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# STOCHASTIC METHODS IN PROTECTIVE STRUCTURE DESIGN: AN INTEGRATED APPROACH

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Protective structures designed to withstand the effects of conventional (nonnuclear) munitions are built primarily according to deterministic design procedures. Real world variabilities in site characteristics, structural attributes like strength and stiffness, and weapon delivery characteristics are generally not accounted for in current design schemes. This report shows the feasibility of developing a balanced design tool which: (1) takes into account the natural random variability of quantitative design parameters; (2) provides a framework for assessing the uncertainty in nonrandom issues such as modeling and boundary conditions assumptions; and (3) accommodates flexibility in modeling various structural response physics caused by evolutions in weapons environment. This report presents results illustrating the advantages of considering variability in the design process. A proposed integrated design system appears feasible which would explicitly show how: information from design handbooks, research findings, and expert knowledge, and stochastic method can be collected together in one place. Design tools, (continued)

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which are capable of considering the influence of natural variability in materials and loads on cost and survivability, will have tremendous value to the Air Force in their planning cycles for new hardened facilities and for necessary field modifications to existing structures.

## PREFACE

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# SECTION I

# INTRODUCTION

## A. OBJECTIVE

The research objective of this SBIR Phase I effort was to demonstrate the feasibility of developing a more balanced and effective tool for designing protective structures to conventional weapons effects which: (1) can take into account the natural random variability of quantitative design parameters; (2) can assess uncertainty in nonrandom issues such as modeling and boundary condition assumptions; and (3) can accommodate flexibility in modeling structure behavioral changes (different physics of response) caused by evolutions in the weapons environment.

The flowchart in Figure 1 illustrates, conceptually, the scope of the SBIR project goal to develop an integrated design system. The areas within the dashed zone highlight the part of the overall problem which was addressed in Phase I to show feasibility of the overall project. Figure 1 indicates that the overall design issue involves more than just the manner in which the uncertainty in the pertinent design variables is quantified (be it random or nonrandom), but it also involves the way in which the results of the numerical analyses are interpreted (automated reasoning) and with the larger problem of incorporating nonnumeric information (expertise, judgment, local design practices, etc.) into the design.

## B. BACKGROUND

### 1. The Design Problem

Protective structures designed to withstand the effects of conventional (nonnuclear) munitions are built primarily according to deterministic design procedures. These procedures assume precise knowledge about the parameters that play a significant role in the structure's final design. Real-world variabilities in site characteristics, structural attributes like strength and stiffness, and weapon delivery characteristics are generally not accounted for in current design schemes. In fact, current design procedures are overly conservative in that, to ensure high confidence in sustaining the facilities' mission, they presume a "worst-case scenario" in selecting values for design parameters. For example, some typical "worst-case" presumptions would involve underestimating structural strength, overestimating the loading imparted to the structure, ignoring complex details like three-dimensional and nonlinear effects, overestimating joint stiffness, and ignoring issues that typically are not quantitative in nature. Examples of the latter would include construction quality control, weather conditions during construction, ad-hoc construction changes and short-cuts, and the validity of the design procedure to emulate real-world situations.

The problem is compounded by the fact that the loading is caused by a weapon environment whose effects on a structure are highly variable, and where significant changes in some parameters can require a complete change in the design model. For example, peak interface pressure at a soil-structure interface is inversely proportional to the distance from the explosive source. But at some point when the explosive source gets very close to a structural element, the response behavior can change from a forced-vibration problem to a wave-propagation, breaching, or penetration problem. A simple Monte-Carlo simulation of the range versus pressure function would completely miss this change in behavior unless the correct phenomena are captured within the design framework.

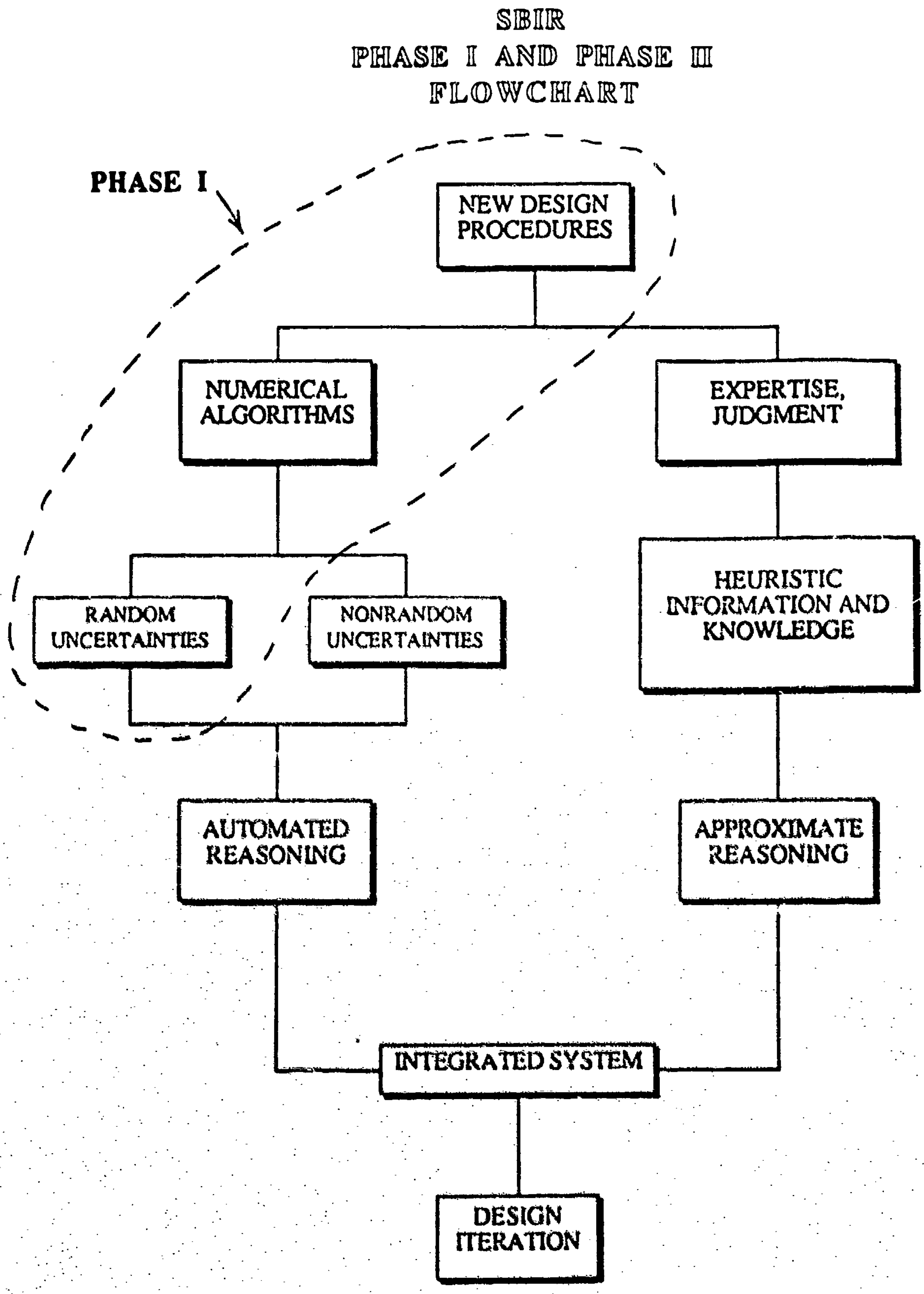


Figure 1. Project Scope

Furthermore, current design procedures are based on a deterministic set of parameters describing loadings based on standard and documented weapons configurations. If the problem of interest requires knowledge about potential future weapons configurations, or if the configuration is not documented, then the designer's only recourse is to interpolate between, or, worse, extrapolate beyond existing situations.

Based on the problem described here it is evident that a balanced (in the sense of accommodating various sources of data, knowledge, and design methods) and effective design tool is needed. This new tool would address random and nonrandom variability and would be flexible in use and adaptable to changes in user requirements.

## 2. Extent of Uncertainty in Design

Consider a typical problem of designing a facility to survive a conventional weapons threat which might be vaguely specified, to be built in a NATO-based country, and to be sized according to an unknown budget. The designer must consider several real-world "nonalgorithmic" issues. Some of these issues might include: local material availability, constructability (things are built according to different practices in Germany and Turkey, for example), local restrictions on architectural, esthetic, planning, zoning and environmental concerns, and built-in flexibility to future threats. It is easy to ignore these non-algorithmic issues, but the development of a new design algorithm should be adaptive to changing requirements if it is to achieve acceptability.

Another issue is that there are at least five types of designs which meet a specified threat. These design types are: (1) hardened (usually for nuclear threats), (2) protective (usually for a direct impact from a conventional threat), (3) semi-hardened (usually designed to some stand-off distance from a conventional threat), (4) splintering (expected fragment damage), and (5) collateral (damage based on proximity to a higher-priority target). It is not uncommon for a client (USAFE, NATO, etc.) to want a structure to be designed to survive a combination of the threats mentioned above.

After decidingwhich design type is to be considered for a particular structure (or a modification to an existing structure, for that matter), one must consider the variability (uncertainty) in the parameters of the threat, the loading mechanism, the structural response, and the survivability of the structure contents. It is also necessary to estimate the uncertainty in the design algorithm itself (e.g, an empirical relation, a finite element model, boundary conditions, etc.), which currently is rarely considered.

In the determination of the threat there are numerous options and, within each threat option, tseveral parameters should be considered in terms of their own uncertainty [Reference 1]. There are several kinds of projectile weapons such as small arms, direct fire weapons, armor piercing (AP) solid shot and capped projectiles, HE shells, mortar shells (such as the large Soviet 240 mm used against R/C structures), and grenades. Several kinds of bombs exist: general purpose (GP), AP and semi-AP, fuel-air explosives, light-case, fire and incendiary, special-purpose (chemical), and dispenser/cluster types. Rockets and missiles exist as tactical and battlefield (U. S. LANCE, Soviet SCUD and FROG) types. Special purpose weapons such as fuel-air munitions, shaped-charges, cratering charges, and heat-gas types are prevalent. And if these categories aren't enough to contend with, the new and evolving weapon threats are more accurate and "smarter". Smart weapons include such features as target hardness sensors which alter the fuzing option according to target rigidity, and damage mechanism sensing where a two-stage weapon will first penetrate with a shape-charge, then detonate at depth with an HE warhead. The newer threats make it increasingly important to consider the synergistic effects of airblast, fragmentation, breaching, spall, etc. as the expected miss distance decreases for these threats.

Another factor to be considered is the uncertainty in the propagation of these weapon effects through the medium surrounding the structure. For above-ground structures this would include a characterization of the airblast, the fragmentation and thermal (and/or chemical) patterns, penetration mechanisms, and the near-surface ground shock propagation. For buried structures this would include a characterization of coupling issues, ground shock, cratering, ejecta, and sub-surface munitions fragmentation characteristics. For example, for airblast the uncertainty in explosive type, in cube-root scaling, in blast-wave phenomena, and in the nature of pressure increases internal to the structure all should be considered.

After propagation effects are considered the resulting loads on the structure must be computed. This is a critical point in a typical design, because the structural loads are a function, not only of the propagation path of the disturbance (airblast, shock, penetration) but also, of the physics of the phenomenon. The physics of a pressure-force relation are different from those of a fragment-penetration relation and the specification in the load environment should accommodate changing physics due to the uncertainty in some parameters; for example, in the miss-distance.

The structural response and associated internal component response is generally decoupled from the specification of the external loads in most design algorithms. This is a mistake since it ignores the interaction effect. The assumption of a flexural response of a slab due to an air-blast pressure might be appropriate for a deterministic design, but in the stochastic case the variability in the miss-distance could produce a situation where a direct-shear failure near the slab support is the governing failure mechanism [Reference 2]. In this phase of the design, special attention needs to be focused on the fact that most design criteria in Europe and the Pacific basin do not account for dynamic phenomena directly, but rather use design "factors" to account for such things as dynamic material properties, higher-mode response, and the relation between the frequency content of the disturbance and the frequency and non-linearity of the response. The response of internal objects is also based on assumptions of rigid-body response without consideration due to the uncertainty in local behavior, joint behavior, and the influence of damage locations on rigid-body behavior.

### 3. Stochastic Methods

Figure 2 shows there are three general classes of probabilistic approaches used in the treatment of stochastic processes. The first class is related to direct statistical simulation. This procedure, often called the Monte Carlo (direct) method, simply takes the deterministic design algorithm, assumes several of the parameters to be described by any of a family of probability distributions (Gaussian, Beta, exponential, Poisson, lognormal, etc.), usually assumes independence among the components of the system, and generates (through a step-by-step invocation of a random number generation) a probability distribution of the output parameters of the design process. Moreover, direct simulation often ignores the variability in the uncertainty of a parameter with time and it can account for the correlation among components (as opposed to parameters) of the design system if suitable correlation information is available [Reference 3]. This method is powerful but can consume considerable computational time on complex problems involving hundreds or more degrees of freedom. Monte Carlo simulation is intuitively appealing, in the sense that its process does not alter the structure of the deterministic design algorithm. In fact the mean values of Monte Carlo simulations generally converge to the quantities of a deterministic analysis. However, these simulations make extensive use of some limiting

## STOCHASTIC PROCEDURES

**CLASS 1**

- Direct Monte Carlo Simulation
- Correlated Simulation (PASS)

**CLASS 2**

- First-Order, Second Moment
- Partial Difference
- Point Estimates by Probability Moments
- Response Surfaces

**CLASS 3**

- Random Differential Eqns. (Random and Stochastic)
- Random Difference Eqns.
  - Stochastic FEM
  - Finite differences
- Approximate Models
  - Markov Chains
  - Renewal Processes
  - Queuing models

Figure 2. Classical Stochastic Procedures

assumptions on physical behavior. Despite these limitations, direct simulation provides a convenient benchmark for the comparison of other proposed probabilistic methods, especially in the absence of precise analytic solutions as benchmarks.

The second class of methods shown in Figure 2 can be generically described as point estimate procedures. These methods attempt to describe the underlying uncertainty in a design by exercising the design model a limited number of times (or at a few points in the n-dimensional Euclidean space described by the "n" parameters of the model) to yield some information about the first and second moments (generally, the mean and variance) of the process. If one only knows the moments of the basic parameters, these linear methods allow the propagation of these moments through the design process. Examples of these methods include the *first-order, second-moment* (FOSM) methods, partial difference methods, the *point estimates by probability moments* (PEPM) methods, and the response-surface methods [Reference 4]. The other methods make assumptions of independence and the general Gaussian character of design results because of the strong influence of the Law of Large Numbers (for problems with numerous random variables).

The third class of stochastic methods (see Figure 2) could be described, generally, as random differential equations. A subset of these consists of the stochastic differential equations based on their association with "white-noise" processes. This class of methods is predicated on the assumption that the design paradigm is known well enough to be described analytically with differential calculus. These methods are general in the sense that they can accommodate random parameters, random boundary conditions, and random forcing functions [Reference 5]. Their disadvantage is that exact solutions exist only for exceedingly simple physical systems (simple linear, first-order equations) [Reference 6].

One approach to the solution of this class of methods is to approximate the differential equations with difference equations. There are numerous approaches to the solution of these equations, including stochastic finite-element methods, finite difference methods, markov chains (discrete and continuous), renewal processes (Poisson and Markov), queuing models and others. Previous studies [Reference 5] have shown that finite difference solutions to general random equations are valuable because of their ability to provide solutions to non-linear equations and because exact solutions do not exist for most problems. Furthermore, the error due to replacing the differential operator with the difference approximation algorithm is small compared to the sample-to-sample variation (which is usually due to the noise in the random number generation scheme). Although these difference methods do require a significant sample size for the computation of the actual response process (the sample paths of the equations), the calculation of the first and second moments of the response is less laborious.

In summary, numerous methods are available for the solution of protective structure design problems. Several of these methods are reviewed as a function of their class in the attached table. These classes of methods differ according to the desired sophistication of the solution, to the difficulty of the design algorithm (i.e., a simple equation, an empirical relation, the exact differential formulation, or a finite element approximation), and to the uncertainty characterization of the parameters of the design.

## C. SCOPE

This Phase I feasibility study considers three main points that are considered in the development of stochastic procedures to account for random variability in design. These points are: 1) the identification of the kind of uncertainty, 2) the characterization of this uncertainty and, 3) the propagation of this uncertainty through the design process. The first point is quite a difficult task given that so many variables and changing requirements make the identification process a continuously evolving task. The second point emphasizes that not all uncertainty can be correctly characterized as being random. Many of the issues which are important in the design process are not random, such as construction quality, local design standards, budget and footage constraints and many others. The third point discussed above has to do with the formalisms used to numerically describe the uncertainty in the design parameters and the design model (i.e., design algorithms) and to computationally propagate this uncertainty through the design. Once the uncertainty in the final design outcome is determined, due to the random variability in the parameters and procedures, it is possible to address the critical issue concerning the relationship between construction or rehabilitation cost and expected survivability.

## SECTION II

## PHASE I RESEARCH

### A. TECHNICAL TASKS

Task 1 - Review current stochastic procedures for their applicability to protective structure design. This task will review currently available stochastic models. Develop and specify an n-dimensional domain of possible loadings, geologies, interaction-effects, structures, failure modes, and design procedures and the possible uncertainty ranges for all these parameters. This domain will then be "down-sized" for focus within the Phase I effort. Identify what parameters and procedures are random and which might be nonrandom. For the non-random uncertainties consider possible characterizations.

Task 2 - Illustrate some of the statistical/stochastic procedures to verify accuracy, complexity, efficiency, and validity. Select the candidate design paradigms for Phase I from the results of Task 1. Conduct comparison studies and critique the approaches.

Task 3 - Extend the procedures from Task 2 to address a specific design problem associated with conventional weapons effects. Develop the framework for an integrated design approach for conventional weapons protective facilities. *Focus on the utility to the user and the ability to conduct design-cost trade-off studies.*

### B. RESULTS AND FEASIBILITY

In the following discussion of the Phase I research results and the demonstration of the feasibility of the development concept, the results of the three research tasks mentioned above are presented. In addition, the significance of these results in determining the feasibility of the overall goal of developing a balanced and integrated design tool is also discussed.

#### 1. Task 1 - Review Procedures and Develop Uncertainty Space

##### a. Stochastic Methods

The designs encountered in this Phase I research were restricted to the Class 1 and Class 2 procedures shown in Figure 2. The procedures characterized as Class 3 were particularly difficult because of the special structure required for the particular problems they solve. For example, stochastic differential equations are used to analyze problems that have a white noise input process. Also the Markov chains require problems which show one-step dependency. Because of the special problem-structure required of these methods reference is made to them here only for completeness. For some special problems and situations the Class 3 procedures may be used [Reference 5].

