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Some Aspects of Nonneutral Plasma Physics

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This memo considers several aspects of nonneutral plasma physics including the use of cooled nonneutral plasmas as sources for high brightness electron beams, the molecular dynamic or Monte Carlo simulation of large isolated plasmas, and the adiabatic compression of toriodal plasma clouds.

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SOME ASPECTS OF NONNEUTRAL PLASMA PHYSICS

1 Introduction

The field of nonneutral plasmas has recently emerged as one of the most fascinating areas of plasma physics. In the American Physical Society Division of Plasma Physics, it has already been recognized by both a Maxwell Award and an Excellence in Plasma Physics Award. Most of the work in nonneutral plasmas involves plasmas in Penning traps, that is traps where the plasma is confined radially by a magnetic field and axially by an electrostatic potential. Two of the most interesting aspects of nonneutral plasmas are that they can exist in thermal equilibrium, and that they can be cooled. The thermal equilibrium plasmas depend on the axial confinement and cylindrical symmetry to give a rigid rotor equilibrium. The plasmas can either be cooled naturally, for instance by electron cyclotron radiation; or with human intervention, as in the case of laser cooling of ion plasmas. As these plasmas cool, both theory and experiment show that they form liquid and crystal states. The crystal states may be the only case in nature where the crystal structure is profoundly influenced by the geometric configuration of the plasma. In fact simulations show that crystals with as many as 10^4 particles (the largest number that can be economically simulated) are greatly influenced by the geometric configuration and are profoundly different from infinite crystals. Many other rf cooling schemes are also possible for both electron and ion plasmas. Although nonneutral plasmas exist in many places, we focus on two here, the pure electron plasmas at University of California at San Diego (UCSD)[1], and the laser cooled ion plasmas at the National Institute of Standards and Technology (NIST)[2] in Boulder Colorado. The former are large systems in that they contain typically 10^{10} electrons or more; the latter are small in that they contain as little as 10^4 ions or fewer, sometimes as few as one. In addition, there is another type of nonneutral plasma, a pure electron plasma in a toroidal configuration[3]. This configuration was investigated at AVCO in the late 1960's. While such a plasma cannot be at thermal equilibrium due to the fact that a poloidal rigid rotor is topologically impossible, these plasmas have confined much more charge than thermal equilibrium plasmas. Furthermore, their confinement limit has been identified, and even within this limit, the confinement time is not so short. This memo looks into various aspects of nonneutral plasmas with an emphasis on what experiments might be done at NRL and what additional theory can be developed here.

First we look into whether the cooled electron plasmas can be used as ultra high brightness electron beam sources. We find that it is theoretically possible that such a plasma can give rise to a beam with orders of magnitude higher brightness than what conventional hot cathode sources can do. However average current is limited, especially if electron cyclotron radiation is utilized as the cooling scheme. Experiments to investigate this could be undertaken at NRL at farily low cost. Second we look into ways in which large condensed plasmas can be analyzed one to two orders of magnitude more cost effectively, but by assuming symmetries in the molecular dynamic simulations. Finally we look into theoretical methods of analyzing the adiabatic compression of toroidal electron plasmas. The inductive charging scheme used at AVCO inherently involves adiabatic compression. We show how this can be modeled by calculating a succession of adiabatically linked equilibria.

2.Nonneutral Plasmas as Ultra High Brightness Electron Beam Sources:

There are now electron Penning traps at many institutions in the world. These trap a pure electron plasma in thermal equilibrium. The electrons are magnetically confined in the transverse direction. In the longitudinal direction, electrons are confined by electrostatic potentials. Let us consider maximum confining DC fields and potentials to be about 50 kG and about 50 kV. There are several simple scaling relations which describe these traps[4]. First of all, the electrons must always have a density below the Brillouin density, $\omega_{pe}^2 < \Omega_{ce}^2/2$. That is

$$n < 5 \times 10^4 B^2 \qquad (1)$$

in order for there to be radial confinement. Usually the density is below the Brillouin density. However in the smaller ion traps, the Brillouin density has been achieved[5], and preliminary experiments to achieve it in the larger electron traps are underway. If the trapped plasma is to be the source of an electron beam that propagates field free, it would be necessary to have the maximum density. However if a magnetized beam is acceptable for the application envisioned, a density below the Brilloiun density would be acceptable.

