RL-TR-91-279
Final Technical Report
November 1991

THE SURFACE IMPEDANCE SYNTHESIS TECHNIQUE

University of Dayton

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	November 1991	Final Jul 88 - Jul 89

4. TITLE AND SUBTITLE
THE SURFACE IMPEDANCE SYNTHESIS TECHNIQUE

5. FUNDING NUMBERS
C - F30602-88-D-0027,
Task N-8-5310
PE - 63003F
PR - 7402
TA - 01
WU - P1

6. AUTHOR(S)
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8. PERFORMING ORGANIZATION REPORT NUMBER
UDR-TR-89-119

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Rome Laboratory (ERPT)
Griffiss AFB NY 13441-5700

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
RL-TR-91-279

11. SUPPLEMENTARY NOTES
Rome Laboratory Project Engineer: Kenneth R. Siarkiewicz/ERPT/(315) 330-7642

Prepared for Syracuse University

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

A synthesis technique is discussed in the solution of inverse electromagnetic scattering problems. The technique generates possible surface profiles for a scatterer from a desired scattering pattern. Then, the surface impedance distribution on a particular surface profile can be determined to produce the desired scattering pattern. Limitations on the specifications of the scattered field are also discussed.

14. SUBJECT TERMS
EM Scattering, Electromagnetic, Surface Impedance

15. NUMBER OF PAGES
44

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-18
298-102

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SECTION 1

INTRODUCTION

Although the number of investigations into the inverse electromagnetic scattering problem has been limited in comparison with the amount of total research completed in diffraction theory, a synthesis method [1, 2] was developed in an extension of the research to reduce radiation losses from surface waves during propagation around bends of the surface [3, 4]. The synthesis method determines the impedance boundary conditions for a physical structure produced from the specification of a desired scattering pattern [5, 6] and has been applied successfully to solving the inverse (i.e. synthesis) electromagnetic scattering problem [7, 8, 9, 10]. Further research provided constraints on the specifications of the scattered field to insure solvability of the inverse diffraction problem [11]. This present study will investigate the possibility of additional constraints on the physical realizability of an scatterer subjected to an incident field to produce a desired scattering pattern.

In the following section a complete derivation of the synthesis method for determining the surface impedance distribution on an scatterer resulting from a required scattering pattern occurs in order to validate its application to inverse electromagnetic scattering problems. This surface impedance

synthesis technique includes the determination of a family of solutions for a representative profile of the scatterer for a desired scattering pattern as well as the calculation of the surface impedance for a particular profile. The energy balance relationship is shown to be fundamental not only to the technique itself but also to restrictions on the technique in solving inverse diffraction problems. Because the mathematical formulation involves approximating a desired shape with fictitious sources that directly influence the scattered field, both single and multiple source configurations will be developed. The representation of the scattered field requires an appropriate selection of the amplitude factor as well as the order of the Hankel function of the second kind so that the azimuthal power flow dominates the radial power flow but does not become too reactive.

In order to implement the technique easily and to allow flexibility in its application, the calculation of the power flow lines for determining possible surface profiles of the scatterer and for displaying various closed contours is essential. The example in the last section shows the tremendous flexibility in the development of an scatterer and in the possible application of other constraints such as material properties.

SECTION 2

SURFACE IMPEDANCE SYNTHESIS TECHNIQUE

The surface impedance synthesis technique essentially involves two major steps. First, with the specification of a desired scattering pattern, the power flow lines are generated for a general scatterer characterized by required shape features as width and length. If such scatterers exist to produce the desired scattering pattern, a family of closed power contours will result and represent possible surface profiles of the scatterer. Next, the surface impedance distribution is determined for a particular surface profile. Although an interdependence exists between the impedance boundary conditions and the surface profile, the technique allows for flexibility in potential designs in both surface impedance and structural variations.

For a generalized, two-dimensional case that is homogenous along the z axis, i.e. the partial derivative with respect to z is zero, let a plane wave, incident upon an arbitrarily shaped scatterer containing no energy sources, be represented as $u^i(\rho,\phi) = e^{-jk\rho\cos\phi}$ where $u^i(\rho,\phi)$ can be either the electric or magnetic field. The total electromagnetic field consists of the sum of the incident and scattered fields taking the form

$$u(\rho,\phi) = u^i(\rho,\phi) + u^s(\rho,\phi) \tag{1}$$

The power flow lines of the Poynting vector for the total electromagnetic field will be coincident with the power flow

lines of the Poynting vector for the incident field at large distances from the scatterer. However, at distances approaching the surface of the scatterer, the structure of the power flow lines becomes more complex. Surfaces passing through the power flow lines do not exhibit a flux of active energy and, therefore, have only a reactive impedance. All source-free profiles having a purely reactive surface impedance will satisfy the following relation for the conservation of power or Poynting Theorem:

$$\mathrm{Re} \oint \mathbf{S} \cdot d\mathbf{s} = 0 \qquad (2)$$

where the complex Poynting vector $\mathbf{S}$ equals $1/2\ \mathbf{E} \times \mathbf{H}^*$ and $d\mathbf{s}$ is the outward unit normal of the surface. However, the surface profile must also enclose all singularities of the scattered field to be physically realizable. Satisfying the condition for conservation of power does not alone ensure physical realizability of the closure condition.

Further investigation into the specification of the scattered field provides an additional constraint which will allow the conservation of power to be the sufficient condition required to ensure closure of the power flow lines. Although the scattered field in both the direct and inverse diffraction problems in two-dimensional cases can be expressed as a series of outgoing cylindrical traveling waves, the scattered field in the latter can be represented by a finite number of terms whose complex coefficients must be determined in order to produce the specified scattering pattern. The outgoing cylindrical traveling waves are represented by the Hankel function of the second kind

and order υ for a $e^{j\omega t}$ time dependency. These waves are attenuated by the factor $\rho^{-\frac{1}{2}}$ as the real argument increases to infinity and, thus, satisfy the radiation condition.

$$H_\upsilon^{(2)}(k\rho) \approx [2/(\pi k\rho)]^{\frac{1}{2}} e^{-jk\rho} j^{\upsilon+\frac{1}{2}} \text{ as } k\rho \to \infty \tag{3}$$