The Phase I research considered the following four stochastic methods, as they are representative of both Class 1 and Class 2 methods and they are readily adapted to design-type problems, where parametric uncertainty plays a key role.

* Monte Carlo simulation
* Partial Derivative
* Point Estimates by Probability Moments (PEPM)
* Response Surfaces

The last three methods are all considered as first order, second moment methods (FOSM) as described in Reference 4. The Direct Monte Carlo method involves a large number of theoretical realizations of the design model and the results are generally analyzed in terms of the mean and variance of parameters of interest within the model. The partial derivative method (also known as the Taylor's series expansion method) involves the expansion of the design equation in a power series about the mean value of a parameter of interest. This method is valid only for analytically described processes (such as an equation) and for small excursions of the variables away from their mean values. The first and second moments of this parameter are then computed directly from the polynomial describing the expansion.

The PEPM method is based on the assumption that the first three moments of all independent variables are known prior to the determination of the first two moments (mean and variance) of the dependent variable. This method approximates the true density function of a random variable as Dirac-delta functions located one standard deviation (points) from the mean for all the variables. These delta functions therefore approximate the actual random realizations of the process at these points. The response surface method is similar to the PEPM method in that it approximates the true response at only a few discrete points in the solution space, but the response surface method can sample a random variable at more than two discrete points. Both the PEPM and response surface methods can be used to estimate the correlation among random variables and, unlike the partial derivative method, are both well suited to problems which do not require analytic solutions but rather require numerical treatments.

The four stochastic methods are compared numerically to one another for a specific design example in Task 2. A comparison of the value of each of the methods and their impact on the overall design goal is also assessed in Task 2.

b. N-Dimensional Space of Parameters

The variability in all of the parameters discussed in the review section above could be mathematically equivalent to an n-dimensional space of parameters and issues for consideration in the design. This n-dimensional space can be loosely described as the mapping of numerous variables onto the domain of design parameters. For example the following equation,

$$\phi = \beta\,(w_1, w_2, \ldots, a_1, a_2, \ldots, p_1, g_2, \ldots, R_1, R_2, \ldots, \alpha_1, \alpha_2) * \delta\,(\eta, \varphi, \ldots, \kappa)$$

can represent this n-dimensional mapping, as the function $\phi$, which is specified in terms of a functional on the algorithmic parameters of the design paradigm, $\beta$, and a functional on the non-algorithmic issues of the design paradigm (local codes, local constructability, etc.),, $\delta$. For the algorithmic parameters uncertainty can be considered in the threat, w, the airblast, a, the penetration, p, the ground shock, g, the various pertinent response modes, R, and internal component (equipment, people) responses, $\alpha$. The uncertainties on the nonalgorithmic parameters, $\eta$, $\varphi$, $\kappa$, (which are <u>not</u> be considered in Phase I) could be prescribed with a logic such as evidence theory or fuzzy set theory, and could be incorporated in a general probabilistic framework similar to works reported earlier [Reference 7].

Obviously the design problem can be made hopelessly intractable by considering all possibilities in the n-dimensional domain of design parameters and issues.

The task in Phase I was to specify a reasonably reduced scope, or subset, of this large n-dimensional space and address the parameters of the subset to show feasibility of the approach. Figure 3 provides an overview by showing six major design areas involved in determining the survivability of protective structures to conventional munitions effects. Many of the parameters in these six areas have been discussed above. Each of the six areas shown in Figure 3 are detailed in the form of six tables in Appendix A. These tables (Tables A.1 to A.6 in Appendix A) provide the pertinent variables of importance in the design of protective facilities and the kind of uncertainty generally apparent in these variables.

The first "box" in the flowchart in Figure 3 involves the choice of a design model. The type of model introduces some uncertainty because the model is the engineer's abstraction of the real world and his choice of a model is based on certain implicit assumptions about the validity of such things as material behavior and boundary conditions. Model selection is primarily nonrandom where the uncertainty is a combination of ignorance, judgment, imprecision and the use of linguistic values. For Phase I, the models were deterministic, hence there was no attempt to assess modeling uncertainty. This issue, however, will be addressed in Phase II. The flowchart in Figure 3 also highlights the parameters addressed in Phase I. By focusing Phase I at just the random uncertainties (stochastic treatment) portion of the numerical design algorithms the feasibility of the overall (Phases I and II) approach has been demonstrated by tackling perhaps the "toughest" part of the flowchart.

A significantly reduced domain of the uncertainty space of the "n-dimensional domain" was developed for Phase I to illustrate the overall feasibility of the Phase I ideas. Certainly, not all of the parameters, design models, and various failure modes have to be addressed in Phase I to demonstrate feasibility. In Phase I the "n-dimensional" character of the uncertainty space was reduced to a more physically intuitive "Three-dimensional space"(see Figure 4). Each of the three axes in this figure represents a scale of uncertainty for the three issues considered important for demonstrating feasibility in Phase I. The first axis needs no explanation, and represents the uncertainty in the parameters of a design. Examples of parameters would include airblast shock velocity, standoff distance, weapon explosive yield, concrete strength, steel rebar strength, wall thickness, etc. and four possible methods to address parameter uncertainty have been discussed.

The second axis in Figure 4, called model sophistication, illustrates the uncertainty involved in choosing among different design modeling procedures, each having a different level of sophistication and different assumption requirements. The four models chosen for Phase I are shown in Figure 5:

* Simple static design algorithm (Fig. 5a)
* Dynamic Single-degree-of-freedom (SDOF) model (Fig. 5b)
* Dynamic Four-degree-of-freedom (4-DOF) model (Fig. 5c)
* Dynamic continuum model (Fig. 5d)

The third axis in Figure 4 involves an attempt to account for various model physics in the same design paradigm. As discussed in the review section, sometimes the physics of the design algorithm changes for the same structure when one or more of the parameters in the model change to certain magnitudes. For example, if the standoff distance between a weapon and a structure approaches zero, the structural response physics will change from a forced-vibration response to a penetration response to a breaching failure. For this effort three model physics are explored:

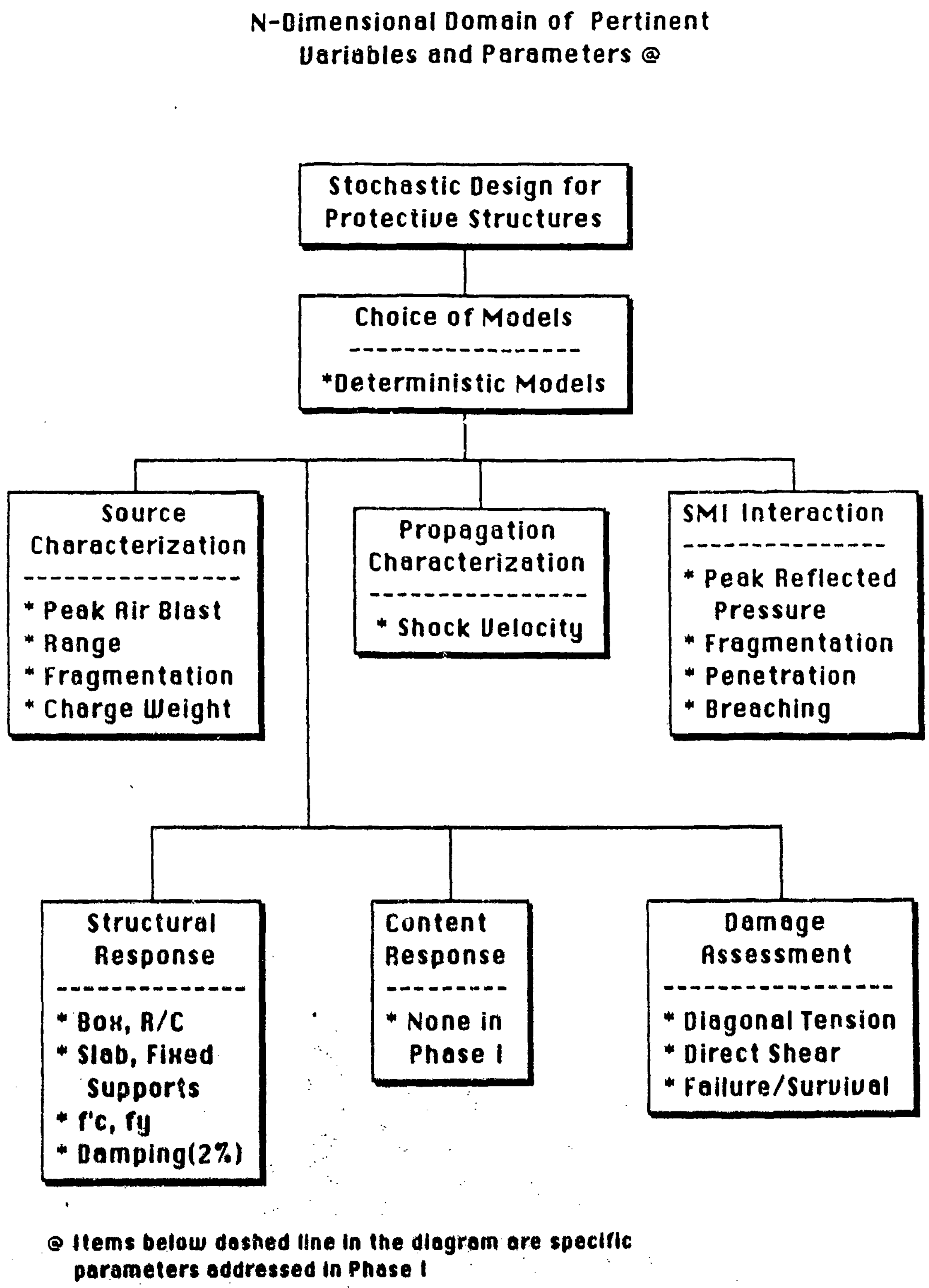


Figure 3. N-Dimensional Domain of Pertinent Variables and Parameters

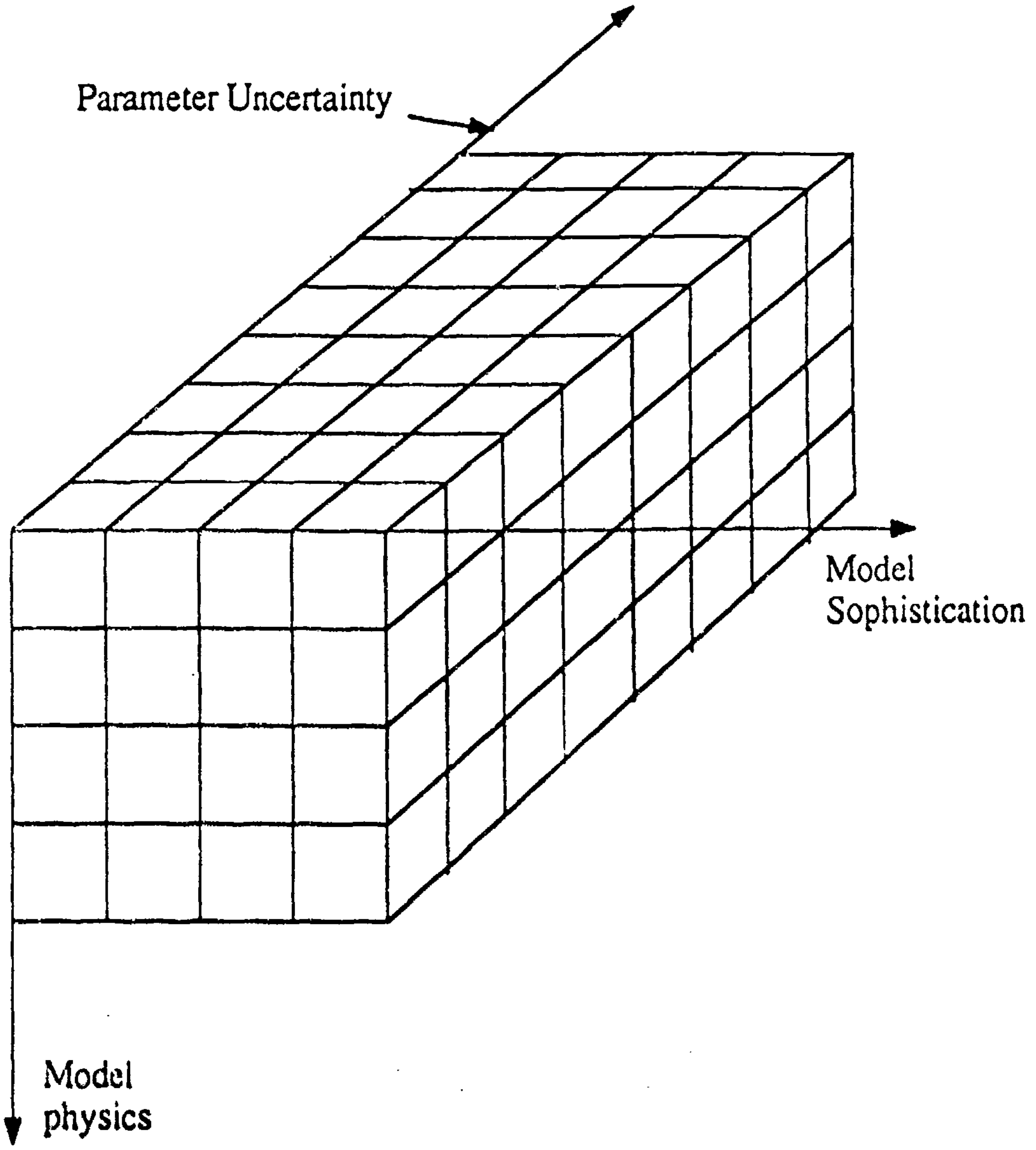


**Figure 4. Reduced Three-Dimensional Uncertainty Space for Phase I**

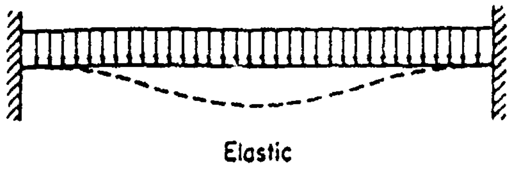


Figure 5a. Elastic Static Model

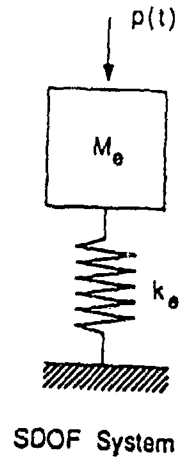


Figure 5b. Dynamic SDOF Model

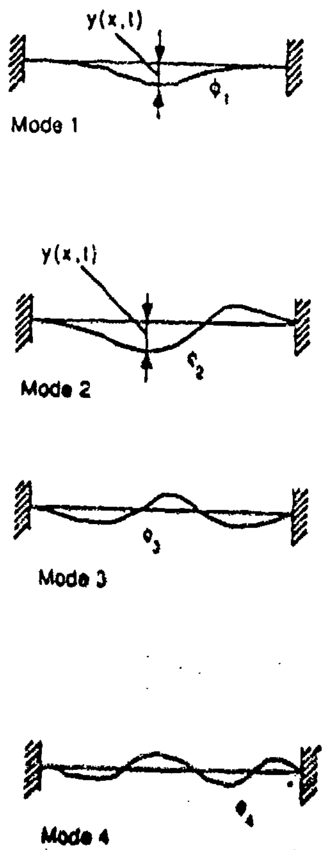


Figure 5c. 4-DOF Model (Normal Mode Approach)

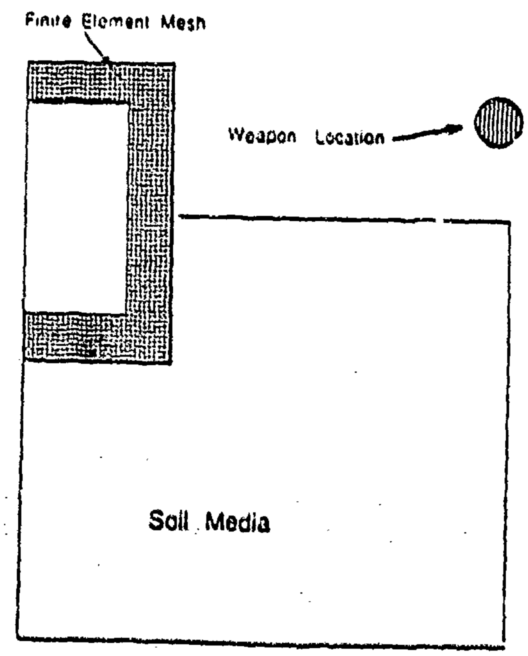


Figure 5d. MDOF Model (or continuum model)

**Figure 5. Four Design Models used in Phase I Research**

* forced vibration failure from airblast
* penetration failure (perforation and spall damage from fragmentation)
* breaching failure (from a near-direct hit)

The uncertainty space shown in Figure 4 can still be quite large. Figure 6 shows how, for a given set of parameters, the two-dimensional space of model sophistications and model physics could be visualized as different combinations of models in the analysis. This space can be thought of as a two-dimensional plane through the solid in Figure 4 for a fixed parameter. In Figure 6, the complexity is reduced somewhat by choosing 4 models of sophistication (n=4) and 3 physics models (m=3), as discussed above.

To further reduce the computational burden, but to keep the essential and pertinent features of the two-dimensional space, the following series of simulations was developed. In the description of these computations (simulations) a great deal of emphasis is placed on the use of empirical design relations whenever possible. These empirical relations are just as useful as sophisticated relations in showing the feasibility of the Phase I work and, more important, are primarily the types of relations normally encountered in real design situations.

c. Series 1 Simulations

Use the ACI shear-strength criteria [References 8 and 9] for a hypothetical beam model and compare static strength results for the four stochastic methods discussed above. The ACI criteria will involve several parameters which shall be considered as random variables. The comparison of the four stochastic methods will be conducted by comparing the resulting probability density functions (PDFs) of shear strength. This will illustrate some features of the parameter uncertainty space as well as some differences in the stochastic methods.

d. Series 2 Simulations

Using the recently successful direct shear-strength criteria [Reference 10] and one stochastic method (Monte Carlo simulation), compare probability of failure results for the four degrees of model sophistication for a specific design example. This will highlight uncertainties introduced when using different kinds of models. The specific design example is a reinforced concrete one-way wall section from a typical, partially buried Air Force facility, as provided by AFESC/RDC. This wall section is shown in Figure 7, and is modeled as a beam, which constitutes a slice out of a one-way slab. The loading on this wall will be modeled as a free-air burst of a 1000-pound General Purpose (GP) bomb. The dynamic load will be assumed to act uniformly across the wall height and it will be modeled in the time-domain as an initially peaked triangular loading. The peak pressure of the load triangle will be the peak reflected pressure, denoted $P_r$, and the duration of the load will be a function of the air-shock speed and the wall height. The peak reflected pressure is determined from the charge weight and range of the weapon and the load duration, $t_d$, is determined from the relation, $t_d = 4*S/U$, where S is the wall height in feet and U is the shock velocity in feet per millisecond. Both $P_r$ and $t_d$ are functions of the charge weight and miss-distance (range), and can be determined from Figure B.1 in Appendix B. In these computations the physics of forced vibration from airblast will be used.

