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# Convergence Performance of Adaptive Detectors, Part 1

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13. ABSTRACT (Maximum 200 words) Convergence results for a mean level adaptive detector (MLAD) are presented. The MLAD consists of an adaptive matched filter (for spatially correlated inputs) followed by a mean level detector (MLD). The optimal weights of the adaptive matched filter are estimated from one batch of data and applied to a statistically independent batch of nonconcurrent data. The threshold of the MLD is determined from the resultant data. Thereafter, a candidate cell is compared against this threshold. Probabilities of false alarm and detection are derived as functions of the threshold factor, the order of the matched filter, the number of independent samples per channel used to calculate the adaptive matched filter weights, the number of samples used to set the MLD threshold, and the output signal-to-noise power ratio of the optimal matched filter. A number of performance curves are shown and discussed.				
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## CONTENTS

1.	INTRODUCTION .....	1
2.	MEAN LEVEL ADAPTIVE DETECTOR DESCRIPTION (MLAD) .....	2
3.	PROBABILITY OF FALSE ALARM .....	6
4.	DETECTION PROBABILITY .....	7
5.	DETECTION PROBABILITY: RALEIGH SIGNAL .....	10
6.	RESULTS .....	12
7.	SUMMARY .....	19
8.	REFERENCES .....	19



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# CONVERGENCE PERFORMANCE OF ADAPTIVE DETECTORS, PART 1

## 1. INTRODUCTION

The matched filter detector (MFD) is a commonly used in deciding whether a desired signal is present or not. Figure 1 shows a schematic of the MFD. The output of  $N$  sensors is input to the MFD. If the covariance matrix  $R_{xx}$  of the inputs  $x_1, x_2, \dots, x_N$  is known a priori and the desired signal can be represented by the an  $N$ -length vector  $s$ , then the matched filter weights  $a_1, a_2, \dots, a_N$  are given by  $\mathbf{a} = R_{xx}^{-1} s$ , where  $\mathbf{a} = (a_1, a_2, \dots, a_N)^T$  and  $T$  denotes transpose [1]. The output of the matched filter is  $\mathbf{a}^H \mathbf{x}$ , where  $\mathbf{x} = (x_1, x_2, \dots, x_N)^T$  and  $H$  denotes conjugate transpose. This output is square-law detected and compared against a threshold. A detection is declared if this threshold is exceeded.

For a known covariance matrix, threshold, and signal-to-noise power ratio, the detection probability  $P_D$  and false alarm probability  $P_F$  have been derived [1]. In some applications the covariance matrix is not known a priori and is estimated. The matched filter weighting is then determined by using what has been termed the sampled matrix inversion (SMI) algorithm [2]. Convergence results for the output noise power residue of an adaptive matched filter that uses the SMI are given in Refs. 2 to 4.

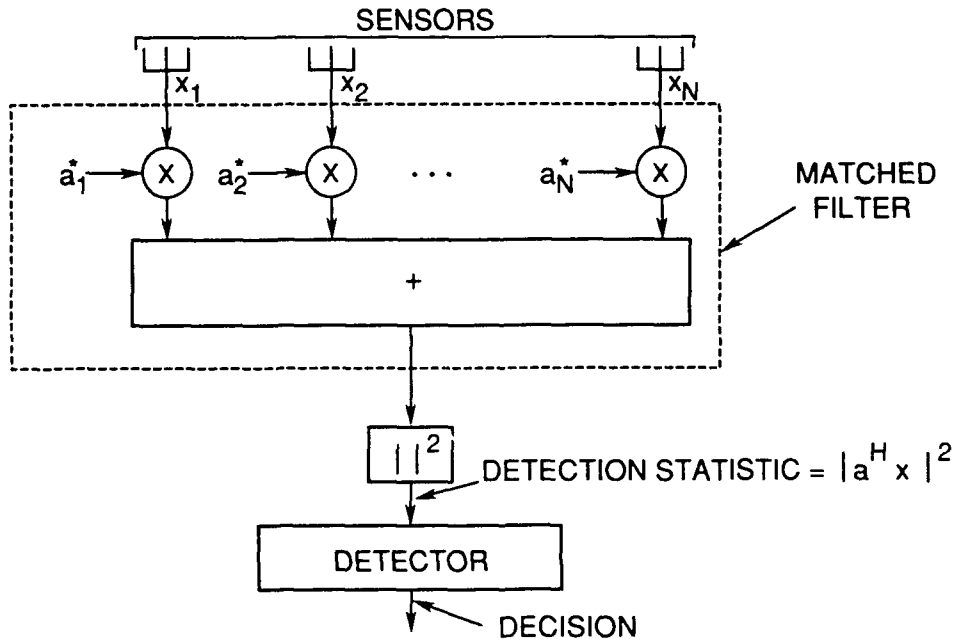


Fig. 1 — Matched filter detector

Kelly [5] derived an adaptive detector under the Gaussian assumption by using the maximum likelihood (ML) estimator for the unknown parameters of the likelihood ratio test, i.e., the unknown covariance matrix and the unknown signal amplitude. This detection scheme is known as the generalized likelihood ratio test [1]. The desired signal's amplitude was assumed to be nonrandom. Input data consisted of a primary data vector of length  $N$  that might contain the desired signal and a number of secondary data vectors that do not contain the desired signal.  $R_{xx}$  is estimated from these secondary data vectors by using the SMI algorithm. Reference 5 presents convergence results for  $P_D$  and  $P_F$ . Expressions for  $P_D$  and  $P_F$  were derived that are a function of the number of statistically independent secondary data vectors, the number of input channels  $N$ , the detector threshold, and the input signal-to-noise power ratio. Note that  $P_F$  did not depend on  $R_{xx}$  (a statistical measure of the external noise environment). Hence this detector exhibited the desirable constant false alarm rate (CFAR) property of having the  $P_F$  be independent of the covariance matrix. References 6 and 7 contain additional research in this area.

Here we consider a different form of CFAR adaptive detection that employs a mean level detector (MLD) [8,9]. For this detection scheme (as in Kelly [5]), a fixed number of secondary data vectors that do not contain the desired signal are used to estimate  $R_{xx}$ . A number of primary data vectors are processed through the matched filter, and square law is detected. Thereafter, one of the resultant outputs is selected as a candidate for detection; the remaining output powers are averaged and multiplied by an arbitrary number to form the threshold. Also in this report, we address the random desired signal. In particular, we present results for the Rayleigh target model. Formulas for  $P_D$  and  $P_F$  are derived for what we term as the mean level adaptive detector (MLAD), and again we show that this detector exhibits the CFAR property of the  $P_F$  being independent of the input covariance matrix.

The pertinent assumptions for this analysis are the following:

- (1) Input noises are complex zero-mean stationary Gaussian random variables (RV). The real and imaginary parts of a given input noise sample are independent and are identically distributed (IID). An RV with these characteristics is called a circular Gaussian process.
- (2) Input noise samples are temporally statistically independent.
- (3) The secondary data is statistically independent of the primary data.
- (4) The desired signal is present in the candidate primary data vector. It is not in the secondary data or the primary data vectors used to form the threshold.

Assumptions (1) through (4) were also used in Ref. 5.

## 2. MEAN LEVEL ADAPTIVE DETECTOR DESCRIPTION (MLAD)

Figure 2 shows a schematic of MLAD. A batch or block of input data (called secondary input data) is used to calculate the adaptive weights. On each of the  $N$  input channels, we measure  $K$  temporally independent samples.