Secondly, the confining electric potential must be high enough to confine the radial self potential. Thus, if the pure electron plasma has number density n and extends out to a maximum radius r_m, the confining potential must exceed $\pi e n r_m^2$. It is interesting that the maximum charge confined is not constrained, only the charge per unit length[4]. Thus in a straight system, with maximum magnetic field and and confining potential, the maximum charge which can be confined is proportional to the length of the system. Specifically,

$$Q(coul) < 10^{-10}L(m)V(volts) \qquad (2)$$

Thus an electrostatic potential of 5×10^4 V can confine about five microcoulomb in system one meter long, which we will consider to be the maximum practical length for the electron plasma. Experimentally, the confinement time for the plasma is given roughly by[6,7]

$$t(s) \approx 3.2 \times 10^2 B^2(G)/n(cm^{-3})L^2(m) \qquad (3)$$

where L is the length of the system. The confinement times were for an electron temperature of 1 eV. A fascinating thing about this plasma is that is is in complete thermal equilibrium, and it can be cooled by cyclotron radiation. This cooled plasma could be a source for an extremely bright electron beam. The electron temperature e folding time is about

$$\tau(s) = 4 \times 10^8/B^2 \qquad (4)$$

so that in the 50 kg field, the plasma cools from about 10^6 degrees Kelvin to about 50 degrees Kelvin in a time of 1-2 seconds. (It is possible that that this time could be considerably shortened by using other cooling schemes analogous to laser cooling for the ions. These would require rf sources tuned precisely to either the transverse or longitudinal electron frequency[8]. We will not consider them here, as they are much more complicated than simple cyclotron cooling, but we note that they open the possibility faster cooling and thereby higher average currents.) Thus if such a plasma can be produced, cyclotron cooled and extracted at a rate of about once every 2 seconds, the average current would be 2.5µA. However the transverse temperature of the plasma would be certainly as low as 50 degrees Kelvin, and possibly as low as 4°K. This is an extraordinarily low transverse temperature for an electron for an electron beam. The instantaneous current depends on how the electron beam is extracted. Let us imagine that one of the end potentials is suddenly switched to a polarity that electrons are extracted. If the extraction potential were 1000 eV, the trap would be emptied in about 100 nsec, so that the peak current would be 50 Amps. If a beam of this current, velocity and transverse temperature had important applications, for instance in free electron lasers, the nonneutral plasma could be a potential source for them. For instance the the beam brightness b, an important parameter by which free electron laser beams are measured, is defined as

$$b = I/\pi^2 \varepsilon^2 \qquad (5)$$

where I is in kiloamps and ε, the beam emittance is in $cm^2\text{-}rad^2$. To evaluate the maximum density, let us take the confinement time to be equal to the cooling time of 2s. Then the maximum density is

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4×10^{11}. The cross sectional area is 1 cm^2 for the 5µC charge cloud we have specified. Thus the current density is about 50 A/cm^6 and the transverse temperature is 4°K, for a brightness of about 2x10^9 if the parallel velocity is 2×10^{10}cm/s. To underscore this parameter, contrast it with the brightness of a beam generated by a thermionic electron gun, for instance a SLAC klystron beam. There the current is about 5 A/cm^2 and the lowest possible transverse temperature would be the emitter temperature, about 0.3 eV. At the same parallel velocity, this beam has a brightness of about 2x10^5. Thus the best beam one could generate from a nonneutral plasma confined in a Penning trap has a brightness four orders of magnitude greater than the SLAC klystron beam.

The density for the beam we specified is Below the Brillouin density, so the applications for this beam would be those for which a magnetized beam would be acceptable. The beam can be manipulated toward the Brillouin limit by electrostatically compressing axially at constant B, radially expanding by reducing B at constant V, or a combination of both[9]. Hence this sort of electron beam source could be very interesting for applications where brightness is the most important requirement. Furthermore, a simple experiment would be relatively straightforward to perform at NRL, as the Penning traps are not expensive to build, and there are several superconducting magnets in the Plasma Physics Division.

3. Simulations Schemes for Large Condensed Plasmas

One of the most amazing aspects of nonneutral plasma physics is that these plasmas can be cooled while remaining in thermal equilibrium. As we have seen, pure electron plasmas in Penning traps can be cooled by cyclotron radiation cooling. However pure ion plasmas can be cooled to much lower temperatures by laser cooling[10]. The idea here is to tune the laser to a frequency just below the ionic transition frequency. Then when the atom moves toward the laser, it is Doppler shifted into resonance, absorbs a laser photon and slows up. Since it reradiates isotopically, the net effect is a slowing. However when the ion moves away from the laser, the frequency is Doppler shifted further out of resonance, and there is no interaction. In this way, a precisely tuned laser can be used to cool the plasma. For the plasmas in the NIST experiments, temperatures of *milli-Kelvin* are common. For a plasma of number density n and temperature T, there is a single parameter which characterizes the plasma, the parameter

$$\Gamma = e^2/aT; \qquad a = (3/4\pi n)^{1/3} \qquad (6)$$

This is related to the more conventional plasma parameter of number of particles in a Debye sphere by $1/n\lambda_D^3 = 4\pi\sqrt{3}\Gamma^{3/2}$.