Let the scattered field be expressed in a standing wave representation with respect to the angle ϕ as a single term as

$$u^s(\rho,\phi) = (A_\upsilon \cos\upsilon\phi + B_\upsilon \sin\upsilon\phi) H_\upsilon^{(2)}(k\rho) e^{j\beta} \tag{4}$$

where A_υ and B_υ are complex amplitudes and β is the initial phase. Because the scattered field has a dependence in angle ϕ which must be periodic and single valued, the order υ must be an integer. If the relative phase between the complex amplitudes is not an integer multiple of 2π, the conservation of power relation becomes a sufficient condition for the closure of the power flow lines. Furthermore, if the complex amplitudes have equal magnitudes and a relative phase difference of $(2n+1)\ \pi/2$, the scattered field can then be expressed as

$$u^s(\rho,\phi) = C_\upsilon e^{j\upsilon\phi} H_\upsilon^{(2)}(k\rho) e^{j\beta} \tag{5}$$

which reduces in the far field to the form

$$u^s(\rho,\phi) = C_\upsilon e^{j\upsilon\phi} e^{j\beta} [2/(\pi k\rho)]^{\frac{1}{2}} e^{-jk\rho} j^{\upsilon+\frac{1}{2}} \tag{6}$$

This traveling wave representation with respect to the angle ϕ produces a family of closed contours about the singularities of the scattered field in the near field; whereas, the standing wave representation produces both closed and open power flow lines.

If, for instance, a physical surface coincided with any of the family of the closed power flow lines, the power is predominately azimuthal.

In applying the surface impedance synthesis technique to inverse diffraction problems, the scattered field must also satisfy the structure of the desired scattering pattern in the far field. From this requirement the scattered field may be represented as

$$u^s(\rho,\phi) = A\mathbf{F}(\phi)e^{j\beta}\,[2/(\pi k\rho)]^{\frac{1}{2}}\;\; e^{-jk\rho}\; j^{+\frac{1}{2}} \tag{7}$$

where A is an amplitude factor, $\mathbf{F}(\phi)$ is the normalized complex scattering pattern, and β is the initial phase. This normalized complex scattering pattern is further expanded into amplitude and phase components as

$$\mathbf{F}(\phi) = F(\phi)e^{j[\psi(\phi)-\psi(0)]} \tag{8}$$

where $F(\phi)$ is the normalized amplitude scattering pattern and $[\psi(\phi) - \psi(0)]$ is the normalized phase scattering pattern.

If the scatterer is represented as a single fictitious source, the complex coefficient C_υ is easily calculated by comparing equations (6) and (7). However, if a configuration using multiple fictitious sources represents the scatterer, the scattered field must be expressed in the form

$$u^s(\rho,\phi) = \sum_{n=1}^{N} C_{\upsilon_n} e^{j\upsilon_n\phi_n}\;\; H_{\upsilon_n}^{(2)}(k\rho_n)\;\; e^{j\beta} \tag{9}$$

where N is the number of sources. For an increased number or

nonlinear orientation of the sources, the mathematical formulation becomes more complicated. With the implementation of the Gram-Schmidt process (Appendix) [12], the complex coefficients are determined for these cases.

After a closed power flow contour has been chosen as a possible surface profile for the scatterer, the surface impedance distribution is determined by calculating the ratio of the tangential components of the electric and magnetic fields.

2.1 AMPLITUDE FACTOR LIMITATIONS

The amplitude factor A cannot be chosen arbitrarily and must be selected judicously in order to provide a realizable solution to the inverse diffraction problem. The constraints on the amplitude factor are determined by using the Poynting Theorem modified to allow for the possibility that the scatterer also has absorption characteristics. Thus,

$$\text{Re} \oint \mathbf{S} \cdot d\mathbf{s} + P^A = 0 \tag{10}$$

where P^A is the total power absorbed. The complex power includes the total power scattered P^S, the power P^{IS} due to the interaction between the incident and scattered fields, and the intrinsic power P^I of the incident field, so that

$$\text{Re} \oint \mathbf{S} \cdot d\mathbf{s} = P^S + P^{IS} + P^I \quad . \tag{11}$$

It can be shown that the real part of the complex Poynting vector may be expressed in terms of the total field $u(\rho,\phi)$ in the form

$$\text{Re } \mathbf{S} = -q \, \text{Im}[u^*(\rho,\phi) \nabla u(\rho,\phi)] \tag{12}$$

where the constant q accounts for the type of electromagnetic wave and the properties of the medium.

$$q = \begin{cases} \dfrac{1}{2\omega\epsilon} & \text{for TE waves} \\ \dfrac{1}{2\omega\mu} & \text{for TM waves} \end{cases} \tag{13}$$

Substitution for the complex Poynting vector in terms of the total field into the Poynting Theorem yields

$$-\, q\ \mathrm{Im} \int_0^{2\pi} u^*(\rho,\phi)\ \frac{\partial u(\rho,\phi)}{\partial \rho}\ \rho\ d\phi + P^A = 0 \tag{14}$$

Assuming that no sources are enclosed by the surface of integration, expanding the Poynting Theorem expression with the components of the total field, and evaluating each resulting term in the far field, simplifies equation (10) to the form

$$P^S + P^{IS} + P^A = 0 \tag{15}$$

where $$P^S = \frac{2qA^2}{\pi} \int_0^{2\pi} F^2(\phi)\, d\phi \tag{16}$$

and $$P^{IS} = q\ \mathrm{Im} \left\{ 4j u^s(\rho,0) [2/(\pi k\rho)]^{-\frac{1}{2}}\ e^{jk\rho}\ j^{-\frac{1}{2}} \right\} \tag{17}$$

The optical theorem is then obvious by the rearrangement of terms and the division of the total powers by the magnitude of the incident field's Poynting vector,

$$\sigma_s + \sigma_A = -\frac{1}{k} \operatorname{Im}\left\{4ju^s(\rho,0)[2/(\pi k\rho)]^{-\frac{1}{2}} e^{jk\rho} j^{-\frac{1}{2}}\right\} \qquad (18)$$

where σ_s and σ_A represent the total scattering cross section and total absorption cross section, respectively.

