Comparison of Analysis and Experiment for Dynamics of Low-Contact-Ratio Spur Gears

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OF LOW-CONTACT-RATIO SPUR GEARS

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ABSTRACT

Low-contact-ratio spur gears were tested in the
NASA gear-noise rig to study gear dynamics including
dynamic load, tooth bending stress, vibration, and
noise. The experimental results were compared with a
NASA gear dynamics code to validate the code as a
design tool for predicting transmission vibration and
noise.

Analytical predictions and experimental data for
gear-tooth dynamic loads and tooth-root bending stress
were compared at 28 operating conditions. Strain
gage data were used to compute the normal load between
meshing teeth and the bending stress at the tooth root
for direct comparison with the analysis. The computed
and measured waveforms for dynamic load and stress
were compared for several test conditions. These are
very similar in shape, which means the analysis suc-
cessfully simulates the physical behavior of the test
gears.

The predicted peak value of the dynamic load
agrees with the measurement results within an average
error of 4.9 percent except at low-torque, high-speed
conditions. Predictions of peak dynamic root stress
are generally within 10 to 15 percent of the measured
values.

INTRODUCTION

In a helicopter, a geared transmission is a very
efficient device for converting the high-speed, low-
torque power output of a gas turbine engine to the low-
speed, high-torque output required to drive the rotor
blades. However, transmission gear noise (which has
been measured at over 100 dB) is a major source of hel-
icopter cabin noise. This noise causes adverse health
effects and disrupts communication. The NASA/Army hel-
icopter transmission-noise-reduction research project
was initiated to solve this problem.

Gear vibration is simulated by many computer
codes: DANST (Lin et al., 1989a, 1989b, 1987a, 1987b
and 1986), developed through NASA grants at the Univer-
sity of Cincinnati and Memphis State University;
GEARDYN (Boyd and Pike, 1987 and Pike, 1981), deve-
loped under NASA contract; and GRD (Kahraman et al.,
1990, Zakrajsek et al., 1990) developed from a NASA
grant at Ohio State University. (Other work is summar-
ized in Lim and Singh, 1989.)

The NASA gear-noise rig was built to satisfy an
acute need to verify these codes with experimental
data taken under carefully controlled conditions. An
experimental facility was needed which could identify
and develop advanced concepts, such as new gear-tooth
forms, for helicopter transmission-noise reduction.
The data and validated computer codes resulting from
the test program will provide a technology base for
future, quiet, transmission designs.

The goal of the test program is to verify predic-
tions of the gear dynamics code DANST (dynamic anal-
ysis of spur gear transmissions) for both the tooth
loads and the bending stress at the gear-tooth root.
This paper complements and extends the work of Ozkul
(1989 and 1987). Ozkul compared measurements made on
a four-square fatigue rig with stress predictions from
finite element analysis and from the dynamics code
GEARDYN.

APPARATUS

The gear noise rig (Fig. 1) measures the
vibration, dynamic loads, and noise of a geared transm-
ission. It features a simple gearbox (Fig. 2) contain-
ing a pair of parallel axis gears straddle mounted and
supported by rolling element bearings. A 150-kW
(200-hp) variable-speed electric motor powers the rig
at one end, and an eddy-current dynamometer applies
power-absorbing torque at the other end. The gearbox
adapts for testing various configurations of gears,
bearings, dampers, and supports. The test rig is loca-
ted in an acoustically treated room to allow more accu-
rate sound measurements. Test gear parameters are
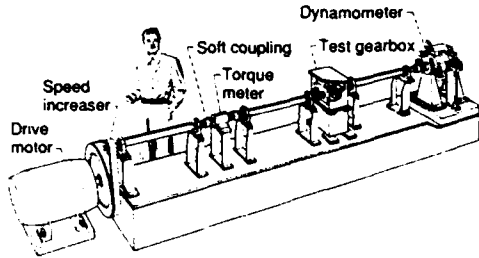shown in Table I; profile traces of the gears are
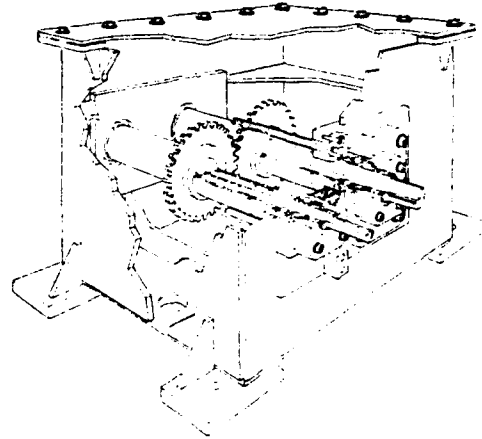shown in Fig. 3.