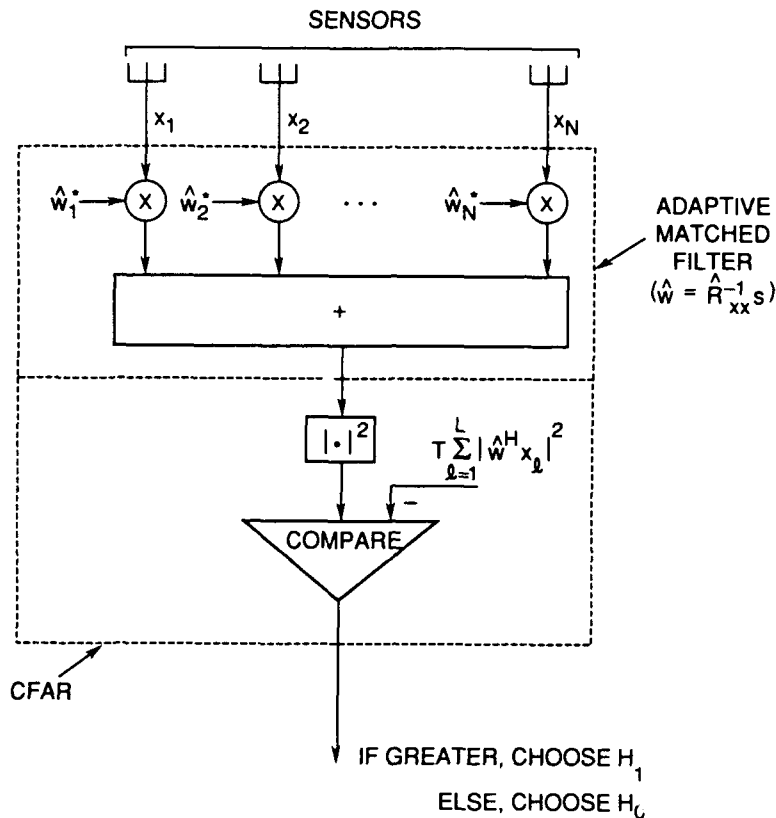


Fig. 2 — Mean level adaptive detector

Define

$X = N \times K$  matrix of secondary input data. The  $n$ th row represents the  $K$  temporally independent samples on the  $n$ th channel. The samples in the  $k$ th column are assumed to be time-coincident;

$s =$  desired steering vector of length  $N$ ; and

$\hat{R}_{xx} = N \times N$  estimated the input covariance matrix.

The optimal estimate  $\hat{w}$  of the optimal  $N$ -length weighting vector is given by [2]

$$\hat{w} = \hat{R}_{xx}^{-1} s, \quad (1)$$

where

$$\hat{R}_{xx} = (XX^H)^{-1}. \quad (2)$$

Equations (1) and (2) are essentially the SMI algorithm for computing the matched filter, or Wiener weights.

This optimal estimate is then applied to another temporally independent set of data called the primary input data. The primary input vectors are of length  $N$  and their elements are assumed to be temporally independent. Let

- $\mathbf{x}$  = candidate primary input data vector of length  $N$ ;
- $\mathbf{x}_l$  =  $l$ th MLD primary input data vector of length  $N$ ;
- $L$  = number of MLD primary input data vectors; and
- $T$  = MLD threshold constant.

The MLAD rule is given mathematically as

$$\begin{array}{c} H_1 \\ |\hat{\mathbf{w}}^H \mathbf{x}|^2 > \\ < \\ H_0 \end{array} T \sum_{l=1}^L |\hat{\mathbf{w}}^H \mathbf{x}_l|^2, \quad (3)$$

where  $|\cdot|$  denotes magnitude,  $H_0$  is the hypothesis that no desired signal is present, and  $H_1$  is the hypothesis that a desired signal is present. Note that with this detector, the standard CFAR procedure of normalizing the candidate primary test statistic is by the average of the estimated power of the other primary test statistics (i.e.,  $1/L$  has been incorporated into  $T$ ). The probabilities of false alarm and detection probabilities are defined as

$$P_F = Prob \left\{ |\hat{\mathbf{w}}^H \mathbf{x}|^2 > T \sum_{l=1}^L |\hat{\mathbf{w}}^H \mathbf{x}_l|^2 \mid H_0 \right\}, \quad (4)$$

and

$$P_D = Prob \left\{ |\hat{\mathbf{w}}^H \mathbf{x}|^2 > T \sum_{l=1}^L |\hat{\mathbf{w}}^H \mathbf{x}_l|^2 \mid H_1 \right\}. \quad (5)$$

We now introduce a matrix transform on the input channels that does not change  $P_D$  or  $P_F$ , but greatly simplifies the analysis. Let  $R_{xx}$  be the  $N \times N$  covariance matrix of the input channels. Assume that the matrix is nonsingular. There exist an  $N \times N$  matrix  $A$  [2], which (1) spatially whitens the  $N$  input channels, (2) normalizes each input channel to have noise power equal to one, and (3) places all of the signal energy in the first channel such that the transformed signal vector  $\tilde{\mathbf{s}}$  is given by

$$\tilde{\mathbf{s}} = A\mathbf{s} = (u, 0, 0, \dots, 0)^T, \quad (6)$$

where  $u$  represents the transformed desired signal voltage.

Define

- $Z = AX = N \times K$  matrix of transformed input data. Each sample is temporally and spatially independent with variance equal to one;

$z_l = Ax_l = l$ th transformed MLD primary data vector; and

$z = Ax =$  candidate primary input data vector.

It is straightforward to show that the optimal estimate  $w$  of the transformed data is given by

$$w = (A^H)^{-1} \hat{w}. \tag{7}$$

Figure 3 shows a schematic of the transformed MLAD rule. It is given by

$$\begin{matrix} H_1 \\ |w^H z|^2 > T \sum_{l=1}^L |w^H z_l|^2 \\ H_0 \end{matrix} \tag{8}$$

