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Final Technical Report for

Grant AFOSR-88-0032

ANALYSIS, ESTIMATION AND CONTROL FOR PERTURBED AND SINGULAR SYSTEMS AND FOR SYSTEMS SUBJECT TO DISCRETE EVENTS

for the period

1 October 1987 through 30 September 1991

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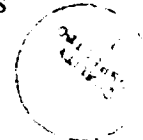
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I. Summary

In this report we summarize our accomplishments in the research program supported by Grant AFOSR–88–0032 over the period from October 1, 1987 to September 30, 1991. The basic scope of this program is the analysis, estimation, and control of complex systems with particular emphasis on (a) multiresolution modeling and signal processing; (b) the investigation of theoretical questions related to singular systems; and (c) the analysis of complex systems subject to or characterized by sequences of discrete events. These three topics are described in the next three sections of this report. A full list of publications supported by Grant AFOSR–88–0032 is also included.

The principal investigator for this effort is Professor Alan S. Willsky, and Professor George C. Verghese is co-principal investigator. Professors Willsky and Verghese were assisted by several graduate research assistants as well as additional thesis students not requiring stipend or tuition support from this grant.

We feel that our work under this grant has been highly successful not only in producing a significant number of results and publications but also in both uncovering new research questions and directions for future investigations and in providing advanced technology presently being transitioned into significant Air Force programs at both the 6-2 and 6-3 levels. In total the publications resulting from research supported by Grant AFOSR-88-0032 include 29 papers that have appeared or been submitted to journals, 11 journal papers presently in preparation, 32 papers presented at conferences, 4 S.M. theses, and 5 Ph.D. theses. We anticipate that several additional papers will also be written. In addition, Prof. Willsky has been invited to give numerous lectures on the results of these efforts, including plenary lectures on our work on wavelet transforms, data fusion, and multiresolution statistical signal processing given at the 1991 IEEE-AES International Conference on Systems Engineering held in Dayton, Ohio in August and the 1991 SIAM Workshop on Linear Algebra and Signal Processing held in Minneapolis, MN in September. Further evidence of Prof. Willsky's leadership position in this particular field can be found in his serving as one of the guest editors (along with Dr. I. Daubechies of AT&T Bell Labs and Prof. S. Mallat of The Courant Institute) of the special issue of the IEEE Transactions on Information Theory on Wavelet Transforms and Multiresolution Signal Analysis. Furthermore, there have been and continue to be a number of applications of Prof. Willsky's research to a variety of Air Force programs, ranging from application of his work on failure detection to the design of fault-tolerant control systems for the Advanced Tactical Fighter to application of our recently developed multiresolution signal and image processing methods to surveillance and reconnaissance problems at both Rome and Wright Laboratories.

II. Multiresolution Modeling and Signal Processing

In this section we describe that portion of our proposed research program dealing with multiresolution signal and image analysis. In the past few years we have had considerable success in our efforts to develop the theoretical foundation and methodology for multiresolution stochastic signal processing by providing a stochastic modeling and filtering framework complementary to the wavelet transform in much the same way that that the framework of stationary stochastic processes and recursive dynamic models complement the Fourier transform. The result of this effort is a new class of stochastic models for signals in one or several dimensions (e.g., images) that explicitly display their multiresolution characterization and that have led to the development of efficient and optimal algorithms for several signal processing and reconstruction problems, identifying explicitly the role of scale-domain filtering and the wavelet transform. As shown in [32,42,66,72–74], a special example of this class of models yields processes which closely resemble so-called fractal "1/f" noise processes, but the full richness of the models described in [54,55,66,72–74] and the range of problems to which they can be applied appears to be substantially greater than even this important class of applications.

At this point it is useful to identify both the conceptual reasons for examining multiresolution methods and some specific military contexts in which such reasons might apply. To begin, it is important to realize that there are three distinct ways in which multiresolution features enter into a signal or image analysis problem. First, the phenomenon under investigation may possess features and physically significant effects at multiple scales. For example, fractal models have often been suggested for the description of natural scenes, topography, ocean wave height, textures, etc. Also, anomalous broadband transient events or spatially-localized features can naturally be thought of as the superposition of finer resolution features on a more coarsely varying background. Second, whether the underlying phenomenon has multi-resolution features or not, it may be the case that the data that has been collected is at several different resolutions. For example the resolutions of sensors operating in different bands — such as IR, microwave, and various band radars — may differ. Furthermore, even if only one sensor type is involved, measurement geometry may lead to resolution differences (for example if two sensors with good angle but poor range resolution view a phenomenon from different perspectives or if radar-produced terrain data is gathered from radars flying at different altitudes). Third, whether the phenomenon or data have multi-resolution features or not, the signal analysis *algorithm* may have such features.