Evaluation of the scattered field produced by the scatterer due to a field incident upon it from the π direction relative to a body-centered coordinate system yields the following expression for the forward scattering direction in the far field

$$u^s(\rho,0) = A\mathbf{F}(0)e^{j\beta}[2/(\pi k\rho)]^{+\frac{1}{2}} e^{-jk\rho} j^{+\frac{1}{2}} \qquad (19)$$

For the forward scattering direction ϕ=0, the normalized complex scattering pattern reduces to the normalized amplitude scattering pattern, $\mathbf{F}(0) = F(0)$. Thus, the scattered field becomes

$$u^s(\rho,0) = AF(0)e^{j\beta}\,[2/(\pi k\rho)]^{+\frac{1}{2}} e^{-jk\rho} j^{+\frac{1}{2}} \qquad (20)$$

Substituting into the Poynting Theorem for the total scattered power and the total interaction power produces a quadratic equation in A. Solving for the amplitude factor results in the expression

$$A = \frac{\pi\left[-F(0)\cos\beta \pm \sqrt{F^2(0)\cos^2\beta - \frac{P^A}{2\pi q}\int_0^{2\pi} F^2(\phi)\,d\phi}\right]}{\int_0^{2\pi} F^2(\phi)\,d\phi} \qquad (21)$$

By applying the Poynting Theorem with the inclusion of

absorption to the solution of inverse diffraction problems, constraints have been developed for the specification of the amplitude factor for a desired scattering pattern. The amplitude factor cannot be specified arbitrarily but is dependent upon the normalized amplitude scattering pattern in the forward scattering direction F(0), the total power absorbed P^A, and the initial phase β. However, the amplitude factor is not dependent upon the phase of the desired scattering pattern.

In order to understand more fully the constraints on the amplitude factor, the following discussion developes the criteria required for the appropriate selection of the amplitude factor. The criteria applies to all inverse diffraction problems in that it is independent upon the desired scattering pattern specified. First, the bounds on the amplitude factor will be determined. Then, the effects of the total power scattered and total power absorbed as well as the initial phase will be explained.

From the expression for the amplitude factor, the minimum and maximum constraints that bound its values are easily observed to occur whenever there is no total power absorbed. Then, the minimum value of the amplitude factor is determined by taking the positive sign preceding the radical. However, this value obviously becomes zero which represents the trivial case where no scatterer even exists. Similarly, the maximum value of the amplitude factor is determined by taking the negative sign preceding the radical. Thus, for no total power absorbed the amplitude factor has the form

$$A = \frac{-2\pi F(0)\cos\beta}{\int_0^{2\pi} F^2(\phi)\,d\phi} \quad . \tag{22}$$

Although the absolute maximum value is strongly dependent upon the desired scattering pattern characteristics, the relative maximum value applicable to all inverse diffraction problems is only dependent upon the initial phase. Because the amplitude factor must be positive, the initial phase must be within the range of $\pi/2$ to $3\pi/2$ such that its cosine is negative. Therefore, the relative maximum value of the amplitude factor occurs whenever the initial phase equals π.

$$A_{max} = \frac{2\pi F(0)}{\int_0^{2\pi} F^2(\phi)\,d\phi} \quad . \tag{23}$$

Observation of the general expression for the amplitude factor also shows that the maximum value allowable for the total power absorbed is determined by the radicand in equation (21). So,

$$P_{max}^{A} = \frac{2\pi q\ F^2(0)}{\int_0^{2\pi} F^2(\phi)\,d\phi} \quad . \tag{24}$$

The amplitude factor under the conditions of maximum total power absorbed and initial phase of π then has the form

$$A_{P^A_{max}} = \frac{\pi F(0)}{\int_0^{2\pi} F^2(\phi)\,d\phi} = \frac{A_{max}}{2} \tag{25}$$

Now that some fundamental terms and constraints have been established, the general expression for the amplitude factor may be manipulated to yield

$$A = \frac{\pi F(0)}{\int_0^{2\pi} F^2(\phi)\,d\phi} \left[-\cos\beta \pm \sqrt{\cos^2\beta - \frac{P^A}{2\pi q F^2(0)} \int_0^{2\pi} F^2(\phi)\,d\phi} \right] \tag{26}$$

where several quantities can be easily recognized. Using equations (24) and (25), the amplitude factor then takes the form

$$A = A_{P^A_{max}} \left[-\cos\beta \pm \sqrt{\cos^2\beta - P^A/P^A_{max}} \right] \tag{27}$$

In order to continue to investigate the selection criteria for the amplitude factor in the generalized sense, two additional factors must be introduced. First, because the total power absorbed is bounded, it may be represented as

$$P^A = \gamma\ P^A_{max} \qquad \text{for } 0 \le \gamma \le 1 \tag{28}$$

Second, the total power absorbed to the total power scattered can be represented as

$$P^A = \alpha\ P^S \tag{29}$$

which results in the following upon substitution for the power quantities

$$\alpha = \gamma \left[\frac{A_{P^A max}}{A} \right]^2 \quad . \tag{30}$$

However, the ratio of the terms related to the amplitude factor is dependent upon the ratio of the total power absorbed to the maximum total power absorbed.

$$\alpha = \frac{\gamma}{\left[-\cos\beta \pm \sqrt{\cos^2\beta - \gamma} \right]^2} \quad . \tag{31}$$

Likewise, this expression may be rearranged to solve for the ratio of the total power absorbed to the maximum total power absorbed in terms of the ratio of the total power absorbed to the total power scattered as well as the initial phase.

$$\gamma = \frac{4\alpha \cos^2\beta}{(1+\alpha)^2} \quad . \tag{32}$$

With the establishment of these ratios, the selection criteria for the amplitude factor follows a straight forward, yet flexible procedure. The interdependence of the ratio of the total power absorbed to the maximum total power absorbed, the ratio of the total power absorbed to the total power scattered, and the initial phase is shown in the three-dimensional plot in figure 1. The maximum total power absorbed is achieved whenever the initial phase is π and the total power absorbed equals the

total power scattered. Furthermore, as the total power absorbed is reduced from its maximum, the ratio of the total power absorbed to the total power scattered can become large for any valid initial phase. Basically figure 2 is a contour plot of figure 1 for levels chosen arbitrarily as 0.2, 0.4, 0.6, and 0.8 for the ratio of the total power absorbed to the maximum total power absorbed. The sections of the contours that are not solid represent values that were calculated with the negative sign preceding the radical. As an example, if the desired ratio of the total power absorbed to the total power scattered is 3.0, as the ratio of the total power absorbed to the maximum total power absorbed becomes smaller, the initial phase can assume a greater range of values. However, in this case the 0.8 ratio of the total power absorbed to the maximum total power absorbed would not achieve the desired ratio of 3.0 regardless of the initial phase. Furthermore, if the ratio of the total power absorbed to the maximum total power absorbed is limited to a maximum value, the initial phase would differ from π by a greater amount. Likewise, if the properties of the surface limit the initial phase that could be imposed onto the surface, the ratio of the total power absorbed to the maximum total power absorbed is also bounded.