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Fig. 1.—Gear noise rig.

Fig. 2 —Test gearbox

TABLE I. - TEST GEAR PARAMETERS

Gear type	Standard involute, full-depth tooth
Number teeth	28
Module, mm (diametrial pitch in.$^{-1}$)	3.175(8)
Face width, mm (in.)	6.35(0.25)
Pressure angle, deg	20
Theoretical contact ratio	1.64
Driver modification amount, mm (in.)	0.023(0.0009)
Driven modification amount, mm (in.)	0.025(0.0010)
Driver modification start, deg	24
Driven modification start, deg	24
Tooth-root radius, mm (in.)	1.35(0.053)
Gear quality	ACMA class 13
Nominal (100 percent) torque, N-m (in.-lb)	71.77(635.25)
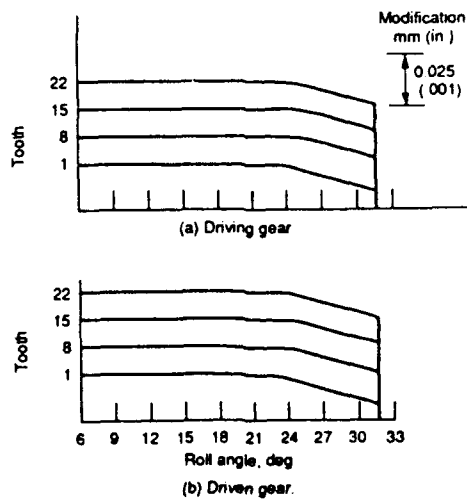
Fig. 3.—Test gear profile traces.

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A poly-V belt drive was used as a speed increaser between the motor and input shaft. A soft coupling was placed on the input shaft to reduce the torque fluctuation at the belt rotation frequency that was caused by a nonuniformity at the belt splice. The low coupling stiffness, 362 N-m/rad (3204 lb-in./rad), must be considered in modeling the input side of the rig.

INSTRUMENTATION AND TEST PROCEDURE

General-purpose, constantan foil, resistance strain gages (gage length, 0.38 mm (0.015 in.)) were installed in the tooth-root fillets on both the loaded (tensile) and unloaded (compression) side of two adjacent teeth on the output (driven) gear (Fig. 4). To
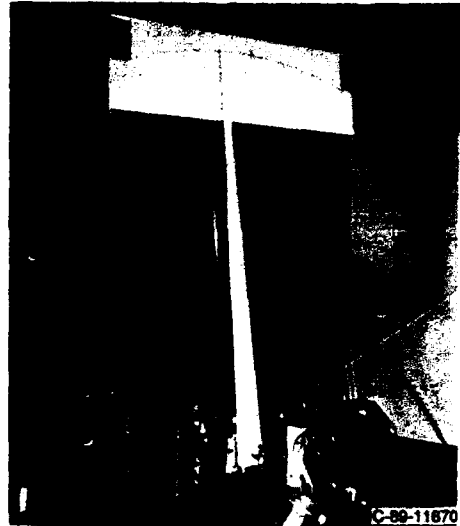

Fig. 5.—Strain gage calibration apparatus.

used to develop an influence matrix which allows computation of both normal and frictional forces acting between mating teeth. The static strain readings were taken with the instrumented gear acting as both the driven and driving gear. This provided strain gage data for the frictional force operating both toward the pitch point and away from the pitch point. For each gage, the mean of the two readings (from driving and driven gear) is the strain gage output caused by the normal component (without friction) of the force between gear teeth. The difference between the reading taken with the gear acting as the driven gear and the mean reading is the strain caused by the frictional force only. The computational procedure will be explained more fully in another report (Rebbechi et al., 1991). An example of the static strain gage calibration data from single-tooth loading is shown in Fig. 6(a).

