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The proposed research is divided into two phases. The first introduces the PUTD (Pseudo-Uptriangular-Decomposition) to reduce the governing equations of motion of articulated mechanical systems. This investigation proposes a new method, which allows the constrained systems to operate in the presence of singularities. This is achieved by a regularization technique which makes use of a new representation of the kinematical and geometrical constraint equations at singular positions. This method of stability analysis is compared with the asymptotic stability presented by Baumgarte. The PUTD is extended to accommodate the dynamics of such systems. An illustration of the utility and effectiveness of the method proposed is shown through a two arm planar robot undergoing large motions and driven through singularities. The driving torques are then compared to check for discontinuities and jerks.
The second phase of the research project set the stage for the testing of the proposed method when the articulated structures are composed of flexible bodies.

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SUMMARY

The proposed research is divided into two phases. The first intorduces the PUTD (Pseudo-Uptriangular-Decomposition) to reduce the governing equations of motion of articulated mechanical systems. Performance of such systems is based greatly on the assumptions and models used to generate the proper control algorithms. This investigation proposes a new method, which allows the constrained systems to operate in the presence of singularities. This is achieved by a regularization technique which makes use of a new representation of the kinematical and geometrical constraint equations at singular positions. This method of stability analysis is compared with the asymtotic stability presented by Baumgarte. The PUTD is extended to accommodate the dynamics of such systems. An illustration of the utility and effectiveness of the method proposed is shown through a two arm planar robot undergoing large motions and driven through singularities. The driving torques are then compared to check for discontinuities and jerks.

The results show clearly that without the regularization and stability of the dynamics of the system through the proposed method, large peaks for driving forces and discontinuities of the velocities and accelerations are attained. The latter could hamper seriously the performance and the mission of the system.

The second phase of the research project set the stage for the testing of the proposed method when the articulated structures are composed of flexible bodies. The complete matrix formulation of the equations of motion is presented based on the finite element, modal analysis, Kane's equations and the PUTD method. Exploitation of the pipelining feature of the IBM 3090's vector processor by implementing the code on this machine and subjecting it to suitable vectorization in the computationally intensive areas. This cuts down the CPU time drastically needed for the dynamic simulation of multibody dynamical systems.

2. Introduction

The handling of the constraints in multibody dynamics could be done in two different ways. First by introducing the so-called Lagrange undetermined multipliers then the dynamics of the system is found by solving the equations of motion together with the constraint equations which are expressed at the acceleration level. This approach leads to solving more equations than needed, hence it is computationally expensive. The second alternative is to reduce the governing equations by eliminating the undetermined multipliers through pre-multiplication by a matrix, orthogonal complement to the constraint Jacobian matrix. This approach is best suited for the case when the constraint forces are no object in the analysis.

At special configurations, the constraint Jacobian matrix may become less than full rank, hence serious difficulties arise in extracting the orthogonal complement array and inverting the generalized mass matrix. Therefore at such instances, the constraint equations need to be modified in such a way to avoid singularities in the numerical method of solution. We propose a new representation of the constraints when this situation occur. These modified equations should be valid at the neighborhood of the special configuration where the Jacobian matrix changes its rank, and therefore they are useful only if the system is in motion when the special configuration occurs. In addition, to avoid the accumulation of the numerical errors in integration, Baumgarte's method [1] of numerical stabilization is also included.

The systematic reduction of the equations of motion could be achieved through several approaches [1-6]. Those methods while their computational schemes differ, their objective is to extract the orthogonal complement array to the Jacobian constraint matrix. In this paper we propose to address the issue of numerical stability resulting from the constraints when they become less than full rank. The intention of this research is to highlight and further demonstrate the utility of the method proposed through a comprehensive analysis of a planar robot in motion passing through a singular position.

This report is divided into several sections. The first is summary. The second section is the Introduction. The development of the equations of motion for constrained mechanical systems and Baumgarte's stability method form the subsections one and two of section 3.

In section four we will present a new method for the stabilization problem when some rows of the constraint Jacobian matrix vanish at the neighborhood of singular position, and when some of them become linearly dependent. An illustrative example is given in section 4. The stability of the flexible systems is discussed in section 5. Section 6 forms the conclusions and the future directions.

3. Theoritical Development

3.1 Equations Formulations for Constrained Systems

The governing equations of motion of a multibody system could be obtained using Kane's equations

$$f_l + f_l^* = 0 \qquad l = 1, 2, \ldots, 6N \tag{1}$$

where f_l is the generalized active force array and is given by

$$f_l = V_k F_k - \omega_k T_k \tag{2}$$

whereas f_l^* defines the generalized inertia force array

$$f_l^* = V_k F_k^* - \omega_k T_k^* \tag{3}$$

in both equations (2) and (3) V_k and ω_k are the corresponding partial velocity (see Amirouche and Jyia, 1987) associated with the mass center velocity and angular velocity of B_k in a reference frame R. In equation (2) F_k and T_k are the $\bar{n}_{om}$ components of the vector force $\bar{F}_k$ and the torque $\bar{T}_k$. (Note that a force acting on a body B_k could be replaced by a force and moment acting at the mass center of B_k)

In equation (3), however, the F_k^* and T_k^* are the $\bar{n}_{om}$ components of the inertia forces $\bar{F}_k^*$ and $\bar{T}_k^*$, where

$$\bar{F}_k^* = -m_k \bar{a}_k \tag{4}$$

and

$$\bar{T}_k^* = -\bar{I}_k \cdot \bar{\alpha}_k - \bar{\omega}_k \times (\bar{I}_k \cdot \bar{\omega}_k) \tag{5}$$

In equation (4) m_k denotes the mass of body B_k, and $\bar{a}_k$ is its mass center acceleration. $\bar{I}_k$ is the inertia dyadic of B_k relative to its center mass and expressed with respect to the $\bar{n}_{om}$ components in R; $\bar{\omega}_k$ is the angular velocity of B_k and $\bar{\alpha}_k$ its corresponding angular acceleration.

If a multibody system is subject to some constraints which might result from mechanical joints, closed loops or prescribed motions, then equation (1) becomes

$$f_l + f_l^* + B_{li}\lambda_i = 0 \qquad l = 1, 2, \ldots, 6N \tag{6}$$

where λ_i is the so-called Lagrange multipliers and B_{li} is the transpose of the constraint Jacobian matrix (see Huston and Wang,1986). Equation (6) is simply

$$f_l + f_l^* + f_l^c = 0 \qquad l = 1, 2, \ldots, 6N \tag{7}$$

where f_l^c are the generalized constraint forces.

The constraint equations could be expressed at the velocity level as

$$By = G \tag{7.a}$$

where G is a function of time, y defines the generalized speeds. Further differentiation of eq(7.a) yields

$$B\dot{y} + \dot{B}y = \dot{G} \tag{7.b}$$

In the dynamic simulation of multibody systems, equation (7.b) is most useful since it forms a set of m differential equations which can be combined with equation (6). To bring the equations of motion to a minimum dimension, we premultiply equation (6) by the orthogonal complement to matrix **B**. This results in an n-m equations, which together with the m equations given by equation (7.b) yields the governing equations of motion.

Orthogonal Complement Matrix. If there are m constraint equations in a mechanical system which has n generalized coordinates, then the constraint Jacobian matrix B will be an $m \times n$ matrix. Consider the transpose of the constraint matrix denoted as B^T, it is known that an $n \times m$ matrix B^T can be reduced to an upper triangular form, say U, either by Gauss-elimination row operations [12] or by Pseudo- uptriangular decomposition method [2]. B^T and U are row equivalent with same rank and the row equivalence transformation can be expressed as

$$PB^T = U \tag{8}$$

where P is an $n \times n$ nonsingular matrix obtained by applying the same row operations which transform B^T to U.

Let I_1 and I_2 denote matrices composed of respectively, the first m columns, and the last $(n-m)$ columns of the n^{th} order identity matrix, therefore, I_2 is orthogonal to I_1. Since the columns of U are a linear combination of the columns of I_1, I_2 is also orthogonal to U. Then, using equation(8), we obtain

$${I_2}^T U = {I_2}^T P B^T = 0 \tag{9}$$

or

$${I_2}^T P B^T = C B^T = 0 \tag{10}$$

where $C = I_2^T P$ is a $(n-m) \times n$ matrix. Since P is nonsingular, the $n-m$ rows of C are linearly independent. Therefore in R^n dimensional space, the $n-m$ rows of C form a basis of vectors orthogonal to the m columns of the transpose of constraint matrix, i.e. C is a complement matrix to B^T.

Premultipling equation (6) by C and applying equation (10), a reduced form of the equations of motion of a constrained multibody system is obtained as

$$K_p + K_p^* = 0 \tag{11}$$

where

$$K_p = C f_l \qquad K_p^* = C f_l^* \tag{12}$$

in equation (11), we have $n - m$ reduced equations. In order to solve for the dynamics of the system, we must combine the m constraint equations at their acceleration level with those of equation (11) to obtain the time history response of the system. More details are given in next section.

3.2 Baumgarte Stability Method

In order to solve the governing equations, we usually represent the constraints at their acceleration level. For instance, if we have m constraint equations (holonomic), they could be represented by

$$h_i(q_1, q_2, \cdots q_n, t) = 0 \qquad i = 1, 2 \cdots m \tag{13}$$

the q's are the generalized coordinates and t denotes time. Equation (13) is said to be a representation of the constraints at the position level. Further differentiation will lead to its representation at the velocity level, where

$$\dot{h}_i = \sum_{l=1}^{n} \frac{\partial h_i}{\partial q_l}\frac{dq_l}{dt} - \frac{\partial h_i}{\partial t} = \sum_{l=1}^{n} b_{il} y_l - g_i = 0 \tag{14}$$

where b_{il} is the (i,l) element of the $m \times n$ constraint matrix $\mathbf{B}$ defined by

$$\mathbf{B} = [b_{il}] = \begin{bmatrix} \frac{\partial h_1}{\partial q_1} & \frac{\partial h_1}{\partial q_2} & \cdots & \frac{\partial h_1}{\partial q_n} \\ \frac{\partial h_2}{\partial q_1} & \frac{\partial h_2}{\partial q_2} & \cdots & \frac{\partial h_2}{\partial q_n} \\ \vdots & & & \\ \frac{\partial h_m}{\partial q_1} & \frac{\partial h_m}{\partial q_2} & \cdots & \frac{\partial h_m}{\partial q_n} \end{bmatrix} \tag{15}$$

y_l is the l^{th} element of the matrix $\mathbf{Y}$ defined by

$$\mathbf{Y} = \begin{bmatrix} \frac{dq_1}{dt} & \frac{dq_2}{dt} & \cdots & \frac{dq_n}{dt} \end{bmatrix}^T, \qquad y_l = \frac{dq_l}{dt} \tag{16}$$

and g_i is the i^{th} element of the $m \times 1$ matrix $\mathbf{G}$ given by

$$\mathbf{G} = \begin{bmatrix} -\frac{\partial h_1}{\partial t} & -\frac{\partial h_2}{\partial t} & \cdots & -\frac{\partial h_m}{\partial t} \end{bmatrix}^T \tag{17}$$

If we further differentiate equation(14), we can get the acceleration representation of the constraints as

$$\ddot{h}_i = \sum_{l=1}^{n} b_{il}\dot{y}_l + \sum_{l=1}^{n} \dot{b}_{il} y_l - \dot{g}_i = 0 \tag{18}$$

one thing we should keep in mind is that $h_i = 0$ implies $\dot{h}_i = 0$, $\ddot{h}_i = 0$, but $\ddot{h}_i = 0$ (which is what we use in equation (18)) does not necessarily yield $\dot{h}_i = 0$ and $h_i = 0$. Therefore it is important to know that during the integration of the equations of motion we must try to satisfy all constraint conditions. In the event we don't. It is possible to accumulate numerical errors which might cause some serious stability and control problems. To overcome this difficulty, Baumgarte introduced a method to assure 'asymptotic stability' by replacing $\ddot{h}_i = 0$ with

$$\ddot{h}_i + \alpha_i \dot{h}_i + \beta_i h_i = 0 \qquad i = 1, 2, \cdots m \tag{19}$$

where $\beta_i = 0$ for nonholonomic constraints, α_i and β_i arbitrary constants chosen to sufficiently fast decay of the errors (Baumgarte, 1972). For example, let us consider the constraint equations $\ddot{h}_i = 0$ where $h_i = 0$ is a holonomic constraint. In addition, assume that, in the process of the integration of the equations of motion after the n^{th} step, the computer yields the values $h = \delta$ $\dot{h} = \epsilon$ which deviate from the exact values $h = 0$, $\dot{h} = 0$. According to the differential equation $\ddot{h} = 0$, the computer should produce the value $h = \epsilon t - \delta$. Thus the holonomic constraint is not satisfied in a linearly stable fashion. It is unlikely that subsequent numerical errors will compensate for this behavior.