After the three terms - the ratio of the total power absorbed to the total power scattered, the ratio of the total power absorbed to the maximum total power absorbed, and the initial phase - have been specifically determined or at least bounded, the amplitude factor can easily be determined. In

figure 3, the amplitude factor normalized by the amplitude factor at which the total power absorbed is a maximum is shown as a function of the initial phase for arbitrarily selected values of the ratio of the total power absorbed to the maximum total power absorbed. If a range of values for the ratio of the total power absorbed to the maximum total power absorbed is given for a specific initial phase, the possible values for the amplitude factor will be bounded. Likewise, if a range of values for the initial phase is given for a specific ratio of the total power absorbed to the maximum total power absorbed, the amplitude factor will again be bounded. Therefore, the amplitude factor can not be arbitrarily specified. In summary, even though the amplitude factor is directly dependent upon the normalized amplitude of the scattering pattern in the forward scattering direction, the total power absorbed, and the initial phase, the selection criteria for the amplitude factor is more easily understood whenever the power ratio terms are introduced and is then applicable to all inverse diffraction problems.

2.2 HANKEL FUNCTION ORDER LIMITATIONS

The surface impedance synthesis technique requires that the order of the Hankel function, used in representing the behavior of the scattered field, must be an integer to insure the solution to be single valued for its ϕ dependence. Otherwise, the order can be arbitrarily selected. However, any physical limitations, e.g. the desired size of the scatterer, must also be considered. These limitations would inherently be different for each problem.

In order for the desired scattering pattern to result from a plane wave incident upon an scatterer generated by the surface impedance synthesis technique, the radiation loss due to the implementation of the surface impedance onto the surface profile must be minimized. The ratio of the radial component to the azimuthal component of the power flux density can be expressed in the form

$$\frac{\text{Re } S_r}{\text{Re } S_\phi} = \frac{2}{\pi \upsilon [J_\upsilon^2(k\rho) + N_\upsilon^2(k\rho)]} \tag{33}$$

where $J_\upsilon(k\rho)$ and $N_\upsilon(k\rho)$ are the Bessel and Neumann functions, respectively, which form the Hankel function $H_\upsilon^{(2)}(k\rho)$. Because of the inversely proportional relationship, as the magnitude and order of the Hankel function increases, the ratio decreases. Moreover, as the argument of the Hankel function decreases relative to its order, i.e. $k\rho < \upsilon$, the absolute value of the Neumann function increases greatly. A similar but weaker constraint on the argument restricts its maximum value to the first, positive zero of the Neumann function, i.e. $k\rho < y_{\upsilon,1}$. Therefore, the appropriate selection of the order determines the maximum radius for which the azimuthal component dominates the radial component making the radiation loss insignificant. As the order of the Hankel function increases, the radius of the region where the power flow lines are closed also increases. Although the availability of a greater number of possible solutions would

then seem to exist for larger orders, the energy stored near the surface of the scatterer increases as the radius decreases. Therefore, the use of higher orders of the Hankel function is primarily to accommodate the requirement for a greater geometrical cross section of the scatterer.

SECTION 3

EXAMPLE

As a simplified example, assume that the surface profile of an scatterer and its associated surface impedance distribution must be determined for the desired scattering pattern characterized by the far-field, normalized amplitude and phase patterns for a right circular cylinder having a 0.2 wavelength radius and its longitudinal axis along the z direction. The scatterer is assumed to have no absorption characteristics and an initial phase of π. The analytical procedure follows two distinct steps. First, the family of closed power flow lines has to be generated to represent the possible solutions for the surface profile. Second, the surface impedance distribution along the selected surface profile must then be determined.

Although previous research discussed a method to produce possible surface profiles first by generating equal phase surfaces and then by generating surfaces perpendicular to them to produce the power flow lines, this method has not been used because it seemed too cumbersome in its approach and allowed for the possible introduction of error by the additional set of calculations. A more straight forward approach was adopted whereby the real part of the complex Poynting vector was calculated for each point of a grid whose center represented the origin of the body-centered coordinate system for the scatterer. From the x-y grid with the power values, both two-dimensional and three-dimensional representations of the power flow lines can be

generated.

Without any physical limitations required in the example, the scatterer would be initially represented by a single fictitious source. Likewise, the Hankel function used in representing the behavior of the scattered field was assumed to be the first order. The three-dimensional plot of the power magnitude for a square grid of 1.2 wavelengths in figure 4 shows the near symmetry of the problem. Because the power magnitude increases greatly as the radius of the scatterer approaches zero, the maximum power magnitude was limited to 800 to allow the structure at the lower power magnitude levels to be observed. Moreover, the influence of the phase of the desired scattering pattern is not readily apparent until the two-dimensional contour plot is observed in figure 5 for arbitrarily chosen power magnitude levels. Using the first positive zero of the order of the Neumann function to limit the maximum radius of the region of possible solutions, the closed power flow lines within 0.35 wavelengths represent possible surface profiles where radiation loss is insignificant. The normalized surface impedance distributions for the power magnitude levels of 100,200,300,400, and 500 are shown in figure 6. As the surface profile decreases, the power magnitude level increases, and the magnitude of the surface impedance required to produce the desired scattering pattern increases. This situation is strongly similar to the reduction of an antenna's aperture and the occurrence of superdirectivity.

