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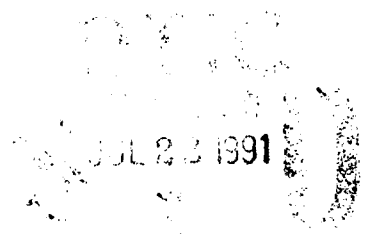
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A MODEL OF A RANGE-ANGLE OF ARRIVAL SENSOR SYSTEM FOR AN ACTIVE PROTECTION SYSTEM

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This report describes a performance model for a sensor system that processes range and angle of arrival information to locate a target. Problems associated with noncolocated sensing elements are discussed. The model provides the covariance structure of a location estimate for a specific geometry.

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1. Introduction

To defeat an incoming projectile, an active armor protection system must be able to estimate the trajectory of the incoming projectile. Many sensor systems have been developed using angle of arrival (AoA) information from two separate receivers. As a rule of thumb the error associated with these systems is a function of range to the target over the separation of the two receivers. As this ratio becomes large, the estimate loses its value. On a tank the maximum separation distance between receivers is less than four meters. An AoA system installed on a tank would begin to lose its effectiveness at ranges in excess of eight meters. Obviously, eight meters does not allow enough reaction time for an active protection system to respond.

H. Bruce Wallace of the Ballistic Research Laboratory (BRL) has proposed that range information also be utilized in order to have a worthwhile target location estimate. There is a large class of potential sensors fitting this description. These can be classified according to the type of electronic processing they use and the parameters that are important for that type of processing. The goal here is to find a general model that allows for various levels of detail in the performance evaluation of this class of sensor systems.

This paper develops a performance model for a system using AoA and range information to estimate the location of a target. The ideas used to derive and validate the performance model are presented, and then a procedure for evaluating specific sensor systems is discussed.

First the two dimensional case is examined in detail. Then the results of applying the same ideas to the three dimensional case are presented. Both models were validated through the use of a simulation. The procedure used to validate the three dimensional model is presented. The next two sections present some electronic models from the literature associated with the accuracy of angle of arrival and range measurements. Finally, a method for the evaluation of a particular system is given.

2. XY Covariance Model

In this section a performance model for a system processing one range measurement and one AoA measurement is presented. The measurements are from a polar coordinate system but the system performance is to be in the Cartesian coordinate system. The focus of the discussion is on how the measurement errors effect target estimation in the Cartesian plane. The relationship between an xy location and an AoA range coordinate is straight forward.

$$x = R \cos \theta + x_o \qquad (1)$$

$$y = R \sin \theta + y_o$$

when R is the range to the target and θ is the AoA to the target. Without any loss of generality, we assume that $x_o = 0$ and $y_o = 0$.

A common approach for relating measurement errors to the xy domain is through the partial derivatives of the location with respect to the measured quantities (Reference 1 and 2). In the x direction the changes caused by range measurement are $\Delta R \cos \theta$. Changes caused by increasing the angle move x closer to the origin by the amount $R \sin (\Delta \theta) \sin(\theta)$. In order to derive the model, several assumptions will be made. Later it will be shown that the assumptions are statistically reasonable.

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Perturbations in x due to measurement errors are described by

$$x + \Delta x = (R + \Delta R) \cos (\theta + \Delta \theta).$$

After expanding the cosine term we have:

$$(R + \Delta R) \cos (\theta + \Delta \theta) = R \cos \theta \cos \Delta \theta - R \sin \theta \sin \Delta \theta +$$

$$\Delta R \cos \theta \cos \Delta \theta - \Delta R \sin \theta \sin \Delta \theta.$$

Since $\Delta \theta$ is small we will assume $\cos \Delta \theta = 1$; also since $\Delta \theta$ is small, we assume $\sin \Delta \theta = \Delta \theta$; and finally we assume $\Delta \theta \, \Delta R = 0$. So we now have:

$$(R + \Delta R) \cos (\theta + \Delta \theta) \approx R \cos \theta + \Delta R \cos \theta - R \Delta \theta \sin \theta.$$