Analyzing the properties of such a plasma analytically is an extremely difficult task which few are up to[11]. Fortunately however a series of numerical approaches have been developed which give a great amount of insight. There are two basic approaches that have been utilized in the literature. These are Monte-Carlo[12] and molecular dynamic[4] simulation schemes. The basic dilemma of these simulations is how one simulates a large system. One solution is to simulate a system with an assumed periodicity; a second is to simulate a small (say 10^3-10^4 particles) isolated system. The initial work in this area seems to be Brush, Salin and Teller, who worked out a Monte-Carlo simulation scheme for an infinite periodic system.

In Ref 12, an infinite periodic system with from 32-500 particles in a cell was simulated. The assumed symmetry was cubic. The Monte Carlo simulation worked in the following way. The atoms were placed randomly in the unit cell with a uniform density of the opposite charge of such value as to provide overall neutrality. Then

the electrostatic energy of the configuration was calculated, including the charges in all other cells of the lattice. Since the summation over cells converges slowly, an Ewald summation scheme was used to speed convergence. Then one particle was moved and the electrostatic energy recalculated. If the energy was reduced, the move was accepted. If the energy was increased by an amount ΔE, the moved was accepted with with probability exp-ΔE/T. The simulation was run until a steady state was reached. From the state of the plasma, the multi-electron distribution function was calculated, and from this, the partition function as a function of temperature was derived. This then allowed the thermodynamic properties to be calculated.

Brush Salin and Teller found that for Γ less than one, the system behaved basically as a plasma. However when Γ got above 2, the system behaved quite differently in that the correlation function was not monotonically decreasing as a function of separation. This is characteristic of a liquid where the particles have short range correlation with one another. As Γ increased to about 125, another transition is observed to a periodic lattice, a solid. The simulation has difficulty determining whether the lattice is face centered cubic or body centered cubic. The latter is energetically favored, but the energy difference is in the fifth decimal place. However the simulation has no trouble seeing that the lattice is not simple cubic, where the energy difference is in the third decimal place.

Instead of a Monte Carlo simulation scheme, one could also envision a molecular dynamic simulation where the dynamics of the particles are followed in the self potentials including the potentials from other lattice elements. This would be a microcanonical ensemble in the statistical mechanics sense, where the system is isolated. The Monte Carlo simulation is a canonical ensemble in the statistical mechanics sense, where the system is in contact with a heat reservoir of temperature T. These two approaches give the same results as long as the number of particles is sufficiently large, typically more than about 50.

A natural concern with either approach is whether the assumed periodicity is consistent with the actual energetically favored periodicity. For instance if the actual energetically favored periodicity were hexagonal, as for graphite, it would be difficult for the simulation to find this state if the periodicity of the simulation

were cubic. One is pounding a round peg into a square hole. Not only must the geometry be consistent, but the number of particles must be consistent also. For instance if the energetically favored state has a cubic periodicity with 7 particles per element, but the simulation had 100 particles in the periodicity cell, one would not find the energetically favored state either. It is difficult to know what one would find in such circumstances, but clearly the larger the number of particles in the basic periodicity cell, the closer either simulation will be able to come to whatever the favored state is.

Another way to simulate the system without either of these difficulties is to simulate an isolated system. The number of particles the computer can track within the budget is then the largest system that can be simulated. Since the isolated, laser cooled nonneutral plasmas produced at NIST typically have fewer than 10^4 particles, this is perfect for such a simulation. These simulations have been done with both Monte Carlo and molecular dynamic schemes, with the latter being more prevalent.

These simulations find that the crystal structure at large Γ is strongly influenced by the finite geometry. At Γ somewhat over 100, the crystal structure forms in cylindrical or spherical shells, but with diffusion along the shells. This is called a smectic crystal. At Γ somewhat above 200, the motion along the shells freezes out, and along each shell, one has what looks like would want to be a hexagonal structure if it were not constrained by the geometry.

An important issue is how one can simulate larger plasmas. While the NIST group now produces plasmas with about 10^4 ions, their next series of experiments will probably be with 10^6 ions and more. This is too large a number to simulate economically with a molecular dynamics or Monte Carlo simulation. A possible scheme would be to use a molecular dynamics simulation with an assumed periodicity in the θ direction. That is one directly simulates a wedge of plasma and assumes that every wedge is alike. The simulation could be broken up into a large number of cells in θ and forces from other cells could be summed up either directly if there are not too many cells, say less than 12, or else with some form of modified Ewald summation for a larger number of cells. In this way one could economically simulate much larger systems, say 10^5-10^6 particles. The issue is whether the energetically favored

