REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

AD-A242 014

...n is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, ...ting and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this ...icing this burden to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson ...nd to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

2. REPORT DATE

3. REPORT TYPE AND DATES COVERED
FINAL, 01 JUL 88 to 01 JUN 91

MATHEMATICAL MODELING IN PLASMA PHYSICS

5. FUNDING NUMBERS
AFOSR-88-0232
61102F, 2304/A9

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AFOSR-TR-91-0845

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
AFOSR/NM
Bldg 410
Bolling AFB DC 20332-6448

10. SPONSORING/MONITORING AGENCY REPORT NUMBER
AFOSR-88-0232

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release;
distribution unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The goal of the conducted research was to find improved mathematical models for the physical processes that occur in radio frequency discharges. Existing models of the plasma sheath have been studied and new ones have been developed for the bounded plasma problem. Numerical algorithms and analytical formulas have been found for solving the equations governing these models, and physical characteristics of the sheath have been obtained.

91-13678

91 10 22 068

14. SUBJECT TERMS

15. NUMBER OF PAGES

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std Z39-18

Mathematical Modelling in Plasma Physics

Final Technical Report

Period covered: Jul 1, 1988–Jun 30, 1991

The proposed research project consisted of three parts. The goals of the first part of the project (see Proposal, p.8, Part A) have been achieved in the first year (July 1, 1988–June 30, 1989):

1. The investigator has developed an algorithm for solving Godyak's [1] nonlinear dynamic model of an electrode sheath.

2. Using the developed algorithm, the investigator has computed the sheath characteristics for various gases in the collisionless case.

3. The numerical results have been compared with the analytic solution of the problem that was stated in [1], and the two sets of values have been found to match perfectly.

4. The computed results for mercury agree with those obtained experimentally by Godyak [1].

5. The results described above were included in the paper "Smooth plama-sheath transition in a hydrodynamic model", IEEE Trans. Plasma Sci., 18, 1, 1990, by the investigator and V. Godyak.

The investigations that produced the results 1.–5. have led to new insights into plasma-sheath modelling. We have discovered that Godyak's model from [1] is accurate only for certain gases and certain ranges of parameters. Our main goal in the second year of the investigation (June 1, 1989–June 30, 1990) was to create a new modified model that would overcome these limitations. This goal has been achieved:

1. Godyak's model [1] has been midified in such a way that the electron and ion conductivity currents in the sheath could be taken into account.

2. The investigator has developed an algorithm for solving the new nonlinear dynamic model of an electrode sheath.

3. Using the developed algorithm, the investigator has computed the sheath characteristics for mercury vapor for different collision parameters.

4. For the collisionless case, the system has been solved analytically.

5. The computed results for mercury agree with those obtained experimentally by Godyak [1].

6. The results described above were included in the paper "Dynamic model of the electrode sheath in symmetrically driven rf discharges", Phys. Rev. A, 42, 4, 1990, by the investigator and V. Godyak.

In order to check the accuracy of the new model more experimental data are needed. During the second and third years of the investigation, V. Godyak has conducted a number of experiments at GTE Laboratories in Waltham, MA for argon. In the next future a new algorithm will be developed to solve the new model numerically, and the experimental and numerical data will be compared. The results that will be obtained in this direction are an extension of Part A of the proposed research project and will be contained in future papers to be written with V. Godyak.

In the third year of the investigation (June 1, 1990–June 30, 1991), we started to work on the second part of the proposed research project (see Proposal p.8, Part B) which is concerned with the capacitance in the sheath. The following has been achieved:

1. Analytical formulas for the computation of high and low frerquency capacitances for any bounded plasma model have been found. These formulas show the relation between the capacitance and the ion movement in the sheath.

2. It has been observed that the difference between high and low frequency capacitances is maximal for the vacuum sheath at high sheath voltages.

3. The relationship between low frequency capacitance and the initial voltage applied to the electrode has been studied for different collision parameters.

4. The results of the conducted study are included in the paper "Sheath capacitance at low and high frequencies" by the investigator and V. Godyak. This paper will appear in the Proceedings of the XX International Conference on Ionized Gases, Pisa, Italy, July 1991.

In the future we are planning to study the capacitance of the sheath for intermediate frequencies. The results that will be obtained in this direction will be contained in future publications to be written with V. Godyak.

The following is the list of articles that have been published or submitted for publication by the investigator during the period covered:

1. "Smooth plasma-sheath transition in a hydrodynamic model" (with V. Godyak), IEEE Transaction on Plasma Science, 18, 1, 159–168, 1990.

2. "One-to-oneness of the solution map in retarded functional differential equations",

J.Diff.Eq., 85, 2, 201-213, 1990.

3. "Dynamic model of the electrode sheaths in symmetrically driven rf discharges" (with V. Godyak), Phys. Rev. A, 42, 4, 2299-2312, 1990.

4. "Sheath capacitance at low and high frequencies" (with V. Godyak), Proceed. XX Int. Conf. on Phenomena in Ionized Gases, Pisa, Italy, July, 1991.

5. "The local stability theorem for a class of retarded functional differential equations", subm. to J. Diff. Eq., 1991.

During the period covered, the investigator gave the following presentations:

1. "Smooth plasma-sheath transition in a hydrodynamic model", invited lecture, Air Force Base, Hanscom Fields, Massachusetts, June 1988.

2. "On an equidistant spectral method", SIAM Annual Meeting, Minneapolis, Minnesota, July 1988.

3. "Numerical observation of homoclinic orbits and chaos", invited lecture, McMaster University, Ontario, Canada, October 1988.

4. "Onset of chaos in differential delay equations", invited lecture, Air Force Base, Hanscom Fields, Massachusetts, March, 1989.

5. "One-to-oneness of the solution map in retarded functional differential equations", invited speaker, International Conference on Differential Equations and Control Theory, Dayton, Ohio, June, 1989.

6. "On the geometry of attractors in Functional Differential Equations on compact manifolds", invited lecture, University of Cologne, West Germany, September, 1989.

7. "Mathematical modelling in Plasma Physics", invited lecture, Center for Dynamical Systems and Nonlinear Studies, Georgia Institute of Technology, Atlanta, Georgia, May, 1990.

8. "A moving boundary problem in Plasma Physics", SIAM Conference on Dynamical Systems, Orlando, Florida, May 1990.

References

1. Godyak, V. A., Soviet RF Discharge Research, Delphic Ass. Inc., Falls Church, 1986.