By virtue of the initial conditions $h(0) = 0$, $\dot{h}(0) = 0$, equation(19) leads to $\ddot{h} = 0$ as before, such that the new constraint is analytically equivalent to $\ddot{h} = 0$. But from the numerical point of view, the situation is different. In order to achieve asymptotical stability,

$$\alpha_i > 0 \tag{20}$$

and

$$\alpha_i^2 - 4\beta_i \leq 0 \tag{21}$$

must be satisfied. We usually choose [Baumgarte, 1972]

$$\alpha_i^2 = 4\beta_i \tag{22}$$

Note that from equation (20-21), we have an infinite set of α's and β's which could be used in equation (19).

3.3 Proposed Stability in the Presence of Singularities

a) Regularization of Vanishing Constraints

It is common in the course of a mechanical systems motion that for B_{sl}, $l = 1, \cdots n$, the S^{th} constraint equation coefficients may become all zero at some instantaneous special configurations. In this case, the procedures we mentioned in previous section fail in determining the orthogonal complement array because the rank of B becomes less than m. In addition the augmented system inertial matrix becomes singular.

Since the generalized constraint forces due to the S^{th} constraint, $\lambda_s B_{sl}$ $l = 1, \cdots n$, are zero, the constraint do not have any effect on the system at that configuration and one possible procedure is deleting the S^{th} row of B, finding the corresponding orthogonal complement matrix C', and then proceeding with the normal coordinate reduction technique. As we mentioned before the constraint at the acceleration level could be expressed as

$$\dot{h}_i = B_{il}\dot{y}_l + \dot{B}_{il}y_l - \dot{g}_i = 0 \qquad i = 1, \cdots m \tag{23}$$

where y denotes the generalized speeds. Suppose for the $i = s$, $B_{sl} = 0$, then we can write

$$\dot{h}_s = \dot{B}_{sl}y_l - \dot{g}_s = 0 \tag{24}$$

Equation(24) must be satisfied by selecting the initial conditions for y_l such that

$$\dot{B}_{sl}y_l = \dot{g}_s \tag{25}$$

in order to achieve the consistency of the equations.

When the S^{th} constraint equation is removed in this manner, the resulting equations can be used in the neighborhood of the special configuration. The drawback of this approach is that the assumption of zero constraint forces in the neighborhood results in fast deviation of the simulation from the constrained behavior.

To capture the effects of the actual constraint forces in the neighborhood of the vanishing constraint, we differentiate further the S^{th} constraint equation. This will provide further information on the constraint, hence a modified form of the constraint equation will be used instead of its deletion. To illustrate this approach, consider the time derivative of equation (23), when $i = s$

$$\ddot{h}_s = B_{sl}\ddot{y}_l + 2\dot{B}_{sl}\dot{y}_l + \ddot{B}_{sl}y_l - \ddot{g}_s = 0 \quad l = 1, \ldots n \tag{26}$$

To simplify the representation of equation(26), we express $\dot{B}_{sl}$ as

$$\dot{B}_{sl} = \frac{\partial B_{sl}}{\partial q_h}\dot{q}_h + \frac{\partial B_{sl}}{\partial t} \tag{27}$$

Since

$$\dot{q}_h = T_{hj} y_j \tag{28}$$

where T is a matrix relating $\dot{q}$ to y. In the light of the above, we can write equation(27) as

$$\dot{B}_{sl} = \phi_{slj} y_j + \psi_{sl} \tag{29}$$

where

$$\phi_{slj} = \frac{\partial B_{sl}}{\partial q_h} T_{hj} \tag{30}$$

and

$$\psi_{sl} = \frac{\partial B_{sl}}{\partial t} \tag{31}$$

Similarly, $\dot{g}_s$ can be expressed as

$$\dot{g}_s = \mu_{sl} y_l + \tau_s \tag{32}$$

where

$$\mu_{sl} = \frac{\partial g_s}{\partial q_h} T_{hl}, \qquad \tau_s = \frac{\partial g_s}{\partial t} \tag{33}$$

Further differentiate of equation (29) and (32) gives the $\ddot{B}$ and $\ddot{g}$ terms as

$$\ddot{B}_{sl} = \dot{\phi}_{slj} y_j + \phi_{slj} \dot{y}_j + \dot{\psi}_{sl} \tag{34}$$

$$\ddot{g}_s = \dot{\mu}_{sl} y_l + \mu_{sl} \dot{y}_l + \dot{\tau}_s \tag{35}$$

Substituting equations (29), (34) and (35) into equation (26), we get

$$B_{sl}\ddot{y}_l + G_{sl}\dot{y}_l + J_s = 0 \qquad l = 1, \ldots, n \tag{36}$$

where

$$G_{sl} = 2\dot{B}_{sl} + \phi_{spl}y_p - \mu_{sl} \quad l, p = 1, \ldots, n \tag{37}$$

and

$$J_s = \dot{\phi}_{slp}y_p y_l + \dot{\psi}_{sl}y_l - \mu_{sl}y_l - \dot{r}_s \tag{38}$$

The usefulness of equation (36) stems from the assumption that, when the elements of the constraint B_{sl}, $l = 1, \ldots, n$ are zero at the special configuration and small at its neighborhood, $B_{sl}\ddot{y}_l$ is negligible compared to the other terms in equation(36), so dropping that term in the equation will yield

$$\ddot{h}'_s = G_{sl}\dot{y}_l + J_s = 0 \tag{39}$$

It is seen that G_{sl} and J_s are, in general, functions of q_l, y_l and t. Equation(39) is linear in accelerations $\dot{y}_l$ and has the form of the constraint at the acceleration level. If one is interested in computing the generalized constraint forces at the neighborhood of singularity, then all we have to do is replace in $F^c = \lambda B^T$, the S^{th} constraint by

$$B_{sl} = G_{sl} \tag{40}$$

In most cases, G_{sl} doesn't vanish while B_{sl} does. The governing equations of motion will then be subject to the constraints given by equation (39).

To further assure the numerical stability of the constraints, we can make use of Baumgarte technique where equation (39) takes on the following form

$$G_{sl}\dot{y}_l + J_s + \alpha_s(\dot{B}_{sl}y_l - \dot{g}_s) + \beta_s(B_{sl}y_l - g_s) = 0 \tag{41}$$

which reduces to

$$G_{sl}\dot{y}_l + J_s + \alpha_s(\dot{B}_{sl}y_l - \dot{g}_s) - \beta_s g_s = 0 \tag{42}$$

where

$$G_{sl}\dot{y}_l = -J_s - \alpha_s(\dot{B}_{sl}y_l - \dot{g}_s) + \beta_s g_s \tag{43}$$

and

$$S_s = -J_s - \alpha_s(\dot{B}_{sl}y_l - \dot{g}_s) + \beta_s g_s \tag{44}$$

Note that $h_s = 0$ is automatically satisfied since $B_{sl} = 0$, $l = 1, \ldots, n$.

b) Regularization of Linearly Dependent Constraints

In a similar fashion to the case of vanishing constraints, the linear dependency causes B to be less than full rank. In this case, it is known that an $n \times m$ matrix B^T can be reduced to Gaussian form U^* by Gauss elimination row operations, where the rows $1, \ldots, r$ are the nonzero rows of U^*, and the leftmost nonzero entry of row i occurs in column k_i, $i = 1, \ldots, r$ then $k_1 < k_2 < \cdots < k_r$.

B^T and U^* are row equivalent with same rank r and the row equivalence transformation can be expressed as

$$PB^T = U^* \tag{45}$$

where P is the corresponding $n \times n$ nonsingular matrix.

It is seen from the definition of U^* that the columns with indices k_i, $i = 1, \ldots, r$ are linearly independent columns, whereas the remaining $m - r$ columns whose indices are denoted as d_i, $i = 1, \ldots, m - r$, can be expressed as linear combinations of the former set. The d_i^{th} columns of B^T can also be written as linear combinations of the k_i^{th} columns of B^T. To show this, we can write equation(45) as

$$Pb^j = u^j \qquad j = 1, \ldots, m \tag{46}$$

where b^j and u^j denote the j^{th} columns of B^T and U^*, respectively. For the columns of U^*, we have

$$u^{d_j} = Z_{ji} u^{k_i} \quad i = 1, \ldots, r \quad j = 1, \ldots, m - r \tag{47}$$

where $Z_{ji}, i = 1, \ldots, r$ are the linear combination constants for the d_j^{th} column. Substitution of equation (46) into equation (47) for u^{d_j} and for each u^{k_i} leads to

$$Pb^{d_j} = u^{d_j} = Z_{ji} u^{k_i} = Z_{ji} P b^{k_i} \tag{48}$$

so,

$$b^{d_j} = Z_{ji} b^{k_i} \quad i = 1, \ldots, r \quad j = 1, \ldots, m - r \tag{49}$$

representing equation (49) in a matrix form, we obtain

$$B_{d_j l} - Z_{ji} B_{k_i l} = 0 \tag{50}$$

we can write the constraint equations in these following forms

$$\dot{h}_{d_j} = B_{d_j l} y - g_{d_j} = 0 \tag{51}$$

$$\dot{h}_{k_i} = B_{k_i l} y - g_{k_i} = 0 \tag{52}$$

$$\ddot{h}_{d_j} = \dot{B}_{d_j l} y + B_{d_j l} \dot{y} - \dot{g}_{d_j} = 0 \tag{53}$$

$$\ddot{h}_{k_i} = \dot{B}_{k_i l} y + B_{k_i l} \dot{y} - \dot{g}_{k_i} = 0 \tag{54}$$

we can use equation (54) to generate our constraint equations at the acceleration level, but we can't use equation (53) because $B_{d_j l}$ is linearly dependent to $B_{k_i l}$, as shown in equation (50). Since

$$\dot{h}_{d_j} - Z_{ji}\dot{h}_{k_i} = [B_{d,l} - Z_{ji}B_{k_i}]y - [g_{d_j} - Z_{ji}g_{k_i}] = 0 \tag{55}$$

and using equation (50), equation (55) becomes

$$\dot{h}_{d_j} - Z_{ji}\dot{h}_{k_i} = -[g_{d_j} - Z_{ji}g_{k_i}] = 0 \tag{56}$$

its differentiated form yields

$$\ddot{h}_{d_j} - Z_{ji}\ddot{h}_{k_i} = (B_{d,l} - Z_{ji}B_{k_i})\dot{y} + (\dot{B}_{d,l} - Z_{ji}\dot{B}_{k_i,l})y - (\dot{g}_{d_j} - Z_{ji}\dot{g}_{k_i}) = 0 \tag{57}$$

making use of equation (50) once more, equation (57) reduces to

$$\ddot{h}_{d_j} - Z_{ji}\ddot{h}_{k_i} = (\dot{B}_{d,l} - Z_{ji}\dot{B}_{k_i,l})y - (\dot{g}_{d_j} - Z_{ji}\dot{g}_{k_i}) = 0 \tag{58}$$