Recalling that $x = R \cos \theta$ we have

$$\Delta x \approx \Delta R \cos \theta - R \, \Delta \theta \sin \theta.$$

By a similar argument it can be shown that

$$\Delta y \approx \Delta R \sin \theta + R \Delta \theta \cos \theta.$$

Note that taking partials of both the x and y values in Equation 1 results in the above set of equations. Thus, the terms we have ignored correspond to higher order differentials. We will assume the measurement errors have the following properties

$$E (\Delta R) = 0$$

$$E (\Delta \theta) = 0$$

$$E (\Delta R^2) = \sigma^2_R \qquad (2)$$

$$E (\Delta \theta^2) = \sigma^2_\theta$$

$$E (\Delta R \, \Delta \theta) = 0.$$

Perturbations of x and y, can be expressed in terms of measured quantities as :

$$\Delta x = \Delta R \cos \theta - \Delta \theta R \sin \theta$$

$$\Delta y = \Delta R \sin \theta + \Delta \theta R \cos \theta$$

$$(\Delta x)^2 = (\Delta R)^2 \cos^2 \theta + (\Delta \theta)^2 R^2 \sin^2 \theta - 2 \Delta R \, \Delta \theta R \sin \theta \cos \theta \qquad (3)$$

$$(\Delta y)^2 = (\Delta R)^2 \sin^2 \theta + (\Delta \theta)^2 R^2 \cos^2 \theta + 2 \Delta R \, \Delta \theta R \sin \theta \cos \theta$$

$$\Delta x \, \Delta y = \Delta R^2 \sin \theta \cos \theta - \Delta \theta^2 R^2 \sin \theta \cos \theta - \Delta R \, \Delta \theta R \sin^2 \theta + \Delta R \, \Delta \theta R \cos^2 \theta .$$

Taking the expectations of Equation 3 and combining this with Equations 2, we get the following expressions for the variance and covariance of the target location.

$$E (\Delta x) = 0$$

$$E (\Delta y) = 0$$

$$E (\Delta x^2) = \sigma^2_R \cos^2 \theta + \sigma^2_\theta R^2 \sin^2 \theta \qquad (4)$$

$$E (\Delta y^2) = \sigma^2_R \sin^2 \theta + \sigma^2_\theta R^2 \cos^2 \theta$$

$$E (\Delta x \, \Delta y) = .5 \left[\sigma^2_R \sin 2\theta - \sigma^2_\theta R^2 \sin 2\theta \right]$$

These equations describe the error matrix for the system in terms of the xy coordinate system. In matrix notation, the covariance matrix of the estimated position would be

$$\begin{bmatrix} E\,(\Delta x^2) & E\,(\Delta x\,\Delta y) \\ E\,(\Delta x\,\Delta y) & E\,(\Delta y^2) \end{bmatrix} = \begin{bmatrix} \sigma^2_x & \rho\sigma_x\sigma_y \\ \rho\sigma_x\sigma_y & \sigma^2_y \end{bmatrix}$$

By rotating the coordinates through the angle α defined by

$$\tan 2\,\alpha = \frac{2\,E\,(\Delta x\,\Delta y)}{E\,(\Delta x^2) - E\,(\Delta y^2)}$$

we decouple the system and find the major and minor axis. (This is derived in Appendix A).

From Appendix A we have

$$\text{Major Axis} = \frac{1}{2}\left\{ E\,(\Delta x^2) + E\,(\Delta y^2) + ((E\,(\Delta x^2) - E\,(\Delta y)^2)^2 + 4\,E\,(\Delta x\,\Delta y)^2)^{1/2}\right\}$$

$$(5)$$

$$\text{Minor Axis} = \frac{1}{2}\left\{ E\,(\Delta x^2) + E\,(\Delta y^2) - ((E\,(\Delta x^2) - E\,(\Delta y)^2)^2 + 4\,E\,(\Delta x\,\Delta y)^2)^{1/2}\right\}$$

The following equalities were used to rewrite Equation 5.

