with those that also use cyclic crosscorrelation measurements, studied the problem of implementing these algorithms using FFTs, investigated the effects of some departures from the idealized models used to derive the algorithms, and initiated the study of higher-order cyclostationarity for TDOA estimation involving signals with weak second-order cyclostationarity.

i. The Least Squares Approach to Exploiting Cyclostationarity for TDOA Estimation

Using idealized models for the interference/noise-corrupted signals received at two platforms, we calculated the cyclic correlations and cyclic spectra to reveal their dependence on the TDOA. We then used these results to set up various ways of minimizing sums of squared errors between measurements of the cyclic correlations (or cyclic spectra). This least-squares approach led to the SPECCORR, SPECCOA, SPECCON, CLP and CPD algorithms. Preliminary derivations and performance evaluations are reported in [3] - [7] and our final results are presented in the comprehensive two-part paper [1], [8].

ii. The Maximum-Likelihood Approach to TDOA Estimation

By studying the maximum-likelihood joint signal-detector and TDOA-estimator for a weak signal in white Gaussian noise, we found that the SPECCOA algorithm is a partial but much simpler implementation of the relatively complicated weak-signal maximum-likelihood TDOA estimator. Similarly, the relatively simple single-receiver spectral-correlation-exploiting detector was shown in [8] to be a partial implementation of the single-receiver weak-signal maximum-likelihood detector. Generalizing on this, we have shown in [2] that the weak-signal maximum-likelihood joint detector and TDOA

estimator is made up of a multiplicity of components that can be partitioned into a variety of detectors and estimators, each of which is optimal in either a maximum-likelihood sense or a maximum-SNR sense. The results of extensive simulations that compare the performances of some of these alternative detectors and estimators were then obtained and it was concluded that the performance of the relatively complex maximum-likelihood joint detector/estimator, which requires a two- or three-dimensional search over phase and timing parameters (for PSK signals, these include the carrier phase [for BPSK only], chip timing, and time-difference-of-arrival), can be closely approximated in many cases (for data record lengths that are normally used in weak-signal detection and source location) with much simpler suboptimum detectors/estimators that avoid all parameter searches for detection, and reduce the search to one dimension for TDOA estimation.

iii. Performance of Signal-Selective TDOA Estimators

Because of the considerable difficulty in obtaining (non-asymptotic) analytical results on performance (RMSE) of TDOA estimators for the highly corruptive environments of interest in this study, we chose to use simulations to evaluate RMSE for a variety of signal and noise/interference environments. In [9], we graphed statistical samples of the TDOA estimation functions whose peak-locations provide the TDOA estimates. This clearly revealed the noise and interference rejection capabilities of our signal-selective algorithms, and provided a qualitative assessment of their performance. We then averaged over many Monte Carlo trials to obtain graphs of RMSE versus averaging time in order to quantify performance. The results presented in [11] show that new signal-selective algorithms exhibit excellent robustness for BPSK signals in a wide range of interference and noise environments and operating conditions. Results not reported show the same level of performance for QPSK signals and

experimentation with other signal types such as FSK suggests that comparable performance is attainable in those cases where the signals of interest exhibit substantial spectral correlation. These new algorithms are tolerant to both interfering signals and noise, and they can outperform conventional algorithms that achieve the Cramer-Rao lower bound on variance for stationary signals because the signals considered here are nonstationary (cyclostationary) and the algorithms exploit the nonstationarity to discriminate against noise and interference. The most important issue regarding signal type and corresponding performance that has been discovered so far is the strength of the signal's cyclic feature (or spectral correlation feature) to be exploited. For example, bandwidth-efficient digital signals can have relatively weak keying-rate features in the sense that the spectral coherence function is close to unity over only a relatively small band. This limits the band over which the linear phase-vs.-frequency characteristic of the spectral correlation functions can be used, thereby limiting reliability of the estimate of the slope of this line. Although this can in principle always be compensated for by increasing the collection time, there are of course practical limits to this.