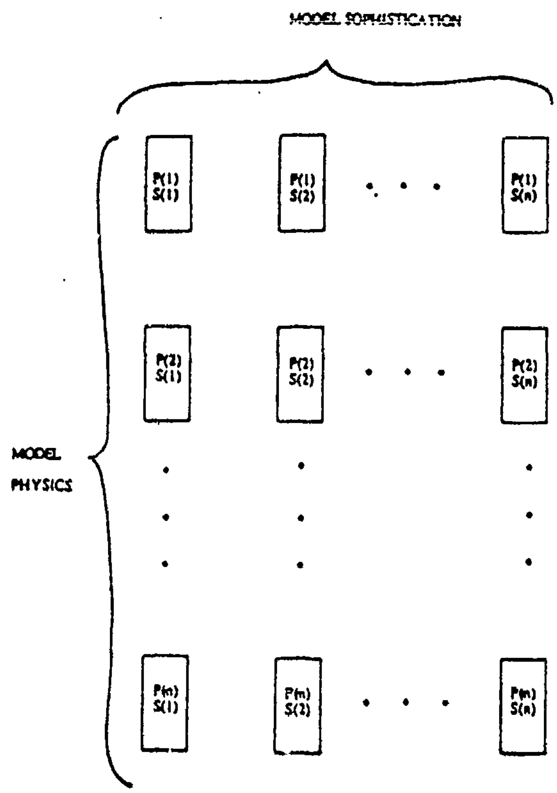


Figure 6. 2D Space of Potential Models

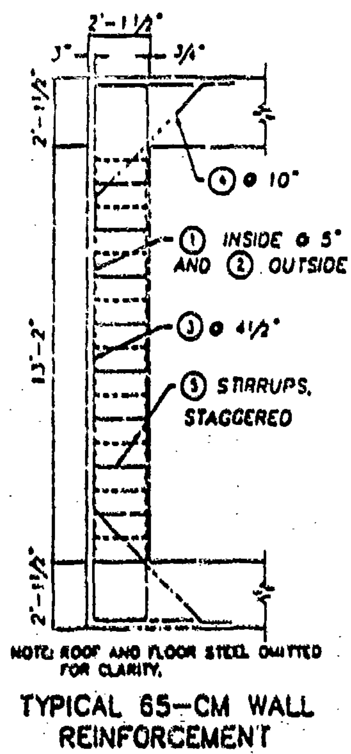


Figure 7. Specific Design Problem for Phase I Simulations

e. Series 3 Simulations

Using one stochastic procedure (Monte Carlo simulation) compare probabilities of failure to the specific wall section, shown in Figure 7, for the three different model physics mentioned above. Two of the physics, forced vibration and breaching, will compare pressure-induced shear stress in the structure to the direct shear failure criteria. The physics of penetration will compare penetration depth from perforation or spalling to the thickness of the wall. These comparisons, which are based on simple empirical design algorithms, will serve to illustrate the necessity of considering different physics in the stochastic design approach.

f. Nonrandom Uncertainties

In the consideration of non-random uncertainties it becomes obvious that numerous assumptions in the design process are candidate nonrandom variables. For example, when the load distribution on a slab is modeled assumptions are made about the spatial distribution (uniformly distributed or some other well-posed shape) of the blast pressure or the ground shock, or about the boundary conditions (fixed, simple, etc.) of the structural element. Some treatment of the influence of the variability of these kinds of uncertainty is important in design, but because the uncertainty in not random it would seem that stochastic treatment is unwarranted. However, there are procedures such as evidence theory, fuzzy set theory, and a rule-based approach that seem particularly appropriate for these kinds of uncertainties. The ability to incorporate nonrandom uncertainty in a balanced, integrated design framework is essential if realistic variability in needed in the modeling process. Such an incorporation of random and nonrandom uncertainty in an integrated system has already been developed by this project's Principal Investigator on another project [Reference 11]. Because of this previous demonstration, the risk involved in integrating random and nonrandom uncertainty is negligible. That it is beneficial to a designer is an issue which will be discussed in the Phase II proposal.

## 2. Task 2 - Illustrate Stochastic Procedures

a. Stochastic Simulations

In *Series 1* calculations were compared for four stochastic methods to compute the variability of the strength of a hypothetical reinforced concrete beam with stirrups regularly spaced along the beam. The beam system is represented by the ACI Committee 426 Design equation:

$$V_{ACI} = B * D * [2 * (F_c)^{1/2} + r * F_y]$$

where $V_{ACI}$ = the shear strength of the cross-section ($V_{ACI}$ was divided by the nominal cross-sectional area to get the shear strength per unit area), B = width of beam, D = effective depth of beam, i.e., depth to tensile flexural reinforcement, $F_c$ = concrete strength in psi, $F_y$ = yield strength of stirrup rebars in psi, $r = A_V/(B*S)$ where $A_V$ is the total cross-sectional area of the stirrups (2 x bar area), and S is the longitudinal spacing of the stirrups.

The shear strength is random because the parameters B, D, $F_c$ and $F_y$ are random (the reinforcement ratio, r, was set to be deterministic to keep the number of random variables to 4). The four stochastic methods used to compute the random variation of the shear strength, again are:

(1) Direct Monte Carlo method, in which a large number of theoretical realizations of the beam are generated, and their shear strengths analyzed for mean and variance.

(2) Partial derivative method, in which the system dependence and variability are approximated by the first and second partial derivatives. The mean and variance of the shear strength are related directly to the mean and variance of the parameters through these derivatives.

(3) Point estimate for probability moment method, or PEPM, in which point estimates of the shear strength are made within the parameter variation space. Approximate expressions for the mean and variance of shear strength are computed based on the point estimates [Reference 12].

(4) Response surface, in which an approximation to the shear strength response within the parameter variation space is obtained, based on a limited number of point estimates of the strength. This method is also called the statistical sensitivity, or factorial sampling method.

The shear strength problem addressed in Phase I is defined in Figure 8, which shows a schematic of the hypothetical beam cross-section investigated, and in Table 1 which lists the parameters and their uncertainties considered in the exercise. The results of this *Series I* study are summarized in Table 2, which compares the mean and standard deviation of the shear strength computed by the four methods. Typical histograms and probability curves are shown for the Monte Carlo and 4-factor Response surface methods in Figure 9 and Figure 10 displays a comparison of the means and standard deviations for all four methods. For this system, which is fairly well-behaved and continuous, the results obtained by the four methods compared very well. There is no significant difference and, hence, one method is as good as the others. Given this situation, the most expedient method is the PEPM method which requires only 16 point estimates of the shear strength plus some very simple arithmetic operations.

b. Task 2 Findings

As seen for a simple design computation, the four stochastic methods provide similar results, so that those used in an integrated design framework might include an assessment not only of accuracy (just illustrated) but also the complexity, efficiency, and validity. It is entirely possible that each of the four methods could be more appropriate for the type of design encountered, so that a generalized design paradigm might want to include all four. In fact, the software for each method is so trivial that to have each available in a generalized design software creates no special burden.

The Monte Carlo method is certainly the most flexible, and most intuitive. Its power and utility is especially useful for simple empirical formulations. The other three first order, second moment procedures attempt to do two things: simplify the design functional relationship, and propagate the moments through the design based on the simplified functional relationship. The PEPM method occupies an intermediate position between the partial derivative and the response surface methods; point estimates are computed as in the latter, and used in a manner which can be loosely interpreted as the finite difference equivalent of the former. On the other hand the PEPM method is unique in the sense that it does not attempt to simplify the design functional relationship first. An equation between the moments of the response and the point-estimates is sought using the mathematical convenience of the Dirac delta function.

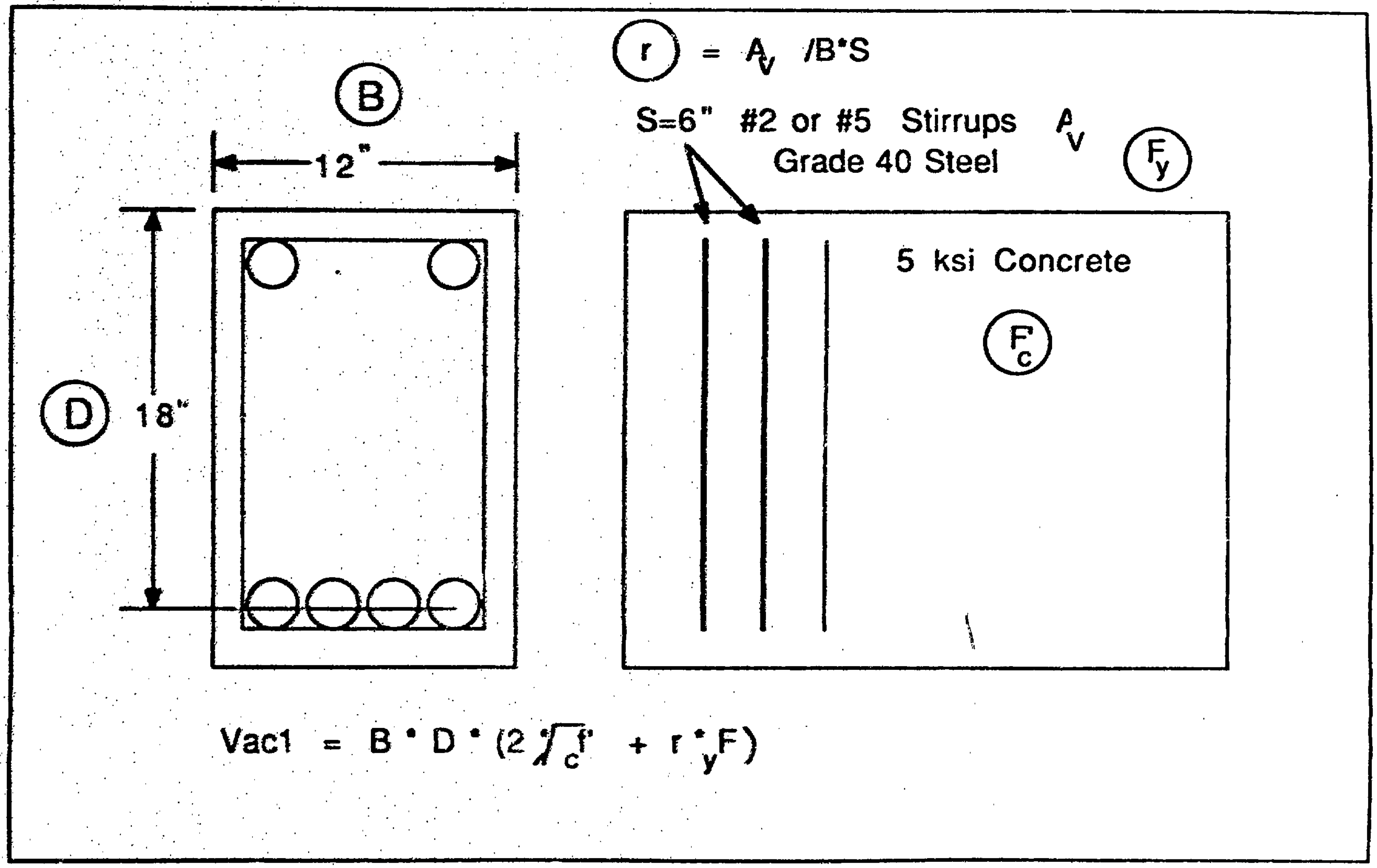


Figure 8. Hypothetical Beam Shear Properties

## Table 1. Hypothetical Beam

PARAMETERS AND UNCERTAINTIES

Geometry/Reinforcement Location

| | | mean | standard deviation | distribution |
|---|---|---|---|---|
| D | eff.depth | 18"-3/16" | 1/2" | normal |
| B | width | 12"+3/32" | 3/16" | normal |
| s | spacing | 6" | 17/32" | normal (assmed det.) |

Stirrup Geometry and Properties

| | | mean | standard deviation | distribution |
|---|---|---|---|---|
| Av | cs area #2 | .05 sq.in | 0. | assm. determ. |
| | cs area #5 | .31 sq.in | 0. | assm. determ. |
| r=2*Av/(B*s) | | | | |
| | #2 | .00139 (ND) | 0. | assm. determ. |
| | #5 | .0086 (ND) | 0. | assm. determ. |
| Fy | grade40 steel | 48.8 ksi | .107 (cov) | beta |
| | | | 5.22 (sig) | assm. normal |

Concrete Properties

| | | mean | standard deviation | distribution |
|---|---|---|---|---|
| Fc' | 5ksi concrete | 4.028 ksi | .18 (cov) | trun. normal |
| | | | 0.6 (sig) | assm. normal |

Table 2. Hypothetical Beam

COMPARISON OF RESULTS
(Shear Strength of Beam Cross-Section in ksi)

| Method | mean | standard deviation |
|---|---|---|
| direct monte carlo* | 4.03168 | 0.323485 |
| partial deriv. (fosm) | 4.07060 | 0.327372 |
| pepm (Rosenblueth) | 4.07060 | 0.327371 |
| response surf.* | 4.02307 | 0.325738 |