$$E\,(\Delta x^2) + E\,(\Delta y^2) = \sigma^2_R \sin^2\theta + \sigma^2_\theta R^2 \cos^2\theta + \sigma^2_R \cos^2\theta + \sigma^2_\theta R^2 \sin^2\theta$$

$$= \sigma^2_R (\cos^2\theta + \sin^2\theta) + \sigma^2_\theta R^2 (\cos^2\theta + \sin^2\theta)$$

$$= \sigma^2_R + \sigma^2_\theta R^2$$

$$E\,(\Delta x^2) - E\,(\Delta y^2) = \sigma^2_R (\cos^2\theta - \sin^2\theta) + \sigma^2_\theta R^2 (\sin^2\theta - \cos^2\theta)$$

$$= \sigma^2_R \cos 2\theta - \sigma^2_\theta R^2 \cos 2\theta$$

$$E(\Delta x\,\Delta y) = .5\,(\sigma^2_R \sin 2\theta - \sigma^2_\theta R^2 \sin 2\theta)$$

Thus the major axis and minor axis are defined by :

$$\frac{1}{2}\,((\sigma^2_R + R^2\sigma^2_\theta) \pm ((-\sigma^2_R + \sigma^2_\theta R^2)^2 \cos^2 2\theta + (\sigma^2_R - \sigma^2_\theta R^2)^2 \sin^2 2\theta)^{1/2})$$

$$= \frac{1}{2}\,((\sigma^2_R + R^2\sigma^2_\theta) \pm (\sigma^2_R - R^2\sigma^2_\theta))\,.$$

The final simplification of the above equation results in principal components of σ^2_R and $R^2\sigma^2_\theta$. In the case of uncorrelated measurement errors, the axes describing the covariance structure are directly related to the measurement errors.

For the two dimensional case, performance can be discussed in terms of x and y or in terms of principal components. Although the ideas are the same in three

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dimensions the relationships are more complex and it is difficult to give a purely trigonometric explanation.

3. XYZ Covariance Model

In three dimensions a covariance model can be derived from range, azimuth angle, and elevation measurements. The elevation angle, θ, is measured from the positive z axis and the azimuth angle, ϕ, is measured from the x axis toward the positive y axis. Note that conventional notation is being used and θ has a different meaning in this section than its previous meaning. Using the same simplification arguments as for the XY Covariance Model the following set of measurement error transformation equations is obtained.

$$\Delta x = \Delta R \sin\theta \cos\phi + \Delta\theta R \cos\theta \cos\phi - \Delta\phi R \sin\theta \sin\phi$$

$$\Delta y = \Delta R \sin\theta \sin\phi + \Delta\theta R \cos\theta \sin\phi + \Delta\phi R \sin\theta \cos\phi$$

$$\Delta z = \Delta R \cos\theta - \Delta\theta R\sin\theta$$

From these equations the covariance model can be found, the quantities of interest are as follows.

$$\sigma^2_X = \sigma^2_R \sin^2\theta \cos^2\phi + \sigma^2_\theta R^2 \cos^2\theta \cos^2\phi + \sigma^2_\phi R^2 \sin^2\theta \sin^2\phi$$

$$\sigma^2_Y = \sigma^2_R \sin^2\theta \sin^2\phi + \sigma^2_\theta R^2 \cos^2\theta \sin^2\phi + \sigma^2_\phi R^2 \sin^2\theta \cos^2\phi$$

$$\sigma^2_Z = \sigma^2_R \cos^2\theta + \sigma^2_\theta R^2 \sin^2\theta$$

$$\sigma_{XY} = \sigma^2_R \sin^2\theta \sin\phi \cos\phi + \sigma^2_\theta R^2 \cos^2\theta \sin\phi \cos\phi - \sigma^2_\phi R^2 \sin^2\theta \sin\phi \cos\phi$$

$$\sigma_{XZ} = \sigma^2_R \sin\theta \cos\theta \cos\phi - \sigma^2_\theta R^2 \sin\theta \cos\theta \cos\phi$$

$$\sigma_{YZ} = \sigma^2_R \sin\theta \cos\theta \sin\phi - \sigma^2_\theta R^2 \sin\theta \cos\theta \sin\phi$$

By finding the eigenvalues and eigenvectors associated with this matrix, the principal components and orientation can be found.