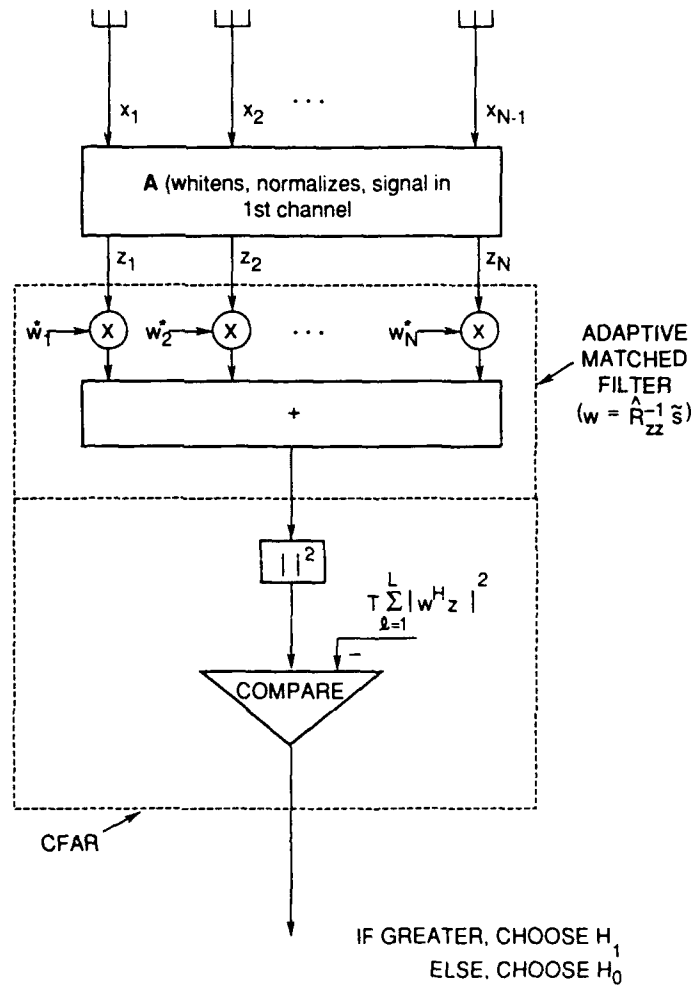


Fig. 3 — Transformed mean level adaptive detector



By substituting for  $\mathbf{w}$ ,  $\mathbf{z}$ , and  $\mathbf{z}_l$  with  $(A^H)^{-1} \hat{\mathbf{w}}$ ,  $A\mathbf{x}$ , and  $A\mathbf{x}_l$ , respectively, we find that Eq. (3) results. Hence, the equivalence of the two decision rules is proven. Thus

$$P_F = Prob \left\{ |\mathbf{w}^H \mathbf{z}|^2 > T \sum_{l=1}^L |\mathbf{w}^H \mathbf{z}_l|^2 \mid H_0 \right\}, \quad (9)$$

and

$$P_D = Prob \left\{ |\mathbf{w}^H \mathbf{z}|^2 > T \sum_{l=1}^L |\mathbf{w}^H \mathbf{z}_l|^2 \mid H_1 \right\}. \quad (10)$$

We see from Eq. (8) that an arbitrary scaling factor multiplying both sides of the decision rule does not change the rule. Henceforth, we set the first element of  $\mathbf{w}$  equal to one and define

$$\mathbf{w} = (1, w_2, w_3, \dots, w_N)^T. \quad (11)$$

Finally we note that as  $K \rightarrow \infty$ , then  $w_n \rightarrow 0$ ,  $n = 2, 3, \dots, N - 1$ . This is so because the Weiner weights are effectively achieved after the transformation by  $A$  [4]. Hence the adaptive weights computed after this transformation are perturbations about their optimal values, which are zero.

### 3. PROBABILITY OF FALSE ALARM

In this section, we derive  $P_F$  and show that the adaptive detection scheme discussed in Section 2 does indeed exemplify the qualities of a CFAR processor, i.e., the  $P_F$  is independent of the external noise environment. To this end, define

$P(F|\mathbf{w}) =$  probability of false alarm conditioned on knowing  $\mathbf{w}$ .

$$v = \frac{\mathbf{w}^H \mathbf{z}}{\|\mathbf{w}\|}, \quad (12)$$

and

$$v_l = \frac{\mathbf{w}^H \mathbf{z}_l}{\|\mathbf{w}\|}, l = 1, 2, \dots, L, \quad (13)$$

where  $\|\mathbf{w}\| = \sqrt{\mathbf{w}^H \mathbf{w}}$ . For  $\|\mathbf{w}\| > 0$  and under  $H_0$ , it is straightforward to that  $v$  and  $v_l$ ,  $l = 1, 2, \dots, L$ , are IID-circular Gaussian processes with zero mean and variance equal to one. The decision rule given by Eq. (8) can be rewritten as

$$\begin{array}{c} H_1 \\ |v|^2 > T \sum_{l=1}^L |v_l|^2 \\ < \\ H_0 \end{array} \quad (14)$$

For the above decision rule, it is well known [8] that

$$Prob \left\{ |v|^2 > T \sum_{l=1}^L |v_l|^2 \right\} = \frac{1}{(1+T)^L}. \quad (15)$$

Thus

$$P(F|\mathbf{w}) = \frac{1}{(1+T)^L}, \quad (16)$$

and

$$P_F = \frac{1}{(1+T)^L}. \quad (17)$$

We note that  $P_F$  is a function only of the arbitrary threshold  $T$ , and  $L$  is the number of MLD samples.

#### 4. DETECTION PROBABILITY

Here we derive the detection probability  $P_D$  associated with the MLAD. Under  $H_1$ ,

$$\mathbf{z} = (u + n_1, n_2, n_3, \dots, n_{N-1})^T, \quad (18)$$

where  $\mathbf{n} = (n_1, n_2, \dots, n_{N-1})^T$  is an additive Gaussian noise vector with zero mean and element variance equal to one and  $u$  is the desired signal voltage through the matrix transform  $A$ . Under  $H_1$ , define

$$P(D|\mathbf{w}, u) = \text{probability of detection conditioned on knowing } \mathbf{w} \text{ and } u.$$

Note we are assuming that the desired signal is not present in the CFAR primary data vectors. Equation 13 defines  $v_l$ ,  $l = 1, 2, \dots, L$ . Also set

$$v = \frac{\mathbf{w}^H \mathbf{z}}{\|\mathbf{w}\|} = \frac{\mathbf{w}^H \tilde{\mathbf{s}}}{\|\mathbf{w}\|} + \frac{\mathbf{w}^H \mathbf{n}}{\|\mathbf{w}\|} = \frac{u}{\|\mathbf{w}\|} + \frac{\mathbf{w}^H \mathbf{n}}{\|\mathbf{w}\|}. \quad (19)$$

We set

$$u_0 = \frac{u}{\|\mathbf{w}\|}, \quad (20)$$

and

$$v' = \frac{\mathbf{w}^H \mathbf{n}}{\|\mathbf{w}\|}. \quad (21)$$

Now  $v$  is a circular Gaussian process with a mean equal to  $u_0$  and variance equal to one. As before,  $v_l$  are IID, circular Gaussian processes with zero mean and variance equal to one. They are also independent of  $v$ .

The decision rule given by Eq. (8) can be rewritten as

$$|u_0 + v'|^2 \underset{H_0}{\overset{H_1}{>}} T \sum_{l=1}^L |v_l|^2. \quad (22)$$

Set

$$r = |u_0 + v'|^2, \quad (23)$$

and

$$z = T \sum_{l=1}^L |v_l|^2. \quad (24)$$

Let the unknown phase of  $u$  be uniformly distributed between  $[0, 2\pi)$ . Under the IID Gaussian assumption, Ref. 1 shows that the probability density function (PDF) of  $r$ , denoted by  $p_r(r)$ , is given by

$$p_r(r) = e^{-(r+|u_0|^2)} I_0(2|u_0|\sqrt{r}), \quad (25)$$

where  $I_0$  is the modified 0th-order Bessel function of the first kind. The distribution of  $z$  is the  $\chi^2$  distribution with PDF given by

$$p_z(z) = \frac{1}{T^L(L-1)!} z^{L-1} e^{-z/T}. \quad (26)$$

Now

$$\text{Prob}\{r > z\} = \int_0^\infty \int_z^\infty p_r(r) p_z(z) dr dz, \quad (27)$$

and

$$P\{D | \mathbf{w}, u\} = P\{D | u_0\} = \text{Prob}\{r > z\}. \quad (28(a))$$

Define

$$q_0 = \frac{1}{\|\mathbf{w}\|^2}, \quad (28(b))$$

where the quiescent ( $K = \infty$ ) output noise power of the 1st channel  $z_1$  is equal to 1. Thus,

$$u_0 = u\sqrt{q_0}. \quad (29)$$

Brennan and Reed [3] showed that  $q_0$  has the following PDF.