Note how the $\dot{y}$ term is not explicit in equation (58). If we further differentiate equation(57), we get

$$h'''_{d_j} - Z_{ji}h'''_{k_i} = (B_{d,l} - Z_{ji}B_{k_i,l})\ddot{y} + 2(\dot{B}_{d,l} - Z_{ji}\dot{B}_{k_i,l})\dot{y} + (\ddot{B}_{d,l} - Z_{ji}\ddot{B}_{k_i,l})y - (\ddot{g}_{d_j} - Z_{ji}\ddot{g}_{k_i}) = 0 \tag{59}$$

equation (59) reduces further to

$$h'''_{d_j} - Z_{ji}h'''_{k_i} = 2(\dot{B}_{d,l} - Z_{ji}\dot{B}_{k_i,l})\dot{y} + (\ddot{B}_{d,l} - Z_{ji}\ddot{B}_{k_i,l})y - (\ddot{g}_{d_j} - Z_{ji}\ddot{g}_{k_i}) = 0 \tag{60}$$

by dropping the term that multiplies $\ddot{y}$ by virtue of equation (50). The representation of the constraint equations given by equation (53) are now represented by equation (60), as

$$\ddot{h}'_{d_j} = 2(\dot{B}_{d,l} - Z_{ji}\dot{B}_{k_i,l})\dot{y} + (\ddot{B}_{d,l} - Z_{ji}\ddot{B}_{k_i,l})y - (\ddot{g}_{d_j} - Z_{ji}\ddot{g}_{k_i}) = 0 \tag{61}$$

equation (61) and equation (54) are linearly independent and form a consistent set of constraint equations to be used when the linear dependency of the constraints occurs. This stability procedure could be further enhanced by applying Baumgarte's method to stabilize the numerical error due to the integration of the equations of motion. Employing Baumgarte technique we further obtain a form of the constraint as

$$\begin{aligned} &2(\dot{B}_{d_j l} - Z_{ji}\dot{B}_{k_i l})\dot{y} + (\ddot{B}_{d_j l} - Z_{ji}\ddot{B}_{k_i l})y - (\ddot{g}_{d_j} - Z_{ji}\ddot{g}_{k_i}) \\ &\quad +\alpha_{d_j}[(\dot{B}_{d_j l} - Z_{ji}\dot{B}_{k_i l})y - (\dot{g}_{d_j} - Z_{ji}\dot{g}_{k_i})] + \beta_{d_j}[-(g_{d_j} - Z_{ji}g_{k_i})] = 0 \end{aligned} \tag{62}$$

writing the above equation in a compact form we get

$$G_{d_j}\dot{y} + S_{d_j} = 0 \tag{63}$$

where

$$G_{d_j} = 2(\dot{B}_{d_j l} - Z_{ji}\dot{B}_{k_i l}) \tag{64}$$

and

$$\begin{aligned} S_{d_j} \;=\; & (\ddot{g}_{d_j} - Z_{ji}\ddot{g}_{k_i}) - y[(\ddot{B}_{d_j l} + \alpha_{d_j}\dot{B}_{d_j l}) - Z_{ji}(\ddot{B}_{k_i l} + \alpha_{d_j}\dot{B}_{k_i l})] \\ & +(\alpha_{d_j}\dot{g}_{d_j} + \beta_{d_j}g_{d_j}) - Z_{ji}(\alpha_{d_j}\dot{g}_{k_i} + \beta_{d_j}g_{k_i}) \end{aligned} \tag{65}$$

4. Procedure Verification and Applications

4.1 Two-Arm Robot with Specified Motion

Consider a two-bar linkage, $L_1 = l$, $L_2 = \frac{l}{2}$, $m_2 = m$, $m_1 = 2m_2 = 2m$, as shown in figure 1. Assume that point p follows the horizontal line $y = \frac{l}{2}$ with a constant velocity along the $-x$ axis. We know that for $q_1 = 0$, $q_2 = \pi$, the system becomes singular. This is a perfect example to check our stability method and further compare it to Baumgarte technique. In the sequel we will show how our stability is completely independent of the asymptotic stability due to the ill representation of the constraints at the acceleration level.

The constraint equations for the problem given are

$$h_1(q_1, q_2, t) = l sin q_1 + \frac{l}{2} sin(q_1 + q_2) - \frac{l}{2} = 0 \tag{66}$$

$$h_2(q_1, q_2, t) = l cos q_1 + \frac{l}{2} cos(q_1 + q_2) + vt - x_0 = 0 \tag{67}$$

where x_0 denotes the initial position of the x coordinate. Their first order differentiated forms yield

$$\dot{h}_1 = lC_1 y_1 + \frac{l}{2} C_{12}(y_1 + y_2) = 0 \tag{68}$$

$$\dot{h}_2 = -lS_1 y_1 - \frac{l}{2} S_{12}(y_1 + y_2) + v = 0 \tag{69}$$

where $S_1 = sin q_1$, $S_{12} = sin(q_1 + q_2)$, $C_1 = cos q_1$, $C_{12} = cos(q_1 + q_2)$, $y_1 = \dot{q}_1$, $y_2 = \dot{q}_2$. Using the above equations and expressing the constraint equations in a matrix form, we obtain

$$\begin{bmatrix} lC_1 + \frac{l}{2}C_{12} & \frac{l}{2}C_{12} \\ -lS_1 - \frac{l}{2}S_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 \\ -v \end{bmatrix} \tag{70}$$

differentiating equations (??)(??) once more, we get the constraints at the acceleration level as

$$\ddot{h}_1 = (lC_1 + \frac{l}{2} C_{12})\dot{y}_1 + \frac{l}{2} C_{12}\dot{y}_2 - (lS_1 y_1^2 + \frac{l}{2} S_{12}(y_1 + y_2)^2) = 0 \tag{71}$$

$$\ddot{h}_2 = (-lS_1 - \frac{l}{2} S_{12})\dot{y}_1 - \frac{l}{2} S_{12}\dot{y}_2 - (lC_1 y_1^2 + \frac{l}{2} C_{12}(y_1 + y_2)^2) = 0 \tag{72}$$

which can be written in matrix form as

$$\begin{bmatrix} lC_1 + \frac{l}{2}C_{12} & \frac{l}{2}C_{12} \\ -lS_1 - \frac{l}{2}S_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} lS_1 y_1^2 + \frac{l}{2}S_{12}(y_1 + y_2)^2 \\ lC_1 y_1^2 + \frac{l}{2}C_{12}(y_1 + y_2)^2 \end{bmatrix} \tag{73}$$

introducing Baumgarte method, equation (73) takes on the following form

$$\begin{bmatrix} lC_1 + \frac{l}{2}C_{12} & \frac{l}{2}C_{12} \\ -lS_1 - \frac{l}{2}S_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} =$$

$$\begin{bmatrix} lS_1 y_1^2 + \frac{l}{2}S_{12}(y_1 + y_2)^2 - \alpha[lC_1 y_1 + \frac{l}{2}C_{12}(y_1 + y_2)] - \beta(lS_1 + \frac{l}{2}S_{12} - \frac{l}{2}) \\ lC_1 y_1^2 + \frac{l}{2}C_{12}(y_1 + y_2)^2 - \alpha[-lS_1 y_1 - \frac{l}{2}S_{12}(y_1 + y_2) + v] - \beta(lC_1 + \frac{l}{2}C_{12} + vt - x_0) \end{bmatrix} \tag{74}$$

From equation (70,73 or 74) the Jacobian constraint matrix is seen to be

$$\mathbf{B} = \begin{bmatrix} lC_1 + \frac{l}{2}C_{12} & \frac{l}{2}C_{12} \\ -lS_1 - \frac{l}{2}S_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \tag{75}$$

From Kane's equation, i.e. $F^* + F + B^T\lambda = 0$, we write the equations of motion for the system as

$$\begin{bmatrix} \frac{3}{2}ml^2 + I_1 + I_2 & \frac{1}{4}ml^2C_2 + I_2 \\ \frac{1}{4}ml^2C_2 + I_2 & \frac{1}{16}ml^2 + I_2 \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} + \begin{bmatrix} -\frac{1}{4}ml^2 y_2(y_1 + y_2)S_2 - 2mglS_1 \\ \frac{1}{4}ml^2 y_1^2 S_2 + \frac{1}{4}mglS_{12} \end{bmatrix}$$

$$= \begin{bmatrix} lC_1 + \frac{l}{2}C_{12} & -lS_1 - \frac{l}{2}S_{12} \\ \frac{l}{2}C_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \tag{76}$$

The governing equations of motion for the two bar system are given by equation (73 and 76) for the case when Baumgarte method is not used and by equation (74 and 76) with Baumgarte representation of the constraints.

We can easily deduce, from equation (73 and 74), when singularity occurs, i.e. $q_1 = \frac{\pi}{2}$, $q_2 = \pi$, the first row of $\mathbf{B}$ vanishes. Employing the method proposed, the new representation of the constraints is given below.

$$\begin{bmatrix} -3lS_1 y_1 - \frac{3l}{2}S_{12}(y_1 + y_2) & -\frac{3l}{2}S_{12}(y_1 + y_2) \\ -lS_1 - \frac{l}{2}S_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} lC_1 y_1^3 + \frac{l}{2}C_{12}(y_1 - y_2)^3 \\ lC_1 y_1^2 + \frac{l}{2}C_{12}(y_1 + y_2)^2 \end{bmatrix} \tag{77}$$

Introducing Baumgarte's method, equation (77) becomes

$$\begin{bmatrix} -3lS_1y_1 - \frac{3l}{2}S_{12}(y_1+y_2) & -\frac{3l}{2}S_{12}(y_1+y_2) \\ -lS_1 - \frac{l}{2}S_{12} & -\frac{l}{2}S_{12} \end{bmatrix} \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} =$$

$$\begin{bmatrix} lC_1y_1^3 + \frac{l}{2}C_{12}(y_1+y_2)^3 - \alpha[lC_1y_1 + \frac{l}{2}C_{12}(y_1+y_2)] - \beta(lS_1 + \frac{l}{2}S_{12} - \frac{l}{2}) \\ lC_1y_1^2 + \frac{l}{2}C_{12}(y_1+y_2)^2 - \alpha[--lS_1y_1 - \frac{l}{2}S_{12}(y_1+y_2) + v] - \beta(lC_1 + \frac{l}{2}C_{12} + vt - x_0) \end{bmatrix} \tag{78}$$

Assume the two-bar links are slender rods, and

$$m_1 = 2\ kg, \quad m_2 = 1\ kg, \quad l = 1\ m,$$

$$I_1 = \frac{1}{12}m_1l^2 = 0.167\ kg - m^2 \quad I_2 = \frac{1}{12}m_2(\frac{l}{2})^2 = 0.0208\ kg - m^2$$

the initial angular positions for link 1 and link 2 are, respectively, 1.047198 rad and 4.41466 rad, while the initial angular velocities are given to be 1.4252 rad/s and -3.51724 rad/s.

As we approach singularity we should experience the necessary jumps if one proceed with the standard technique where the constraints are not eliminated. To test this problem we have run a simulation and found some very interesting results. The cases are labelled as with and without the modified **B**. The first one being the **B** at hand and its corresponding constraint forces and the second one corresponds to the proposed stability method with the modification of **B**. In addition, we have run our simulation for the previous two cases with and without Baumgarte technique to further see whether there were any gains in the stability procedure. Figure 2 and figure 3. display the constraint forces at the neighborhood of singularity. It is very clear that the stability method proposed (solid line) assures smoothness of motion. The constraint forces do not experience any jumps as is the case when **B** is kept unchanged.

This is further illustrated by figure 4. where the jumps in velocity is seen to yield large peaks. The latter could hamper the system performance. In figure 5-6-7 the same simulation is repeated introducing Baumgarte technique. We can easily conclude that at singular position Baumgarte technique doesn't provide any stability. This is quite excepted since the **B** matrix if kept unchanged the vanishing of the constraint will cause the jumps we are seeing. The results also show that the stability provided by the method in this paper is not altered when adding Baumgarte technique to it. We believe that the combination of Baumgarte technique and the proposed method herein could serve as a unique feature in conducting simulations of articulated mechanical systems.

4.2 Discussion

In this investigation we introduced a stability method needed to regulate the motion when we operate in the presence of singularity. The constraint forces (driving forces) are shown that they can be kept smooth and continuous by using the proposed technique in handling the constraints at singular positions. Detecting and modifying their corresponding constraint equations could be done during the process of the evaluation of the orthogonal complement array to **B**. Baumgarte technique is seen that it can be extended to incorporate stability method proposed to further provide a more robust control forces in the dynamics of constrained multibody systems. We envisage some important finding if this method is applied to flexible multibody systems.