There are two strong motivations for considering multi-resolution algorithmic structures, especially in imaging contexts, motivated by the two principal manifestations of the at least superficially daunting complexity of many image processing problems The first and more well-known of these is the use of multi-resolution algorithms to combat the computational demands of such problems by solving coarse (and therefore computationally simpler) versions and using these to guide (and hopefully speed up) their higher resolution counterparts. Multigrid relaxation algorithms for solving partial differential equations are of this type as are a variety of computer vision algorithms, as well as the electromagnetic imaging algorithm that provided some of the motivation for the methods on which the work in [54,55,66,72–74] is based. The second, and equally important reason, is that a multi-resolution formalism allows one to exercise very direct control over "greed" in image reconstruction. In particular, many imaging problems are, in principle, ill-posed in that they require reconstructing more degrees of freedom then one has elements of data. In such cases one must "regularize" the problem in some manner, thereby guaranteeing accuracy of the reconstruction at the cost of some resolution. Since the usual intuition is precisely that one should have higher confidence in the reconstruction of lower resolution features, we are led directly to the idea of reconstruction at multiple scales, allowing the resolution-accuracy tradeoff to be confronted directly. Surprisingly this concept has not been as widely developed as one might expect. This work also points to another feature that such a framework allows, namely the use of *spatially-varying* resolutions, in which the resolution is matched both to the phenomenon being reconstructed and the available data. For example, in surface reconstruction from irregularly sampled measurements, one might expect that it would be best to use a spatially-varying resolution in which finer detail was reconstructed near the points at which samples were available.

The analytical setting for the theory of multiscale statistical signal modeling and processing that we have developed, which is rich enough to address all of these issues, has as its starting point the structure suggested by the wavelet transform, which we now briefly review. For simplicity we describe everything for 1-D signals but the extension to images in two or more dimensions introduces only notational rather than mathematical complexity.

The multiscale representation of a continuous signal f(x) consists of a sequence of approximations of that signal at finer and finer scales where the approximation of f(x) at the mth scale is given by

$$f_m(x) = \sum_{n=-\infty}^{+\infty} f(m,n)\, \phi(2^m x - n) \tag{1}$$

As $m \to \infty$, the approximation consists of a sum of many highly compressed, weighted, and shifted versions of the function $\phi(x)$ whose choice is far from arbitrary. In particular, in order for the $(m+1)^{st}$ approximation to be a refinement of the mth, we require that $\phi(x)$ be exactly representable at the next scale:

$$\phi(x) = \sum_n h(n)\, \phi(2x - n) \tag{2}$$

Furthermore, in order for (1) to be an orthogonal series, $\phi(x)$ and its integer translates must form an orthogonal set. The function $h(n)$ must satisfy several conditions for this and several other properties of the representation to hold. In particular $h(n)$ must be the impulse response of a quadrature mirror filter (QMF).

By considering the *incremental detail* added in obtaining the $(m+1)^{st}$ scale approximation from the mth, we arrive at the *wavelet transform*. Such a transform is based on a single function $\psi(x)$ that has the property that the full set of its scaled translates $\left\{ 2^{m/2}\psi\left(2^m x - n\right)\right\}$ form a complete orthonormal basis. It can be shown that ϕ and ψ are related via an equation of the form

$$\psi(x) = \sum_n g(n)\, \phi(2x - n) \tag{3}$$

where $g(n)$ and $h(n)$ form a *conjugate mirror filter* pair, and the evolution from one scale to the next is of the form

$$f_{m+1}(x) = f_m(x) + \sum_n d(m,n)\, \psi(2^m x - n) \tag{4}$$

Thus $f_m(x)$ is simply the partial orthonormal expansion of $f(x)$, up to scale m, with respect to the basis defined by ψ and the wavelet transform of $f(x)$ consists of the set of coefficients $\{d(m, n)\}$ at all scales m and all translational offsets m.

