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SMALL SAMPLE DESIGN ALLOWABLES FROM PAIRED DATA SETS

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ABSTRACT

This paper identifies an acceptable statistical procedure for obtaining design allowable values from a small set of material strength data. The allowable represents a material design number defined as the 95% lower confidence bound on the specified percentile of the population of material strength data. The percentiles are the first and tenth for the A and B allowables. The proposed method reduces the penalties commonly associated with small sample allowable computation by accurately maintaining the definition requirements and reducing variability in the estimate. Application of very small samples will obviously reduce costs in testing and manufacturing which is the primary motivation for this study.

In the evaluation process five methods were considered for computing the design allowable. Three of these methods involved certain statistical distribution assumptions while the other two were nonparametric procedures. The latter methods introduced a pooling process such that the small sample was combined with a larger, previously obtained sample.

Monte Carlo studies showed that the nonparametric procedures are the most desirable for computing the design allowable value.

CONTENTS

1 Introduction

The A or B statistically based design allowable value is a statistic which is less than the first or tenth percentile of the population with probability .95. That is, the value is a 95% lower tolerance limit for the percentile. In Figures 1A and 1B, a graphical display is shown for the B allowable value probability density function for sample sizes of n equal to 10 and 50 from a standard normal population. The dotted vertical lines indicate the tenth percentile of the population and the probability that the allowable is less than or equal to the tenth percentile is .95 for the design allowable value probability density function. The graphical display of the allowable value density functions show much less dispersion for $n = 50$ than for $n = 10$. Therefore, small samples will usually result in lower allowable values. In,[1, 2, 3, 4, 5], various procedures are described for determining the statistical design allowable values.

The motivation for the work described in this paper resulted from a need by the aircraft industry to obtain a less conservative, statistically based material design value from a small sample of composite material strength data. Here, 'conservative' is to be interpreted to mean 'excessively low', which corresponds to a design engineer's use of the word. Statistical conservatism, that is a confidence exceeding the nominal level of .95, need not be present for 'engineering conservatism' to be a problem.

The use of small samples reduces the amount of testing and consequently the manufacturing cost of composite aircraft structures. For example, in order to qualify a composite material to be used in the manufacture of a commercial aircraft, the FAA,[6] requires property values for tension, compression, and shear tests subjected to the enviromental conditions: hot-wet, cold-dry, and room temperature for three separate batches of material. In the development of a composite tail section by one of the major aircraft companies the cost of testing was more than 20 million dollars. In addition to the cost, excessively conservative allowable values can also result in an over-design situation, since the value often provides information in determining a structural design.

In order to avoid the penalty associated with using small samples in the tolerance limit computation, a procedure is introduced in this paper involving pooling a large

[1] Military Handbook 17B, Army Materials Technology Laboratory, Polymer Matrix Composites, Volume 1, Guidelines, 1988.

[2] Neal, D. M., Vangel, M. G., and Todt, F., "Determination of Statistical Based Composite Material Properties" in Engineered Materials Handbook, Composites, C.A. Dostal, ed., American Society of Metals Press, Metals Park, Ohio, Vol. 1, 1987.

[3] Neal, D. M., Vangel, M. G., "Statistical Based Material Properties - A Military Handbook-17 Perspective", MTL TR 90-5, U.S. Army Material Technology Laboratory, Watertown, Massachusetts 02172-0001, 1990.

[4] Neal, D. M. and Spiridigliozzi, L., "An Efficient Method for Determining the 'A' and 'B' Design Allowables", ARO Report 83-2, U.S. Army Laboratory Command, Army Research Office, P.O. Box 12211, Research Triangle Park, North Carolina 27709-2211, 1983.

[5] Shyprykevich, P.,"The Role of Statistical Reduction in the Development of Design Allowables for Composites", Test Methods for Design Allowables for Fibrous Composites: 2nd Vol., ASTM STP 1003, pp. 111-135, 1989.

[6] Soderquist, Joseph, National Resource Specialist for Composites (FAA), Private Conversation

sample with a smaller one in order to obtain the allowable value. This is done in order to reduce the inherent variability that occurs from applying the smaller data set alone.

In the pooling process the larger data set should be obtained from prior available test results or from less expensive tests. Ideally, both samples should be from the same material, test, and enviromental conditioning process. In the pooling process it is assumed that for a given material (eg., graphite-epoxy) there are similar classes of failure modes.

In order to avoid the uncertainties involved in identifying a statistical model from a small sample when computing the allowable value, this paper introduces two nonparametric methods (Ferguson,[7] and the Modified Hanson-Koopmans,[8]) In applying the Bayesian nonparametric method, the larger set represents the prior and the smaller one the empirical data. In the Modified Hanson-Koopmans method an ordered array of strength measurements is obtained from the pooled data sets. The tolerance limit is determined from a specific ratio of ordered values multiplied by a factor determined from the sample size of the pooled data.

The Reduced Ratio Method,[9] another procedure for computing small sample design allowables, was also evaluated. This method is commonly used by the aircraft industry. For example, a U.S. helicopter company routinely uses this method for obtaining allowables from six specimens tested in tension at 180°F. In the analysis an additional, previously obtained sample of at least thirty room temperature tension test results are included in order to reduce variability in the allowable estimate.

2 Determination of Allowable Values Nonparametric Bayesian Method

The nonparametric Bayesian,[7] allowable value is obtained from the following. Let $\{x_i\}_1^n$ represent the current empirical data which the allowable value is to represent and $\{t_j\}_1^m$ the larger prior data set obtained from previous test results.

In the analysis the cummulative density function (CDF) of the prior (larger data set) is written as

$$F_0(t) = \alpha((-\infty, t])/\alpha(R) \tag{1}$$

where $\alpha(R)$ is the sample size and $\alpha((-\infty, t])$ represents the number of values less than t from $\{t_j\}_1^m$. The CDF of the smaller sample $\{x_i\}_1^n$ is

$$F_n(t \mid x_1, x_2, ..., x_n) = \sum_{i=1}^{n} \delta_{x_i}((-\infty, t])/n \tag{2}$$

[7]Ferguson, T. S., "A Bayesian Analysis of Some Nonparametric Problems", Annals of Statistics, Vol. 1, No. 2, 209-230, 1973.

[8]Vangel, M. G., "Lower Tolerance Limits for Log-Convex Distributions", to be published.

[9]Metallic Materials and Elements for Aerospace Vehicle Structures, MIL-HDBK-5C, 15 September 1976, pp. 9-14.

where n is the sample size and the sum over i of $\delta_{x_i}(t)$ is equal to the number of x_i values less than or equal to t. For example,

if $t = 1, 2, 3, 4, 5$
and $x = 6, 7, 8, 9, 10$
then $F_n(5 \mid 6, 7, 8, 9, 10) = 0.$
If $t = 11, 12, 13, 14, 15$
then $F_n(11 \mid 6, 7, 8, 9, 10) = 1.$

The posterior distribution for $\{x_i\}_1^{\infty}$ is then written as

$$\hat{F}_n(t \mid x_1, x_2, ..., x_n) = P_n F_0(t) + (1 - P_n)F_n(t \mid x_1, x_2, ..., x_n), \tag{3}$$

where

$$P_n = \frac{\alpha(R)}{\alpha(R) + n}. \tag{4}$$

An example of a Bayes estimate for $x = 1$ when

$t = 1, 2, 3, 4, 5$
and $x = 1, 2, 3, 4, 5$ is
$\hat{F}_n = P_n F_0 + (1 - P_n)F_n = (.5)(.2) + (.5)(.2) = .2.$

3 Nonparametric Tolerance Limit on the Bayesian Quantile Estimate

The allowable value as described previously is a tolerance limit on the quantile estimates. The process for obtaining that limit is shown in this section. Initailly, a random sample F(Y) of size $M = \alpha$(R) + n is assumed independent of the mixture of the prior and empirical data sets shown in Equation 3. By ordering a sample of Y_1,Y_2,...,Y_M values, the probability density function for Y_i, $1 \leq i \leq M$ can be written as a Beta distribution,

$$f_{z_i}(z) = \frac{\Gamma(M) z^{uM-1}(1-z)^{(1-u)M-1}}{\Gamma(uM)\Gamma((1-u)M)}, \tag{5}$$

where $z_{(i)} = \mathrm{F}(Y_{(i)})$ and $i = uM$ with u representing the CDF value corresponding to the i^{th} ordered number. The tolerance limit Y^* for Y_q is

$$P(Y_q \geq Y^*) = 1 - \alpha = P[F(Y_q) \geq F(Y^*)] \tag{6}$$

where Y_q is the $100q^{th}$ percentile of Y. Since

$$P(Y \geq Y_q) = \int_0^q \frac{\Gamma(M) z^{uM-1}(1-z)^{(1-u)M-1}}{\Gamma(uM)\Gamma((1-u)M)} dz \tag{7}$$

from Equation 5, a 1 - α tolerance limit on Y_q can be obtained by solving for u from the following. In the case of the B allowable computation, $\alpha = .05$ and $q = .10$, Equation 7 can be written as

$$\int_0^{.10} \frac{\Gamma(M) z^{uM-1}(1-z)^{(1-u)M-1}}{\Gamma(uM)\Gamma((1-u)M)} dz = .95. \tag{8}$$

See Table I for tabulation of u and M values that satisfy Equation 8.