5. Stability and Control of Large Scale Flexible Articulated Systems

5.1 Equations of motion based on recursive formulation

The inter-connection between two flexible bodies B_k and B_j is shown in Figure 8. N^k and N^j are the floating reference frames for B_k and B_j located outside the bodies, with respect to which the associated elastic deformations are evaluated. These are transformed to the inertial reference frame R, through the local reference frames n^k and n^j for the two bodies located at Q_k and Q_j. The rigid body rotations/translations of the bodies are with reference to n^{k^*}. The bodies could be discretized using suitable finite elements, based on the geometric configuration, each element i having its local reference frame n^{k^i}. Further details of the relative and floating reference frames and the the different associated vectors has been explained in the reference [11]. Concepts of the indices of reference arrays have been incorporated in the kinematical equations, which were shown to improve the computational efficiency.

The mathematicl equations describing the vectors q and d, which describe the position of a particular point on the body and the body vector respectively, and the angular velocity $\bar{\Omega}$, could be written in terms of the shape function matrices denoted as $\mathcal{N}$ and $\mathcal{M}$ (for the linear and rotational elastic deformation), which includes the necessary transformations as :

$$q^{ki} = \mathcal{N}^{ki}\bar{u}_e^{ki} + \mathcal{N}^k\bar{u}_e^k \tag{79}$$

$$d^k = \mathcal{N}^{k\bullet}\bar{u}_e^{k\bullet} - \mathcal{N}^j\bar{u}_e^j \tag{80}$$

$$^{n^j}\bar{\Omega}^{n^{k\bullet}} = \mathcal{M}^{k\bullet}\dot{\bar{u}}_e^j - \mathcal{M}^j\dot{\bar{u}}_e^j \tag{81}$$

$$^{n^k}\bar{\Omega}^{N^k} = -\mathcal{M}^k\dot{\bar{u}}_e^k \tag{82}$$

where $\bar{u}_e$ represents the identified and extracted vector, containing the exact kinematical quantities associated either with the element or the flexible body.

The generalized speeds $\mathcal{Y}$ are written as :

$$\mathcal{Y} = \left[\ \Omega^T, \dot{\Upsilon}^T, \dot{\eta}^T \ \right]^T \tag{83}$$

where Ω, $\dot{\gamma}$, $\dot{\eta}$ are the vectors of the components of the angular velocity of the bodies in the system, derivatives of the components of translational velocities and the derivatives of the components of modal co-ordinates of the flexible bodies in the system, respectively. The relationship between the derivatives of Euler angles $\dot{\theta}_i$ or the Euler parameters $\dot{\Theta}_i$ are given by :

$$\dot{\theta}_i^k = \bar{D}\Omega_i^k \tag{84}$$

$$\dot{\Theta}_i^k = \bar{E}\Omega_i^k \tag{85}$$

where $\bar{D}$ and $\bar{E}$ are the related transformation matrices.

The use of component mode sysnthesis lies in expressing rotational elastic deformation $\bar{\varphi}$ and translational elastic deformation $\bar{\tau}$, in terms of nodal-modal-tranformation matrix χ

and the modal co-ordinate vectors η, associated with the selected mode shapes as :

$$\bar{\varphi}^{ki} = \mathcal{M}^{ki} \bar{\chi}_e^{ki} \eta_e^{ki} \tag{86}$$

$$\bar{\tau}^{ki} = \mathcal{N}^{ki} \bar{\chi}_e^{ki} \eta_e^{ki} \tag{87}$$

where the subscript e refers to the index of reference array $edof$ [12].

A more detailed explanation of the kinematical matrix representation could be seen in the reference [12]. Let us now discuss in brief, the kinematical equations involved herein.

The partial velocity vectors $\bar{\varsigma}$ and $\bar{\xi}$ associated with the rigid and flexible motions respectively, could be utilized to express the angular velocities of any body k of the multi-body system in R as :

$$\bar{\Omega}^k = \bar{\varsigma}^k \Omega + \bar{\xi}^k \dot{\eta} \tag{88}$$

$$\bar{\Omega}^{k\bullet} = \bar{\varsigma}^j \Omega + \bar{\xi}^{k\bullet} \dot{\eta} \tag{89}$$

where

$$\bar{\varsigma}^k = \bar{\varsigma}^j + \bar{S}^{k\bullet} a^k \tag{90}$$

$$\bar{\xi}^k = \bar{\xi}^j + \bar{S}^j \mathcal{M}^{k\bullet} \bar{\chi}_{eb} - \bar{S}^k \mathcal{M}^k \bar{\chi}_{eb} \tag{91}$$

Once more k and k^* refers to the unit vectors, fixed at the inter-connected elements of the adjacent bodies B_k and B_j (see Figure 8). The subscripts eb refers to the indices connected with both the reference arrays $edof$ and $bdof$ [12]. The velocity and acceleration of an arbitrary point, in a similar fashion could be written as :

$$V^{ki} = \gamma^{ki} \Omega + \nu^k \dot{\Upsilon} + \beta^{ki} \dot{\eta} \tag{92}$$

$$A^{ki} = \gamma^{ki} \dot{\bar{\Omega}} + \nu^k \ddot{\Upsilon} + \beta^{ki} \ddot{\eta} + \dot{\gamma}^{ki} \bar{\Omega} + \dot{\nu}^k \dot{\Upsilon} + \dot{\beta}^{ki} \dot{\eta} \tag{93}$$

where γ, ν and β are again the partial velocity arrays, associated with Ω, $\dot{\Upsilon}$ and $\dot{\eta}$ respectively. ν is same as the partial angular velocity matrix. γ and β could be written in a recursively computed form as:

$$\gamma^{ki} = \gamma^k + \bar{q}^{ki} \nu^k \tag{94}$$

$$\beta^{ki} = \beta^{k} + \left[\bar{\mathcal{S}}^{k} \left[\mathcal{N}^{ki} \bar{\chi}_{eb}^{ki} - \mathcal{N}^{k} \bar{\chi}_{eb}^{k} \right] + \bar{q}^{ki} \bar{\xi}^{k} \bar{\chi} \right] \tag{95}$$

where

$$\gamma^{k} = \gamma^{j} + \Upsilon^{k} \nu^{j} + \bar{d}^{k} \nu^{j} \tag{96}$$

and

$$\beta^{k} = \beta^{j} - \left[\bar{\Upsilon}^{k} \bar{\xi}^{j} - \bar{d}^{k} \bar{\xi}^{j} \right] \bar{\chi} + \mathcal{S}^{j} \mathcal{N}^{k*} \bar{\chi}_{eb}^{k*} - \bar{\mathcal{S}}^{j} \mathcal{N}^{j} \bar{\chi}_{eb}^{j} \tag{97}$$

It is worth mentioning at this point the recursive nature of the computation of kinematical quantities as indicated by the superscripts j and k for the lower and upper bodies respectively. The process of computation starts at the lowest body level, where all the quantities are known and through subsequent recursive substitution into the expressions, the quantities required ultimately for the point under consideration are evaluated.

The details of the equations of motion in a matrix representation form based on Kane's equations , is given in reference [12], which after simplification yields :

$$\mathcal{M}\dot{\mathcal{Y}} - \mathcal{P} - \mathcal{Q} - \lambda\mathcal{J} = \mathcal{F} \tag{98}$$

where $\mathcal{M}$, $\mathcal{Q}$, λ, $\mathcal{J}$, $\mathcal{F}$ and $\mathcal{P}$ represent the co-efficients matrices for the generalized co-ordinates accelerations, quadratic velocity vector, the c dimensional vector of undetermined multipliers, constraint jacobian matrix, generalized external forces and stiffness due to strain energy of the flexible bodies in the system respectively. These matrices are given by $\mathcal{M} =$

$$\begin{bmatrix} \sum\sum\int \rho\gamma^{ki^T}\gamma^{ki}\partial V/\partial\mathcal{Y} dV & \sum\sum\int \rho\gamma^{ki^T}\nu^{ki}\partial V/\partial\mathcal{Y} dV & \sum\sum\int \rho\gamma^{ki^T}\beta^{ki}\partial V/\partial\mathcal{Y} dV \\ \sum\sum\int \rho\gamma^{ki^T}\nu^{ki}\partial V/\partial\mathcal{Y} dV & \sum\sum\int \rho\nu^{ki^T}\nu^{ki}\partial V/\partial\mathcal{Y} dV & \sum\sum\int \rho\nu^{ki^T}\beta^{ki}\partial V/\partial\mathcal{Y} dV \\ \sum\sum\int \rho\gamma^{ki^T}\beta^{ki}\partial V/\partial\mathcal{Y} dV & \sum\sum\int \rho\nu^{ki^T}\beta^{ki}\partial V/\partial\mathcal{Y} dV & \sum\sum\int \rho\beta^{ki^T}\beta^{ki}\partial V/\partial\mathcal{Y} dV \end{bmatrix} \tag{99}$$

where ρ is the mass density of the material used, $\sum\sum$ is the summation of the volume of the element V over all the elements of a body and all the bodies in the system.

$$\mathcal{Q} = \begin{bmatrix} \sum\sum\int_V^{ki} \rho\gamma^{ki^T} \left[\dot{\gamma}^{ki}\Omega + \dot{\nu}^k\dot{\Upsilon} + \dot{\beta}^{ki}\dot{\eta} \right] \\ \\ \sum\sum\int_V^{ki} \rho\nu^{ki^T} \left[\dot{\gamma}^{ki}\Omega + \dot{\nu}^k\dot{\Upsilon} + \dot{\beta}^{ki}\dot{\eta} \right] \\ \\ \sum\sum\int_V^{ki} \rho\beta^{ki^T} \left[\dot{\gamma}^{ki}\Omega + \dot{\nu}^k\dot{\Upsilon} + \dot{\beta}^{ki}\dot{\eta} \right] \end{bmatrix} \tag{100}$$

$$\mathcal{F} = \begin{bmatrix} \sum\sum\int_{S_{k_i}} \gamma^{ki^T} f^{ki} dS - \int \gamma^{ki^T} b^{ki} dV \\ \\ \sum\sum\int_{S_{k_i}} \nu^{ki^T} f^{ki} dS - \int \nu^{ki^T} b^{ki} dV \\ \\ \sum\sum\int_{S_{k_i}} \beta^{ki^T} f^{ki} dS - \int \beta^{ki^T} b^{ki} dV \end{bmatrix} \tag{101}$$

where f^{ki} and b^{ki} are the surface traction and body forces associated with the global boundary of the finite elements and their weights respectively.

and

$$\mathcal{P} = \begin{bmatrix} 0 \\ 0 \\ \chi^T \left[K - G \right] \chi\eta \end{bmatrix} \tag{102}$$

where K and G are the block diagonal matrices, whose diagonal sub-matrices are K^k and G^k, being refered to the structural stiffness and geometric stiffness for body K.

5.2 Derivation of the Constraint equations :

Use of partial velocity arrays in the recursive formulation of the the dynamical equations of motion, renders a noteworthy benefit of the automatic generation of the constraint equations. Basically, the various constraints could be broadly classified as:

1. Closed loops

2. Prescribed motions

3. Contact between inter-connected bodies.

Closed loops are a common type of constraint, found in various mechanisms. The joining point, where the closed loop occurs, could be reached in two ways, from the fixed inertial reference frame. Denoting these two ways by p and q, the constraint equation could be written in a compact form as :

$$\begin{bmatrix} \begin{bmatrix} \gamma_i^p - \gamma_i^q \end{bmatrix} & \begin{bmatrix} \nu_i^p - \nu_i^q \end{bmatrix} & \begin{bmatrix} \beta_i^p - \beta_i^q \end{bmatrix} \end{bmatrix} \begin{bmatrix} \Omega \\ \dot{\Upsilon} \\ \dot{\eta} \end{bmatrix} = 0 \tag{103}$$

The specification of the kinematical quantities such as speeds and angular velocities form the second prescribed motions. T'le later are based on the linear and angular velocity, as a function of time, say, $G(t)$ and $h(t)$, where the constraint equation take the following form :

$$\begin{bmatrix} \gamma^{ri} & \nu^{r} & \beta^{ri} \end{bmatrix} \begin{bmatrix} \Omega \\ \dot{\Upsilon} \\ \dot{\eta} \end{bmatrix} = G(t) \tag{104}$$

$$\begin{bmatrix} \varsigma^{r} & \bar{\xi}^{r} \end{bmatrix} \begin{bmatrix} \Omega \\ \dot{\eta} \end{bmatrix} = h(t) \tag{105}$$

where r refers to the position vector C^r of a point A, which has the specified velocity, in local reference frame of body K.