Figure 1. Interdependence of the power ratios and the initial phase

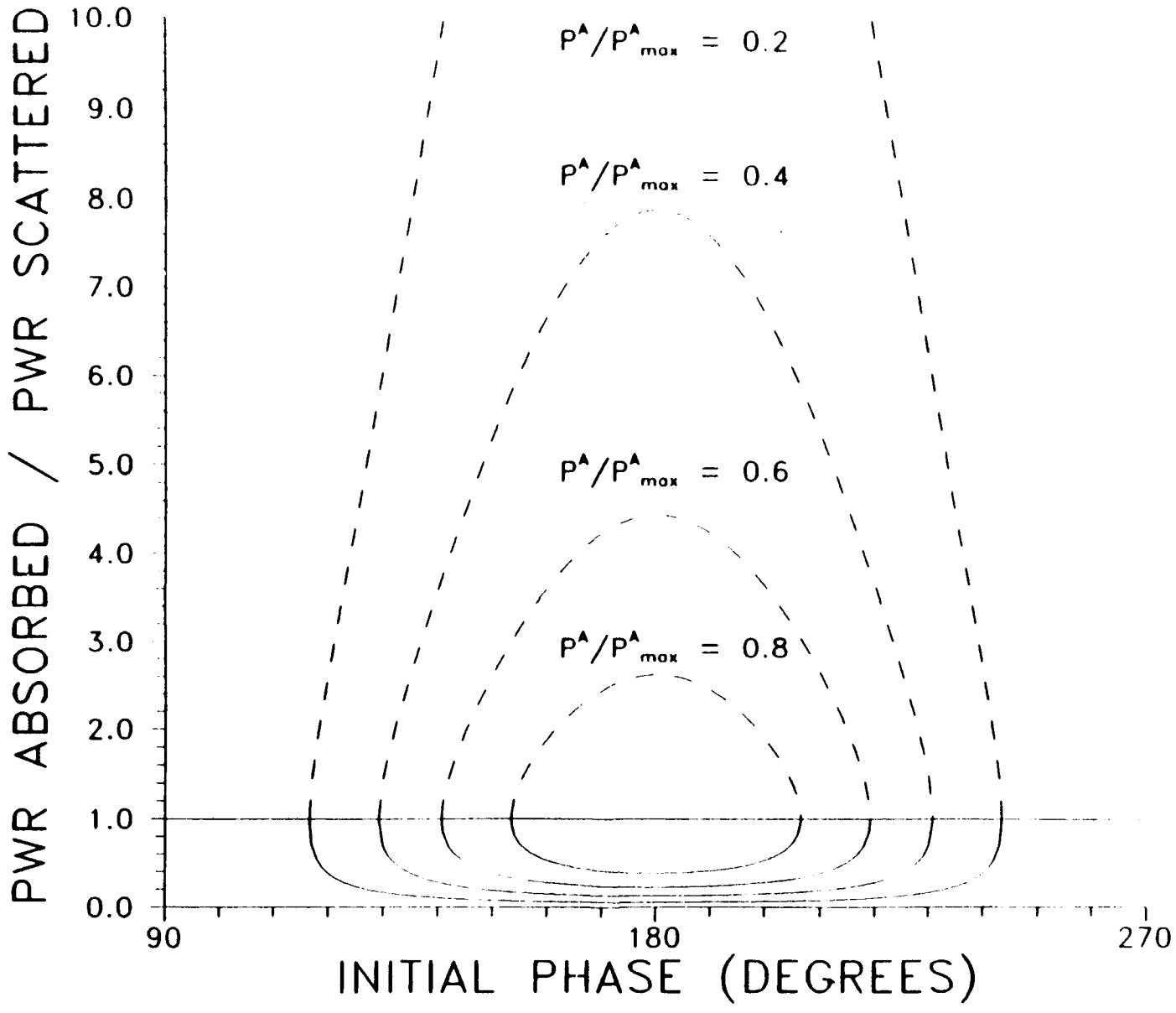

Figure 2. Interdependence of the power absorbed/power scattered ratio and the initial phase for various power absorbed/maximum power absorbed ratios

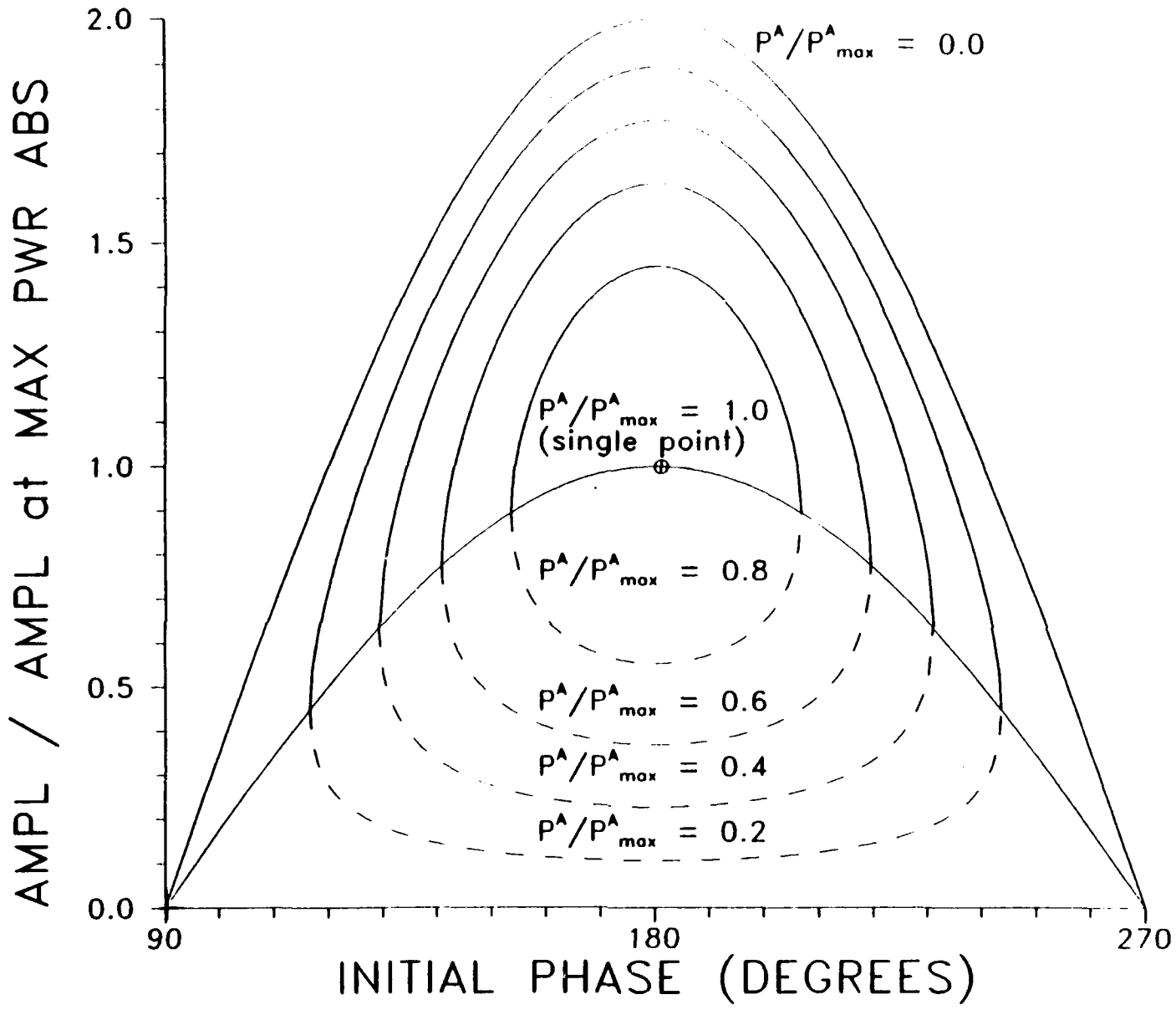

Figure 3. Interdependence of the normalized amplitude factor and the initial phase for various power absorbed/maximum power absorbed ratios