4. Noncolocated Sensors

The detection elements of a sensor system are not usually located at the same position. When the target is far away the errors associated with assuming colocation are negligible. For an active protection system the distance to the target will become small, hopefully not too small, and problems will result from assuming colocation. In this section, the effects of separating the range and AoA detectors will be discussed. The two topics of this section are an estimation technique and a general method for finding the covariance structure around a target location.

Let (x,y,z) be the position of the range sensor.

Let (0,0,0) be the position of the interferometer.

The distance from the interferometer to the target, K, has the following xyz components

$$(K \sin\theta \cos\phi, \ K \sin\theta \sin\phi, \ K \cos\theta)$$

The sum of the squares in the x, y, and z directions should equal the square of the measured range. We proceed by finding K in terms of R.

$$R^2 = (Ksin\theta\,cos\phi - x)^2 + (Ksin\theta\,sin\phi - y)^2 + (Kcos\theta - z)^2$$

$$= K^2sin^2\theta\,cos^2\phi - 2Ksin\theta\,cos\phi\,x + x^2$$

$$+ K^2sin^2\theta\,sin^2\phi - 2Ksin\theta\,sin\phi\,y + y^2$$

$$+ K^2cos^2\theta - 2Kcos\theta\,z + z^2$$

In this case the coefficient for K^2 is one. K is found by using the quadratic formula.

$$K = \frac{-b \pm (b^2 - 4c)^{.5}}{2}$$

where $c = x^2 + y^2 + z^2 - R^2$
and $b = -2zcos\theta - 2xsin\theta\,cos\phi - 2ysin\theta\,sin\phi$.

The next goal is to find a general method for determining the covariance structure around a target location. The measurement error structure can be determined a priori from performance knowledge of the particular sensors. The typical case is uncorrelated measurements between the sensors; and leads to a diagonal matrix describing the error structure in the measurement space. The problem is to find a representation of this error structure in a different reference system. The associated theory is discussed by Dempster (3). The procedure is to find representations of the reference basis in terms of the measurement error basis. The matrix P is formed by concatenating these column vectors. A vector in the reference frame, R, has the representation PR in the measurement frame. To find the covariance structure in the reference frame consider any vectors, R_1 and R_2, from this space. To find the correlation between the two vectors, the inner product is taken; however, the inner product is defined in the measurement space by the measurement covariance matrix, Σ. The following equation illustrates the process.

$$<R_1, R_2> = <PR_1, PR_2>$$

$$= (PR_1)^t\Sigma PR_2$$

$$= R^t_1 P^t\Sigma PR_2$$

From this, the covariance in the reference space can be seen to be $P^t\Sigma P$. Unfortunately, in the typical case it is not possible to find P directly. The matrix Q ($=P^{-1}$) can be found by finding the representations of the measurement basis in terms of the reference basis, and concatenating these as columns. By finding the inverse of Q, the desired matrix P is found. Notice if Q is ill conditioned, P will include some large values. If P is an orthonormal set, then the process is a rotation and P^t is unified with $P^{-1} = Q$.

The process will be illustrated for the previously discussed three dimensional case. The errors associated with each measurement will be described in sequence. Errors in the azimuth measurement, ϕ, result in perturbations in the xy plane

perpendicular to the line defined by ϕ. The normalized vector in this direction is $(-\cos\phi, \sin\phi, 0)$. The vector associated with the elevation measurement, θ, is perpendicular to the ray going through the target and contained in the plane defined by the xy direction to the target with z unrestricted. Recall that θ is measured from the positive z axis toward the xy plane. The normalized vector is $(-\cos\theta\sin\phi, -\cos\theta\cos\phi, \sin\theta)$. The direction of the range measurement error is along the ray connecting the range sensor with the target and is $(\sin\theta\sin\phi, \sin\theta\cos\phi, \cos\theta)$. It is straightforward to verify that these form an orthonormal set. Concatenating these vectors as columns forms the matrix Q, or in this case P^t. The magnitudes of the measurement errors need to be placed on the diagonal of a matrix. The variance of the error associated with ϕ is $\sigma^2_{\phi}(R\sin\theta)^2$, where R is the distance to the target. The range error's magnitude is σ^2_R. Finally, the variance associated with errors in measuring θ is $(R\sigma_{\theta})^2$. It can be verified that this gives the same result as Section 3 when the matrices $P^t \Sigma P$ are multiplied. This approach is general in that it can be used for any nondegenerate situation.