Solving for u in Equation 8 determines the lower tolerance limit of the CDF of sample size M where the i^{th} ordered value is equal to uM. Obtaining a lower ordered CDF value from Equation 3 that is approximately equal to a u determines the 1 - α tolerance limit of the q^{th} quantile of the posterior CDF for a sample size M.

An example of this would be if there were only prior data $\{t_j\}_1^{30}$ and a B allowable value is required where

$$t = 5, 6, 7, 8, 12, 16, 20, 25, ..., 40 \text{ and}$$
$$F_0(t) = .033, .066, .099, ..., 1.0,$$

then $M = 30$ and $u = .034$ from Table I. The allowable value t_j is determined from the approximate solution of $u \approx F(t)$ resulting in $t_1 = 5$; therefore, the first ordered value of the prior represents the B allowable value, which is the same as the nonparametric quantile sign test,[10] result, when the sample size is 30.

4 The Nonparametric Modified Hanson-Koopmans (MHK) Procedure

A nonparametric procedure (MHK),[8] for estimating the allowable value is introduced for any sample size greater than or equal to 2. The method is a modification of Hanson-Koopmans,[11] process. The modification has reduced the conservatism in computing property values when compared with the original method.

The method involves the following. Let $x_1,...,x_n$ be the order statistics of an independent and identically distributed sample from a continuous distribution F. Assume that $\bar{F}$ is log-convex, that is -log F(x) is a convex function. The class of log-convex functions includes a large enough group of distributions so that the following procedure involving log-convex functions can be considered nonparametric for most purposes.

The Hanson-Koopmans lower tolerance limits are of the form

$$T_{rs} = kx_r + (1 - k)x_s, \tag{9}$$

where $r < s$ and $k \geq 1$. The tolerance limit T_{rs} can be negative, even if the distribution F is zero for any negative values. A practical solution to this problem is to apply the Hanson-Koopmans approach to the log of the data x, that is,

$$T^*_{rs} = k \log x_r + (1 - k) \log x_s, \tag{10}$$

and then obtain by exponentiation the following

$$\tilde{T}_{rs} = e^{k \log x_r} + e^{(1-k) \log x_s} = x_r \left(\frac{x_r}{x_s}\right)^k. \tag{11}$$

[10] Conover, W. J., "Practical Nonparametric Statistics", John Wiley and Sons, 1980, p. 111.

[11] Hanson, D. L. and Koopmans, L. H., "Tolerance Limits for the Class of Distributions with Increasing Hazard Rates", Annals of Mathematical Statistics, Vol. 35, 1964.

For most distributions of interest, $\tilde{T}_{rs}$ still provides conservative tolerance limits, although technically $\tilde{T}_{rs}$ is valid for a class of distributions smaller than the log-convex class corresponding to T_{rs}.

In order to determine the B allowable value, the r, s, and k values are obtained for a given n in Table II. Tables are also available for the A allowable in Reference 8.

5 Allowable Computation for Normal and Weibull Models

The following small, single sample, data set allowable computation procedures were included for comparison purposes. This comparison is made with respect to the results obtained from the other methods described in this paper.

The normal PDF is

$$f_N(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}, \tag{12}$$

where μ and σ are the mean and standard deviation. The normal allowable is

$$A_N = \bar{X} - K_A s, \tag{13}$$

where K_A is a factor obtained from Reference 1 and $\bar{X}$ and s are the sample mean and standard deviation.

The Weibull allowable computation is as follows. The Weibull PDF is

$$f_W(x) = \frac{\beta x^{\beta-1}}{\alpha^\beta} e^{-(x/\alpha)^\beta}, \tag{14}$$

where β and α are the shape and scale parameters and the Weibull allowables can be written as

$$A_W = \hat{\alpha}[-\log(P_A)]^{1/\hat{\beta}}, \tag{15}$$

where the P_A's are tabulated in Reference 3 with $\hat{\alpha}$ and $\hat{\beta}$ being the maximum likelihood estimates for α and β obtained from an algorithem also shown in Reference 3.

6 The Reduced Ratio Method (RRM)

The Reduced Ratio Method,[9] determines an allowable value for a smaller data set $\{S_i\}_1^n$ by introducing an indirect computation procedure involving a larger, previously obtained set of data, $\{L_j\}_1^m$.

The first step is to determine the mean of L, that is $\bar{L} = \frac{1}{m}\sum_{i=1}^m L_j$. The second step requires obtaining the ratios $R_1=S_1/\bar{L}$, $R_2=S_2/\bar{L}$, ..., $R_n=S_n/\bar{L}$ and the mean ($\bar{R}$) of the R_i's. The reduced mean, $\bar{R}^*$ is then obtained from

$$\bar{R}^* = \bar{R} - t_{(.95)} V_R/\sqrt{n}, \tag{16}$$

where $t_{(.95)}$ is the .95 quantile of the t distribution for n - 1 degrees of freedom and V_R is the standard deviation of the R_i's. The next step is to compute an allowable (L_B) from the L sample using some single sample procedure such as described in the previous section. After obtaining L_B the allowable S_B for S is determined as follows

$$S_B = L_B \bar{R}^*. \tag{17}$$

7 The Pooling Process

The pooling process, as previously mentioned, requires combining a smaller data set S (the one represented by the allowable) with a larger set L obtained from prior test results. In the MHK process the objective is to represent S with a combined data set of S and L with sample size $m = n_S + n_L$. In the Bayes method the prior is represented by L and the empirical data by S.

If both the means and variances of S and L are known to be equal, then the pooling process can be easily justified. Unfortunately, this is seldom the case. Therefore, the following transformation is suggested. Let L_i and S_i be the data from sets L and S respectively and define the new data sets S* and L* by

$$S_i^* = \frac{S_i - \bar{S}}{\bar{S}} \tag{18}$$

and

$$L_i^* = \frac{L_i - \bar{L}}{\bar{L}}, \tag{19}$$

where $\bar{L}$ and $\bar{S}$ are the data set means. This procedure involves reducing the mean of S and L to a common mean of zero for S* and L*. In addition, the transformed data sets, S* and L*, have standard deviations equal to the CV's of S and L. Schematics of this transformation are shown in Appendices A and B.

It is suggested that an equality of variance test between S* and L* be made in order to determine if an excessively large difference in variance exists. The Siegel-Tukey nonparametric rank sum method,[12] proved effective in testing for equality of variance although for small samples (less than ten), the test on equality of variance will result in a certain amount of uncertainty.

8 Allowable Values for S* from Pooled Data

8.1 Bayes Solution

In the Bayes application let the smaller sample x (newly obtained data) of size n_S be represented by the S* values and the larger sample t (the prior) with n_L values by L*. Initially, u in Equation 8 is obtained from Table I for M equal to the combined sample sizes of S* and L* in order to determine the allowable for S*. CDF values are

[12] Siegel, S. and Tukey, J. W., "A Nonparametric Sum of Ranks Procedure for Relative Spread in Unpaired Samples", Journal of American Statistical Association, September, 1960.

determined from Equation 3 where $t = L^*$ and $x_i = S_i^*$ $i = 1,2,...,n$. Equating the CDF value that corresponds to u determines the ordered (uM) value of $\hat{F}_n$. Inverting $\hat{F}_n$ so that the corresponding ordered test result is obtained then determines the allowable value S_B^*.

