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13. ABSTRACT (*Maximum 200 words*)

The research centered around three topics in computational fluid dynamics. These were: schemes for the time-dependent Navier-Stokes equations, improved methods for the steady Navier-Stokes equations, and domain decomposition methods. (In the original proposal, the domain decomposition methods were referred to as overlapping grid methods.) All of these topics are coupled to the development faster algorithms for solving the systems of indefinite linear equations that arise from discretizations of the Navier-Stokes equations. Significant results have been obtained in all of these areas.

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Computational & Theoretical Aspects of Nonlinear Partial Differential Equations

Final Report

John C. Strikwerda
Bradley J. Plohr

April 8, 1991

U.S. Army Research Office

Grant number DAAL03-87-K-0028

University of Wisconsin - Madison

Computational & Theoretical Aspects of
Nonlinear Partial Differential Equations

Research results are discussed for Prof. Strikwerda. Prof. Plohr is now on the faculty at SUNY Stony Brook, and he has not submitted information on his research.

Statement of Research Objectives.

The research of Prof. Strikwerda discussed in the original proposal centered around three topics in computational fluid dynamics. These were: schemes for the time-dependent Navier-Stokes equations, improved methods for the steady Navier-Stokes equations, and domain decomposition methods. (In the original proposal, the domain decomposition methods were referred to as overlapping grid methods.) All of these topics are coupled to the development faster algorithms for solving the systems of indefinite linear equations that arise from discretizations of the Navier-Stokes equations. Significant results have been obtained in all of these areas, and Prof. Strikwerda has done work in several other areas as well.

Summary of Significant Results.

For the steady Navier-Stokes equations, Prof. Strikwerda and his student, Carl Scarbnick, developed far-field boundary conditions for the Navier-Stokes equations. These boundary conditions provide for a great reduction in the size of the computational domain without sacrificing accuracy. These results are contained in the dissertation of Carl Scarbnick. A separate publication of these results is planned. These boundary conditions were tested by computing the flow past a circular cylinder and the flow past a square. The computation of the flow past a square used domain decomposition to place a cartesian grid near the square and a polar coordinate system was used farther from the square.

For the time-dependent Navier-Stokes equations, a new class of schemes has been developed and tested [9]. These schemes have been shown to be second-order accurate in a variety of test problems, including tests with domain decomposition.

The method of domain decomposition has been developed for both the time-dependent and steady Navier-Stokes equations, [6][9]. The fundamental concepts of domain decomposition are the same for both of these equations. The method used in this work interpolates the velocity from one domain to the boundary of any overlapping domains. Because the pressure is not interpolated, the pressure on each sub-domain is determined only up to an additive constant, and there is no relation between the constants on the different subdomains. This method has the great advantage that there are no global constraints that must be imposed on the interpolation methods or on the boundary data in order to obtain a solution.

Discretizations of both the time-dependent and steady Navier-Stokes equations give rise to indefinite linear or nonlinear systems of difference equations. There have been several improvements in improving the speed at which the these systems can be solved. These linear systems are of the form

$$\begin{pmatrix} A & G \\ D & 0 \end{pmatrix} \begin{pmatrix} w \\ p \end{pmatrix} = \begin{pmatrix} f \\ g \end{pmatrix}$$

where D is approximately the transpose of G and A is positive definite. The first improvement was to use a multigrid method to solve the system [9]. Although this method was a significant improvement over earlier methods it still had the difficulty that the pressure variables, p in the above system, converged too slowly.

A significant recent development has been the development of a new fast method. The method depends on solving the system,

$$DA^{-1}Gp = DA^{-1}f - g$$

for the pressure, p, and then solving for the velocity. w. The equation for w is

$$Aw = -Gp + f.$$

The advantage of this method is that, on appropriate spaces, the operator $Q = DA^{-1}G$ is nonsingular and close to being symmetric and positive definite. Moreover, the eigenvalues of Q are bounded independently of the mesh spacing. Thus, the conjugate gradient method can be used to solve for p, and the number of iterations is independent of the mesh. Moreover, for the steady Stokes equations the number of iterations is rather small, approximately 10 for a square region. The essential idea for this approach was suggested to Prof. Strikwerda by Profs. Wahlbin and Schatz at MSI.

The inversion of the matrix A can be done by several methods since A is usually symmetric and positive definite. Either a preconditioned conjugate gradient method or a multigrid method can be used. The number of iterations to invert A can thus be made to be $O(h^{-1/2})$ or $O(\ln h^{-1})$, where h is a mesh spacing parameter. Since the number of iterations to invert Q is independent of h, the overall work is nearly optimal.