Fig. 4.—Strain gage installation on test gear.
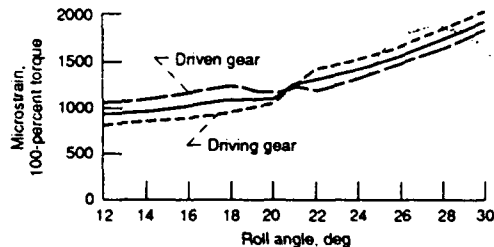
(a) Single-tooth loading
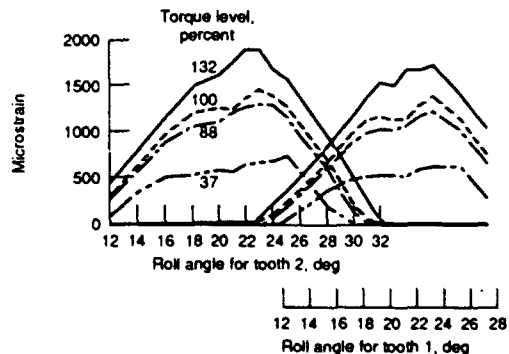
(b) With standard mating gear.

Fig. 6.—Static strain gage (calibration) data.

measure maximum tooth bending stress, the gages were placed at the 30° tangency location (Cornell, 1980). A wheatstone bridge circuit was used for signal conditioning for the static calibration.

Strain gages were calibrated under static torque conditions with a special calibration gear. On this calibration gear, the teeth adjacent to the loaded tooth were ground away to ensure single-tooth contact. The roll angle of the test gear was measured with a large protractor (Fig. 5). The calibration data was

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Static strain gage data were also taken with the strain gage gear mated with a standard gear. The curves of the mean strain (caused by the normal force alone) show readings from four different torques ranging from 32 to 132 percent of the nominal torque (Fig. 6(b)).

For dynamic measurements, constant-current amplifiers were used as signal conditioners. The data were collected by a 14-bit analog-to-digital data acquisition system and stored on computer disk. By simultaneously recording a 1/rev signal from an optical encoder adjusted so its signal was produced at a known roll angle of the gear, we obtained accurate rotation data. At least 500 data samples/rev for at least 6 rev of the gear were taken at each test condition. This data was digitally resampled at either 1000 or 2000 samples/rev (depending on speed) and then synchronously averaged.

To compare dynamic data with the analytical results from DANST, strain gage readings were recorded for 28 test conditions, including speeds of 800, 2000, 4000 and 6000 rpm with torques of 16 to 110 percent of the nominal torque of 71.8 Nm (635 in.-lb). Measured strain values were converted to stress using Hooke's law. The value assumed for Young's modulus is 203 GN/m^2 (30x10^6 psi).

ANALYTICAL MODELING PROCEDURE

Computer program DANST employs 4 torsional degrees of freedom. These degrees of freedom represent the input (motor), the two gears, and the output (load). (See Lin, 1989(b) for details.) Equivalent mass and stiffness elements were calculated to represent the input and output elements of the rig. Table II shows the rig-modeling data. The gear profiles are modeled as perfect involutes from the low point of contact to the start of modification; the tooth tips are modified with linear tip relief as specified in Table I.

RESULTS AND DISCUSSION

The normal tooth load (dynamic force) was computed from strain gage readings taken from both loaded and unloaded sides of the tooth with the influence matrix described previously. Figure 7((a) to (d)) shows four examples of experimental dynamic load data superimposed on analytical predictions to allow direct comparison of experiment and analysis. The load data is plotted in terms of the gear roll angle. Since the instrumented gear is a driven gear, tooth contact starts at the tooth tip (at 31.4°) and ends near the

tooth root (10.3°). The similarity of the analysis and experiment waveforms shows that the analysis accurately simulates the physical behavior of the test gears. In Fig. 7(b), the analysis successfully predicts loss of tooth contact which occurs near the pitch point (at 20.85°). At these low-torque levels, the profile modifications (which are optimized for high-torque) cause strong periodic transmission errors which increase the dynamic load. Munro (1989) describes this effect and cites instances where the teeth lose contact as illustrated in Fig. 7(b).

The static (0 rpm) tooth force from DANST is superimposed on the dynamic and experimental values in Fig. 7(c). The effect of load sharing is seen by comparing the single-tooth contact zone (where the force is constant) to the double-tooth contact zones near the tooth tip and root.