In a similar fashion, the third type of constraint mentioned above, is defined by the number of points used to describe the contact between a gear/shaft and a gear/gear type of bodies. It will be shown in the following sections of the paper, that the jacobian matrices associated with the generalized co-ordinate derivatives, could be generated automatically, for the cases of both the rigid and flexible gears/shafts.

The Holonomic and Non-holonomic constraints at the velocity level could be written in a compact form as :

$$\mathcal{J}\mathcal{Y} = G(t) \tag{106}$$

where, $\mathcal{J}$ is the jacobian constraint matrix of order c by n, c being the number of the constraints in the system and n, the number of generalized co-ordinates.

Differentiating the above equation once more and combining it with equation (98) after the elimination of the undetermined multipliers, we get

$$\mathcal{L}\dot{\mathcal{Y}} = \mathcal{R} \tag{107}$$

where

$$\mathcal{L} = \left[\mathcal{C}^T \mathcal{M} \right] \tag{108}$$

and

$$\mathcal{R} = \left[\mathcal{C}^T \left[\mathcal{F} - \mathcal{P} - \mathcal{Q} \right] \right] \tag{109}$$

$\mathcal{C}$ in the above equation is the orthogonal-complement to the jacobian matrix $\mathcal{J}$, and is obtained by Psuedo-uptriangular Decomposition method. Subsequent numerical integration of the governing equations of motion (107), would yield the time history of the generalized speeds or generalized co-ordinates.

5.3 Vectorization and its computer implementation

The above developed algorithmic procedure is implemented on the various supercomputers and mainframes.

The code by its nature of dynamical equations derived on the basis of flexibility and finite element approach, constitute a substantial part of long vectors/arrays. This feature makes it a potential candidate for the exploitation of the pipelining technique of modern Vector-Processor hardware in supercomputers. This technique allows the overlapping of instructions to a sequence of operands simultaneously, as compared to a single operand in a scalar processor. Thus, speedup is achieved by a kind of micro-parallelism, in which, different stages of pipeline work simultaneously on different data. In general, performance increases with the vector length upto a limit governed by the section size of the Vector-Pipe. The section size refers to the number of elements a Vector-Pipe could hold at a particular point of time. (The section size varies with the machine, like it is 256 in IBM-3090 and 64 in the CRAY).

Effort was made to improve the efficiency of execution of the implemented code by reducing the CPU time, by vectorizing the code. This was carried out both at the compiler option level (outside the code) and at the Do-loop level (inside the code) in the computationally intensive areas. In particular, the concepts of promoting the scalar variables to vectors in the left hand side of the expressions, was found to be more effective in certain computationally intensive areas, which improved the overall speed of execution of the code.

5.4 Proposed Stability and Control for Systems Operating in the Presence of Singularity

In our future work, we intend to extend the developments of phase 1 of this project for large class of multibody systems including those with the elastic bodies. The validity of the developed method presented in phase 1 will give us a new insight on the basic control research problems for articulated structures undergoing quick maneuvers and operating in the presence of singularities.

6 Conclusions and Future Directions

We have demonstrated under the proposed contract how the PUTD could be extended to handle singularities in the dynamics of articulated structures. A stability method was developed to regularize the vanishing and linearly dependent constraints that are time variant, through a new representation making use of higher order derivatives. The continuity of the motion is demonstrated through an example of two arm robot driven through singularity. Our goal is to extend this development and results to realistic models of structures with flexible bodies.

The second phase has been initiated where an all purpose code was developed and subject to vectorization in supercomputers to test its ability to achieve real time simulations. We believe that our efforts, if continued will lead to some major contributions in the control and stability of constrained multibody systems. In particular quick maneuvers of space structures and basic ground research of robotics could benefit greatly from the implementation of these algorithms.

