RL-TR-91-278
Final Technical Report
November 1991

(2)

A THEORY OF INVERSE OPERATORS FOR MULTIPLE EXCITATIONS

University of Dayton

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave Blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	November 1991	Final Sep 88 - Sep 89

4. TITLE AND SUBTITLE
A THEORY OF INVERSE OPERATORS FOR MULTIPLE EXCITATIONS

5. FUNDING NUMBERS
C - F30602-88-D-0027, Task N-8-5322
PE - 61101F
PR - 7402
TA - 01
WU - P2

6. AUTHOR(S)
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8. PERFORMING ORGANIZATION REPORT NUMBER
UDR-TR-89-115

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Rome Laboratory (ERPT)
Griffiss AFB NY 13441-5700

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
RL-TR-91-278

11. SUPPLEMENTARY NOTES
Rome Laboratory Project Engineer: Kenneth R. Siarkiewicz/ERPT/(315) 330-7642

Prepared for Syracuse University

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words)

In this report we consider the solution to integral equations that arise in the theory of electromagnetic scattering. We develop an interative solution valid not for just one excitation but multiple excitations. We first consider a matrix equation obtained as a projection of the integral equation to finite dimensional space. After each iteration an orthogonal vector is determined and from it a corresponding component of the inverse operator. Thus, after each iteration a best estimate to the inverse operator is determined. This may be used to obtain the estimated solution to an arbitrary number of excitations. It is shown that the outer product of the vectors needs to be computed to compute the inverse operator. Since this is an expensive operation, an alternative approach is determined wherey solution to multiple excitations is determined without computing the outer products. This theory, developed in N-dimensional space, is then generalized to infinite dimensional space. This leads to a definition and a computational procedure for inverse integral operator. The theory is illustrated by computing the induced currents on a square cylinder for several excitations.

14. SUBJECT TERMS
Electromagnetics, EM Scattering, Method of Moments

15. NUMBER OF PAGES
48

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UL

TABLE OF CONTENTS

LIST OF FIGURES

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SECTION 1

INTRODUCTION

The techniques that are available to compute scattered fields from arbitrary targets may be classified as a) analytical techniques and b) numerical techniques. The analytical techniques are largely based on asymptotic theory and are valid when the target size is sufficiently large compared to a wavelength. These techniques are limited to relatively simple targets and are limited to situations where the requisite diffraction coefficients are available.

The numerical techniques, based on extensive use of the digital computer, have a wide applicability and the target configuration is arbitrary. These techniques formulate the problem either as an integral equation or differential equation subject to boundary conditions to be solved. We consider in this report, the solution of integral equations.

The integral equations may be solved directly as is done in [1] without explicitly reducing it to a matrix equation. However, with this method, each time the excitation is changed, the solution process must be started all over again. Thus, when there are a large number of excitations to be considered, as is often the case, presently available direct solutions are inefficient. A more common approach is to take a projection of the integral equation into a finite dimensional space reducing it to a matrix equation [2]. This matrix equation may be solved by several conventional methods [3]. An advantage of this method is the availability of the inverse operator. Once computed, this may be used repeatedly on any number of excitations simply and effectively. This approach is clearly among the best to solve problems with multiple excitations.

As the electrical size of the target becomes large, the order of the matrix becomes large and obtaining the inverse operator becomes numerically inefficient. In addition, round-off error tends to vitiate the computations as the order of the matrix becomes large. Iterative techniques have been used to overcome this problem [7] - [11]. However, iterative methods whether based on Lanczos iteration [12] or conjugate gradients [13] are limited to only one excitation. If the excitation changes, the problem must be solved in its entirety again.

In this report we set ourselves the task of devising an iterative scheme that may be used with not just one excitation but an arbitrary number of excitations. That is, in effect, we seek to compute an inverse operator iteratively.

Let us consider a solution of the matrix equation,

$$Ax = F \tag{1.1}$$

where A is an NxN matrix. Let A be self adjoint. The matrix A may be considered to be a collection of N-Vectors, j^{th} vector being simply the j^{th} column of the matrix. Equation (1.1) may be put in the form,

$$< A_j, x > = V_j \quad , \quad j = 1,2 \quad N \quad . \tag{1.2}$$

A geometrical interpretation of equation (1.2) is to find the vector x such that its projections along the columns of the matrix ($< A_j, x >$) equal the components of the excitation. This problem is best solved by setting up an orthogonal co-ordinate system. This is accomplished by starting from an arbitrary initial vector x_o and computing a sequence of linearly independent set of vectors, x_o, Ax_o, A^2x_o .... $A^N x_o$. This set of vectors is called the Krylov sequence of vectors and each member of this sequence is computed by operating

on the preceding vector by the matrix A. That is, Krylov sequence may be computed iteratively.

The Krylov sequence is not orthogonal. These vectors may be orthogonalized by the Gram-schmidt procedure [3]. However, this procedure is numerically inefficient. Lanczos [12], [6] had shown that it is only necessary to enforce orthogonality of a vector from Krylov sequence to the two preceding orthogonal vectors; orthogonality to the rest of the vectors is assured. Thus, a set of A-orthogonal vectors are computed iteratively, satisfying the relation.

$$< q_i, Aq_j > = < Aq_i, q_j > = \Lambda_i \delta_{ij} \quad . \quad (1.3)$$

It is shown in this report that the inverse operator may be thought of as consisting of N-components. Each component matrix is shown to be related to one of the A-orthogonal vectors. For instance, the j^{th} orthongonal vector q_j leads to a matrix given by, $\Lambda_j^{-1} q_j q_j^H$. Thus, as each new A-orthogonal vector becomes available, the inverse operator may be up-dated. This leads to an interative computation of the inverse operator. However, these computations involve computing the outerproduct $q_j q_j^h$. Such a computation is numerically expensive. A technique is shown whereby solution to multiple excitations is obtained but without explicitly computing the outer product.

The theory developed for the finite dimensional operator is then extended to infinite dimensional space. This leads to the definition of the inverse integral operator. This inverse operator is related to a set of orthogonal functions in the same way the matrix inverse operator is related to the q-vectors.

In section-2 we present the detailed development of the theory of the inverse operator. Section-3 presents a numerical implementation of the theory and a few examples. Section-4 contains a discussion of the results and some suggestions for further research.

SECTION 2

THEORY

Here we consider the solution of Integral Equations (IE). In the field of electromagnetic scattering Fredholm's integral equations of both the first and second kind arise. The electric field integral equation (EFIE) is an integral equation of the first kind while the magnetic field integral equation (MFIE) is of the second kind. The general form of these IE's is given below:

$$g = Kf \tag{2.1}$$

$$f = g + Kf \tag{2.2}$$

K is a linear integral operator. Equation (2.2) may be rearranged so that it assumes the form of an integral equation of the first kind:

$$\begin{aligned} g &= f - Kf \\ &= (I-K)\, f \\ &= K'\, f \end{aligned} \tag{2.3}$$

Where K' is a new operator given by

$$K' = I - K$$

Equation (2.3) is an integral equation of the first kind. The kernel of this new IE involves a dirac delta function. Thus, without loss of generality, we consider only operator equations of the first kind.

The integral equation may be thought of as a problem in infinite dimensional space. By using an appropriate technique such as the method of moments [2], it is possible to take a projection of the IE (eq. 2-1) from infinite dimensional to an N-dimensional space. This procedure reduces the IE

to a finite dimensional matrix operator equation, of the form shown below.

$$Ax = F \tag{2.4}$$

Where the operator A is an NxN matrix.

We will develop the theory in terms of the finite dimensional operator A and eventually obtain the solution to the IE as the limit of the solution to equation (2.4) as $N \to \infty$. The infinite dimensional space is considered to be a limiting case of N-dimensional space.

Let X and Y be two vectors defined by

$$X = [\, x_1,\ x_2\ \ .....x_n]'$$

$$\text{and}\quad Y = [\, y_1,\ y_2\ \ .....y_n]'$$

The prime is a transpose operator. Then, the inner product of two vectors is defined by,

$$< X,Y > = X^H\, Y$$

The superscript H indicates Hermitian operation and X^H is obtained from X by conjugating and transposing the elements of X.

$$< X,Y > = X^H\, Y = \sum_{k=1}^{N} \overset{*}{x}_k\, y_k \tag{2.5}$$

where $\overset{*}{x}_k$ is the complex conjugate of x_k. The outer product of two vectors, x and y is an NxN matrix Z, given by

$$Z = X\, Y^H \quad . \tag{2.6}$$

An operator is said to be self-adjoint if the following identity holds.

$$< AX,\ Y > = < X,\ AY > \tag{2.7}$$

Without loss of generality we assume the operators K and A to be self-adjoint. The impedance matrix that is obtained in electromagnetic problems is not self adjoint. However, the problem may be reformulated to yield an operator that is self adjoint.

$$ZI = V \tag{2.8}$$

Multiplying both sides of equation (2.7) by Z^H, we obtain,

$$(Z^H Z) I = Z^H V \quad . \tag{2.9}$$

It may easily be seen that the operator $Z^H Z$ is self adjoint.

2.1 A-ORTHOGONAL VECTORS

We now determine a sequence of vectors, q_1, q_2, q_2 .... q_n starting from an arbitrary initial vector x_o. These vectors are A-orthogonal vectors. That is,

$$< Aq_i, q_j > = \Lambda_i \delta_{ij} \quad . \tag{2.10}$$

Since the operator A is self-adjoint,

$$< Aq_i, q_i > = < q_i, Aq_j, > = \Lambda_i \delta_{ij} \quad . \tag{2.11}$$

These vectors are obtained by A-orthogonalizing the Krylov sequence of vectors. Starting from an initial arbitrary vector, x_o, the Krylov sequence is obtained as x_o, Ax_o, A^2x_o ..... , $A^{N-1} x_o$. Note that each vector of this sequence is obtained from the preceding vector by operating on it once with the operator A.