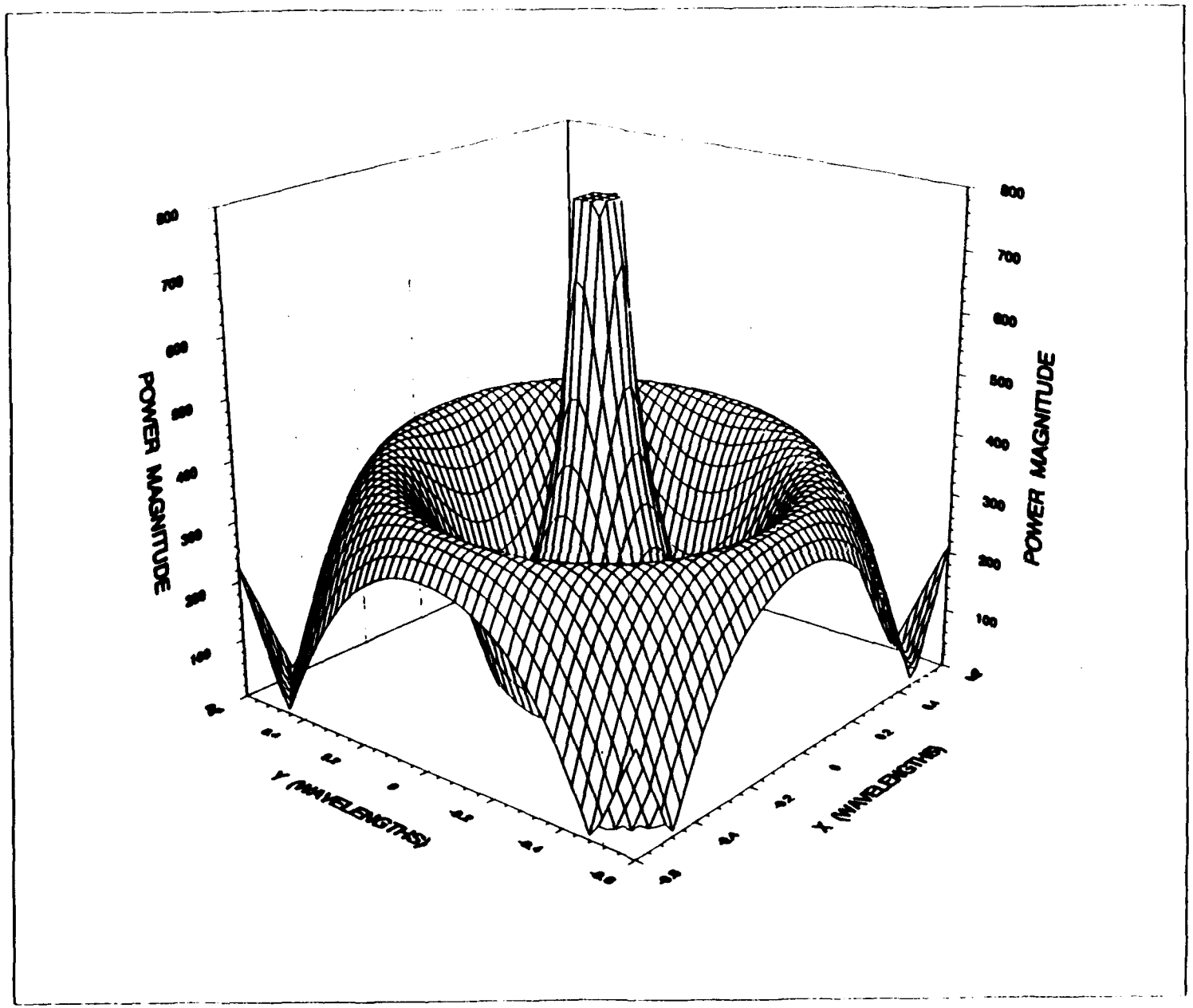

Figure 4. Power flow surface for a single source profile (ν=1) for a normalized scattering pattern from a 0.2λ radius right circular cylinder and initial phase of π