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* based on 100 samples

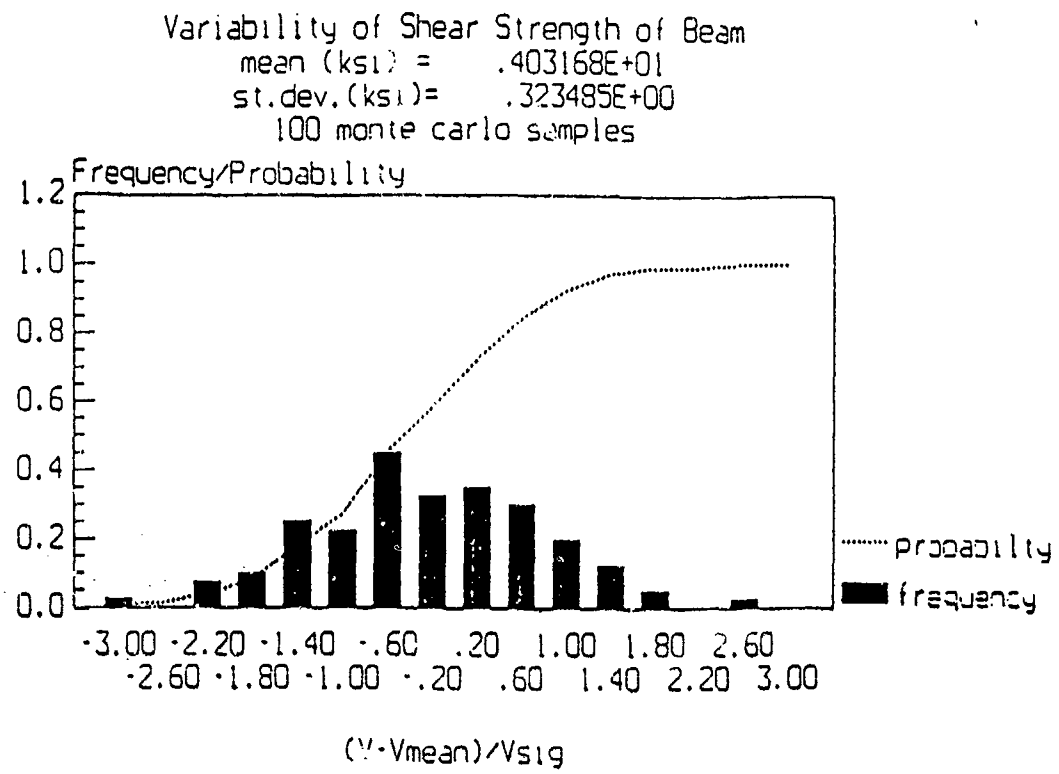



Figure 9a. Histogram and Probability for Monte Carlo Methoa

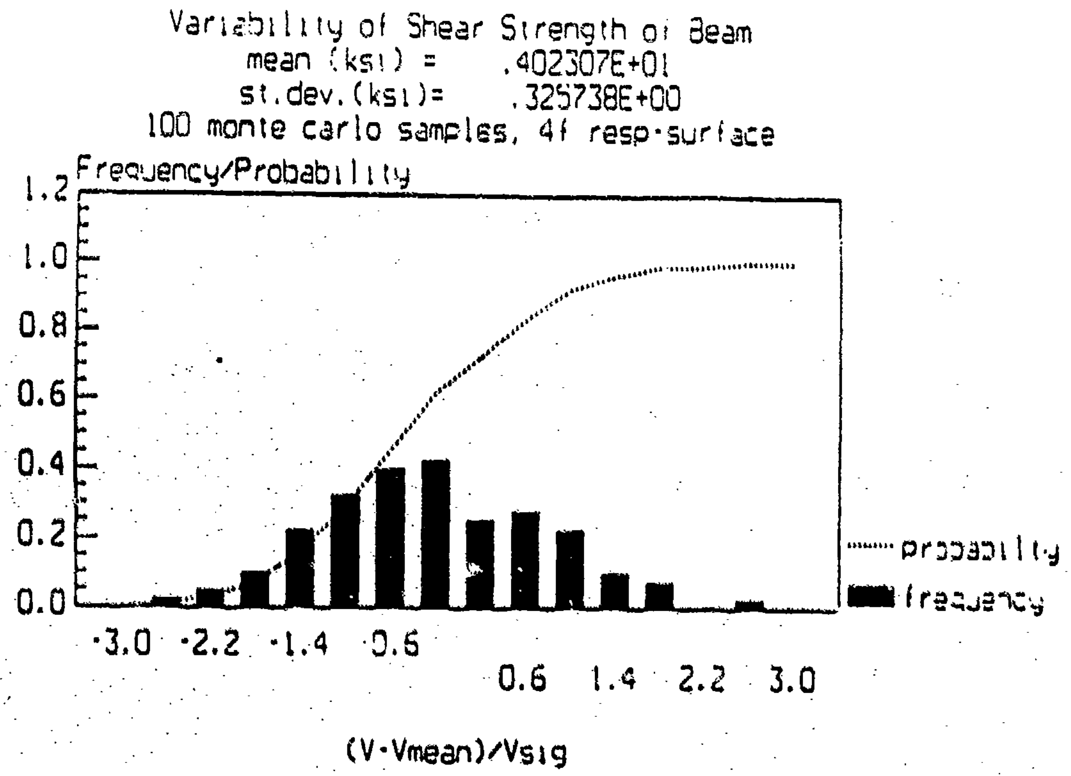



Figure 9b. Histogram and Probability for Response Surface Method

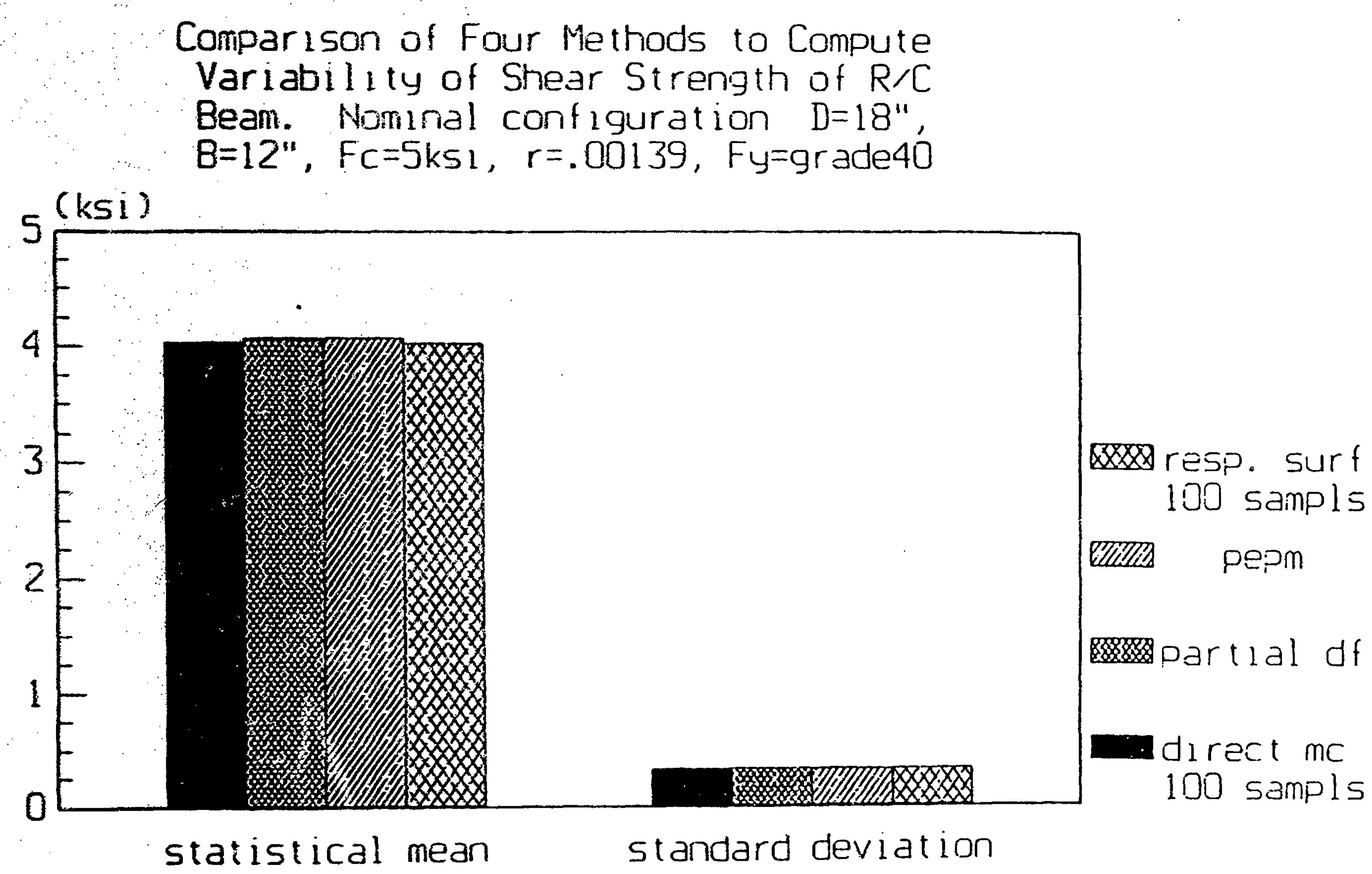



Figure 10. Graphical Comparison of 4 Stochastic Methods

The major difference between the PEPM and the response surface methods is in the end-products the methods provide. In the response surface method, the end-product is an approximation to the true response function of the design within a limited region of interest. The moments of the response are then computed based on this approximate response, using numerical means. In the PEPM method, the probability moments of the response, and not the response itself, is the only product. Hence, no new information on the response other that the statistics of its uncertainty is derived.

The demand for more realistic engineering design modeling and the availability of sophisticated computer techniques renders the partial derivative method less useful, except for the most simple, perhaps preliminary, designs. The PEPM and response surface methods become attractive in more complicated design situations because of their compatibility with modern numerical techniques such as normal-mode decomposition or finite element methods. In the final analysis, the efficiency and validity of the PEPM and response surface methods when compared to direct Monte Carlo simulation will be predicated on the hardware available to the designer, since these three methods differ only in the degree of number crunching required for a particular design. It is suggested that any balanced design framework include all four methods described for purposes of flexibility and comparability. Moreover, it is suggested that the design framework would use the Monte Carlo procedure as the default procedure for assessing the influence of uncertainty in design for user convenience.

### 3. Task 3 - Design Problem Case Study

#### a. Series 2 Simulations

In *Series 2* simulations probability of direct-shear failure is compared for four different kinds of model sophistication. In these calculations the Monte Carlo simulation is used to sample from the probability density functions of all the random variables of the static model and the three dynamic models. Failure in each Monte Carlo loop is defined to occur when the maximum shear-stress (in the time-domain) induced in the beam from the loading exceeds the shear-strength. The simulation simply counts the proportion of cycles in which a failure is indicated. Since the simulation is for dynamic models the maximum shear-stress can occur at any time during or after the triangular load is applied to the beam. The beam is assumed to be fixed at the supports and to exhibit linear properties through the simulation process.

For direct shear the following formulation [Reference 10] is used for direct-shear resistance, $V_u$:

$$V_u = [\, 8 * (f'_c)^{1/2} + 0.8\rho_s f_y] \;<\; 0.35 f'_c \text{ (psi)}$$

These strength relations are functions of the concrete compressive strength, $f'_c$, the percentage of longitudinal steel, $\rho_s$, and the yield strength of the steel, $f_y$, in the slab. In the dynamic simulations the strength properties of the concrete and steel are enhanced by 30% to crudely account for strain rate effects. All these parameters can be random. Table 3 shows the variability of each of the random variables in the static and dynamic models. Again these models are abstractions of the actual wall design shown in Figure 7. In the case of Gaussian (Normal) random variables the mean and standard deviation are expressed and in the case of Beta-distributed random variables the end-points (min, max) are specified along with the two parameters of the Beta: α and β. In the Beta

## Table 3. Random Variables for Simulations

### General Gaussian Random Variables

| Parameter; units | Mean | Standard Deviation |
|---|---|---|
| Slab Thickness (inches) | 25.5 | 1.0 |
| Unit Thickness (inches) | 5.0 | 0.5 |
| Concrete Strength (psi) | 4028 | 600 |
| Steel Strength (psi) | 48800 | 5220 |
| Charge Weight (pounds) | 555 | 55.5 |

### Static Random Variables

| Parameter | Type | Values |
|---|---|---|
| Stirrup steel ratio | Deterministic | 0.0026 |
| Longitudinal steel ratio | Deterministic | 0.0066 |
| Beam Length | Normal | Mean = 156 in.; $\sigma$ = 12 in. |
| Miss-distance (range) | Beta | 4 ft$<R<$40 ft; $\alpha = 2$, $\beta = 2$ |

### Dynamic Random Variables

| Parameter | Type | Values |
|---|---|---|
| Damping | Deterministic | 2% |
| Longitudinal steel ratio | Deterministic | 0.0066 |
| Mass Density | Deterministic | 0.0002247 #-sec$^2$/in$^4$ |
| Miss-distance (range) | Beta | 4 ft$<R<$40 ft; $\alpha = 2$, $\beta = 2$ |
| Wall height | Normal | Mean = 15.25 ft; $\sigma$ = 1.52 ft. |

### Penetration and Breaching Random Variables

| Parameter | Type | Values |
|---|---|---|
| Casing Thickness | Deterministic | 0.5 in. |
| Bomb Weight | Deterministic | 1000-# |
| Casing inside diameter | Deterministic | 17.8 in. |
| Longitudinal steel ratio | Deterministic | 0.0066 |
| Miss-distance (range) | Beta | 0$<R<$40 ft; $\alpha = 2.2$, $\beta = 1.8$ |
| Fragment weight ($W_f$) | Beta | 0.2 oz$<W_f<W_{fmax}$; $\alpha$=1, $\beta$=3 |

distribution, when $\alpha=\beta$ the distribution is symmetric; when $\alpha=\beta=1$ the distribution becomes the uniform distribution; and when $\alpha<\beta$ the distribution is skewed to the right.

For the static Monte Carlo simulation Figure 11a shows a comparison of probability density functions (PDFs) for the ACI shear strength ($V_{ACI}$) vs. the static shear response. The area where the PDFs overlap indicates the relative likelihood of failure; i.e., where the response exceeds the resistance. Figure 11b shows the same comparison as Figure 11a except the PDFs are for the direct shear strength ($V_u$) vs. the static shear response. In both Figures 11a and 11b the static shear response is simply a calculation of the shear at the support of a fixed-fixed beam subjected to a static pressure normal to the axis of the beam. This pressure is simply assumed to be the peak pressure from the weapon at a given range, assumed to act statically on the beam. In both Figures 11a and 11b the histograms from response are generally higher than those for resistance, hence these particular curves indicate a beam geometry and material properties that will usually fail for most miss distances of the weapon (see Table 3 under static parameters). Figure 12 displays the probability of failure ($P_f$) curves for the ACI and direct-shear ($V_u$) criteria. As can be seen, a significant decrease in miss-distance (range) for the same $P_f$ can be realized by using the less-conservative direct shear criteria. This translates to a significant cost savings for the same survivability if the designer elects to use a less-conservative shear-failure criterion.

In the dynamic simulations, Figure 13 plots the histograms (PDFs) of direct-shear resistance (strength) vs. shear response. Again, the overlapping area between the two PDFs gives an indication of the probability of failure. Also seen in the curves is the fact that the variance in the resistance is much "tighter" than the variance in the response. This is because the response is very sensitive to the variance in the miss-distance (range) which is significant as seen in Table 3. Figure 14 shows the same kinds of PDFs of direct shear resistance vs. shear response for a continuum model with seventeen (17) degrees of freedom. Again, the same phenomena is seen in Figure 14 as in Figure 13. Finally, Figure 15 compares the probability of failure ($P_f$) curves for the four models: (1) static response, (2) SDOF response, (3) 4-DOF response and (4) continuum model with many degrees of freedom. From a design point-of-view the static calculation is the least conservative (for a given range, the static simulation provides the lowest $P_f$) for this particular problem, but the interesting feature is that there is not much significant difference between the simulations among the dynamic models. More specifics of dynamic models as specified in terms of a normal-mode superposition solution approach is given in Appendix B. The power and efficiency of the normal-mode approach cannot be overemphasized for a design tool. The same code can be used to calculate one, two, or any number of modes of a given structure up to the limit of the discretization in the model (i.e., the number of elements in the model).

b. Series 2 Findings

It may be to premature to make many conclusions about these *Series 2* comparisons, but the case for a simpler design model is compelling, at least in preliminary stages. In fact, the uncertainty in major variables such as the miss-distance or the strength criterion obscures the minor differences between models of varying degrees of sophistication.