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LIST OF FIGURES

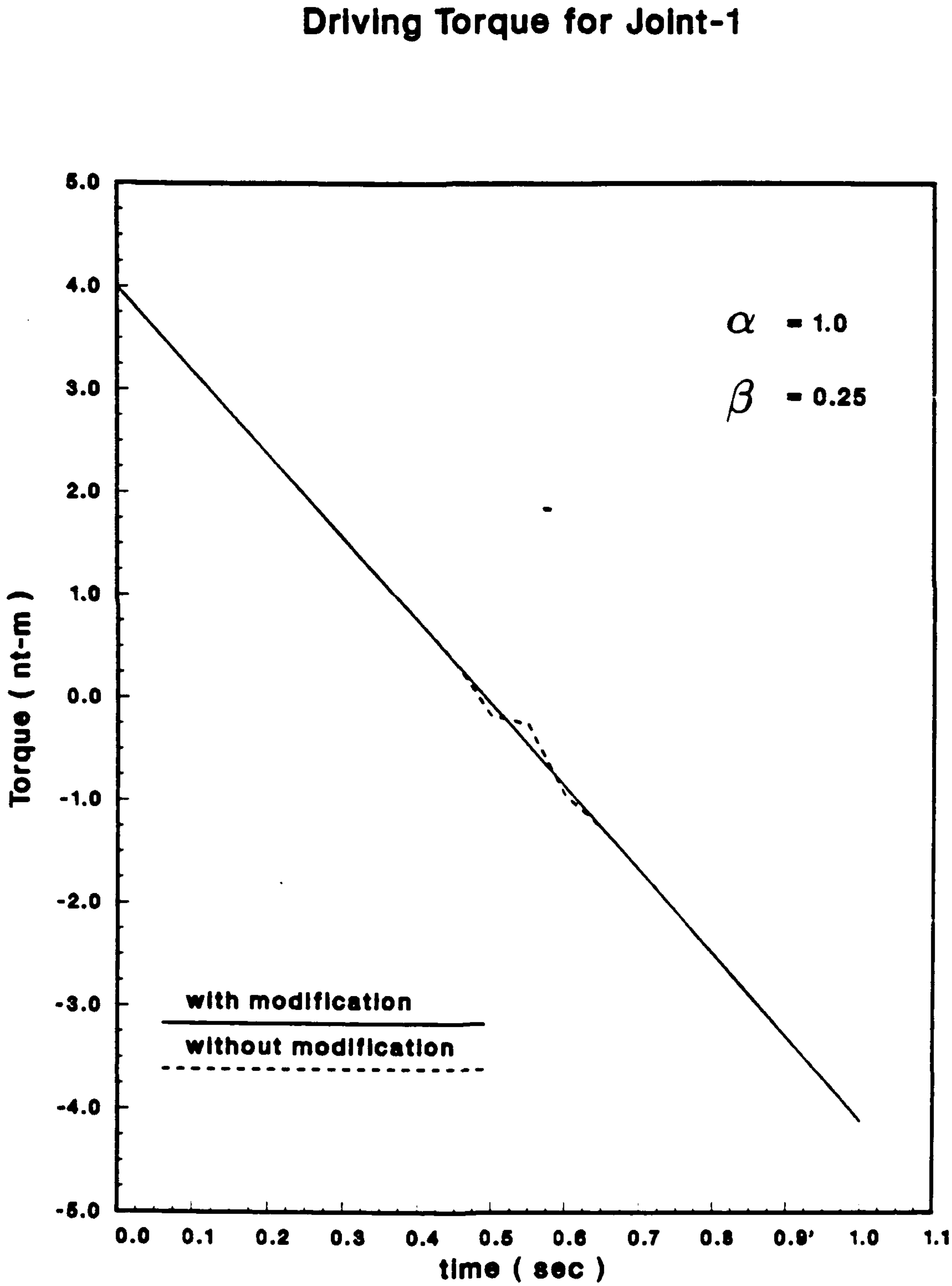

Figure 5: Introducing Baumgarte technique on C.F-1

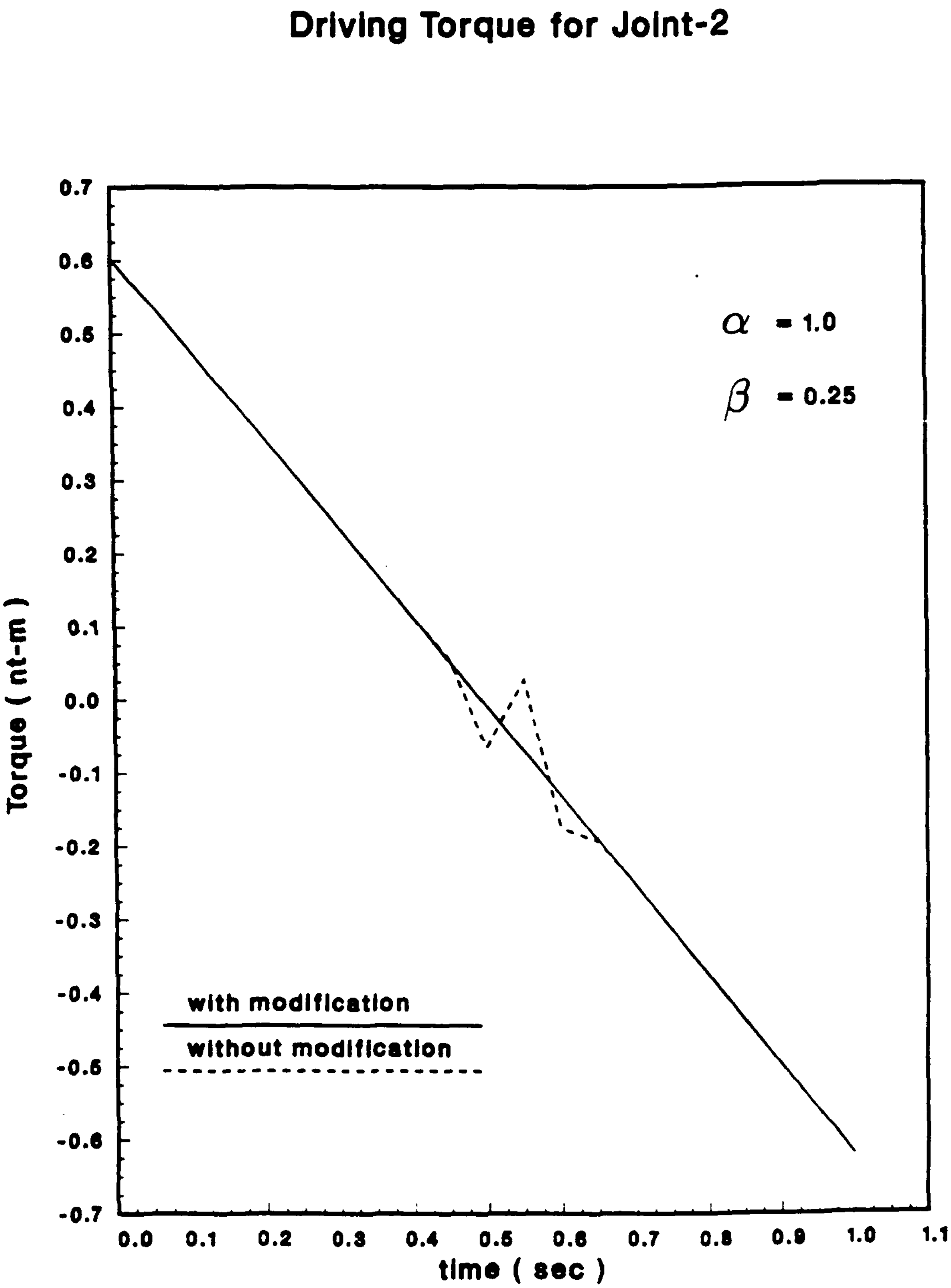

Figure 6: Introducing Baumgarte technique on C.F-2

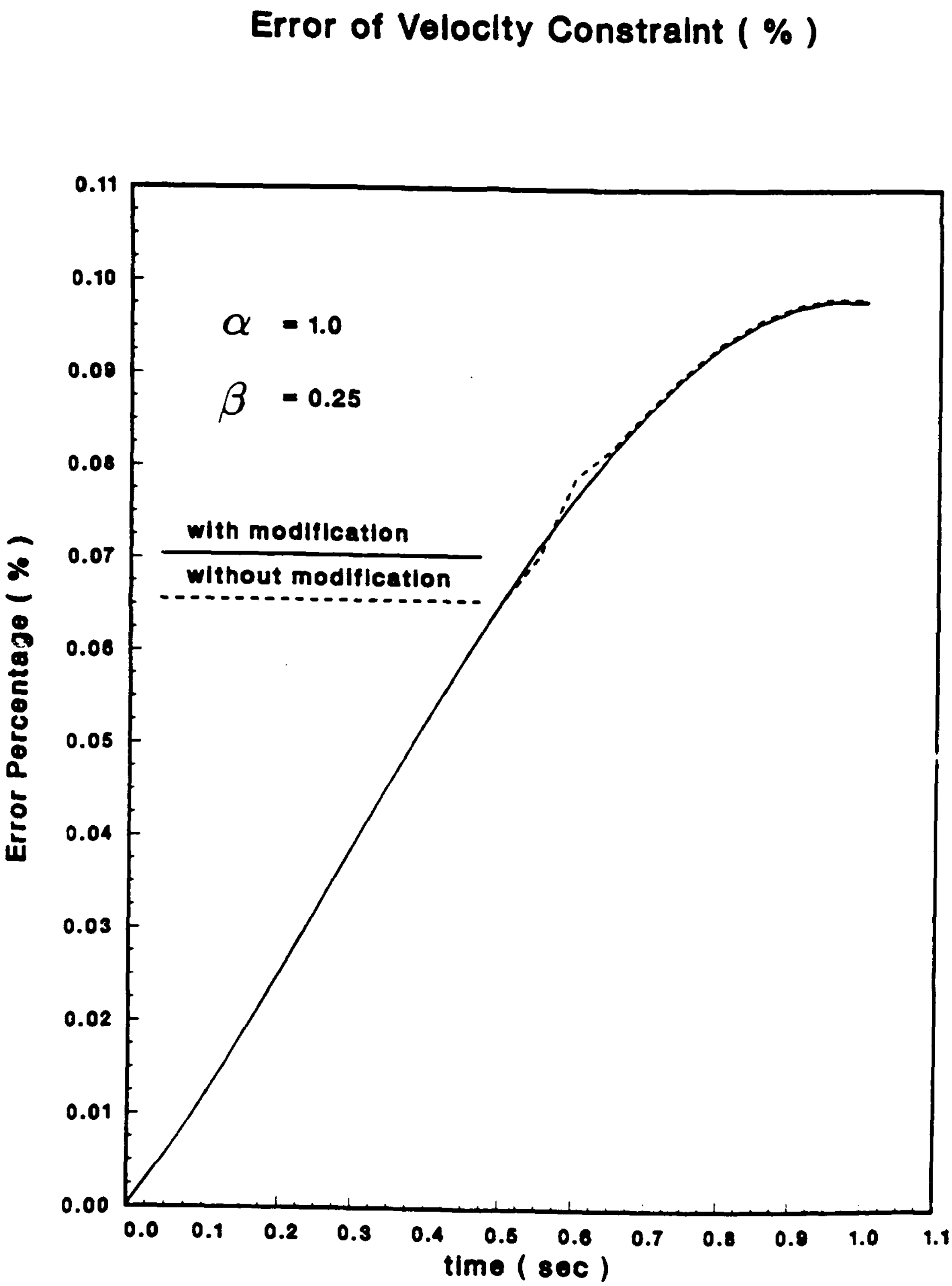

Figure 7: Introducing Baumgarte technique on velocity constraint

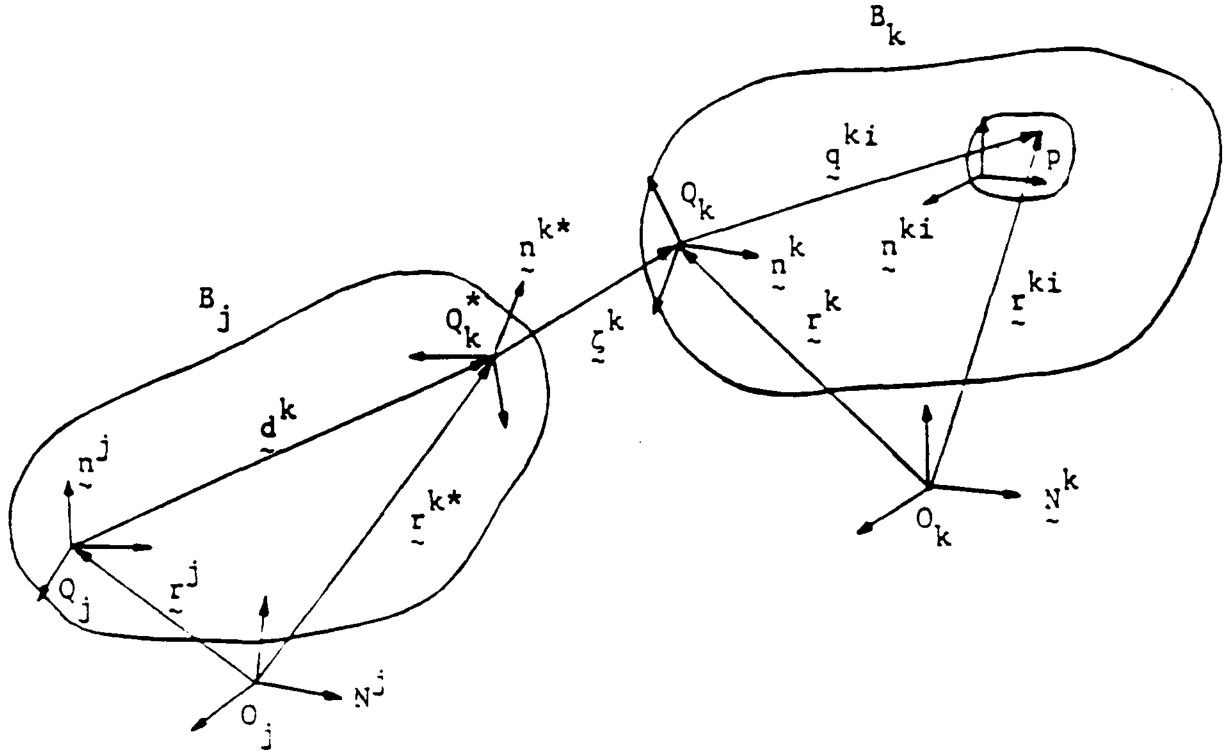

Figure 3. Labelling position vectors in adjacent bodies.