The case where the range sensor is not colocated with the interferometers will be discussed next. The interferometer measurements are the same as discussed above. The range measurement direction is found by normalizing the ray between the range sensor and the target. If D is the distance to the target and (x_R, y_R, z_R) are the respective x, y, and z distances, then the normalized vector is found by dividing each of these quantities by D. Note in this case the set of normalized vectors will rarely form an orthonormal set and thus Q^{-1} must be calculated to find P. It is worth noting that as the target gets closer to the sensors, the projection of the range measurement into the space defined by the two angle measurements will increase. This increase in collinearity becomes pronounced as the distance to the target approaches the separation distance of the sensors.

5. Model Validation Effort

Both the xy and the xyz models were compared with simulation data to verify their performance. Using a Gaussian random number generator, errors for range and azimuth or for range, azimuth, and elevation were generated. These errors were added to the true values and then the estimated position was calculated from the corrupted values. Using ten thousand such points, the covariance of the target position, S, was calculated and then compared to the covariance predicted by the model, Σ. The code designed to perform the simulation is included as Appendix B.

As a first test, the determinants were compared to see if they were in agreement. A test based on the asymptotic distribution of the sample covariance was used for this. The assumption under the null hypothesis is that

$$\sqrt{n}(\frac{\det(S)}{\det(\Sigma)} - 1)$$

is distributed as N(0,2p)

where

        p is the dimension of the matrix
        n is the degrees of freedom.

In this case, a nonrejection of the null hypothesis indicates that global error associated with each covariance structure is similar; however, an acceptance of the model based upon a determinant test is not conclusive. These tests have no sensitivity to the orientation of the covariance structure, and will not detect certain differences in the magnitude of the major components of the matrix. For example, a diagonal matrix with components (1 ,1) has the same determinant as the diagonal matrix with components (.1, 10) or the matrix with row one elements (5, 3) and row two elements (3, 2).

The statistical properties of a covariance structure are described by the Wishart distribution. Dempster(3) discusses the related theory. The use of this distribution in the three dimensional case leads to six independent tests, each one checking a separate component of the covariance matrix. Note that the off-diagonal terms are based on estimates of the diagonal terms and thus have more uncertainty associated with them. The procedure used was as follows:

1. Calculate the error structure from the model.
2. Calculate the error structure from the simulated data.
3. Find the normalizing transformation based on the model.
4. Apply the normalizing transformation to the result of step 2.
5. Test the resulting matrix to see if it is statistically equivalent to the identity matrix (see Dempster(3)).

In each of the cases investigated the model and the simulated data produced covariance structures that were statistically the same.

6. AoA Errors

The two models included in this section are based entirely on thermal noise and should be used as the best case situations for AoA errors. When additional sources of error are modeled it is usually correct to take their root mean square with the error due to thermal noise. Dr. Alexander in (2) gives the following two equations for relating electronic parameters to σ_θ the angle measurement error.

For pulsed AoA processing the thermal noise, σ_{th} , of a phase interferometer is given by

$$\sigma_{th} = \frac{(360/2\pi)C}{2\pi f d \cos\theta 4 (S/N)_{INTG}}.$$

For an amplitude monopulse the thermal noise is

$$\sigma_{th} = \frac{25.4\,\theta_B}{(S/N_{INTG})^{1/2}}.$$

where,
θ is the angle form boresight to the target.
c   is the propagation velocity (M/S).
f   is the RF carrier (Hz) .
d   is the spacing between the receiving antennas.
θ_B is the antenna half power beam width (deg).