8.2 The Modified Hanson-Koopmans Method

The nonparametric,[8] solution for obtaining allowable values involves pooling the values from S^* and L^* and letting the combined ordered array of values be x in Equation 11 with sample size $n = n_S + n_L$. Let this value be denoted S_B^*(in place of $\tilde{T}_{rs}$). This method is very simple to apply yet provides results for any sample size greater than 2.

9 Transformation Procedure in Determining Allowable

The allowable value for S^* is not sufficient since S and L were the original data sets involved in the analysis and their magnitudes differ from S^* and L^*. Therefore, the following transformation is required:

$$S_B = S_B^* \bar{S}_{.95} + \bar{S}_{.95} \tag{20}$$

where S_B is the required allowable value for the small sample S. The $\bar{S}_{.95}$ values represent the lower 95% confidence value for the mean of the S values. The purpose in using $\bar{S}_{.95}$ instead of $\bar{S}$ is to adjust for the variability in estimating the mean $\bar{S}$ of the small sample S. This variability in $\bar{S}$ directly effects the computation of the allowable S_B. This often results in S_B values being greater than the pth percentile of the population of S more than 5% of the time. This is counter to the requirement for an allowable value as described in the introduction.

10 Results and Discussions

10.1 Coverage Rates from MHK, Bayes, and RRM

In Tables III, IV, and V the coverage rate results are tabulated from the application of the MHK, Bayes, and RRM procedures, as functions of the coefficient of variations (CV(i)) for both the small sample S and the large sample L. The coverage rate represents the percent of values less than the 10% pt. (B allowable) or the 1% pt. (A allowable) of a population of values representing the data set. The data was obtained by randomly selecting values from either a normal or Weibull distribution with the specified CV's.

The mean and standard deviation are identified as: m(1) and s(1) for the Larger sample L and m(2) and s(2) for the smaller sample S. The sample sizes are usually n(1) = 30 and n(2) = 6 for the large and small data sets respectively. CV(1) and CV(2) have similar representation for the two samples.

In Table III results from randomly selected values obtained from normal distributions with sample sizes of 30 and 6 show that for differences in the CV's less than 20% an acceptable coverage rate can be obtained from all methods since the rates are greater than 95%. The MHK and Bayes methods provide acceptable results even for a 60% difference in the CV values although they fail to obtain the desired 95% minimum. The RRM coverage results with 40% differences in CV's fail to provide acceptable coverage as shown in both the A and B allowable computation. The A allowables could not be computed using the Bayes method since an amount of data much greater than 36 would be required. The A allowable tables for u and M have not been computed because of the excessively large data set requirements. When CV(1) = .12 and CV(2) = .10, greater variability in L than S, the coverage is much greater than required, therefore, resulting in potentially over conservative estimates for the S allowable. This will usually be the case when CV(1) > CV(2).

The MHK and Bayes methods' ability to provide acceptable coverage when the CV's are .16 for the small sample and .10 for the large sample shows that the methods are quite robust with respect to differences in the spread of the data sets. In actual engineering application it is unlikely that the material being considered in the design (small sample) would have a variability 60% greater than that of the previously tested, similar material (large sample).

In Table IV, the small sample data set was randomly selected from a Weibull distribution where the shape and scale values were computed so that they were equivalent to the tabulated mean and CV's. The larger data set was obtained from a normal distribution. The results are similar to those in Table III for the MHK and Bayes methods. The Table IV RRM results show a reduction in the coverage when compared with those in Table III, an example is the 78.8% coverage for the A allowable in Table III compared to 48% in Table IV for differences in the CV's of only 40%.

These results indicate that the RRM is sensitive to the statistical model assumption in representing the test data while the MHK and Bayes methods are much less sensitive. Since MHK and Bayes are nonparametric methods, this robustness to the model assumption could be expected.

In Table V data was obtained from normal distributions with CV's of .10 and .16 for L and S respectively. The coverage percent and range of allowable values are tabulated with respect to increasing sample sizes of both L and S for the RRM and MHK procedures. Results show that increasing sample size for L with constant small sample size for S of 6 causes the RRM process to perform poorly since the coverage is reduced from 86.6% to 73%. The only advantage is the reduction in the range of the allowable from 17 to 14 which is not very significant. Increasing the sample size of S from 6 to 15 also shows a somewhat unsatisfactory result since a 81% to 72.8% reduction in the coverage occurs. These coverage reductions are the inherent weakness in the method which is vulnerable to situations where L has a much smaller CV than S. The range reduction from 15 to 10 could be considered an improvement since there is less spread in the allowable estimate. Unfortunately, this advantage is removed because of the coverage loss. This implies that many more (much greater then 5%) allowable values will be greater than the 10% pt. of the population of material strength measurements. *This situation could result in an overly optimistic allowable value and therefore a potential under-design situation.*

MHK results provide reasonably acceptable coverage for all the combinations of sample size for both L and S. That is, results show, at least for the cases considered, that the method is robust to a variety of sample sizes for both L and S. The range of the allowables is affected by the sample sizes particularly for the case MHK(15,6) vs. MHK(30,15). The MHK method can provide a smaller range on the allowable but will not make significant improvements on the coverage capability when the sample sizes are increased. In the results for MHK(60,6) and MHK(15,6) the coverage is 88% and 92.8% showing that increasing the sample size of L can reduce the coverage. This is the result of sample L's increased influence in the allowable computation which the analyst should be aware of when applying the MHK method. It is suggested that the ratio of sample sizes n(2)/n(1) should not be any smaller than .2.

10.2 A Comparison Study: Single Sample Vs. Two-sample Allowable Computation

In Figures 2 through 5 a comparison is made between the multi-sample methods (MHK, RRM, and Bayes) and the single sample Weibull and normal methods with respect to the coverage percentage and the spread in the allowable estimates. In Figures 2 and 3 results were obtained by using a random selection of data from normal distributions. The N(6) and W(6) designations represent results from applying 6 data values to the Normal and Weibull allowable computation procedures. MHK(36) results are for the Modified Hanson-Koopmans method using a single sample with 36 data values from the S population distribution. CV's of .10 and .14 are introduced for L and S in order to represent a possible difference in the spread of the two data sets. The ordinate values (Δ) shown in the figure represent the 95^{th} percentile value of the allowable simulation results. Ideally, the values should be located on the dotted line for optimum coverage. Values above the line indicate that coverage has not been achieved. Those below the line provide the coverage. This can also identify an excessively low allowable value. In the second part of the figure the vertical dotted lines represent the spread in the allowable estimates (1 to 99 percent of all the data from the simulation results).

The Figure 2 results show that the MHK and Bayes methods can provide an almost optimum computed B allowable. The RRM approach fails to provide the coverage since results show an 87% rate. Normal distribution for single sample (S) of 6 provided reasonably good coverage as expected since the data was originally obtained from a normal model. The Weibull results were overly conservative, possibly, because an incorrect model was assumed for the data (normal). MHK(36) results were excellent as expected since the 36 values applied to the model were all from the normal distribution representing the data sample S.

Evaluation of the models' capabilities with respect to spread in the allowables showed the two-sample methods' allowable values to have much less variability than those of the single data set methods.

The results in Figure 3 are similar to those in Figure 2 except that the A allowable was computed. The Bayes method was omitted since a very large data set would have been required. A spread in excess of 50 was determined from applying the single sample normal analysis with the 1% point showing an allowable of -12. This result can discourage the engineers from using statistical procedures for obtaining design allowables. *In this case, the single sample method, although statistically correct, provides a design number that is incorrect from an engineering perspective.* This result has been the primary motivating factor in the authors' examination of alternate small sample procedures. The results from MHK and RRM show a more reliable range of values for the allowable.