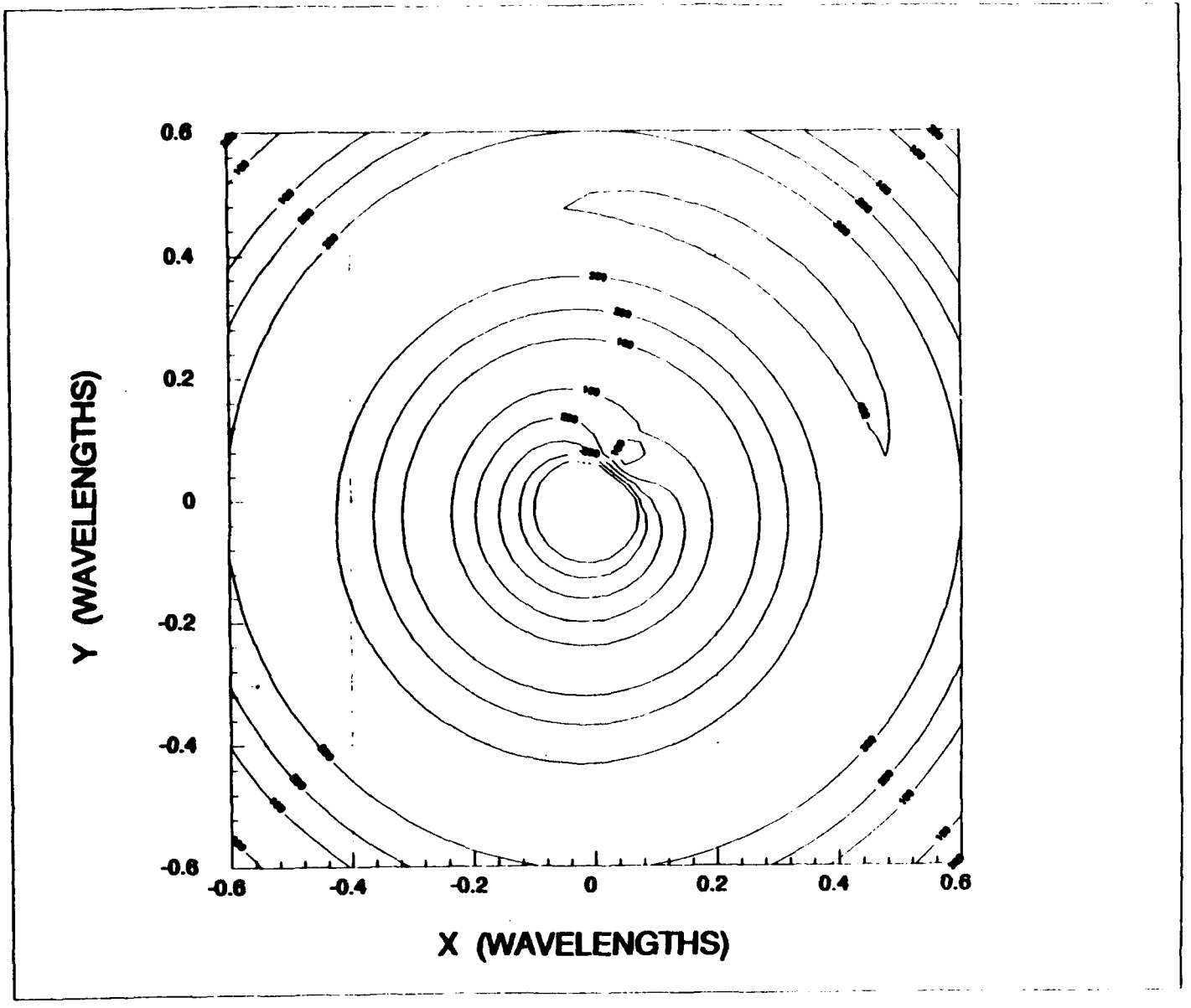

Figure 5. Power flow contours for a single source profile (ν=1) for a normalized scattering pattern from a 0.2λ radius right circular cylinder and initial phase of π

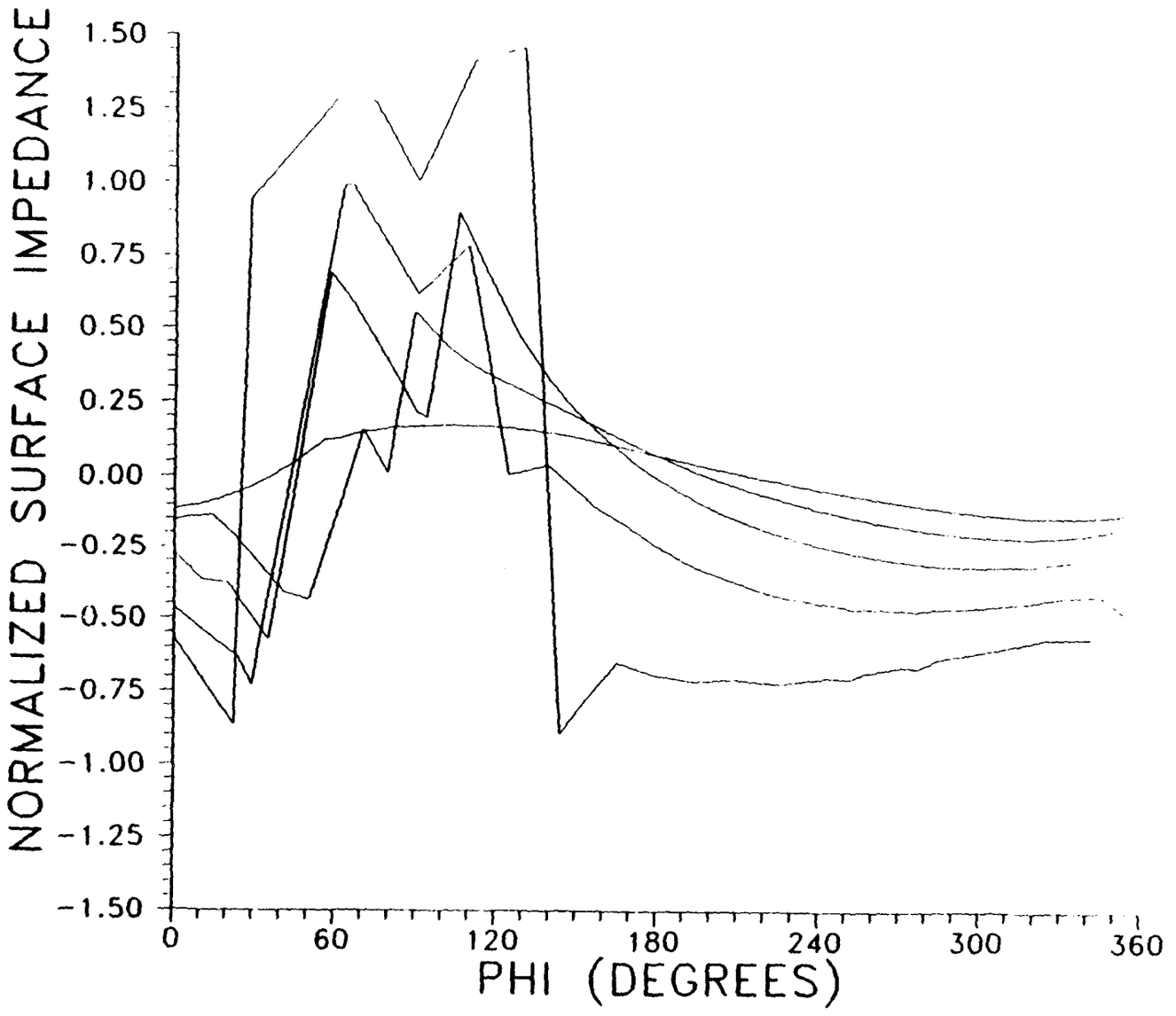

Figure 6. Normalized surface impedance distribution for power magnitude levels of 100, 200, 300, 400, and 500.