The set of Krylov vectors are not A-orthogonal. These may be orthogonalized by a special procedure such as the Gram-Schmidt procedure. However, Gram-Schmidt procedure is numerically very expensive. Furthermore, due to the special properties of the Krylov sequence of vectors, it may be shown

that p^{th} orthogonal vector, q_p, may be obtained by ensuring that $A^p x_o$ is orthogonal to only the <u>two</u> preceding vectors, q_{p-1} and q_{p-2}! (Gram-Schmidt would have required that $A^p x_o$ be orthogonal to <u>all</u> the preceding vectors.) This clearly results in an efficient computational procedure. We now give the details of construction of the q-vectors.

$$q_1 = x_o$$

$$q_2 = Aq_1 - \gamma\, q_1 \; ; \quad \gamma_1 = \frac{< Aq_1,\, Aq_1 >}{< q_1,\, Aq_1 >} \tag{2.12}$$

$$q_{i+1} = Aq_i - \gamma_i\, q_i - \delta_i\, q_{i-1} \quad , \quad i = 2,3 \ldots\ldots N-1 \tag{2.13}$$

$$\gamma_i = \frac{< Aq_i,\, Aq_i >}{< q_i,\, Aq_i >}$$

$$\delta_i = \frac{< Aq_i,\, Aq_{i-1} >}{< q_{i-1},\, q_{i-1} >}$$

The vectors so computed constitute a complete A-orthogonal set as proved in [1]. Note that the computational process may be so arranged that each of these vectors are computed iteratively, one per iteration. The A-orthogonality of these vectors may be expressed, compactly, using matrix notation as follows:

$$Q^T A\, Q = \Lambda \tag{2.14}$$

Where Q^T is the transpose of the NxN matrix Q, Q is a matrix with the q-vectors as its elements. That is,

$$Q = [\, q_1,\, q_2,\, q_3 \ldots\ldots q_n \,] \quad . \tag{2.15}$$

Λ is a diagonal matrix, whose diagonal elements are given by

$$\Lambda_i = < q_i \ , \ Aq_i > \quad . \tag{2.16}$$

2.2 INVERSE OPERATOR

We now obtain explicit expressions for the inverse of the operator in terms of the q-vectors. The matrices Q and Q^H are partitioned as follows.

$$Q = [\ Q_p \ \ Q_{\bar{p}} \ ] \quad . \tag{2.17}$$

Where Q_p is an N x P matrix obtained from the first P A-orthogonal vectors. That is,

$$Q_p = [\ q_1 \ \ q_2 \ \ \cdots \ \ q_p \ ] \quad . \tag{2.18}$$

Q_p is an N x (N-P) matrix defined below.

$$Q_{\bar{p}} = [\ q_{p+1} \ q_{p+2} \ \ \cdots \ q_N \ ] \tag{2.19}$$

Q_p^H and $Q_{\bar{p}}^H$ are matrices obtained from transposing and conjugating Q_P and $Q_{\bar{P}}$. From these matrices, Q^H may be shown to be,

$$Q^H = \begin{bmatrix} Q_p^H \\ \\ Q_{\bar{p}}^H \end{bmatrix} \quad . \tag{2.20}$$

Now, taking the inverse of equation (2.14) gives

$$[\ Q^H \ A \ Q \ ]^{-1} = \Lambda^{-1}$$

$$Q^{-1} \ A^{-1} \ [Q^H]^{-1} = \Lambda^{-1} \tag{2.21}$$

$$A^{-1} = Q \ \Lambda^{-1} \ Q^H \tag{2.22}$$

Note that since Λ is a diagonal matrix, Λ^{-1} is simply another diagonal matrix, whose diagonal elements are Λ_i^{-1}. Thus, equation (2.22) shows the relationship between the inverse operator and the A-orthogonal vectors. However, equation (2.22) implies that the inverse operator may be computed only after all the q-vectors are computed. In fact, the q-vectors are computed iteratively; an additional q-vector is available after each iteration. It can be shown that the inverse operator itself may be obtained iteratively. The diagonal matrix Λ^{-1} may be partitioned as shown below.

$$\Lambda^{-1} = \begin{bmatrix} \Lambda_p^{-1} & 0 \\ 0 & \Lambda_{\bar{p}}^{-1} \end{bmatrix} \tag{2.23}$$

The Λ_p^{-1} and $\Lambda_{\bar{p}}^{-1}$ are diagonal matrices of orders PxP and P x $\bar{P}$ ($\bar{P}$ = N - P). Then, from equation (2.22),

$$A^{-1} = [\ Q_p\ Q_{\bar{p}}\ ] \begin{bmatrix} \Lambda_p^{-1} & 0 \\ 0 & \Lambda_{\bar{p}}^{-1} \end{bmatrix} \begin{bmatrix} Q_p^H \\ Q_{\bar{p}}^H \end{bmatrix} \tag{2.24}$$

$$= [\ Q_p\ Q_{\bar{p}}\ ] \begin{bmatrix} \Lambda_p^{-1} Q_p^H \\ \Lambda_{\bar{p}}^{-1} Q_{\bar{p}}^H \end{bmatrix}$$

$$= [\ Q_p\ \Lambda_p^{-1}\ Q_p^H + Q_{\bar{p}}^H\ \Lambda_{\bar{p}}^{-1}\ Q_{\bar{p}}^H\ ]$$

$$= [\ B_p + B_{\bar{p}}\ ] \quad . \tag{2.25}$$

Where,

$$B_p = [\ Q_p\ \Lambda_p^{-1}\ Q_p^H\ ] \tag{2.26}$$

and

$$B_{\bar{p}} = [\ Q_{\bar{p}}\ \Lambda_{\bar{p}}^{-1}\ Q_{\bar{p}}^H\ ] \tag{2.27}$$

B_p may be thought of as the best estimate of the inverse operator after P

iterations. Indeed, each A-orthogonal vector q corresponds to a matrix component of the inverse operator. By direct expansion of equation (2.26),

$$B_p = [q_1 \Lambda_1^{-1} q_1^H + q_2 \Lambda_2^{-1} q_2^H + ... + q_p \Lambda_p^{-1} q_p^H]$$

$$= \sum_{j=1}^{p} \Lambda_j^{-1} q_j q_j^H \quad . \tag{2.28}$$

Thus, the outerproduct (a matrix) formed from the j^{th} vector q_j is a component of the inverse operator. It must be pointed out that computing an outerproduct is numerically a very expensive operation. Indeed, solution to the original operator problem for multiple inputs may be computed without actually computing the outerproduct and hence the inverse operator!

2.3 SOLUTION WITHOUT EXPLICIT INVERSE OPERATOR

We now consider solution to the operator problem with M-excitations.

$$AX^j = F^j \qquad j = 1,2, ...M \quad . \tag{2.29}$$

Let P iterations of the procedure be carried out resulting in the computation of P A-orthogonal vectors $q_1, q_2, ... q_p$. Then the solution x_p^j corresponding to the j^{th} excitation F^j is given by

$$x_p^j = B_p F^j$$

$$= [\sum_{\ell=1}^{p} \frac{1}{\Lambda^\ell} q_\ell q_\ell^H] F^j$$

$$= \sum_{\ell=1}^{p} \frac{1}{\Lambda^\ell} q_\ell q_\ell^H F_j$$

$$= \sum_{\ell=1}^{p} \frac{1}{\Lambda^\ell} < q_\ell, F^j > q_\ell \tag{2.30}$$

$$= \sum_{\ell=1}^{p} \frac{< q_\ell, F^j >}{< q_\ell, Aq_\ell >} q_\ell \qquad (2.31)$$

From equation (2.31) it is evident that it is only necessary to compute the inner products $< q_\ell, F_j >$ and not the outerproduct $q_\ell \, q_\ell^H$ to obtain the solution. This is clearly much more efficient than computing the inverse operator directly. Even for purposes of storing the inverse, it is best to store the collection of q-vectors, the Q-matrix, rather than A itself. Equation (2.31) may also be written as,

$$x_p^j = \sum_{\ell=1}^{p-1} \frac{< q_\ell, F^j >}{< q_\ell, Aq_\ell >} q_\ell + \frac{< q_p, F^j >}{< q_p, Aq_p >} q_p$$

$$= x_{p-1}^j + \frac{< q_p, F^j >}{< q_p, Aq_p >} q_p \qquad . \qquad (2.32)$$

Thus, after P iterations, as the P^{th} vector becomes available, the solution computed at the end of P-1 iterations is up-dated by the addition of the factor $< q_p, F^j > q_p / < q_p, Aq_p >$. This is analogous to up-dating the inverse operator by the addition of the matrix $\Lambda_p^{-1} \, q_p \, q_p^H$.

2.4 ITERATIVE SCHEME

With the theory in hand, we are now in a position to outline the iterative scheme to be implemented on a digital computer. Let x_o be an arbitrarily chosen initial vector.

Step 1:

$$q_1 = x_o$$

$$\Lambda_1 = < q_1, Aq_1 >$$

$$t_1^j = \frac{1}{\Lambda_1} < q_1, F^j > q_1 \quad , \quad j = 1,2, \ldots M \quad .$$

$$x_1^j = t_1^j \quad , \quad j = 1,2, \ldots M \quad . \tag{2.33}$$

Step 2:

$$\gamma_1 = \frac{1}{\Lambda_1} < Aq_1 , Aq_1 >$$

$$q_2 = Aq_1 - \gamma_1 q_1$$

$$\Lambda_2 = < q_2, Aq_2 >$$

$$t_2^j = \frac{1}{\Lambda_2} < q_2, F^j > q_2 \quad , \quad j = 1,2 \ldots M \quad .$$

$$x_2^j = x_1^j + t_2^j \quad , \quad j = 1,2 \ldots M \quad . \tag{2.34}$$

Step 3: For i = 2, 3 .....

$$\gamma_i = \frac{1}{\Lambda_i} < Aq_i, Aq_i >$$

$$\delta_i = \frac{1}{\Lambda_{i-1}} < Aq_i, Aq_{i-1} >$$

$$q_{i+1} = Aq_i - \gamma_i q_i - \delta_i q_{i-1}$$

$$\Lambda_{i+1} = < q_{i+1}, Aq_{i+1} >$$

$$t_{i+1}^j = \frac{1}{\Lambda^{i+1}} < q_{i+1}, F^j > q_{i+1} \quad , \quad j = 1,2, \ldots M \quad .$$

$$x_{i+1}^j = x_i^j + t_{i+1}^j \quad , \quad j = 1,2 \ldots M \quad .$$

$$\epsilon^j = \| F^j - Ax_{i+1}^j \| \quad , \quad j = 1,2 \ldots M \quad . \tag{2.35}$$

Step 4:

Terminate if ϵ^i is sufficiently small or go back to Step-3.

This procedure computes one A-orthogonal vector for each iteration and uses this vector to update this solution for all M excitations and not just one excitation. Note that the inverse operator A^{-1} is not computed explicitly. As observed earlier, the matrix Q, which is a collection of the A-orthogonal vectors serves the same purpose as A^{-1} but numerically is much more efficient.