In Figures 4 and 5 random samples were obtained from a NASA contractor report,[13] on composite material strength measurements. The figures identify the names of the companies that manufactured the material and the number of specimens tested. In Figure 4, the CV's of .10 and .13 were obtained from unidirectional tension and crossply tension data. The results show that the MHK and Bayes methods are effective in obtaining a desirable allowable estimate. The RRM results are greater than the 10% point and therefore fail to provide an acceptable allowable estimate. The normal and Weibull perform well in obtaining the proper coverage but as shown previously the spread in allowables for N(6) and W(6) is much greater than that of the MHK, Bayes, and RRM results.

In Figure 5, the random samples for both S and L were obtained from 230 data values (composite short beam shear test). The results are similar to those in Figure 4 except that the normal analysis, N(6), fails to provide acceptable coverage and the MHK and Bayes allowables are more conservative (excessive coverage). A relatively good agreement between the coverages can be identifed by comparing MHK(36) and MHK results. A reasonable correlation also exists for the spread in the allowable estimates. This implies that MHK can perform almost as well as if 36 values from S were applied to the MHK analysis instead of only 6 from S and 30 from L.

11 Conclusions

Results from this comparison study show that the nonparametric MHK method is superior in determining small sample design allowables when compared to the the results from the other procedures evaluated in this paper. The allowable values obtained from the MHK method application consistently meet the coverage requirement (95% of values less than a specified percentile of the population of all test data) for a relatively wide spectrum of data sets. The variability of the MHK values is much

[13] Reese, C. and Sorem, J. Jr., "Statistical Distribution of Mechanical Properties for Three Graphite-Epoxy Material Systems", NASA Contract Report No. 165736, 1981.

lower than that of the values resulting from the small, single sample normal or Weibull analysis.

The nonparametric Bayesian method provides acceptable allowable values although this method is limited by the sample size requirements. This limitation prevents the method from being as desirable as the MHK process. Another undesirable feature is the complexity involved in applying the method.

The Reduced Ratio Method, which is currently used by the aircraft industry, is not effective due to its failure in providing the required coverage when there are relatively small differences between the CV's of the prior large data set and the smaller empirical set from which the allowable is obtained. Also, increasing the sample size of empirical data and incorrectly assuming statistical models for the data sets prevents proper coverage.

Application of the small, single sample analysis (Normal and Weibull) results in extremely large variability in the allowable estimate. In addition, the methods fail to provide acceptable coverage when incorrect models are assumed.

The proposed pooling process introduced in this paper provides a desirable method for combining the small and large data sets when there is a difference in their mean values. Application of this process in the MHK and Bayesian analysis results in an effective solution in obtaining economical allowable values.