One of the critical elements in understanding why such a transform might be of value is in the interpretation of (4) as a *dynamical* relationship from one scale to the next. Specifically, if we let f_m and d_m denote vectors containing, respectively, all of the values of $f(m, n)$ and $d(m, n)$ at the mth scale. Then (1) - (4) imply that

$$f_{m+1} = H^* f_m + G^* d_m \tag{5}$$

where H and G are linear transformation computed directly from h(n) and g(n), and H^* and G^* denote their adjoints. Furthermore a direct consequence of the construction is that (5) can be reversed. Specifically

$$f_m = Hf_{m+1}, \quad d_m = Gf_{m+1} \tag{6}$$

The *wavelet analysis equation* (6) describes an efficient fine-to-coarse dynamic structure for computing the wavelet transform: at each scale, we transform the signal approximation f_{m+1} at the current scale in order to "peel off" the component, d_m, at that scale and to compute the approximation, f_m, at the next, coarser scale. Furthermore, it is usually the case that h(n) and g(n) are localized — e.g., they are of finite extent — so that the H^* and G^* are extremely sparse. This leads to very efficient and highly parallelizable procedures for computing wavelet transforms. Similarly the *wavelet synthesis equation* (5) describes a dynamical relationship for constructing signals from coarse to fine scales. This also can be computed efficiently but it has another even more significant role here.

As we have argued, for the wavelet transform to be of significant value in problems of signal and image analysis, we require not only the simplicity of its computation but also the simplification it brings to various problems of signal analysis. In particular, one would expect the transform to be of considerable value if the statistical description of the transform coefficients {d(m, n)} is dramatically simpler than that of the original signal. The key to identifying such a signal class is the observation that (5) describes a dynamic system in scale with d_m as the input. If this input is uncorrelated from scale to scale, then the wavelet analysis equation (6) performs a *scale-by-scale whitening* of the signal, resulting in the desired simplification. While this is an important observation, it is not completely satisfactory. In particular, if we think of scale as a time-like dynamic variable, then (5) represents a first-order system. As we know, first-order systems driven by white noise yield a comparatively small class of processes which can be broadened dramatically if we allow higher-order dynamics. Also, in sensor fusion problems one wishes to consider collectively an entire set of signals or images from a suite of sensors. In this case one is immediately confronted with the need to use higher-order vector models in which the actually observed signals may represent samples from such a model at *several* scales, corresponding to the differing resolutions of individual sensors.

These motivations, together with the interpretation of wavelet transforms as dynamic models in scale, has led over the past three years [32,44,66,72–74] to a substantial generalization of (5), (6) to *vector state models in scale*:

$$x(m+1) = A(m) \; x(m) + B(m) \; w(m) \tag{7}$$

$$y(m) = C(m) \, x(m) + v(m) \qquad\qquad (8)$$

where $x(m)$ is a vector process representing higher-order multiscale characteristics and/or the joint description of the scale dependent character of a set of sensor signals. Here $y(m)$ represents the "observed signal" at the mth scale (i.e., those sensor signals with the corresponding resolution), $v(m)$ and $w(m)$ are white noise processes in scale, and $A(m)$, $B(m)$, and $C(m)$ are operators describing the scale-to-scale evolution. An alternate form for such multiscale models as higher-order multiscale autoregressive processes has also been completely developed [33,54,55,66].

Several important observations must be made about this model. First, by allowing A, B, and C to depend on m, we can capture a wide variety of scale-dependent effects. The most obvious of these is $C(m)$ which can be used to select signals at their appropriate scales. In addition scale variations in $A(m)$ and $B(m)$ can be used to capture statistical variations from scale to scale. For example, geometric dependence on m allows one to capture so-called *self-similar* stochastic phenomena, while other variations, especially in $B(m)$ allow one to isolate critical scales in multi-channel signals and images (e.g., as is often done heuristically in modeling atmospheric turbulence). Furthermore, by defining the structure of these operators carefully, we can retain all of the advantages of wavelet transforms. Specifically, as discussed in [66] the localized nature of $h(n)$ and $g(n)$ imply a *lattice* structure on the coefficient $f(m,n)$, where $f(m,n)$ at one scale is connected to $f(m+1,n)$ if the value of $f(m+1,n)$ depends on $f(m,n)$. By maintaining this lattice structure in (7), (8) (which corresponds to considering multichannel wavelet transforms or to higher-order models for the wavelet coefficients), we find that the use of a multichannel wavelet transform again leads to a whitening of the data and model in that the transform yields a set of *decoupled* dynamic systems (in scale) for the different multiscale components of the full set of signals.