$$p_{q_0}(q_0) = \frac{K!}{(N-2)!(K-N+1)!} (1-q_0)^{N-2} q_0^{K-N+1}, \quad 0 \leq q_0 \leq 1. \quad (30)$$

Set  $q = \sqrt{q_0}$ . It is straightforward by using elementary probability theory to show that

$$p_q(q) = \frac{K!}{2(N-2)!(K-N+1)!} (1-q^2)^{N-2} q^{2(K-N)+1}, \quad 0 \leq q \leq 1. \quad (31)$$

Thus the joint PDF of  $|u|$  and  $q$ , which are assumed to be independent random variables, is given by

$$p_{|u|,q}(|u|, q) = p_{|u|}(|u|) p_q(q). \quad (32)$$

At this point, we note that

$$P(D|w, |u|) = P(D | |u_0|) = P(D | |u|, q). \quad (33)$$

Thus knowing the PDF of  $|u_0|$  is not necessary since we have the joint PDF of  $p_{|u|,q}(|u|, q)$ .

Finally the detection probability is found as

$$P_D = \int_0^\infty \int_0^1 P(D | |u|, q) p_{|u|,q}(|u|, q) dq d|u|, \quad (34)$$

or

$$P_D = c \int_0^\infty \int_0^1 \int_0^\infty \int_y^\infty e^{-\left[r + \frac{|u|^2}{q^2}\right]} I_0 \left( 2 \frac{|u|^2}{q} \sqrt{r} \right) \cdot y^{L-1} e^{-y/T} (1-q^2)^{N-2} q^{2(K-N)+1} p_{|u|}(|u|) dr dy dq d|u|, \quad (35)$$

where

$$c = \frac{K!}{2(N-2)!(K-N+1)!} \frac{1}{T^L (L-1)!}. \quad (36)$$

We note that the signal parameters are contained in  $P_{|u|}(|u|)$  and that the power of  $|u|$  is computed by using the input signal parameters, the desired signal vector, the matrix transform  $A$ , and the

fact that the output noise power residue of the matched filter ( $K = \infty$ ) is normalized to one. The quantity  $|u|^2$  is actually the output signal-to-noise ratio of the matched filter for a constant input signal amplitude.

### 5. DETECTION PROBABILITY: RAYLEIGH SIGNAL

In this section, we derive an expression for  $P_D$  for the special case when the envelope of desired input signal  $u$  is Rayleigh distributed. We define  $v_l$  as done by Eq. (13) and  $v$  as done by Eq. (19). Implicit in the assumption that  $|u|$  is Rayleigh distributed is the fact that  $u$  is a circular Gaussian process with zero mean and variance equal to  $\sigma_u^2$ . Thus  $v$  is circular Gaussian with zero mean and

$$\sigma_v^2 = \frac{\sigma_u^2}{\|\mathbf{w}\|^2} + 1. \quad (37)$$

The decision rule given by Eq. (8) can be rewritten as

$$\begin{array}{l} H_1 \\ |v|^2 > T \\ < T \\ H_0 \end{array} \sum_{l=1}^L |v_l|^2. \quad (38)$$

We define  $z$  as in Eq. (24) and

$$r = |v|^2. \quad (39)$$

The PDF of  $r$  is given by

$$p_r(r) = \frac{1}{\sigma_v^2} e^{-r/\sigma_v^2}. \quad (40)$$

Steenson [8] shows that for the assumed PDF,

$$P(D|\mathbf{w}) = \text{Prob}\{r > z\} = \left[1 + \frac{T}{\sigma_v^2}\right]^{-L}. \quad (41)$$

Again set  $q_0 = 1/\|\mathbf{w}\|^2$ . Thus Eq. (41) becomes

$$P(D|\mathbf{w}) = P(D|q_0) = \left[1 + \frac{T}{\sigma_u^2 q_0 + 1}\right]^{-L}. \quad (42)$$

We define

$$\left(\frac{S}{N}\right)_{opt} = \sigma_u^2 = \text{optimal signal-to-noise power ratio of the test statistic } z_1.$$

We note that  $(S/N)_{opt}$  is also the optimal signal-to-noise power ratio of the output of the matched filter where the optimal linear weight is given by  $w_{opt} = R_{xx}^{-1} s$ . The PDF of  $q_0$  is given by Eq. (30). It follows that

$$P_D = \frac{K!}{(N-2)!(K-N+1)!} \int_0^1 \left[ 1 + \frac{T}{q_0 \left[ \frac{S}{N} \right]_{opt} + 1} \right]^{-L} (1-q_0)^{N-2} q_0^{K-N+1} dq_0. \quad (43)$$

By using Eq. (17),

$$T = P_F^{-1/L} - 1. \quad (44)$$

This expression substituted into Eq. (43) results in

$$P_D = c \int_0^1 \left[ \frac{q_0 + a}{q_0 + b} \right]^L (1-q_0)^{N-2} q_0^{K-N+1} dq_0, \quad (45)$$

where

$$a = \left[ \frac{S}{N} \right]_{opt}^{-1}, \quad (46)$$

$$b = \left[ \frac{S}{N} \right]_{opt}^{-1} P_F^{-1/L}, \quad (47)$$

and

$$c = \frac{K!}{(N-2)!(K-N+1)!}. \quad (48)$$

Furthermore, if we set  $q_1 = q_0 + b$ , then

$$P_D = c \int_b^{b+1} (q_1 + a - b)^L (1 + b - q_1)^{N-2} (q_1 - b)^{K-N+1} q_1^{-L} dq_1. \quad (49)$$

If we expand the integrand of the above integral by using the binomial formula, a finite series expansion in term of powers of  $q$  results. These may be integrated yielding the following:

$$P_D = (-1)^{L+K+N+1} c \alpha^L (1+b)^{N-2} b^{K-N+1}$$

$$\cdot \sum_{l=0}^L \sum_{h=0}^{N-2} \sum_{m=0}^{K-N+1} \binom{L}{l} \binom{N-L}{n} \binom{K-N+1}{m} (-1)^{l+m+n} \frac{F(l+m+n-L, b)}{\alpha^l (1+b)^n b^m} \quad (50)$$

where  $(\cdot)$  is the binomial coefficient of its arguments,

$$\alpha = \left[ \frac{S}{N} \right]_{opt}^{-1} (P_F^{-\frac{1}{L}} - 1), \quad (51)$$

and  $F(\cdot, \cdot)$  is a function defined as

$$F(i, b) = \int_0^{1+b} q_1^i dq_1 = \begin{cases} \frac{(1+b)^{i+1} - b^i}{i+1}, & i \neq -1 \\ \ln \left( 1 + \frac{1}{b} \right), & i = -1. \end{cases} \quad (52)$$

We note that as  $K \rightarrow \infty$ , then  $q_0 \rightarrow 1$ . Thus the quiescent  $P_D$  denoted by  $P_D^{(q)}$  is given by

$$P_D^{(q)} = \left[ 1 + \frac{T}{\frac{\sigma_u^2}{\sigma_{min}^2} + 1} \right]^{-L}. \quad (53)$$

By using Eq. (44) and  $(S/N)_{opt} = \sigma_u^2 / \sigma_{min}^2$ , then

$$P_D^{(q)} = \left[ 1 + \frac{P_F^{-\frac{1}{L}} - 1}{\left[ \frac{S}{N} \right]_{opt} + 1} \right]^{-L}. \quad (54)$$

## 6. RESULTS

In this section, we present some results on the detection probability  $P_D$  of the MLAD vs the independent parameters: the probability of false alarm  $P_F$ ; the steady-state signal-to-noise output power ratio of the matched filter  $(S/N)_{opt}$ ; the number of independent samples per channel  $K$  used to calculate the sample covariance matrix; the order of the adaptive matched filter  $N$ ; and the number of samples  $L$  used to set the mean level threshold. We found that the solution for  $P_D$  given by Eq. (50) though exact is numerically unstable for computer evaluation. Hence the integral solution given by Eq. (43) was evaluated.