Figure 1A

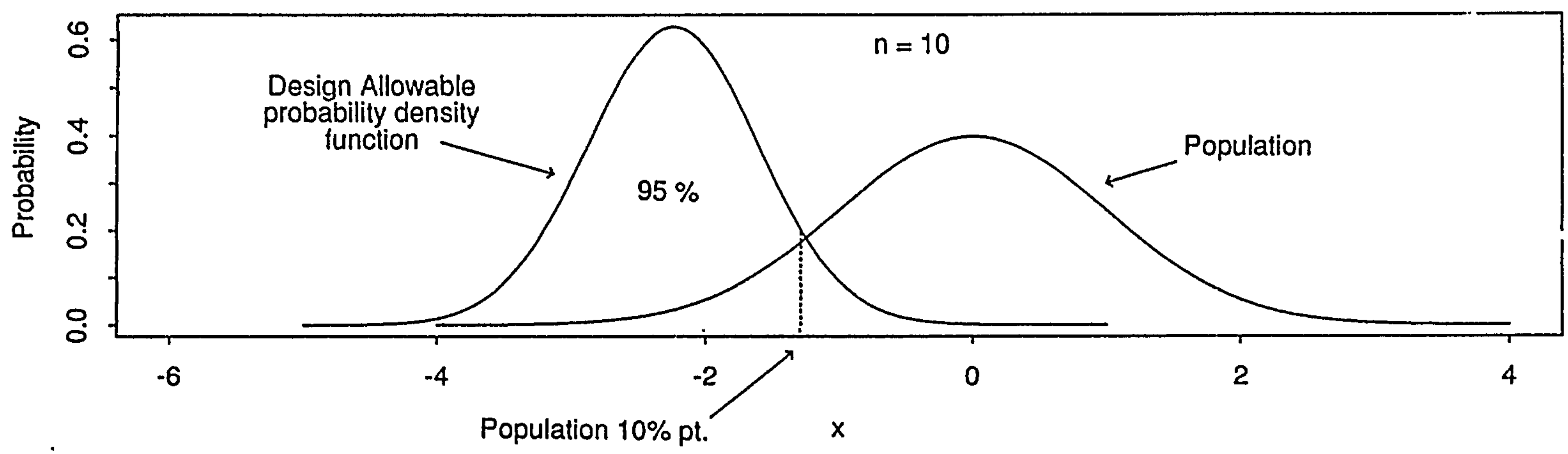


Figure 1B

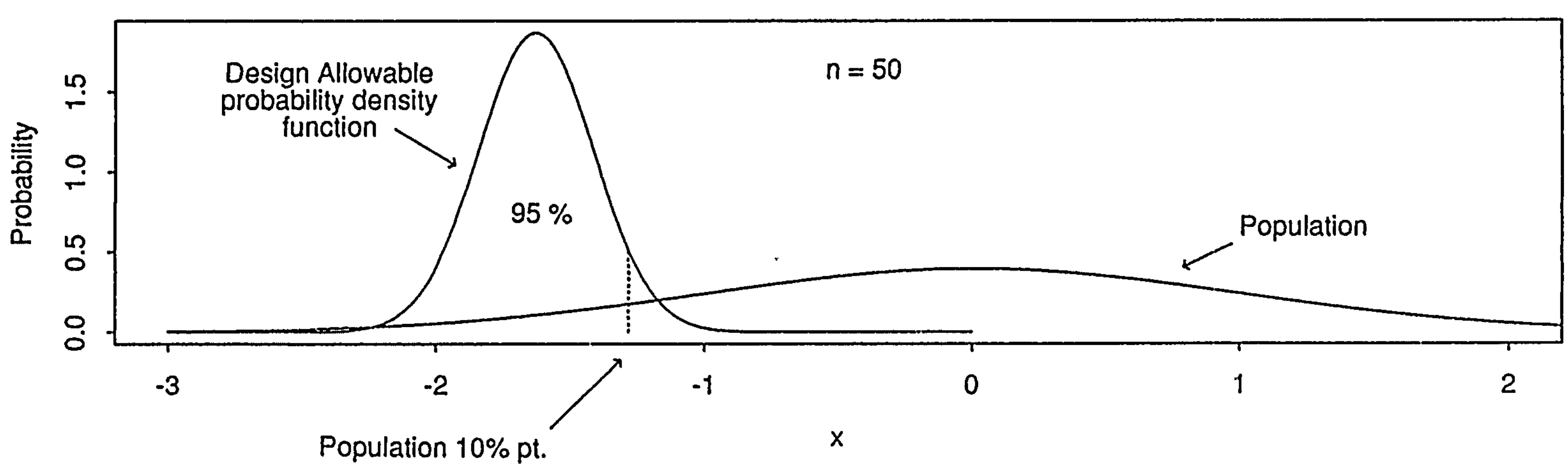


Figure 1. Tolerance Limit for N(0,1) Population

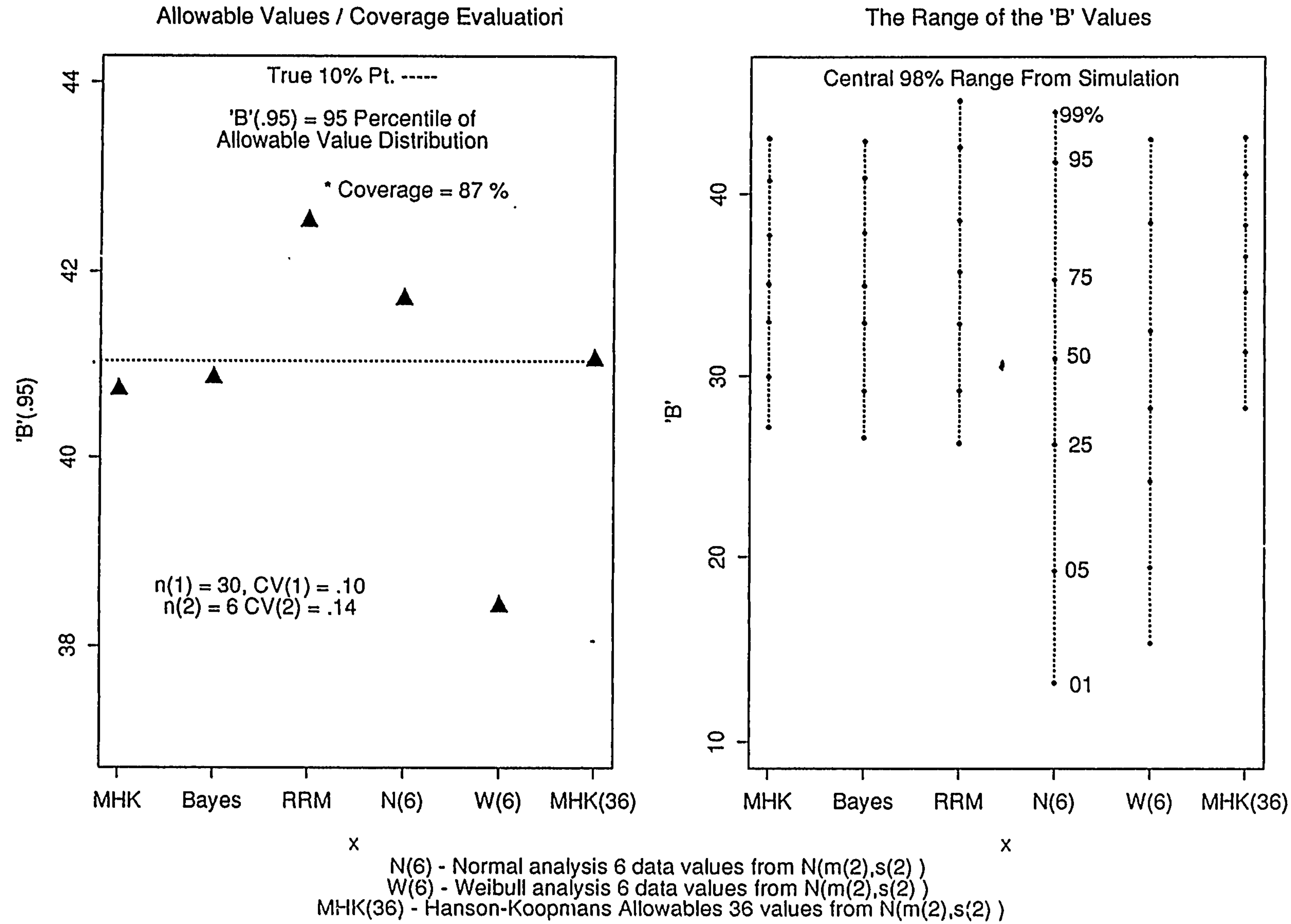


Figure 2. Monte Carlo Study - Allowables Versus Coverage/Range of Allowables

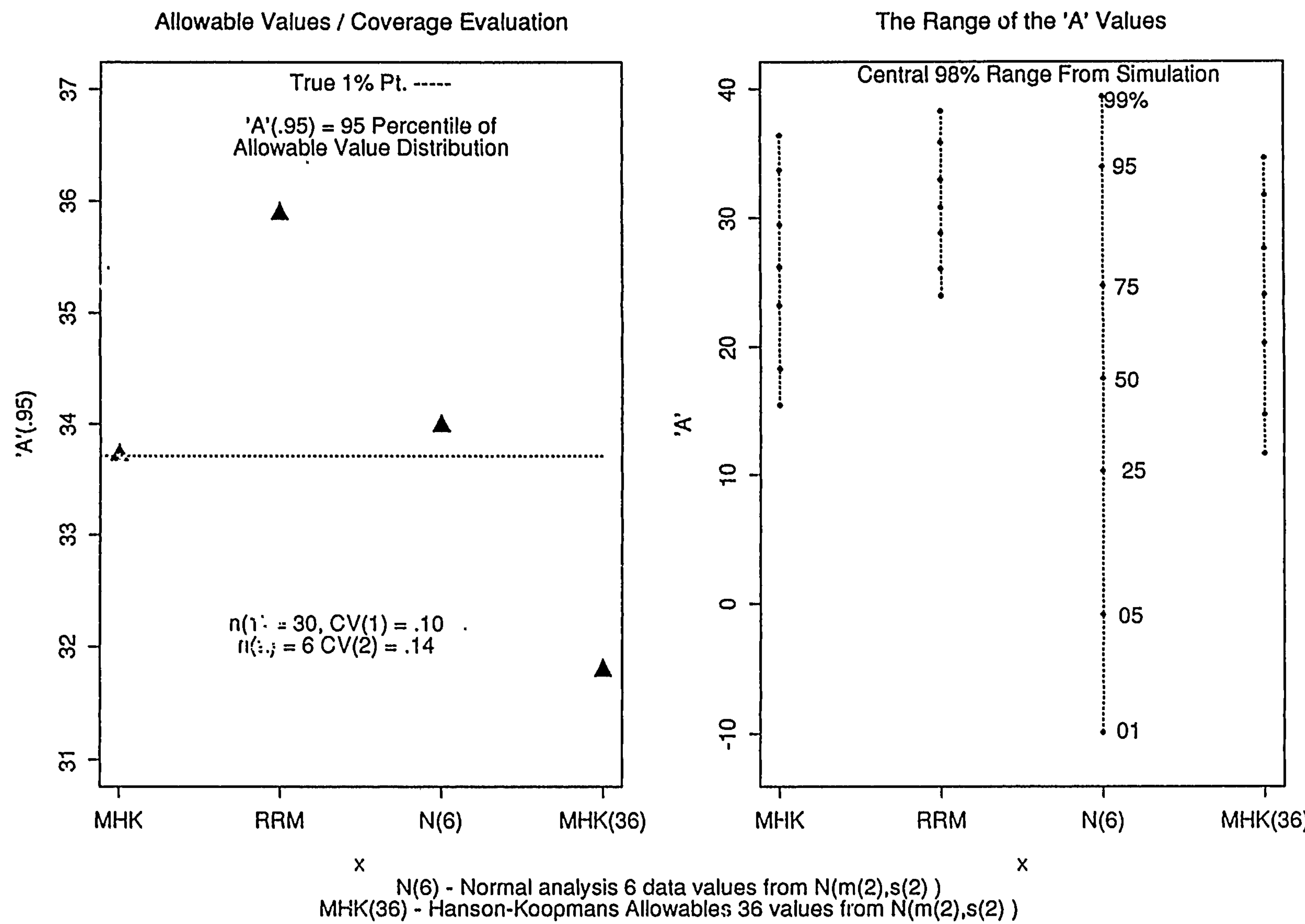