In Fig. 8, the peak value of the dynamic load is compared with DANST predictions for 28 operating conditions. The test conditions included speeds of 800 to 6000 rpm and torques from 16 to 110 percent of the gear design torque. Except at the two lowest torque levels tested (16 and 31 percent of nominal), the maximum dynamic load prediction agrees with the measurement results within an average error of 4.9 percent. The analysis overestimates dynamic loads for the two lowest torque levels plotted with solid circles in Fig. 8. For example, in the waveform illustrated in Fig. 7(b), the peak load is overestimated by 93 percent.

The damping factors used in DANST were 0.1 for the gear mesh and 0.05 for the structural damping in the connecting shafts. (These are typical values used by other investigators.) An increased mesh-damping factor of 0.2 was tried which did not significantly reduce the high-dynamic effect predicted at low-torque and high-speed conditions.

The analysis may have overestimated the dynamic tooth loads at the two lowest torque levels because (1) DANST does not consider secondary effects such as the smoothing and blending effects of load fluctuations from the motor and belt drive, and (2) these test conditions may require more numerical iterations or a tighter convergence tolerance than the program currently allows.

Strain gage data were also used to compute the bending stress at the tooth root for comparison with DANST predictions. Figure 9((a) to (d)) shows sample plots of tooth-root stress as a function of roll angle. These plots are the same four test conditions illustrated in the gear-tooth force plots of Fig. 7. Once

TABLE II. - TEST RIG MODELING PARAMETERS

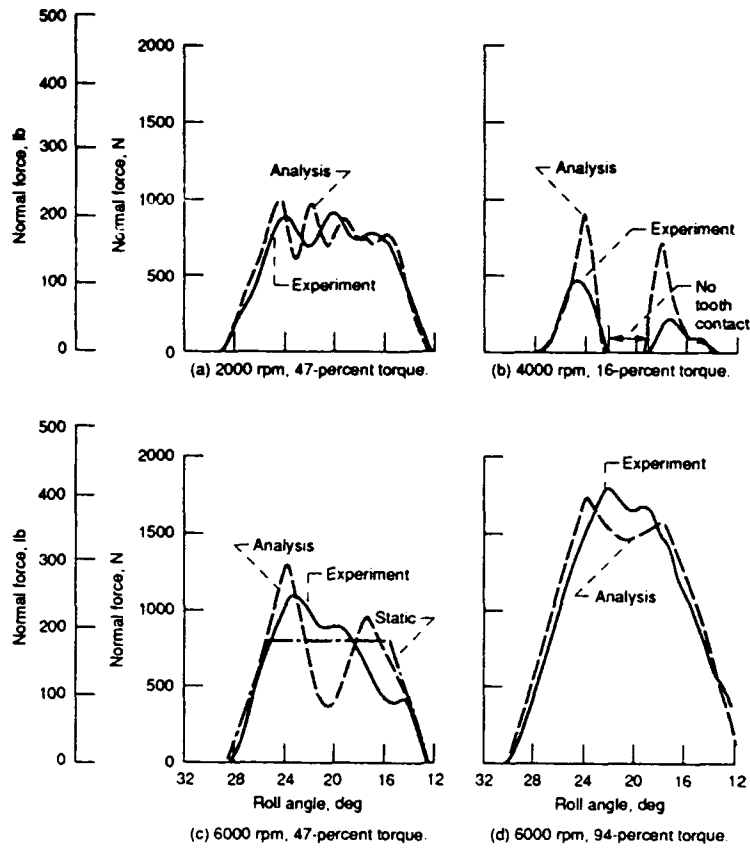|	
Input inertia, J_1, kg-m^2 (lb-s^2-in.)	0.0237(2.10)
Gear inertia, J_2, J_3, kg-m^2 (lb-s^2-in.)	0.0000364(0.00322)
Load inertia, J_4, kg-m^2 (lb-s^2-in.)	0.085(7.5)
Input stiffness, K_1, N-m/rad (lb-in./rad)	341(3017)
Gearbox stiffness, K_2, N-m/rad (lb-in./rad)	6158(54 500)
Load stiffness, K_3, N-m/rad (lb-in./rad)	12 700(112 300)
Natural frequencies (eigensolution), Hz	6.56, 52.5, 1220
DANST natural frequencies, Hz	6.73, 645, 5821