We set  $K = MN$  where  $M$  is a positive integer and use  $M$  as an independent parameter called the order factor. Plots of  $P_D$  vs  $(S/N)_{opt}$  and  $M$  for  $P_F = 10^{-6}, 10^{-10}$  and various  $N$  are shown in Figs. 4 through 13. We note that for these figures, we have set  $L = K - 1$ . This might be a logical choice for the number of samples used to set the threshold since all the samples except the candidate

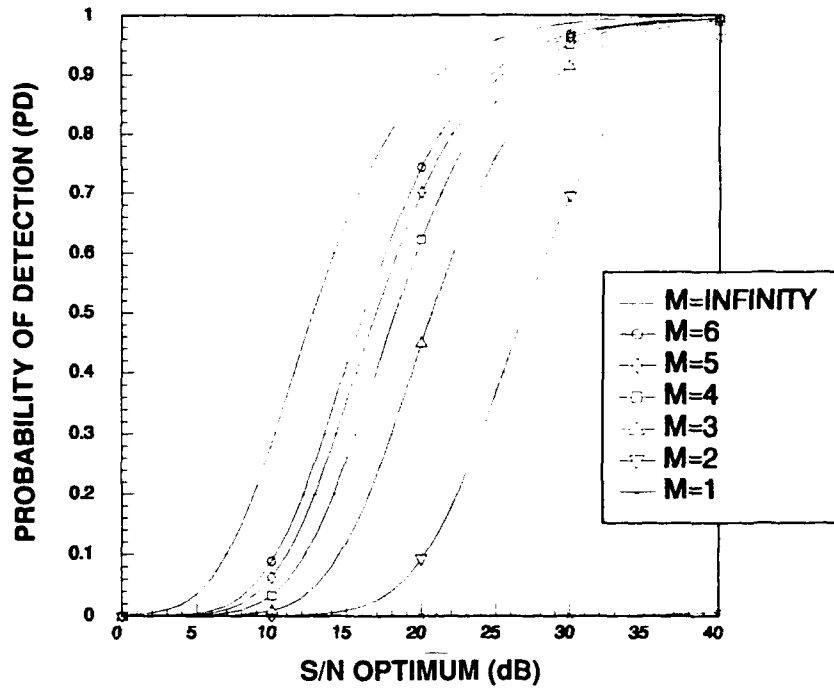


Fig. 4 -  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-6$ ,  $N = 2$

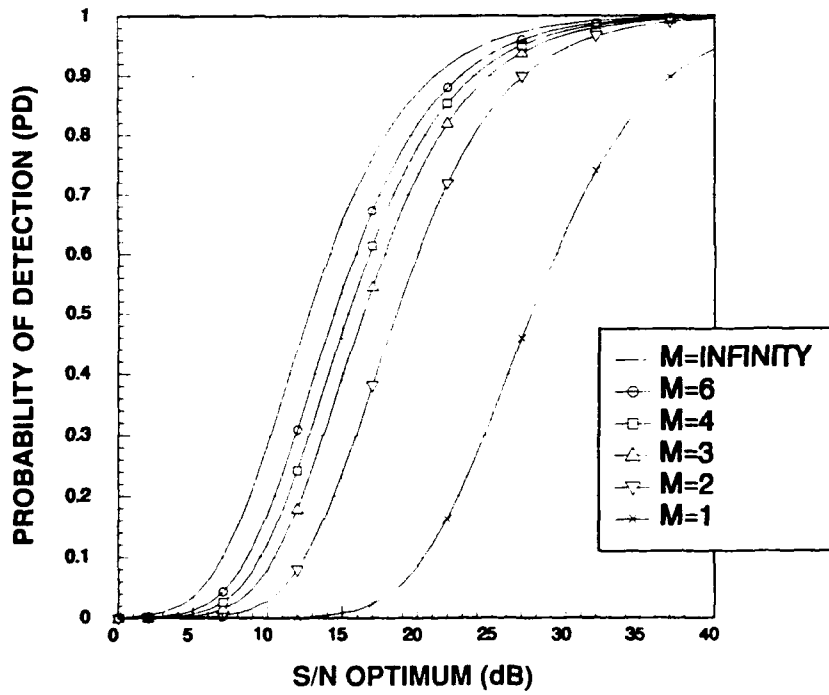


Fig. 5 -  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-6$ ,  $N = 5$



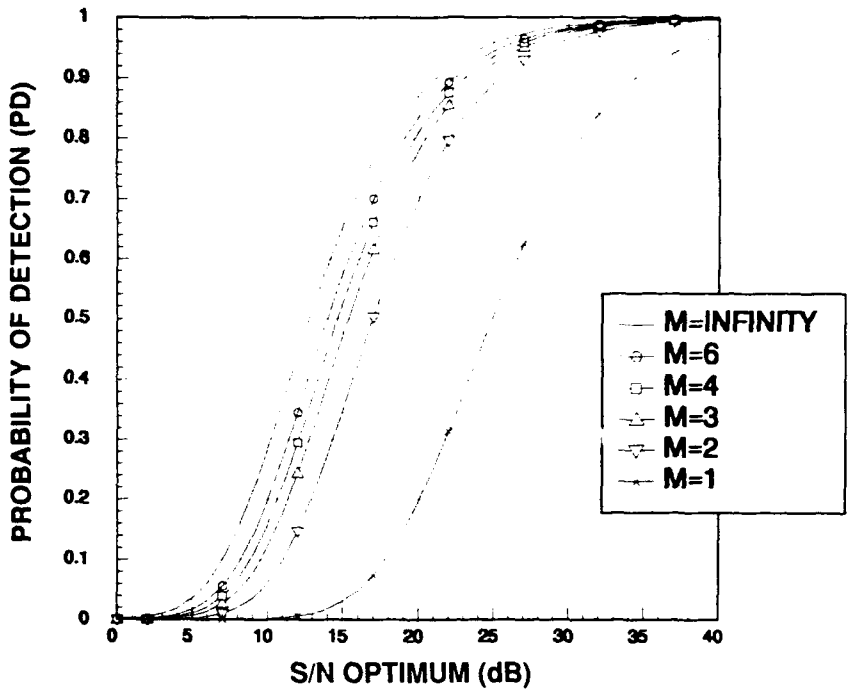


Fig. 6 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-6$ ,  $N = 10$

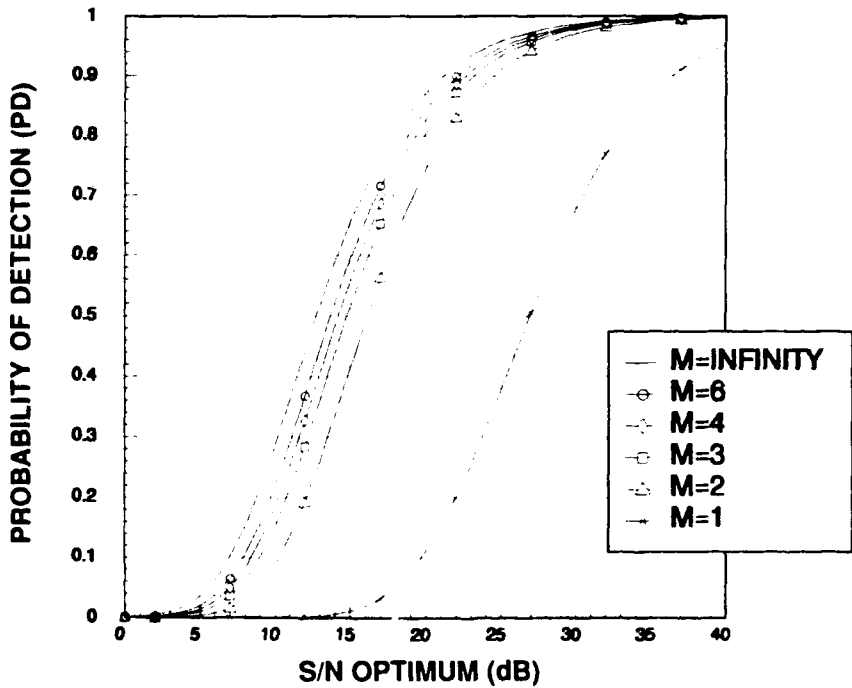


Fig. 7 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-6$ ,  $N = 30$