S/N_{INTG} is the integrator output of signal to noise ratio.

7. Range Errors

Errors in range depend on the ability to measure the time of arrival of a given pulse. The range error is $\sigma_r = C/2 \, \sigma_t$ where σ_t is the time error. The time error is dependent on pulse type, the following are suggested by Skolnik(4):

For a rectangular pulse

$$\sigma_t \approx \left(\frac{\tau}{4 \, \beta \, E/N_0} \right)^{1/2}$$

where τ is the pulse width

β is the bandwidth

E is the received signal energy

N_0 is the noise per unit bandwidth.

For a trapezoidal pulse

$$\sigma_t = \left(\frac{T_2^2 + 3 \, T_1 \, T_2}{6 \, E/N_0} \right)^{1/2}$$

T_1 is the time duration of the pulse

T_2 is the rise and fall time of the top of the pulse.

For a triangular pulse

$$\sigma_t = \frac{2 \, T_2}{\sqrt{12} \, (2 \, E/N_0)^{1/2}}.$$

For a Gaussian pulse of the form $s\,(t) = \exp\left(-\frac{1.384 t^2}{\tau^2}\right)$

$$\sigma_t = \frac{1.18}{\pi \, \beta \, (2 \, E/N_0)^{1/2}}.$$

If the pulse has the form $\dfrac{\sin(\pi \, \beta \, \tau)}{\pi \, \beta \, \tau}$

then

$$\sigma_t = \frac{\sqrt{3}}{\pi \, \beta \, (2 \, E/N_0)^{1/2}}.$$

For Continuous Wave, the error is

$$\sigma_R = \frac{C}{4 \, \pi \, \Delta f \, (2 \, E/N_0)^{1/2}}$$

where Δf is the difference between the two frequencies.

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The specific electronic model will vary depending on the method used to extract information from the signal. The models included above only represent some of the common techniques. The important thing is that they relate specific electronic or geometric parameters to the measurement errors associated with either range or angle of arrival.

8. Analysis Method

In using hese models to analyze the performance of a system, the following steps must be taken.

1.   Choose a AoA error model and assign values to the parameters.

2.   Choose a range error model and assign values to the parameters.

3.   Use the range and AoA errors as input to the xyz location model.

By following this procedure at a number of different points a system's performance can be presented as a function of target location.

9. Conclusion

This application is typical of the error analysis approach used in many engineering studies. In this case , the extra step of checking the statistical validity of the model was included. The dominant feature of this approach to system analysis is to start with the measurement errors and follow them as they propagate through the system and degrade the ideal system performance.

The models presented herein can be used to evaluate the performance of many range - angle sensor systems. The performance can be based on specific electronic parameters such as frequency, pulse shape, or on more general specifications such as three degree angle with five percent range errors. This work could be continued by designing a software package that includes the selection of the possible options.

This model may be used to simulate a stream of observations or to provide information about the error associated with an observation. The performance of filtering techniques is enhanced by knowledge of error associated with given observations. In evaluating candidate systems for an active armor protection system this model, covers systems processing range and AoA information.

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Appendix A
Principle Components for Two Dimensions

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APPENDIX A

We wish to find the angle α of rotation in the coordinate system to decouple the system.

$$\Delta x' = \Delta x \cos \alpha + \Delta y \sin \alpha$$

$$\Delta y' = -\Delta x \sin \alpha + \Delta y \cos \alpha$$

In terms of the new coordinate system we have

$$E(\Delta x') = 0$$

$$E(\Delta y') = 0$$

$$E(\Delta x'^2) = E(\Delta x^2) \cos^2 \alpha + 2 E(\Delta x)(\Delta y) \sin \alpha \cos \alpha + E(\Delta y^2) \sin^2 \alpha$$

$$= E(\Delta x^2) + E(\Delta y^2) - E(\Delta x^2) \sin^2 \alpha - E(\Delta y^2) \cos^2 \alpha$$