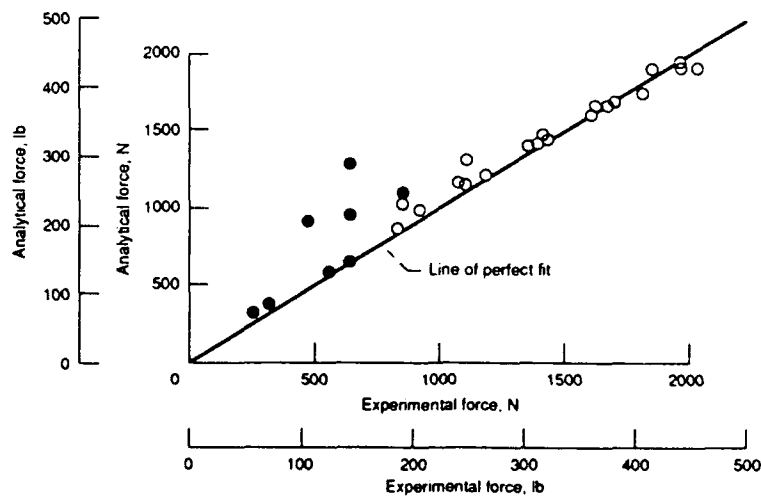Fig. 7.—DANST force predictions compared to strain gage data.

(a) 2000 rpm, 47-percent torque.

(b) 4000 rpm, 16-percent torque.

(c) 6000 rpm, 47-percent torque.

(d) 6000 rpm, 94-percent torque.

Fig. 8.—DANST predictions for peak force compared to experimental data.
(data from two lowest torque levels shown as solid symbols)

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(a) 2000 rpm, 47-percent torque.

(b) 4000 rpm, 16-percent torque.

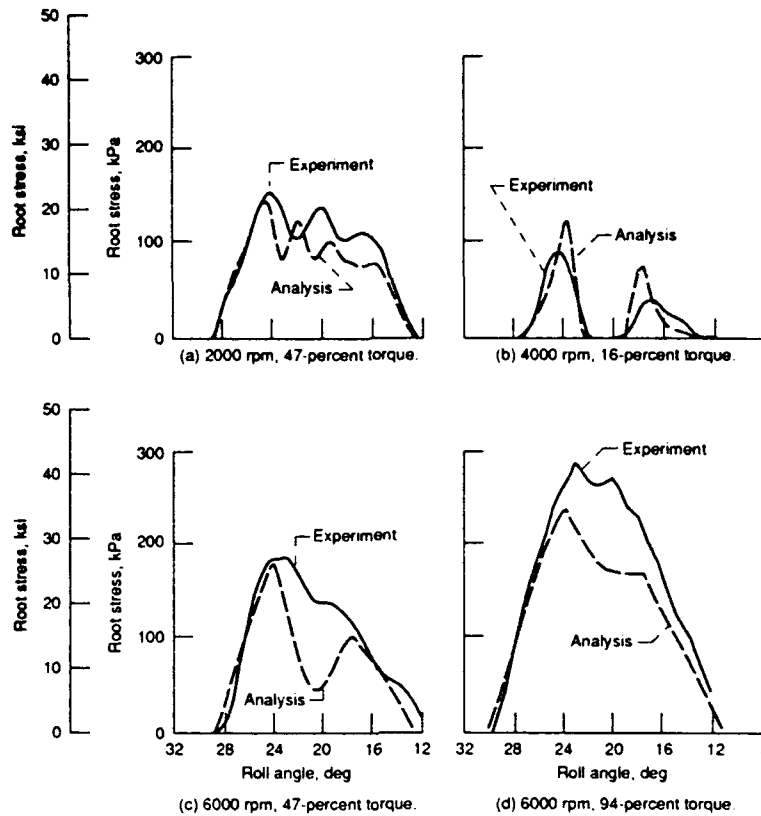(c) 6000 rpm, 47-percent torque.

(d) 6000 rpm, 94-percent torque.

Fig. 9.—DANST stress predictions compared to strain gage data.

again, the waveforms of the analytical and experimental data are very similar except at low torque where the stress is overestimated.