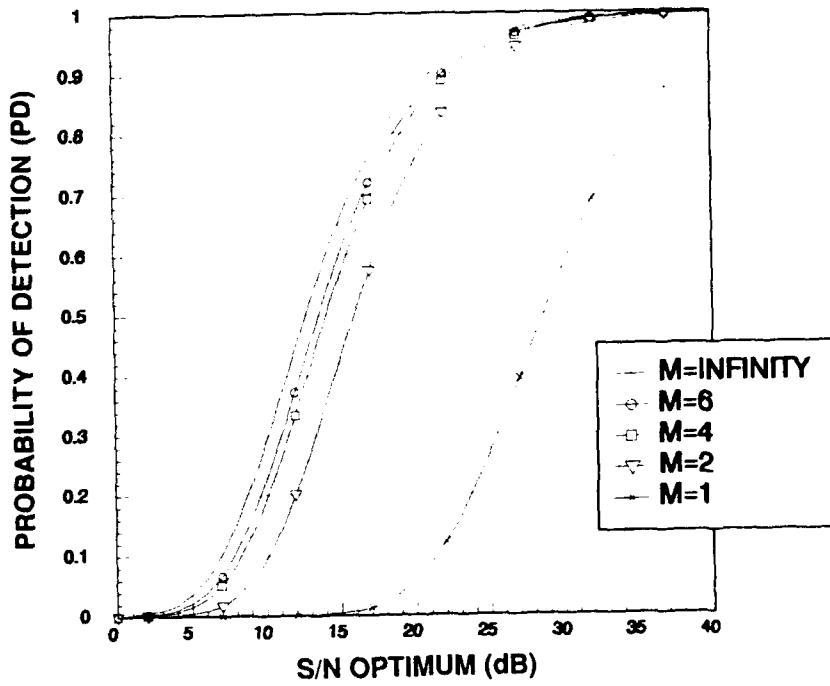


Fig. 8 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-6$ ,  $N = 50$

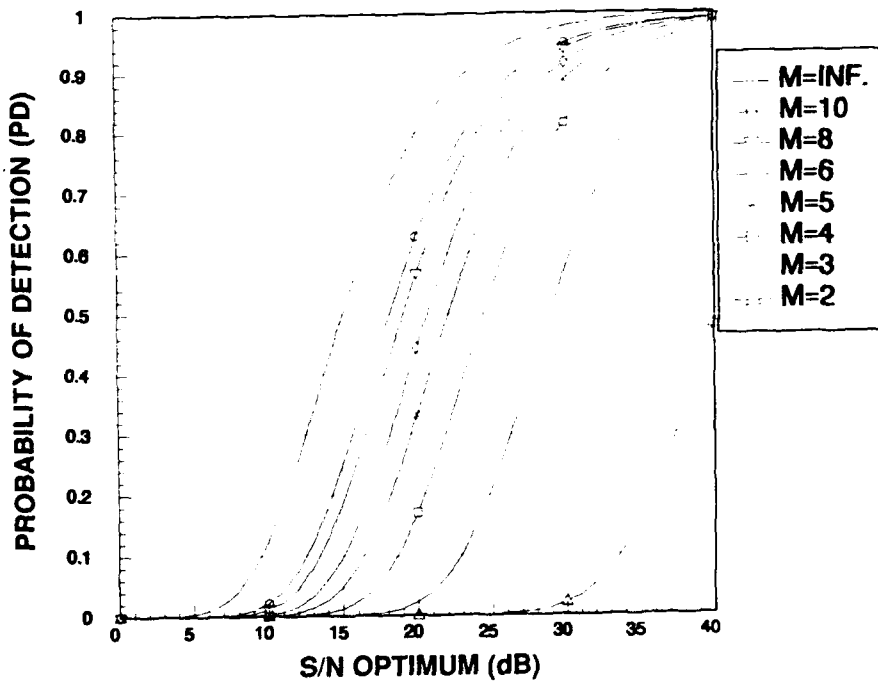


Fig. 9 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-10$ ,  $N = 2$

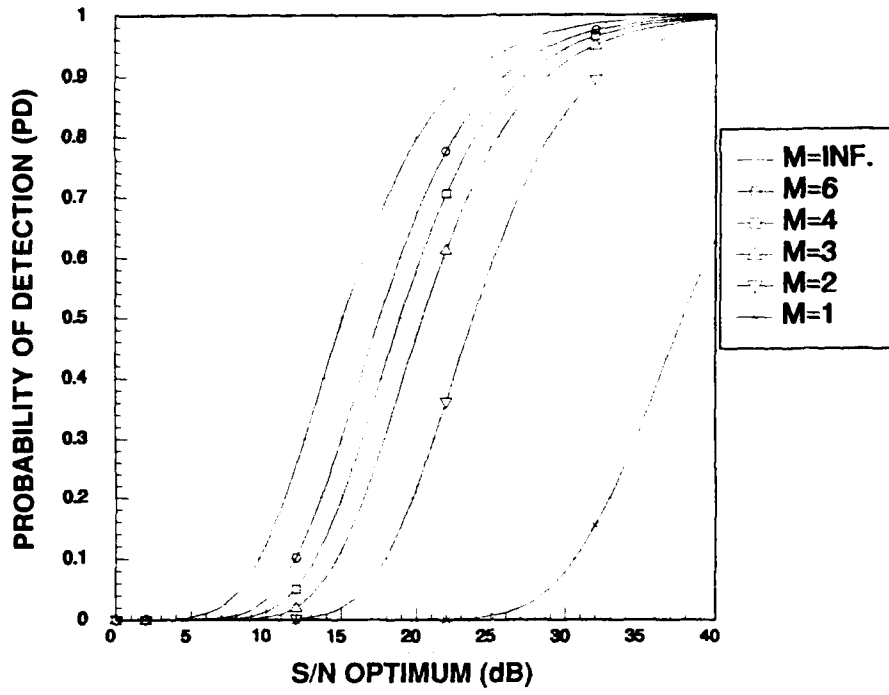


Fig. 10 -  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-10$ ,  $N = 5$

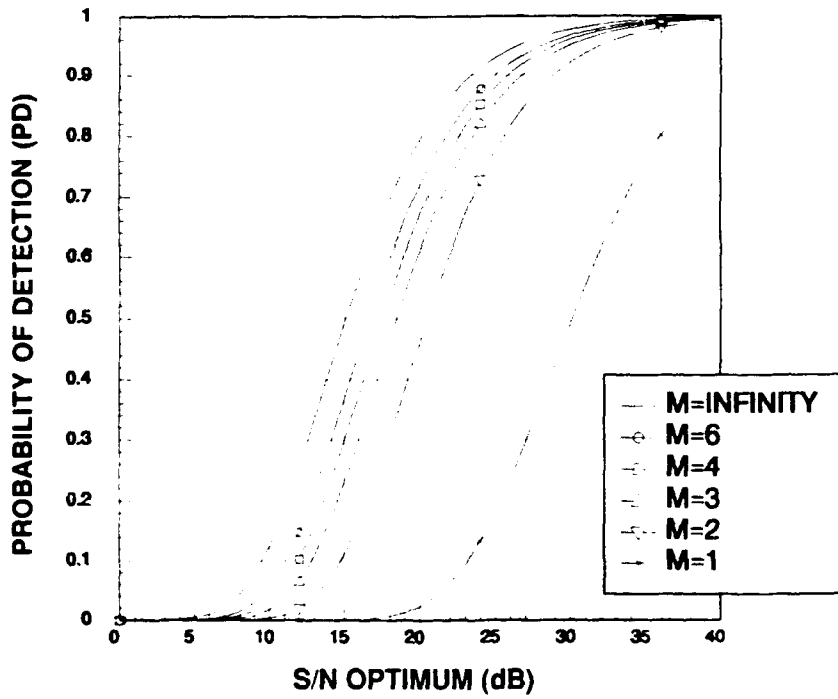


Fig. 11 -  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-10$ ,  $N = 10$

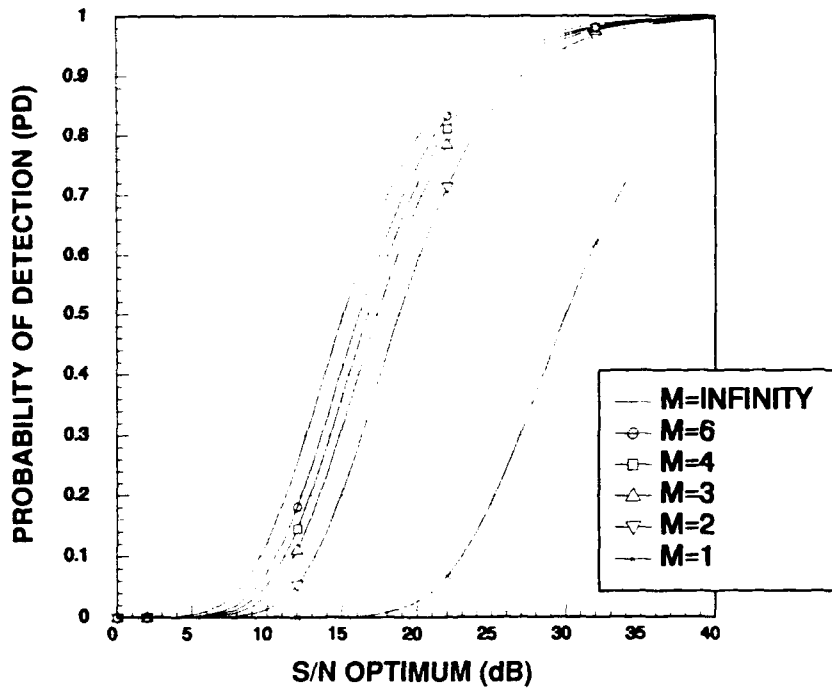


Fig. 12 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1, D = 10, N = 30$

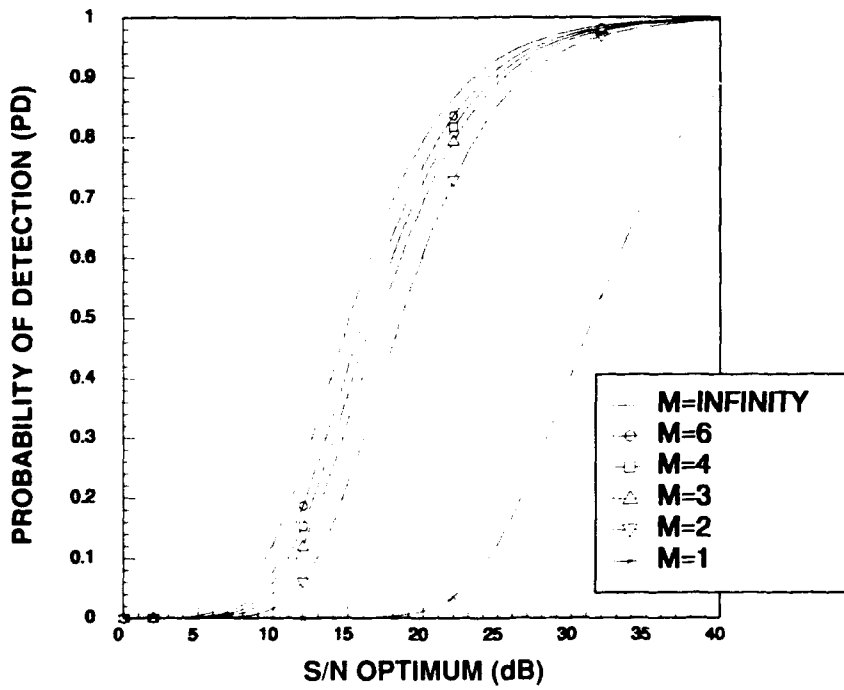


Fig. 13 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1, D = 10, N = 50$

primary input data in a given batch are used to set the threshold. Note that as  $M \rightarrow \infty$ , then  $L \rightarrow \infty$ , and that [1]

$$P_D^{(g)} = P_F \left[ 1 + \left( \frac{S}{N} \right)_{opt} \right]^{-1} \quad (55)$$

Kelly [5] defines the (S/N) loss of an adaptive detector as the difference of required (S/N) to obtain a given  $P_D$  between a steady state ( $M = \infty$ ) detector and transient state ( $M$  finite) with all other independent parameters being equal. Define  $M_{3dB}$  to be the order factor such that (S/N) loss is nearest to 3 dB. We make the following observations from Figs. 4 through 13.

- (1) The MLAD is slower to converge to its optimal  $P_D$  ( $M = \infty$ ) for lower ordered matched filters. For example, for most  $P_D$ 's (0.1 to 0.9),  $P_F = 10^{-6}$ , if  $N = 2$ , then  $M_{3dB} = 6$ ; if  $N = 10$ , then  $M_{3dB} = 3$ .
- (2) There are diminishing returns in convergence by using a larger order factor.
- (3) Convergence slows for decreasing  $P_F$ . For example, for most  $P_D$  (0.1 to 0.9) and  $N = 2$ , if  $P_F = 10^{-6}$ ,  $M_{3dB} = 6$ , if  $P_F = 10^{-10}$ ,  $M_{3dB} = 10$ .