Figure 3. Allowables Versus Coverage/Range of Allowables

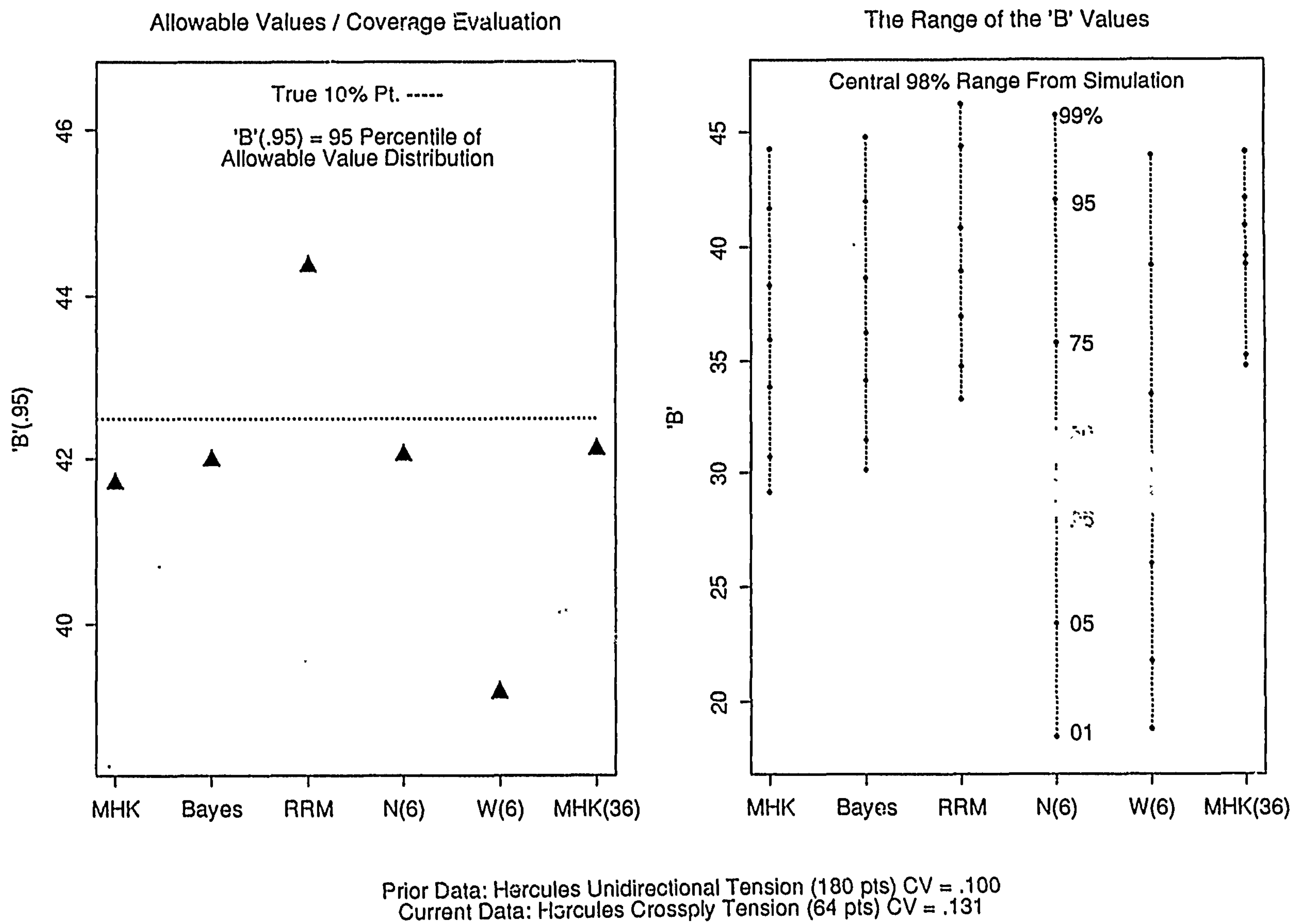
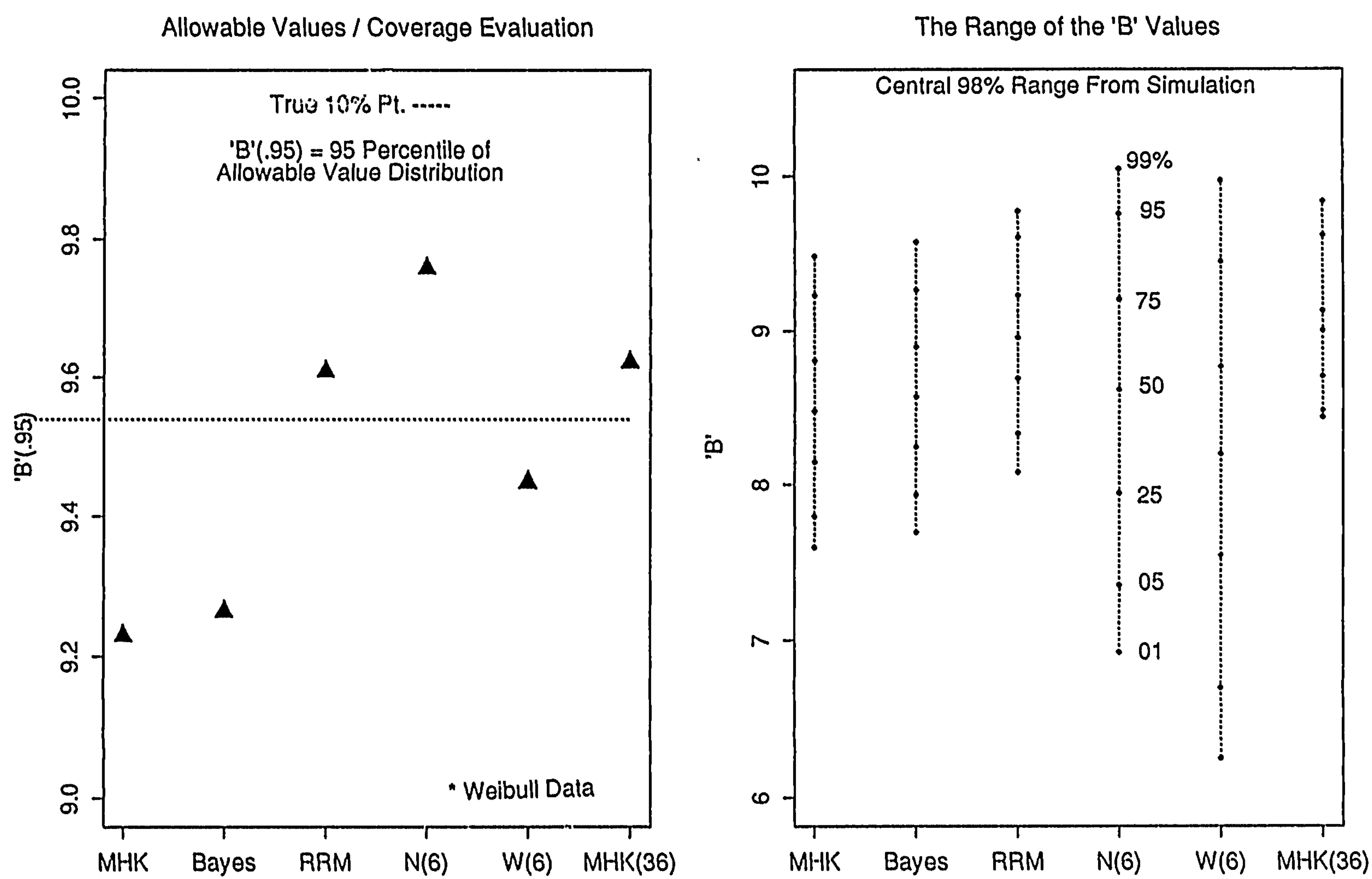


Figure 4. Allowables Versus Coverage/Range of Allowables



Prior Data: Narmco Shortbeam Shear (230 pts) CV = .0425
Current Data: Narmco Shortbeam Shear (230 pts) CV = .0425