Figure 10 shows a comparison for the peak bending stress at the 28 test conditions previously described. The analysis generally underestimated the peak measured stress by 10 to 15 percent, which indicates that improvement is needed for calculation of stress concentration. The stress concentration is computed by using the modified Heywood method (Cornell, 1980) with the assumption that tooth-root geometry is created by a standard hob.
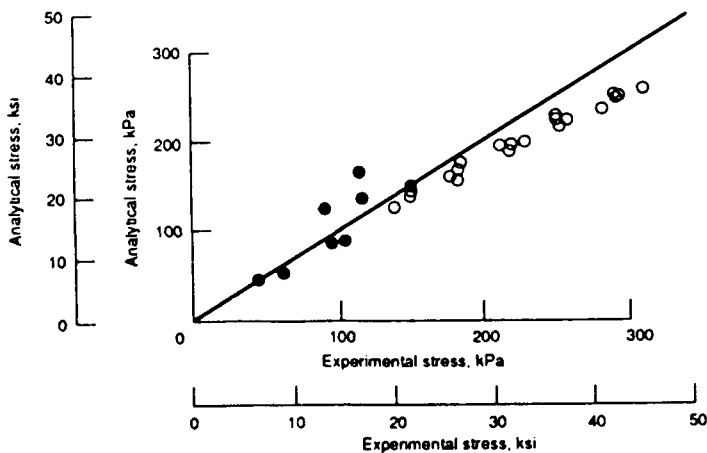

Fig. 10.—DANST predictions for peak stress compared to experimental data (data from two lowest torque levels shown as solid symbols).

An exact correlation between the peak load (force) data in Fig. 8 and the peak stress data of Fig. 10 cannot be made because the stress depends on both the magnitude of the load and on its position (height) on the tooth.

The total gearbox stiffness (input shaft to output shaft) was measured and the inertia of the test gears was computed to allow the calculation of natural vibration frequencies (assuming 4 degrees of freedom). Table II lists this information and the natural frequencies as calculated by DANST. DANST (which calculates its own values) does not use these gearbox stiffness and gear inertia values. The measured gearbox stiffness is much lower than the gear-mesh stiffness calculated in DANST because the measured stiffness includes the effects of gear shaft lateral flexibility, as well as the effects of gear shaft torsional flexibility which are not included in the DANST model.

The accuracy of the results from DANST depends on the accuracy of modeling the tooth profile, on estimates for the rig inertia and stiffness, and on the validity of the simple 4 degrees-of-freedom lumped-parameter system model. Even with these simplifications, the analysis predictions are in very good agreement with experimental data.

CONCLUSIONS

Low-contact-ratio spur gears with linear profile modifications were tested in the NASA gear noise rig to study dynamic load and tooth bending stress. The experimental results were compared at 28 operating conditions with the NASA gear dynamics code DANST to validate the code as a design tool for predicting

transmission vibration and noise. The following con-
clusions were obtained:

1. The computed waveforms for gear-tooth loads
and bending stress compare very well with experimental
results. This indicates the analysis simulates the
physical behavior of the test gears.
2. Peak dynamic load predictions agree with the
measurement results within an average error of 4.9 per-
cent except at low-torque, high-speed conditions.
3. The analysis generally underestimated the root
stress by 10 to 15 percent. This may be due to underes-
timating the stress concentration at the tooth root.

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16. Abstract

Low-contact-ratio spur gears were tested in the NASA gear-noise-rig to study gear dynamics including dynamic load, tooth bending stress, vibration, and noise. The experimental results were compared with a NASA gear dynamics code to validate the code as a design tool for predicting transmission vibration and noise. Analytical predictions and experimental data for gear-tooth dynamic loads and tooth-root bending stress were compared at 28 operating conditions. Strain gage data were used to compute the normal load between meshing teeth and the bending stress at the tooth root for direct comparison with the analysis. The computed and measured waveforms for dynamic load and stress were compared for several test conditions. These are very similar in shape, which means the analysis successfully simulates the physical behavior of the test gears. The predicted peak value of the dynamic load agrees with the measurement results within an average error of 4.9 percent except at low-torque, high-speed conditions. Predictions of peak dynamic root stress are generally within 10 to 15 percent of the measured values.

17. Key Words (Suggested by Author(s))

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