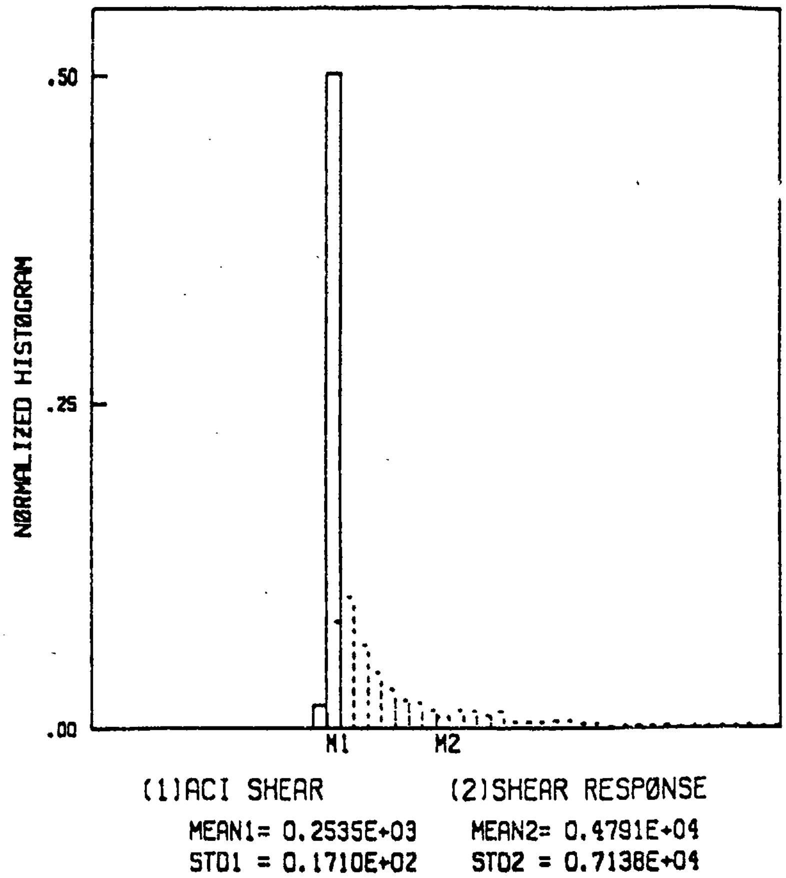



Figure 11a. Static Monte Carlo Simulation for ACI Shear and Shear Response

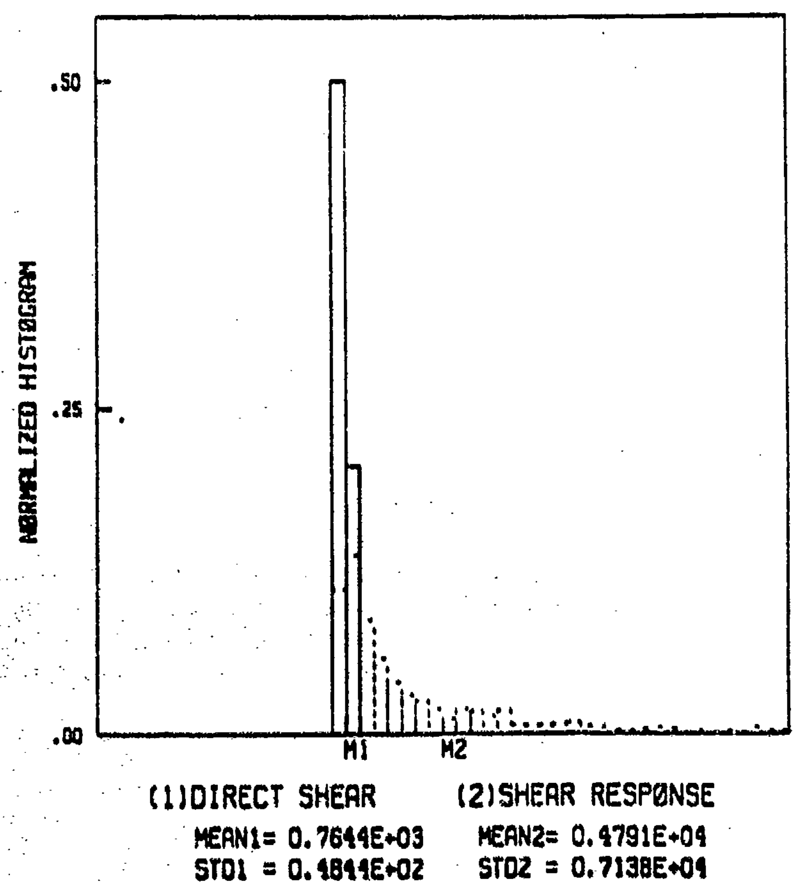



Figure 11b. Static Monte Carlo Simulation for Direct Shear and Shear Response

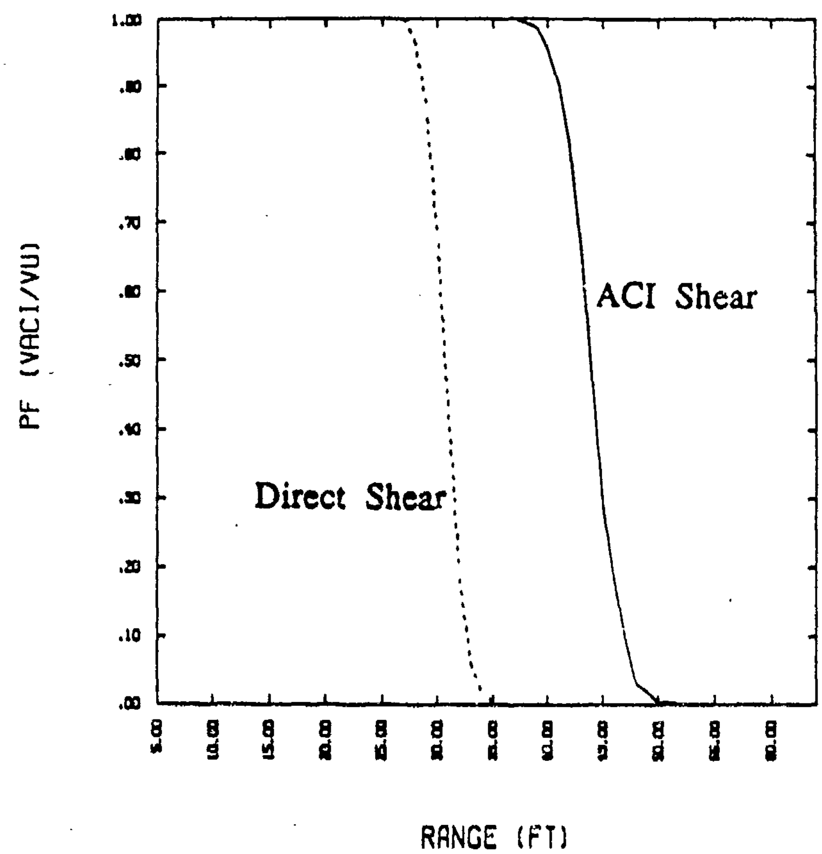



**Figure 12. Comparison of Shear Failure Criteria**

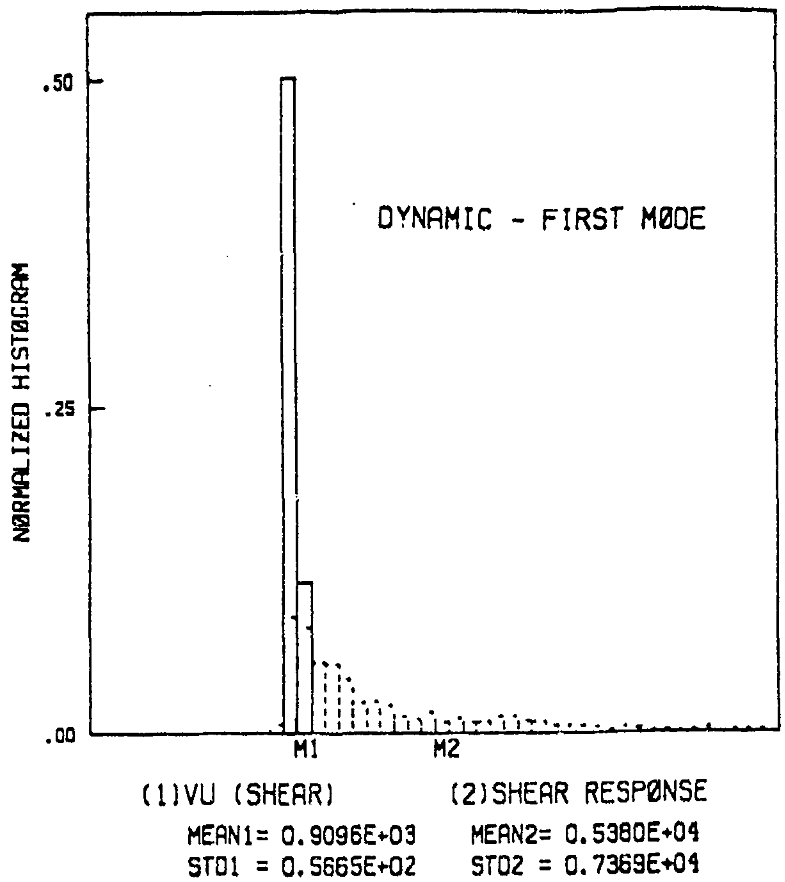



Figure 13. Simulation for Dynamic SDOF Response vs. Direct Shear Criteria

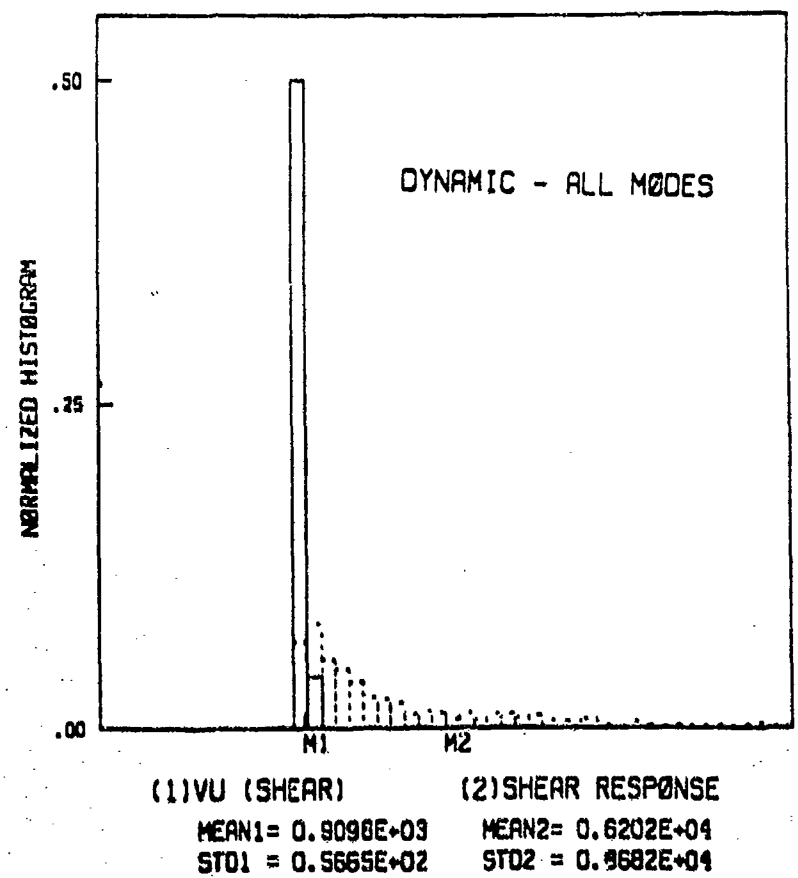



Figure 14. Simulation for Dynamic 17-DOF Response vs. Direct Shear Criteria

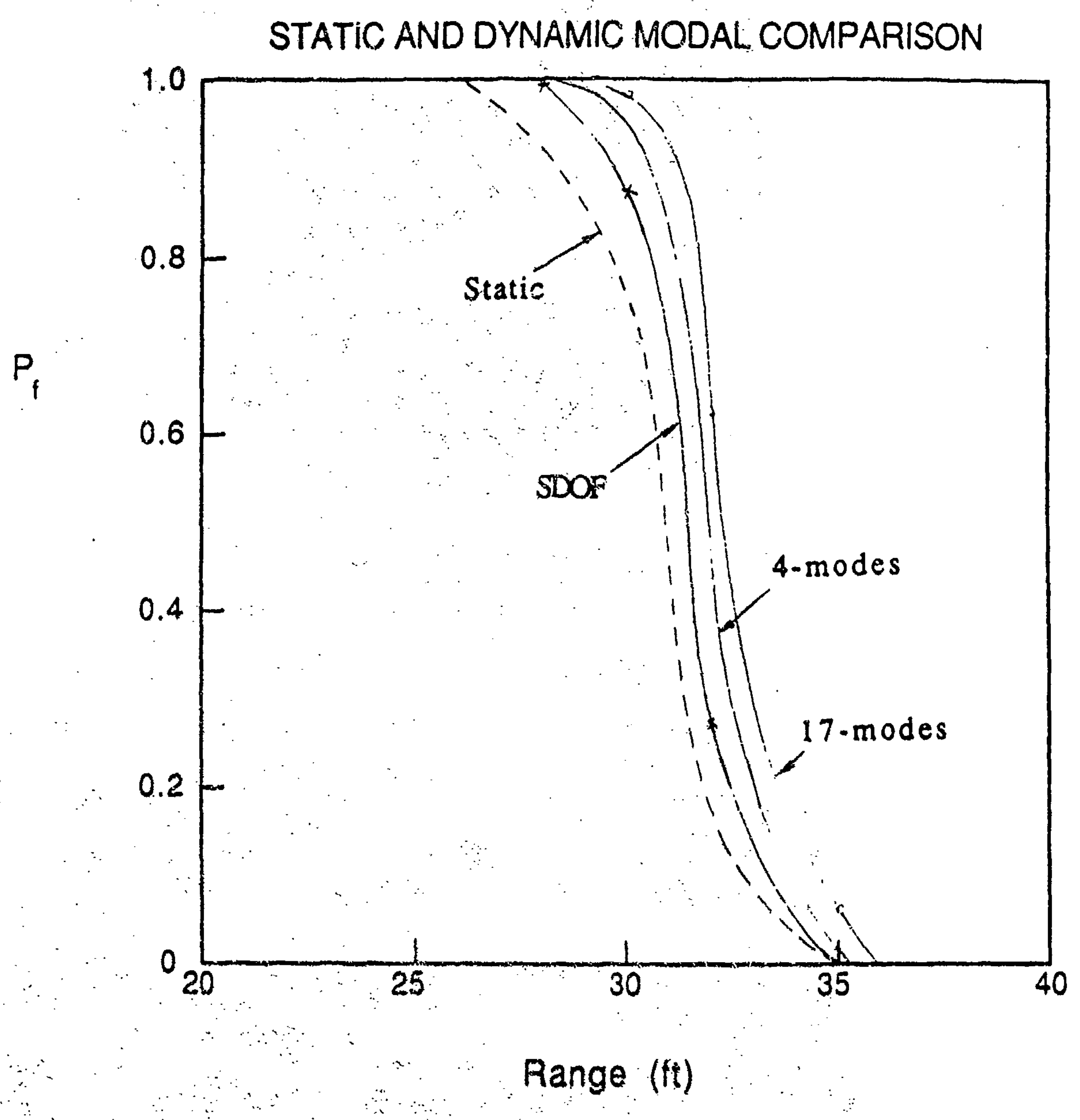



Figure 15. Comparison of Probability of Failure Curves for (1) Static Response, (2) Dynamic SDOF Response, (3) Dynamic 4-DOF Response, and (4) Dynamic 17-DOF Response

c. Series 3 Simulations

In *Series 3* simulations probability of direct-shear failure is compared for three different kinds of model physics. In these calculations the Monte Carlo simulation is used to sample from the probability density functions of all the random variables of the three physics models. These models all attempt to address the question of whether different physical responses can govern the probability of failure if a framework is develop to exercise them. The three physics (response modes) models are: (1) failure from forced-vibration from airblast in direct shear (this model has been addressed in the Series 2 simulations); (2) failure from penetration of bomb casing fragments; and (3) failure from breaching (punch-through) of the R/C wall due to a near-direct hit. In the stochastic analysis the range of detonation of the 1000-pound GP bomb from the wall slab is a random variable. If a single realization of this variable makes it very close to the wall, penetration or fragmentation may govern. If a realization of the random variable for range makes it far from the wall slab, a forced-vibration shear failure may govern. Table 3 displays all the random variables and their distribution type and statistical parameters. Details on the empirical and theoretical developments of the last two response modes (penetration and breaching) are provided in Appendix B.

Figure 16 shows the PDFs of the equivalent resisting radius and equivalent breaching radius of the breaching physics model, as developed in Appendix B. This punch-through phenomenon essentially computes the radius of a disk of the wall slab that is necessary to cause breaching ($r_L$) and compares it to the radius of a disk that is sufficient to resist failure in direct shear ($r_R$) and then compares the two. Failure in breaching occurs when $r_R > r_L$. Figure 17 shows the PDFs of wall thickness and the penetration depth of a casing fragment in the penetration model. Figure 18 shows the probability of failure ($P_f$) curves for perforation ($P_p$) and for spalling ($P_s$). As developed in Appendix B, $P_f = P_p + P_s$. These curves are quite erratic because of the crude number of iterations in the Monte Carlo simulation. Note in Figure 18 that the perforation curve approaches asymptotically a value for $P_f = 0.38$ when the range approaches 20 feet. This is because the fragment velocity is considered to approach a maximum value (the break-up velocity) at R = 20 feet (see Appendix B). Finally, Figure 19 compares the probability of failure curves of the three physics models.

d. Series 3 Findings

Figure 19 reveals some very interesting phenomena relating to the need for design models to account for uncertainties in weapons and structural parameters. From a design point of view the physics of breaching would impose the least conservative constraint on structural requirements while the physics of forced-vibration from airblast would impose the highest constraints (i.e., for this example design the wall would fail due to airblast before it would fail due to breaching). Failure due to penetration also seems to be abnormally high at larger ranges (ranges above 35 feet; scaled ranges above about $\lambda$ = 4). This is because the probability of a given fragment even hitting the wall was not addressed in the simulation. This uncertainty is a function not only of miss-distance, but also the azimuthal location between the bomb and the structural element. Perhaps the biggest conclusion from Figure 19 is the fact that synergistic effects between the various modes of physics are clearly revealed but have not been accounted for explicitly in the physics models! For example, at a range of about 30 feet fragmentation and airblast would produce combined effects on a wall and at a range of about 15 feet fragmentation, airblast, and breaching would all come into play. Not only should uncertainties be accounted for in this case, but research into the true physics of synergistic effects is warranted.

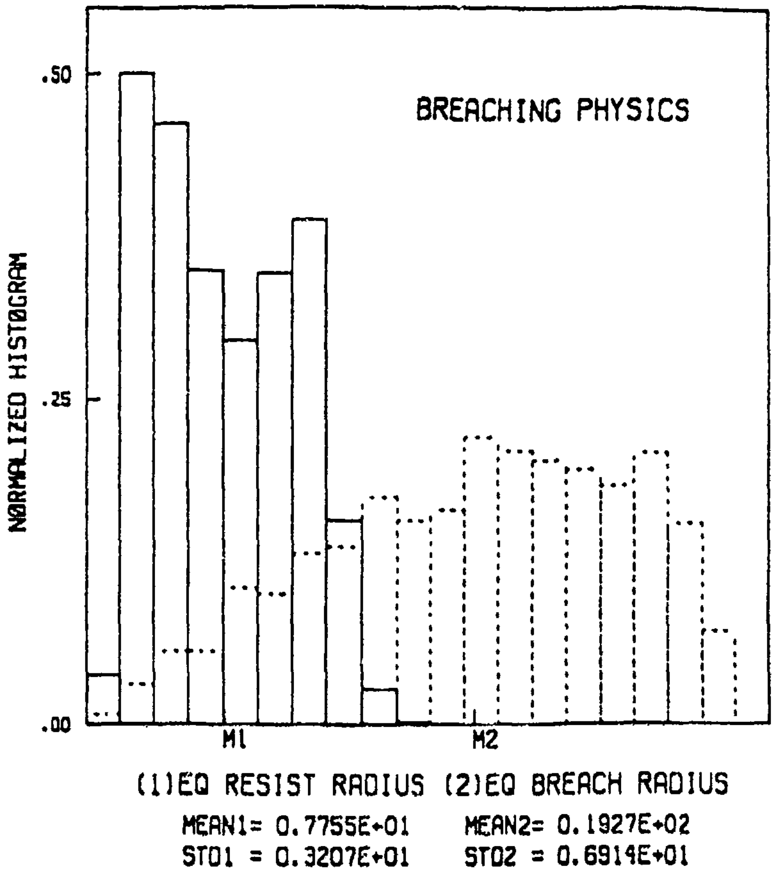



Figure 16. Comparison of Resisting and Breaching Radii for Model Slab

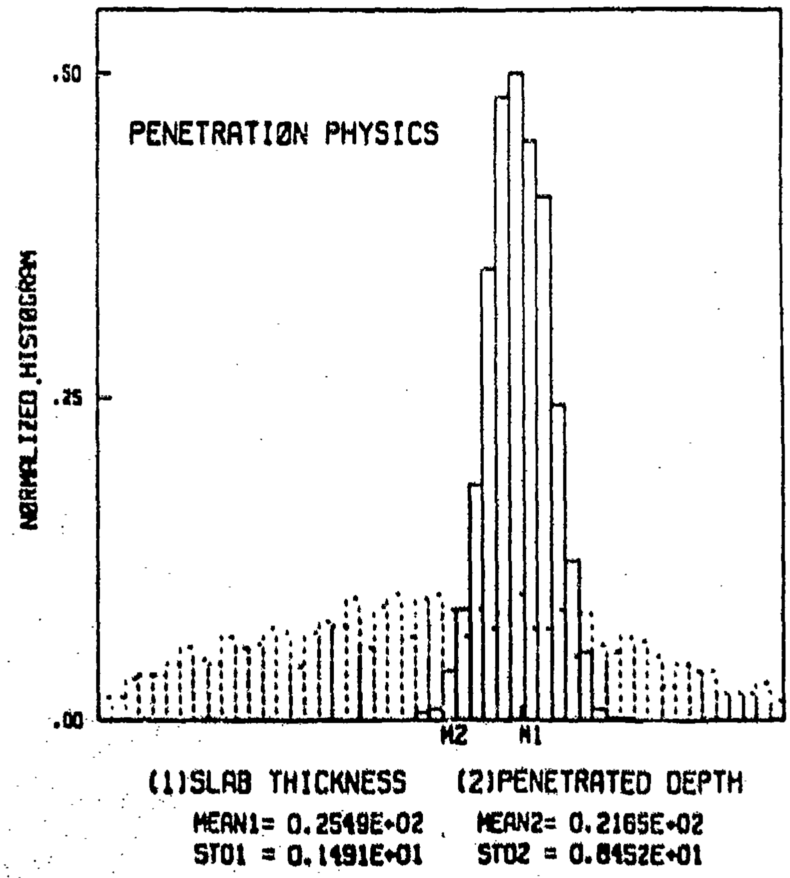



Figure 17. Comparison of Penetration Depth vs. Slab Thickness

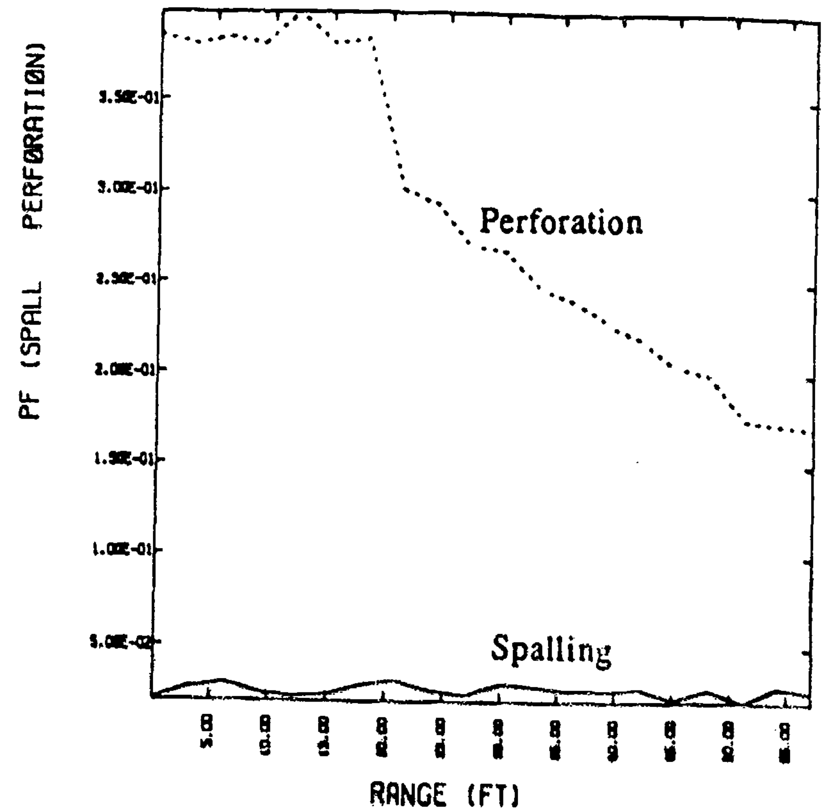



Figure 18. Probability of Failure Curves for Perforation and Spalling

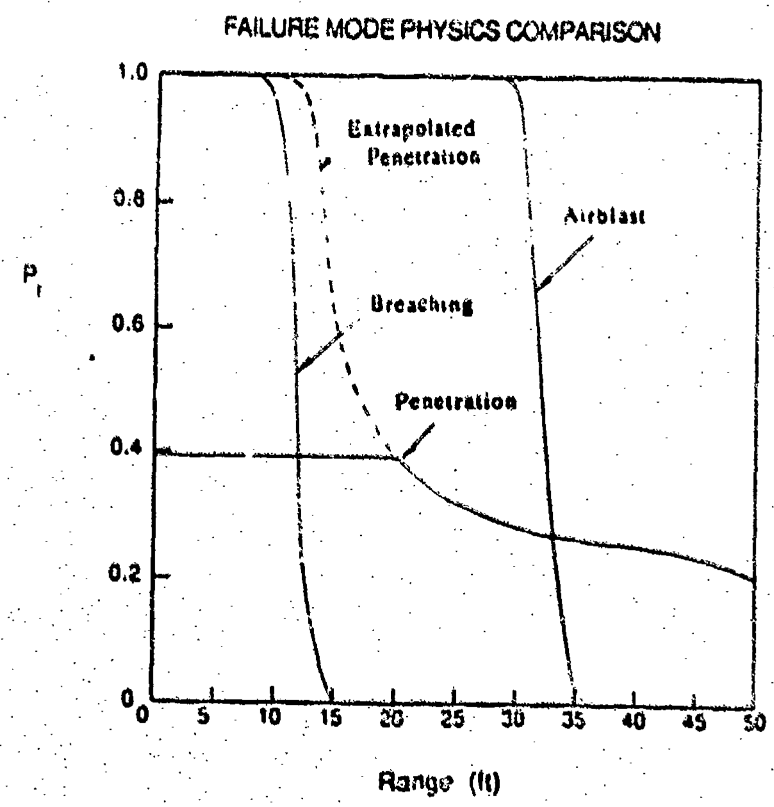



Figure 19. Comparison of Pf curves for (1) Airblast, (2) Penetration and (3) Breaching failure modes

### e. Integrated System for Protective Structure Design (ISPSD)

Some exploratory work has been done on the feasibility of an Integrated System for Protective Structure Design (ISPSD). There appears to be no specific technical reason which, on its own merits, would prevent the development of an ISPSD. Indeed, the development of this system with capabilities to gather, in one framework, design information, codes and practices may provide an important impetus to significant savings in design time and construction costs and in realistic expectations of survivability to advancing threats. The production of such a system is feasible since most of the technical problems have been solved and demonstrated in two previous studies on projects related to commercial software. One of these [Reference 13] combines conventional numerical codes with advanced symbolic codes (such as expert systems) within the same framework on a microcomputer. The other study [Reference 14] uses a novel graph-structure consisting of nodes where "evidential-knowledge" (such as design rules-of-thumb, local code provisions, etc.) is manipulated in an automated reasoning framework (see Figure 1).