SECTION 4

CONCLUSIONS

The surface impedance synthesis technique has been successfully applied to solving inverse electromagnetic scattering problems. Equally important, the technique provides insight into limitations on possible solutions. For instance, the amplitude factor of the scattered field cannot be arbitrarily specified but is dependent upon the normalized amplitude scattering pattern in the forward scattering direction, the total power absorbed, and the initial phase. Similarly, an interdependence exists between the ratio of the total power absorbed to the maximum total power absorbed, the ratio of the total power absorbed to the total power scattered, and the initial phase.

The synthesis technique has the flexibility to be applied to simple or complex scatterers of various sizes. For any given situation, as the size of the scatterer is reduced, the magnitude of the surface impedance distribution on the surface profile increases. Hence, the analogy to the reduction of an antenna's aperture implies a superdirectivity concept for scatterers.

THE APPENDIX

The Gram-Schmidt process was used to calculate the complex coefficients used in the expansion of the scattered field represented by the original source functions. The process generates a set of orthonormal functions from the original source functions. The desired function used is the product of the amplitude factor and the normalized complex scattering pattern. Actual data or a functional form would be available for the desired function.

In general terms, it can be represented as an infinite summation of the original source functions as

$$f_d(\phi) = \sum_{n=1}^{\infty} c_n \, u_n(\phi)$$

where c_n are complex coefficients and $u_n(\phi)$ are the original source functions. The estimate of the desired functions depends upon the number of sources used to model the scattered field from the scatterer.

$$f_e(\phi) = \sum_{n=1}^{NS} C_n \, u_n(\phi)$$

where C_n are complex coefficients and NS is the number of sources. Therefore, the desired function can be approximated by its estimate in the form

$$f_d(\phi) \approx \sum_{k=1}^{NS} f_k \, U_k(\phi)$$

where $U_k(\phi)$ are the orthonormal functions . The functional notation showing the angular dependence will be omitted at this point for simplicity.

Let a set of orthogonal functions be expressed in terms of the original source functions and its associated orthonormal functions.

$$U_1' = u_1 \text{ and } U_1 = U_1'/d_1$$

$$U_2' = u_2 + a_{21}U_1 \text{ and } U_2 = U_2'/d_2$$
$$\text{with } < U_2', U_1 > = 0$$

$$U_3' = u_3 + a_{32}U_2 + a_{31}U_1 \text{ and } U_3 = U_3'/d_3$$
$$\text{with } < U_3', U_2 > = 0 \text{ and } < U_3', U_1 > = 0$$

$$U_k' = u_k + \sum_{j=1}^{k-1} a_{kj}U_j \text{ and } U_k = U_k'/d_k$$

$$\text{with } < U_k', U_{k-1} > = 0, \ \ldots \ , < U_k', U_1 > = 0$$

where the angular brackets represent the inner product such that

$$< f,g > = \int_a^b f(\phi)g^*(\phi)d\phi$$

and d_k represents the magnitude of the function U_k' such that

$$d_k = < U_k', U_k' >^{\frac{1}{2}}$$

The coefficients of the orthonormal funcitons must be determined so that the appropriate inner products between the orthogonal and orthonormal functions are zero. Using the inner product relationship again in order to calculate the a_{kj} coefficient

through the $a_{k(k-1)}$ value results in the expressions

$$< U_k', U_i > = < [u_k + \sum_{j=1}^{k-1} a_{kj} U_j], U_i >$$

with $< U_k,' U_i > = 0$ for $i = 1, \ldots, k-1$

$$0 = < u_k, U_i > + < \sum_{j=1}^{k-1} a_{kj} U_j, U_i >$$

$$0 = < u_k, U_i > + \sum_{j=1}^{k-1} a_{kj} < U_j, U_i >$$

but $< U_j, U_i > = \delta_{ji}$

because of the properties of orthonormality so that

$$\sum_{j=1}^{k-1} a_{kj} \delta_{ji} = - < u_k, U_i >$$

$$a_{ki} = - < u_k, U_i >$$

Therefore, the set of orthogonal functions can be generated from

$$U_k' = u_k + \sum_{j=1}^{k-1} a_{kj} U_j$$

and the set of orthonormal functions from

$$U_k = u_k + \sum_{j=1}^{k-1} a_{kj} U_j$$

where $a_{kj} = - < u_k, U_j >$

If each orthonormal function is represented in the form

$$U_j = \sum_{\ell=1}^{j} \omega_{j\ell} u_\ell$$

then the set of orthonormal functions can be expressed in terms of the original source functions as

$$U_k = \left[u_k + \sum_{j=1}^{k-1} a_{kj} \sum_{\ell=1}^{j} b_{j\ell}\, u_\ell \right] / d_k$$

Expanding this expression and combining appropriate coefficients yields

$$U_k = \sum_{n=1}^{k} b_{kn}\, u_n$$

where $$b_{kn} = \sum_{j=n}^{k-1} \frac{a_{kj}}{d_k d_n} \cdot b_{jn}$$

Now, recall the estimate of the desired function

$$f_d \approx \sum_{k=1}^{NS} f_k\, U_k$$

Before substituting for the set of orthonormal functions, the f_k coefficients will be determined again by applying the inner product.

$$< f_d,\ U_j > = < \sum_{k=1}^{NS} f_k\, U_k,\ U_j >$$

$$< f_d,\ U_j > = \sum_{k=1}^{NS} f_k\, \delta_{kj}$$

$$f_j = < f_d,\ U_j >$$

Then, the estimate of the desired function becomes

$$f_d \approx \sum_{k=1}^{NS} f_k\, U_k$$

$$f_d \approx \sum_{k=1}^{NS} f_k \sum_{n=1}^{k} b_{kn}\ u_n$$

$$f_d \approx \sum_{k=1}^{NS} \sum_{n=k}^{NS} f_n\ b_{nk}\ u_k$$

resulting in the simplication

$$f_d \approx \sum_{n=k}^{NS} c_k\ u_k$$

where $c_k = \sum_{n=k}^{NS} f_n\ b_{nk}$

and the set of complex coefficients for

$$f_e = \sum_{n=1}^{NS} C_n\ u_n$$

easily has a one-to-one correspondence, i.e. $C_n = c_n$. Thus, all coefficients required for the expansion of the estimate of the desired function in terms of the original source functions can be determined by implementing the Gram-Schmidt process.

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