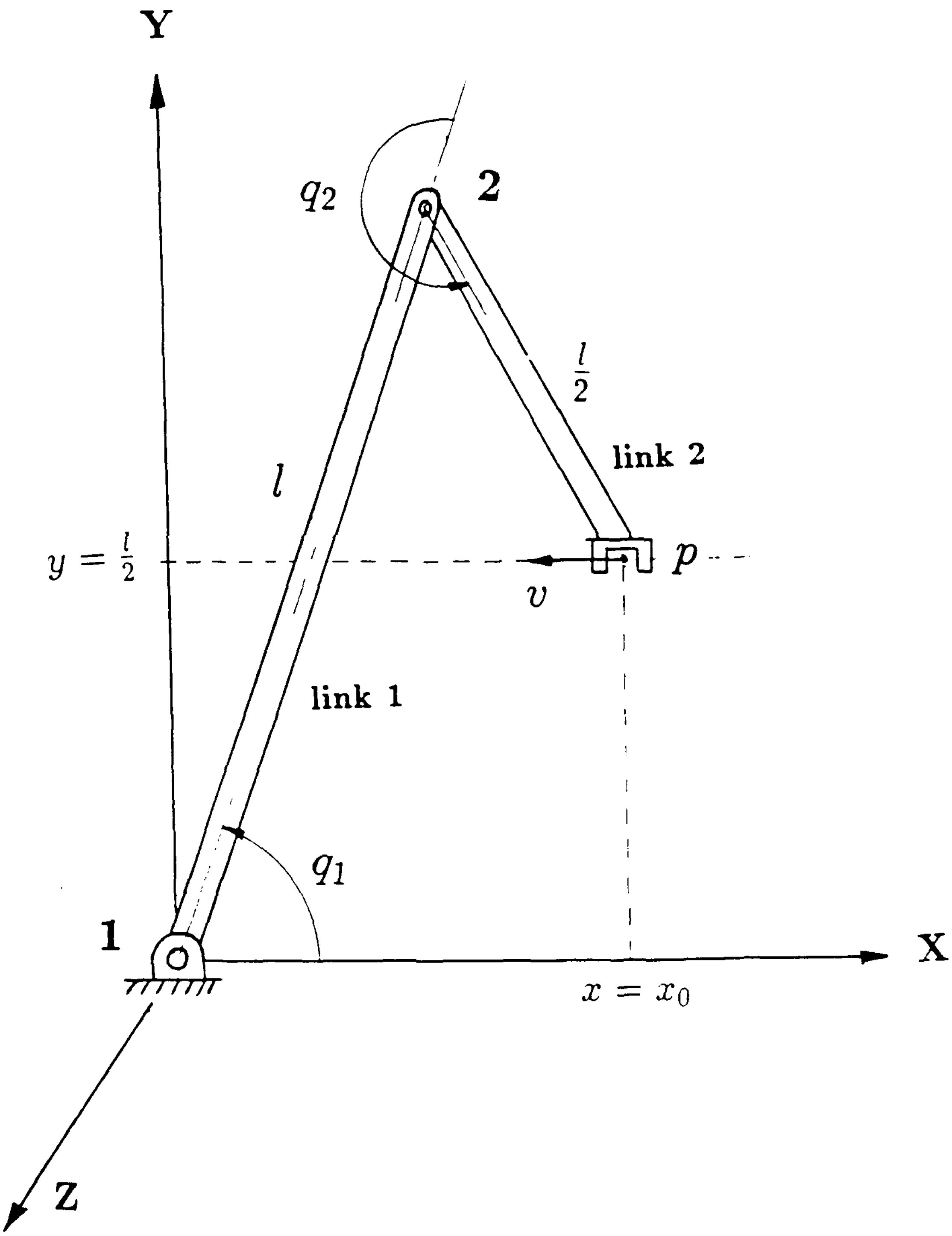

Figure 1 Two Link Manipulator Undergoes A Constrained Motion

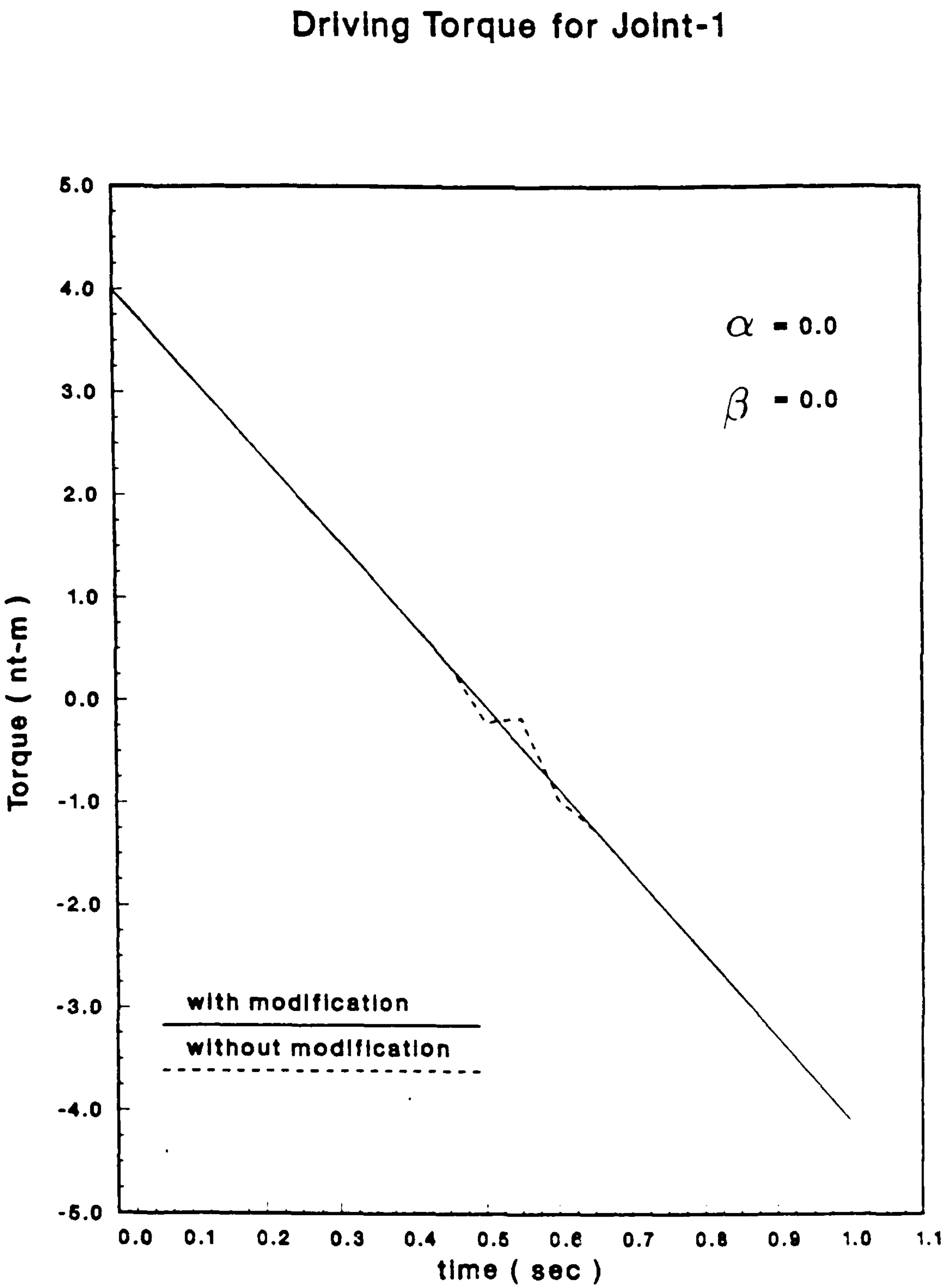

Figure 2: Effect of modification on constraint force-i

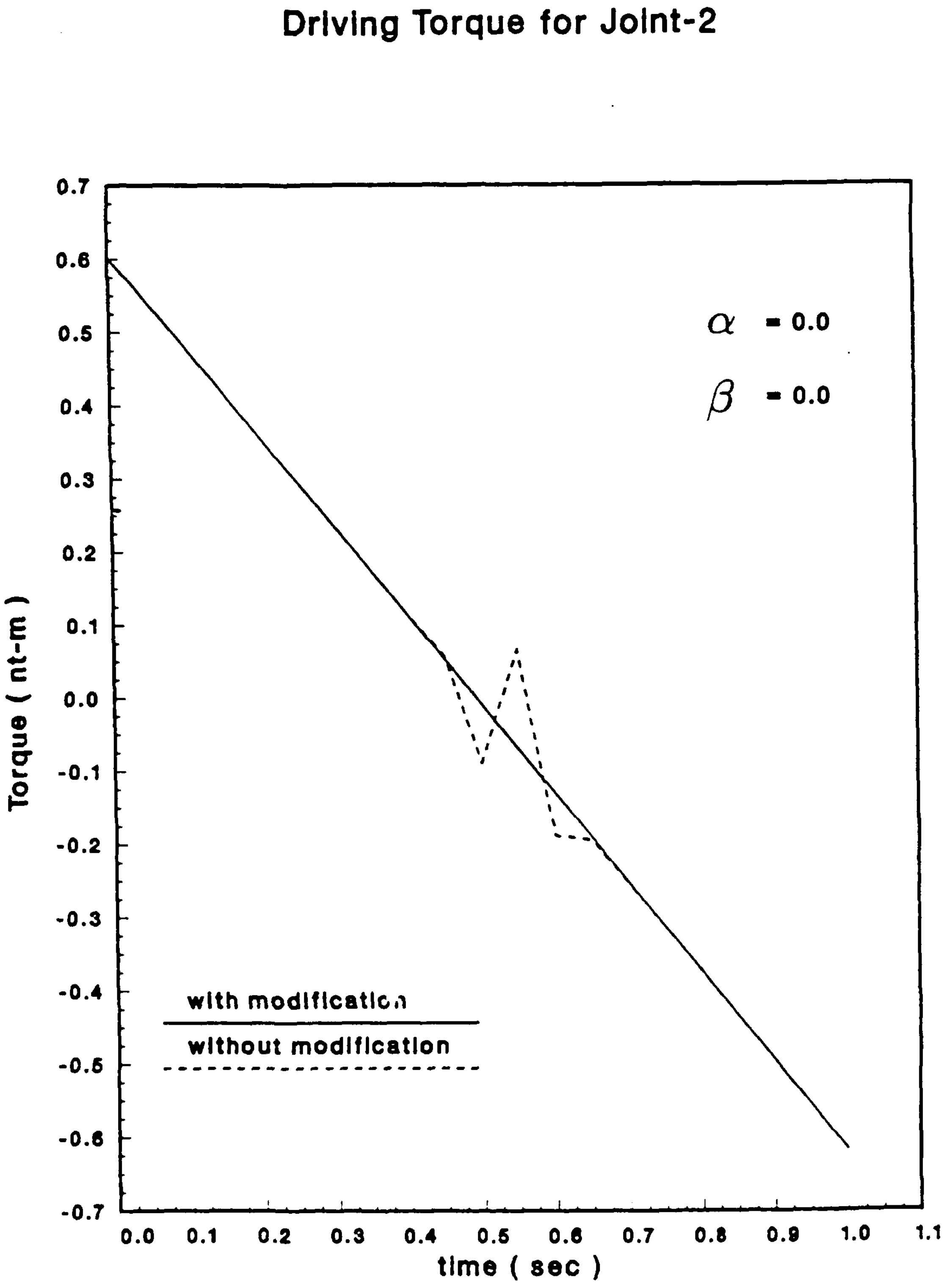

Figure 3: Effect of modification on constraint force-2

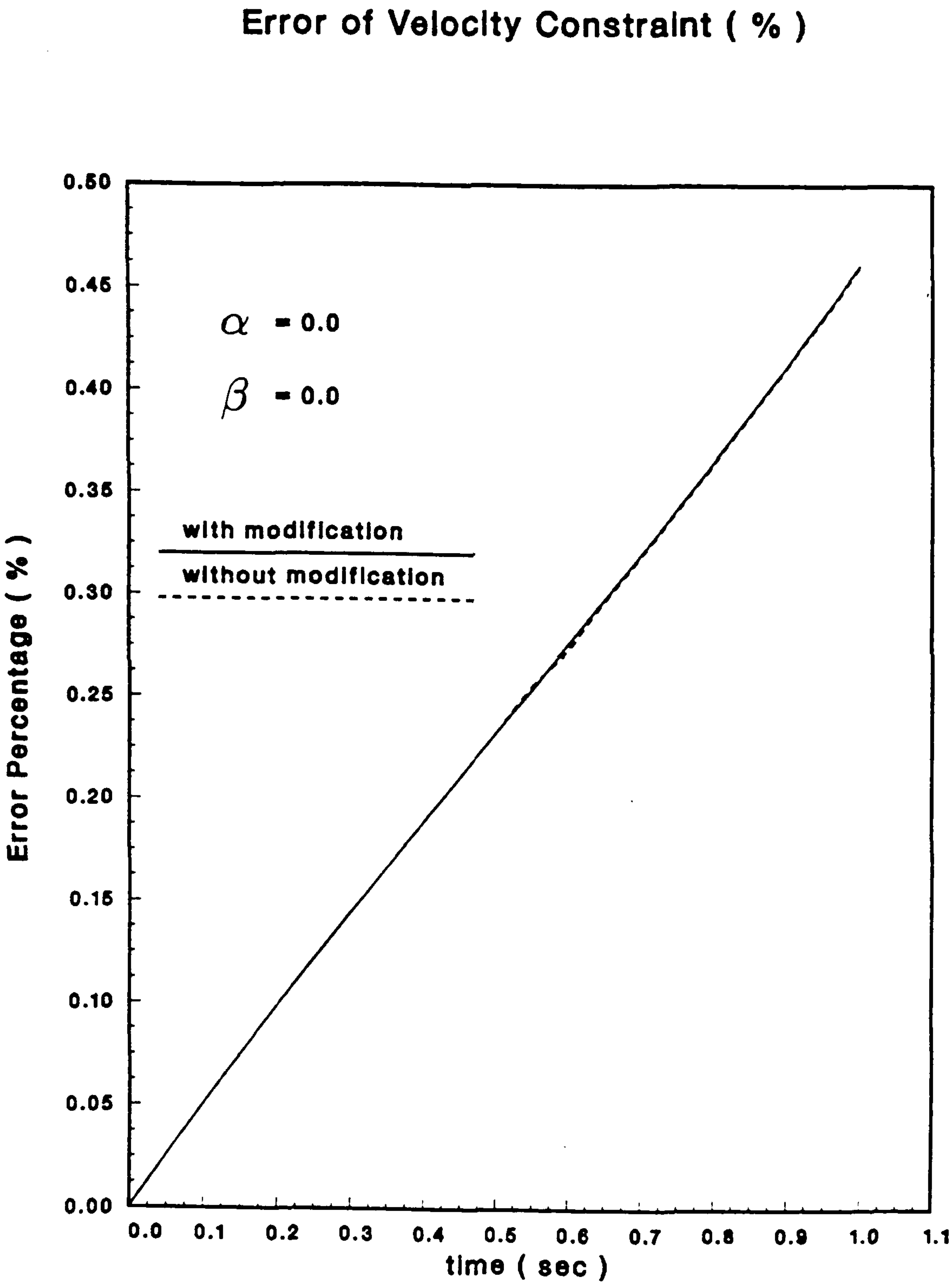

Figure 4: Effect of modification on velocity constraint