The Integrated System for Protective Structure Design (ISPSD) will be briefly described in this Phase I report in a graphical representation (fault-tree format) which portrays a number of "slides" which serve as the interface between a microcomputer-based system and the user. More details of this approach are provided in a previous Phase I progress report [Reference 15] and will be proposed as part of the Phase II proposal plans. These "slides" would be interlaced with "PC-screen image" dialog sessions which interactively guide the user through the ISPSD (these interactive dialog sessions are omitted here for brevity, but appear as examples in [Reference 15]).

The following graphical representation shows possible approaches to the ISPSD. Handbook and design codes give procedures and equations to use for the design, but they do not give the many fine points associated with the design procedures and equations. An integrated system methodology (ISPSD) can enhance the codes' procedures by interjecting expert experience and proper usage which are not explicitly stated in the algorithms. The power of the system is further enhanced by making stochastic techniques (such as the engineering programs developed for *Series I* simulations) an integral part of the system, so that the variability of the design strength can be determined. Furthermore, it gives some idea as to the magnitude of the difference between the analytic predictions and the actual strength and variability of real beams.

The following "slides" for a protective structure design (called PSDesign) follow only one of the many paths possible in a full integrated system (the subject of Phase II) and this path is indicated by the heavy line in the flow diagrams. The heavy line in this example is intended to lead to the design of beams against diagonal tension, by designing the web reinforcement. This is done to highlight the calculations conducted on "Series I" simulations. Figure 20 shows a schematic where the user is questioned about the type of structural element to be designed, and when beam is selected is questioned further about the failure mode of interest. In a general and flexible system a "help-facility" or tutorial would be available for novice or infrequent users. If the user selects "A" for a diagonal tension mode in Figure 20 he then is exposed to "web-design" issues as shown in Figure 21, and the design session continues on through "B", Figure 22 on web-reinforcement, then to "C", Figure 23 for diagonal tension, then finally to "D" in Figure 24 where parametric uncertainty is addressed.

In a previous Phase I progress report [Reference 15] (and in the proposed Phase II work) it was shown how: (1) information from design hand-books, (2) findings of recent Air Force research efforts, (3) any expert knowledge, and (4) stochastic methods, could be collected together in one place. A short tutorial could be used to bring the novice user up to date. For the infrequent user, the design procedures guide him along paths which he may have forgotten. For the experienced user, the system offers another level of sophistication in the form of the stochastic module; uncertainties in the design strength and how it relates to real-world behavior are important in protective structures design but yet not readily obvious from the design equations. The unique feature of an ISPSD, as previously described, is that it can incorporate many sources of information and different kinds of information about the design process.

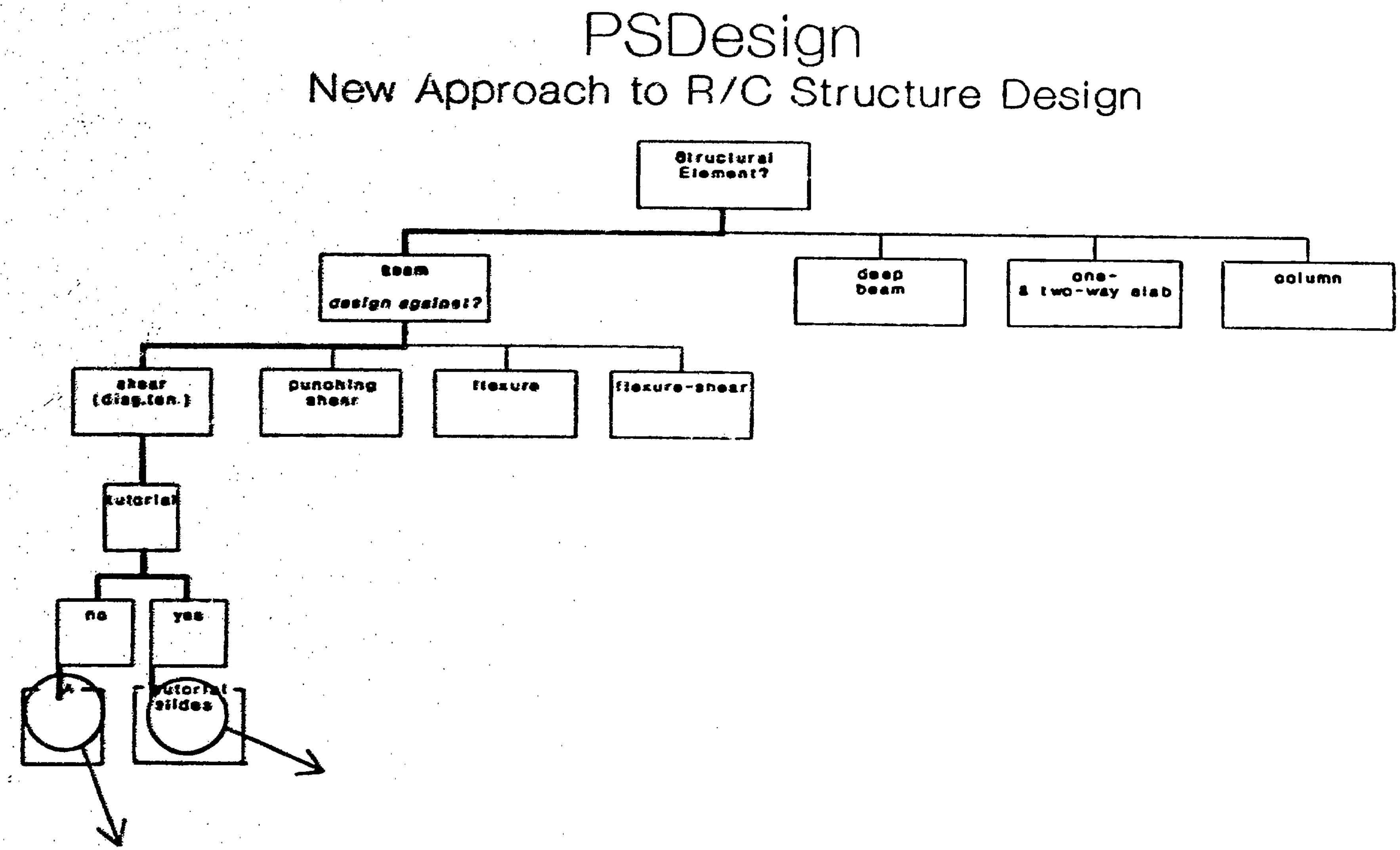



Figure 20. PSDesign: New Approach to R/C Structure Design

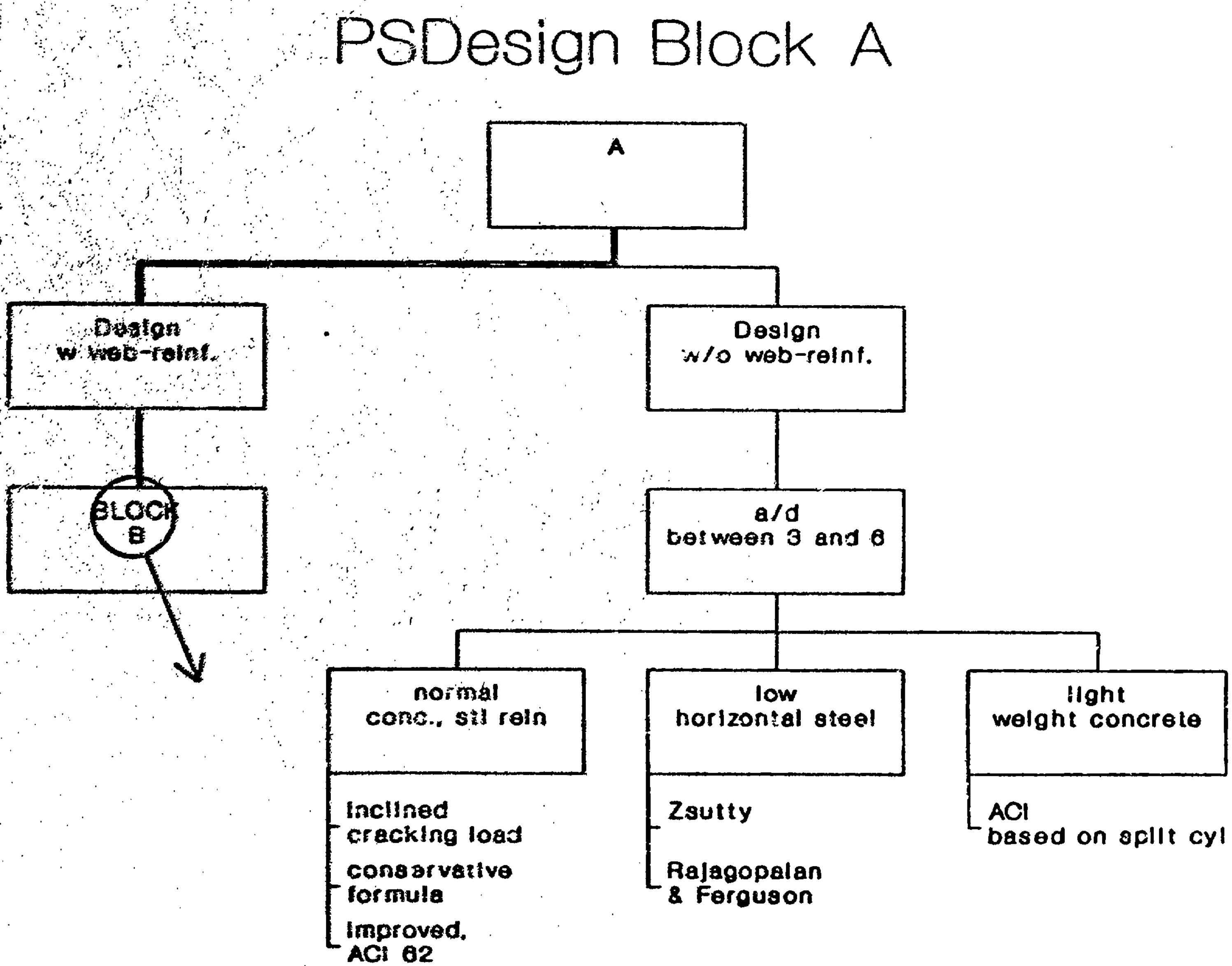



Figure 21. PSDesign: Block A

# PSDesign Block B

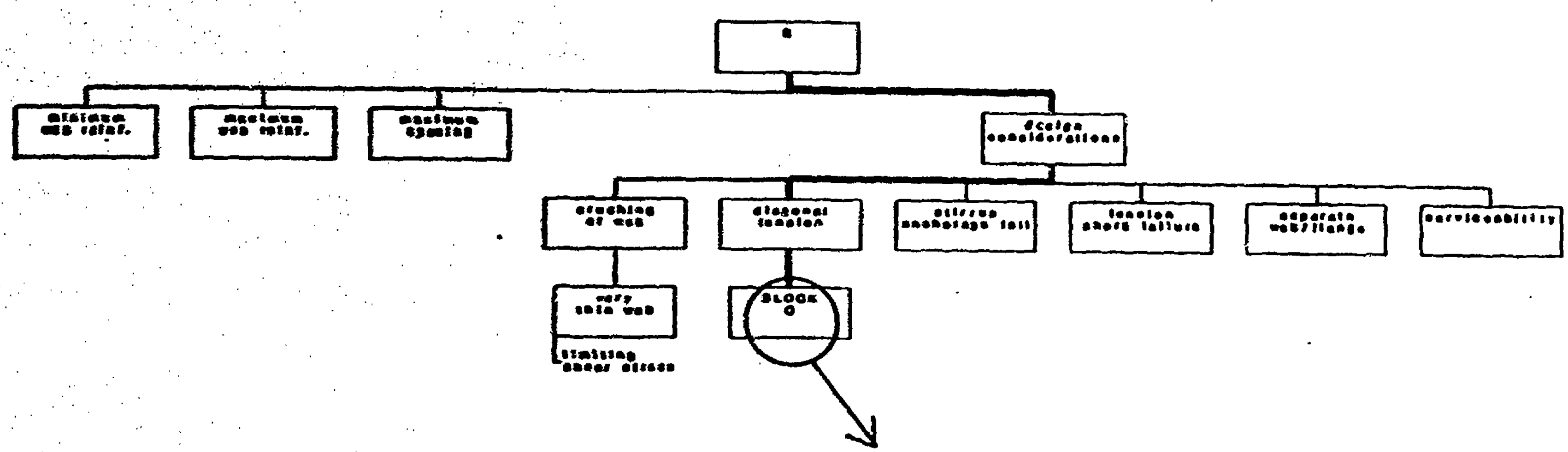



Figure 22. PSDesign: Block B

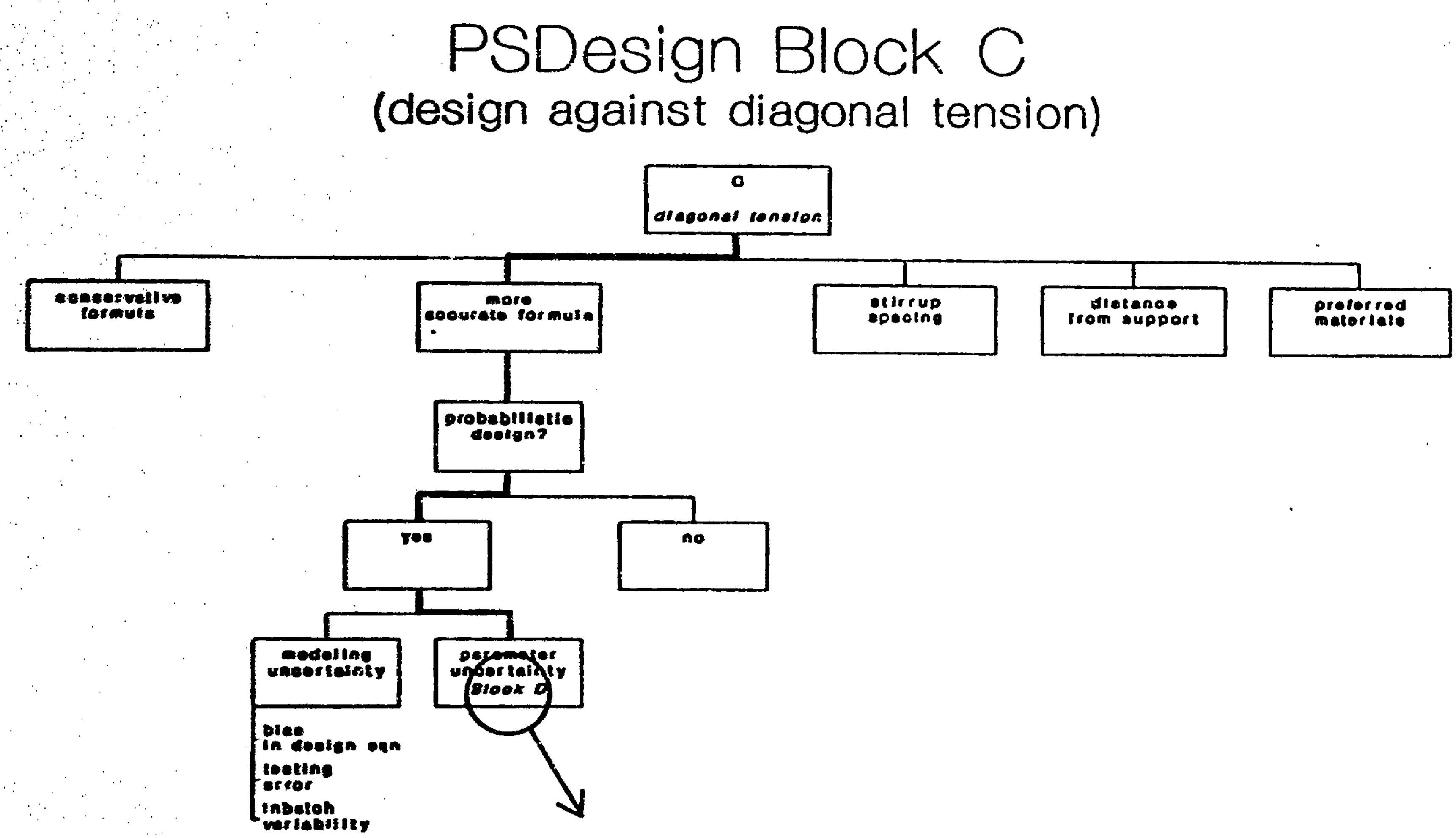



Figure 23. PSDesign: Block C (design against diagonal tension)