2.5 INVERSE INTEGRAL OPERATOR

The theory of the inverse operator has been developed in N-dimensional space. We now let the number of dimensions, N, approach infinity so that the matrix operator becomes integral operator. It is worth remembering that the matrix operator which becomes integral operator was originally obtained as a projection of the integral operator into finite dimensional space. That is,

$$\text{as} \quad N \to \infty \ , \quad A \to K \tag{2.36}$$

The vectors of finite dimensional space become functions in infinite dimensional space. That is,

$$q_i \to \xi_i \tag{2.37}$$

The inner product in infinite dimensional space of two functions f and g may be defined as,

$$< f, g > = \int \overset{*}{f} g \, d\ell \tag{2.38}$$

As before, the operator K is self adjoint if,

$$< K f, g > = < f, Kg > \tag{2.39}$$

We now consider the solution of the integral operator equation, for multiple

excitations:

$$K\, fj = g^j \quad , \qquad j = 1,2 \ldots M \quad . \tag{2.40}$$

As in the case of finite dimensional space, the functions ξ_i are computed such that they satisfy the orthogonality condition.

$$< \xi_i \ , \ K\xi_j \ > = \Lambda_i \ \delta_{ij} \tag{2.41}$$

The difference is that, in N-dimensional space the number of q-vectors are finite in number, equal to N but in infinite dimensional space, ξ_i are infinite in number. However, the inverse operator K^{-1} is related to the ξ_i in a manner analogous to the way A^{-1} is related to q_i.

In N-dimensional space, the solution after P-iterations is given by equation (2.31):

$$x_p^j = B_p \, F^j = \sum_{\ell=1}^{p} \frac{< q_\ell, \ F^j >}{< q_\ell, \ Aq_\ell >} \, q_\ell \quad .$$

Generalizing this equation to infinite dimensional space.

$$f_p^j = K_p^{-1} \, g^j = \sum_{\ell=1}^{p} \frac{< \xi_\ell, \ g^j >}{< \xi_\ell \ K\xi_\ell >} \, \xi_\ell \quad , \qquad j = 1,2, \ \ldots \ M \tag{2.42}$$

Thus, obtaining an iterative solution to integral equation with multiple excitations consists of the following steps.

Step 1:

Determine K-orthogonal functins ξ_i starting from an arbitrary initial function f_o and Krylov sequence of functions.

Step 2:

These ξ_i are computed iteratively, ie, one function during each iteration. This function may be used to compute the update for the

solution as given by equation (2.42). The inverse operator may be expressed as follows:

$$K^{-1} = \sum_{\ell=1}^{\infty} \frac{\xi_\ell \, \xi_\ell^H}{< \xi_\ell, K\xi_\ell >} \tag{2.43}$$

The outerproduct of two functions $\xi_\ell \, \xi_\ell^H$ results in a new operator. The operation of this operator on a function f is defined by:

$$(\xi_\ell \, \xi_\ell^H) \, f = \xi_\ell < \xi_\ell, \, f > = < \xi_\ell, \, f > \xi_\ell \tag{2.44}$$

The effect of this operation is to transform the function f into a scalar multiple of ξ_ℓ.

As may be seen from equation (2.43), this inverse operator depends only on ξ's and the operator K and is independent of excitation. We now give an iterative scheme for the solution of equation (2.40).

2.5.1 ITERATIVE SCHEME FOR INVERSE INTEGRAL OPERATOR

The computational scheme for the solution of the integral equation with multiple right hand sides is given here.

Step 1. Choose an arbitrary initial function f_o. Then,

$$\xi_1 = fo \; ; \; \Lambda_1 = < \xi_1, \, K\xi_1 >$$

$$f_1^j = \frac{1}{\Lambda_1} < \xi_1, \, g^j > \xi^1 \quad , \qquad j = 1,2 \ldots M \quad . \tag{2.45}$$

Step 2.

$$\gamma_1 = \frac{1}{\Lambda_1} < K\xi_1, \, K\xi_1 >$$

$$\xi_2 = K\xi_1 - \gamma_1 \xi_1$$

$$\Lambda_2 = < \xi_2, K\xi_2 >$$

$$f_2^j = x_1^j + \frac{1}{\Lambda_2} < \xi_2 , g^j > \xi_2 \quad , \qquad j = 1,2, \ldots M \quad . \tag{2.46}$$

Step 3.

$$\gamma_i = \frac{1}{\Lambda_i} < K\xi_i , K\xi_i >$$

$$\delta_i = \frac{1}{\Lambda_{i-1}} < K\xi_i , K\xi_{i-1} >$$

$$\xi_{i-1} = K\xi_i - \gamma_i \xi_i - \delta_i \xi_{i-1}$$

$$\Lambda_{i+1} = < \xi_{i+1}, K\xi_{i+1} >$$

$$f_{i+1}^j = f_i^j + \frac{1}{\Lambda_{i+1}} < \xi_{i+1}, g^j > \xi_{i+1} \quad , \qquad j = 1,2, \ldots M \quad .$$

$$\epsilon^j = \| g^j - K f_{i+1}^j \| \quad , \qquad j = 1,2, \ldots M \quad . \tag{2.47}$$

Step 4.

If ϵ^j is not sufficiently small, go back to Step-3.

It may be noted that in order to obtain the solution to the multiple excitation problem, it is only necessary to store at most <u>three</u> ξ's at any given time. However, if the inverse operator were to be stored to be used at a later time on more excitations, then all the ξ's computed need to be stored.

2.6 DEGENERACY

The iterative scheme described in Sections 2.3 and 2.5.1 may terminate prematurely without the error becoming sufficiently low if the initial vector x_0

or f_o are deficient. The initial vector x_o must contain components along all the eigenvectors of the operator A. Otherwise the Krylov sequence ceases to produce new linearly independent vectors after a certain number of iterations. When this happens, it is not possible to compute a new A-orthogonal vector leading to a premature termination of the iterative scheme. Similar comments apply for the case of continuous operator as well. In such cases of degeneracy, the iteration may be re-started using the following procedure.

Let x_o^1 be the initial vector. Let P A-orthogonal vectors $q_1, q_2 \ldots q_p$ be computed before degeneracy occurred. Then the iterative procedure may be continued from another vector x_o^2. However, x_o^2 must be A-orthogonal to all the preceding P vectors. This may be ensured in the following way. Define q_{p+1} such that,

$$q_{p+1} = x_o^2 - \sum_{\ell=1}^{p} \frac{1}{\Lambda_\ell} < x_o^2, Aq_\ell > q\ell \qquad (2.48)$$

q_{p+1} may be shown to be A-orthogonal to all the preceding q-vectors. That is,

$$< q_{p+1}, Aq_j > = 0 , \qquad j = 1,2 \ldots P \quad . \qquad (2.49)$$

$$< q_{p+1}, Aq_j > = < x_o^2, Aq_j > - \sum_{\ell=1}^{p} \frac{1}{\Lambda_\ell} < x_o^2, Aq_\ell > < q_\ell, Aq_j > \quad ,$$

$$j = 1,2, \ldots P.$$

$$= < x_o^2, Aq_j > - \sum_{\ell=1}^{P} \frac{1}{\Lambda_\ell} < x_o^2, Aq_j > \Lambda_j \, \delta_{\ell j}$$

$$= < x_0^2, Aq_j > - \frac{1}{\Lambda_j} < x_o^2, Aq_j > \Lambda_j$$

$$= 0 \quad . \qquad (2.50)$$

With q_{p+1} defined this way, the iteration may be continued from Step-3 of Section 2.4. However, it must be noted that equation (2.48) implies that P A-orthogonal

vectors are stored and are available. It is possible to devise a scheme whereby it is not necessary to store the A-orthogonal vectors. Two vectors x_o^1 and x_o^2 are initially chosen and the iteration is started with x_o^1 as given in section 2.4. In addition, however, after each iteration, x_o^2 is modified so that it is A-orthogonal to q-vector that is computed. During ℓ^{th} iteration, then, x_o^2 is replaced by,

$$x_o^2 \rightarrow x_o^2 - \frac{1}{\Lambda_\ell} < x_o^2, q_\ell > q_\ell \qquad (2.51)$$

This procedure ensures that x_o^2 is always A-orthogonal to all the available q's. Of course, similar procedure may be used for the continuous operator case as well.

SECTION 3

NUMERICAL ANALYSIS

The numerical implementation of the theory described in Section-2 is discussed here. We present here the numerical considerations that result in the most efiicient code as well as the numerical difficulties that may arise and some prescriptions to overcome these problems. Some numerical examples are presented.

3.1 NUMERICAL CONSIDERATIONS

a) A significant part of the computation consists in computing q_{i+1} from Aq_i by subtracting the projections of q_i and q_{i-1}. This may be accomplished by computing γ_i and δ_i and computing,

$$q_{i+1} = Aq_i - \gamma_i q_i - \delta_i q_{i-1} \tag{3.1}$$

as described in Section 2.6.1, Step-3. A better way to accomplish this is to carry out the recursive procedure described below [3], [4].

$$q_{i-1} = Aq_i - \gamma_i q_i \tag{3.2}$$

$$\gamma_i = < Aq_i, Aq_i > / \Lambda_i \tag{3.3}$$

$$q_{i+1} = q_{i+1} - \delta_i q_{i-1} \tag{3.4}$$

$$\delta_i = < q_{i+1}, Aq_{i-1} > / \Lambda_i \tag{3.5}$$

The projections $\gamma_i q_i$ and $\delta_i q_{i-1}$ are removed one at a time. In a finite precision arthmetric, this leads to a more stable computation.

b) The impedance matrix Z that comes up in electromagnetic problems is not self adjoint. However, the problem may be reformulated to yield a new matrix operator that is self adjoint. That is, 'the original matrix equation,

$$Z\ I = V \tag{3.6}$$

is pre-multiplied by the matrix Z^H to obtain,

$$(Z^H\ Z)\ I = Z^H\ V \quad . \tag{3.7}$$

Equation (3.7) may be put in the form

$$Ax = F \quad . \tag{3.8}$$

Where,

$$A = Z^H\ Z \quad . \tag{3.9}$$

Multiplication of a matrix by another matrix is numerically a very expensive operation. In fact, the numerical scheme may be arranged so that one never needs to carry out the operation in equation (3.9). The entire iterative scheme discussed in section 2 is based on a matrix operating on a vector, AX. The operation $(Z^H\ Z)\ x$ may be carried out in two steps as follows. Let,

$$Y = (Z^H\ Z)\ x \tag{3.10}$$

then, $$Y_1 = Zx \tag{3.11}$$

and $$Y = Z^H\ Y_1 \tag{3.12}$$

That is, the result required in equation (3.10) is obtained by implementing equations (3.11) and (3.12). Both of these equations consist of matrix operating on a vector, a numerically efficient operation.

c) In an infinite precision arithmetic, the q-vectors computed using the procedure in section 2.5 will always be A-orthogonal. However, in a finite precision arithmetic, as invariably is the case, the q-vectors slowly lose

their orthogonality. Such a loss of orthogonality completely destroys the validity of computations and thus must be monitored and must be prevented from happening. In this investigation, we monitor the loss of global orthogonality by computing a constant "C" as defined below, after computing q_{p+1}.

$$C = < Aq_1 \ , \ q_{p+1} > \tag{3.14}$$

If C were to be more than a prescribed value, $\in$, (say 0.05), q_{p+1} is deemed to have lost orthogonality and is rejected; The iteration is restarted from a new vector x_o^2 after ensuring that it is orthogonal to all the preceding P number of q-vectors.