We note that these trends were also observed by Kelly [5] for his adaptive detection algorithm.

Since, in general, the number of samples  $L$  used to set the mean level detection threshold is arbitrary, we present two sets of curves (Figs. 14 and 15) where  $L$  is not related to  $K$ . For these curves,  $L = 10$ . Note that Eq. (54) and not Eq. (55) is used to evaluate  $P_D^{(g)}$ .

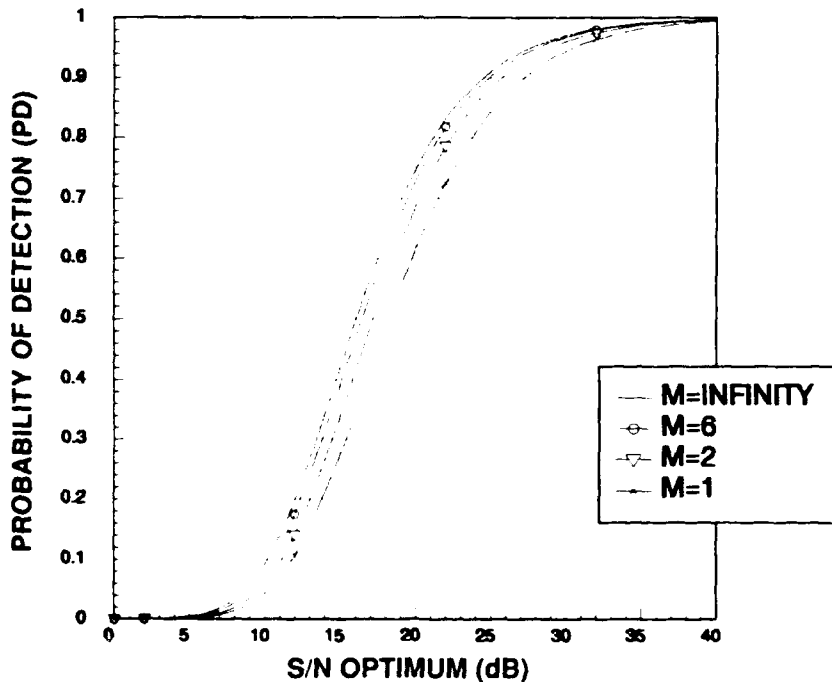


Fig. 14 -  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 10^{-6}$ ,  $N = 2$ ,  $L = 10$

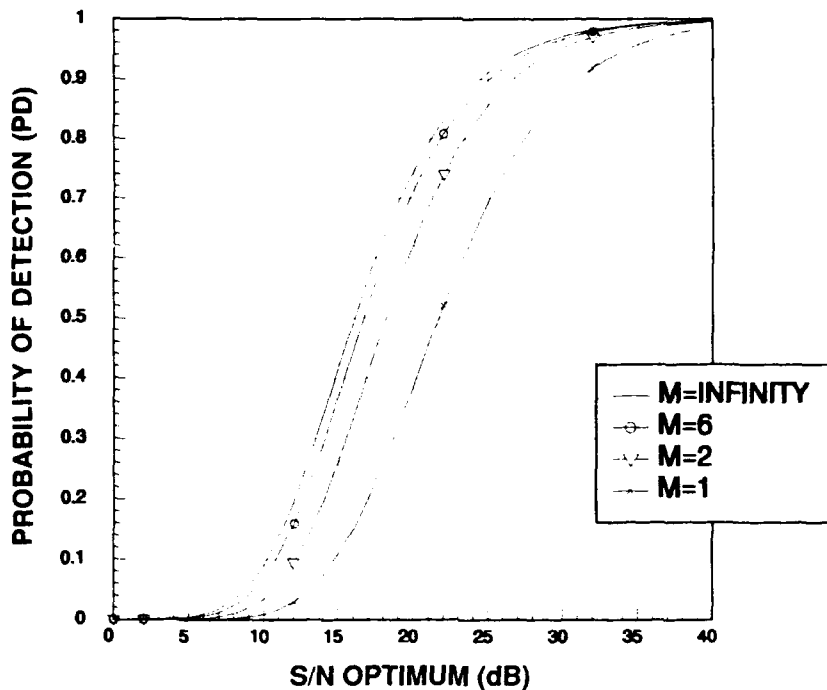


Fig. 15 —  $P_D$  vs  $(S/N)_{opt}$  for  $P_F = 1.D-6$ ,  $N = 5$ ,  $L = 10$

## 7. SUMMARY

Convergence results for a mean level adaptive detector (MLAD) have been presented. The MLAD consists of an adaptive matched filter (for spatially correlated inputs) followed by a mean level detector (MLD). The optimal weights of the adaptive matched filter are estimated from one batch of data and applied to a statistically independent batch of nonconcurrent data. The threshold of the MLD is determined from the resultant data. Thereafter a candidate cell is compared against this threshold. Probabilities of false alarm and detection were derived as a function of the threshold factor, the order of the matched filter, the number of independent samples-per-channel used to calculate the adaptive matched filter weights  $K$ , the number of samples used to set the MLD threshold  $L$ , and the output signal-to-noise power ratio of the optimal matched filter. A number of performance curves were shown and discussed. It was shown for the particular case when  $L = K - 1$ , the MLAD is slower to converge to its optimal value for lower-ordered matched filters, there are diminishing returns in convergence performance when using more independent samples per channel, and convergence slows for increasing probability of false alarm.

## 8. REFERENCES

1. H. L. Van Trees, *Detection, Estimation, and Modulation Theory, Vol. 1* (John Wiley and Sons, New York, 1968).
2. I.S. Reed, J.D. Mallett, and L.E. Brennan, "Rapid Convergence Rate in Adaptive Arrays," *IEEE Trans. AES* 10, 853-863 (1974).

KARL GERLACH

3. L.E. Bernnan and I.S. Reed "Digital Adaptive Arrays with Weights Computed from and Applied to the Same Sample Set," in Proceedings of the 1980 Adaptive Antenna Symposium 1, Dec. 1980; also technical report RADC-TR-80-378, Rome Air Development Center, Griffiss Air Force Base, Rome, NY.
4. K. Gerlach and F.F. Kretschmer, "Convergence Properties of Gram-Schmidt and SMI Adaptive Algorithms," *IEEE Trans. AES* **26**(1), 44-57 (1990).
5. E.J. Kelly, "An Adaptive Detection Algorithm," *IEEE Trans. AES* **22**(1), 115-127 (1986).
6. R. Nitzberg, "Detection Loss of the Sample Matrix Inversion Technique," *IEEE Trans. AES* **20**(6), 824-827 (1984).
7. I.S. Reed and L.E. Brennan, "Analysis of Constant False Alarm Rate Sidelobe Canceller Criterion," Adaptive Sensors, Inc., Internal Report, May 1985.
8. B.O. Steenson, "Detection Performance of a Mean-Level Threshold," *IEEE Trans. AES* **4**(4), 529-534 (1980).
9. G.M. Dillard, "Mean-Level Detection of Nonfluctuating Signals," *IEEE Trans. AES* **10**(6), 795-799 (1974).