iv. Comparison of Auto- and Cross-Correlation Methods for Signal-Selective TDOA Estimation

The CPD method of TDOA estimation, which uses the cyclic autocorrelation (or cyclic auto-spectrum) and the SPECCOA method, which uses the cyclic cross correlation (or cyclic cross-spectrum) were compared in [10]. The CPD has the advantage of requiring only a very low-capacity cross-link, whereas the SPECCOA method requires a cross-link with bandwidth equal to that of the signal. However, the CPD algorithm produces ambiguous TDOA estimates because these estimates are given modulo $1/\alpha$, where α is the cycle frequency (e.g., chip rate). The CPD method also produces considerably larger RMSE

(e.g., 10 dB, that is, 20 dB in MSE, larger for collects ranging from 1024 to 8192 for a BPSK signal in noise with 0 dB SNR). Nevertheless, this ambiguity and reduced accuracy might be acceptable when the capacity of the cross-link must be small.

It was also shown in [10] that for both methods studied, RMSE of TDOA is independent of the distance between sensors.

v. **Implementation and Sources of Degradation for TDOA Estimation**

We have developed algorithms for implementing all methods of TDOA estimation that we have introduced. These algorithms, reported in [9], utilize FFTs to obtain computational efficiency and they use the spectral-smoothing method of obtaining reliable estimates of spectral correlation functions. Sources of degradation that were studied and reported on in [9] include inappropriate spectral-smoothing window-width, insufficient averaging time, error in knowledge of cycle frequency, and propagation-channel mismatch. It was found that the amount of data often needed to obtain acceptable TDOA estimates is more than adequate to obtain sufficiently accurate estimates of cycle frequencies with no prior knowledge.

vi. **Comparison of Performance of Array-Based AOA Estimation and TDOA Estimation**

Our simulations with a 5-sensor array showed in [11] that using multiple pairs of sensors in the M-SPECCOA algorithm provides relatively little improvement in performance over that attainable using the single pair of sensors with the greatest separation in the SPECCOA algorithm. Furthermore, for close sensors, e.g., a sensor array on a single platform, the array-based AOA estimation algorithm, Cyclic MUSIC provides much more accurate AOA estimates than the multiple-sensor-pair TDOA-based algorithm M-SPECCOA; e.g., for collect of 4096, SPECCOA produces RMSEs of 15° and 2° for SNRs

of 0 dB and 10 dB, whereas Cyclic MUSIC produces RMSEs of 0.7° and 0.2° in the same situations. It is estimated that SPECCOA would produce an RMSE as small as 0.2° for an SNR of 10 dB if either the collector or the sensor separation were increased by a factor of 10. For widely separated sensors, e.g., on separate platforms, it is clear the SPECCOA can outperform Cyclic MUSIC. Also, although CPD has an RMSE that is about a factor of 10 greater than that produced by SPECCOA, it too should outperform Cyclic MUSIC if it is used with widely separated platforms. However, the ambiguity in the CPD estimate of AOA must be resolved by some other means. On the other hand, when used in conjunction with SPECCOA or CPD, Cyclic MUSIC can be useful (e.g., to help resolve ambiguity) if each of two (or more) platforms contains a sensor array.

vii. Exploitation of Higher-Order Cyclostationarity

For bandwidth-efficient signals, like some PSK, CPFSK, and partial-response encoded signals with small (or zero) excess bandwidth, the degree of second-order cyclostationarity can be relatively weak (or zero). We have shown with preliminary simulations of CPFSK that signal-selective TDOA estimation can be performed by exploiting higher-order cyclostationarity (4-th order in the simulations). Motivated by this, we have initiated the development of a general theory of the time-domain and frequency-domain characteristics of higher-order cyclostationarity [12] - [14].

c. Conclusions

In this research project, we have developed and simulated methods that exploit cyclostationarity, which is exhibited by most communication and telemetry signals, to estimate the locations of emitters by estimating time-difference-of-arrival at two or more reception platforms. Exploitation of cyclostationarity renders the new methods signal-selective, which enables them to perform well

even in the presence of temporally and spectrally overlapping interference and noise, regardless of the proximity of the interfering emitters to the emitters of interest. Future work should attempt to analytically characterize the performance of these new algorithms and pursue even better-performing algorithms.