Figure 5. Allowables Versus Coverage/Range of Allowables

Table I. M and u Values for Bayesian Basis Value Computation

M	u	M	u	M	u	M	u
1	0.021953	51	0.044804	101	0.057686	151	0.064302
2	0.017855	52	0.045192	102	0.057856	152	0.064395
3	0.016529	53	0.045565	103	0.058023	153	0.064514
4	0.016140	54	0.045937	104	0.058188	154	0.064609
5	0.016199	55	0.046301	105	0.058352	155	0.064717
6	0.016516	56	0.046648	106	0.058517	156	0.064814
7	0.016997	57	0.046996	107	0.058670	157	0.064912
8	0.017590	58	0.047339	108	0.058837	158	0.065010
9	0.018264	59	0.047673	109	0.059006	159	0.065099
10	0.018996	60	0.048011	110	0.059156	160	0.065193
11	0.019769	61	0.048318	111	0.059313	161	0.065273
12	0.020570	62	0.048642	112	0.059454	162	0.065382
13	0.021391	63	0.048945	113	0.059619	163	0.065462
14	0.022223	64	0.049255	114	0.059761	164	0.065555
15	0.023060	65	0.049563	115	0.059914	165	0.065658
16	0.023897	66	0.049848	116	0.060051	166	0.065734
17	0.024729	67	0.050144	117	0.060192	167	0.065822
18	0.025554	68	0.050421	118	0.060344	168	0.065910
19	0.026368	69	0.050695	119	0.060480	169	0.065996
20	0.027171	70	0.050968	120	0.060628	170	0.066108
21	0.027959	71	0.051238	121	0.060754	171	0.066192
22	0.028734	72	0.051506	122	0.060883	172	0.066277
23	0.029491	73	0.051771	123	0.061031	173	0.066384
24	0.030233	74	0.052034	124	0.061162	174	0.066449
25	0.030959	75	0.052284	125	0.061292	175	0.066530
26	0.031666	76	0.052530	126	0.061420	176	0.066613
27	0.032361	77	0.052773	127	0.061547	177	0.066705
28	0.033033	78	0.053017	128	0.061679	178	0.066789
29	0.033695	79	0.053244	129	0.061802	179	0.066872
30	0.034339	80	0.053479	130	0.061933	180	0.066934
31	0.034967	81	0.053702	131	0.062065	181	0.067007
32	0.035577	82	0.053932	132	0.062179	182	0.067098
33	0.036172	83	0.054160	133	0.062293	183	0.067176
34	0.036754	84	0.054375	134	0.062430	184	0.067258
35	0.037328	85	0.054600	135	0.062553	185	0.067333
36	0.037884	86	0.054808	136	0.062667	186	0.067418
37	0.038420	87	0.055017	137	0.062784	187	0.067486
38	0.038952	88	0.055221	138	0.062894	188	0.067569
39	0.039461	89	0.055435	139	0.063010	189	0.067628
40	0.039964	90	0.055634	140	0.063128	190	0.067720
41	0.040459	91	0.055831	141	0.063245	191	0.067794
42	0.040944	92	0.056024	142	0.063344	192	0.067871
43	0.041409	93	0.056215	143	0.063459	193	0.067952
44	0.041864	94	0.056417	144	0.063550	194	0.068022
45	0.042314	95	0.056599	145	0.063666	195	0.068103
46	0.042751	96	0.056781	146	0.063763	196	0.068178
47	0.043182	97	0.056960	147	0.063899	197	0.068237
48	0.043596	98	0.057153	148	0.063985	198	0.068315
49	0.044009	99	0.057332	149	0.064101	199	0.068388
50	0.044413	100	0.057502	150	0.064197	200	0.068459

Table II. Modified Hanson-Knoopmans Constants for Basis Value

n	r	s	k
2	1	2	35.177
3	1	3	7.859
4	1	4	4.505
5	1	4	4.101
6	1	5	3.064
7	1	5	2.858
8	1	6	2.382
9	1	6	2.253
10	1	6	2.137
11	1	7	1.897
12	1	7	1.814
13	1	7	1.738
14	1	8	1.599
15	1	8	1.540
16	1	8	1.485
17	1	8	1.434
18	1	9	1.354
19	1	9	1.311
20	1	10	1.253
21	1	10	1.218
22	1	10	1.184
23	1	11	1.143
24	1	11	1.114
25	1	11	1.087
26	1	11	1.060
27	1	11	1.035
28	1	12	1.010
29	1	--	1
30	2	12	1.373
31	2	12	1.344
32	2	12	1.315
33	2	13	1.270
34	2	13	1.245
35	2	13	1.221
36	2	13	1.197
37	2	13	1.174
38	2	13	1.151
39	2	13	1.129
40	2	13	1.108
41	2	14	1.083
42	2	14	1.064
43	2	14	1.045
44	2	14	1.027
45	2	14	1.009
46	2	--	1

Table III. Simulation Results/Computing Allowable Value
Coverage Rate (%) Versus CV Differences Normal - Normal Distributions

CV		Coverage Rate (%)				
		'B' Allowables			'A' Allowables	
CV(1)	CV(2)	MHK	Bayes	RRM	MHK	RRM
.10	.10	99.0	99.2	98.6	99.4	99.0
.10	.12	97.0	98.0	94.8	98.4	94.4
.10	.14	95.4 (94.6)*	95.6 (93.8)*	86.6 (84.2)*	95.8 (92.8)*	78.8 (72.8)*
.10	.16	91.8	92.6	81.0	89.4	59.4
.10	.18	88.2	89.2	72.2	83.4	41.2
.10	.20	83.2	83.0	64.2	72.6	24.6
.12	.10	99.8	99.8	99.6	99.8	100

CV(i) = s(i)/m(i) , i = 1,2 m(1) = 200 , m(2) = 50

Assumed Distributions are N(m(1),s(1)), N(m(2),s(2)) = Normal distribution for prior and current data sets respectively

MHK - Modified Hanson-Koopmans
Bayes - Nonparametric Bayes (Ferguson)
RRM - Reduced Ratio Method (Mil-5)

Sample size n(1) = 30(prior), n(2) = 6(data) for cases except ( )*
* sample size n(1) = 60, n(2) = 6

Table IV. Simulation Results/Computing Allowable Values
Coverage Rate (%) Versus CV Differences Normal - Weibull Distributions

CV		Coverage Rate (%)				
		'B' Allowables			'A' Allowables	
CV(1)	CV(2)	MHK	Bayes	RRM	MHK	RRM
.10	.10	98.6	99.2	97.8	99.6	88.6
.10	.12	98.0	98.4	90.8	98.4	68.6
.10	.14	94.0 (94.2)*	94.2 (------)*	82.2 (90.6)*	94.4 (96.0)*	48.0 (63.6)*
.10	.16	89.0	89.6	73.4	89.2	29.0
.10	.18	84.8	86.8	65.0	82.0	19.6
.10	.20	76.0	76.6	57.4	69.0	14.0
.12	.10	99.8	99.8	99.4	99.6	98.2

CV(i) = s(i)/m(i) , i = 1,2   m(1) = 200 , m(2) = 50

Distributions N(m(1),s(1)), W(a(2),b(2))
where N and W are Normal and Weibull models
for prior and current data sets respectively

a(2) = shape parameter and b(2) = scale
determined for prescribed CV in columns 1 and 2