$$+ 2 E(\Delta x \, \Delta y) \sin \alpha \cos \alpha .$$

Recall that

$$\sin \alpha \cos \alpha = \frac{1}{2} \sin 2\alpha$$

$$\sin^2 \alpha = \frac{1 - \cos 2\alpha}{2}$$

$$\cos^2 \alpha = \frac{1 + \cos 2\alpha}{2}$$

Then

$$E(\Delta x'^2) = \frac{1}{2}(E(\Delta x^2) + E(\Delta y^2)) + \frac{1}{2}(E(\Delta x^2) - E(\Delta y^2)) \cos 2\alpha$$

$$+ E(\Delta x \, \Delta y) \sin 2\alpha .$$

Similarly it can be shown that

$$E(\Delta y'^2) = \frac{1}{2}(E(\Delta x^2) + E(\Delta y^2)) - \frac{1}{2}(E(\Delta x^2) - E(\Delta y^2)) \cos 2\alpha$$

$$- E(\Delta x \, \Delta y) \sin 2\alpha$$

and

$$E(\Delta x' \, \Delta y') = -E(\Delta x^2) \sin \alpha \cos \alpha + E(\Delta x \, \Delta y)(\cos^2 \alpha - \sin^2 \alpha)$$

$$+ E(\Delta y^2) \sin \alpha \cos \alpha$$

$$= \frac{1}{2}\left[E(\Delta y^2) - E(\Delta x^2)\right] \sin 2\alpha + E(\Delta x \, \Delta y) \cos 2\alpha .$$

The covariance term will be zero if

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$$\tan 2\alpha = \frac{2\,E\,(\Delta x\,\Delta y)}{E\,(\Delta x^2) - E\,(\Delta y^2)}\ .$$

Note that for this angle

$$\sin 2\alpha = \frac{2\,E\,(\Delta x\,\Delta y)}{(4\,E\,(\Delta x\,\Delta y)^2 + (E\,(\Delta x^2) - E\,(\Delta y^2))^2)^{1/2}}$$

$$\cos 2\alpha = \frac{E\,(\Delta x^2) - E\,(\Delta y^2)}{(4\,E\,(\Delta x\,\Delta y)^2 + (E\,(\Delta x^2) - E(\Delta y^2))^2)^{1/2}}\ .$$

Using these relations it can be shown that the major and minor axis are defined by

$$1/2\,(E\,(\Delta x^2) + E\,(\Delta y^2) \pm ((E\,(\Delta x^2) - E(\Delta y^2))^2 + 4\,E\,(\Delta x\,\Delta y)^2)^{1/2})\ .$$

Appendix B
Validation Program

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APPENDIX B

```c
#include <math.h>
#include "ranvar.h"
#include "stat.h"

main()
{
float range,r,r_sd,r_var,r_sq;
float theta,th,theta_sd,theta_var;
float psi,p,psi_sd,psi_var;
float x,y,z,x_sum,y_sum,z_sum,x_var,y_var,z_var;
float xsq,ysq,zsq,xy,xz,yz;
float r_sim,th_sim,psi_sim;
float sinth,costh,sinpsi,cospsi,sin2th,cos2psi,cos2th,sin2psi;
float cov_xy,cov_xz,cov_yz;
float x2_mod,y2_mod,z2_mod,xy_mod,xz_mod,yz_mod;
float data_det,model_det,minus,plus,z_test,prob;

int i,n,seed;

/* initialize the variables for this run */
range=50;
r_sd=.05*range;        /* five percent range */
r_var=r_sd*r_sd;
r_sq=range*range;
theta=M_PI/4;
theta_sd=5*2*M_PI/360;   /*five degree error elevation */
theta_var=theta_sd*theta_sd;
psi=M_PI/4;
psi_sd=5*2*M_PI/360;   /* five degree error azimuth */
psi_var=psi_sd*psi_sd;

n=10000;            /* number of replications */
seed=23719;          /* random number seed */

xsq=0;
ysq=0;
zsq=0;
x_sum=0;
y_sum=0;
z_sum=0;
xy=0;
xz=0;
yz=0;

for (i=0;i<n;i++)
```

```
{
/* rv_gauss is a gaussian random number generator
   the first two sections find the measured value
   and calculate the position

   cnf is the cumulative normal distribution function
   and is accurate to six decimal places
*/
th = theta+theta_sd*rv_gauss(seed);
r=range+r_sd*rv_gauss(seed);
p=psi+psi_sd*rv_gauss(seed);

z=r*cos(th);
x=r*sin(th)*cos(p);
y=r*sin(th)*sin(p);

x_sum+ =x;
y_sum+ =y;
z_sum+ =z;

xsq+ =x*x;
ysq+ =y*y;
zsq+ =z*z;
xy+ =x*y;
xz+ =x*z;
yz+ =y*z;
}  /* end of replication loop */
x=x_sum/n;
y=y_sum/n;
z=z_sum/n;

/* the following values are the covariance elements based on
   the simulations data
*/
x_var=(xsq-x*x_sum)/(n-1);
y_var=(ysq-y*y_sum)/(n-1);
z_var=(zsq-z*z_sum)/(n-1);
cov_xy=(xy-x*y_sum)/(n-1);
cov_xz=(xz-x*z_sum)/(n-1);
cov_yz=(yz-y*z_sum)/(n-1);

/* the next section uses the model to find the predicted covariance
   structure
*/
sinth=sin(theta);
sin2th=sinth*sinth;
costh=cos(theta);
```