3. Application of the New Algorithms

For location of ground-based emitters using one or more ground-based platforms, a position estimate can be obtained by intersecting two lines of bearing obtained from AOA estimates obtained at each of the two platforms, or by intersecting two ISO-TDOA hyperbolas obtained from two pairs of platforms (e.g., using 3 platforms) or from one moving pair of platforms at two different times, or by intersecting one ISO-TDOA hyperbola and one line of bearing. Also, if TDOA can be measured only modulo some value (e.g., the chip interval, as is the case with the CPD algorithm), then the ambiguity can be resolved by intersecting two families of ISO-TDOA hyperbolas and one line of bearing.

With a small antenna array on a platform, we can use algorithms such as cyclic MUSIC or CLS to obtain high-resolution signal-selective estimates of the AOA's of multiple cochannel signals with the same cycle frequency (eg., chip rate). This requires no cross-link between platforms. To intersect the two lines of bearing corresponding to the AOA estimates from two platforms corresponding to a single emitter requires the transmission of only a single number (an AOA) from one platform to the other.

With only a single antenna on each of the two platforms, we can use the CPD algorithm to obtain a family of ISO-TDOA hyperbolas corresponding to the desired TDOA estimate plus all integer multiples of the reciprocal of the cycle frequency (the chip rate). This requires transmission of only a very small number, say less than 10, of complex numbers (values of the cyclic autocorrelation for the cycle frequency equal to the chip rate and 10 samples of the lag parameter ranging from zero to the chip interval) from one platform to the other.

Finally, by using only a pair of sub-bands separated by the cycle frequency of the received data, a low-capacity cross-link can be used to enable the measurement of

a cross-cyclic spectrum over a sub-band, which can be used by the SPECCOA algorithm to obtain a reduced-accuracy estimate of TDOA, but without ambiguity. This estimate could be used to resolve the ambiguity in the more accurate TDOA estimate obtained using the CPD algorithm.

However, when maximum accuracy is desired, intersection of two ISO-TDOA hyperbolas obtained from the SPECCOA algorithm or the PP-SPECCON algorithm operating on the full band of the signal, is indicated.

When multiple cochannel emitters sharing the same cycle frequency are to be located, problems of resolving adjacent peaks in the TDOA estimation function produced by SPECCOA can occur if emitters are too close together. (CPD and PP-SPECCON will not work at all if there is more than one signal with the same cycle frequency.) However, if a small array of antennas is available on each platform, then *spatial separation methods* can be used. Although, the cyclic MUSIC algorithm will provide estimates of the AOAs of multiple cochannel signals with the same cycle frequency, another approach which could yield more accuracy is to use the SCORE algorithm to extract the individual signals of interest, and then use SPECCOA on the individual signals. This requires the *proper pairing of signals from the two platforms*, which is possible with SPECCOA.