It is important to note that this decoupling is critically dependent upon one important feature not explicitly highlighted in (7), (8). Specifically, for the wavelet transform to decouple the scale components of such a model, it must be the case that the model (7), (8) is translation- invariant, i.e. that all variation is only in scale. For example, this requires that variances and correlation functions of quantities such as $f(m,n)$ and $d(m,n)$ depend only on m, and not on n. Thus the modeling of localized or transient phenomena or of spatially-varying textures fall outside the class for which wavelet transforms make life simple. Also, at least as important a fact is that translation invariance requires that the measurements be translation-invariant--i.e. if a measurement is taken at a particular scale and at a particular location (i.e. a particular (m,n) pair), then measurements must be

available at all translational locations at that scale (i.e. at (m,n') for all n'). The implications of this are that pure wavelet transform-based methods can't be used if either windowed or irregularly-sampled data are considered. The former of these is actually a much lesser worry. Specifically, if the only cause of translation-varying behavior is windowing, one can apply heuristic, approximate methods as in [75] of periodic data repetition or one can use the optimal adjusted wavelet transform described in [72,74] which produces decoupled dynamics by modifying the wavelet transform near the window edges.

The issue of irregular data, however, requires a drastically different approach, and in our work [33,49,66,73] we have developed two alternative methods. One of these involves multiscale iterative processing very similar to the structure of celebrated multigrid algorithms which have revolutionized methods for the efficient, highly parallel solution of partial differential equations. The other involves a pyramidal processing structure in which fine level data is fused in a fine-to-coarse sweep, followed by an optimal coarse-to-fine reconstruction/interpolation step. Not only do these methods, together with the wavelet transform based approach, indicate how one should think about the optimal fusion of information from heterogeneous suites of sensors but they also provide the basis for extremely efficient algorithms. In particular these three processing structures provide optimal Wiener and Kalman smoothing procedures in *scale* that are far *more* efficient and highly parallelizable than their time series counterparts We refer to [32,49, 72–74] for more detailed discussion of these issues.

Thus we see that the model class (7), (8) is extremely attractive from a computational perspective and for the apparent promise of providing a natural framework for capturing multiscale phenomena and measurements. The remaining critical question then is: Are there large and meaningful classes of space-time stochastic processes that can be adequately modeled as in (7), (8)? Happily the answer to this is already known to be a resounding "yes", although there is much unknown territory to be explored here. In particular, as discussed in [72,75], wavelet transforms — while not performing exact whitening for large classes of standard process models — come sufficiently close that the discrepancy is negligible both in the sense that it is small compared to unavoidable errors in the modeling of the original process, and in the sense that the performance of algorithms based on neglecting these discrepancies yield impressive results in terms of performance and efficiency.

As we have indicated, one of the algorithmic structures we have developed has a pyramidal structure, in which a fine-to-coarse recursion is followed by a coarse-to-fine step. In these algorithms the variables being reconstructed at any scale essentially correspond to wavelet transform approximations at that scale. Recently we have begun

work on the development of similarly structured pyramidal parallel processing algorithms for a different class of statistical signal processing problems in which the variables at different "scales" represented *decimated* versions of the quantity to be estimated. The motivation for their study comes directly from the issue of computational complexity for multidimensional processing problems. Specifically, if we measure complexity in terms of required storage, we know that the complexity of a standard recursive filter equals the order of the filter or equivalently the dimension of its state, which represents in essence a set of initial conditions that are propagated forward in time to summarize all that is needed to determine the effects of past inputs on future outputs. In two dimensions, we see that things are quite different since the counterpart of propagating initial conditions forward is that of propagating *boundary* conditions inward and outward. Not only does this lead to a new notion of recursion in multiple dimensions (see [13,14,20,39, 67–70,76]) but also to the fact that complexity in 2-D depends upon the size of the boundary, which in turn depends upon the size of the data region to be analyzed. This suggests the idea of partitioning the data into many, smaller, subregions, with parallel processing performed in each subregion outward toward subregion boundaries, followed by an interprocessor data exchange across boundaries and a final set of parallel inward processing steps within each subregion. Algorithms of this type have been developed previously by us and by others in the 1-D case for estimation of standard state space models. In 1-D, of course, the set of regional boundaries consists of a set of points, representing a decimated version of the original process. In this earlier work this fact was used to develop approaches to the data interchange step that propagated boundary information sequentially from one processor to the next. This idea, of course, doesn't work in 2-D since, for example, the set of boundaries of a rectangular subdivision don't have any natural order that allows us to specify a useful sequential form for the data interchange step. However, in our recent work [69,70,76] we have developed an alternate approach that appears to have considerable promise in 1-D <u>and</u> in multiple dimensions. In particular in the 1-D case, rather than performing the data exchange step sequentially, and all at once, we do it by <u>further</u> decimation steps-- i.e. in parallel we can merge information between disjoint pairs of neighboring intervals. For example, if we have partitioned our data with boundary points $t_1 < t_2 < t_3 < ...$, we can fuse the two intervals $[t_1, t_2]$ and $[t_2, t_3]$ to form $[t_1, t_3]$, the two intervals $[t_3, t_4]$, and $[t_4, t_5]$, to form $[t_3, t_5]$, etc., leading to a second <u>decimated</u> partition $t_1 < t_3 < t_5 < ...$ In essence this is nothing more than another outward processing step in which we grow small intervals outward to form larger ones. Thanks to the Markovian nature of 1-D state models, all processing on disjoint data intervals can be performed in parallel, and the fusion of subintervals involves the merging of information about interval