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```
cos2th = costh*costh;
sinpsi = sin(psi);
sin2psi = sinpsi*sinpsi;
cospsi = cos(psi);
cos2psi = cospsi*cospsi;

x2_mod = r_var*sin2th*cos2psi;
x2_mod + = theta_var*r_sq*cos2th*cos2psi;
x2_mod + = psi_var*r_sq*sin2th*sin2psi;

y2_mod = r_var*sin2th*sin2psi + theta_var*r_sq*cos2th*sin2psi;
y2_mod + = psi_var*r_sq*sin2th*cos2psi;

z2_mod = r_var*cos2th + theta_var*r_sq*sin2th;

xy_mod = r_var*sin2th*cospsi*sinpsi + theta_var*r_sq*cos2th*cospsi*sinpsi;
xy_mod- = psi_var*r_sq*sin2th*cospsi*sinpsi;

xz_mod = r_var*sinth*costh*cospsi-theta_var*r_sq*sinth*costh*cospsi;
yz_mod = r_var*sinth*costh*sinpsi-theta_var*r_sq*sinth*costh*sinpsi;

/*perform determinate test
*/
plus = x_var*y_var*z_var;
plus + = cov_xy*cov_yz*cov_xz*2;
minus = cov_xz*cov_xz*y_var;
minus + = cov_xy*cov_xy*z_var;
minus + = cov_yz*cov_yz*x_var;
data_det = plus-minus;

plus = x2_mod*y2_mod*z2_mod;
plus + = 2*xy_mod*xz_mod*yz_mod;
minus = xy_mod*xy_mod*z2_mod;
minus + = y2_mod*xz_mod*xz_mod;
minus + = x2_mod*yz_mod*yz_mod;
model_det = plus-minus;

z_test = sqrt(n-1) * (data_det/model_det - 1);
z_test / = sqrt(2*3);
prob = cnf(z_test);

printf("0);
printf("***************_____*****************");
printf(" Range : %f   theta : %f   psi : %f",range, theta,psi);
printf("0d(range) = %f sd(theta) = %f sd(psi) = %f",r_sd,theta_sd,psi_sd);
printf("0imulated Var(X) = %f model value was %f",x_var,x2_mod);
printf("0imulated Var(Y) = %f model value was %f",y_var,y2_mod);
```

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```
printf("0imulated Var(Z) = %f model value was %f",z_var,z2_mod);
printf("0imulated Cov(XY) = %f model value was %f",cov_xy,xy_mod);
printf("0imulated Cov(XZ) = %f model value was %f",cov_xz,xz_mod);
printf("0imulated Cov(YZ) = %f model value was %f",cov_yz,yz_mod);
printf("0 value of %f with probability of %f",z_test,prob);
}
```

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