The original method used by Prof. Strikwerda to solve the Navier-Stokes equations was based on successive-over-relaxation and the number of iterations was proportional to $O(h^{-1})$, but also the constant of proportionality is very large. This method required hundreds of iterations for many problems. Thus the new 'Q' method is a very great improvement over other methods.

This work has been done with graduate student Dongho Shin. He has extended the method to the nonlinear Navier-Stokes equations and used it in the context of domain decomposition.

Other Research

In addition to the research mentioned in the grant proposal Prof. Strikwerda has done research on several other topics.

With Prof. Amarnath Mukherjee, of the University of Pennsylvania, formerly a student in the Computer Sciences Department at the University of Wisconsin. Prof. Strikwerda has been working on differential equations to model strategies for congestion control in computer networks [4][11]. The partial differential equation model has helped to analyze local area network congestion control protocols. This work has resulted in two papers.

With Prof. Bruce Wade of the University of Wisconsin-Milwaukee, formerly a student of Prof. Strikwerda and a post-doctoral researcher at MSI, Prof. Strikwerda has worked on extensions of the Kreiss matrix theorem [2][8]. The Kreiss matrix theorem is a significant result in theoretical numerical analysis and has been the subject of much research. The theorem consists of a set of equivalent conditions that a family of matrices must satisfy in order to have all the powers uniformly bounded. Profs. Wade and Strikwerda have added a series of conditions between two of the conditions, the power boundedness condition and the resolvent condition, that serve to clarify the relation between them. A paper describing this work has appeared.

Prof. Strikwerda also was involved with water wave computations to support the theories developed by Profs. Richard Meyer and Jean-Marc Vanden-Broeck [10]. This work has resulted in a paper being submitted to the journal Wave Motion.

Independently, Prof. Strikwerda has obtained a beautiful result in the theory of initial-boundary value problems for finite different schemes [12]. The result concerns estimates of solutions at a point as functions of time. It is shown under what conditions the norm in time of a solution at a point can be estimated by the initial data. This result has an impact on the theory of initial-boundary value problems and these estimates are important in domain decomposition problems. A paper on this result has been submitted to Mathematics of Computation.

Publications

This list is of publications of Prof. Strikwerda for the period April 1987 to February 1991, the period covered by this grant.

1. A Numerical Method for the Incompressible Navier-Stokes Equations in Three-dimensional Cylindrical Geometry, (with Y. Nagel), J. Comp. Phys., 78, (1988) pp.64–78.
2. An Extension of the Kreiss Matrix Theorem, (with B.A. Wade), SIAM J. Numerical Analysis, 25, (1988), pp.1272–1278.
3. Numerical Solution of Forced Convection Heat Transfer in He II, (with A. Kashani and S.W. Van Sciver), Numerical Heat Transfer, Part A, 16, (1989) pp.213–228.
4. Evaluation of Retransmission Strategies in a Local Area Network Environment, (with A. Mukherjee and L.H. Landweber), SIGMETRICS Performance Evaluation Review, 17, (1989), pp.98–107. Proceedings of ACM SIGMETRICS and PERFORMANCE '89, International Conference on Measurement and Modeling of Computer Systems, Berkeley, CA. (1989)
5. Regularity Estimates up to the Boundary for Elliptic Systems of Difference Equations, (with B.A. Wade and K.P. Bube), SIAM J. Numerical Analysis, 27, (1990), pp292–322.
6. A Domain Decomposition Method for Incompressible Viscous Flow, (with C. Scarbnick), Computer Sciences Tech. Report #896, December 1989. To appear in SIAM J. on Scientific and Statistical Comput.
7. Simultaneous Analysis of Flow and Error Control Strategies with Congestion-Dependent Errors, (with A. Mukherjee and L.H. Landweber), Proceedings of ACM SIGMETRICS Conference, 1990, Boulder, Colorado.
8. Cesàro Means and the Kreiss Matrix Theorem, (with B.A. Wade), J. on Lin. Alg. and Appl., 145, (1991) pp.89–106.

9. A New Class of Schemes for the Time-Dependent Stokes Equations, Finite Difference Methods for Elliptic Systems, Proc. Eighth Army Conf. on Appl. Math. and Comp. Sci. (1990)

10. Notes on Wave Attenuation on Beaches, (with R.E. Meyer and J.-M. Vanden-Broeck), submitted to Wave Motion.

11. Analysis of Dynamic Congestion Control Protocols– A Fokker-Planck Approximation (with A. Mukherjee), CS. Tech. Report 1003, February 1991, submitted to IEEE Transactions on Communication.

12. On the Folded Leapfrog Example: Estimates at a point for solutions of finite difference schemes, Computer Sciences Tech. Report #1007, February 1991, submitted to Mathematics of Computation.

Textbook

Finite Difference Schemes and Partial Differential Equations, published by Wadsworth & Brooks/Cole Press, 1989.

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