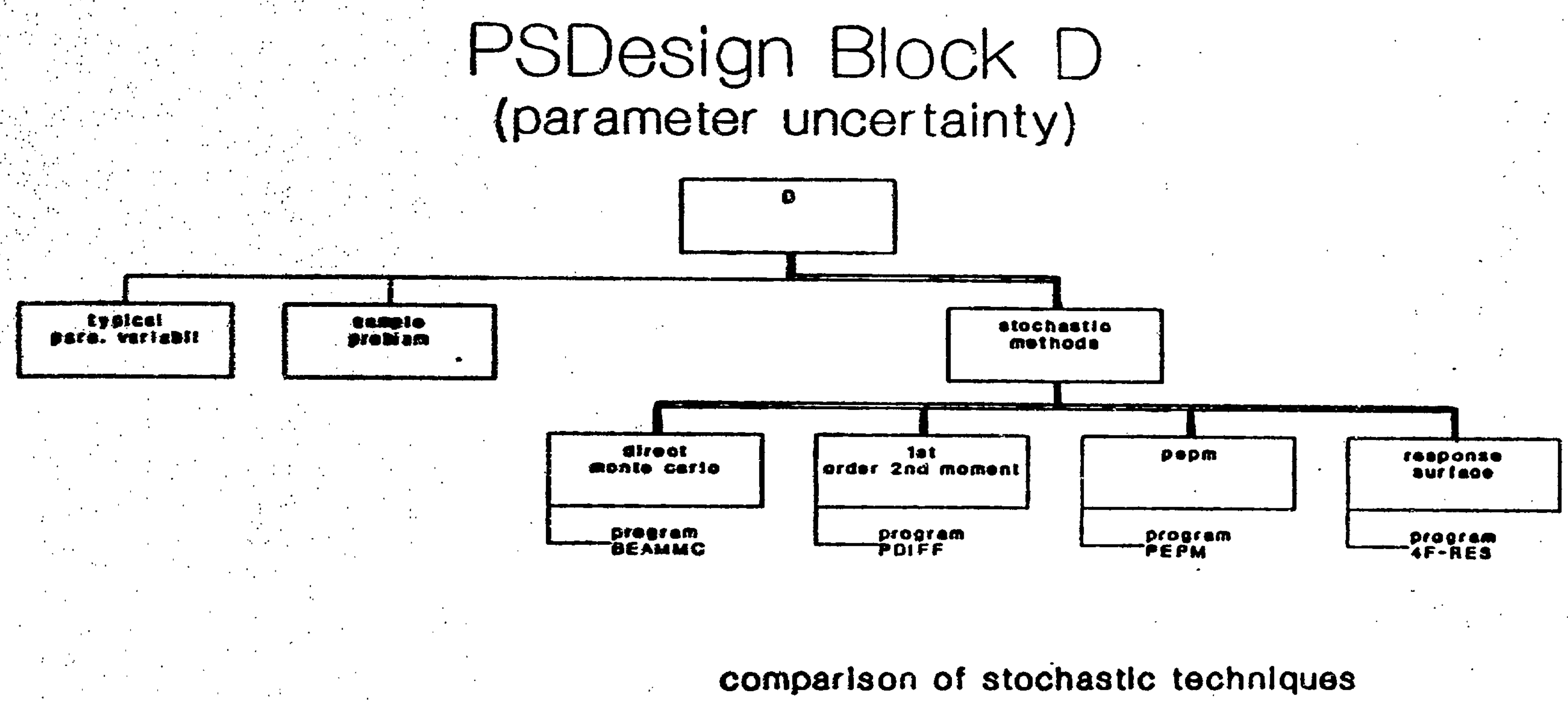



Figure 24. PSDesign: Block D (parameter uncertainty)

## SECTION III

## FUTURE R & D (PHASE II)

The framework of the proposed ***integrated system for protective design*** (ISPSD) would include the effects of variability in random parameters as well as the variability in non-random parameters. The integrated approach should be user-friendly, should minimize the need for knowledge in stochastic processes, should allow for design-cost trade-off studies, should accommodate English-language or menu-driven input (as opposed to formatted numerics), should conform to local design standards and codes, and should provide for mid-run explanations to the user. The focus on this framework will be on its utility to designers at the Air Base level, or contractor level, who are not experts in stochastic design or in computer software or in assessing different kinds of uncertainty and their impact on a final design.

The ISPSD (which will be called PSDesign in Phase II) will be predicated on the notion that a design will have four primary constraints, as shown in the schematic in Figure 25. Typically, a designer is given the location of the new facility, its approximate budget, and an idea of the floor space requirements due to the facilities' function. The conventional weapon threat is something that may not be given directly to the designer, depending on the circumstances. Perhaps some broad loading environments are all that a designer sees, but threat is mentioned for generality because of its importance as a real design constraint and because a flexible ISPSD should be able to accommodate advances in the enemy threat. Then the design problem is: enhancing the survivability of the facility while minimizing costs (this could also mean minimizing construction or post-attack rehabilitation time, for the Air Force situation).

In designing structural elements to the loading environments from conventional weapons, assumptions concerning the strength of materials and the influence of combined loading states create large uncertainties. These assumptions are typical of those made under conventional civil designs as recommended by the AISC and ACI and as adopted by the U. S. armed services' design manuals. The AISC and ACI design formulae typically come from a hybrid of simple analyses and tests conducted under "similar" conditions to those expected in the field. The problem here is that data under conventional weapons environments are not typical of those implied in the use of the AISC and ACI codes. Hence, the ISPSD will also accommodate "local" code practices and more advanced information from Air Force research laboratories. This information would generally be in a non-numeric form, similar to expert knowledge.

From the results discussed in this report for Tasks 2 and 3, probability of failure (or survival) curves, such as those shown in Figure 19 for the effects of different response modes (physics), can be compared directly to see which parameters effect facility hardness the most. However, to tie these curves in with design constraints such as cost, each critical design parameter or issue should be weighted by cost (and perhaps by its influence on the other constraints in Figure 25). The proposed ISPSD would then be capable of determining an overall facility cost vs. survivability relation. This relation is the goal of the designer, who could then relate to the decision-maker the balance between cost and survivability.

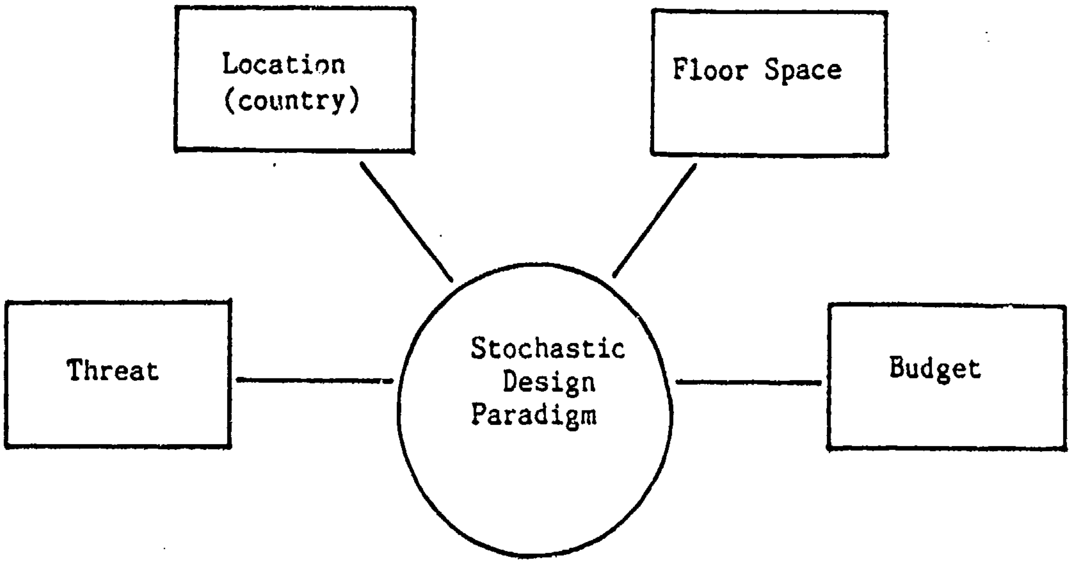



**Figure 25. Constraints on the Design Problem**

## SECTION IV

## CONCLUSIONS AND RECOMMENDATIONS

Stochastic simulations for a simple design example show that uncertainty in major variables such as the miss-distance or the strength criterion tends to obscure the minor differences between models of varying degrees of sophistication. Other results show that, from a design point of view, the physics of breaching would impose the least conservative constraint on structural requirements while the physics of forced-vibration from airblast would impose the highest constraints. Failure due to penetration also seems to be abnormally high at larger ranges (ranges above 35 feet; scaled ranges above about $\lambda = 4$). This is because the probability of a given fragment even hitting the wall was not addressed in our simulation. This uncertainty is a function not only of miss-distance, but also the azimuthal location between the bomb and the structural element.

Perhaps the biggest conclusion is the fact that synergistic effects between the various modes of physics are clearly revealed but have not been accounted for explicitly in the physics models! For example, at a range of about 30 feet fragmentation and airblast would produce combined effects on a wall and at a range of about 15 feet fragmentation, airblast, and breaching would all come into play. Not only should uncertainties be accounted for in this case, but research into the true physics of synergistic effects is warranted and worthy of closer scrutiny in future design packages.

Some exploratory work has been concluded on the feasibility of an Integrated System for Protective Structure Design (ISPSD). There appears to be no specific technical reason which, on its own merits, would prevent the development of an ISPSD. Indeed, the development of this system with capabilities to gather, in one framework, design information, codes and practices may provide an important impetus to significant savings in design time and construction costs and in realistic expectations of survivability to advancing threats. The production of such a system is feasible since most of the technical problems have been solved and demonstrated in two previous projects using commercial software.

This report has discussed the considerable breadth and depth of variability in parameters and models involved in protective structure design. The technical overview can take on an added perspective when the importance of how protective structure design, and a consideration of the design uncertainties, is related to the overall air base operability issue. There are many general philosophical, but realistic, issues associated with protective structure design on an air base overseas which could be resolved through the use of an ***integrated design system***. One question to ask is "Given the uncertainty in the threat, the propagation of effects through air and soil, the structural response mode, and the behavior of components, how does one decide to expend funds on a particular design? In other words how does one know where to put the money to enhance survivability? Does one buy tougher materials, bury the structure underground, use a different geometry? The answer to these questions, at the air base level, may also help to understand decisions in the research community about which programs to fund in support of the operational Air Force needs in the area of survivability and operability.

An integrated design approach, such as that described here, could benefit the commercial design community since the methods will reduce design and construction costs by optimizing structural performance under the influence of uncertain loadings and the uncertain degradation of properties with time. These reduced costs will be balanced with improved assessments of the risks of hazardous load environments.

# Appendix A

## N-DIMENSIONAL DOMAIN OF PERTINENT VARIABLES AND PARAMETERS

There are six major design areas involved in determining the survivability of protective structures to conventional munitions effects. These areas are the: Source Characteristics, Propagation Characterization, Soil-Media Interaction (SMI), Structural Response, Content Response, and Damage Assessment. The tables shown below for each of these six areas details the pertinent variables of importance in the design of protective facilities and the kind of uncertainty generally apparent in these variables. The uncertainty is assessed as being random or nonrandom. For random uncertainties a specific distribution type is suggested and for nonrandom uncertainties potential quantification methods (evidence theory, fuzzy set theory, interval theory) is suggested. The tables also point out the parameters that were considered in the Phase I effort.

The choice of a design model could also be considered a major design area. Certainly the type of model introduces some (often significant) uncertainty because the model is the engineer's abstraction of the real world and his choice of a model is based on certain implicit assumptions about the validity of such things as material behavior and boundary conditions. Model selection is primarily nonrandom where the uncertainty is a combination of ignorance, judgment, imprecision and the use of linguistic values. For Phase I the models were deterministic.

Table A.1 Source Characteristics

| Environment: Parameter | Uncertainty | Type/Method |
|---|---|---|
| *Air Burst* | | |
| HOB | Random | Gamma |
| Peak Air Blast* | Random | Normal |
| Range* | Random | Beta |
| Fragmentation* | Random | Beta |
| *Surface Burst* | | |
| Contact; Proximity | Random | Gamma |
| Peak Airblast | Random | Normal |
| Range | Random | Normal |
| Fragmentation | Random | Bivariate-Normal |
| *Buried Burst* | | |
| DOB | Random | Beta |
| Peak Ground Stress | Random | Normal |
| Range | Random | Normal |
| Media-Coupling | Nonrandom | Evidence Theory |
| Impact Velocity | Random | Beta |
| Angle of Incidence | Random | Beta |
| Yaw | Random | Beta |
| Weapon Types | Uncertainty | Type/Method |
| *Projectiles, bombs, special purpose, rockets/missiles* | | |
| Charge weight* | Random | Normal |
| Accuracy | Random | Bivariate-Normal |
| Fuzing | Nonrandom | Evidence Theory |
| Slenderness Ratio | Determinate | |
| Shape | Determinate | |

*Items addressed in Phase I

## Table A.2 Propagation Characteristics

| Environment: Parameter | Uncertainty | Type/Method |
|---|---|---|
| *Air* | | |
| Shock Velocity* | Random | Normal |
| Density | Determinate | |
| Shock Conditions | Nonrandom | Fuzzy Theory |
| *Surface and Buried* | | |
| Media Coupling | Nonrandom | Evidence Theory |
| Soil type (rock, etc.) | Nonrandom | Fuzzy Theory |
| Density | Random | Normal |
| Porosity | Random | Normal |
| Stiffness (load/unload) | Random | Beta |
| Water Content | Nonrandom | Evidence Theory |
| Layering | Nonrandom | Fuzzy Theory |
| Shock Conditions | Nonrandom | Fuzzy Theory |

*Items addressed in Phase I

## Table A.3 Structure-Media Interaction (SMI)

| Environment: Parameter | Uncertainty | Type/Method |
|---|---|---|
| *Air* | | |
| Reflected Pressure* | Random | Normal |
| Fragmentation* | Random | Beta |
| Dynamic pressure | Random | Normal |
| Penetration* | Random | Beta |
| *Surface or Buried* | | |
| Radiation damping | Nonrandom | Fuzzy Theory |
| Water content | Nonrandom | Evidence Theory |
| Layering | Nonrandom | Evidence Theory |
| Soil type | Nonrandom | Fuzzy Theory |
| density, stiffness..... | Random | Various |

*Items addressed in Phase I

**Table A.4 Structural Response**

| Environment; Parameter | Uncertainty | Type/Method |
|---|---|---|
| *Geometry* | | |
| Box* | Deterministic | |
| Arch | Deterministic | |
| Dome | Deterministic | |
| Cylinder | Deterministic | |
| Hybrid | Nonrandom | Interval Th |
| *Material* | | |
| Reinforced Concrete* | Random | Various |
| Steel | Random | Log-Normal |
| Composites | Random | Log-Normal |
| *Constitutive Models* | | |
| Compressive strength* | Random | Normal |
| Yield strength* | Random | Normal |
| Bulk modulus | Random | Normal |
| Failure surface | Nonrandom | Fuzzy Theory |
| Cap Model | Nonrandom | Interval Theory |
| Friction | Nonrandom | Interval Theory |
| Damping* | Nonrandom | Fuzzy Theory |
| *Structural Elements* | | |
| Plates, slabs* | Deterministic | |
| Columns | Deterministic | |
| Beams | Deterministic | |
| Rings | Deterministic | |
| Shells | Deterministic | |
| Solids | Deterministic | |
| Membranes, cables | Deterministic | |
| *Boundary Conditions* | | |
| Fixed* | Nonrandom | Interval Theory |
| Hinged | Nonrandom | Interval Theory |
| Free | Deterministic | |
| Impedance | Nonrandom | Evidence Theory |
| *Structural Systems* | | |
| Joints | Nonrandom | Various |
| Fenestrations (windows, etc) | Nonrandom | Various |
| 3D effects | Nonrandom | Various |

*Items addressed in Phase I

| Table A.5 Content Response* | | |
|---|---|---|
| Environment: Parameter | Uncertainty | Type/Method |
| *Rugged Equipment* | | |
| Acceleration | Random | Normal |
| *Sensitive Equipment* | | |
| Velocity | Random | Uniform |
| Acceleration | Random | Beta |
| *Humans* | | |
| Acceleration | Nonrandom | Evidence Theory |
| Displacement | Nonrandom | Fuzzy Theory |

*Not addressed in Phase I

| Table A.6 Damage (Survivability) Assessment | | |
|---|---|---|
| Environment: Parameter | Uncertainty | Type/Method |
| | All of these are nonrandom uncertainties subject to imprecision, ignorance and the reliance on some linguistic variables | |
| *Response Modes* | | |
| Flexure | | |
| Diagonal Tension* | | |
| Direct Shear* | | |
| Membrane Tension | | |
| Penetration | | |
| Acceleration, etc. | | |
| *Failure Levels* | ditto | |
| None | | |
| Slight | | |
| Moderate | | |
| Severe | | |
| Co'lapse | | |
| Failure/Survival* | | |
| *Repair Considerations* | | |
| Repair Time | ditto | |
| Cost | | |
| Facility Value | | |
| Mission Critical | | |

*Items addressed in Phase I

# *Appendix B*

## DYNAMIC MODAL ANALYSIS AND BREACHING/PERFORATION PHYSICS

## Dynamic Modal Analysis

Equation of Motion

For a structural system with a stiffness matrix [K], mass matrix [M], proportional damping matrix [C], and applied forces, f(t), the equations of motion for dynamic equilibrium are, in matrix form,

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{f(t)\} \qquad (1)$$

where a "dot" over a symbol denotes one derivative with respect to time.

Modal Superposition

The normal mode solution is obtained from the corresponding undamped homogeneous form of equation (1) through the following development,

$$[M]\{\ddot{X}\} + [K]\{X\} = 0$$
$$(-\omega^2 [M] + [K])\{X\} = 0$$
$$-\omega^2 [M]\{X\} = [K]\{X\} \qquad (2)$$

Eq (2) is the standard eigenvalue form and its solution produces $\omega_i^2$ (natural frequencies) for each mode and the modal matrix [Φ] (matrix whose columns are the normal mode shapes). Using a normalized [Φ], that is

$$[\Phi]\,[\Phi]^T = [I]$$

Eq (1) can be rewritten as

$$[\Phi]^T[M][\Phi]\,[\Phi]^T\{\ddot{X}\} + [\Phi]^T[C][\Phi]\,[\Phi]^T\{\dot{X}\} + [\Phi]^T[K][\Phi][\Phi]^T\{X\} = [\Phi]^T\{f(t)\} \qquad (3)$$

A theorem proves that,

$$[\Phi]^T[M]\,[\Phi] = [M_m]$$
$$[\Phi]^T[C]\,[\Phi] = [C_m]$$
$$[\Phi]^T[K]\,[\Phi] = [K_m]$$

are all diagonal matrices, hence, their use uncouples the equations of motion.