boundaries alone (rather than values at interior points). The result is precisely a fine-to-coarse estimation step in which we produce estimates of a succession of increasingly decimated versions of the process based on data in intervals of increasing size. This is then followed by a coarse-to-fine interpolation step which results in the full exchange of boundary information among all processors. As discussed in [69], this procedure has precisely the same pyramidal structure as our multiscale estimation algorithms in [32,49,66,73], (which thus are perfectly matched to hypercube computer architectures), although the statistical models and bases for the two algorithms are apparently very different. While there are a number of open issues related to the 1-D case-- in terms of understanding more deeply the statistical differences between the approaches, of developing system theoretic results analogous to those in [49,73], and of extending the methodology to more general digital filtering applications (leading to new digital filter structures)-- it is in two and higher dimensions that we expect to devote most of our attention. In particular, we expect that it will be possible to develop analogous methods for the optimal estimation of 2-D local and Markov random field models, although there will necessarily be differences, both in the inward/outward recursions (which must have varying dimension to reflect changes in boundary lengths) and in the boundary merging step in which we fuse together boundary information about two neighboring rectangles in order to obtain an estimate of the outer boundary of the larger, rectangle resulting from their merger.

We have also had success in another related area of research, namely the multiple time scale analysis of singularly perturbed systems and processes. This work also involves the examination of processes at multiple resolutions, where successive levels of aggregation occur as we look at increasingly longer time scales. Our work in this area, which has had a considerable history [1,3,6,7,15,18,19], has been motivated by the desire to develop analytic and quantitative methods capable of dealing with discrete-state systems of considerable complexity. In many systems of this type--such as large-scale C^3 systems, flexible manufacturing systems, and interconnected power systems-- there are extremely large numbers of states and transition events with probabilities of occurrence that vary over orders of magnitude. For example in many such systems faults represent comparatively rare events (compared to the events corresponding to normal operation such as successful message transmission or completion of the manufacture of a part). Moreover, in many cases, not only are there several orders of magnitude difference among transition probabilities but also there may be critical event sequences that involve the occurrence several rare events. For example, a rare fault may, most probably, be correctly detected and accommodated; however on a rare occasion a misclassification of the fault or a large

delay in detection may occur. Such cascades of events can be found in many complex systems such as in crashes of distributed communication networks and large-scale blackouts. Obviously there are strong motivations for developing methods of analysis for complex systems in order to characterize such behavior and devise effective system-wide monitoring systems capable of detecting the onset of such event sequences in a timely enough fashion to institute corrective action. However the sheer complexity of such systems makes it necessary to seek far more efficient methods than simple enumeration of all of the possible event sequences in order to isolate the "weak links", i.e., the least rare of a very large number of possibilities. Developing methods to deal with this complexity has been the driving force in this portion of our work.

One of the models that we have studied in detail in this portion of our work has been that of a perturbed finite-state Markov process, whose probability distribution, p(t), evolves according to

$$\dot{p}(t) = A(e)p(t)$$

where $A(\varepsilon)$ is an infinitessimally stochastic matrix. The parameter ε represents a small variable that makes explicit the different orders of magnitude of various events (so that transition rates of order $1, \varepsilon, \varepsilon^2$, etc. may be present). Briefly stated our contributions to date include the following:

•The development of hierarchical aggregation methods for this model capturing the critical events at each time scale and performing successive aggregation to obtain simpler models. Specifically using singular perturbation methods we show in [1] (and in [6] for discrete-time chains) how we can construct a sequence of far simpler models, capturing events that have a significant probability of occurring at increasingly long time scale (of orders $1, 1/\varepsilon, 1/\varepsilon^2$...) and blurring out the detail of events (and aggregating the corresponding states) that occur at shorter ti e scales. Together this set of models allows us to construct an asymptoticall accurate description of the full transition behavior of the original system using a set of far simpler models. Results of this type have been available for some time, motivated by applications such as queuing and computer systems. However, prior to our work all previous work required conditions of "near decomposability" which completely leaves out the possibility of sequences of rare events that are characteristic of catastrophic behavior, such as blackouts and distributed system crashes. In our work [1,3,15,19] we have been able to overcome this restriction completely by identifying the crucial role of so-called "almost transient states", i.e. states that are entered after the occurrence of a rare event and from which recovery is quite likely but not absolutely certain (since a second rare event may occur prior to recovery).

•We have developed some results of the same type for semi-Markov processes [7]. A key here is that rare events can occur for *two* reasons: the presence of rare transitions and the presence of holding time distributions with heavy tails. The latter aspect had not been considered at all in any previous work, and its presence

leads to a far richer class of behavior (e.g. without it, the first step of the aggregation process leads immediately to a Markov rather than semi-Markov process).

- We have also developed some initial results [8,15] on the structural analysis of perturbed Markov chains. Specifically, it is possible to develop a completely integer-based algorithm (keeping track of <u>orders</u> of ε) that determines the structure of the multiple time scale models corresponding to an $A(\varepsilon)$ which is also specified structurally (in terms of the orders of magnitude of transitions) This result separates completely the determination of the critical time scales and the structure of the corresponding models from the numerical problem of computing the coefficients of the corresponding transition probabilities.

- In [19] we present some initial results on the use of our methodology to evaluate performance measures for complex processes. In particular in [19] we examine the situation in which certain events in the full process are to be counted (corresponding in [19] to the completion of a product in a manufacturing system). Two critical points to make are: (1) while such events appear as individual counts at the original process level, the count of their occurrence may appear as a *random variable* at subsequent levels at which the states associated with the event have been aggregated; and (2) just as the work in [1] requires more care with ε-dependent quantities, the work in [19] requires slightly more than in [1]. Specifically, in many situations the event count we wish to make corresponds to counting occurrences of any of a <u>set</u> of transition events in our process. For example, counting failed message transmissions in a communication network requires counting such events when the network is operating normally, *and* when it is operating in a degraded mode; since these modes of operation represent different states, we must count transitions from both. This requirement demands some care, since a low rate of occurrence from a state with high probability is of the same importance as a high rate of occurrence from a state with low probability.

We have also made considerable progress in our study of multiresolution modeling for control in an interesting class of discrete-event, continuous-time systems, namely power electronic circuits. These circuits are usually modeled as interconnections of linear, time invariant circuit components and ideal switches. The switching events are determined by the relationship between time varying control inputs and periodically varying clocking waveforms. In steady state, the behavior of a power convertor is periodic. To design controllers that regulate departures from steady state, we need appropriate dynamic models.

The main focus of our research so far has been on averaged models, [34-37], [47], [62]. This focus reflects the fact that in many power circuits - high frequency PWM converters in particular- we are interested in controlling the *local average* of circuit waveforms, not the instantaneous values. If the clocking waveform in the circuit has period T, the local average of interest for a variable x(t) is defined by

$$\overline{x}(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau)d\tau$$

This average is not the one produced by the classical averaging methods developed in nonlinear mechanics.

Using the fact that the derivative of the local average equals the local average of the derivative, and making reasonable approximations, we can obtain continuous-time dynamic models for the averaged variables by taking the local average of dynamic models for the instantaneous variables. It is also possible to obtain these averaged dynamic models directly in circuit form by direct averaging of the instantaneous circuit, [36]. The continous-time averaged models are usually far simpler than the instantaneous switched models to analyze and simulate, and are more fruitful in developing controller designs, [62].

The converter considered in [62] is driven by a periodically varying voltage source, and thus has *two* natural averaging periods associated with it: the long period T_L of the voltage source, and the short period T_S of the clocking or switching waveform. We demonstrate in [62] the advantages of a multiresolution approach that uses averaged models at both these time-scales. The T_S-averaged model is periodic with period T_L, because it is driven by the periodic voltage source. The T_L-averaged model is time invariant, and is obtained by averaging the T_S-averaged model. We are not aware of a similar two-stage procedure being treated in the classical averaging literature, and intend to pursue this possibility.