3.2 NUMERICAL ILLUSTRATIONS

The target is a square cylinder illuminated by a TE plane wave. The angle of incidence is arbitrary. Each angle of incidence corresponds to a different excitation. For purposes of illustration, we consider three angles of incidence ranging from 90° to 270° as shown in Figure-1. We consider three different sizes with W=0.2λ, 0.3λ and 0.4λ. Each of these cylinders is illuminated with ϕ_i = 180°, 225°, 270°.

A standard method of moments code [14] is used to generate the impedance matrix Z and the excitatin vectors V^j, j = 1,2,3. The matrix equation is therefore,

$$Z \ I = V^j \qquad j = 1,2,3 \tag{3.15}$$

This equation is solved using the procedure described in this report iteratively and for all the excitations simultaneously. The currents obtained using this procedure are checked by comparing them to currents obtained using Crout's method [5].

Figures 2-5 show the results of 0.2λ square cylinder. Figures 2-4 show

that the currents obtained using the iterative scheme compare well with the currents obtained using elimination procedure. Figure 5 shows the monotonic decrease of the r•m•s error as iteration progresses. Similar results are shown in figures 6-9 for 0.3λ case and in figures 10-13 for 0.4λ case.

In each of these cases the iteration is started by choosing the initial vector to be

$$x_o = \frac{F^1}{< AF^1,\ F^1 >} \tag{3.16}$$

$$F^1 = Z^H V^1 \tag{3.17}$$

The vector F^1 is the right hand side of the transformed matrix equation corresponding to $\phi_i = 180°$. Examination of figures 5, 9 and 13 show that error decreases fastest f the $\phi_i = 180°$ case. The rate of convergence seems to depend on the initial choice of the vector and the best choice seems to be the right hand side of the matrix equation itself.

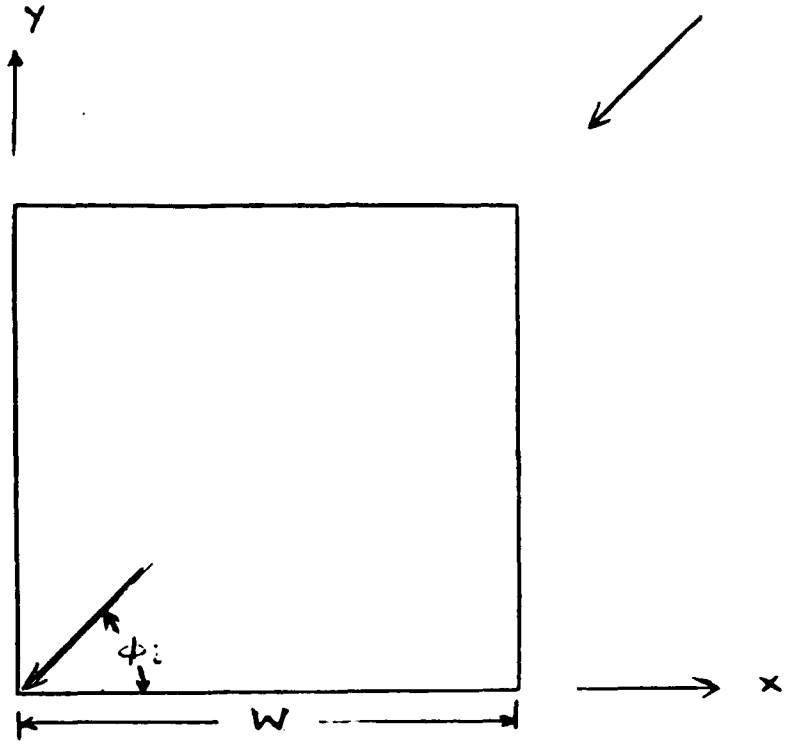

Figure 1. Geometry of square cylinder.

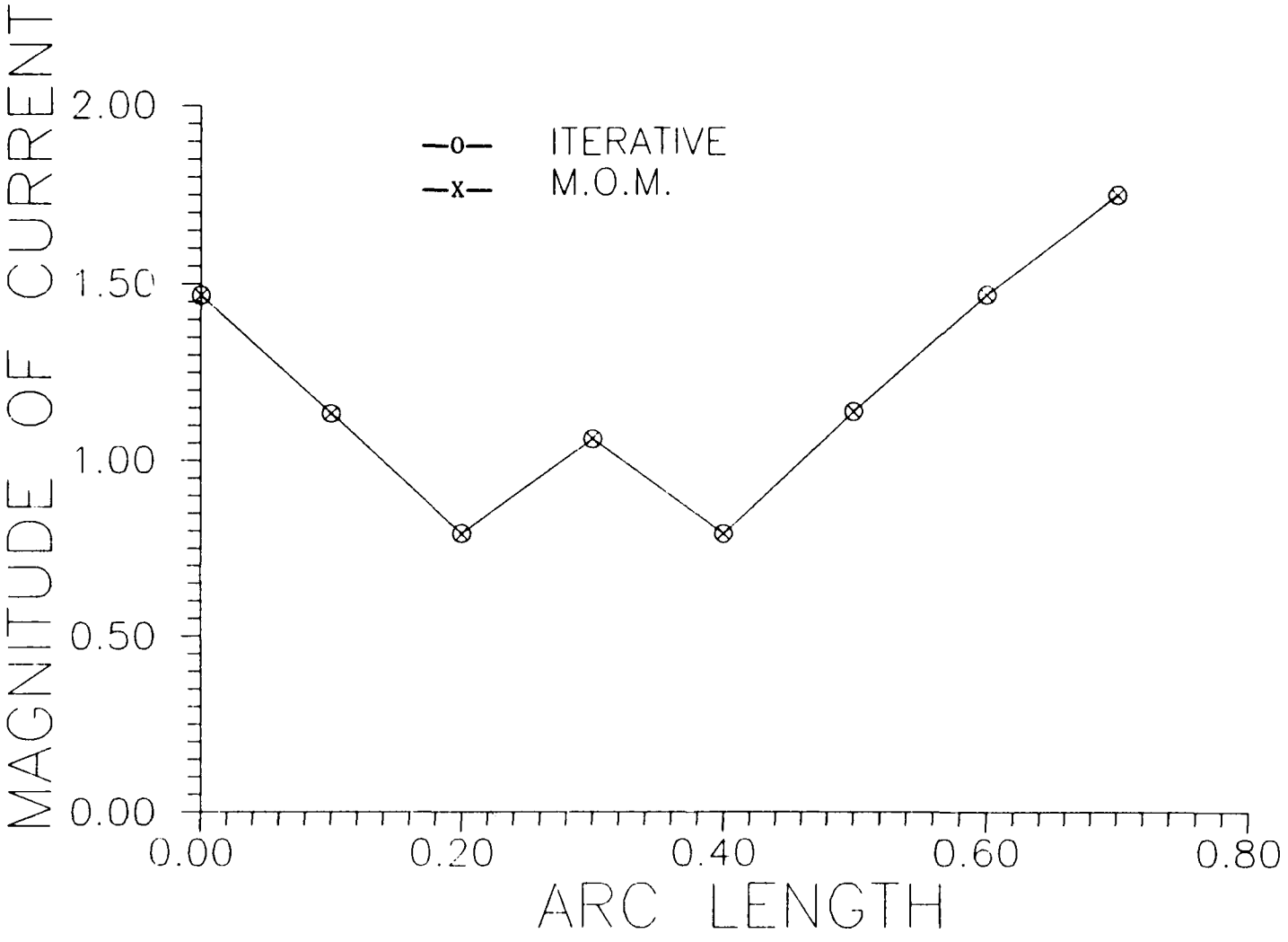

Figure 2. Currents on a perfectly conducting square cylinder, w=0.2λ, PHI=180°.