periodicity will fit into the assumed periodicity. This is a difficult question to answer. However for Γ' between about 100 and 200, there is no periodicity in the θ direction anyway, so the assumed periodicity would most likely describe the diffusive motion along the shells. The configuration of the simulation is such that any structure in the radial direction could be modeled. However the much larger number of particles would now allow one to more accurately calculate the effects of finite radial extent on the smectic crystals formed. As Γ gets above 200, the motion along the cylindrical shells crystalizes also, and the simulation is further constrained. The outermost shells, having more particles are less constrained than the inner shells, so these would probably be reasonably accurately described, while the innermost shell, containing a very small fraction of the total number of the particles, are less likely to be described correctly. In any case, there does not appear to be any other simple, economical model for simulating million ion laser cooled charge clouds. Thus the imposition of an assumed an assumed θ periodicity could be an important advance to the simulation capability of nonneutral plasmas.

4. Model for Adiabatic Compression of Toroidal Nonneutral Plasmas

The total charge contained by a nonneutral plasma in a Penning trap is constrained by the steady state Voltage that can be applied to it. A formula for the maximum charge is given by Eq.(2) and we assumed a maximum charge of 5 μC. This is in excess of what has been achieved in the laboratory up to now. A way to maximize the charge contained is to use a toroidal system. This experiment, called HIPAC< was done by AVCO in the late sixties. In a first experiment, they confined a charge of 50μC for 60 μs[3] and later 1ms[13] in a toroidal vessel of radius 46 cm with a magnetic field of 1.5 kG and containing an electron number denstiy of 4x10[9].. The loss mechanism was identified as an ion resonance instability. The confinement time is only an order of magnitude below that observed experimentally denoted by Eq.(3). Thus, if the ion resonance instability could be eliminated by better vacuum conditions, or more magnetic compression to remove the plasma from the walls, one might expect to do much better regarding charge confined and containment time. This is particularly interesting in that for toroidal configuration, thermal equilibrium is not possible since a rigid rotor in the poloidal direction is topologically forbidden. A new series of experiments on such a plasma is now being set up in India[14].

In its most basic form, the theory for a single equilibrium is not difficult. The key is that electrons move along equipotential surfaces due to the ExB drift. Since the drift speed is proportional to $\nabla\phi$, and the separation between potential surfaces is proportional to $1/\nabla\phi$, one can determine the density in terms of the flux around the equipotential surface. The result gives a Poisson's Equation

$$\nabla^2\phi = Q(\phi)/r^2 \qquad (7)$$

where Q is proportional to the poloidal flux. One factor of r on the right comes from the radial dependence of the toroidal field; the other comes from the toroidal length of the equipotential surface as a function of r. When this equation was first derived[15], there was very little experience in solving Grad-Shrafranov equations, and little was done with it. Numerical schemes to solve it are now routine so that information concerning equilibria can be obtained[16]. However one does not only desire single equilibria; one would like to be able to solve for adiabatically connected equilibria as well. For

instance in HIPAC, an inductive charging scheme was used, whereby toroidal field lines coming into the vessel passed by an emitting filament and carried electrons in with them. After the device was filled up and the filament turned off, the plasma could be compressed by further increasing the toroidal field. The plasma will then pull away from the walls and increase in density.

When the plasma compresses, the toroidal flux enclosed by a drift surface is an adiabatic invariant. A means of calculating a sequence of adiabatically connected equilibria has been developed previously with regard the the NRL modified betatron[17,18]. There, the total toriodal flux enclosed by the beam was used as an adiabatic invariant, and this made the formulation much simpler than what it would have been had we assumed the flux within each drift surface was the adiabatic invariant. One can then use a similar approach to perform the calculation for series of adiabatically connected equilibria in toroidal electron rings. For the first equilibrium, where the electron cloud fills the chamber, choose a $Q(\phi)$ and calculate the equilibrium of the electron cloud out to the wall. Note that Q specifies the density only as a function of ϕ, a quantity which must be solved for self consistently. Thus it does not specify in any way either the total charge contained, or even the density as a function of position.

Next parameterize the function $Q(\phi)$ in such a way that there the initial specification corresponds the first equilibrium. Then one parameter multiplying Q to specify adiabatically connected equilibria is an overall constant. The other parameter is a cutoff potential above which the $q(\phi)=0$. (Recall that as the plasma is compressed, the electrostatic potential of the plasma decreases.)..Then the two parameters are adjusted with respect to one another so that the new equilibrium has the same number of electrons as the initial one. Now note however, that the magnetic field has never been introduced as a parameter for the specification of any subsequent equilibria. Thus once an electron conserving equilibrium has been calculated, the magnetic field can also be specified so that toroidal flux is conserved in the entire electron cloud. In this way, a succession of adiabatically connected equilibria can be calculated.

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