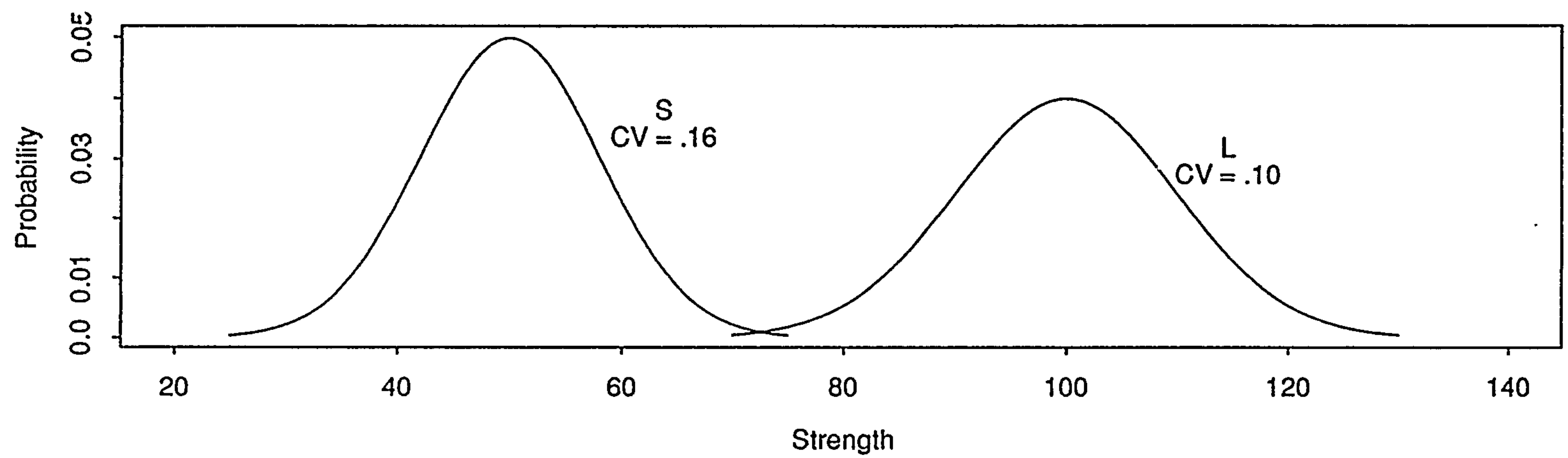
* sample size n(1) = 15, n(2) = 6

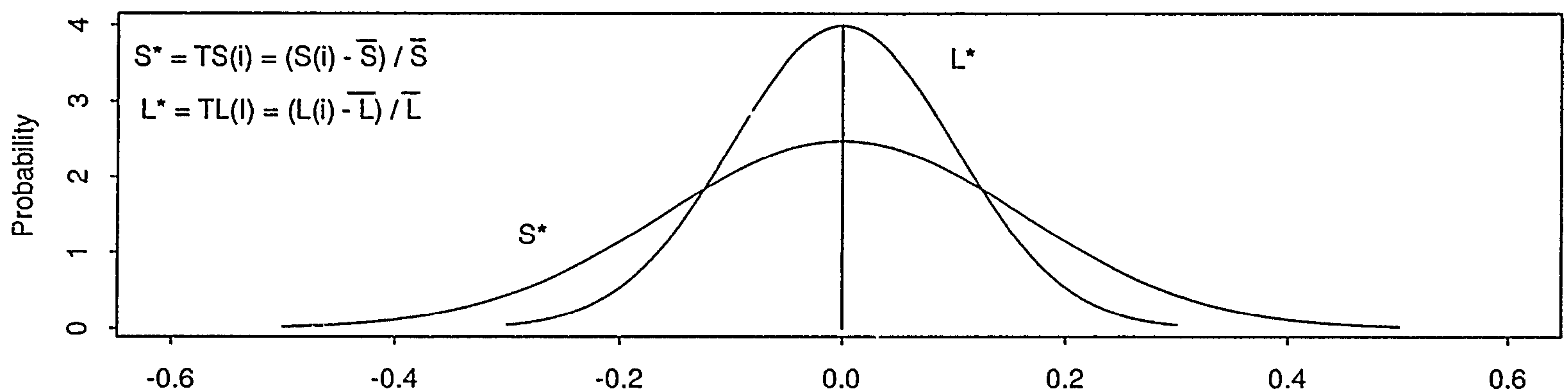
Table V. Range and Coverage (%) Versus Sample Size/Methods
Normal Distributions

Method (n(1), n(2))	Range (%) of 'B' Allowable			Coverage (%) 'B' Allowable
	01	50	99	
RRM (15,6)	27.19	35.55	44.28	86.6
RRM (30,6)	29.10	36.80	44.58	81.0
RRM (60,6)	30.32	37.52	44.70	73.0
RRM (30,15)	33.42	38.23	43.84	72.8
MHK (15,6)	20.49	32.95	42.87	92.8
MHK (30,6)	25.12	34.51	43.25	91.8
MHK (60,6)	27.40	35.19	42.98	88.4
MHK (30,15)	29.06	36.17	42.07	90.0

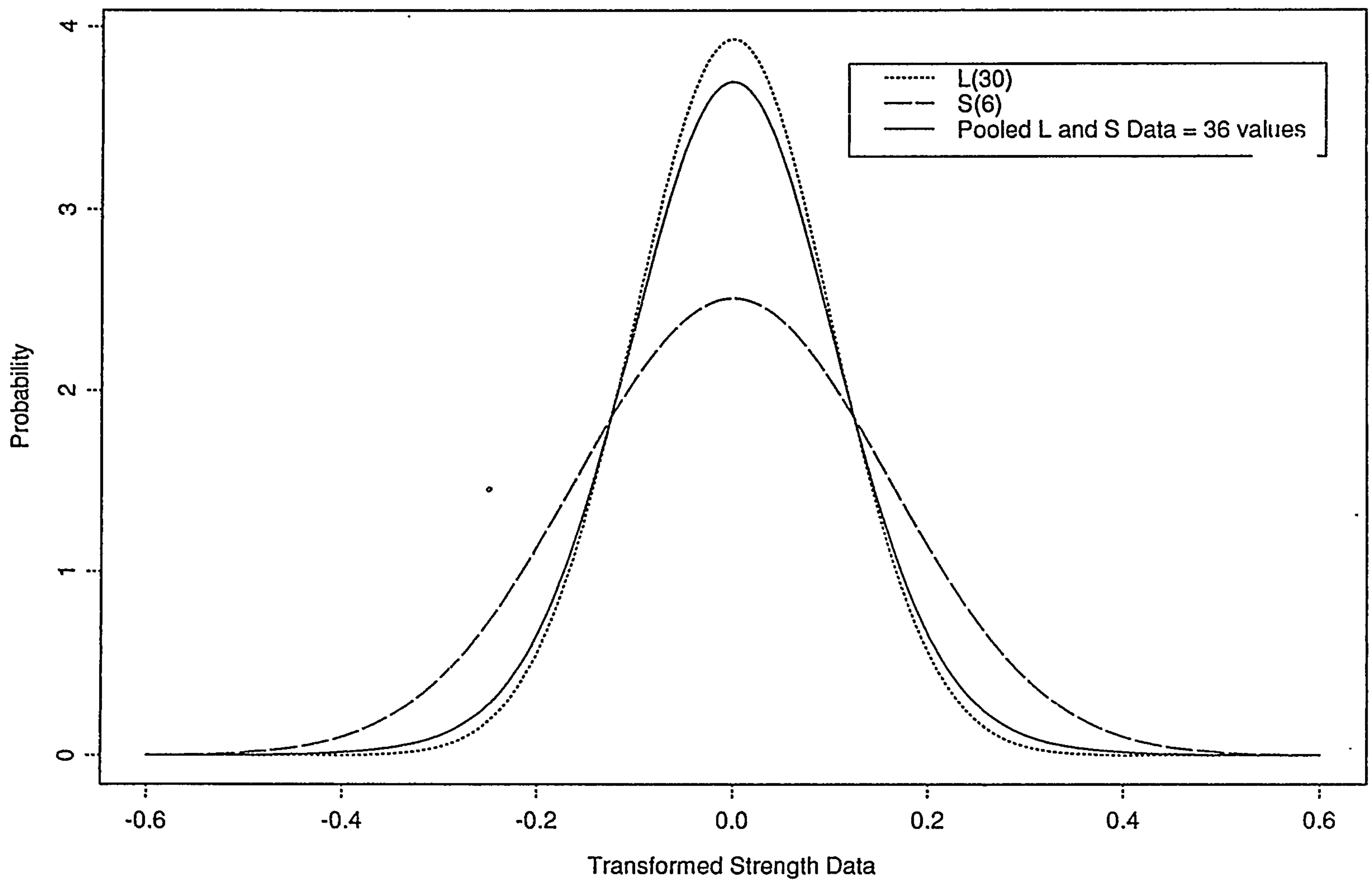
n(1) = L sample size CV(1) = .10
n(2) = S sample size CV(2) = .16

Appendix A. Transformation Process for Pooling PDF's of Small Data Set(S) and Large Data Set(L)





Appendix B. PDF's for Both Pooled and Single Data Sheets



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SMALL SAMPLE DESIGN ALLOWABLES FROM PAIRED DATA SETS -
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SMALL SAMPLE DESIGN ALLOWABLES FROM PAIRED DATA SETS -
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Key Words

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Reduced ratio method
Pooling data

This paper identifies an acceptable statistical procedure for obtaining design allowable values from a small set of material strength data. The allowable represents a material design number defined as the 95% lower confidence bound on the specified percentile of the population of material strength data. The percentiles are the first and tenth for the A and B allowables. The proposed method reduces the penalties commonly associated with small sample allowable computation by accurately maintaining the definition requirements and reducing variability in the estimate. Application of very small samples will obviously reduce costs in testing and manufacturing which is the primary motivation for this study. In the evaluation process five methods were considered for computing the design allowable. Three of these methods involved certain statistical distribution assumptions while the other two were nonparametric procedures. The latter methods introduced a pooling process such that the small sample was combined with a larger, previously obtained sample. Monte Carlo studies showed that the nonparametric procedures are the most desirable for computing the design allowable value.