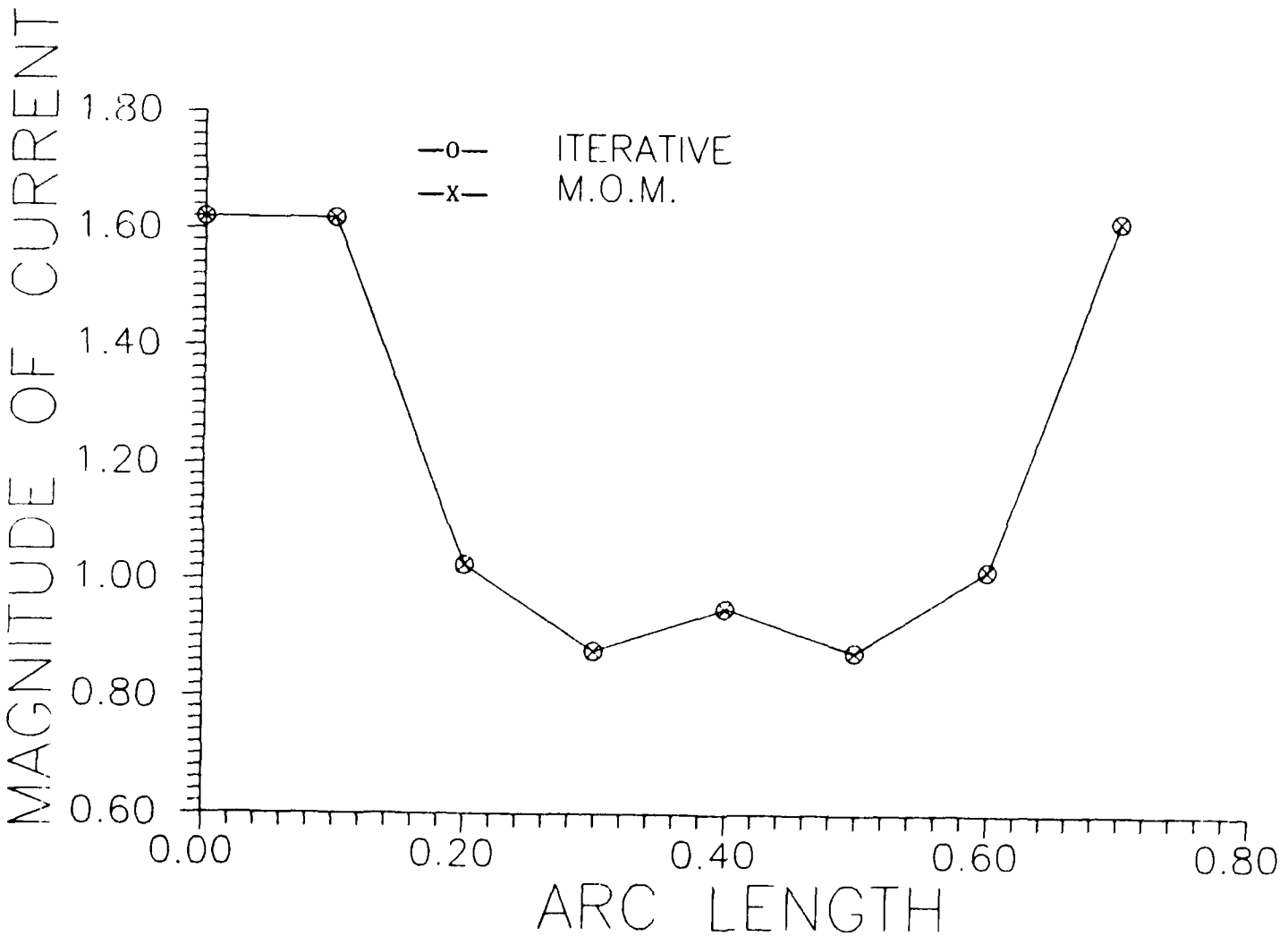

Figure 3. Currents on a perfectly conducting square cylinder, w=0.2λ, PHI=222.5°.

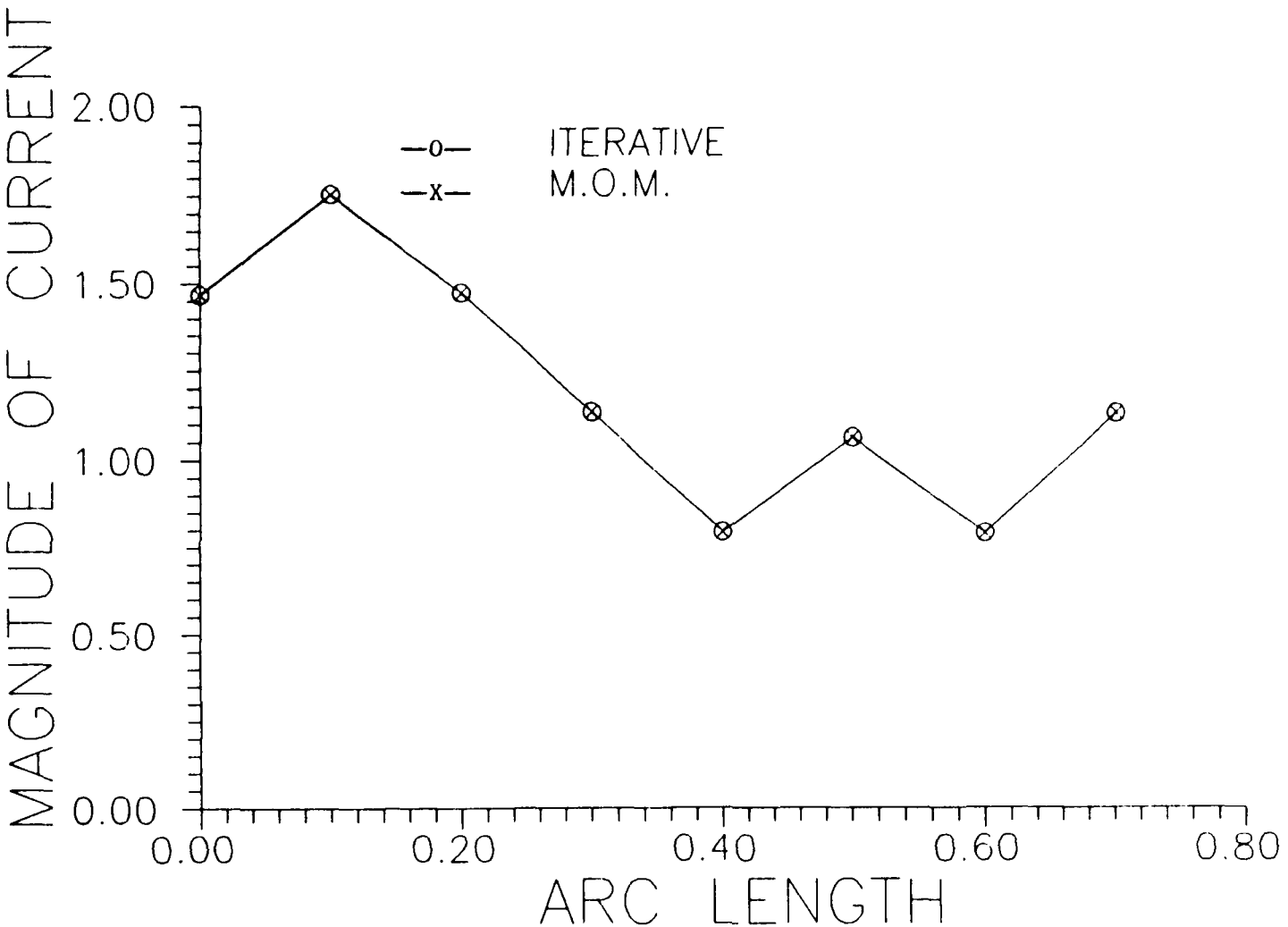

Figure 4. Currents on a perfectly conducting square cylinder, w=0.2λ, PHI=227°.

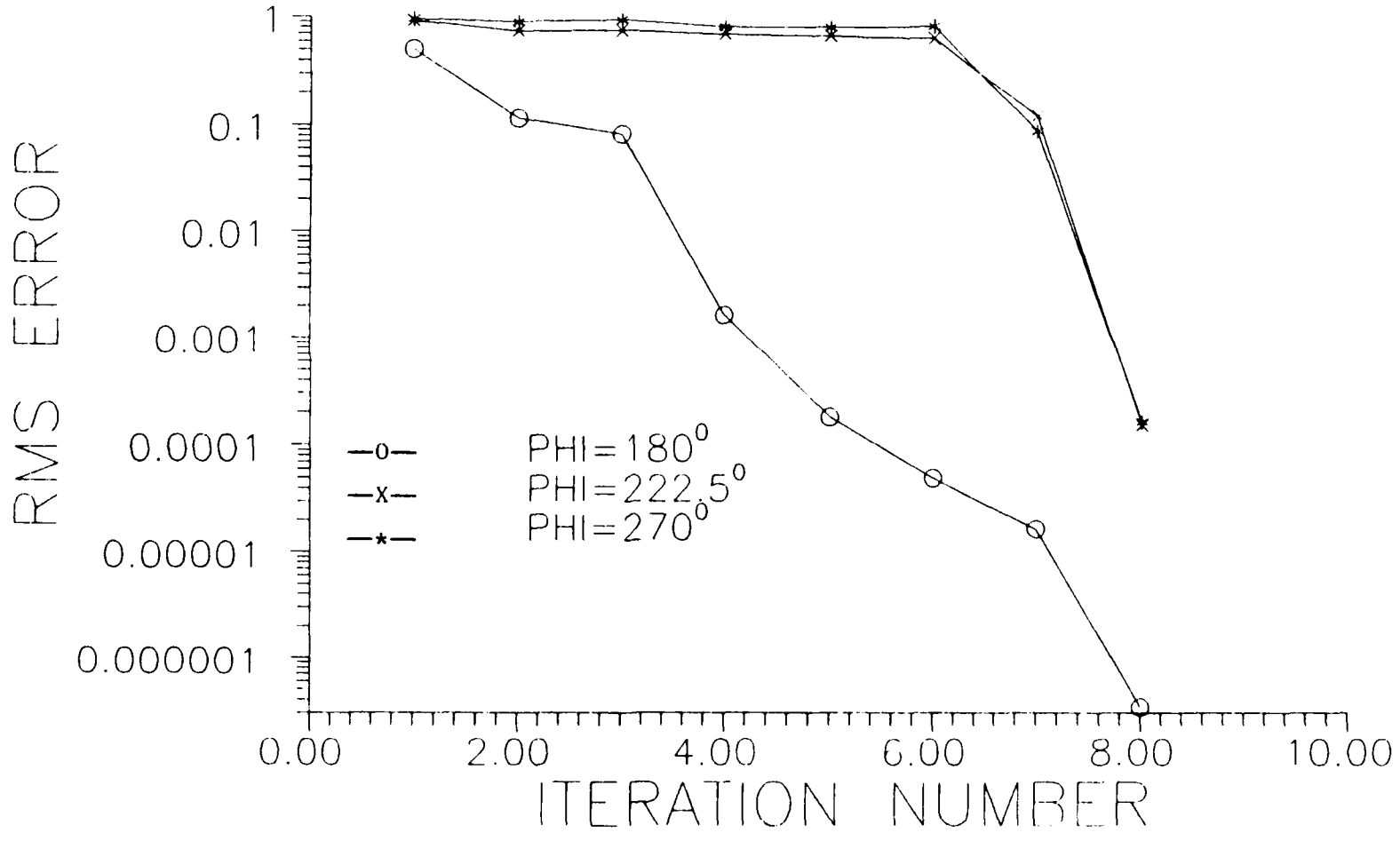

Figure 5. RMS Error Vs Iteration Number for three different excitations of square cylinder, w=0.2λ.