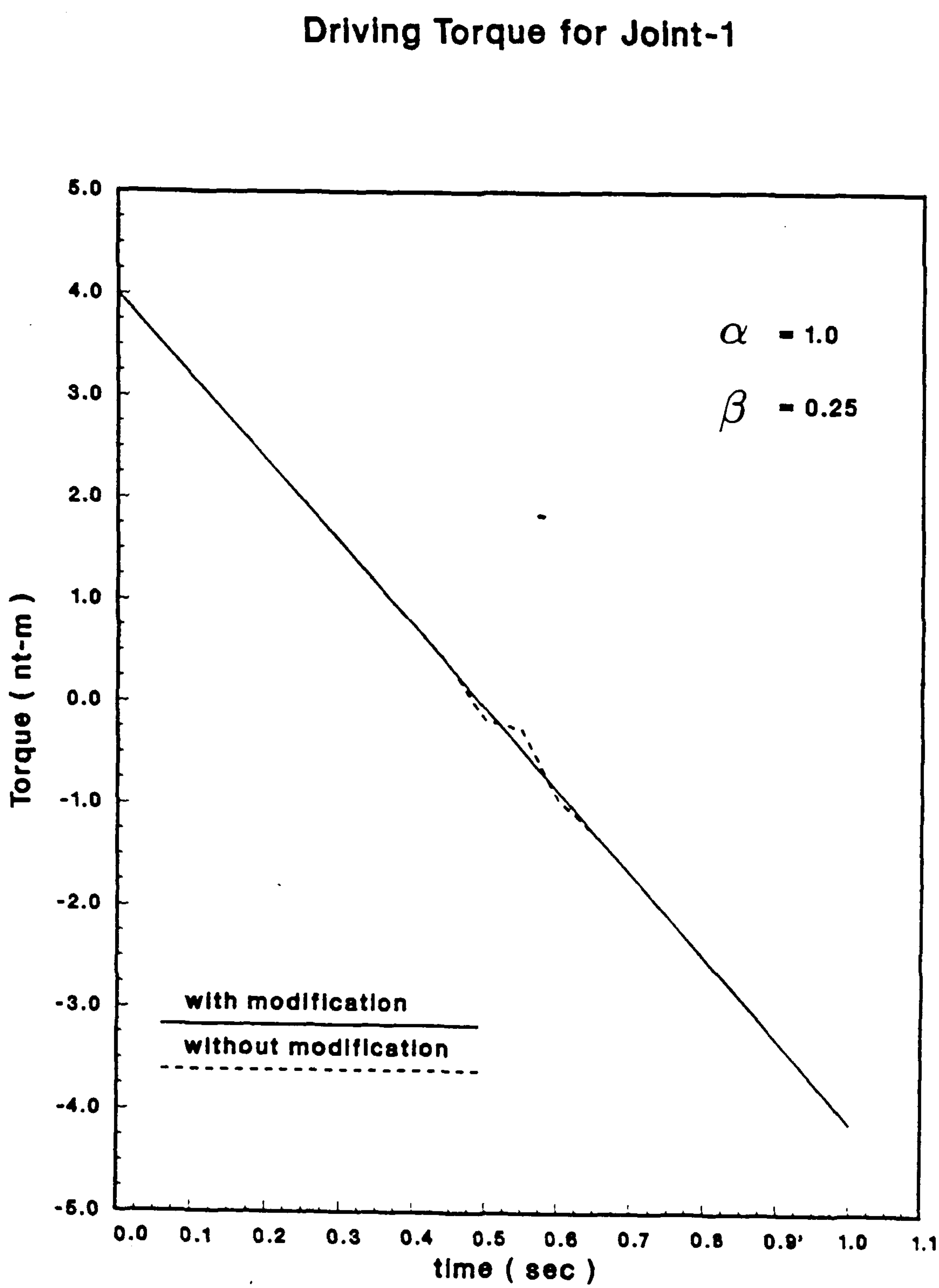

Figure 5: Introducing Baumgarte technique on C.F-1

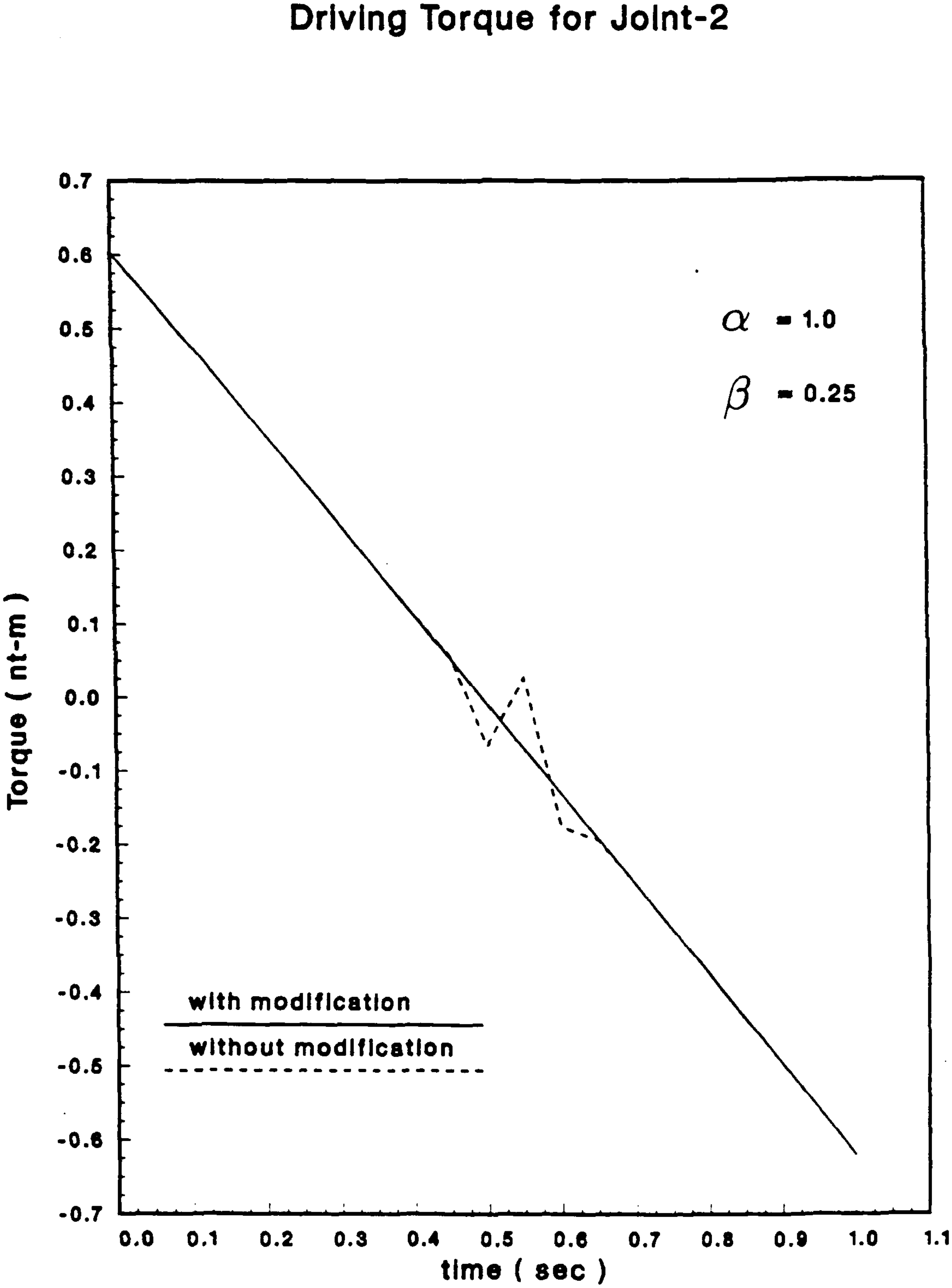

Figure 6: Introducing Baumgarte technique on C.F-2

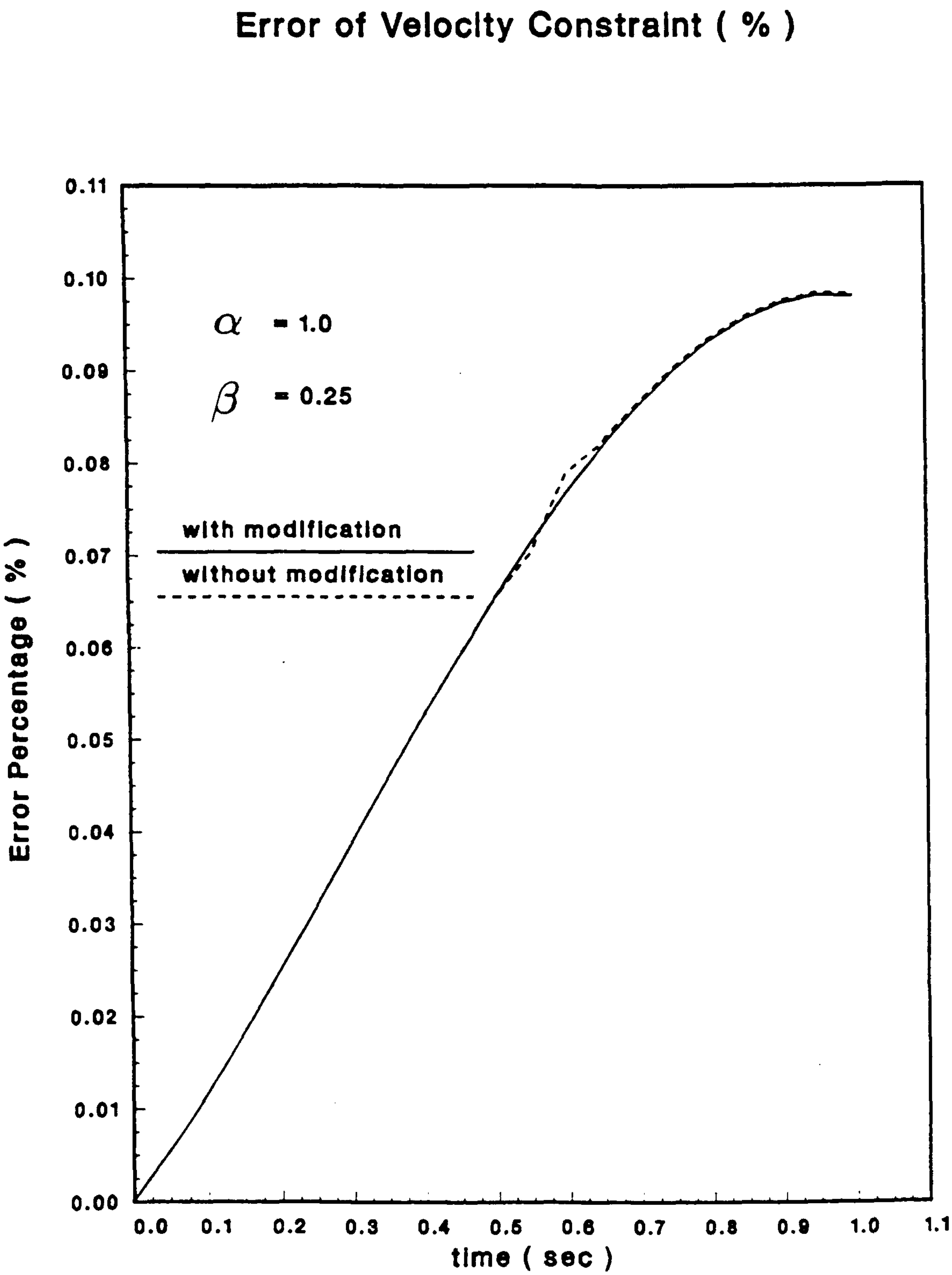

Figure 7: Introducing Baumgarte technique on velocity constraint

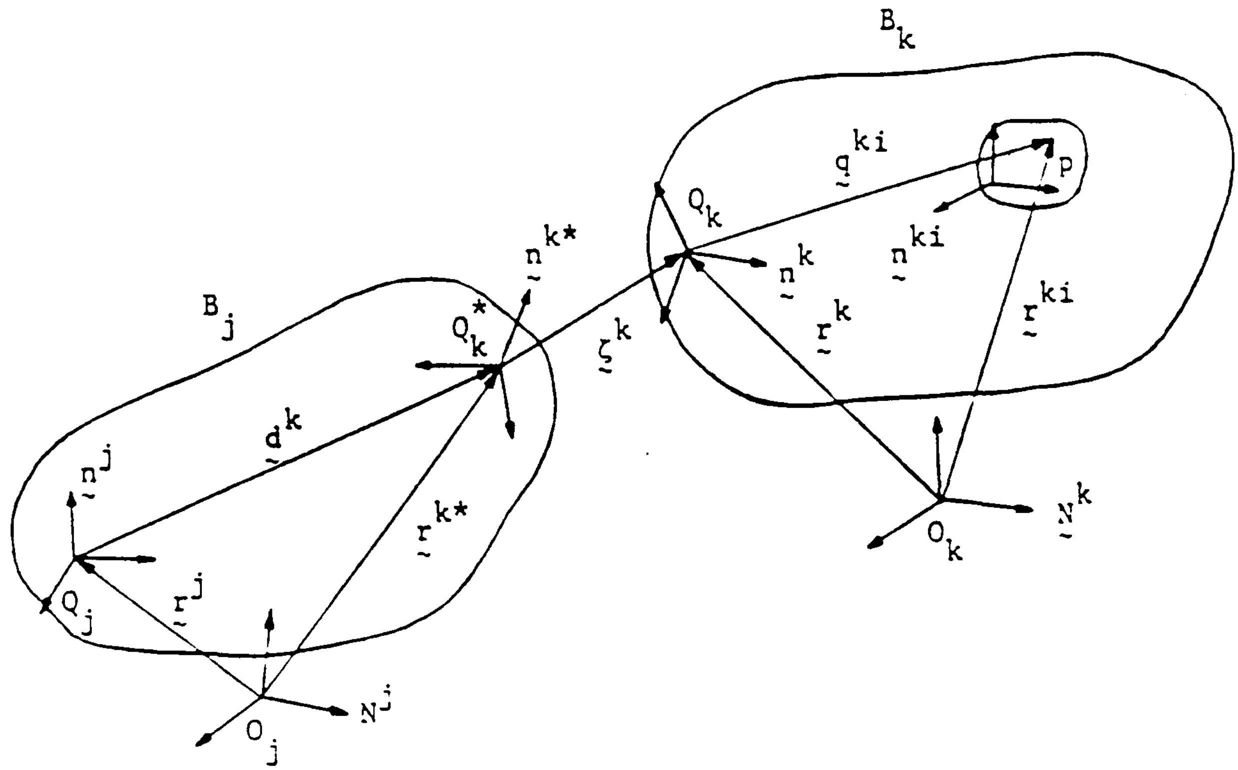

Figure 2. Labelling position vectors in adjacent bodies.