C. LIST OF PUBLICATIONS AND TECHNICAL REPORTS *

a. Journal and Conference Papers

1. S. V. Schell and W. A. Gardner. SIGNAL-SELECTIVE DIRECTION FINDING FOR FULLY CORRELATED SIGNALS. Sixth Multidimensional Signal Processing Workshop, September 5-7, 1989, Pacific Grove, California (17 page poster paper, 2 page abstract published).
2. S. V. Schell and W. A. Gardner. SIGNAL-SELECTIVE HIGH-RESOLUTION DIRECTION-FINDING IN MULTIPATH. Proceedings of the International Conference on Acoustics, Speech, and Signal Processing, Albuquerque, New Mexico, April 1990, pp. 2667-2670.
3. S. V. Schell and W. A. Gardner. CRAMER-RAO LOWER BOUND FOR PARAMETERS OF GAUSSIAN CYCLOSTATIONARY SIGNALS. Proceedings of the 1990 International Symposium on Information Theory and Its Applications, Waikiki, Hawaii, November 27-30, 1990, pp. 255-258.
4. S. V. Schell and W. A. Gardner. DETECTION OF THE NUMBER OF CYCLOSTATIONARY SIGNALS IN UNKNOWN INTERFERENCE AND NOISE. Proceedings of the Twenty-Fourth Annual Asilomar Conference on Signals, Systems, and Computers, Pacific Grove, CA, November 5-7, 1990. (invited paper)
5. S. V. Schell and W. A. Gardner. PROGRESS ON SIGNAL-SELECTIVE DIRECTION FINDING. Proceedings of the Fifth ASSP Workshop on Spectrum Estimation and Modeling, Rochester, NY, October 10-12, 1990, pp. 144-148.
6. S. V. Schell. EXPLOITATION OF SPECTRAL CORRELATION FOR SIGNAL-SELECTIVE DIRECTION FINDING. Ph.D. Dissertation, Department of Electrical Engineering and Computer Science, University of California, Davis, December 1990.
7. W. A. Gardner and C. M. Spooner. HIGHER-ORDER CYCLOSTATIONARITY, CYCLIC CUMULANTS, AND CYCLIC POLYSPECTRA. Proceedings of the 1990 International Symposium on Information Theory and Its Applications, Waikiki, Hawaii, November 27-30, 1990, pp. 355-358.
8. W. A. Gardner and C. K. Chen. SIGNAL-SELECTIVE TIME-DIFFERENCE-OF-ARRIVAL ESTIMATION FOR PASSIVE LOCATION OF MANMADE SIGNAL SOURCES IN HIGHLY CORRUPTIVE ENVIRONMENTS, PART I: THEORY AND METHOD. IEEE Transactions on Signal Processing. (in press)
9. C. K. Chen and W. A. Gardner. SIGNAL-SELECTIVE TIME-DIFFERENCE-OF-ARRIVAL ESTIMATION FOR PASSIVE LOCATION OF MANMADE SIGNAL SOURCES IN HIGHLY CORRUPTIVE ENVIRONMENTS, PART II: ALGORITHMS AND PERFORMANCE. IEEE Transactions on Signal Processing. (in press)

* Submitted or published during the period of this ARO contract and under sponsorship from this ARO contract.

10. S. V. Schell and W. A. Gardner. THE CRAMER-RAO LOWER BOUND FOR DIRECTIONS OF ARRIVAL OF GAUSSIAN CYCLOSTATIONARY SIGNALS. IEEE Transactions on Information Theory. (submitted)
11. S. V. Schell and W. A. Gardner. ESTIMATING THE DIRECTIONS OF ARRIVAL OF CYCLOSTATIONARY SIGNALS - PART I: THEORY AND METHODS. IEEE Transactions on Signal Processing. (submitted)
12. S. V. Schell and W. A. Gardner. ESTIMATING THE DIRECTIONS OF ARRIVAL OF CYCLOSTATIONARY SIGNALS - PART II: PERFORMANCE. IEEE Transactions on Signal Processing. (submitted)
13. W. A. Gardner and C. M. Spooner. WEAK-SIGNAL DETECTION AND SOURCE LOCATION: SIMPLIFICATIONS OF THE MAXIMUM-LIKELIHOOD RECEIVER. IEEE Transactions on Signal Processing. (submitted)
14. W. A. Gardner and C. M. Spooner. COMPARISON OF AUTO- AND CROSS-CORRELATION METHODS FOR SIGNAL-SELECTIVE TDOA ESTIMATION. IEEE Transaction on Signal Processing. (in press)