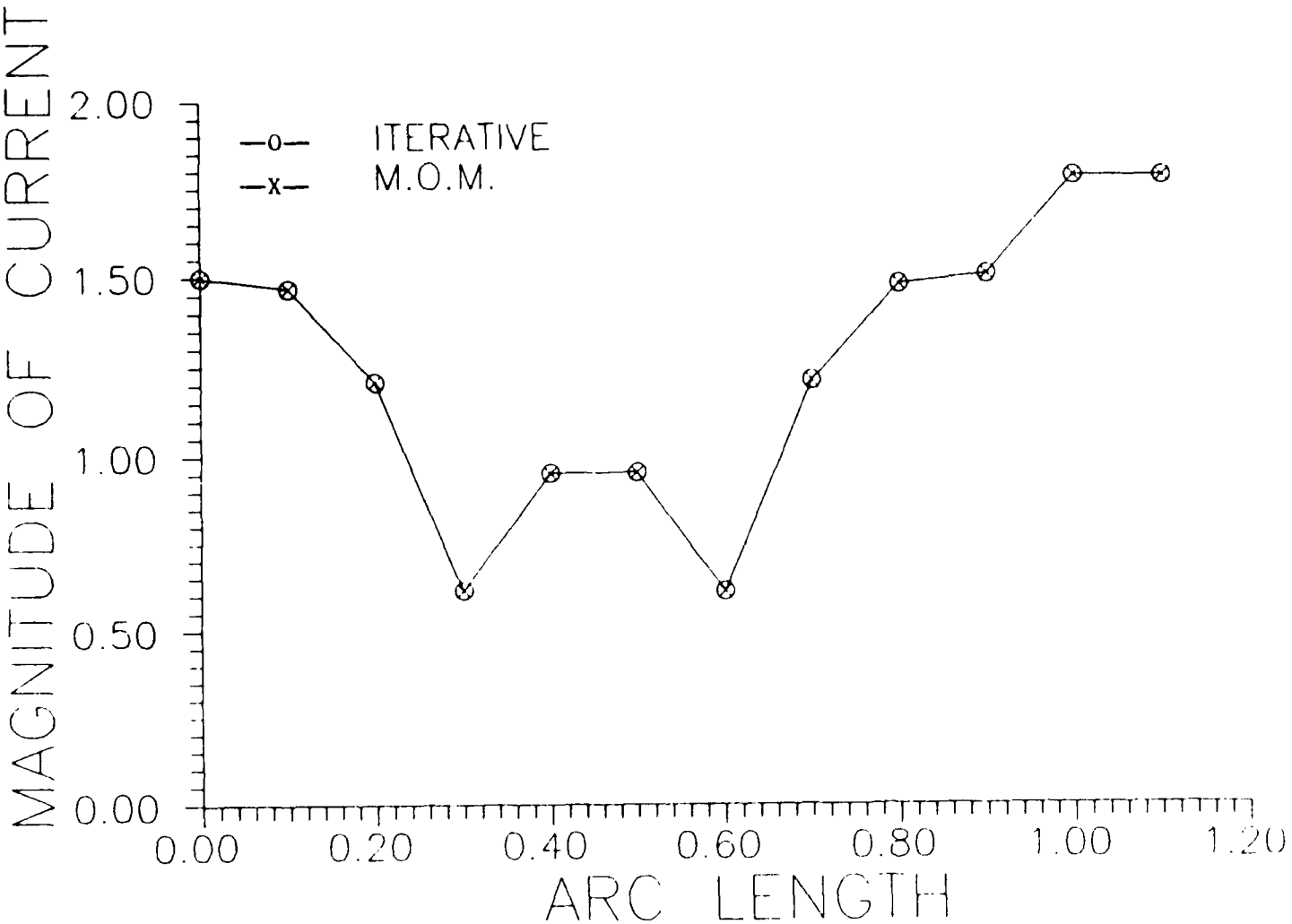

Figure 6. Currents on perfectly conducting square cylinder, w=0.3λ, PHI=180°.

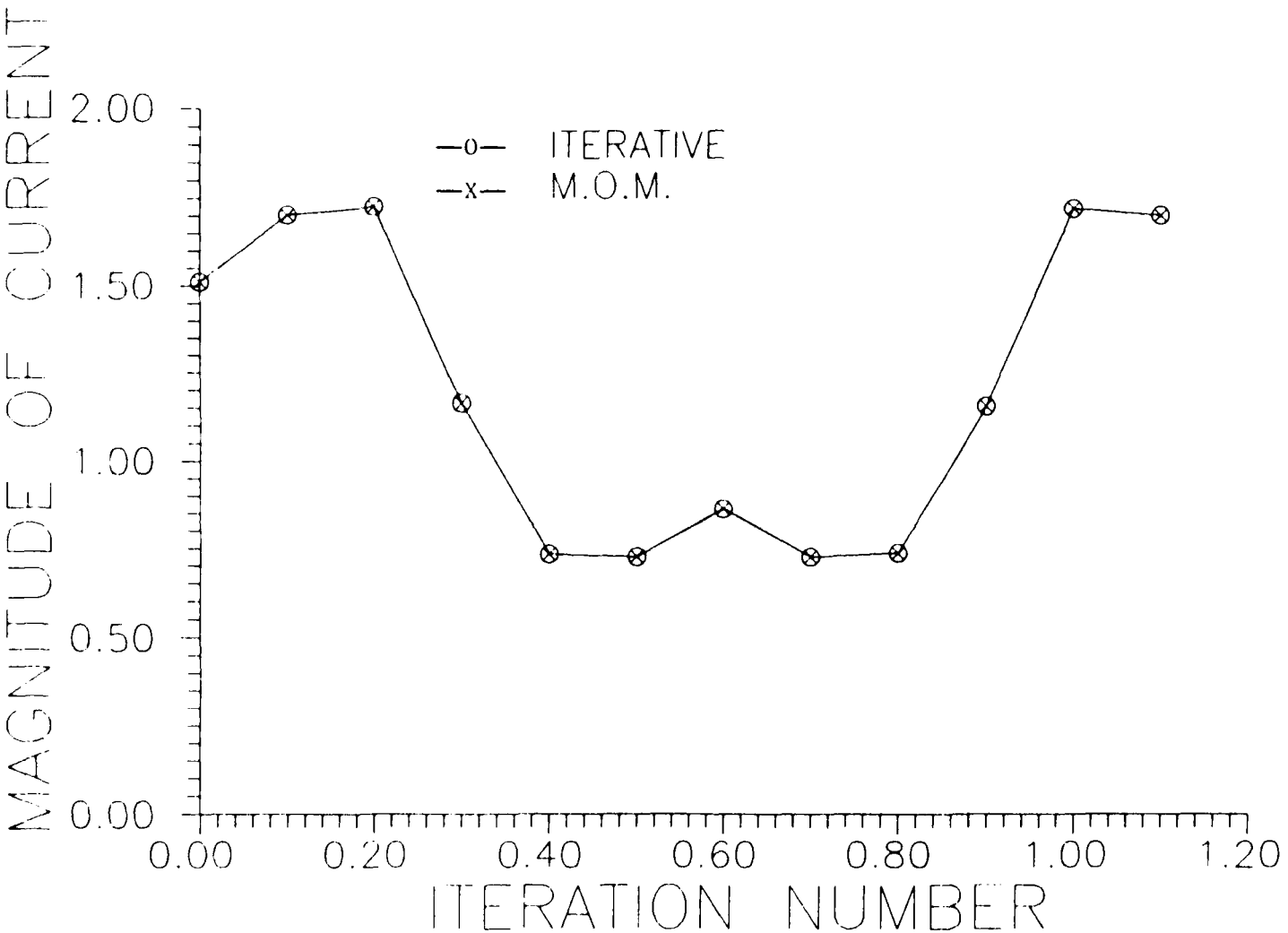

Figure 7. Currents on a perfectly conducting square cylinder, w=0.3λ, PHI=222.5°.

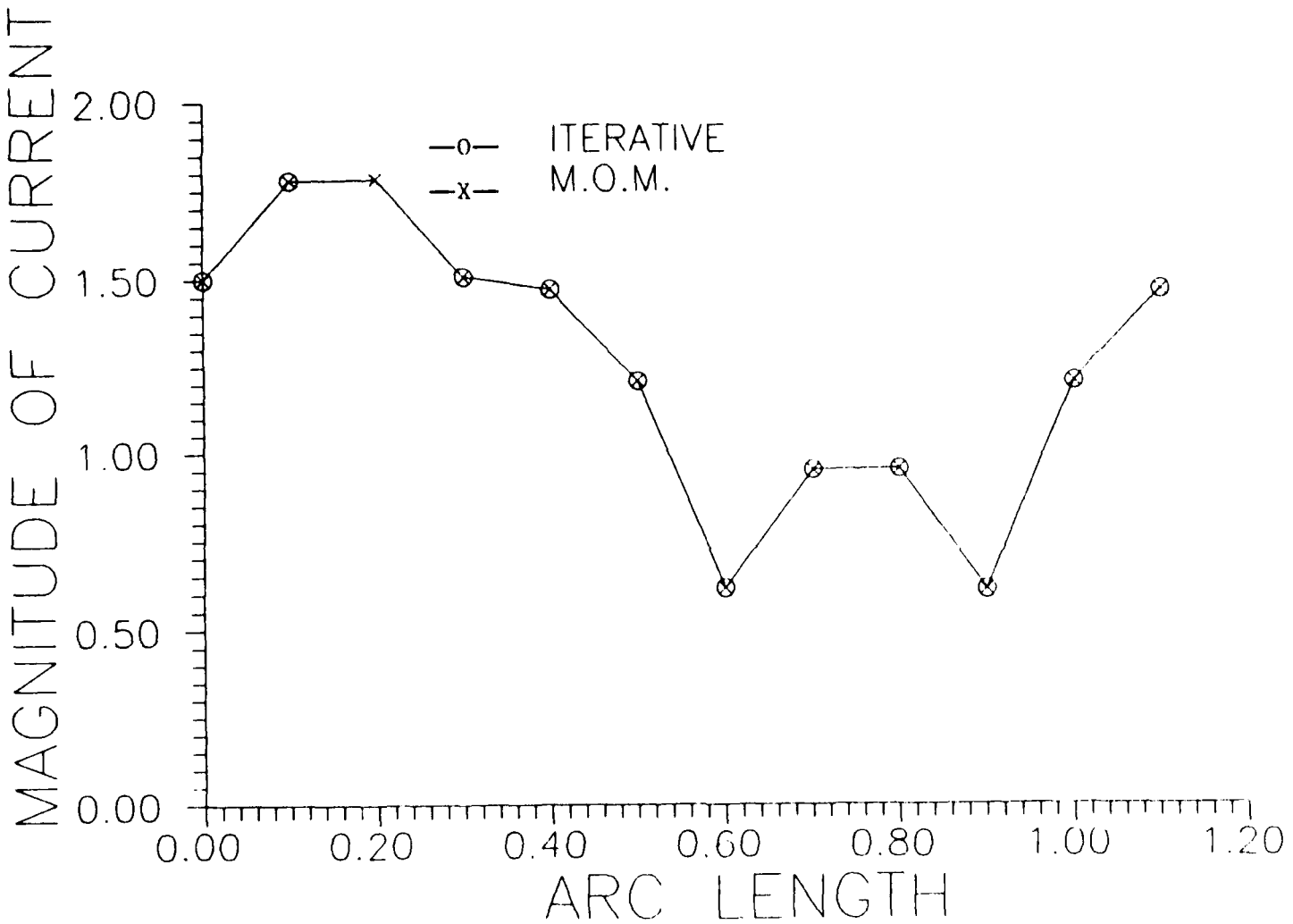

Figure 8. Currents on a perfectly conducting square cylinder, w=0.3λ, PHI=227°.