The time invariant controllers designed in [62] are derived using the T_L-averaged model. By using the T_S-averaged model instead, we obtain periodically varying controllers that enable much faster recovery from transients, [63]. We are now studying the possibility of tuning the parameters in the T_S-controller on the basis of computations that involve the T_L-model. Such hierarchical control based on aggregation at successively larger time scales is an important theme in both the multiresolution and discrete-event aspects of our research.

In many other situations in power electronics, it is not the local average but the *component at the switching frequency* (or some other frequency) that is of primary interest. This is the case with so-called resonant converters, for instance. Also, even with PWM converters, we often wish to refine the predictions of an averaged model by computing the "ripple', which is the switching-frequency component. This motivates the definition of the local ω-component as the local average of $x(t)e^{-j\omega t}$. If $x(t)$ is periodic, this is just the Fourier series coefficient at the frequency ω. With this definition, we can again obtain

dynamic models for the local ω-component from the instantaneous dynamic models. The value of this approach for resonant converters is demonstrated in [28].

III. Systems Subject to Discrete Events

In this section we describe that portion of our research dealing with the qualitative analysis of systems subject to discrete events and in particular with the development of feedback control concepts and a servomechanism theory for such systems. Problems of this type arise in contexts that are completely described at a discrete level (such as a military C^3 system) as well as in systems that involve the interaction of continuously-evolving dynamics with event-driven elements (as in the integrated intelligent control of an advanced fighter).

Our work in this area, described in [30,31,43,44,45,46,71] has focused on developing a number of control concepts for a particular class of discrete-event dynamic systems (DEDS). Specifically, the motivation for our work on DEDS came from a desire (a) to investigate the concepts of reliability, resiliency, and error recovery in DEDS; and (b) to develop counterparts in the DEDS framework to standard control system concerns such as stabilization and tracking. In order to begin to understand these issues we chose as our initial focus of study the class of systems introduced in the initial investigations of control for DEDS. Specifically we focused attention on control systems modeled as nondeterministic finite-state automata with intermittent event observations. These models are defined over the quintuple

$$G = \left(X, \Sigma, \Phi, \Gamma, \Xi\right)$$

where X is the finite set of states, with $n=|X|$, Σ is the finite set of possible events, $\Phi \subset \Sigma$ is the set of controllable events, $\Gamma \subset \Sigma$ is the set of observable events, and $\Xi \subset \Sigma$ is the set of tracking events. Also, let $U = 2^\Sigma$ denote the set of admissible control inputs consisting of a specified collection of subsets of Σ. The dynamics defined on G are as follows, where $\overline{\Phi}$ denotes the complement of Φ:

$$x[k+1] \in f(x[k], \sigma[k+1])$$

$$\sigma[k+1] \in \left(d(x[k]) \cap u[k]\right) \cup \left(d(x[k]) \cap \overline{\Phi}\right)$$

Here, $x[k] \in X$ is the state after the kth event, $\sigma[k] = \Sigma$ is the $(k+1)$st event, and u[k] Œ U is the control input after the kth event. The function $d : X \rightarrow 2^\Sigma$ is a set-valued function that specifies the set of possible events defined at each state (so that, in general, not all

events are possible from each state), and the function $F : X \times \Sigma \rightarrow X$ is also set-valued, so that the state following a particular event is not necessarily known with certainty.

Our model of the output process is quite simple: whenever an event in G occurs, we observe it; otherwise, we see nothing. Specifically, we define the output function $h : \Sigma \rightarrow \Gamma \cup \{\varepsilon\}$, where ε is the "null transition" by

$$h(\sigma) \begin{cases} \sigma & \text{if } \sigma \in \Gamma \\ \varepsilon & \text{otherwise} \end{cases}$$

Then, our output equation is

$$\gamma[k + 1] = h(\sigma[k + 1])$$

The set Ξ, which we term the tracking alphabet, represents events of interest for tracking purposes. This formulation allows us to define tracking over a selected alphabet so that we do not worry about listing intermediary events that are not important in tracking. We use $t : \Sigma^* \rightarrow \Xi^*$, to denote the projection of strings over σ into Ξ^*.