b. Book Chapters

15. S. V. Schell and W. A. Gardner. HIGH-RESOLUTION DIRECTION FINDING. Chapter in *Handbook of Statistics, Vol., 10, Signal Processing*, edited by N. K. Bose and C. R. Rao, Amsterdam: Elsevier, 1992.
16. W. A. Gardner. CYCLOSTATIONARY PROCESSES. Chapter 12 of *Introduction to Random Processes with Applications to Signals and Systems*, (Secs 12.8.2, 12.8.3, 12.8.4), New York: McGraw-Hill, 1990.

c. Technical Reports

17. S. V. Schell. OVERVIEW OF SIGNAL PROCESSING SOFTWARE. (Includes Mac disk). Delivered to Technical Liaison Dr. Stephan Rhodes. U. S. Army Communications Electronics Command Center for Signals Warfare.
18. C. K. Chen, S. V. Schell and W. A. Gardner. EXPLOITATION OF POLARIZATION IN SIGNAL PROCESSING. Delivered to Technical Liaison Dr. Stephan Rhodes. U. S. Army Communications Electronics Command Center for Signals Warfare.
19. W. A. Gardner, S. V. Schell and C. M. Spooner. A COMPARISON OF ARRAY-BASED AOA ESTIMATION AND TDOA ESTIMATION, SSPI Tech. Rept. 91-1, May 1991. Delivered to Technical Liaison Dr. Stephan Rhodes. U. S. Army Communications Electronics Command Center for Signals Warfare.
20. W. A. Gardner, EXPLOITATION OF CYCLOSTATIONARITY OF FREQUENCY-HOPPED SIGNALS, Delivered to Technical Liaison Dr. Stephan Rhodes. U. S. Army Communications Electronics Command Center for Signals Warfare.

D. SCIENTIFIC PERSONNEL SUPPORTED

William A. Gardner: Principal Investigator

Chih-Kang Chen, Ph.D: Engineer

Randy S. Roberts, Ph.D: Engineer

Stephan V.Schell, Ph.D: Engineer (Received Ph.D degree in December 1990)

Chad M. Spooner, M.S: Engineer (Will receive Ph.D in 1992)

E. **HONORS AND AWARDS***

(Principal Investigator):

- 1989 Received the Stephen O. Rice Prize Paper Award in Communication Theory from the IEEE Communications Society for 1988 paper on signal interception.
- 1989-1991 Five invited papers on cyclostationarity at the Twenty-third, Twenty-fourth, and twenty-fifth Asilomar Conferences on Signals, Systems, and Computers.
- 1990 Invited paper on cyclostationarity at IEEE ASSP Fifth Workshop on Spectrum Estimation and Modeling.
- 1990 Invited Guest Editor of a Special Issue of the *IEEE Signal Processing Magazine* on cyclostationarity, April 1991.
- 1990 Elected to Fellow of the IEEE for contributions to the theory of cyclostationary signals.
- 1991 Invited coauthor of chapter on high-resolution direction-finding in the *Handbook of Statistics*, Vol. 10, edited by N. K. Bose and C. R. Rao.
- 1991 Invited member of International Scientific Committee for Program for U.S.- Poland workshop on nonstationary (particularly cyclostationary) processes.
- 1991 Invited by NSF to organize a U.S. workshop on exploitation of cyclostationarity to be held in 1992.

(Participants):

- 1991 Stephan V. Schell received the *Anil K. Jain Prize for Best Ph.D Dissertation* from the Department of Electrical and Computer Engineering.

* During the period of this ARO contract and associated with the research conducted under this ARO contract.

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10. W. A. Gardner and C. M. Spooner, "Comparison of auto- and cross-correlation methods for signal-selective TDOA estimation," *IEEE Trans. Sig. Proc.*, 1992. (In press)
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