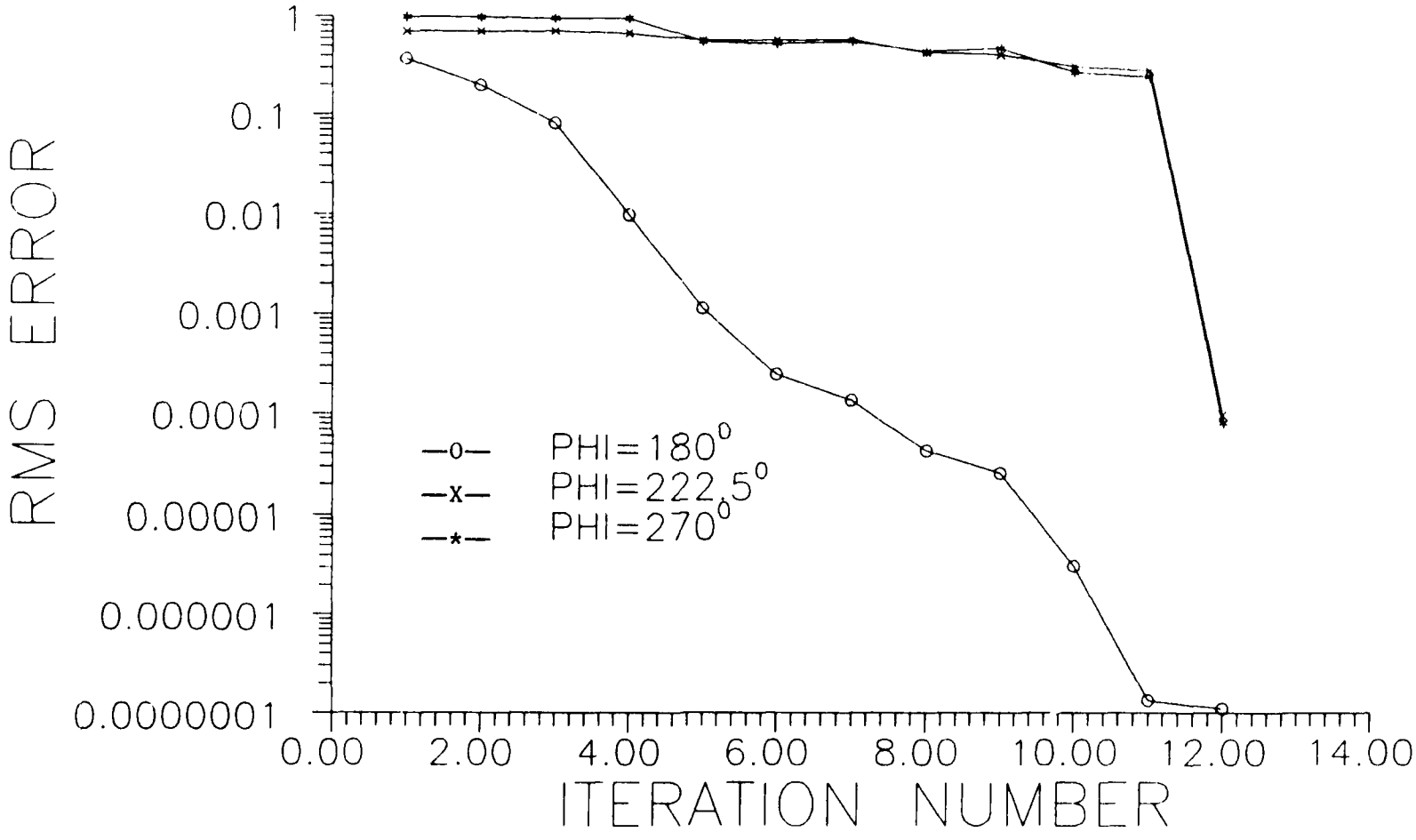

Figure 9. RMS Error Vs Iteration Number for three different excitations of square cylinder, w=0.3λ.

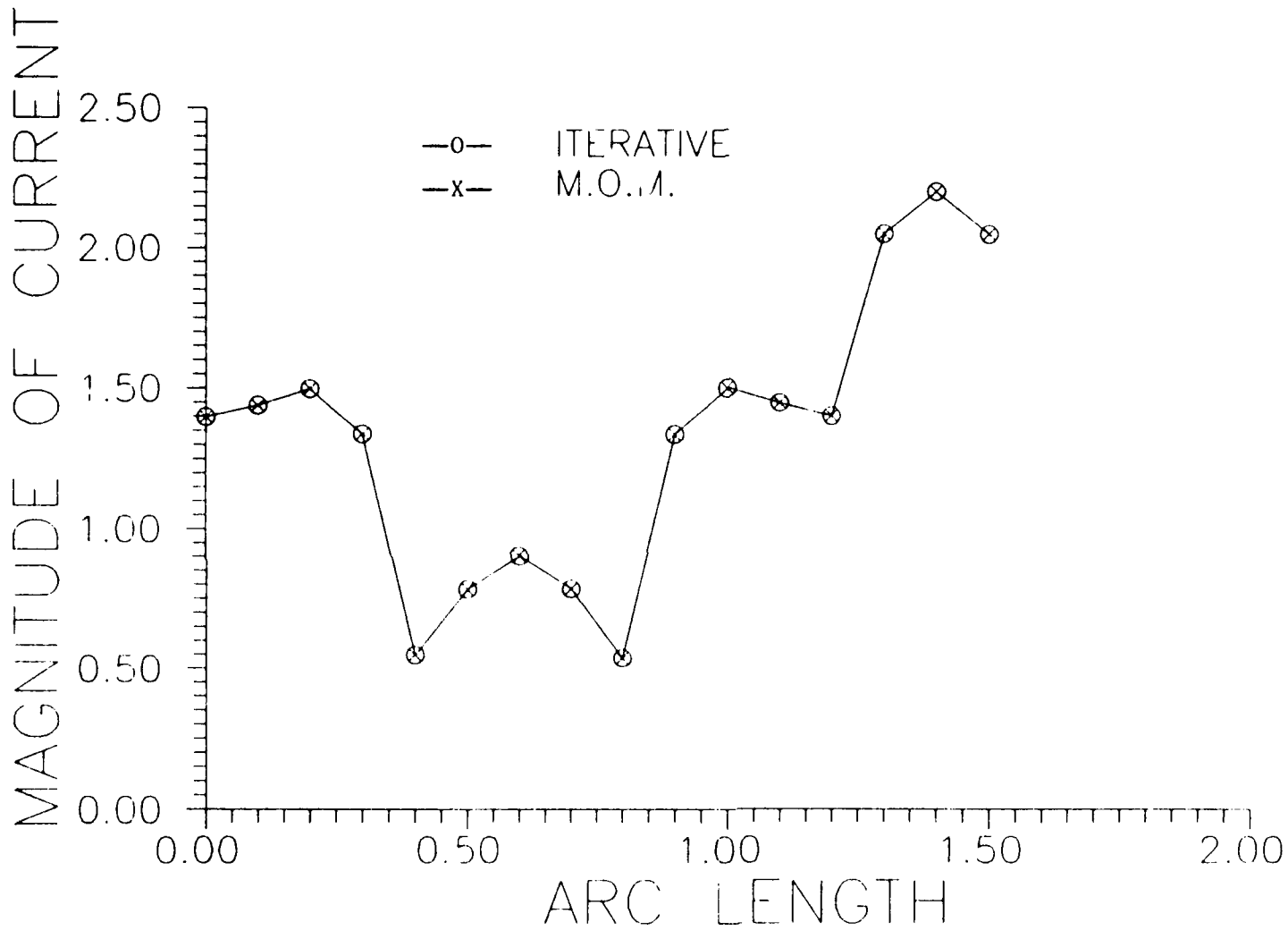

Figure 10. Currents on a perfectly conducting square cylinder, w=0.4λ, PHI=180°.

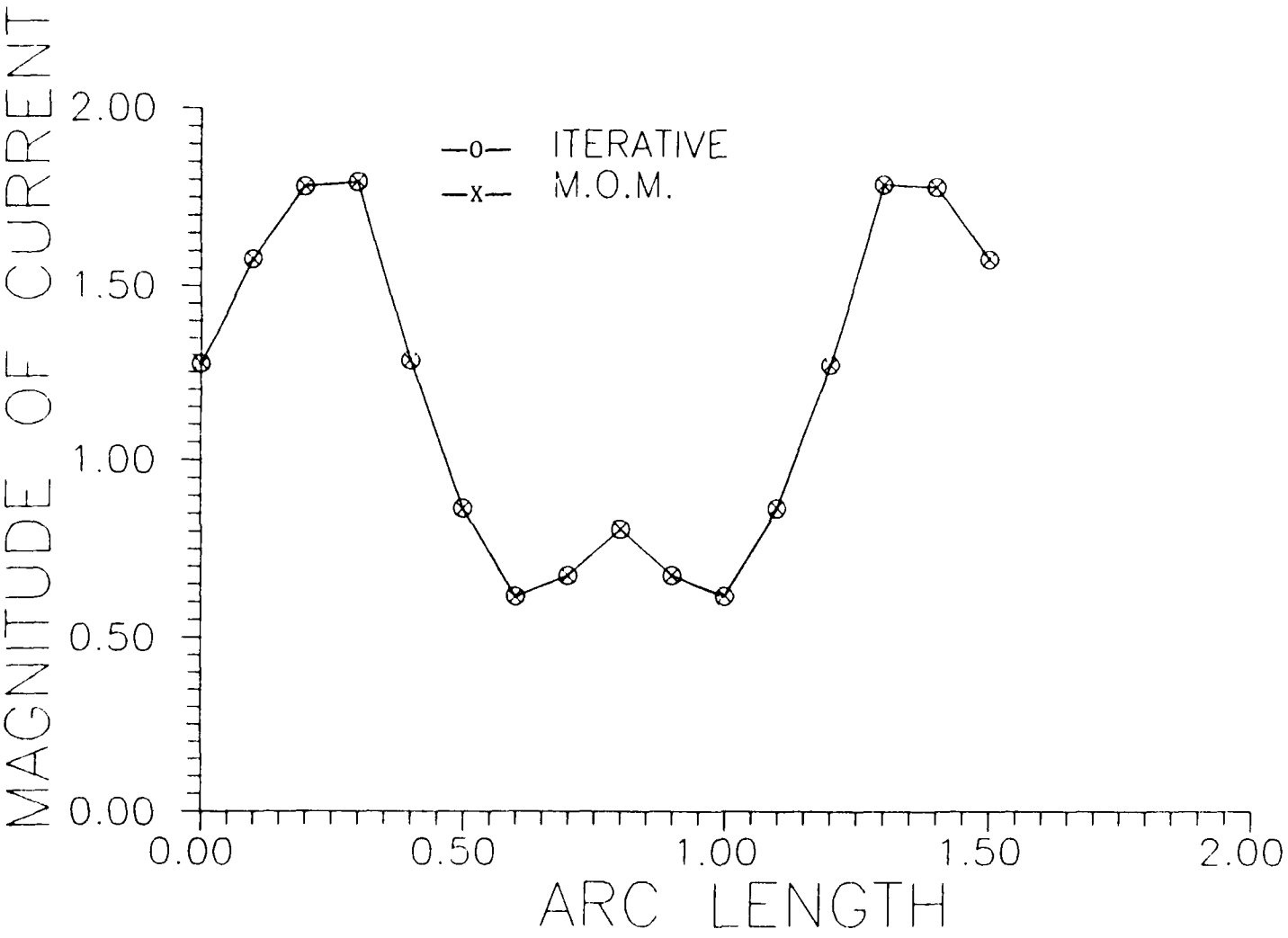

Figure 11. Currents on a perfectly conducting square cylinder, w=0.4λ, PHI=222.5°.