In our work to date on this class of models we have accomplished the following:

(1) *The development of a theory of stability and stabilization by state feedback.*
     This represents our first attempt to capture the concept of error recovery for
     DEDS. Specifically we assume that we are given a set of states $E \subset X$
     which can be thought as "good" states from which the DEDS can begin to
     perform desired operations. For example in a manufacturing system E
     might consist of start-up states for the production of different parts.
     Because of either normal operation (e.g. part production) or errors (e.g.
     during the production cycle) the system makes excursions out of E. It is
     then of interest to make sure that the system returns to E so that desired
     operation can continue (if the excursion is simply due to the normal
     production cycle) or be recovered (if an error occurred). We have
     developed tests for what we term E-stability and E-stabilizability via state
     feedback and algorithms for constructing stabilizing state feedback laws.
     An important point to note is that these concepts seem to be of central
     importance in DEDS control. In particular they are necessary for the
     analysis in all of the problems we have considered.

(2) *The development of a theory of observability and observer design for
     DEDS.* The model of DEDS observation we have described includes what

we feel is an essential feature. Specifically in complex systems the sensed information often concerns events rather than states, and furthermore these observations are often received sporadically and asynchronously, as only a subset of the full range of events is observed. While our model also is rich enough to capture regular event (and state) information, this more general notion includes an extremely important characteristic: between observable events DEDS evolution is not visible, and thus even if we know the state at some point, the possible occurrence of unobservable events leads to uncertainty in state knowledge at least until the occurrence of the next observable transition. In our work we have analyzed the natural DEDS observer for such a system in which the state of the observer is the set of states in which the system might have been immediately following the last observable transition. In this case observability—defined as stability of the observer, in the sense of (1), with respect to the singleton sets—corresponds to being able to determine the system state at intermittent points in time. In addition, we have shown that in this case the observer is robust to errors—i.e. the observer will recover correct operation after a burst of observation errors corresponding to false alarms (i.e. indication of observable events when none occurred), missed detections (i.e. no observation indicated when an observable event has occurred) or incorrect identifications (an observable event being incorrectly identified as a different event).

(3) *The synthesis of (1) and (2) to develop a theory of output stabilization.* There are two major differences between this theory and that for standard control problems. First, because of the sporadic nature of observations, observability and stabilizability do *not* guarantee stabilizability by output feedback (this highlights the problem of synchronization of state knowledge and control action). Secondly, there are two notions of output stabilization. In the weaker of these we design a compensator that is guaranteed to drive the system through E at intermittent points in time, but we may never know exactly when the system is in E. In the stronger notion, we design a compensator that guarantees that at intermittent points in time the system is in E and we know it. The latter problem can be directly formulated as a *state* feedback stabilization problem for the

observer. Again we also have developed notions of error recovery in this context as well.

(4) *The reconstruction of complete event history from the observed events.* This is important in trouble-shooting in complex systems and also in complex estimation and inference problems in which many information sources are being fused. It also provides us with a first setting in which to examine nonresilient behavior analogous to the concept of catastrophic error propagation for convolutional codes. In particular in addition to developing a theory for reconstructing event sequences we have also developed methods for determining if reconstruction can be made robust to error bursts in the sense that correct reconstruction is recovered a bounded number of steps after an error burst.

(5) *Tracking and restrictability of DEDS.* This represents our first examination of command-following for DEDS. We have developed methods for tracking specific sets of strings (by state or output feedback), for restricting event behavior to specified sets of strings, and for identifying sets of starting states for tracking and restriction. This latter specification then provides a constructive method for finding the set E with respect to which we wish to consider stability. Our notion of restrictability is a slight generalization of Wonham and Ramadge's notion of a controllable language. However, in our investigation we examine two other significant extensions which seem to be natural in a control context. The first of these is *eventual restrictability*—i.e. the output tracks the commanded string set *after an initial transient* (much as in the tracking of a step, ramp, or other command input in a classical servo loop). The second is *reliable restrictability*—i.e. the ability to resume correct event sequence restriction after a transient recovery period following a burst of errors or failure events.

(6) *Task following, aggregation, and higher level modeling.* Using the results of (5) we have developed a theory for controlling DEDS so that one of a specified set of *tasks* is performed, where a task is specified as the completion of one of a set of tracking event strings. Doing this in a compatible way requires the development of notions of task independence,

reachability, and observability (so that we know when a task has been completed). This leads directly to the task-level modeling of a DEDS in which "words"—i.e. event sequences—at the lower level are aggregated into single "letters"- representing tasks—at the higher level. This has the potential to reduce model complexity considerably and thus to enhance our ability to consider complex and higher-level control questions.

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