Letting $[\Phi]\{X\} = \{Y\}$, where {Y} is a vector of the principal coordinates, then

$$[M_m][\ddot{Y}] + [C_m][\dot{Y}] + [K_m][Y] = [\Phi]^T\{f(t)\}$$

represents n uncoupled SDOF equations of motions, i.e., for each mode,

$$M_{mi}\ddot{Y}_i + C_{mi}\dot{Y}_i + K_{mi}Y_i = f_i(t) \qquad i = 1, 2, \ldots, n$$

A Newmark integration scheme can be used to obtain the modal displacement, velocity, and acceleration in the principle coordinates, i.e., $Y$, $\dot{Y}$, $\ddot{Y}$, respectively.

Once [Y] is determined it is usually desirable to return to general coordinates [X], with the following operation,

$$[X] = [\Phi][Y].$$

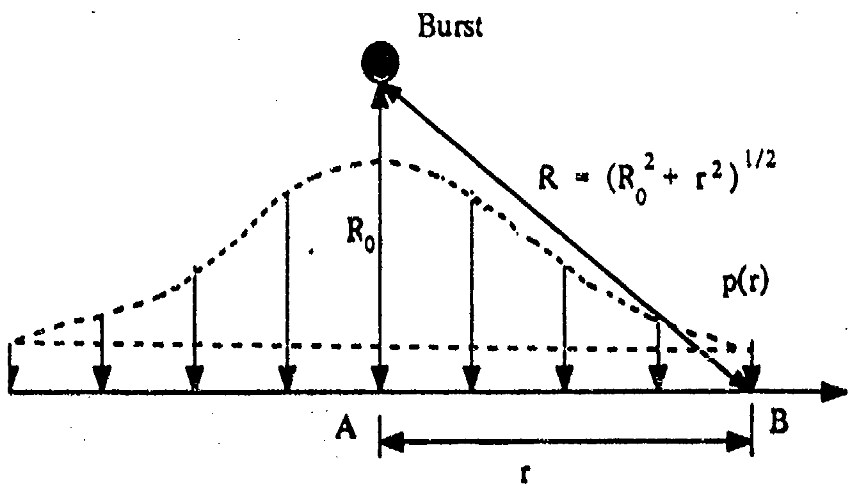
The maximum shear stress along the beam is then determined by

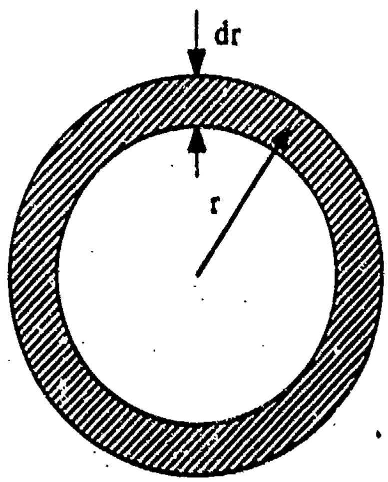
$$V_m(x,t) = -\frac{EI\,[X]'''}{b\,h}$$

where a " prime " is a symbol which denotes differentiation with respect to x, and the product "bh" is the shear area.

## Breaching Analysis

When a conventional munition detonates close to a structural element ( within a scaled range of about 1.5) it imparts a spherical shock front to the element. Hence, the maximum pressure from the detonation on the element face is proportional to the cube of the distance between the burst and the element face. In the schematic shown below,




the pressure at point B is given by

$$p(r) = \left(\frac{R_0}{R}\right)^3 P_r \quad \text{and} \quad R = (R_0^2 + r^2)^{1/2}$$

where $P_r$ is the peak reflected pressure at point A. The value of $P_r$ is a function of R and the charge weight, W, as shown in Figure B.1. For a small annulus of area on the face of the element with a radius of r and a width of dr (see schematic) then the total force on this annulus from the blast pressure is given by

$$\Delta F(r_0) = \left(\frac{R_0}{R}\right)^3 P_r \cdot 2\pi r\, dr$$

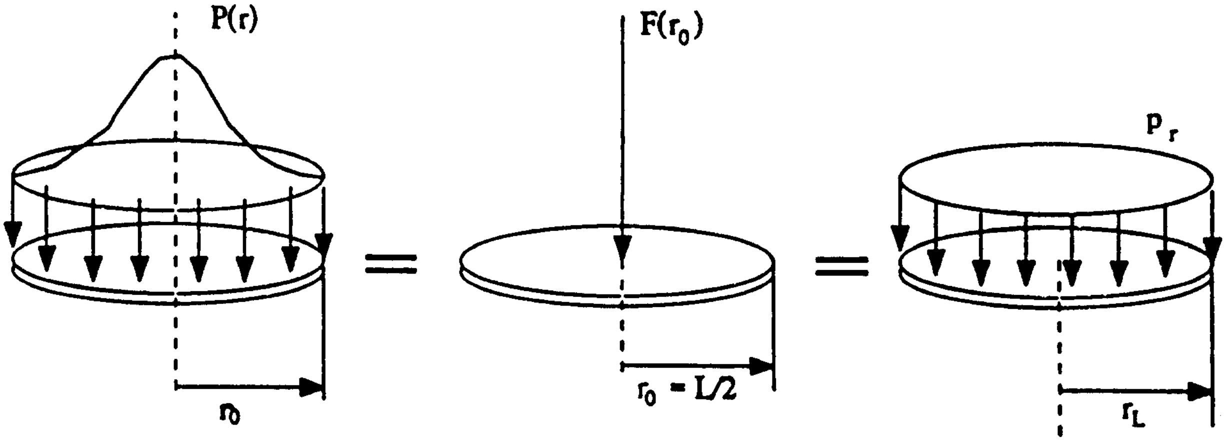
and the total force on an area of the element face of radius $r_0$ is then,

$$F(r_0) = \int \Delta F(r,t) = 2\pi p_r R_0^3 \int_0^{r_0} \frac{r}{R^3}\, dr = 2\pi p_r R_0^3 \int_0^{r_0} \frac{r\, dr}{(R_0^2 + r^2)^{3/2}}$$

which, through the substitution $Y = R_0^2 + r^2$ and $dY = 2r\, dr$, yields

$$F(r_0) = 2\pi R_0^2 P_r \left[ 1 - \frac{R_0}{(R_0^2 + r_0^2)^{0.5}} \right]$$

So $F(r_0)$ is the total force on a circular area of radius $r_0$ as shown in the figure below.



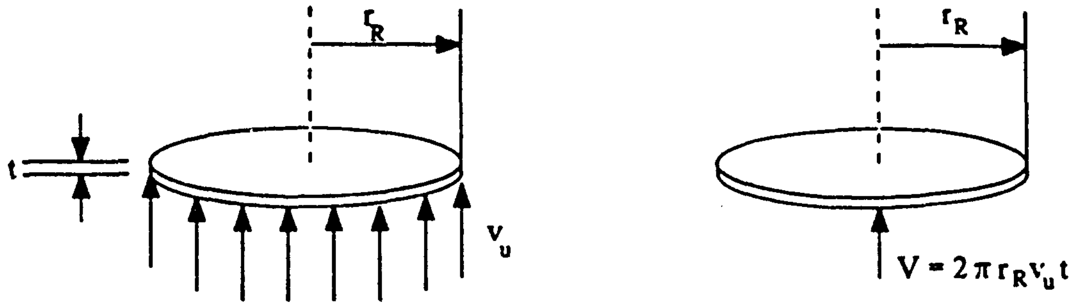

Normally, the parameter $r_0$ is taken to be one-half the slab length (L/2). Now, if the total force $F(r_0)$ on the slab is presumed to be produced by the uniform pressure, $p_r$, the relationship is determined,

$$r_L = \left[ \frac{\phi F(\frac{L}{2})}{\pi p_r} \right]^{1/2}$$

where $r_L$ is the radius of an equivalent circular area with the same force, $F(r_0)$, acting on it and f = 0.57 is a factor accounting for the fact that the peak reflection factor only occurs at normal incidence (where $R = R_0$) and most of the pressure occurs for $R > R_0$. Now if the maximum shear stress capacity of the structural element is set equal to the direct shear stress, $v_u$, given by

$$v_u = 8\,(f_c')^{1/2} + 0.8\,\rho_s\,f_y \leq 0.35\,f_c'$$

where $f_c'$ is in psi, $r_s$ is the longitudinal steel percentage, and $f_y$ is the tensile strength of the steel, and assume this stress acts on the perimeter of a circular "punched" area of the element, as shown below,




then the total resisting shear force acts over an area with a radius given by,

$$r_R = \frac{\phi F(\frac{L}{2})}{2\pi v_u t}$$

where t is the element thickness and $r_R$ is the radius. Finally, <u>failure occurs for the condition</u> $r_R > r_L$.

## Penetration Physics

Penetration physics is comprised of two phenomena: perforation and spalling. The first phenomenon involves a fragment completely passing through a structural element and the second phenomenon involves a fragment being embedded in a structural element and, while not completely passing through an element, it gets close enough to the exit surface to cause spall damage from reflected tension waves. All of the mathematical and empirical relations shown below have been obtained from Reference 16.

### Perforation

For the case of perforation the procedure involves estimating the weight of the largest bomb casing fragment and then estimating the distribution of fragment weights. Then the velocity of each fragment is determined and finally the perforation depth is obtained from information about the weight and velocity of fragments.

The largest fragment weight from a GP bomb is given by

$$W_{f_{max}} = M_A^2 \ln^2 [\frac{8 W_c}{M_A^2}] \quad \text{(ounces)}$$

where $M_A = 0.3 (t_c)^{5/6} (d_i)^{1/3} [1 + \frac{t_c}{d_i}]$ ,

and where the following parameters apply for 1000# GP bombs.

**Parameters for 1000 Pound GP Bomb**

Total weight = 1064 lbs = $W_t$
Actual Charge Weight = 555 lbs (TNT) = W
Casing Inside Diameter = 17.8 inches = $d_i$
Casing wall thickness = 0.5 inches = $t_c$
Casing weight = 0.8 ($W_t$ - W ) = $W_c$ (Pounds)

When $W_{fmax}$ is determined ( this is the maximum fragment weight) then general fragment weight, $W_f$, is given a Beta distribution with a minimum of 2 ounces and a maximum of $W_{fmax}$. The striking velocity $V_s$ of a fragment with weight $W_f$ is given by

$$V_s = V_0 \, e^{(-0.004 R / W_f^{1/3})} \quad \text{in ( ft per second)}$$

where R is the range in feet and where the initial velocity, $V_0$, is given by

$$V_0 = 6940 \left\{ \frac{W/W_c}{1 + 0.5 \, W/W_c} \right\}^{1/2} \quad \text{in (fps)}$$

Also note that $V_s = V_0$ when $R < 20$ feet.

With the striking velocity and fragment weight known the perforation depth in reinforced concrete (with compressive strength, $f'_c$, in psi) is denoted as $X_f$ and given by

$$X_f = (0.162)(10^{-5})(W_f)^{0.4} (V_s)^{1.8} [\frac{5000}{f'_c}]^{0.5}$$

Perforation failure occurs when $X_f$ exceeds the thickness of the reinforced concrete element.

## Spalling

If $X_f$ is less than the element thickness we then check for spall damage. Spall failure will occur if the parameter $C_1$, shown in Figure B.2, falls within the spall band shown in the figure. The parameter $C_1$ is found from,

$$C_1 = \left[ \frac{T_c + 0.348\, W_f^{1/3}}{X_f} - 1 \right] \left[ \frac{C_s}{V_s} \right]^{1/3}$$

where $T_c$ = thickness of the concrete element (inches), and where

$$C_s = 5.16\, (E_c)^{1/2} \text{ in (fps)}$$

$$E_c = 57000\, (f_c')^{1/2} \text{ where } f_c' \text{ is in (psi)}$$

If spall failure does not occur, then the structural element survives penetration failure. Note that the probability of *penetration* is $P_f = P_p + P_s$ where $P_p$ is the probability of *perforation* and $P_s$ is the probability of *spall*.

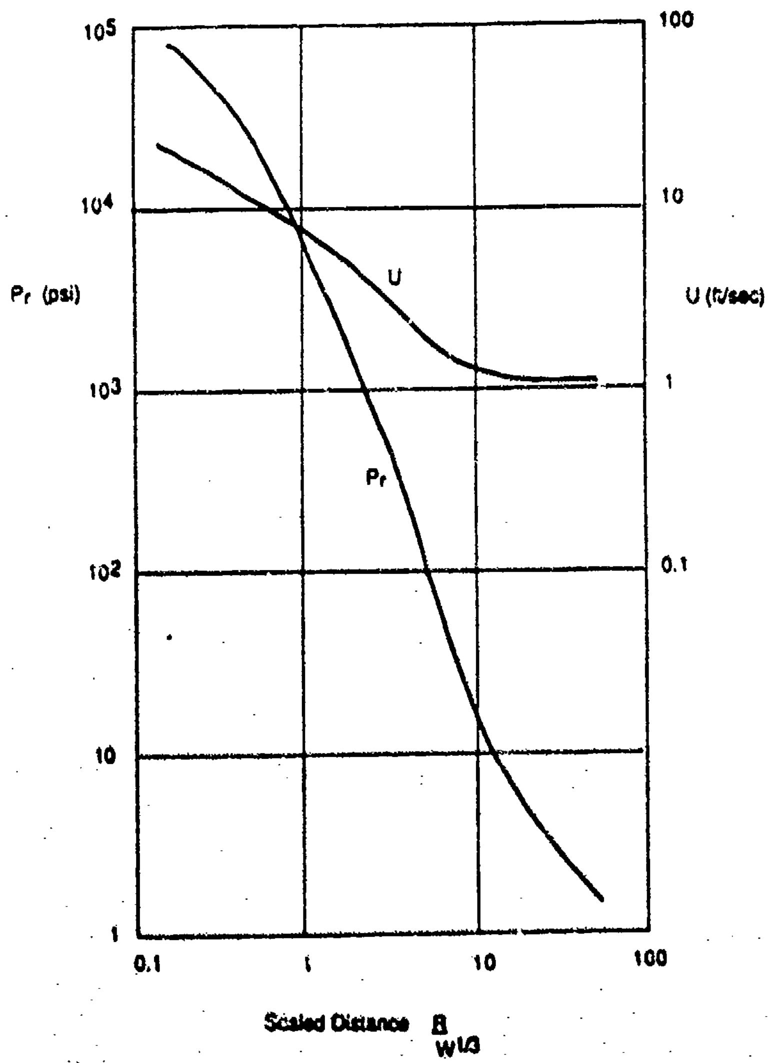



Figure B.1 Peak Reflected Pressure ($P_r$) and Shock Velocity (U) vs. Scaled Range (Crawford, et. al: 1970)

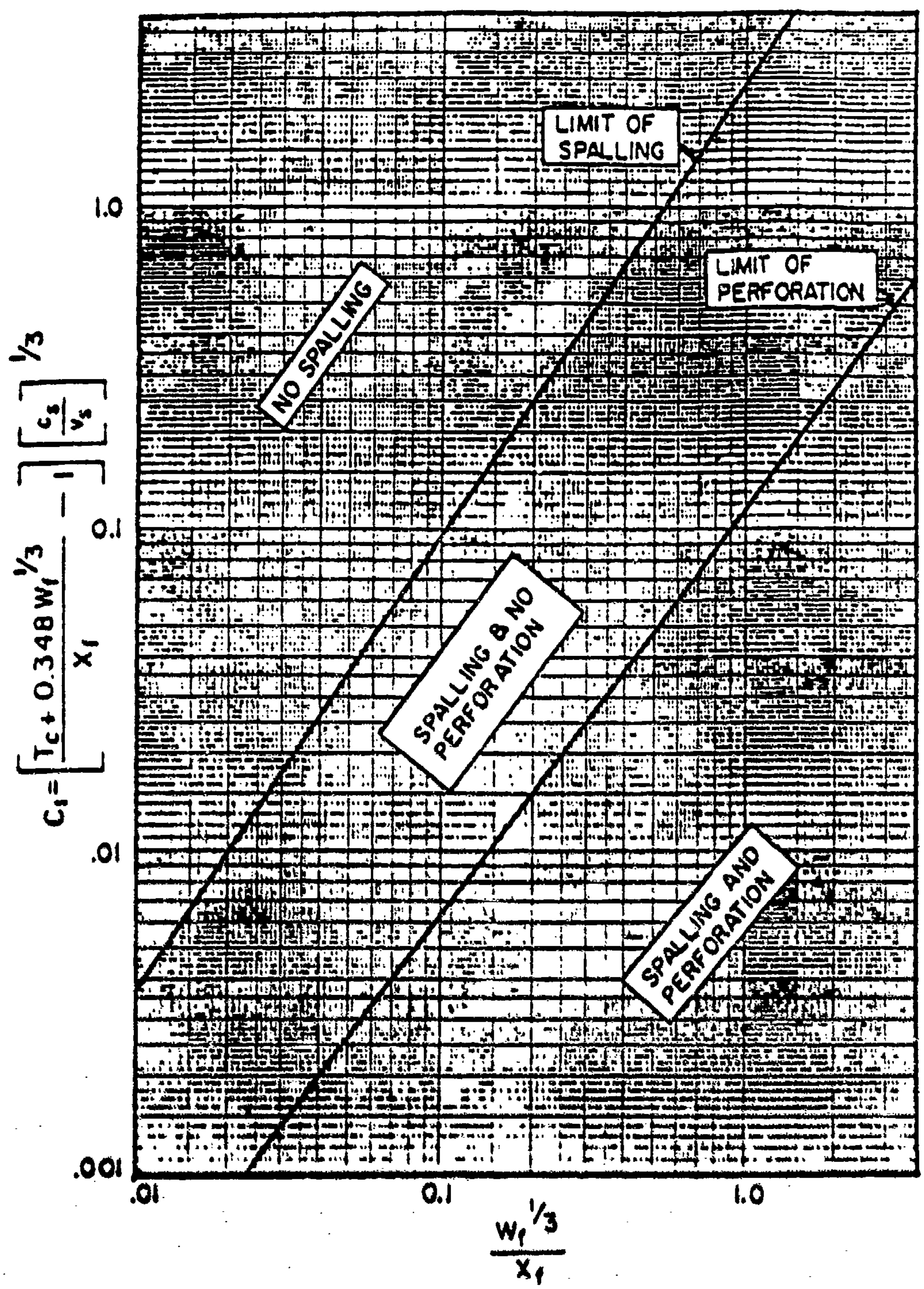



Figure B.2 Limits of Concrete Spalling and Perforation from Fragments (Crawford, et. al: 1970)



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