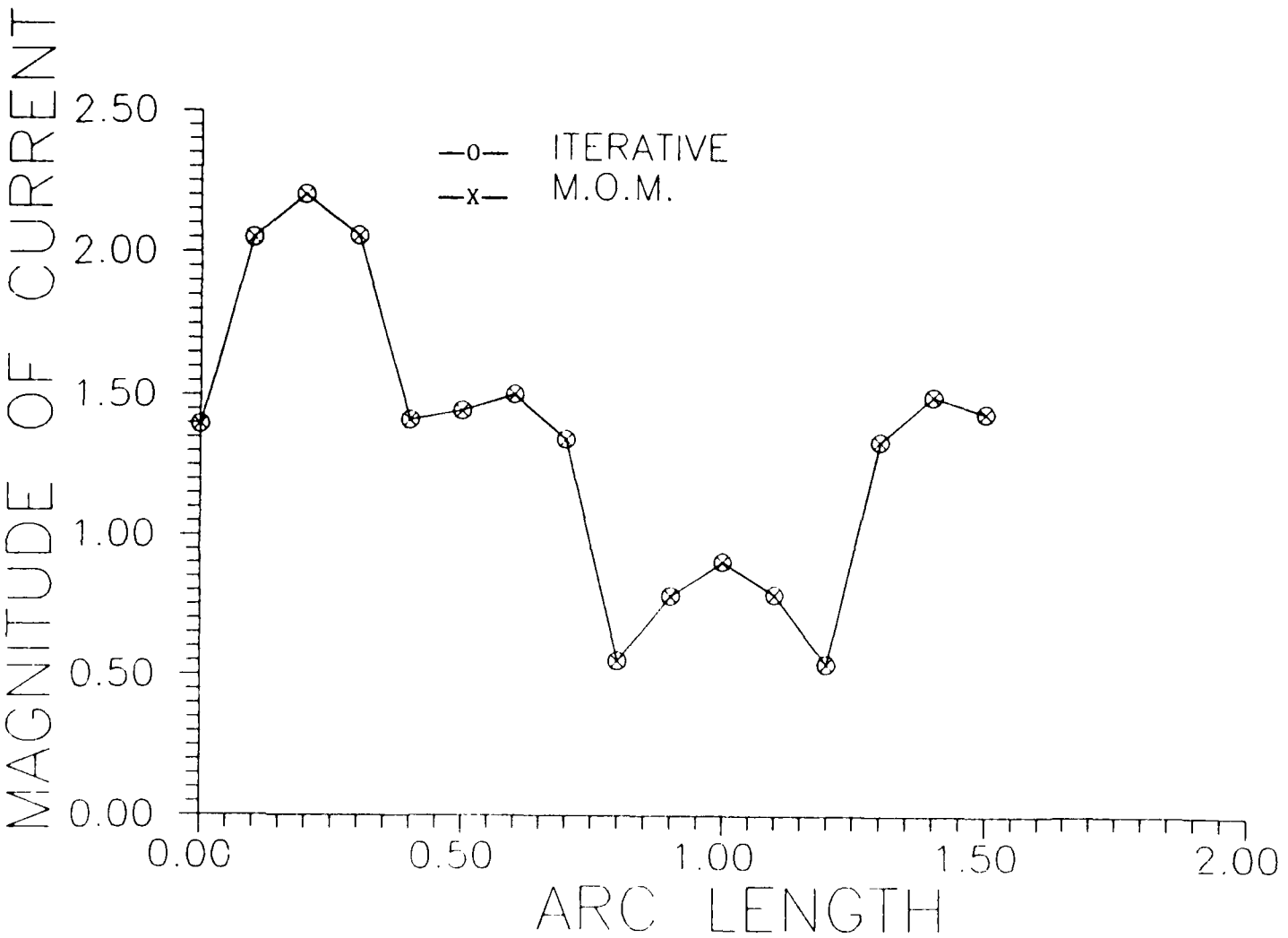

Figure 12. Currents on a perfectly conducting square cylinder, w=0.4λ, PHI=227°.

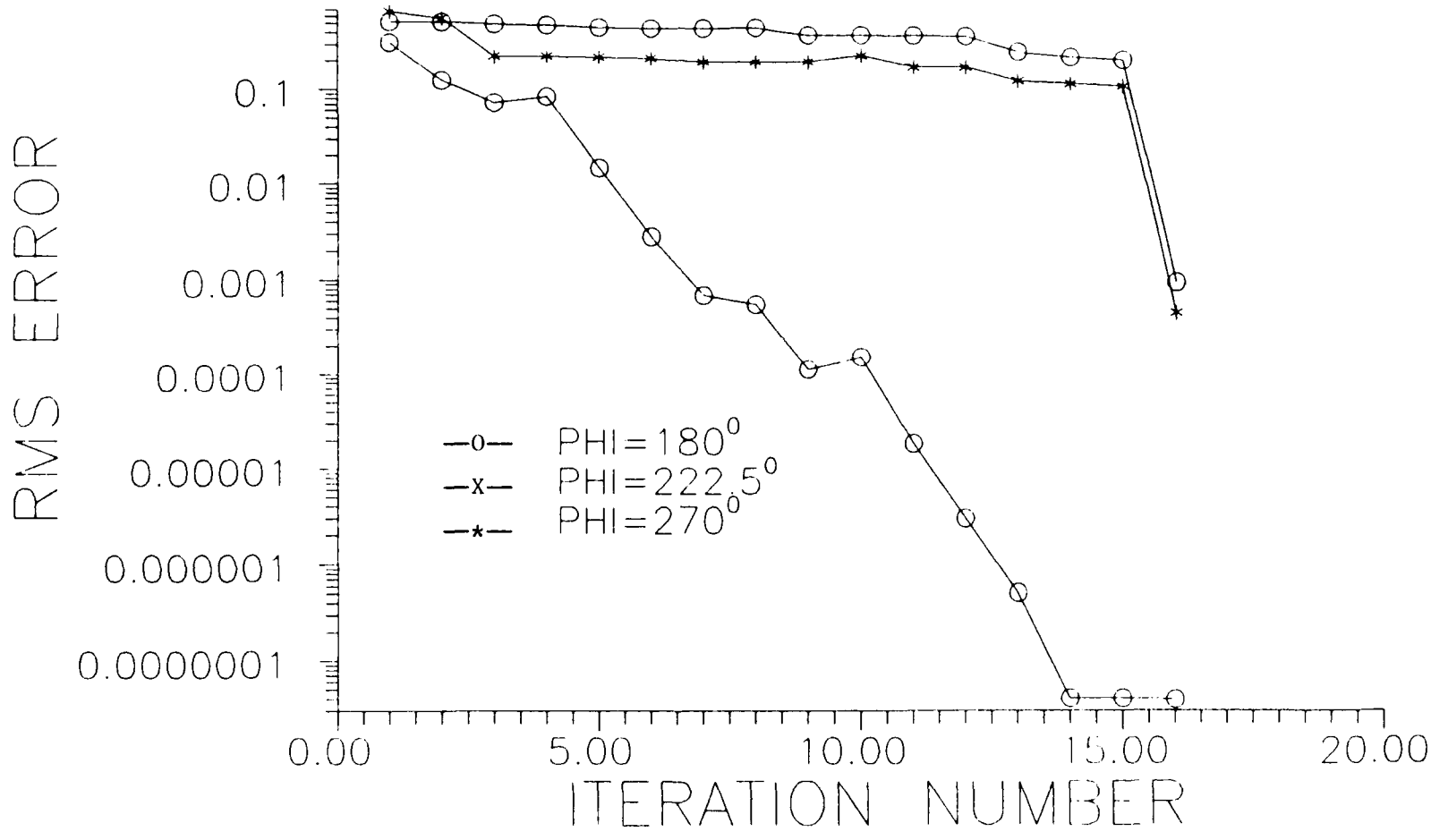

Figure 13. RMS Error Vs Iteration Number for three different excitations of square cylinder, w=0.4λ.

SECTION 4

DISCUSSION

In this report we have developed the theory of inverse matrix operator and inverse integral operator. We have shown that it is possible to compute the solution iteratively for multiple excitations rather than just a single excitation.

While the theory is complete, several numerical problems exist. The basis of the inverse operator is the construction of a proper orthogonal set of vectors. In a finite precision machine, as the iteration progresses, orthogonality is slowly lost. If the vectors were not A-orthogonal, the theory developed is not applicable and the computations become meaningless. Hence, efforts must be made to monitor the orthogonality and when the loss of orthogonality is signaled, the iteration must be re-started.

Paige [15] had shown that loss of orthogonality is not simply because of accumulation of round-off error. Rather it is because of the combined effects of round-off error and convergence of a few eigenvectors. In fact, the losses of orthogonality occurred only along the directions of converged vectors. We recommend therefore the inclusion of selective orthogonalization rather than complete re-orthogonalization which is numerically expensive. As the computation continues, each loss of orthogonality becomes a signal that a new eigenvector is available. This eigenvector is computed and is removed from all later q's as otherwise an image of the eigenvector appears again and again.

Much further work needs to be done to develop the inverse operator theory. It would be very interesting indeed to compute the inverse integral operator; That is, implement the scheme discussed in section 2.6.1. If this were to be successful it completely eliminates the need to explicitly reduce the integral equation to matrix equation.

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